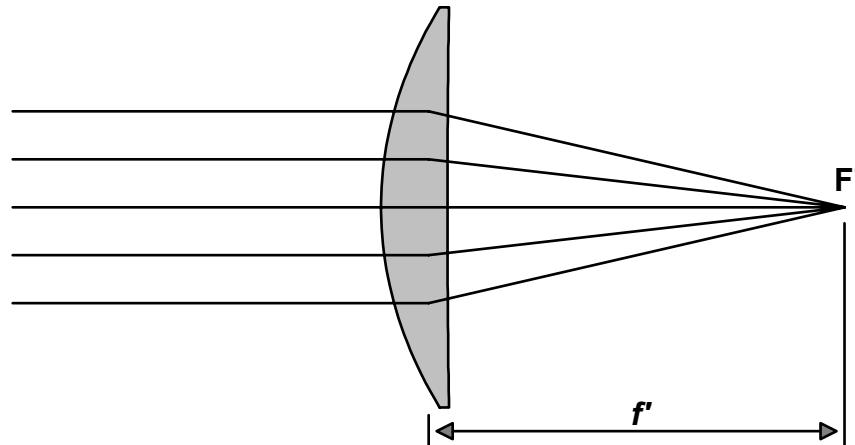


INTRODUCTION TO OPHTHALMIC OPTICS

Darryl Meister, ABOM

James E. Sheedy, OD, PhD



CARL ZEISS VISION

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Darryl Meister, ABOM

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James E. Sheedy, OD, PhD

Pacific University College of Optometry

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1. Introduction to Ophthalmic Optics

The purpose of this workbook is to provide an understanding of ophthalmic lenses (spectacle lenses) and how they are designed to correct vision. In order to provide this understanding, the essential fundamentals of physical, geometrical, and visual optics are presented as they relate to the eye and vision.

This book provides a working and practical background in ophthalmic optics for those working in the field or training to do so. The book has been written primarily for those who work in eye care offices or optical dispensaries. However, the book can be equally useful for those working for ophthalmic manufacturers, for optical laboratories or in sales of ophthalmic products. The text can readily be used in a programmed training course. It could also be particularly useful for anyone preparing to take an examination in ophthalmic optics.

1.1 BACKGROUND

Ophthalmic means “of or pertaining to the eye”. Therefore, **ophthalmic optics** includes any optics that pertain to the eye. Technically, this includes spectacle lenses, the optics of the eye itself, *contact* lenses, *intraocular* lenses, and even refractive surgery and optical instruments used to examine the eye. However, in common usage, “ophthalmic optics” most specifically refers to spectacle lenses, and that is the topic of this book.

In order to provide a practical and useful understanding of ophthalmic optics, it is necessary to learn some fundamental optics and principles. But before we dive into the optics, it is best that the reader have a basic understanding of the ophthalmic industry. If you are already working in the industry, you may choose to skip this section.

1.2 REFRACTIVE ERRORS AND LENSES

Spectacle lenses are the most common method of correcting refractive errors and enabling people to see well. Contact lenses and refractive surgery are other options. Refractive errors include **myopia** or near-sightedness, and **hyperopia** or far-sightedness. Myopia and hyperopia are corrected with **spherical lenses** of minus and plus power respectively. A spherical lens has circular curves. While looking straight at a spherical lens as it is worn before the eye, any radial cross section of the lens (i.e. a cross section that includes the center of the lens—also called a **meridian** of the lens) will have the same lens curvatures and the same power. This is illustrated in Figure 1:1. A result of this is that the rotational position of the lens before the eye is unimportant. In other words, while looking at the wearer and assuming that the lens is properly centered in front of the eye, if the lens were to rotate (say, within

a circular eyeglass frame), it would have no effect upon vision. When prescribing, fitting, or manufacturing spherical lenses the lens must be properly centered before the eye, but the rotational position of the lens before the eye is unimportant. Spherical lenses are specified by their spherical power only.

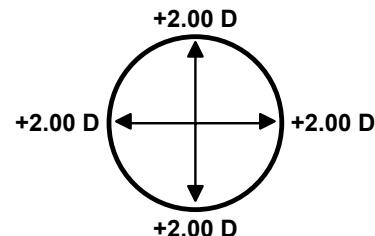


FIGURE 1:1 A spherical lens has the same power in every meridian.

Astigmatism is a non-spherical refractive error and can exist along with myopia or hyperopia. Astigmatism is corrected with a **sphero-cylindrical lens** in which the power is different in different lens meridians, as shown in Figure 1:2. Most commonly, astigmatism is caused by a non-spherical *cornea* of the eye and requires a sphero-cylindrical correction in which the power varies by lens meridian. For sphero-cylindrical lenses, the lens center must be properly located before the eye and its rotational position before the eye is also critical. Sphero-cylindrical lenses are specified by their spherical power, their cylindrical power (effectively the meridional power variation), and the rotational position or **axis** of the lens.

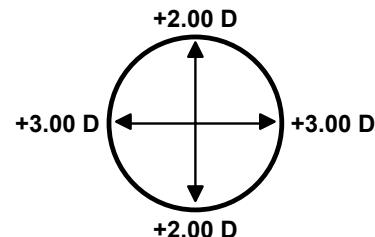


FIGURE 1:2 A sphero-cylindrical lens varies in power from meridian to meridian.

Myopia, hyperopia and astigmatism, by themselves, require correction by **single vision lenses**. Single vision lenses are designed to have a single spherical or sphero-cylindrical power in the lens. **Multifocal lenses** have more than one designed power in the lens. Examples of multifocals include **bifocals**, **trifocals**, and **progressive addition lenses**, as illustrated in Figure 1:3. Multifocal lenses are most commonly prescribed for **presbyopia**, an age-related condition in which people lose the ability to change the focus of their eye to see clearly within arms' length—usually occurring in the mid-to-late forties. Multifocal lenses have a near-power addition to enable people with presbyopia to see clearly when they view near objects.

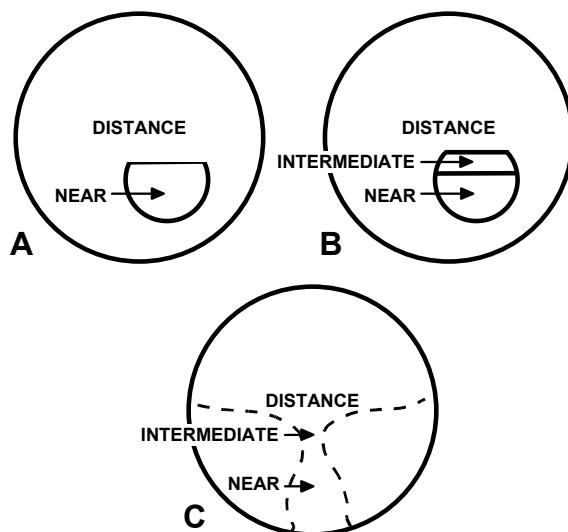


FIGURE 1:3 A) A bifocal lens, B) a trifocal lens, and C) a progressive addition lens.

The power addition is located in the lower portion of the lens since people most commonly look downward when viewing near objects. Obviously, it is important that a multifocal lens be properly centered and properly rotated in front of the eye so that the near power is located downward. Multifocal lenses are specified by spherocylindrical power and axis in the top, or distance portion of the lens, along with the type of multifocal and the **add power** of the multifocal.

1.3 LENS MANUFACTURING

Most spectacle lenses are initially fabricated as round **lens blanks** by an **optical manufacturer**. These lens blanks require further fabrication before they can be inserted into a frame and provided to the patient. The additional fabrication is often completely provided by an **optical laboratory**, which purchases the lens blanks from the manufacturer, completes the fabrication and then sends the glasses to an **optical dispensary** where they are provided to the patient. Some, or even all, of the additional fabrication may be provided at the dispensary location.

The particular manufacturing path taken can depend upon many factors, including the refractive correction, the lens design that has been ordered, the material of the lens, the lens options such as coatings or tints and the mode of operation preferred by the dispenser. Only some of the basic factors are discussed here.

Manufacturers make finished and semi-finished lens blanks. **Finished lens** blanks have finished optical surfaces on the front and back of the lens and have the complete final power that is required for the prescription. A finished lens, unless tinting or coating is first required, is ready to be **edged** (by a machine called an edger) to fit into the frame. Finished lenses are

nearly always single vision lenses. It is usually cost-effective for a laboratory to maintain an inventory of finished lenses that have the most common spherical and cylindrical power combinations. This can also be cost-effective for many dispensaries. When a particular spherocylindrical power is required for the prescription, the finished lens containing that power is selected from inventory, it is properly rotated so that its axis matches that required by the prescription, and then the lens is edged and inserted into the frame.

Semi-finished lens blanks, which do not have a finished back surface, are used for prescriptions for which it is not cost-effective to maintain finished inventory at the laboratory or dispensary. This occurs most often for multifocal lenses and for single vision lenses of higher powers. In the case of multifocals, it becomes nearly impossible to carry all combinations of spherocylindrical and add powers—especially when considering that the multifocal optics require a certain orientation (down) as does the axis of the cylinder correction.

For multifocal lenses, the multifocal optics are on the front lens surface of the semi-finished lens blank which is provided by the manufacturer. At the laboratory, the semi-finished blank which contains the correct curvature and correct multifocal type and add is selected from inventory, the multifocal is properly oriented, and then the back surface is ground and polished to contain the proper spherocylindrical power and axis with respect to the multifocal. This process is referred to as **surfacing** or **finishing** the lens. The lens is now ready for edging and insertion into the frame. Surfacing requires more expensive equipment than edging, so it is less often performed at the dispensing location.

The above descriptions provide only the basics about the supply chain of spectacle lenses. There are many variations dependent upon other lens features or upon the chosen mode of business. However, these descriptions provide an orientation to the industry that will make your upcoming study of ophthalmic optics more meaningful.

1.4 TOPICS

Many of the topics covered in this book are typically presented with rigorous mathematics. Since this text is intended for those in clinical or laboratory careers, the mathematical treatment of the subjects has been minimized in both amount and complexity. However, it is not possible to cover these optical topics without some mathematical treatment. Some fundamental principles and concepts are provided in the appendix at the end of this text. We encourage the reader to peruse this appendix beforehand.

For a more detailed treatment of any of the topics presented here, the reader is also encouraged to review the appropriate references from the works cited list. The basic optical principles discussed throughout this paper will be presented in the following sequence:

- The nature of light
- Refraction and reflection
- Focal power
- Sphero-cylindrical lenses
- Mechanics of lens form
- Ophthalmic prisms
- Visual optics
- Spectacle frames and fitting
- Ophthalmic lenses and design
- Lens materials

2. Nature of Light

Since the time of Plato, scientists have theorized about the exact nature of light. The renowned physicist, Isaac Newton, proposed that light consisted of streams of minute particles that were emitted from a source. He called this the **corpuscular theory of light**. In the late 1600s, Robert Hooke and Christian Huygens each suggested that light emanated from a source in the form of waves, much like ripples across a pond. This has become known as the **wave theory of light**. At least one of these two theories could be applied to explain the various phenomena produced by light. Still, neither of them was able to *completely* explain every aspect of the complex behavior of light.

Scientists broke further ground in the 1800s when Thomas Young was able to substantiate the wave nature of light with his “double-slit experiment.” Another milestone came a few years later when James Maxwell suggested that light was, in fact, a form of **electromagnetic radiation**. Maxwell discovered that certain types of energy traveled through space via waves of electromagnetic radiation, which consisted of oscillating electrical and magnetic fields vibrating perpendicularly to the direction of their **propagation** (or travel). This integrated the wave theory of light with the principles of electromagnetism.

2.1 ELECTROMAGNETIC RADIATION

To understand electromagnetic (EM) radiation, one needs to know that a changing magnetic field produces a changing electrical field perpendicular to it—a phenomenon known as **electromagnetic induction**. Likewise, a changing electrical field produces a changing magnetic field. At some critical speed, *mutual induction* occurs between these changing fields and they regenerate each other indefinitely. All electromagnetic waves—including light—travel at this critical speed, which is approximately 300,000 kilometers per second (186,000 miles per second) in free space. Figure 2:1 depicts these oscillating electric and magnetic fields, which are perpendicular to one another and compose waves of electromagnetic radiation.

So electromagnetic radiation comprises *transverse* waves (as opposed to *longitudinal* waves, like sound), which oscillate about their direction of travel. These waves are *periodic*, since they repeat at regular intervals. They are also *harmonic*, since they can be described by a simple sine function (Keating 452).

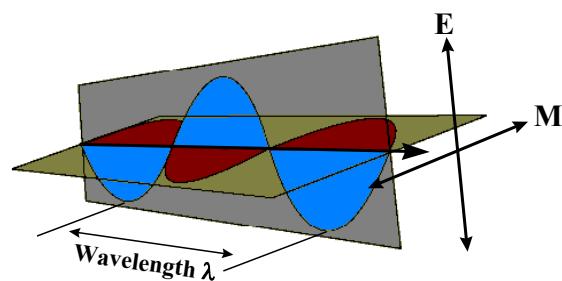


FIGURE 2:1 Electrical and magnetic fields propagating through space as an electromagnetic wave.

Figure 2:2 depicts a progression of such electromagnetic waves, called a **wave train**.

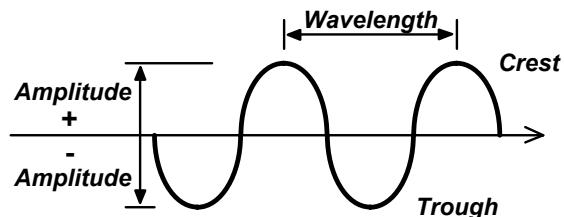


FIGURE 2:2 The electromagnetic wave train is sinusoidal in form, and has repeating crests (*maximum + amplitude*) and troughs (*minimum - amplitude*).

The relationship between the **frequency** f , or the number of vibrations per second of an EM wave train; the **wavelength** λ , which is the distance from one crest to another; and the velocity of light V , in meters per second, is given by

$$\text{EQ. 1} \quad f = \frac{V}{\lambda}$$

Thus, the frequency of an electromagnetic emanation is *inversely proportional* to its wavelength, and vice versa; as the wavelength increases, the frequency decreases. Frequency is given in cycles per second, or Hertz.

We think of **light** as the *visible* portion of the electromagnetic spectrum, which consists of cosmic rays at one end, and radio waves at the other. This region consists of EM radiation ranging from 380 to 760 nanometers (one billionth of a meter) in length. This is only 0.000380 to 0.000760 millimeters! The full EM spectrum is illustrated in Figure 2:3 (Smith & Atchison 6).

White light is composed of all the wavelengths in the visible spectrum. Individual wavelengths within the visible spectrum, by themselves, create different color sensations as shown in Table 1. These are the spectral colors.

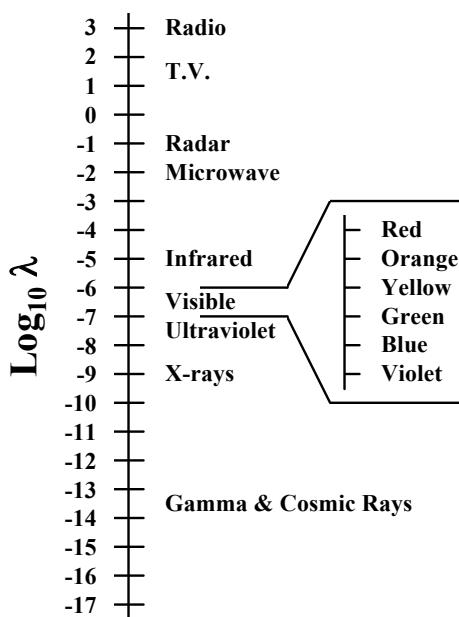


FIGURE 2:3 Electromagnetic spectrum. The wavelengths, in meters, have been expressed in logarithms of the common base 10. The wavelengths of visible light, for instance, lie between 10^{-7} and 10^{-6} m.

TABLE 1 The UV, visible, and IR spectrum

Wavelength (nm)	Color
200 to 380	Ultraviolet
380 to 450	Violet
450 to 490	Blue
490 to 560	Green
560 to 590	Yellow
590 to 620	Orange
620 to 760	Red
760 to 1,000,000	Infrared

Remember that colors with shorter wavelengths, like blue and violet, have higher frequencies. *Ultraviolet (UV) radiation* and *infrared (IR) radiation* are also listed in Table 1, because they are immediately adjacent to the visible portion of the EM spectrum. However, they do not create the sensation of vision and are therefore not classified as light.

2.2 QUANTUM THEORY

Although the electromagnetic wave explanation of light seemed to be the most complete, it still failed to account for certain effects produced by light, like the *photoelectric effect*. In the early 1900s, Max Planck hypothesized that radiation wasn't simply produced in continuous waves of energy by the source, but rather discrete packets of energy that he called **quanta**.

A few years later, Albert Einstein extended Planck's **quantum theory** to light, and called these bundles of energy **photons**. Einstein proposed that light consisted

of streams of these high-speed energy particles. The energy of a photon is *directly proportional* to its frequency; so electromagnetic radiation with a higher frequency also has a higher energy level. The energy content of ultraviolet light, for instance, is greater than that of infrared radiation. In essence, this theory of light was simply another form of the particle theory.

Today we say that light has a *dual nature*, with both particle-like and wave-like properties. Though, for our purposes, the differences are only academic. In summary, we will assume for our purposes that light consists of high-speed particles of energy—or photons—that travel in a wave-like manner (Sears 6).

2.3 SOURCES OF LIGHT

Light is emitted by a **luminous (or primary) source**, which generates the radiation. This radiation is often produced by heat, and such sources include the sun, incandescent light bulbs, fire, etc. Other objects are visible because they reflect light from luminous sources. These objects are called **secondary sources**. For instance, you can see a red car down the road because it is absorbing every color of white light from the sun, except red. The color red is reflected off the car, which then serves as a secondary source for observers (Keating 2).

2.4 INDEX OF REFRACTION

Recall that waves of light travel at a constant velocity of approximately 300,000 km/s in free space. In other transparent media, including lens materials, waves of light will be transmitted at a slower rate. The velocity of light in other media will vary as a function of the **index of refraction** n for that material. The index of refraction n of a transparent medium, which is also called the **refractive index**, is the ratio of the velocity of light in air (V_{AIR}) compared to the velocity of light in the *material* ($V_{MATERIAL}$):

$$\text{EQ. 2} \quad n = \frac{V_{AIR}}{V_{MATERIAL}}$$

Except for air (and vacuums) which has a refractive index of 1, the refractive index of most substances is greater than *unity* ($n > 1$). Water, for instance, has a refractive index of 1.333.

In reality, the refractive index of any material varies slightly as a function of the wavelength. This means that various colors of light will actually have different indices of refraction in the same lens material! This is a result of the fact that colors of light with shorter wavelengths, like violet, travel more slowly through most transparent materials than colors with longer wavelengths, like red. Therefore, violet light has a higher index of refraction than red.

This phenomenon is responsible for the **chromatic dispersion** of white light into its component colors by prisms and also lenses. For now, we will assume an average wavelength of 587.56 nm for all calculations. This *yellow-green* color, produced by the *helium d line*, is the standard **reference wavelength** for ophthalmic optics in the United States. In some countries, 546.07 nm is used as the reference wavelength, which is produced by the *mercury e line*. This means that the power of a lens or prism is based upon the refractive index that the material has for the chosen reference wavelength.

Example

A ray of light travels at a velocity of 200,000 km/s through a particular lens material. What is the index of refraction for that lens material?

$$n = \frac{300,000}{200,000}$$

$$n = 1.500$$

\therefore Index of refraction is 1.500

2.5 CURVATURE

Throughout this workbook, we will rely on the concept of the *curvature* of a surface. The **curvature** R is defined as the angle through which the surface turns in a unit length of arc, which is when the length of the arc equals 1 m. The curvature R is given simply by (Fannin & Grosvenor 24),

$$\text{EQ. 3} \quad R = \frac{1}{r}$$

where r is the radius of curvature of the surface in meters. The unit of measurement for curvature is the reciprocal meter (m^{-1}).

The curvature of a surface is *inversely proportional* to its radius of curvature, and will *increase* in magnitude as the radius *decreases* in magnitude. This is demonstrated in Figure 2:4 with two different circles. The circle with the smallest radius has more curvature, and vice versa.

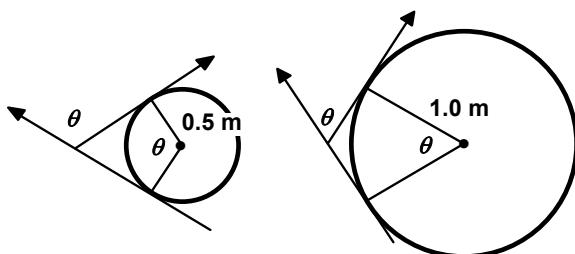


FIGURE 2:4 The angle θ represents the angle turned over a unit length of arc. The circle with the 0.5-m radius has twice as much curvature as the circle with the 1.0-m radius.

Example

A spherical surface has a 0.5-m radius of curvature. What is the curvature of the surface?

$$R = \frac{1}{0.5}$$

$$R = 2$$

\therefore Curvature is 2 m^{-1} .

2.6 WAVE PROPAGATION AND VERGENCE

As described earlier, when light is emitted by a luminous source the radiation travels outward in a wave fashion. This is similar to a pebble creating ripples in a pond. Waves traveling across water are confined to one plane, so they travel outward *circularly*. Light waves from a luminous point source, however, travel in every direction and form **spherical wave fronts**.

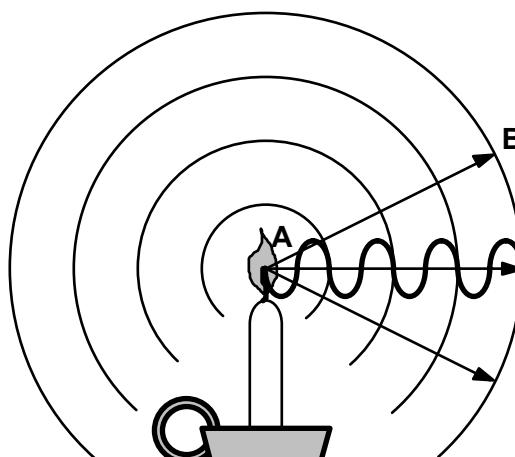


FIGURE 2:5 Point A represents a source with wave trains of light propagating outward in every direction. The *wave front* is the spherical shell that envelopes all of these wave trains at a particular distance from the source. Line AB represents a ray originating from point A and perpendicular to the wave front at point B.

Each wave front is a spherical shell that envelopes all of the wave trains at a particular distance from the source. Hence, the wave fronts also propagate outward from the point source, which serves as their common center of curvature. This process is illustrated in Figure 2:5.

The substance through which these wave fronts travel is referred to as the **medium**. Media can include empty space, air, lens materials, etc. For our purposes, we can represent these wave fronts with simple rays traveling through the media in the direction of the wave trains, as shown in Figure 2:5. Moreover, since these rays originate at the center of curvature of the wave fronts, the rays are also perpendicular to them. When the direction of light is represented using rays we call it **rectilinear propagation**.

These rays, which are used to represent the geometric behavior of the propagation of the wave fronts, *diverge* from such a point source. Furthermore, since the point source serves as the center of curvature of these wave fronts, the distance from the source to a particular wave front serves as the *radius of curvature* of that wave front. As a given wave front becomes more distant from the source, its radius of curvature *increases* in length. This, of course, *decreases* the curvature of the wave front.

We use the term **vergence** to describe the amount of curvature of a given wave front. Unlike regular curvature, however, the vergence of a wave front is measured in units called **diopters**—instead of reciprocal meters. In addition, we use l to represent the linear distance from a particular object or image point. This distance serves as the radius of curvature of the wave front. In air, the vergence L of the wave front—in **diopters** (abbreviated ‘D’)—is equal to the reciprocal of the radius of curvature l of the wave front.

It is given by the simple relationship

$$\text{EQ. 4} \quad L = \frac{1}{l}$$

where l is the distance from the source measured in meters. Or, more simply, *the vergence in air is equal to the reciprocal of the radius of the wave front*.

Note: A point source will produce 1 diopter of vergence at a distance of 1 meter.

It should now be obvious that our formula for vergence is the same formula utilized to calculate regular curvature. Remember that the radius of curvature of a given wave front *increases* in magnitude as the wave train gets farther and farther from the point source. This results in a *decrease* in the magnitude of vergence, which is the curvature of the wave front. Figure 2:6 demonstrates this progressive effect.

The sign (\pm) of the value of l identifies the type of vergence. This is because vergence can be either positive or negative:

- *Positive (+)* values for vergence will produce *convergent* wave fronts that come to a point.
- *Negative (-)* values for vergence will produce *divergent* wave fronts that spread apart (as if from a point).
- *Zero (0)* values for vergence will produce *parallel* wave fronts, with no vergence.

In Figure 2:5 and Figure 2:6, the vergence of the light rays is away from a point source of light; this is called *negative vergence*, or **divergence**. Light rays can also come together to form an image; this is called *positive vergence*, or **convergence**.

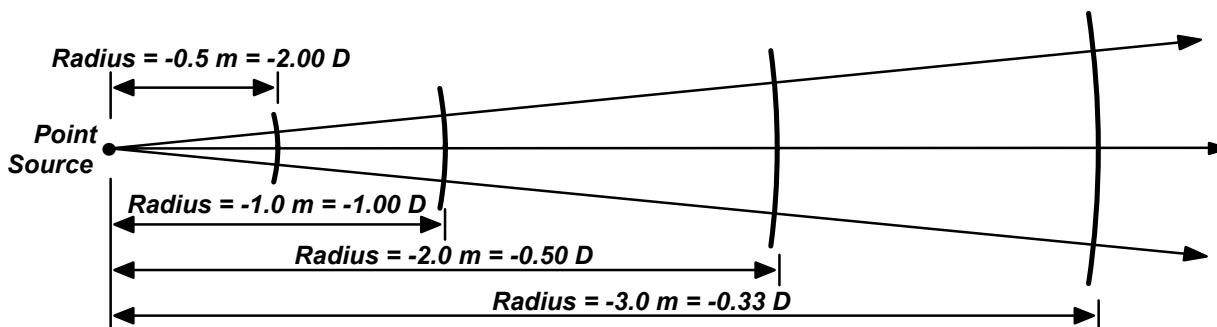


FIGURE 2:6 Wave fronts diverging from a point source eventually become more and more parallel the farther they get from the source. The rays representing these wave fronts, which are perpendicular to them, also become increasingly parallel. The vergence L approaches 0 as the distance l approaches *optical infinity* (∞). For practical purposes, this distance is typically considered to be at 6 m (20 ft) and beyond. The actual vergence past 6 m will be less than 0.17 D. This concept will be discussed in more detail in the next section.

Wave fronts become progressively flatter as the distance from their center of curvature increases. As the radius of the wave front becomes infinitely long, the vergence approaches zero. The rays used to represent these flatter wave fronts also become increasingly parallel as they diverge. Rays emanating from point sources located beyond **optical infinity** (∞), which is approximately 6 m (or 20 ft), will become effectively parallel with each other. That is to say that the wave fronts beyond this distance will have an insignificant amount of divergence. To illustrate this concept, consider Table 2 below which shows the vergence values of wave fronts from increasingly distant objects.

TABLE 2 Vergence of light over various distances

½ m	1 m	5 m	10 m	1000 m
2.00 D	1.00 D	0.20 D	0.10 D	0.001 D

2.7 SIGN CONVENTION

Before we go any further we need to say a few things about the sign convention used in this and most other ophthalmic optics textbooks. This sign convention, as diagrammed in Figure 2:7, includes (Loshin 8):

- Light rays are generally depicted as traveling from left to right for consistency.
- The path that light rays travel in one direction, including the image and object points, is the same path that the light rays would travel coming from the opposite direction. Light rays are *reversible*.
- The vergence l is typically measured from the wave front to an object or image point.
- Distances measured in the same direction that light travels are *positive* (+). Distances measured in the opposite direction are *negative* (-). For instance, measuring l from the wave front of the diverging light back to the point source in Figure 2:5 produces a *negative* value.

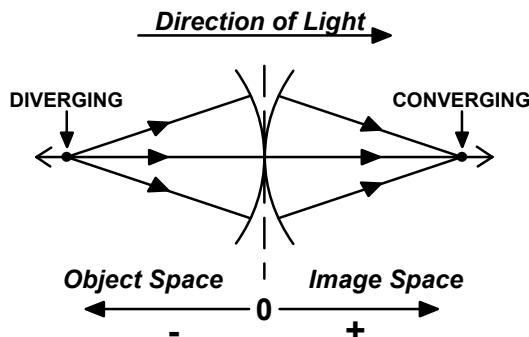


FIGURE 2:7 Optical sign convention.

Rays of light originate—or appear to originate—from every minute *point* of a source referred to as the **object** of the lens. After passing through the lens, these rays

are converged to, or diverged from, either a real or virtual *point* focus. (This process will be described in detail in Section 4.1.) The sum of these points combine to form either a real or virtual **image** of the original object, as illustrated in Figure 2:8. Every object point is associated with an image point; therefore, these points are called **conjugate points**.

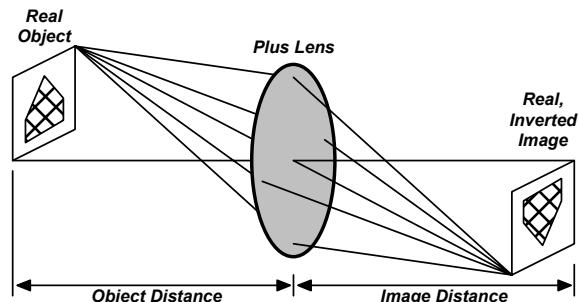


FIGURE 2:8 Conjugate object and image points. This *plus* lens has formed a *real* image of a *real* object. The image is actually *inverted* (rotated 180°), relative to the original object. Real images can be focused upon a screen; virtual images cannot.

The distance of an *object* from the lens or surface is referred to simply as the **object distance** l . Similarly, the distance of the resultant *image* of the object from the lens or surface is referred to as the **image distance** l' . Remember that the image and object points are conjugate.

We also have some sign convention rules to follow for objects and images, as shown in Figure 2:9:

- Objects located in the *object space* (which are to the *left* of the lens for our purposes) are **real**. Objects located in the *image space* (to the *right* of the lens) are **virtual**.
- Images located in the *image space* (which are to the *right* of the lens for our purposes) are **real**. Images located in the *object space* (to *left* of the lens) are **virtual**.

Example

Light is diverging (*negative*) from an object located 2 m away. What is the vergence of the wave front?

$$L = \frac{1}{-2}$$

$$L = -0.50$$

∴ Vergence is -0.50 D (*divergence*)

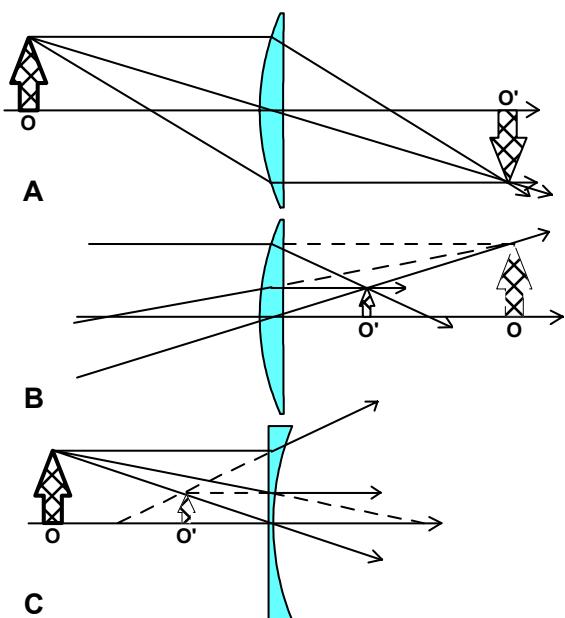


FIGURE 2:9 Common object and image types. A) A *real* image O' formed by a *plus* lens and a *real* object O; B) A *real* image O' formed by a *plus* lens and a *virtual* object O; and C) A *virtual* image O' formed by a *minus* lens and a *real* object O.

Example

A wave front is converging (*positive*) upon an image point located 1/3 m away. What is the vergence of the wave front?

$$L = \frac{1}{\frac{1}{3}}$$

$$L = 3.00$$

∴ Vergence is 3.00 D (*convergence*)

3. Refraction, Reflection, and Prism

When light strikes an object, it may be *transmitted* (allowed to pass through), *absorbed* and converted into heat, and/or *reflected* off of the object. Each of us experiences these phenomena every day. We will look closely at the process of *refraction*, and then briefly consider *reflection*. Both play integral roles in the interaction between light and optical surfaces.

3.1 REFRACTION

When light travels from one medium into another with a different index of refraction, the velocity of the light will change. When going from a lower index medium to a higher index medium that is more dense, such as from air to a piece of glass, the velocity is reduced. If the rays of light are incident upon the glass surface perpendicularly, or *normal* to the surface (at a 90° angle to the surface), the rays will pass through without changing direction.

When rays of light strike a differing medium obliquely, or at an angle, they are **refracted**, or bent, at the interface between the two media. When going from a lower-index medium to a higher-index medium that is more dense, such as from air to a piece of glass, the rays of light are shifted *toward* the **normal**, which is an imaginary line of reference perpendicular to the surface at the point of incidence. This process can be better visualized by considering the *wave* form of light, as shown in Figure 3:1.

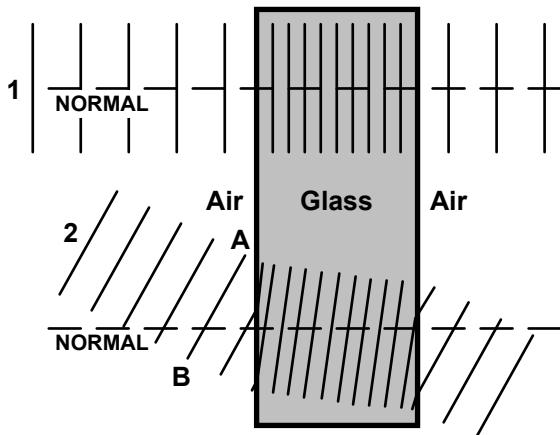


FIGURE 3:1 The wave fronts of wave train 1 enter the glass medium perpendicularly, and are slowed down uniformly. There is no change in direction. The wave fronts of wave train 2 enter the glass obliquely. Side A of the approaching wave fronts strikes the glass before side B, causing side A to slow first. As a result, the wave train is *refracted*, or bent, as it enters the medium.

When going from a higher index medium to a lower index medium that is less dense, such as from a piece of glass to air, the reverse occurs and the rays of light are shifted *away* from the normal of the surface.

Snell's law of refraction is fundamental to the study of optics. It mathematically establishes how much rays of light will be deviated from their original path as they pass through various media (Freeman 18):*

$$\text{EQ. 5} \quad n \cdot \sin i = n' \cdot \sin i'$$

Snell's law tells us that the product of the sine of the **angle of incidence** i , and the refractive index n of the medium containing this angle, is equal to the product of the sine of the **angle of refraction** i' , and the refractive index n' of the medium containing this angle. It also states that the angles of incidence and refraction lie in one plane.

Both of the angle of incidence and the angle of refraction are measured from the *normal*. A third angle, the **angle of deviation** θ , lies between the refracted ray and the direction of its original path. This angle represents the shift of the ray from its original path. Hence, the angle of deviation is equal to the difference between the angles of incidence and refraction. Snell's law is illustrated in Figure 3:2 for another parallel block of glass. From the formula, we can also conclude that:

- If $n > n'$ then $i < i'$
- If $n < n'$ then $i > i'$

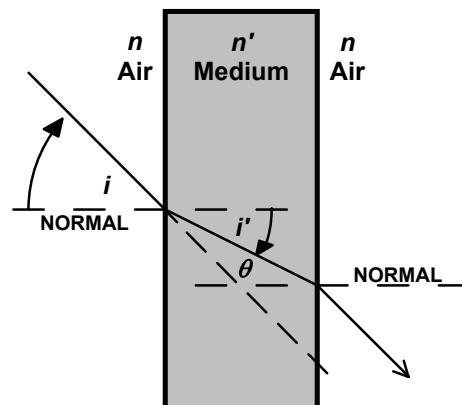


FIGURE 3:2 Snell's law of refraction. Note that $\theta = i - i'$. Since the sides of the glass medium are parallel, the ray emerges parallel to its original path (but displaced).

If both sides of the block of glass (or higher index medium) are completely parallel to each other, as is the case in both Figure 3:1 and Figure 3:2, the rays will exit traveling parallel with, yet slightly displaced from, their original direction. The amount of displacement will depend upon both the index of refraction and the thickness of the medium.

* Snell's law can be proven using either *Huygens' principle* or *Fermat's principle of least time*.

Example

A ray of light strikes a lens material with a 1.523 refractive index, at a 30° angle of incidence in air (which has a refractive index of 1). What is the angle of refraction?

$$1 \cdot \sin 30^\circ = 1.523 \cdot \sin i'$$

$$\sin i' = 0.3283$$

$$i' = 19.17^\circ$$

\therefore Angle of refraction is 19.17° .

3.2 REFLECTION

We will only address the phenomenon of reflection briefly in this workbook. It was mentioned earlier that we see most objects simply because light “bounces off” them. This process is known as **reflection**. Reflection can be described with a simple relationship, as illustrated in Figure 3:3, known as the **law of reflection**. This law simply states that the angle of incidence i is equal to the **angle of reflection r** (Freeman 13):

EQ. 6 $i = r$

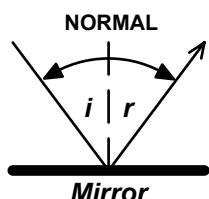


FIGURE 3:3 Reflection of a light ray.

As with refraction, these angles are both measured from the *normal*, the imaginary line of reference perpendicular to the surface at the point of incidence. Imagine a rubber ball bouncing off a wall; the ball will bounce off at the same angle at which it strikes. The same concept applies to the reflection of light.

Reflection occurs when light is incident upon the boundary or interface between two different media (e.g., air and water). Part of the incident light travels back into the first medium. The amount and color of light that is reflected back into the first medium depends upon the nature of the materials, as well as the angle of incidence. There are two principal types of reflections that can occur. When rays of light strike a *rough* surface, like concrete, the uneven surface reflects, or scatters, the light in every direction. This is known as **diffuse reflection**. When rays of light strike a *smooth* and shiny surface, like glass, the surface reflects an image of the source. This is known as **specular reflection**. Both types of reflection are illustrated in Figure 3:4. (Keating 6).

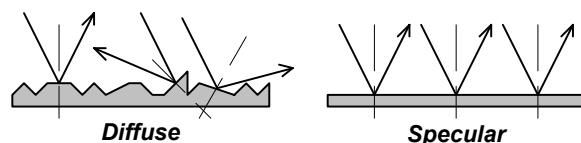


FIGURE 3:4 Rough versus smooth surface reflection.

All transparent materials reflect a certain amount of light. The fraction of incident light reflected from a surface is referred to as the **reflectance ρ** . So how much light is reflected? For light striking a lens perpendicularly in air, the reflectance ρ is given by **Fresnel's formula**:

$$\text{EQ. 7} \quad \rho = \left(\frac{n - 1}{n + 1} \right)^2$$

where n is the refractive index of the material. To express the reflectance ρ as a percentage, simply multiply by 100.

Consequently, as the index of refraction of the lens material *increases* the amount of light reflected *increases*.

Example

Light is incident upon a transparent medium with an index of refraction of 1.500. What is the reflectance?

$$\rho = \left(\frac{1.500 - 1}{1.500 + 1} \right)^2$$

$$\rho = \left(\frac{0.5}{2.5} \right)^2$$

$$\rho = 0.04$$

\therefore Reflectance is 0.04 (or 4%).

Each surface of common glass and plastic lens materials reflects at least 4 to 5% of the light incident upon the lens. Between both surfaces, that is a total reflectance of at least 8%. Conversely, a completely clear lens (with little or no absorption) can only transmit 92% of the light passing through it. Thin coatings can be applied to a lens surface to reduce this reflectance to almost nothing—thereby increasing the transmittance of the lens.

3.3 PRISM

We can now apply the concept of refraction to elements designed to manipulate light. A **prism** is a refracting medium bound by non-parallel sides. Like a triangle, the *thickest* edge of the prism is referred to as the **base**, while the *thinnest* edge of the prism is referred to as the **apex**. This wedge-shaped element changes the direction of light without necessarily changing its vergence.

When light passes through a prism, the rays are deviated toward the *base*. This causes the image to appear displaced towards the *apex*, as illustrated in Figure 3:5.

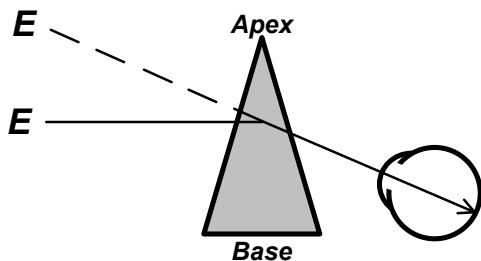


FIGURE 3:5 The ray of light passing through the prism is refracted towards the base, causing the image to appear displaced towards the apex.

Developing an understanding of the action of a prism upon light is essential, in order to comprehend the process of refraction through ophthalmic lenses. Figure 3:6 demonstrates the apparent displacement of an image, as seen through the prism.



FIGURE 3:6 Image seen through a prism.

Before we continue, a few additional terms and concepts need to be defined. The **angle of deviation** is the total displacement of a ray of light passing through the prism from its original direction. The **apical angle** is the angle between the two faces of a prism (towards the apex). The deviation produced by an ophthalmic prism is traditionally measured in units called prism diopters (abbreviated $^{\Delta}$).

The prismatic deviation Δ of an image, in prism diopters, is simply equal to

$$\text{EQ. 8} \quad \Delta = \frac{y_{CM}}{x_M}$$

where y_{CM} is the image displacement, in centimeters, by the prism over a given distance x_M , in meters.

Note: 1 prism diopter represents a displacement of 1 centimeter over a distance of 1 meter. *

* Strictly speaking, however, the prism diopter is not a *true* unit of angular deviation since its deviation changes with greater angles. For instance, 1^{Δ} is equal to 0.57° , while 40^{Δ} —which is $40 \times 1^{\Delta}$ —is equal to 21.80° . However, this is only $38 \times 0.57^{\circ}$.

The prism diopter is illustrated in Figure 3:7. Moreover, the prism diopter can be defined more generally as a deviation of 1 arbitrary unit over a distance of 100 such units. For instance, 1^{Δ} also equals a displacement of 1 inch over a distance of 100 inches.

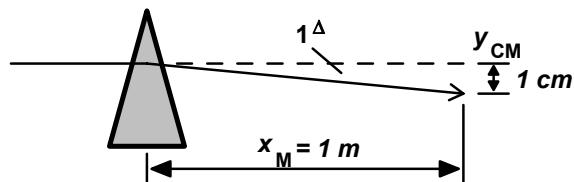


FIGURE 3:7 The prism diopter: 1^{Δ} deviates light 1 cm over a distance of 1 m (or 1 unit over 100 units).

Example

A prism deviates a ray of light 2 cm at a distance of 4 m. What is its prismatic deviation?

$$\Delta = \frac{2}{4}$$

$$\Delta = \frac{1}{2}$$

∴ Prismatic deviation is $\frac{1}{2}^{\Delta}$ (0.5^{Δ}).

The prismatic deviation θ can also be expressed in degrees by converting it with the formula

$$\text{EQ. 9} \quad \theta = \tan^{-1}\left(\frac{\Delta}{100}\right)$$

For small angles, this means that 1° of deviation will be nearly equal to 1.75^{Δ} of deviation.

Example

A certain prism has 6 prism diopters of power. What is its prismatic deviation when expressed in degrees?

$$\theta = \tan^{-1}\left(\frac{6}{100}\right)$$

$$\theta = 3.43^{\circ}$$

∴ Prismatic deviation is 3.43° .

If we know the apical angle of a *thin* prism (which has a small apical angle) we can also determine its approximate prismatic deviation. We will employ a *small angle approximation* of Eq. 5, Snell's law of refraction (Fannin & Grosvenor 82):

$$n \cdot \sin i \approx n \cdot i$$

Essentially, the sine of a *small* angle is approximately equal to the angle itself, when the angle is expressed in radians. We will go systematically through the process of showing a relationship between the apical angle a of

prism, its index of refraction n , and the prismatic deviation θ it produces, in degrees. If you find the math a bit tedious, you can skip over the derivation. Keep in mind, though, that it will give you some insight into how many of the other formulas that you will encounter are derived.

If we assume that the rays of light strike the first surface of the prism in Figure 3:8 perpendicularly, we can ignore the refraction of light at this surface.* We are now only concerned with the refraction of light at the second surface, and the angle of incidence i at this surface, to determine the angle of prismatic deviation θ .

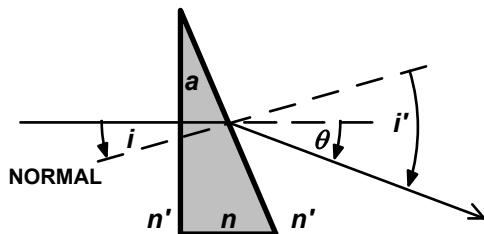


FIGURE 3:8 Relationship between deviation and the apical angle of a prism. Since the normal is perpendicular to the hypotenuse of the triangle (prism), angle $i = a$.

We can now use the following relationships to determine the angle of deviation θ . Recall that the angle of deviation θ is the difference between the angles of refraction i and incidence i' :

$$\theta = i' - i$$

Similarly,

$$i' = \theta + i$$

Now consider that the original ray was perpendicular to the first surface of the prism. Further, the angle of incidence i is measured from the normal to the second surface (which is perpendicular to it). Consequently, the apical angle a is equal to the angle of incidence i :

$$a = i$$

We can now also substitute our earlier relationships for i and i' into the approximation of Snell's law of refraction to give us

$$n \cdot i \approx n' \cdot i'$$

Moreover, $n' = 1$ (since n' is air). We will now substitute angle a for angle i into the expression.

$$n \cdot a = 1i'$$

* In reality, the angle of incidence at the first surface will affect the total amount of deviation. However, this approximation will be suitable for our purposes.

Next, we will plug our earlier relationship for angle i' back into the expression above (once again substituting a for i):

$$n \cdot a = \theta + a$$

Finally, after rearranging and factoring out a , the prismatic deviation θ —in degrees—is given by:

$$\text{EQ. 10} \quad \theta = a(n - 1)$$

Keep in mind that this approximation holds for *thin* prisms only, and will quickly lose accuracy as the apical angle approaches and exceeds 10°.

Example

A prism has an apical angle of 10° and a refractive index of 1.500. What is the prismatic deviation produced by the prism in degrees?

$$\theta = 10(1.500 - 1)$$

$$\theta = 5^\circ$$

∴ Prismatic deviation is 5°.

3.4 REDUCED DISTANCE

You have probably noticed at one time or another that objects in water appear to be closer than they actually are. This apparent image displacement is just another consequence of refraction. This phenomenon is illustrated in Figure 3:9, where a fish is located at a distance t below the surface of the water. The **reduced distance**—or **equivalent thickness**—to the fish is the distance d , and is equal to (Keating 137)

$$\frac{d}{n'} = \frac{t}{n}$$

Since the first medium is generally air ($n' = 1$), we will ignore this value (since $d / 1 = d$) and use

$$\text{EQ. 11} \quad d = \frac{t}{n}$$

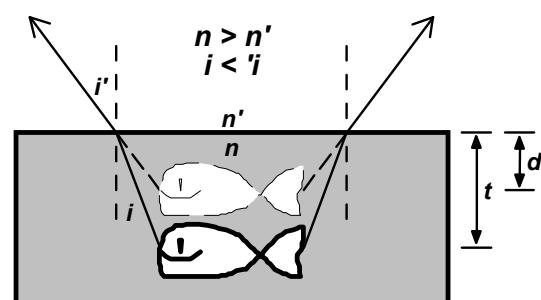


FIGURE 3:9 The fish is located at a distance t from the surface of the water. It appears to be located at the *reduced distance* d , however, because of the refraction of light.

We refer to the apparent depth of the fish as ‘reduced,’ simply because the distance to the image of the fish appears shorter. We may also call this distance ‘equivalent,’ because this is the distance from the fish in *air* that would produce the same vergence as that produced by the fish in *water* at the original distance. We will return to the concept of reduced distance when discussing *thick lenses*.

It should be apparent from Figure 3:9 that the refraction of light at the interface between the water and air effectively changes the *curvature* of the wave front leaving the surface of the water. In the earlier analysis of vergence in Section 2.6, the vergence of light was described in *air*. Our discussion of reduced distance, however, has reiterated the fact that light propagates differently in different media, because of changes in its velocity (i.e. refractive index).

A more general description of vergence, that takes into account the effects of the medium, is given by using the reciprocal of the reduced distance l / n —as opposed to the physical distance l . Our new formula, for any medium, is

$$L = \frac{1}{n/l}$$

EQ. 12 $L = \frac{n}{l}$

where l is the physical distance from an object or image point measured in *meters*.

Of course, when *air* is the medium, $n = 1$; returning us back to the original vergence formula:

$$L = \frac{1}{l}$$

Recall that vergence is measured in diopters.

Example

A person stands above the water looking at an object located 1 m below the surface of a pond ($n = 1.333$). How far from the surface does the object appear to be located (what is the reduced distance)?

$$d = \frac{1}{1.333}$$

$$d = 0.75$$

∴ Reduced distance is 0.75 m.

4. Refractive Power and Lenses

One of the primary applications of an ophthalmic lens is to change the vergence of incident light, typically to compensate for a refractive anomaly of the eye. The ability of a lens to change the vergence of light is referred to as **focal power**. We begin our discussion of *focal power* by first analyzing curved refracting surfaces, since a **lens** is essentially a lens material bounded by two refracting surfaces.

So why employ curved refracting surfaces? Consider Figure 4:1 and Figure 4:2. Prisms, placed base-to-base, are used to bring light rays together (similar to the action of a *convex* surface or lens). In the first example, using only two prisms, the rays are not all brought to a single point focus. In the second example, using a separate prism for each ray, we are able to cause them all to intersect. Obviously, it would take an infinite number of prisms to bring every individual ray to a single focus.

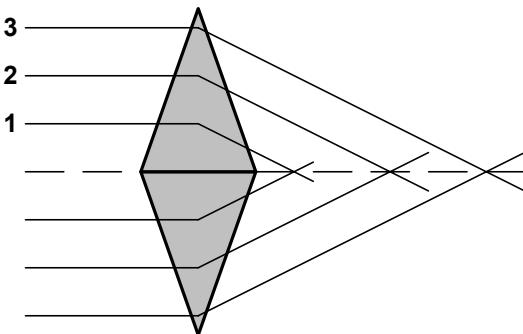


FIGURE 4:1 Rays 1, 2, and 3 strike the prisms at progressively farther distances from the central section. Each ray is deviated by the same amount, so they do not combine to a single point.

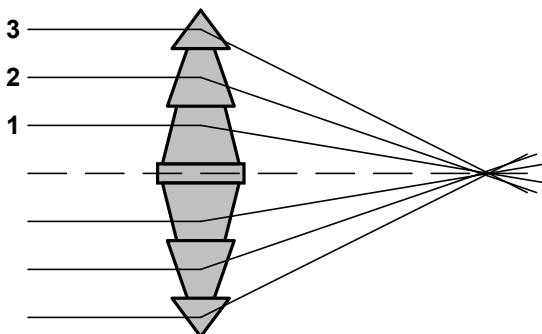


FIGURE 4:2 Rays 1, 2, and 3 strike the prisms at progressively farther distances from the central section. These rays require stronger and stronger prisms (with increasing apical angles) to deviate the light to a single point.

This is exactly what curved surfaces allow us to do, as shown in Figure 4:3. These diagrams can also be redrawn with prisms apex-to-apex, to simulate the divergence of light produced by a minus lens.

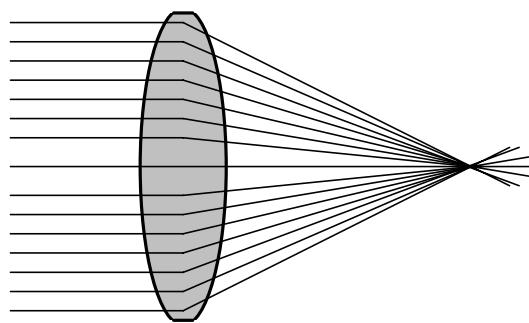


FIGURE 4:3 Each individual ray is brought to a single point focus using curved refracting surfaces (if we neglect aberrations). The slope of a curved refracting surface naturally increases away from the central zone, which is flat. This changing slope acts like a continuously increasing apical angle, or an infinite series of prisms, as shown with this *plus* lens.

4.1 CURVED REFRACTING SURFACES

Lens surfaces are often referred to as **surfaces of revolution**, since they can be described by revolving a plane geometric shape—like a circle or an arc—about an **axis of revolution** that lies within its plane. This creates a three-dimensional surface that we can measure in terms of its curvature. The most common example of these surfaces is the **sphere**, which is produced when a circle is rotated about an axis that passes through its center, as depicted in Figure 4:4. A typical ‘spherical’ ophthalmic lens surface is essentially a section cut from such a sphere.

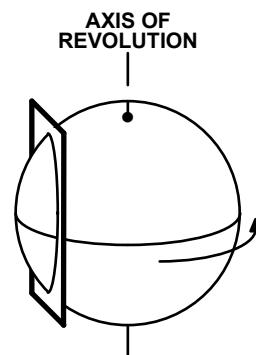


FIGURE 4:4 A lens surface is cut from a section of this spherical surface of revolution. Any point on the surface of a sphere is equidistant from its center of curvature.

A lens surface is simply an *interface* between two media with different indices of refraction. This interface is between air (with a refractive index of 1) and the lens material. The common curvature of this interface, which is determined by the radius of curvature r , and the difference in refractive index between the two media determine how light is affected (or ‘refracted’) as it passes from one medium to the other. For our purposes, we will use n to represent the medium to the *left* of this

interface in *object space*, and n' to represent the medium to the right in *image space*.

The ability of a particular refracting *surface* to change the vergence of incident light is referred to as its **surface power**. If we know the distance of an object from the surface, and its surface power, we can determine the distance at which the image of the object will be formed.

Recall that the *object* distance l is the distance of an *object* from the surface, while the *image* distance l' is the distance of the resultant *image* from the surface. Now consider Figure 4:5, where *diverging* rays of light from the real object point L are intercepted by the lens surface. The object distance from the lens surface is l . Once again, Snell's law of refraction can be simplified to develop a relationship between the refractive indices of the two media (n and n') surrounding the surface, its radius of curvature r , and the image distance l' from the surface that rays of light are brought to a focus at point L' after refraction. This formula, known as the **conjugate foci formula** for lens surfaces, is given by*

$$\text{EQ. 13} \quad \frac{n'}{l'} = \frac{n' - n}{r} + \frac{n}{l}$$

where n is the refractive index of the medium to the *left* of the surface in *object space*, n' is the index of the medium to the *right* in *image space*, l is this *image* distance, l' is the *object* distance, and r is the radius of curvature of the surface (or *interface*). All of these distances are measured in meters.

At this point, we can substitute '*dioptric equivalents*' for the three terms of the conjugate foci formula. For instance, the term on the left side of the equation, n'/l' , represents the *image vergence* L' in diopters. (Remember our preceding discussion on vergence and reduced distance.) The last term on the right side of the equation, n/l , is the *object vergence* L in diopters.

This gives us

$$L = \frac{n}{l} \quad \text{and} \quad L' = \frac{n'}{l'}$$

as the *dioptric equivalents*.

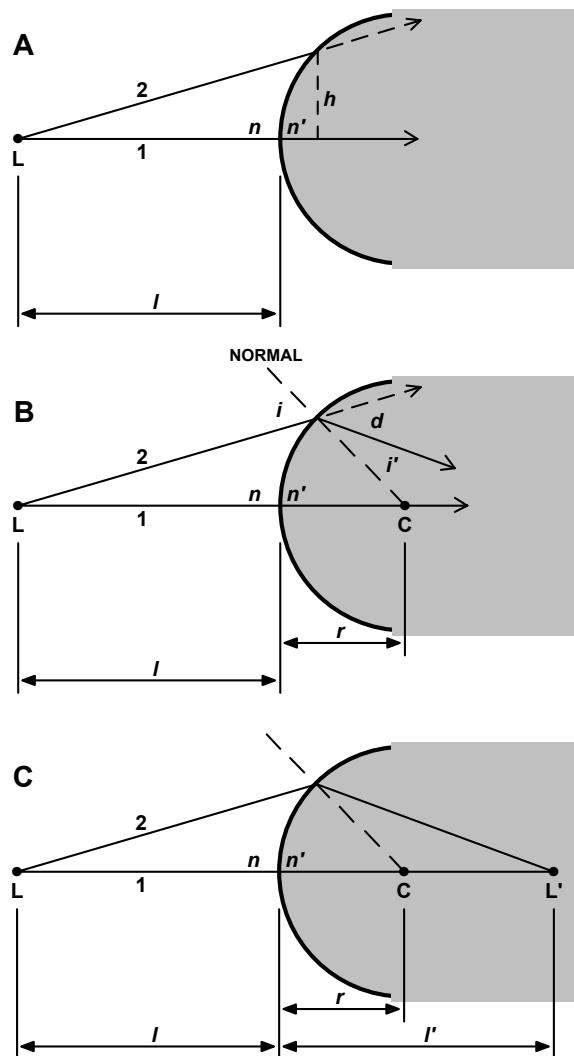


FIGURE 4:5 Refraction of light rays by a spherical surface (assuming $n' > n$). Rays 1 and 2 are diverging from a *real object* point at L, before striking the lens surface. A) Ray 1 strikes the surface perpendicularly, and is not refracted. Ray 2 strikes the surface at the height h . B) The normal to the surface at the height of ray 2 has been drawn through the center of curvature at C. Ray 2, which is parallel to ray 1, strikes the surface at an angle of incidence i . After refraction by the surface, ray 2 is at an angle i' to the normal, and is deflected by the angle of deviation d . C) Finally, ray 2 intersects ray 1 to form a *real image* at point L'. Note that L and L' are conjugate points.

The first term on the right side of the equation, $(n' - n) / r$, is the *surface power* F_S in diopters. This quantifies the ability of the lens surface to change the vergence of incident light. Hence, the basic **surface power formula** is

$$\text{EQ. 14} \quad F_S = \frac{n' - n}{r}$$

* The complete derivation for this formula can be found in Appendix B.

Substituting these ‘dioptric equivalents’ back into the conjugate foci formula (Eq. 13) gives us a much more pleasant-looking equation:

$$\text{EQ. 15} \quad L' = F_s + L$$

This variation of the formula tells us that the image vergence L' produced by a lens surface is simply equal to the sum of the surface power F_s and the object vergence L . Or, more simply, *the image vergence is the net result of the effect that the surface power has on the object vergence.*

You should commit this formula to memory, since it is fundamental to many of the equations that follow.

When referring specifically to lens surfaces in air, we often distinguish between the following three basic types of surface curvatures and power, which are all illustrated in Figure 4:6:

- **Convex curves** (think of the *outside* of a bowl) produce a *positive* (+) surface power, and add *convergence* to incident rays of light.
- **Concave curves** (*inside* of a bowl) produce a *negative* (-) surface power, and add *divergence* to incident rays of light.
- **Plano curves** (*flat*) produce *zero* surface power (0), and do *not* change the vergence of incident rays of light. (The radius of curvature of this surface is infinitely long.)

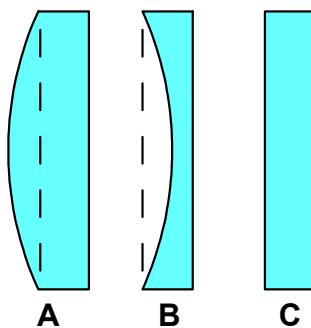


FIGURE 4:6 Lens surfaces. A) A *convex* curve; B) A *concave* curve; and C) A *plano* curve.

It is also helpful to keep in mind that we don’t necessarily have to substitute *all* of the dioptric equivalents into the conjugate foci formula. For instance, equations like the ones below are acceptable:

$$\frac{n'}{l'} = F_s + \frac{n}{l}$$

Or,

$$\frac{n'}{l'} = F_s + L$$

Example

Light from an object 50 cm away (0.5 m) strikes a convex refracting surface with a refractive index of 1.500 and a radius of curvature of 12.5 cm (0.125 m). How far from the surface is the image formed?

First, we will determine the surface power, given a material index with a refractive index of 1.500 surrounded by air with a refractive index of 1:

$$F_s = \frac{1.500 - 1.00}{0.125}$$

$$F_s = 4.00$$

∴ Surface power is +4.00 D.

Next, we will determine the object vergence using Figure 4:7. Since the light is diverging from the object towards the surface, the object distance is negative (using our sign convention from Section 2.6). Since the object is in air, $n = 1$:

$$L = \frac{1}{-0.5}$$

$$L = -2.00$$

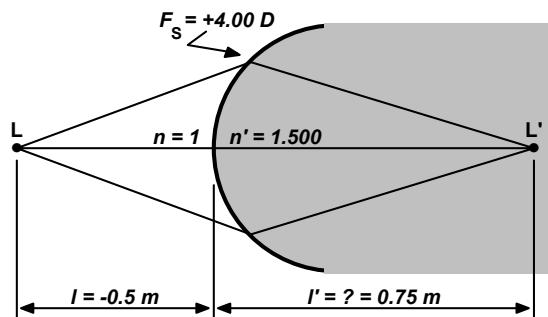


FIGURE 4:7 Conjugate foci for a +4.00 D surface. The object distance is -0.5 m and the image distance is 0.75 m.

Now we can substitute into our conjugate foci formula -2.00 D for the object vergence, 1.500 for the refractive index n' of the lens material, and +4.00 D for the surface power. The only unknown variable now is the image distance l' ,

$$\frac{1.500}{l'} = +4.00 + (-2.00)$$

$$l' = \frac{1.500}{+2.00}$$

$$l' = 0.75$$

∴ Image distance is +0.75 D.

Example

A lens material with a refractive index of 1.500 is bound by a front, concave (*negative radius*) refracting surface with a -100 mm (-0.100 m) radius of curvature. What is its surface power?

$$F_s = \frac{1.500 - 1}{-0.100}$$

$$F_s = -5.00$$

∴ Surface power is -5.00 D.

The curvature of a surface is *inversely* proportional to its radius of curvature; as the radius increases, the curvature decreases. Hence, the surface power is *directly* proportional to both the amount of curvature and the refractive index of the lens material. An increase in either the magnitude of the curvature, or the refractive index, will result in an increase in the magnitude of the surface power.

There are certain conjugate points that have special applications in ophthalmic optics. The *object* point that forms an image located at optical infinity (∞) is referred to as the **primary focal point** F of the surface. In addition, the object distance from the primary focal point to the surface is referred to as the **primary focal length**. To determine the primary focal length f , set the image distance to infinity:

$$\frac{n'}{\infty} = F_s + \frac{n}{f}$$

The n'/∞ term is essentially equal to 0, so it can be dropped from the equation. (Any number divided by infinity ∞ approaches 0.) Hence, the primary focal length f of the surface is related to its surface power by

$$\text{EQ. 16 } F_s = -\frac{n}{f}$$

where F_s is the surface power and f is the *primary* focal length in meters.

The *image* point formed by an object located at optical infinity (∞) is referred to as the **secondary focal point** F' of the surface. The image distance from the secondary focal point to the surface is referred to as the **secondary focal length** f' .

$$\frac{n'}{f'} = F_s + \frac{n}{\infty}$$

Once again, the n/∞ term is dropped.

Hence, the secondary focal length f' is related to its surface power by

$$\text{EQ. 17 } F_s = \frac{n'}{f'}$$

where F_s is the surface power and f' is the *secondary* focal length in meters.

The relationship between the primary and secondary focal points and lengths for *convex* and *concave* lens surfaces are illustrated in Figure 4:8 and Figure 4:9.

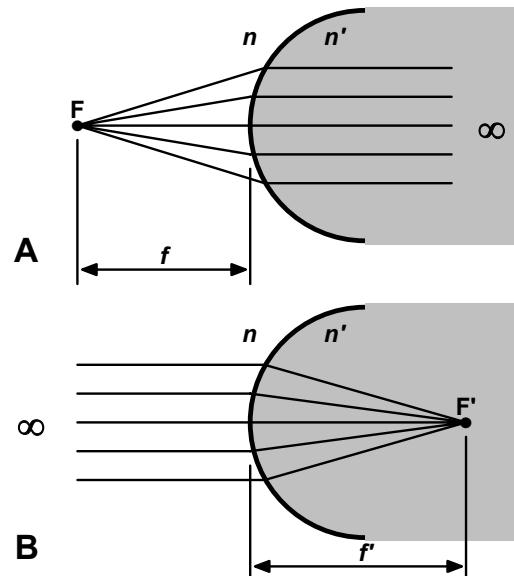


FIGURE 4:8 Refraction at a spherical, *convex* surface ($n' > n$). A) The *primary* focal point F of a surface is the *object* location required to produce an *image* at infinity. B) The *secondary* focal point F' of a surface is the *image* location produced by an *object* at infinity.

For a *concave* surface to have the image at infinity, incoming rays must be convergent. The *primary* focal point is the location where the object would have been formed if the lens had not intercepted the rays and focused them at infinity. This is a *virtual* object. When an object is at infinity and parallel rays hit the surface, in the case of a concave surface the refracted rays are diverging. The point from which they appear to diverge is the *secondary* focal point. This is a *virtual* image.

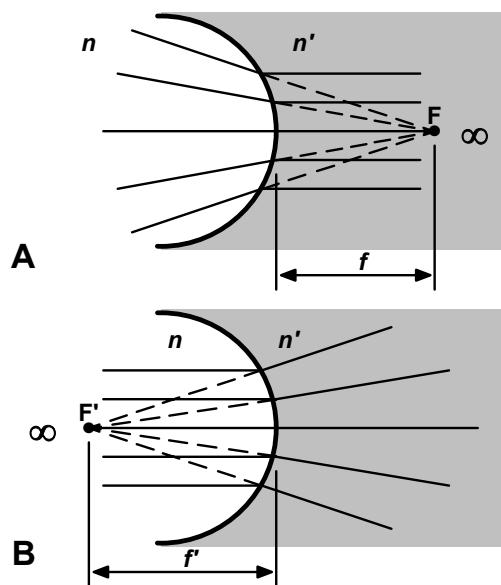


FIGURE 4:9 Refraction at a *concave* surface. A) The *primary* focal point F . B) The *secondary* focal point F' .

4.2 OPTICAL AXIS

Recall that a **lens** is essentially a refractive material bounded by front and back surfaces. Moreover, these optical surfaces may be convex, concave, or plano in curvature—allowing a wide variety of surface combinations. We now know that each surface has a center of curvature (at least one). Also recall that any ray of light passing through the center of curvature of a spherical surface is *normal* (or perpendicular) to that surface. Since the ray is normal to the surface, it will not be refracted.

Before we discuss lenses any further, we should address the term *optical axis*. When discussing prism, it is important to understand the position or location of the optical axis. The **optical axis** is an imaginary line of reference passing through both centers of curvature (C_1 and C_2) of a lens. Since a line passing through the center of curvature of a surface is perpendicular to that surface, the optical axis is *normal* to both the front *and* back surfaces. Moreover, the front and back surfaces are exactly parallel with each other at the two points intersected by the optical axis. The optical axes of some common lens styles are shown in Figure 4:10.

Some lenses do not have a natural optical axis. For instance, lenses with *concentric* surfaces that share a common center of curvature do not have a *single* optical axis but any ray perpendicular to the first surface is also perpendicular to the second.

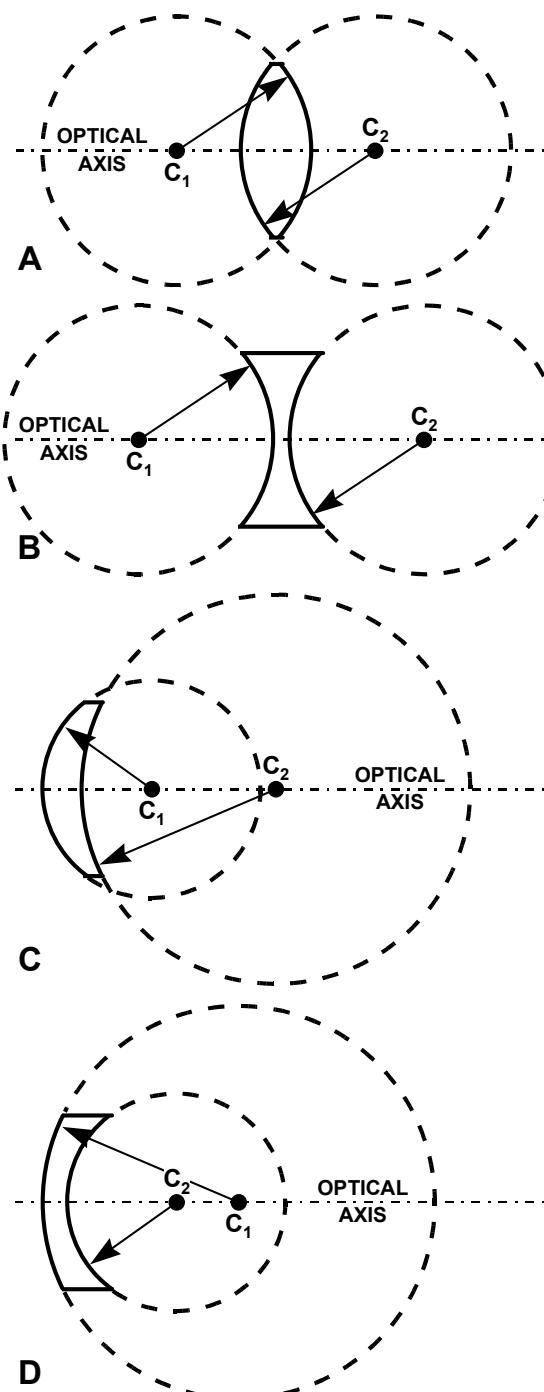


FIGURE 4:10 The optical axis of a lens is the line passing through the centers of curvature of the front and back surfaces (C_1 and C_2). The lens surfaces are parallel with each other at these points. A) Optical axis of lens with two *convex* surfaces, B) a lens with two *concave* surfaces, C) a lens with a *convex* and a *concave* surface, and D) another lens with a *convex* and a *concave* surface.

4.3 THIN LENS POWER

Now that we have looked at individual refracting surfaces, we can look at how these surfaces work together to form a lens. Many of the principles developed for surface power are equally applicable to lenses. The ability of a *lens* to change the vergence of incident light is known as **focal power**. We will first consider ‘thin’ lenses whose center thickness is small and relatively inconsequential. When the two surfaces of such a lens are virtually in contact at the optical axis, we refer to the lens as a **thin lens**. Once the vergence of incident light is affected by the power of the first surface, it is immediately subjected to the effects of the second surface. Consequently, we can ignore the effects of thickness for a ‘thin’ lens. Recall that each surface power is given by (Eq. 14)

$$F_s = \frac{n' - n}{r}$$

For the *front* refracting surface F_1 of a lens, which has air as the medium to the left, the numerator will be $n' - 1$; where n' is the refractive index of the lens material. For the back surface F_2 , which has air as the medium to the right, the numerator will be $1 - n$; where n is the refractive index of the lens material. For our purposes, we will drop the *primed* (') value of n and just use n as the refractive index of the lens material for both the front and back surfaces:

$$F_1 = \frac{n - 1}{r} \text{ and } F_2 = \frac{1 - n}{r}$$

where F_1 is the surface power of the front curve, and F_2 is the surface power of the back curve.

Theoretically, there is no separation between the surfaces of a thin lens. The change in vergence imparted by the first surface is at once followed by a change in vergence at the second surface. In reality, all lenses have some center thickness—particularly *plus* lenses. We will look at the effects of center thickness in the next section. Until then, we can make a slight modification to our conjugate foci formula (Eq. 15) to allow for the effects of both the front *and* back surfaces (F_1 and F_2):

$$\text{EQ. 18} \quad L' = F_1 + F_2 + L$$

The *combination* of the first two terms on the right side of the equation, F_1 and F_2 , is referred to as the *focal power* of the lens in diopters. Hence, the focal power F of a thin lens can be expressed as

$$\text{EQ. 19} \quad F = F_1 + F_2$$

Or, more simply, the *focal power of a thin lens is equal to the algebraic addition of the front F_1 and back F_2 surface powers*. This simple formula is known as the

lensmaker’s formula. Furthermore, when the lens is in air, this formula is equivalent to

$$\text{EQ. 20} \quad F = \frac{n - 1}{r_1} + \frac{1 - n}{r_2}$$

We should now address the three fundamental types of lenses and focal power (there are additional lens types that will be considered later):

- **Plus, positive, or convex lenses** produce a *positive* (+) focal power, and add *convergence* to incident rays of light.
- **Minus, negative, or concave lenses** produce a *negative* (-) focal power, and *divergence* to incident rays of light.
- **Plano lenses** produce *zero* focal power (0), and do *not* change the vergence of incident rays of light.

Example

A lens has a +4.00 D front curve and a -6.00 D back curve. What is its focal power?

$$F = +4.00 + (-6.00)$$

$$F = -2.00$$

∴ Focal power is -2.00 D.

In the example above, the front surface produced +4.00 D of *convergence*, while the back curve produced -6.00 D of *divergence*. This left a net power of $+4.00 + (-6.00) = -2.00$ D of *divergence*.

We can now substitute the focal power F into our conjugate foci formula for thin lenses (Eq. 18):

$$\text{EQ. 21} \quad L' = F + L$$

Note that this formula is the same as Eq. 18, with F substituted for F_1 and F_2 , and describes the ability of a lens to change the vergence of incident light—not just a single surface. For our purposes, the lens will always be surrounded by air ($n = 1$) which means that object and image vergences (L and L') are equal to

$$L = \frac{1}{l} \text{ and } L' = \frac{1}{l'}$$

Or, more simply, *the object vergence L of the light entering the lens is equal to the reciprocal of the object distance l , which is the distance of the object from the lens*.

Similarly, *the image vergence L' of the light exiting the lens is equal to the reciprocal of the image distance l' , which is the distance of the image from the lens*.

This allows us to simplify the conjugate foci formula for thin lenses:

$$\frac{1}{l'} = F + \frac{1}{l}$$

where l' is the image distance from the lens in meters, F is the focal power in diopters, and l is the object distance to the lens in meters.

All measurements of object and image distances for a thin lens are taken from a theoretical reference plane at the center of the lens (since both surfaces are theoretically in contact at this point for a thin lens). An example of the conjugate image and object points for a +4.50 D lens is illustrated in Figure 4:11.

Example

An object is placed 40 cm (0.4 m) in front (*negative vergence* or *divergence*) of a +4.50 D lens. What is the final image distance?

$$\frac{1}{l'} = \frac{1}{-0.4} + 4.50$$

$$\frac{1}{l'} = 2.00$$

$$l' = 0.5$$

∴ Image distance is +0.5 m (50 cm).

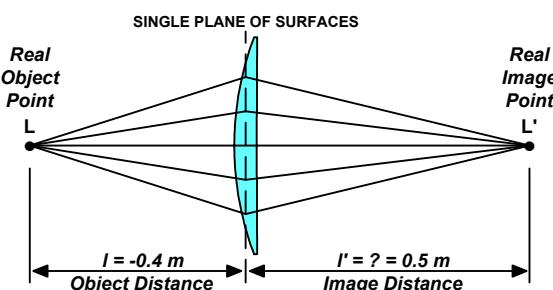


FIGURE 4:11 Conjugate foci for an infinitely thin +4.50 D lens. The length l is the *object* distance, and l' is the *image* distance. These distances are measured from a theoretical reference plane centered within the lens.

As with single surfaces lenses have certain important conjugate points. The *object* point along the optical axis that forms an image located at optical infinity (∞) is referred to as the **primary focal point** F of the lens. In addition, the object distance from the primary focal point to the lens is referred to as the **primary focal length**.

To determine the primary focal length f , set the image distance to infinity:

$$\frac{1}{\infty} = F + \frac{1}{f}$$

Again, the $1 / \infty$ term is essentially equal to 0, and is dropped. Hence, the primary focal length f of the lens is related to its focal power by

$$\text{EQ. 22} \quad F = -\frac{1}{f}$$

where F is the focal power and f is the *primary* focal length in meters.

The *image* point along the optical axis that is formed by an object located at optical infinity (∞) is referred to as the **secondary focal point** F' of the lens. The image distance from the secondary focal point to the lens is referred to as the **secondary focal length** f' .

$$\frac{1}{f'} = F + \frac{1}{\infty}$$

The $1 / \infty$ term is dropped. Hence, the secondary focal length f' of the lens is related to its focal power by

$$\text{EQ. 23} \quad F = \frac{1}{f'}$$

where F is the focal power and f' is the *secondary* focal length in meters.

Or, more simply, *the secondary focal length f' of a lens is equal to the reciprocal of its focal power F* . Moreover, for thin lenses surrounded by air, the primary focal length is equal in magnitude to, yet in the opposite direction of, the secondary focal length:

$$-f = f'$$

Example

A lens has a secondary focal length of 25 cm (0.25 m). What is its focal power?

$$F = \frac{1}{0.25}$$

$$F = +4.00$$

∴ Focal power is +4.00 D.

In ophthalmic optics, we are particularly interested in the secondary focal length of the lens. You should now realize that the reciprocal of the focal power provides the image distance from the lens at which light from an object at infinity will either converge to a *real* point focus for *plus* lenses, or appear to diverge from a *virtual* point focus for *minus* lenses—after refraction through the lens. The image plane that contains all of the image points from such an object is referred to as the **secondary focal plane**; this plane is positioned at the secondary focal point and is perpendicular to the optical axis.

The secondary focal lengths, planes, and points for both a *plus* lens and a *minus* lens are illustrated in Figure 4:12 and Figure 4:13.

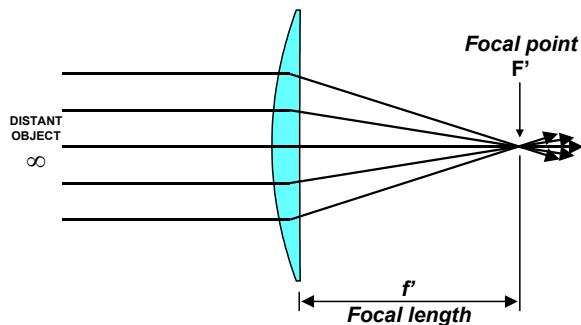


FIGURE 4:12 Cross-sectional view shows parallel rays of light, from a *real* object at infinity (∞), converging to form a *real* point focus at the secondary focal point F' of a *plus* lens. A *real* image is created.

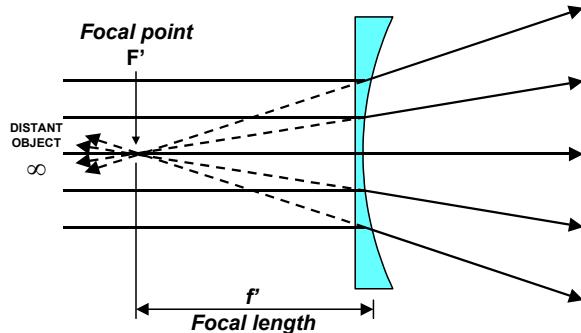


FIGURE 4:13 Cross-sectional view shows parallel rays of light, from a *real* object at infinity (∞), diverging as if from a virtual point focus located at the primary focal point F' of a *minus* lens. A *virtual* image is created.

To reiterate, the secondary focal point is the location—along the optical axis—of the plane in which the image points from an object at infinity will lie. Essentially, the relationship between focal power and focal length is another application of the *vergence* concept. The secondary focal point serves as the center of curvature of the wave fronts that approach the image point, and the secondary focal length is equal to the radius of curvature of the image wave front at the plane of the lens. Hence, focal power is simply the *vergence* of the wave front, from the secondary focal point to the plane of the lens.

This vergence approach is demonstrated in Figure 4:14 and Figure 4:15.

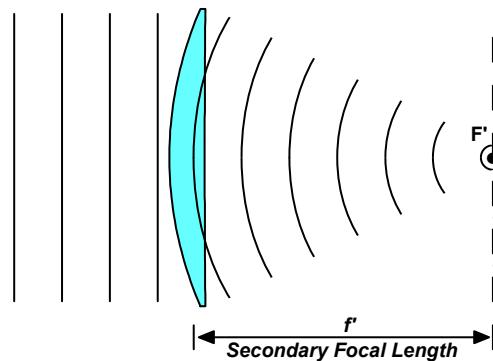


FIGURE 4:14 The action of a *plus* lens upon light can also be described by wave fronts converging to point F' .

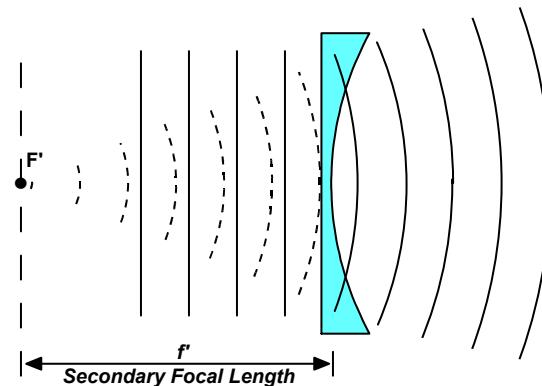


FIGURE 4:15 The action of a *minus* lens upon light can also be described by wave fronts diverging as if from point F' .

The primary focal lengths and points for both a *plus* lens and a *minus* lens are illustrated in Figure 4:16 and Figure 4:17. The secondary focal lengths and points are also shown for comparison.

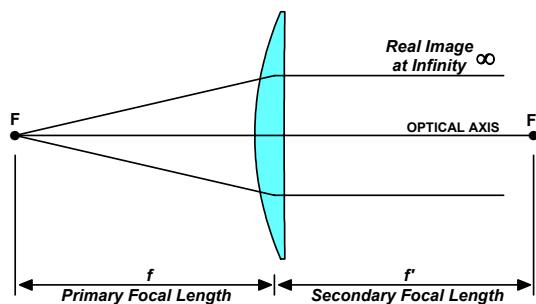


FIGURE 4:16 Cross-sectional view shows rays of light diverging from a *real* object, positioned at the primary focal point F of a *plus* lens, emerging from the lens parallel. A *real* image at infinity (∞) is created.

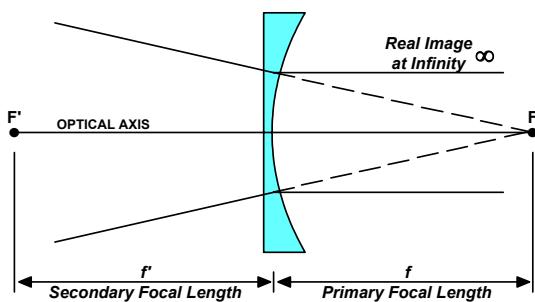


FIGURE 4:17 Cross-sectional view shows rays converging to form a *virtual* object, located at the primary focal point F of a *minus* lens, emerging from the lens parallel. A *real* image at infinity (∞) is created.

Some common diopter-to-focal length equivalents are provided in Table 3. Since the secondary focal length of a lens is equal to the reciprocal of its focal power, we say that they are *inversely* proportional to each other. This means that as the focal *power* increases in magnitude, the focal *length* becomes shorter.

Note: 1 diopter of focal power will focus light at a distance of 1 meter.

TABLE 3 Diopter and focal length equivalents

Focal Power (D)	Focal Length (m)	Focal Power (D)	Focal Length (m)
0.25	4.00	4.50	0.22
0.50	2.00	5.00	0.20
0.75	1.33	5.50	0.18
1.00	1.00	6.00	0.17
1.25	0.80	6.50	0.15
1.50	0.67	7.00	0.14
1.75	0.57	7.50	0.13
2.00	0.50	8.00	0.13
2.25	0.44	8.50	0.12
2.50	0.40	9.00	0.11
2.75	0.36	9.50	0.11
3.00	0.33	10.00	0.10
3.25	0.31	10.50	0.10
3.50	0.29	11.00	0.09
3.75	0.27	11.50	0.09
4.00	0.25	12.00	0.08

Now that we have a working knowledge of how a lens affects the vergence of incident light, we can look at what happens to the image distance as an object approaches the lens. Consider the conjugate points in Figure 4:18 for a +5.00 D lens ($f' = 0.20 \text{ m}$). With the object at optical infinity, the image is formed at the secondary focal length. As the object moves closer, the image moves further away. Once the object reaches the primary focal point, the image is formed at optical infinity. As the object moves even closer, the image becomes virtual (it now lies on the left side of the lens—in *object* space).

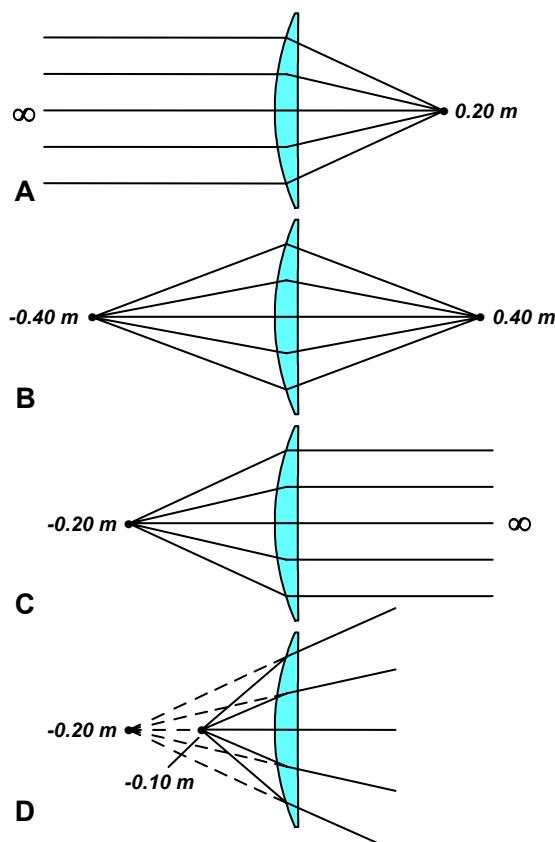


FIGURE 4:18 Conjugate foci for a +5.00 D lens. A) As a point object approaches this plus lens from infinity (∞) the vergence *entering* the lens L is 0 and the vergence *exiting* the lens L' is equal to its focal power F , or $L = 0$ and $L' = +5.00 \text{ D}$. Light is coming to a focus at the *secondary* focal point of the lens. B) Now $L = -2.50 \text{ D}$ and $L' = +2.50 \text{ D}$. C) Now $L = -5.00 \text{ D}$ and $L' = 0$. Light is being rendered parallel from the *primary* focal point of the lens. D) Finally $L = -10.00 \text{ D}$ and $L' = -5.00 \text{ D}$. The image is now *virtual*.

There are certain physical and optical characteristics that are shared by all lenses with *plus* power; similarly, there are equal, yet opposite, characteristics shared by all *minus*-powered lenses. For instance, moving a *plus* lens will make objects viewed through the lens appear to be displaced in the opposite direction of the movement, because of *prismatic* effects that will be discussed later. This phenomenon is known as **against-the-motion** movement. A *minus* lens will exhibit the same type of effect, but in the opposite direction. Objects appear to follow the movement of a *minus* lens; this is known as **with-the-motion** movement. In addition, lenses with *cylinder power* (which will be discussed in Section 5.1) will produce another effect when an object is viewed through them while the lens is rotated. Objects appear to rotate and/or skew as a lens with cylinder is rotated over them; this is referred to as **scissors-motion** movement. These phenomena are illustrated in Figure 4:19 and Figure 4:20.

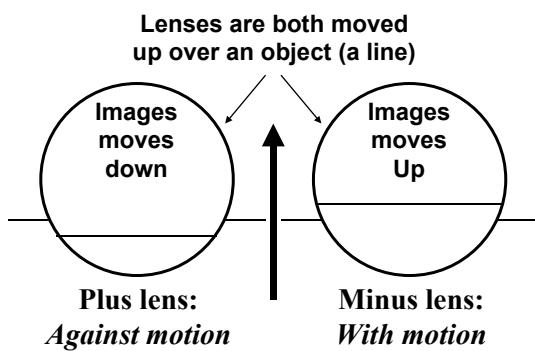


FIGURE 4:19 Plus lenses produce against-the-motion and minus lenses produce with-the-motion image displacement.

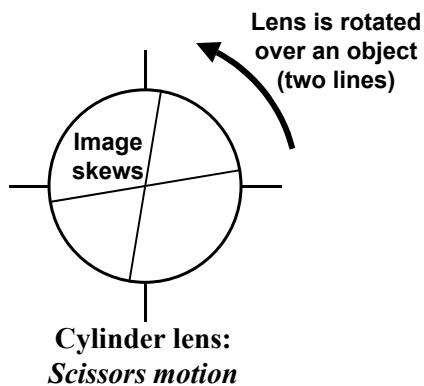


FIGURE 4:20 Lenses with cylinder produce scissors motion, causing images to skew when the lens is rotated.

TABLE 4 Characteristics of plus and minus lenses

Plus-powered Lenses
Add convergence to incident light rays
Are thicker at the center than at the edge
Magnify images, making them appear larger
Show <i>against-the-motion</i> image movement
Minus-powered Lenses
Add divergence to incident light rays
Are thinner at the center than at the edge
Minify images, making them appear smaller
Show <i>with-the-motion</i> image movement

Furthermore, because of the relationship between the front and back surfaces, *plus* lenses are generally thinner at the edge than at the center. Conversely, *minus* lenses are generally thinner at the center than at the edge. A summary of the more notable characteristics of plus- and minus-powered lenses is provided in Table 4.

4.4 BACK VERTEX (THICK LENS) POWER

Unfortunately, the lensmaker's formula (Eq. 19) for *thin* lenses quickly loses accuracy for lens forms of significant thickness or curvature. For *thick* lenses, the vergence of light as it passes through the lens also needs to be taken into consideration. Since we previously assumed that the two surfaces of a thin lens were in contact at the center, we did not take into account these effects. Because of these effects, the power of a **thick lens** is no longer simply equal to the combination of the front and back surface powers.

Another issue to consider is the fact that we assumed a theoretical reference plane centered between the two surfaces of a *thin* lens (at their imaginary contact point). For *thick* lenses, this is no longer practical since the front and back surfaces are separated by an appreciable amount. Consequently, the focal lengths of a thick lens depend upon the reference plane that the focal points are measured from. Further, since the focal power of a lens is equal to the reciprocal of the focal length (Eq. 23), the reference plane will also affect the stated focal power.

For spectacle lenses, the focal lengths of a lens are most easily measured from either the front or back surfaces at the **vertices** (V and V') are the positions on the lens where the optical axis intersects the front and back surfaces. When the focal power of a lens is measured relative to a plane containing one of the vertices—that is, from either the front or back surface—we call the measured value the **vertex power** of the lens.

Further, a thick lens generally produces powers that actually differ between measurements from the front and the back surfaces (or vertices). In Figure 4:21 and Figure 4:22, a comparison is made between the front and back vertex powers (F_N and F_V) of a *thick lens* and an infinitely *thin lens*. Both lenses have the same approximate focal power F of +4.00 D using the lensmaker's formula (Eq. 19):

$$F = F_1 + F_2$$

In ophthalmic optics the *back* vertex power is most commonly used. The **back vertex power** F_V is the vertex power of the lens, produced by an infinitely distant object, as measured from the back vertex V' of the surface.

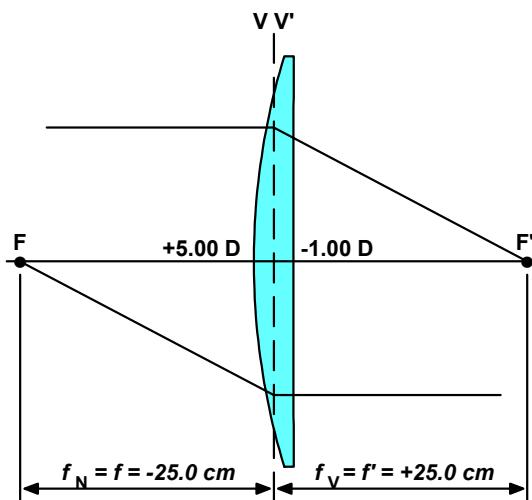


FIGURE 4:21 Thin lens; $F = +4.00 \text{ D}$, $F_V = +4.00 \text{ D}$. Assume that this lens has a zero center thickness (the dimensions in the picture have been exaggerated for clarity). For the *thin lens*, the thickness of the lens can be disregarded, and $F_V = F = F_N$. Recall that the focal lengths are measured from a theoretical reference plane centered within the lens. Moreover, the magnitudes of the focal lengths are equal since $-f_N = f_V$.

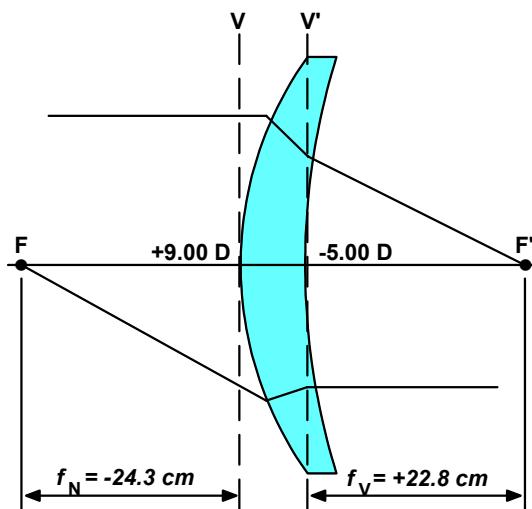


FIGURE 4:22 Thick lens; $F = +4.00 \text{ D}$, $F_V = +4.39 \text{ D}$. Because of the steep front curve and thick center, this *thick lens* will add additional convergence to the wave front as the light passes through to the back surface. The thin lens approximation is no longer accurate, or $F_V \neq F$, $F_N \neq F$, and $F_V \neq F_N$. The *back focal length* f_V is measured from the vertex V' of the back surface to the secondary focal point F' , and the *front focal length* f_N is measured from the vertex V of the front surface to the primary focal point F . Moreover, the magnitudes of the focal lengths are no longer equal since $-f_N \neq f_V$.

The back vertex power F_V of a lens can be calculated if the front and back surface powers (F_1 and F_2), the refractive index n , and the center thickness t in meters are all known. In this situation, the *equivalent thickness* (t / n) of the lens is also considered which is the

vergence of the light passing through the thickness of the lens.

To determine the back vertex power, we need to consider the refraction at each surface of the lens *and* the *equivalent thickness* (or t / n). The thick lens in Figure 4:23 utilizes a convex front curve F_1 and a concave back curve F_2 .

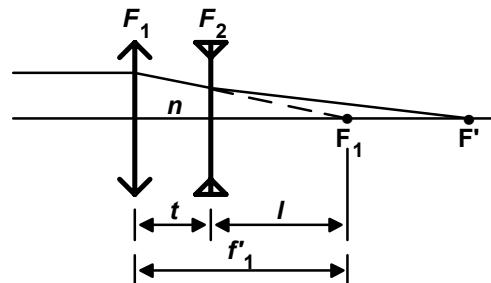


FIGURE 4:23 To determine the *back vertex power* of a thick lens, the vergence of the light, n / l , as it passes through the thickness of the lens must be determined. The reduced thickness of the lens is given by t / n . The vergence of light at the back surface is given by $n / (f_1' - t)$.

We will now determine, step-by-step, how incident light is affected as it passes through a thick lens. Let us first consider the interaction of light from a distant object with the first surface:

$$L' = F_1 + L$$

Since L is equal to 0 (the object is at infinity), the image vergence L' produced by the first surface is simply equal to its surface power F_1 :

$$L' = F_1$$

Now, the image L' of the first surface becomes the new object L of the second surface. The light is now traveling through a medium—the lens material—with a refractive index of n . To determine what the actual object vergence L is, at the plane of the back surface, consider the fact that L is equal to

$$L = \frac{n}{l}$$

From Figure 4:23, we can see that the radius of the wave front striking the back surface, which is the new object distance l , is equal to $f_1' - t$. Therefore, the object vergence L of light incident upon the back surface is given by

$$L = \frac{n}{f_1' - t}$$

where n is the refractive index of the material and t is the center thickness of the lens in meters. Moreover, we

can substitute the term n / F_1 in place of the secondary focal length f'_1 (using Eq. 17) to give us

$$L = \frac{n}{\frac{n}{F_1} - t}$$

Rearranging the terms slightly results in

$$L = \frac{F_1}{1 - \frac{t}{n} F_1}$$

We now know that the *object* vergence at the back surface is L . The next step is to add the change in vergence produced by the back surface power F_2 to determine the final *image* vergence F_V :

$$F_V = L + F_2$$

And finally, substituting for L brings us to the *back vertex power* of the lens, as measured from the plane of the *back* surface (or vertex). Our final formula becomes:

$$\text{EQ. 24} \quad F_V = \frac{F_1}{1 - \frac{t}{n} F_1} + F_2$$

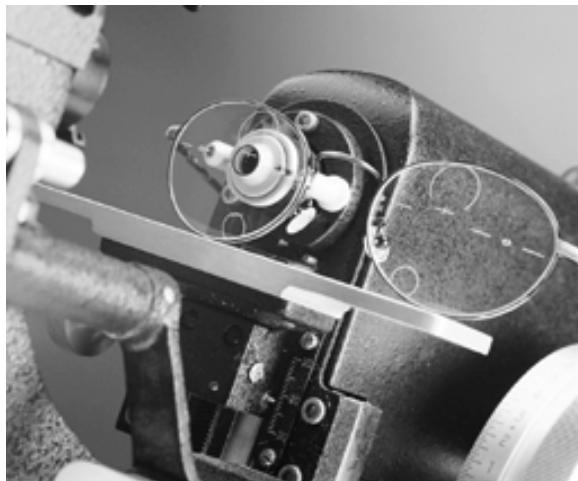


FIGURE 4:24 Measuring the back vertex power of a spectacle lens using a focimeter.

Figure 4:24 demonstrates how the back vertex power of a typical spectacle lens is measured using a **focimeter**, which is a telescope-like device that measures both focal power and prism.

When the quantity $(t / n) \cdot F_1$ approaches 0, the denominator approaches 1 and the first term of the formula reduces to just F_1 . This is the case for thin lenses with flatter front curves (typically *minus-powered* lenses). The result is the lensmaker's formula for thin lenses provided earlier (Eq. 19):

$$F = F_1 + F_2$$

The equations described above are exact formulas. There is an approximate version that is easier to work, however, and is generally accurate enough for our purposes. We can do a binomial expansion on our back vertex power formula, and then drop the 'higher order' terms to give us:

$$\text{EQ. 25} \quad F_V = F_1 + F_2 + \frac{t}{n} F_1^2$$

which uses the same variables and quantities that the exact formula employs (Eq. 24). This formula further illustrates the fact that the power of a *thick* lens is the result of both the *thin* lens power (given by $F_1 + F_2$) and the gain in power caused by the thickness and form of the lens.

The latter is given by the additional term,

$$\frac{t}{n} F_1^2$$

Example

A certain lens has a +5.00 D front curve, a -1.00 D back curve, a center thickness of 4.0 mm (0.004 m), and a refractive index of 1.500. What is the approximate back vertex power?

$$F_V = 5.00 + (-1.00) + \frac{0.004}{1.500} (5.00)^2$$

$$F_V = 5.00 + (-1.00) + 0.07$$

$$F_V = 4.07$$

∴ Back vertex power is +4.07 D.

This is a gain in power of only +0.07 D over the *thin lens* approximation—a much smaller difference than the previous example. Because of the flatter base curve and thinner center thickness, the back vertex power of this lens is nearly equal to the approximate power given by the thin lens formula. For minus lenses, the thin lens formula will often suffice.

The distance from the *back vertex* of the lens to the *secondary* focal point F' is known as the **back focal length** f_V .

The back focal length f_V is equal to the reciprocal of the back vertex power F_V :

$$f_V = \frac{1}{F_V}$$

where f_V is the distance from the back vertex V' of the lens to the secondary focal point F' in meters. Recall

that the secondary focal point is the image point conjugate with an object at optical infinity (∞).

Example

A certain lens has a +9.00 D front curve, a -5.00 D back curve, a center thickness of 7.0 mm (0.007 m), and a refractive index of 1.500. What is the exact *back* vertex power?

$$F_V = \frac{9.00}{1 - \frac{0.007}{1.500}(9.00)} + (-5.00)$$

$$F_V = 9.39 + (-5.00)$$

$$F_V = 4.39$$

\therefore Back vertex power is +4.39 D.

Note that the back vertex power is +0.39 D stronger than our *thin lens* approximation of focal power. This 0.39 D represents the additional gain in vertex power produced by the *form* of the lens (or the front curvature, thickness, and index).

4.5 FRONT VERTEX AND ADD POWER

Although ophthalmic lenses are specified in terms of their *back* vertex power, lenses will also produce a **front vertex power** F_N , or **neutralizing power**, when measured from the *front* vertex. This is the vergence of light from the *primary focal point* F to the *front* vertex V of the lens. The equation for the front vertex power F_N of a lens is given by

$$\text{EQ. 26} \quad F_N = \frac{F_2}{1 - \frac{t}{n} F_2} + F_1$$

This can be derived using a method similar to that for the back vertex power formula (Eq. 24).

The equation for the *front* vertex power F_N is very similar to the equation for the *back* vertex power F_V . Indeed, the only difference is that the front curve has been substituted for the back curve—and vice versa. In fact, solving for the front vertex power equation is equivalent to just flipping the lens around (so as to treat the old front curve as the new back curve) and recalculating for the back vertex power.

Example

Given the lens form from our previous example (with a +9.00 D front, -5.00 D back, and 7-mm center thickness), what is the *front* vertex power?

$$F_N = \frac{-5.00}{1 - \frac{0.007}{1.500}(-5.00)} + 9.00$$

$$F_N = -4.88 + 9.00$$

$$F_N = 4.11$$

\therefore Front vertex power is +4.11 D.

In this example, the front vertex power differs from the back vertex power by more than 0.25 D. This further illustrates the effects of both the position of the reference plane and the form of the lens on measuring focal power.

The distance from the *front* vertex of the lens to the *primary* focal point F is known as the **front focal length** f_N . The front focal length f_V is related to the front vertex focal power F_N by

$$f_N = -\frac{1}{F_N}$$

Recall that with *thin* lenses, the following relationship holds true for the primary and secondary focal lengths of the lens:

$$-f = f'$$

This is typically not the case for *thick* lenses. In most instances,

$$-f_N \neq f'_V$$

Multifocal lenses, which will be discussed in Section 11, reduce the divergence of light from near objects by effectively using a small *plus* lens to render the diverging light more parallel. The additional plus power provided by the lens is referred to as its **add power**, and is generally produced within a small region of the lens referred to as the *near zone* or **segment**. When the segment is on the *front* surface, which is generally the case, the add power is related to the *front vertex power* of the segment. Specifically, the add power is the additional power provided by the segment, itself. This additional power is produced by increasing the surface power of the lens through the segment by either increasing the curvature, as is the case for *plastic* lenses, or by using a higher-index material within the segment, as is the case for *glass* lenses. Some common reference points for a typical *flat-top* bifocal lens are shown in Figure 4.25.

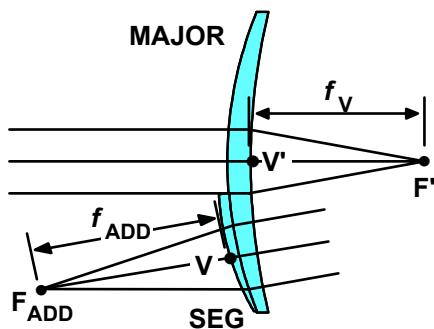


FIGURE 4:25 The *back vertex power* F_V of the distance portion of the lens is measured from the back vertex V' . The *front vertex power* F_{ADD} of the segment ('seg') is measured from the front vertex V .

Multifocal lenses are verified for distance and near powers as follows:

1. To verify the *distance power* of the lens, the *back vertex power* through the distance (or 'major') portion is measured.
2. To verify the *add power* of the lens, the difference between the *front vertex power* through the major portion and the *front vertex power* through the 'segment' is measured.

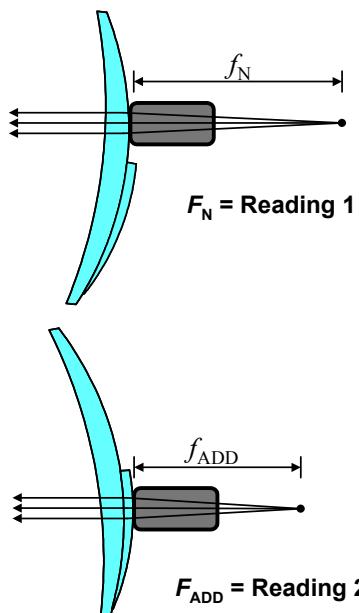


FIGURE 4:26 The *add power* of a multifocal lens is the difference between the front vertex powers of the distance portion (F_N) of the lens and the segment (F_{ADD}): $Add = F_{ADD} - F_N$. This is the difference between the two readings of the focimeter.

To measure the add power accurately, the front surface of the lens—containing the segment—should be placed against the lens stop of your **focimeter** (which is an instrument for measuring focal power). The first reading is taken in the distance portion of the lens and the second reading is then taken within the segment, as

demonstrated in Figure 4:26. The difference between the front vertex power of the segment (F_{ADD}) and the front vertex power of the distance portion (F_N) is the add power:

$$\text{EQ. 27} \quad Add = F_{ADD} - F_N$$

Note that any *cylinder power* is neglected. For thin lenses (like low- and minus-powered lenses), there is little difference between front and back vertex power measurements. Consequently, flipping a minus lens around to measure it from behind won't affect the add power reading very much. *Plus* lenses, however, should always be measured with the front surface against the lens stop, since they generally have appreciable center thicknesses. In addition, some automatic focimeters might compensate for this effect, so refer to its manual if you are in doubt.

We now know that the form and thickness of a lens will affect its focal power. Using the back vertex power to measure focal power allows us to employ a virtually unlimited number of lens forms and thicknesses for a given power. Even if we have to use certain front curve or center thickness values—for various optical or mechanical reasons that will be discussed later—we can still produce the same back vertex power by simply modifying the back curve accordingly. When surfacing a lens, this is known as a **vertex power allowance** (Wray & Jalie 297).

For instance, a lens with a refractive index of 1.500, a front curve of +5.00 D, a back curve of -1.07 D, and a center thickness of 4 mm produces a back vertex power of +4.00 D. The exact same back vertex power (+4.00 D) can be produced using a front curve of +9.00 D and a center thickness of 7 mm if we employ a back curve of -5.39 D.

The amount of *vertex power allowance* required can be determined by calculating the additional gain in power produced by the thickness and form of the lens—at the plane of the back surface. Then, subtract this from the prescription before determining the back curves using the simple lensmaker's formula for thin lenses (Eq. 19). The vertex power allowance A is approximately equal to:

$$\text{EQ. 28} \quad A = \frac{t}{n} F_1^2$$

where A is the vertex power allowance in dipters, F_1 is the front curve, n is the refractive index, and t is the center thickness in meters.

5. Sphero-cylindrical Lenses

Up to this point, only **spherical lenses** have been discussed. Spherical lenses have the same power in all meridians, producing a point focus as described in the earlier sections. However, prescriptions that incorporate a correction for **astigmatism** (which will be described in Section 6.2) require a lens with *different powers* that vary from meridian to meridian. These lenses have a meridian of greatest power and a meridian of least power, called **principal meridians**, which are 90 degrees apart. These lenses are specified by the powers in the principal meridians.

5.1 CYLINDER LENSES

It is possible to produce a lens with two different powers by employing a lens surface with the shape of a *cylinder*. A **cylinder lens** has a cylindrically-shaped surface with no surface power (*plano*) through the *plane meridian*, and maximum surface power 90° away through the meridian of curvature. The *plano meridian* is called the **axis meridian** of the lens, and the curvature meridian is called the **power meridian**.

Instead of creating a *focal point*, like a spherical lens, a cylinder lens will create a *focal line* from an object point at the secondary focal length of the power meridian. This is a result of the fact that refraction only occurs through the power meridian, as illustrated in Figure 5:1. Further, this line focus is perpendicular to the power meridian, and parallel to the axis meridian.

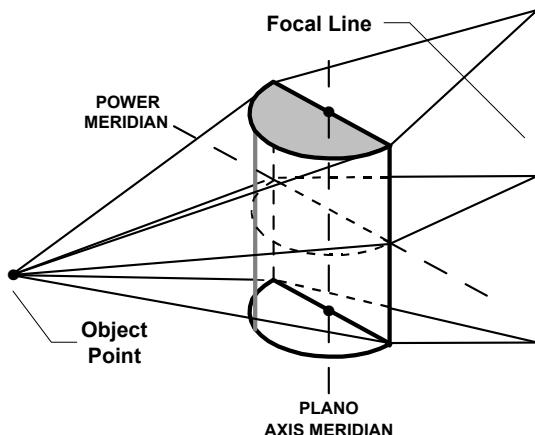


FIGURE 5:1 A cylinder lens focuses rays of light from a *point* object to a *line* image.

Instead of producing with- or against-motion movement like spherical lenses, rotating a cylinder lens will produce a **scissors-motion** movement, which makes objects appear to rotate slightly or to skew. As illustrated in Figure 5:2, there are two types of cylindrical surfaces. A *convex* cylindrical surface is referred to as a **plus cylinder**, and a *concave* surface is referred to as a **minus cylinder**.

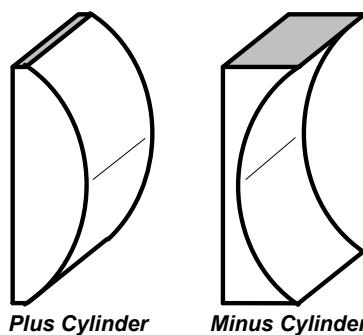


FIGURE 5:2 Plus and minus cylindrical sections.

The orientation of these meridians is critical to produce the desired effect, and a **cylinder axis** corresponding to the *plano axis meridian* needs to be specified. Remember that there is no curvature/power through the axis meridian, and full curvature/power through the power meridian. This axis is noted as being between 1 and 180 degrees, measured counter-clockwise like a protractor. Figure 5:3 is an example showing the orientation of the axis meridian of a cylinder rotated to axis 45°.

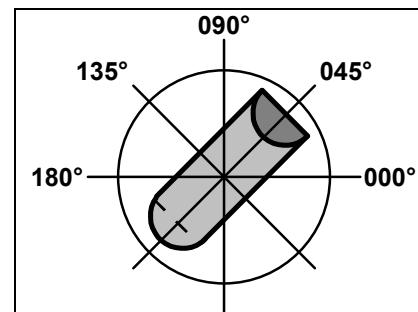


FIGURE 5:3 Cylinder axis notation; axis 45 (as seen when viewing the patient or front of the lens).

The protractor-style axis notation for cylinders is demonstrated for both the right and left lenses, as seen when facing the wearer, in Figure 5:4.

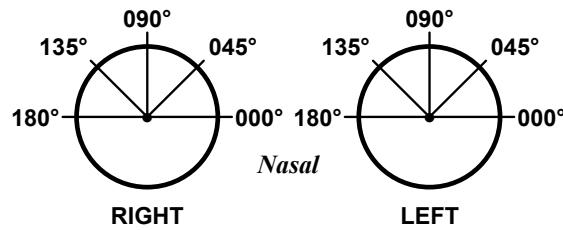


FIGURE 5:4 Cylinder axis notation for both eyes, as seen when facing the spectacle wearer.

The degree symbol (°) is not used in the written prescription to avoid confusion with the number 0. The axis is specified in 1-degree increments.

5.2 TORIC LENSES

Originally, lenses used to produce cylinder power were actually cylindrical in form, having one plane meridian and one meridian of curvature. Modern lens forms with cylinder power, however, will employ a **toric** (or **toroidal**) surface, having *maximum* and *minimum* amounts of curvature. A *toric* surface is another surface of revolution that can be created by rotating a circle or arc about an axis of revolution that does *not* pass through its center of curvature.

The word ‘toric’ comes from the Greek word ‘torus,’ which is the donut-shaped molding at the base of columns. The toric surface often used for spectacle lenses, having the shape of a common *tire*, is illustrated in Figure 5:5. This is the type of toric lens surface typically generated by conventional surfacing equipment. Other toric surface types include the *barrel* and the *capstan*.

The meridian of maximum surface curvature F_{MAX} , is again crossed at right (90°) angles to the meridian of minimum curvature F_{MIN} . The difference in surface powers between these two meridians is the amount of **cylinder power** of that surface. Since toric surfaces have two surface powers at right angles to each other, they produce two different focal powers. Lenses utilizing toric surfaces are usually referred to as **sphero-cylindrical lenses**, since they can produce both a *sphere* power and a *cylinder* power.

Early lenses with cylinder power were produced using the convex outside of a toric surface for the *front* curve. These lenses were **plus-cylinder** in form. Modern lenses are produced using the concave inside of a toric surface for the *back* curve. These lenses are **minus-**

cylinder in form (see Figure 5:5). Minus-cylinder lens forms are preferred over plus-cylinder forms because they improve cosmetics, reduce magnification from one meridian to the other, and are retained more securely in frames. Modern, front side multifocal designs also necessitate the use of minus-cylinder lens forms.

Eyeglass prescriptions can also be written in either *plus-* or *minus-cylinder form*, depending the type of equipment employed during the eye examination. The cylinder form of the prescription does not, however, dictate the actual cylinder form used for producing the lenses. Before the lens can be fabricated from a given prescription, the cylinder form of the prescription must be converted into the desired cylinder form of the lens. Details about this conversion will be discussed shortly.

Lenses containing cylinder power have two **principal meridians** of dioptric power perpendicularly crossed at right (90°) angles. The difference in focal power between the two principal meridians of a lens—as called for by the prescription—is referred to as the **cylinder power** (or **nominal cylinder**) of the lens—or of the prescription. This is the same term given to the difference in surface powers.

The *axis* principal meridian will contain a focal power equal to the **sphere power** F_{SPH} of the prescription, and the *power* principal meridian will contain a focal power F_{CYL} equal to the combined sphere F_{SPH} and cylinder power C of the prescription, so that $F_{CYL} = F_{SPH} + C$. Since the *power* meridian is equal to the combined sphere and cylinder power, a sphero-cylindrical lens can be thought of as lens with two components: a simple spherical component (through all meridians) combined with a simple cylindrical component that is oriented at the axis of the prescription.

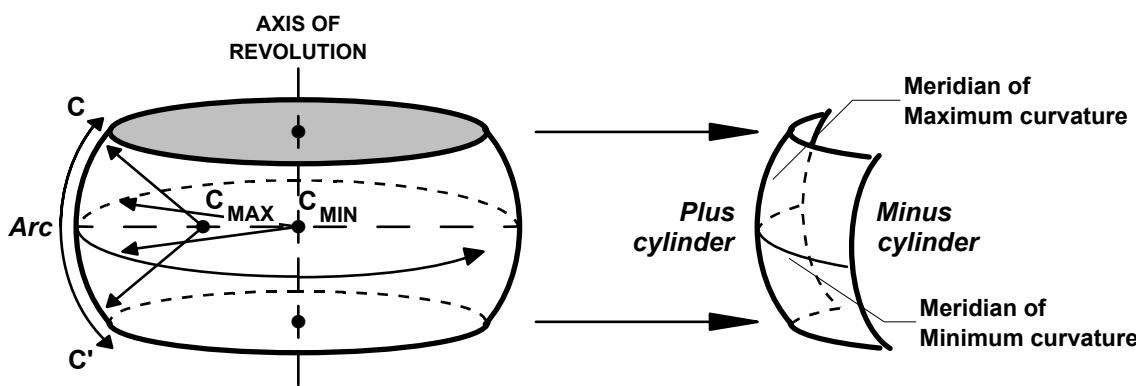


FIGURE 5:5 This *tire* toric surface was created by rotating arc CC', with its center of curvature at C_{MIN} , around an axis of revolution passing through C_{MAX} . Arc CC' represents the *transverse* meridian of *maximum* curvature. A second arc is created, perpendicular to CC', with its center of curvature at C_{MAX} . This is the *equatorial* meridian of *minimum* curvature. If the *convex* side of this toric surface is used as a lens surface, it forms a **plus-cylinder** lens. If the *concave* side is used, it forms a **minus-cylinder** lens.

Once the prescription has been converted—or ‘transposed’—into the correct cylinder form, the *axis* principal meridian (containing the sphere power) will correspond to the meridian of *minimum* curvature. The *power* principal meridian (containing the combined sphere and cylinder power) will correspond to the meridian of *maximum* curvature.

Since the curvature of a toric surface changes from a minimum value in the axis meridian to a maximum value in the power meridian, the edge thickness of a spherocylindrical lens also varies. This change in curvature and thickness is illustrated in Figure 5:6, for a lens made in minus-cylinder form.

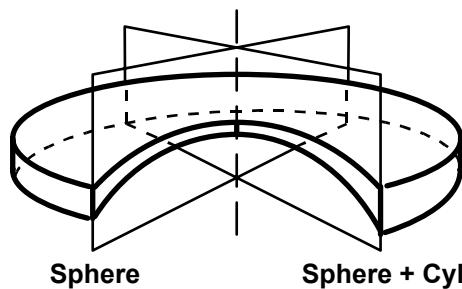


FIGURE 5:6 The edge thickness of a spherocylindrical lens varies from a minimum, through the plane containing the axis (or sphere) meridian, to a maximum 90° away, through the plane containing the power (or combined sphere and cylinder) meridian.

Remember that the orientation of these meridians is critical to produce the desired effect, and an axis corresponding to the *sphere meridian* needs to be specified. The best technique to visualize the powers of the principal meridians of a prescription is to put the prescription on an **optical cross**, as demonstrated in Figure 5:7. Here is the preferred method of writing an ophthalmic lens prescription incorporating cylinder; note that the following two prescriptions are identical in power:

- +2.00 DS +0.50 DC \times 045 (*Plus* cylinder)
- +2.50 DS -0.50 DC \times 135 (*Minus* cylinder)

The abbreviation **D.S.** stands for *diopters of sphere*, and **D.C.** stands for *diopters of cylinder*.

OPTICAL CROSS ANALYSIS

$$\begin{aligned} F_{SPH} &= +2.50 \text{ D} && \text{through } 135^\circ \\ C &= -0.50 \text{ D} && \text{axis } 135^\circ \\ F_{CYL} &= +2.50 + (-0.50) = +2.00 \text{ D} && \text{through } 045^\circ \end{aligned}$$

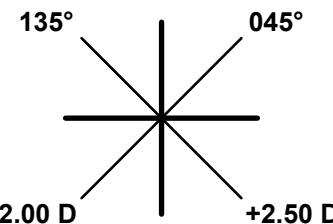


FIGURE 5:7 Optical cross; +2.50 DS -0.50 DC \times 135.

Toric and cylinder lenses produce an **astigmatic** (or non-point) **focus**, as opposed to a single *point* focus. A lens with cylinder power creates two perpendicular focal lines, as illustrated in Figure 5:8 and Figure 5:9. Note that each focal line is perpendicular to the corresponding principal meridian that produces it.

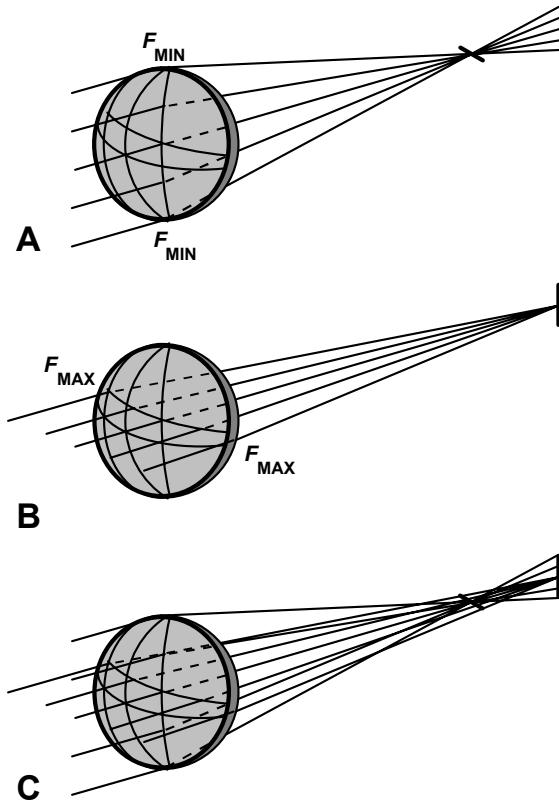


FIGURE 5:8 Astigmatic focal lines of a spherocylindrical (toric) lens made in minus-cylinder form; +3.00 DS -1.00 DC \times 090. A) The *minimum* curvature F_{MIN} of the toric surface lies in the *vertical* meridian (90°) and produces a horizontal focal line perpendicular to this. This meridian provides the *sphere* power F_{SPH} of the lens. B) The *maximum* curvature F_{MAX} lies in the *horizontal* meridian (180°) and produces a vertical focal line perpendicular to this. This meridian provides the combined *sphere and cylinder* power of the lens, so that $F_{CYL} = F_{SPH} + C$. C) The astigmatic focus contains both focal lines and their interval.

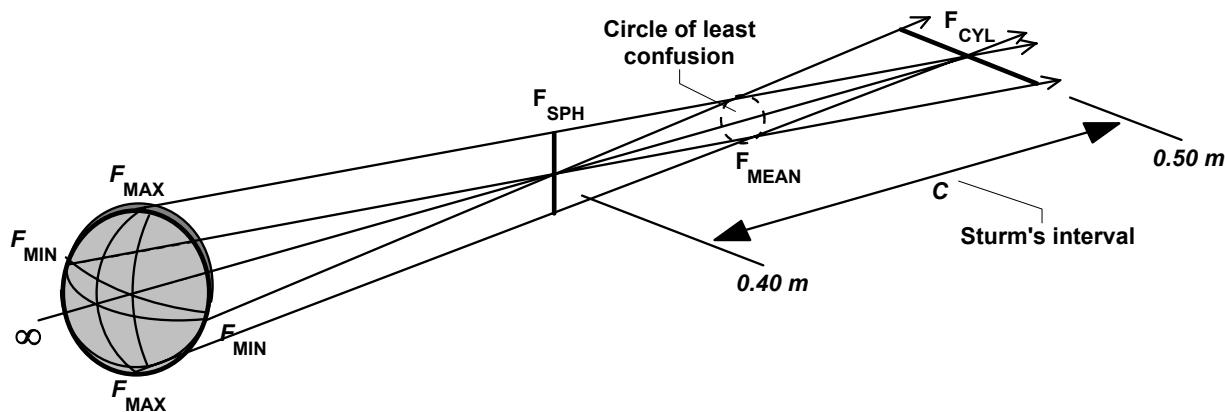


FIGURE 5:9 Astigmatic focal lines of a minus-cylinder toric lens for an object at infinity (∞); +2.50 DS -0.50 DC \times 180. The cylinder power C is the dioptric difference in focal power between the two principal meridians of the prescription, or -0.50 D. The average focal position F_{MEAN} of the lens is the location of the *circle of least confusion* (which will be discussed later).

Example

A rear, concave toric surface has a refractive index of 1.500, an equatorial radius of curvature of 0.1 m, and a transverse radius of curvature of 0.0625 m. What is the cylinder power of this surface?

$$F_{MIN} = \frac{1 - 1.500}{0.1}$$

$$F_{MIN} = -5.00$$

$$F_{MAX} = \frac{1 - 1.500}{0.0625}$$

$$F_{MAX} = -8.00$$

The cylinder C is the difference ($F_{MAX} - F_{MIN}$):

$$C = -8.00 - (-5.00)$$

$$C = -3.00$$

\therefore Cylinder power is -3.00 D.

5.3 FLAT AND TORIC TRANSPOSITION

Prescriptions for spherical lenses obviously need no orientation or cylinder component, since the power is uniform throughout every meridian of the lens. Prescriptions incorporating cylinder power can be written in either of three different forms (the third form exists, but is rarely employed):

1. **Plus-cylinder form:** This prescription style uses the *least plus* (or most minus) principal meridian for the *sphere power*. The cylinder power is a *positive* (+) value.
2. **Minus-cylinder form:** This prescription style uses the *most plus* (or least minus) principal meridian

for the *sphere power*. The cylinder power is a *negative* (-) value.

3. **Crossed cylinder form:** This prescription style, which is rarely used today, expresses the power of both principal meridians as two separate cylinders combined.

Since most modern *lenses* are made in minus cylinder form, *prescriptions* often need to be converted into it. The process of converting between plus and minus cylinder form is referred to as **flat transposition**:

1. Add the sphere power F_{SPH} to the cylinder power C ; the sum is the new sphere power.
2. Change the sign of the cylinder power C ; this is the new cylinder power.
3. Change the axis by 90°; this is the new axis of the sphere power. To change the axis 90°, add 90° to axis values $\leq 90^\circ$, and subtract 90° from axis values $> 90^\circ$.

Example

A prescription is written as +2.50 DS -0.50 DC \times 135 in minus-cylinder form. What is the same prescription in plus-cylinder form?

$$F_{NEWSPH} = +2.50 + (-0.50)$$

$$F_{NEWSPH} = +2.00$$

$$C = -(-0.50) = +0.50$$

$$A = 135 - 90 = 45^\circ$$

\therefore Prescription in plus-cylinder form is:

$$+2.00 \text{ DS} +0.50 \text{ DC} \times 045$$

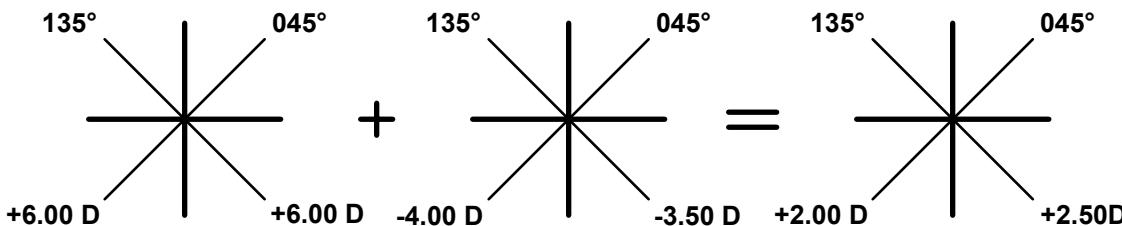


FIGURE 5:10 Optical cross confirmation of tool curves; +2.50 DS -0.50 DC \times 135.

Note that an optical cross made from this new prescription, +2.00 DS +0.50 DC \times 045, will still be identical to the previous one.

Once the *base curve* of a lens has been chosen, the backside surface powers, or **tool curves**, need to be determined to produce the desired prescription. For *thin* lenses, we can employ the now-familiar lensmaker's formula (Eq. 19) to perform a process referred to as **toric transposition**:

1. Transpose the prescription into *minus-cylinder* form, if it contains a cylinder component.
2. Subtract the selected base curve F_1 from the sphere power F_{SPH} to obtain the tool **back base curve** F_{MIN} , so that $F_{MIN} = F_{SPH} - F_1$. If the lens requires cylinder power, so that the back surface will be toric in form, this will be the meridian of minimum curvature.
3. If the lens requires cylinder power, add the nominal cylinder power C to the back base curve F_{MIN} to obtain the tool **cross curve** F_{MAX} , so that $F_{MAX} = F_{MIN} + C$. This will be the meridian of maximum curvature.

In practice, however, the thick lens formula (Eq. 24) should be used for determining the correct tool curves for toric transposition. The tool curves are often written in the following format:

Base Curve

Back Base Curve/Cross Curve

where the *top* half denotes the *front* curve and the *bottom* half denotes the *back* curves.

Example

A +2.50 DS -0.50 DC \times 135 prescription is made using a +6.00 D base curve in minus-cylinder form. What are the tool curves?

$$F_{MIN} = +2.50 - 6.00$$

$$F_{MIN} = -3.50$$

$$F_{MAX} = -3.50 + (-0.50)$$

$$F_{MAX} = -4.00$$

\therefore Final tool curves are:

$$\begin{array}{r} +6.00 \\ \hline -3.50 / -4.00 \end{array}$$

These tool curves can be confirmed using optical crosses, as demonstrated in Figure 5:10. Once the tool curve values have been determined, each tool curve may need to be converted using the appropriate tooling index, as described in 12.1. The final tool curves then need to be rounded to the nearest lap tool increment.

5.4 SPHERICAL EQUIVALENT

Most semi-finished base curves are spherical and not available in toric form. Therefore, discussions pertaining to the selection of base curves for a given prescription will use the *spherical equivalent* for any lens powers with a cylinder component. The **spherical equivalent** F_{MEAN} is simply the *mean*, or average, power of the two principal meridians (F_{SPH} and F_{CYL}). This also provides the location of the **circle of least confusion**, which is the *dioptric mid-point* of the astigmatic line foci; refer to Figure 5:9. The spherical equivalent can be found by either adding half of the cylinder power to the sphere power of the prescription, or by using this equation which uses the powers in the two principal meridians:

$$\text{EQ. 29} \quad F_{MEAN} = \frac{F_{SPH} + F_{CYL}}{2}$$

where all units are expressed in diopters.

The same result can be found by using the values in the prescription—adding half of the cylinder power C to the sphere power F_{SPH} , or

$$\text{EQ. 30} \quad F_{MEAN} = F_{SPH} + \frac{1}{2}C$$

Example

A prescription calls for a +2.50 DS -0.50 DC \times 135 spectacle correction. The powers of the two principal meridians, in minus-cylinder form, are +2.50 D for the F_{SPH} meridian, and $+2.50 + (-0.50) = +2.00$ D for the F_{CYL} meridian. What is the spherical equivalent?

$$F_{MEAN} = \frac{2.50 + 2.00}{2}$$

$$F_{MEAN} = +2.25$$

\therefore Spherical equivalent is +2.25 D.

5.5 OBLIQUE POWER OF A CYLINDER LENS

Recall that the focal power of a lens with a cylinder component will vary in power from one meridian to the other, because of the change in curvature of the toric surface. The principal meridians are 90° apart—or mutually perpendicular—and will differ in power by the value of the cylinder. Sometimes, it is necessary to determine the power of the lens in an *oblique* meridian, somewhere in between the two principal meridians. To determine the approximate focal power F_θ through an oblique meridian of a lens, we can use the following formula:^{*}

$$\text{EQ. 31 } F_\theta = F_{SPH} + C \cdot \sin^2 \theta$$

where θ is the angle between the oblique meridian and the axis of the prescription (which corresponds to the *sphere* meridian), F_{SPH} is the sphere power, and C is the cylinder value.

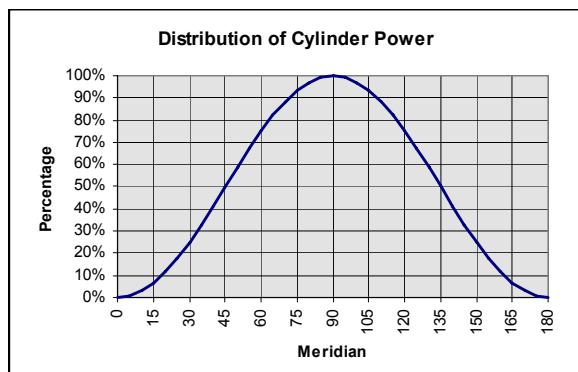


FIGURE 5:11 Distribution of cylinder power by meridian.

Figure 5:11 shows the percentage of cylinder power distribution, through various meridians at a given angle from the sphere axis of the prescription. For instance, at 45° away the distribution is 50%.

By multiplying the cylinder power C by the sine-squared value of the angle θ between the *axis* of the

* This formula actually determines the approximate *circular* curvature in the oblique meridian of a cylindrical or toric surface (this curvature is actually *elliptical*). Rays passing through oblique meridians of a lens with cylinder power are not refracted to a point focus, and are called **skew rays**. The “power” of an oblique meridian is merely an approximation of these rays (Fannin & Grosvenor 77).

prescription and the meridian of regard, the contribution of the cylinder power in that particular meridian can be determined. The contribution is then added to the sphere power F_{SPH} to calculate the total power F_θ in the oblique meridian of the lens.

Remember that a lens with a cylinder component will have no cylinder power through the *axis* principal meridian of the prescription, which is the same as the principal meridian of the sphere power F_{SPH} . Maximum cylinder power ($F_{CYL} = F_{SPH} + C$) is at the *power* principal meridian 90° away. Moreover, the power of the lens through a meridian at 45° away from the axis will also be equal to the spherical equivalent discussed in the preceding section, since this is effectively the same as adding $\frac{1}{2}$ the cylinder power to the sphere power.

Some common percentages for the contribution of cylinder power at various angles from the axis are provided in Table 5. These values are based upon the trigonometric functions of special triangles, and can be easily memorized.

TABLE 5 Common cylinder distribution percentages

$^\circ$	030	045	060	090	120	135	150	180
%	25	50	75	100	75	50	25	0

Like other principles that apply equally to both lenses and surfaces, the sine-squared expression can also be applied to lens surfaces. In fact, this formula is actually based upon the surface curvature of a lens.

Example

A certain lens has a power of +2.50 DS -0.50 DC \times 135. What is the approximate focal power through the vertical (90°) meridian of this lens?

$$\theta = 135 - 90 = 45^\circ$$

$$F_{45} = 2.50 + -0.50 \cdot \sin^2 45^\circ$$

$$F_{45} = +2.50 + (-0.25)$$

$$F_{45} = +2.25$$

\therefore Vertical power is +2.25 D.

6. The Eye and Refractive Errors

Up to this point, ophthalmic lenses have been considered as single optical elements. The purpose of an ophthalmic lens, however, is to correct—or, more accurately, to compensate for—a refractive error of the eye. When an ophthalmic lens is used in this capacity, it becomes an intrinsic component of the complete optical system of the eye. To understand the visual process, the fundamental optics of the eye need to be described.

6.1 OPTICS OF THE EYE

The basic anatomical and optical structures of the human eye are shown in Figure 6:2. The human eye is similar to a camera, as depicted in Figure 6:1; it has a dark interior chamber, a variable aperture (called the **pupil**) to control retinal illuminance, and the **crystalline lens**, which enables adjustable focusing. The main refracting element of the eye is the **cornea**. *Real, inverted* images are formed on a layer of photosensitive tissue, called the **retina**, by the optical elements of the eye (Keating 4).

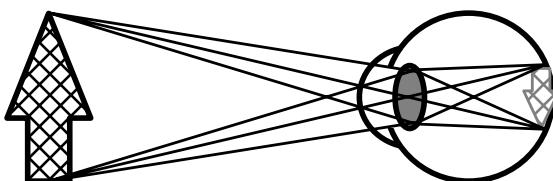


FIGURE 6:1 A *real, inverted* image of an object is formed upon the retina at the back of the eye.

The physical and optical values of a typical human eye have been summarized by various researchers over the years to serve as the basis for theoretical studies of the optics of the eye. Where possible, these values are typically simplified to some extent to allow for workable models. One popular eye model, or **schematic eye**, is the Gullstrand-Emsley schematic eye. Some of the more relevant characteristics of this schematic eye are summarized in Table 6 (Bennett & Rabbatts 250).

TABLE 6 Gullstrand-Emsley Schematic Eye

Parameter	Value
Refractive Indices	
Cornea	1.3760
Aqueous Humor	1.3333
Crystalline Lens	1.4160
Vitreous Humor	1.3333
Axial Separations	
Cornea Thickness	0.50 mm
Depth of Anterior Chamber	3.60 mm
Crystalline Lens Thickness	3.60 mm
Depth of Posterior Chamber	16.69 mm
Surface powers	
Cornea Front	+48.83 D
Cornea Back	-5.88 D
Crystalline Lens Front	+8.27 D
Crystalline Lens Back	+13.78 D
Overall Length	23.89 mm
Equivalent Power	+60.49 D

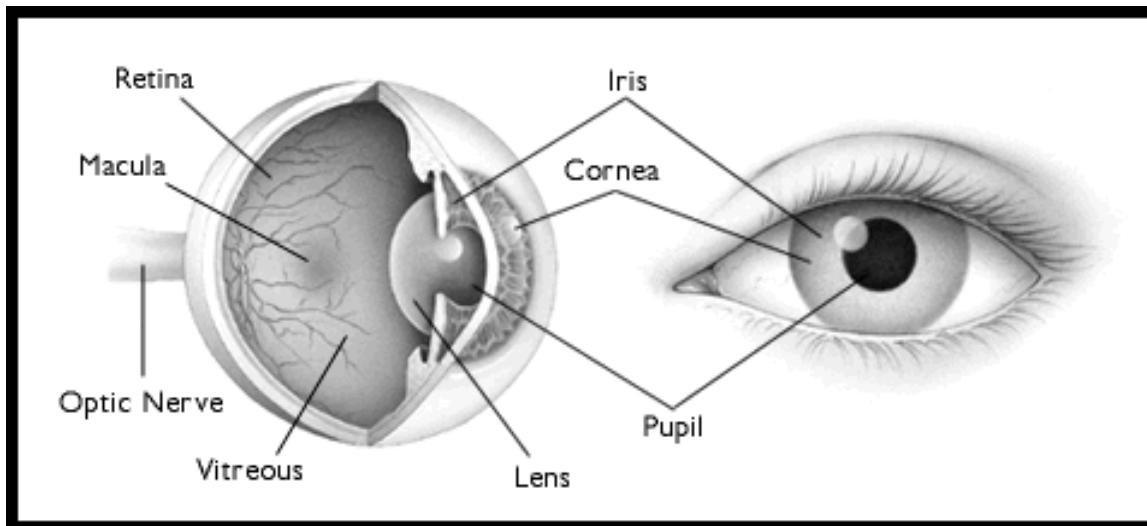


FIGURE 6:2 Basic anatomical and optical structures of the eye, including the cornea, pupil, crystalline lens, and retina. In addition, the *macula*—which is the most sensitive area of the retina—and the *optic nerve* are shown. Reprinted by permission from the American Academy of Ophthalmology.

Fortunately, the optics of the eye, which are quite complex, can be *reduced* to a rather simplified representation referred to as the **reduced eye**. Figure 6:3 is a diagram of this *reduced eye*; which is a single refracting surface, bounded by an index of refraction n' of $4/3$ (≈ 1.333), with a center of curvature located at point N. The center of curvature of the refracting surface is referred to as the **nodal point** (Bennett & Rabbets 19).

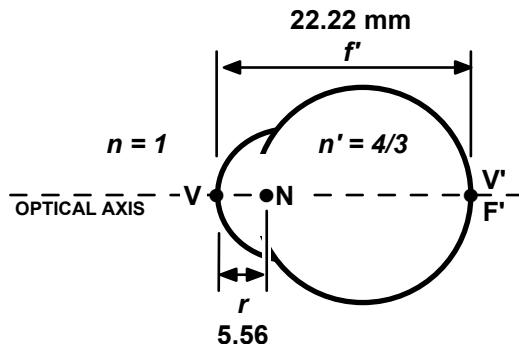


FIGURE 6:3 The reduced eye.

Point V serves as the vertex of the surface. The axial length of the reduced eye is 22.22 mm and the refractive power is 60.00 D, both are close approximations to the real eye. An index of $4/3$ (or ≈ 1.333)—the index of water—was chosen because it approximates the index of the fluids of the eye. A goal is to have the reduced eye properly focused; therefore, the secondary focal distance also needs to be 22.22 mm so that the image of a distant object can be focused on the retina. The focal length f' of the eye is measured from the vertex V to the secondary focal point F'. The radius of the refracting surface of the reduced eye is calculated as follows, using the other values, in order to accomplish proper focus.

$$r = \frac{1.3333 - 1.000}{60}$$

$$r = 0.005555$$

∴ Radius of the refracting surface of the eye is 0.005555 m or 5.56 mm.

The resulting radius is not representative of any real curvature of any surface in the eye (e.g. the cornea). Nor would we expect it to be, since it is a calculated value for a single refracting surface whereas the eye actually has both a cornea and a crystalline lens. The line passing through the vertices V and V' serves as both the **visual axis** (or the *line of sight*) and the **optical axis** of the reduced eye. The visual axis is an imaginary line of reference passing from the object of fixation, through the nodal point, and into the macula of the eye. In reality the eye is not a centered optical system; its

visual axis is not entirely coincident with its optical axis. This will be discussed further in Section 9.2.

6.2 REFRACTIVE ERRORS

Ideally, the secondary focal point F' of the eye should fall upon the retina located at the posterior pole V' of the globe. When this occurs, distant objects will come to a sharp focus on the retina of the eye creating a condition, free from refractive error, called **emmetropia**. This refractive state is illustrated in Figure 6:4. If the secondary focal point fails to coincide with the retina, a refractive error is produced, creating a condition called **ametropia**.

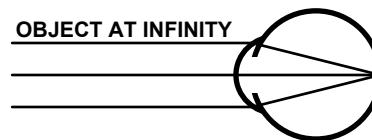


FIGURE 6:4 Emmetropic focus (on the retina).

The state of ametropia can be classified as either **axial** in nature if it occurs because the axial length differs from that of the reduced eye (i.e., the eye is too long or short), or as **refractive** in nature if it occurs because the secondary focal length differs from that of the reduced eye (i.e., the power is too strong or weak). It is also possible to have a combination of axial and refractive ametropia, if both the axial length and secondary focal length differ from those of the reduced eye. It is important to note that an eye can still be emmetropic, while deviating from the reduced eye, as long as the secondary focal point of the eye coincides with the retina.

Ametropia is categorized by the specific type of optical deficiency, or **refractive error**:

- **Hyperopia:** This refractive error occurs when rays of light, emanating from an object at infinity, come to a focus behind the retina. Either the optical elements of the eye are too weak, or the axial length of the eye is too short. Hyperopia is commonly referred to as *farsightedness*, and it is often possible for small amounts to be accommodated into focus by the eye (especially for young patients). Otherwise, *plus* lenses can be used to correct it. This refractive state is illustrated in Figure 6:5.

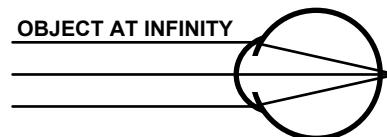


FIGURE 6:5 Hyperopic focus (behind the retina).

- **Myopia:** This refractive error occurs when rays of light from infinity come to a focus in front of the

retina. Either the optical elements of the eye are too strong, or the axial length of the eye is too long. Myopia is commonly referred to as *nearsightedness*, and can be corrected with *minus* lenses. This refractive state is illustrated in Figure 6:6.

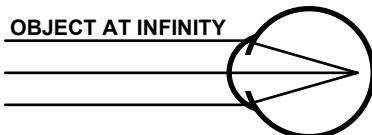


FIGURE 6:6 Myopic focus (in front of the retina).

- **Astigmatism:** This refractive error occurs when rays of light do not come to a focal point, but instead form two focal lines after refraction by the optical elements of the eye. This effect is caused by a non-uniform refraction through the various meridians of the eye, and is similar to that produced by the cylinder and toric lenses described earlier. Astigmatism can be corrected by lenses with *cylinder* power. This refractive state is illustrated in Figure 6:7.

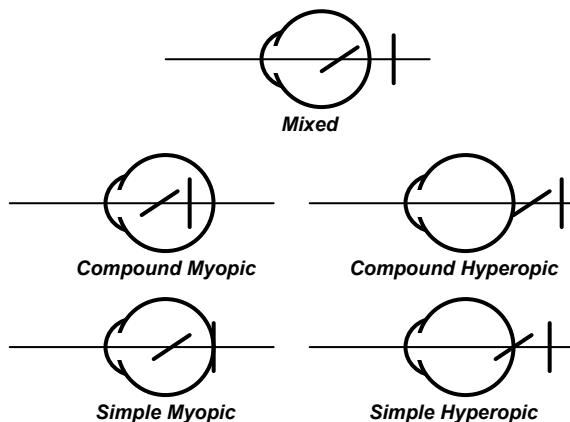


FIGURE 6:7 Types of astigmatism: mixed astigmatism (one focal line in front of and one behind retina), compound myopic astigmatism (both focal lines in front of retina), simple myopic astigmatism (one focal line in front of and one on retina), compound hyperopic astigmatism (both focal lines behind retina), and simple hyperopic astigmatism (one focal line behind and one on retina). The actual orientation of these focal lines will vary—but they will always be at right angles (90° apart).

There are five distinct types of astigmatism, depending upon the combination of refractive errors in the principal meridians of the eye: *mixed astigmatism* (hyperopic in one meridian, myopic in the other), *compound myopic astigmatism* (myopic in both meridians), *simple myopic astigmatism* (myopic in one meridian, emmetropic in the other),

compound hyperopic astigmatism (hyperopic in both meridians), and *simple hyperopic astigmatism* (hyperopic in one meridian, emmetropic in the other). These five types of astigmatism, and their corresponding focal lines, are shown in Figure 6:7.

6.3 ACCOMMODATION

The *crystalline lens* of the human eye can increase the overall focal power up to +70.00 D or more. This process is called **accommodation**, and its function is to bring objects at near into focus. During this process, the crystalline lens becomes more bi-convex; effectively increasing the plus power of the eye.

The *farthest* point from the static eye that objects can be brought into focus, with accommodation completely relaxed, is called the **far point of accommodation**. This is the object point that is conjugate to the retina of the relaxed eye. An object located at this distance will create an image distance equal to the axial length of the eye. Therefore, objects that are located at—or appear to be located at—the far point have focused images on the retina.

The *closest* point from the eye that objects can be brought into focus, with accommodation fully exerted, is called the **near point of accommodation**. Recall that the *crystalline lens* of the eye provides *accommodation*, which is a variable amount of additional *plus* power, to bring objects at near into focus. (The need for plus power can be easily demonstrated using Eq. 21.) The crystalline lens does this by becoming more *bi-convex* in shape, so that the plus power of the eye is increased. This process is illustrated in Figure 6:8.

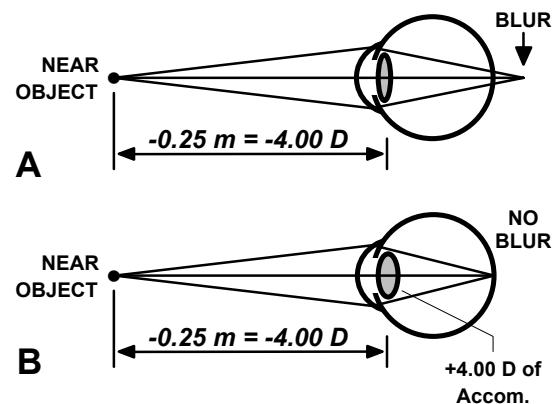


FIGURE 6:8 An object 25 cm in front of the eye (0.25 m) produces -4.00 D of divergence. A) The optical system of an emmetropic eye (with no error) is effectively weak by this amount, with accommodation at rest. B) With +4.00 D of accommodation, however, the image is brought back into focus.

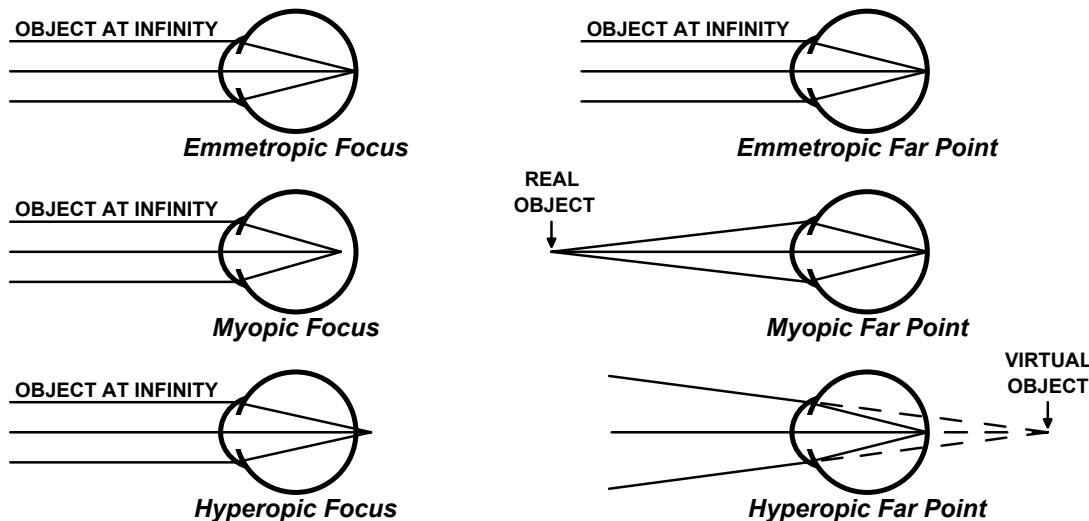


FIGURE 6:9 Refractive errors and their corresponding far points of accommodation. For the *emmetrope*, parallel rays of light focus *on* the retina; the far-point—conjugate to the retina—is also at infinity. For the *myope*, parallel rays of light come to a focus *in front* of the retina; the far point is a real object point in front of the eye, but within optical infinity. For the *hyperope*, parallel rays of light come to a focus *behind* the retina; the far point is a virtual object point lying behind the eye.

As the lens ages, however, it gradually becomes less flexible, and slowly loses its accommodative power (or **amplitude of accommodation**). Figure 6:10 is Donder's graph of the normal amplitudes of accommodation plotted against age for a typical person (Carlson et al. 23).

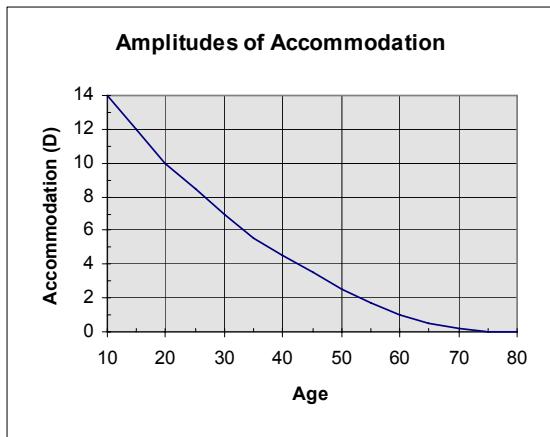


FIGURE 6:10 Reduction in accommodation with age.

As this aging process happens, near point of accommodation begins to recede from the eye. **Presbyopia** occurs once objects can no longer be held at a comfortable distance at near with sustained accommodation. For most people, this happens in their mid-40s. At this point, additional *plus* power is needed for near vision (Bennett & Rabbets 140).

With an *emmetropic* eye, the far point is located at infinity, since the secondary focal length of the eye is equal to its axial length. When this status fails to be achieved, the eye is *ametropic*, and the far point is no

longer located at infinity. For hyperopic refractive errors, the far point is a *virtual* object point located behind the eye. For myopic refractive errors, the far point is a *real* object point located in front of the eye. Astigmatic refractive errors will have two far points corresponding to the refractive errors of the principal meridians of the eye. The secondary focal length and the far point position for both emmetropic and ametropic eyes are shown in Figure 6:9, above.

The total *amplitude of accommodation* is simply the *dioptric difference* between the far and near points of accommodation. For our purposes, we will specify the distance m of the far (remote) and near (proximal) points of accommodation (M_R and M_P) from the plane of the spectacle lens; which is typically about 13.5 mm from the eye. Figure 6:11 and Figure 6:12 show two different ametropic eyes, along with their far and near points of accommodation.

The refractive error M_R of an eye for distance vision, in diopters, is simply the dioptric value of the far point of accommodation m_R . If both the far and near points of accommodation are known, the amplitude of accommodation A can also be quickly determined, since $A = M_R - M_P$. The far and near points of accommodation can be converted into their dioptric equivalents by our common formula for converting back and forth from distances to diopters. This is our familiar formula for vergence:

$$\text{EQ. 32} \quad M = \frac{1}{m}$$

Here, m is the distance from the spectacle plane to either the far or near point M of the eye in meters. Remember to follow the sign convention.

Example

An eye has a *virtual* far point of accommodation 50 cm (0.5 m) behind the spectacle plane, and a near point 25 cm (-0.25 m) in front it. What are the distance refractive error and the amplitude of accommodation?

$$M_R = \frac{1}{0.50}$$

$$M_R = +2.00$$

$$M_P = \frac{1}{-0.25}$$

$$M_P = -4.00$$

$$A = 2.00 - (-4.00)$$

$$A = 6.00$$

\therefore Distance refractive error is +2.00 D and the amplitude of accommodation is +6.00 D.

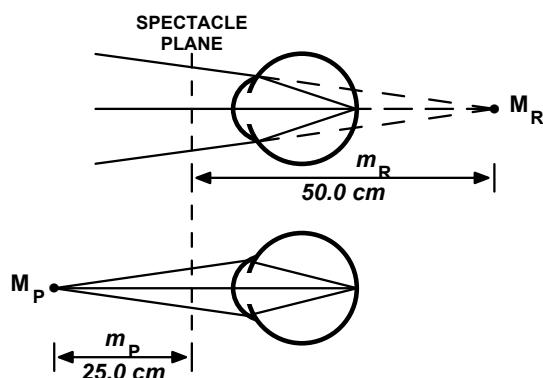


FIGURE 6:11 Eye with +2.00 D of *hyperopia*, and an amplitude of accommodation of +6.00 D. The far point of accommodation M_R is 50 cm *behind* the spectacle plane, while the near point M_P is 25 cm in front of it.

Example

An eye has a *real* far point of accommodation 50 cm (-0.5 m) in front of the spectacle plane, and a near point 20 cm (-0.20 m) in front of it. What are the distance refractive error and the amplitude of accommodation?

$$M_R = \frac{1}{-0.50}$$

$$M_R = -2.00$$

$$M_P = \frac{1}{-0.20}$$

$$M_P = -5.00$$

$$A = -2.00 - (-5.00)$$

$$A = 3.00$$

\therefore Distance refractive error is -2.00 D and the amplitude of accommodation is +3.00 D.

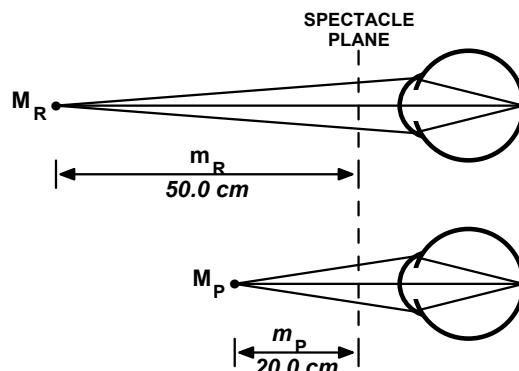


FIGURE 6:12 Eye with -2.00 D of *myopia* and an amplitude of accommodation of +3.00 D. The far point of accommodation M_R is 50 cm in front of the eye, while the near point M_P is 20 cm in front of the eye.

6.4 CORRECTING LENSES AND THE RX

For the *hyperope*, a *plus* lens placed in front of the eye will provide the eye with the additional plus power required to compensate for the fact the refracting elements of the eye are too weak. For the *myope*, a *minus* lens will reduce the plus power of the eye, to compensate for the fact that the refracting elements are too strong.

The *image* produced by an ophthalmic lens becomes the *object* for the optical system of the eye. In the presence of a refractive error, the purpose of an ophthalmic lens is to produce an image at the far point M_R of the eye, which is the ideal focal plane. Therefore, a lens power is utilized that produces a secondary focal length f' equal to the distance m_R of the far point from the spectacle plane, so that the secondary focal point of the lens F' falls upon the far point M_R of the eye. If this requirement is satisfactorily met, objects at infinity will be made conjugate to the retina, providing an artificially emmetropic refractive status—free from error.

This ideal configuration is demonstrated in Figure 6:13 and Figure 6:14 for plus and minus lenses.

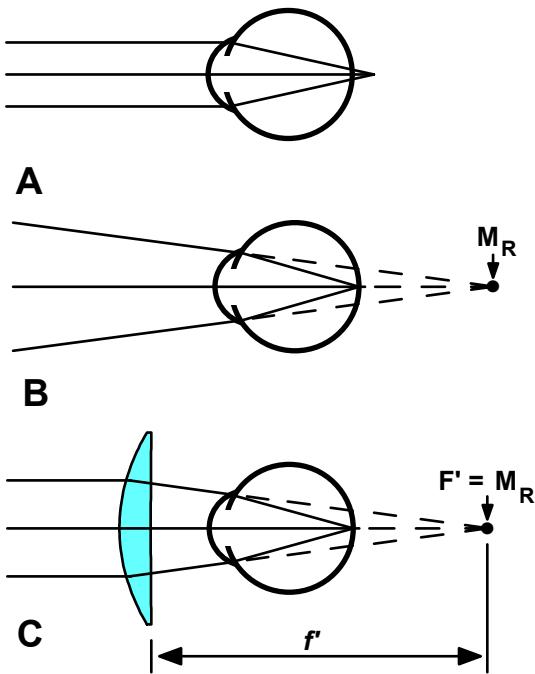


FIGURE 6:13 Plus lens corrections. A) Parallel rays of light from a distant object focus behind the retina of the *hyperopic* eye. B) Rays of light converging to the far point M_R of the eye focus on the retina. C) The corrective *plus* lens converges parallel rays from an object at infinity to form a *real image* at F' , which corresponds to the far point M_R of the eye. This image now becomes a *virtual object* for the eye that is conjugate with the retina, and forms a clear image upon it.

The doctor specifies the optical characteristics a pair of ophthalmic lenses should provide for a given wearer by writing a **spectacle prescription**, or ‘Rx.’ The Rx describes the focal powers needed to correct the refractive errors for distance and/or near vision, as well as any prescribed prism. Spectacle prescriptions are specified by eye, and the following abbreviations apply (Keeney et al. 205):

- **OD** (*Oculus Dexter*) or **RE** represents the wearer’s right eye.
- **OS** (*Oculus Sinister*) or **LE** represents the wearer’s left eye.
- **OU** (*Oculi Uterque*)—when used—represents a prescription suitable for *both* eyes.

Table 7 shows a common format for eyeglass prescriptions, including the *sphere power* (sph), *cylinder power* (cyl), *axis of the cylinder*, *prism magnitude and orientation*, and/or *add power* for the right (OD) and left (OS) eyes.

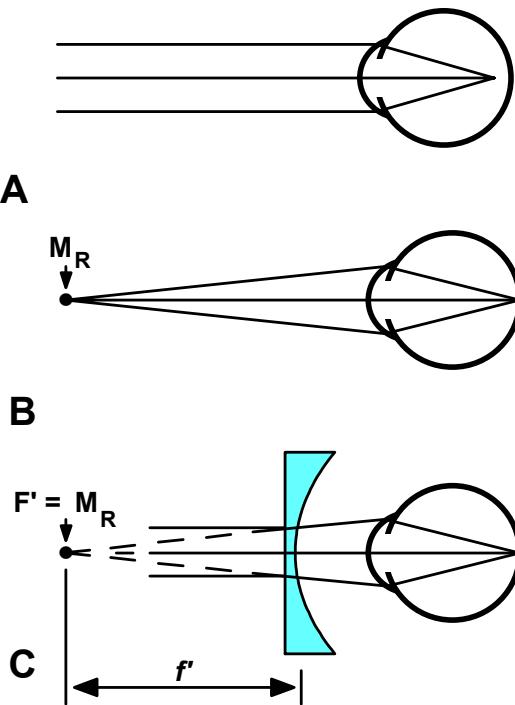


FIGURE 6:14 Minus lens correction. A) Parallel rays of light from a distant object focus in front of the retina of the *myopic* eye. B) Rays of light diverging from the far point M_R of the eye focus on the retina. C) The corrective minus lens diverges parallel rays from an object at infinity to form a virtual image at F' , which corresponds to the far point M_R of the eye. This image now becomes a real object for the eye that is conjugate with the retina, and forms a clear image upon it.

TABLE 7 A typical prescription

Eye	Sph	Cyl	Axis	Prism	Add
OD	-4.00	sph			+2.50
OS	-2.50	-0.50	135		+2.50

The spectacle Rx above depicts a right (OD) eye that requires a -4.00 D lens for a myopia correction. The left (OS) eye requires a -2.50 DS -0.50 DC lens \times 135 for a compound myopic astigmatism correction. Both eyes (OU) require a +2.50 D *add power*. No prism was prescribed.

The *sphere* (sph) *power*, *cylinder* (cyl) *power*, and *axis* describe the necessary focal powers required by the wearer for clear distance vision. The *add power* describes the additional power (if any) needed for clear near vision. The prism power describes the amount of prismatic deviation (if any) needed to provide comfortable binocular vision. To avoid errors in prescription interpretation, refractionists generally adhere to the following conventions and nomenclature when writing spectacle prescriptions:

- Sphere and cylinder powers are generally prescribed and rounded to the nearest 1/4 (0.25) diopter step, since this is the smallest discernible difference (or *just noticeable difference*) for most people. In some *very rare* instances, the prescribed

power might be rounded to 1/8 (0.125) diopter step. In these cases, the last '5' is dropped from the thousandth place and it is written '0.12.'

- The sphere and cylinder powers are both written with two decimal places and at least three significant figures. It is correct to write '0.50 D,' not '0.5 D' or '.50 D.'
- Both the sphere and cylinder powers should be written with the appropriate sign (+ or -) in front of them; they should not be assumed.
- After the *sphere* power, either one of two abbreviations may appear: 'D' for *diopter* or 'DS' for *diopters of sphere*. Occasionally, 'sph' may be written in cylinder power field of the prescription.
- After the *cylinder* power, either one of two abbreviations may appear: 'D' for *diopter* or 'DC' for *diopters of cylinder*.
- When the sphere powers have different signs, the prescription is **antimetropic**. This is rare except in low powers or after certain eye surgeries. When in doubt, this should be confirmed with the refractionist.
- Cylinder powers should be written in the same cylinder notation, either plus cylinder or minus cylinder. If the cylinder power signs are not the same, the prescription should be confirmed with the refractionist.
- Prescriptions with cylinder powers should always have an accompanying axis designation, which describes the orientation of the cylinder.
- Axes for cylinder powers are generally written from 1 to 180 degrees, in 1-degree intervals. The degree symbol ($^{\circ}$) is not used, since this could be confused with a zero (0).
- Except in rare instances, add powers are usually equal. They are also *always* positive (+).

The *add power*, when prescribed, represents the additional plus power required for *near vision*. It is most commonly prescribed for presbyopia in which the natural accommodation is no longer sufficient. It is called an 'add' because it is in *addition* to the distance power—it assumes that the ametropia (refractive error) is already corrected. If the wearer prefers a pair of single vision reading lenses, as opposed to multifocals, you must compute the near vision prescription. To determine what the actual near vision prescription for each eye is, simply add the *add power* to the *sphere power* of the distance prescription, and keep the same cylinder power and axis.

An example is provided, below, for a distance prescription calling for a +2.75 D add power:

Near Rx Conversion	Sph	Cyl	Axis
Original Distance Rx	-2.25	-0.50	180
Plus Add Power	+2.75	↓	↓
Equals Near Vision Rx	+0.50	-0.50	180

A common rule of thumb is that the patient should not have to use any more than half of his or her amplitude of—or available—accommodation for an extended period of time. For a given working distance (often considered to be 40 cm), the add power represents the additional *plus* power required to supplement roughly half of the patient's amplitude of accommodation. For instance, consider a patient with an amplitude of accommodation of 2.00 D. Half of this value would be 1.00 D. For this patient to read comfortably at a distance of 40 cm (which represents -2.50 D of divergence), an add power of 1.50 D would be required (Grosvenor 334).

Up to this point, we have discussed **single vision lenses** designed to provide a correction at a single distance. However, for presbyopic patients that require both near *and* distance corrections, at least two distinct focal powers need to be provided. The most common method of providing additional plus power (i.e. the *add*) at near is with the use of a **multifocal lens**, which is simply a lens with more than one focal power for different distances. Multifocal lenses will be discussed in more detail in Section 11. Figure 6:15 illustrates a typical *flat-top bifocal* lens. The *major portion* provides distance vision, and the *seg* (which is short for 'segment') provides the *add power* for near.

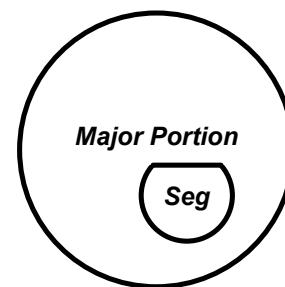


FIGURE 6:15 The flat-top bifocal lens.

6.5 DEFOCUS BLUR

The final image, created by the optical system of the eye, is the **optical image**. If this image does not fall upon the retina (i.e. the image distance differs from the axial length of the eye), the image at the plane of the retina is defocused and blurred. The image at the plane of the retina is referred to as the **retinal image**. When the optical image lies in a different plane than the retinal image because of a refractive error, each point from the object will form a defocused **blur circle** upon the retina.

(assuming that the pupil aperture is circular). Blur circles created by image defocus, which are simply diffuse patches of light that have been intercepted either before or after coming to a focus, are illustrated in Figure 6:16.

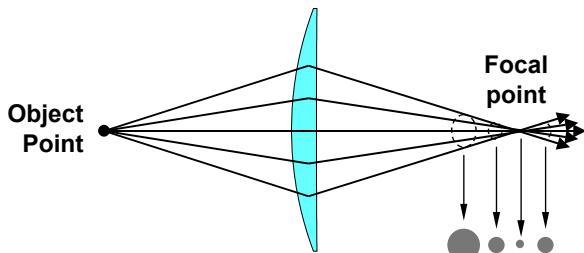


FIGURE 6:16 Blur circles produced by defocus. A screen that intercepts rays of light—refracted by a lens from an object point—will only show a sharp image point at the secondary focal point of the lens. At any other location, a diffuse patch of light—in the shape of the limiting aperture—is produced. For the human eye this aperture is the pupil, which is circular. For a given eyeball, the size of the blur circle is directly proportional to the size of the pupil and the amount of refractive error present (i.e., the distance of the retina from the focal point of the eye).

For lenses that contain cylinder power, this defocused patch of light may take on a variety of shapes, depending upon where the retina lies within the bundle of rays that make up the astigmatic focus. This astigmatic focus is referred to as **Sturm's interval** (see Figure 5:9), and the defocused images produced within it are shown for a typical spherocylindrical lens in Figure 6:17.

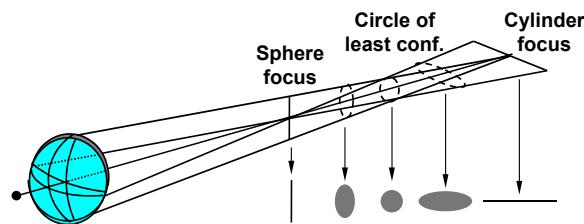


FIGURE 6:17 The shape of the defocused patch of light produced by this spherocylindrical lens varies within Sturm's interval from a vertical line, to vertical ellipses, to a circle, to horizontal ellipses, and then a to horizontal line. Note that a single image point is never formed.

Blur circles are a consequence of the fact that rays of light focusing in front of the retina (as a result of a myopic refractive error) intersect and then begin to diverge afterward, while rays focusing behind the retina (as a result of a hyperopic refractive error) are intercepted before they can intersect or focus. Equal amounts of myopic and hyperopic refractive errors produce blur circles of equal size. The diameter of the blur circle will vary with both the magnitude of the refractive error and the size of the pupil. Figure 6:18 illustrates the blur circles formed by both small and

large *myopic* errors, in which rays of light come to a focus in front of the retina.

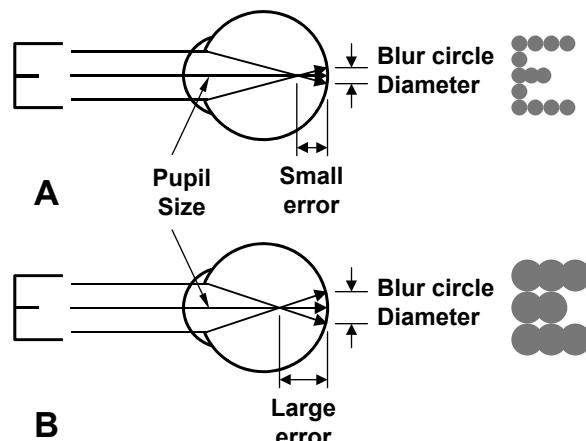


FIGURE 6:18 A) Defocus blur created by a *small* myopic refractive error and B) Defocus blur created by a *large* myopic refractive error. In both instances, light rays come to a focus in front of the retina, diverge, and then form a diffuse patch of light on the retina. For *hyperopic* refractive errors, the retina intercepts the rays before they come to a focus—thereby creating the same diffuse patch of light on the retina. As the pupil *constricts* (or becomes smaller), the size of the blur circle diminishes. As the pupil *dilates* (or becomes larger) the size of the blur circle enlarges.

For a purely astigmatic error, in which the circle of least confusion lies on the retina, the size of the blur circle is equal to the circle of least confusion. Moreover, the blur circle (and circle of least confusion) for a purely astigmatic error is one-half the size of an equivalent spherical error of the same magnitude. This means that the blur produced by spherical errors is more significant than the blur produced by comparable cylindrical errors.

A blurred image can be thought of as a myriad of overlapping blur circles. The larger the blur circle the greater the diffusion of visual information, like color and intensity, and the more blurred the image. Consequently, adjacent parts of the image become less distinct, and the transitions between different areas become more gradual. A comparison showing the effects of blur on an image is provided in Figure 6:19 and Figure 6:20.

Because of the fact that the size of the pupil will also affect the diameter of the blur circle, smaller pupil apertures will increase the **depth of focus** of the eye. This is the amount of refractive error that an eye can tolerate, while maintaining a perceived sharp retinal image. This is why squinting can sometimes improve the sharpness of vision for someone with a refractive error. This is also the basis for the pinhole camera, which produces an image without a lens. The depth of focus of a typical observer under ideal conditions is around ± 0.25 D (Tunnacliffe 56).



FIGURE 6:19 A clear image.



FIGURE 6:20 A blurred image.

6.6 VISUAL ACUITY

The visual performance of the eye is typically rated by its ability to resolve two points as separate or to discern small details. The resolving ability is determined by measuring the observer's **minimum angle of resolution** (MAR). This is the minimum angular separation, subtended at the nodal point N, of two object points that the eye can still distinguish as separate. The angular separation of two object points is illustrated in Figure 6:21. The capability of a person to detect this minimum separation is typically measured by tests of **visual acuity**. An observer's visual acuity is inversely proportional to his or her minimum angle of resolution. Consequently, the smaller the MAR, the greater the resolving power of the eye (Tunnacliffe 123).

The size and spacing of the photoreceptive cells, and the *diffraction** effects of the pupil limit the resolving ability of the eye. A *normal* eye—without any errors of refraction—can typically resolve two objects as separate when their separation subtends a *visual angle* of approximately one minute of arc (0.0833°). This angle is measured at either the entrance pupil or nodal point of the eye. Other factors, like the depth of focus and the ability of the eye to perceive contrast, will also affect visual acuity (Grosvenor 12).

* **Diffraction** is the wave-like bending (or spreading) of light at an aperture, such as the pupil of the eye.

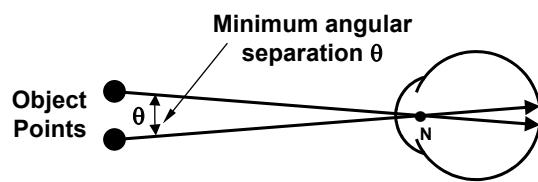


FIGURE 6:21 The minimum angle of resolution for an eye represents the smallest angular separation θ between two object points that will still allow the points to be resolved as separate. These angles are measured in arcminutes (').

Although there are several methods of measuring and specifying visual acuity, **Snellen notation**—also called **Snellen acuity**—is the most common in the United States. This test uses a series of rows of letters, called **optotypes**, which progressively increase in size line by line from the bottom row up. A typical **Snellen chart** is shown in Figure 6:22. The test subject reads these letters from a distance of 20 feet (or 6 meters). Recall that this distance is considered optical infinity.

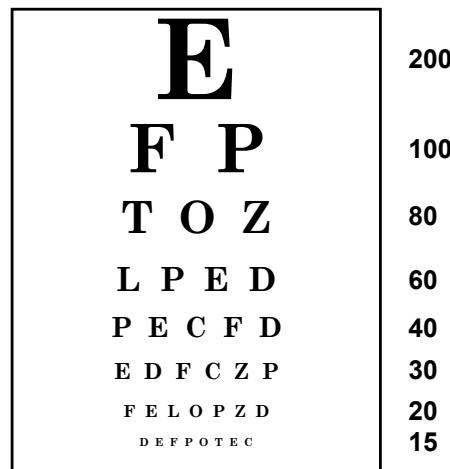


FIGURE 6:22 A typical Snellen chart. The letter in the top row subtends 5' at 200 ft, the letters in the next row down subtend 5' at 100 ft, and so on. (Not to scale.)

Each limb and gap of an optotype subtends 1' of arc at a given distance, for a total angle of 5' of arc, as shown in Figure 6:23.

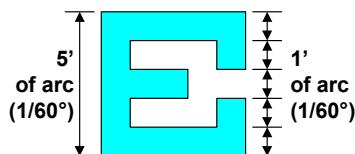


FIGURE 6:23 Optotypes subtend 5' at a specific distance.

Snellen visual acuity VA_{SNEL} is generally written in the form of the **Snellen fraction**,

$$EQ. 33 \quad VA_{SNEL} = \frac{\text{Testing Dist}}{\text{Dist Smallest Letters} = 5'}$$

where the denominator is the distance that the line of smallest letters read subtends an angle of 5' of arc.

Having "20/20" acuity means that, at a testing distance of 20 feet, a person is able to resolve a letter subtending an angle of 5' of arc at 20 ft (6 m). This letter is 8.7 mm tall at 20 ft. Such a person has "normal" vision. Having "20/40" vision means that, at a 20-ft testing distance, a person is only able to resolve a letter that subtends an angle of 5' of arc at a distance of 40 ft (12 m). This letter is 17.5 mm tall. This also means that the 20/40 letter is twice as large as the 20/20 letter, as illustrated in Figure 6:24 for the Snellen 20/20 and 20/40 optotype 'E's.

Since the fraction is based upon the minimum angle of resolution of a "normal" eye, it can also be said that a person with 20/40 acuity can resolve an object at 20 ft that someone else with "normal" acuity can resolve from 40 ft. The required letter height for the 20/20 'E' can be determined by multiplying the tangent of 5' (or 0.083°) by the testing distance (6 m), or $6 \tan 0.083 = 0.0087$ m (or 8.7 mm).

The most standardized system for specifying visual acuity is based upon the logarithm of the minimum angle of resolution. Each successively larger line on charts using this principle, such as the Bailey-Lovie acuity chart, increases geometrically in size by nearly 26% from the bottom row (specifically, a factor increase of $\sqrt[10]{10}$). The **LogMAR acuity** is provided, which is the common (base 10) log unit for the minimum angle of resolution for the smallest line read at a specific testing distance. Normal acuity, which designates a minimum angle of resolution of 1', is equal to a LogMAR score of 0. Each successively larger line on the chart represents a LogMAR increase of 0.1 log units. A conversion table for visual acuity systems is provided for comparison in Table 8 (Benjamin 185).

TABLE 8 Snellen and LogMAR acuity comparison

Snellen	LogMAR	Decimal
20/20	0.0	1.000
20/25	0.1	0.800
20/32	0.2	0.625
20/40	0.3	0.500
20/50	0.4	0.400
20/63	0.5	0.317
20/80	0.6	0.250
20/100	0.7	0.200
20/125	0.8	0.160
20/160	0.9	0.125
20/200	1.0	0.100

The LogMAR acuity VA_{LOG} is given by

$$VA_{LOG} = \log_{10} MAR$$

Another system of describing visual acuity, which is not as common, is referred to as **decimal acuity**. The decimal acuity VA_{DEC} is inversely proportion to the minimum angle of resolution MAR, so that

$$VA_{DEC} = \frac{1}{MAR}$$

It is interesting to note that, since the MAR of the normal observer is considered to be 1', an observer's decimal acuity is also equal to the decimal form of his or her Snellen fraction. For instance, an observer with "20/20" Snellen acuity has an MAR of 1'. This observer has a decimal acuity of $1 / 1' = 1.0$. We can then show that $20 / 20 = 1.0$.

The visual acuity of the eye also varies with the size of the blur circles created by any refractive errors: decreasing as the blur circles increase, since these blur circles affect the resolution of the retinal image. Moreover, a visual acuity measurement can often give an indication of the amount of uncorrected refractive error—and vice versa. For instance, the following regression formula can be used to estimate the Snellen denominator D based upon a refractive error of magnitude E (Bennett & Rabbets 72):

$$EQ. 34 \quad D = 10^{0.5E+1.25}$$

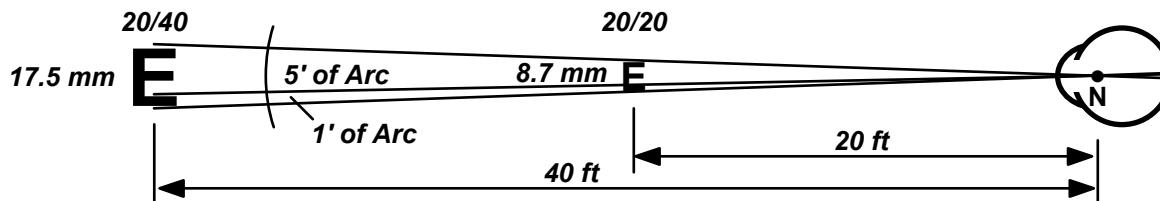


FIGURE 6:24 Heights of Snellen 20/20 and 20/40 'E' at 20 ft and 40 ft, respectively. Note that both letters subtend 5' of arc.

where D is the denominator of the Snellen fraction (Eq. 33) in feet and E is the magnitude of the refractive error in diopters.*

The value of E is equal to the absolute value of the spherical error S :

$$E = |S|$$

This formula (Eq. 33) should only be used as an estimate for predicting visual acuity, since the actual acuity will be affected by the test chart luminance, pupil diameter, etc (Bennett & Rabbets 94). Table 9 shows the predicted visual acuity for a range of refractive error magnitudes, using this formula.

TABLE 9 Refractive error and predicted visual acuity

Refractive Magnitude E	Snellen Acuity	Decimal Acuity
0.12 D	≈ 20/20	≈ 1.000
0.25 D	≈ 20/25	≈ 0.800
0.50 D	≈ 20/30	≈ 0.667
0.75 D	≈ 20/40	≈ 0.500
1.00 D	≈ 20/55	≈ 0.364
1.50 D	≈ 20/100	≈ 0.200
2.00 D	≈ 20/180	≈ 0.111
2.50 D	≈ 20/315	≈ 0.063
3.00 D	≈ 20/560	≈ 0.036

* When determining the value of E for a spherocylindrical error, use this formula to determine the magnitude E of the resulting *power vector*:

$$E = \sqrt{\frac{S^2 + (S + C)^2}{2}}$$

where S is the spherical component and C is the cylindrical component of the prescription.

7. Lens Form and Thickness

Often, it is desirable to know the **finished thickness** of a given lens—which is the final center or edge thickness of the lens after surfacing, fining, and polishing. Patients purchasing new eyewear may want an estimate of the thickness of their new lenses. An idea of the maximum thickness of a lens is also required in order to determine whether a certain lens blank will work for a given power. Like focal power, we will begin our discussion of lens thickness with an analysis of the geometry of curved refracting surfaces.

7.1 SURFACE GEOMETRY

The *height* (or depth) of the curvature of a surface, at a given diameter, is referred to as the **sagitta**, or simply **sag**, of that curve. This value will vary with both the radius of curvature of the surface and the diameter, as illustrated in Figure 7:1. With the help of this diagram, we can show that the sagitta s of a lens surface can be computed for a given diameter \emptyset , and radius of curvature r , using the Pythagorean theorem. This is called the **sag formula**:

$$(r - s)^2 + \left(\frac{1}{2}\emptyset\right)^2 = r^2$$

EQ. 35 $s = r - \sqrt{r^2 - \left(\frac{1}{2}\emptyset\right)^2}$

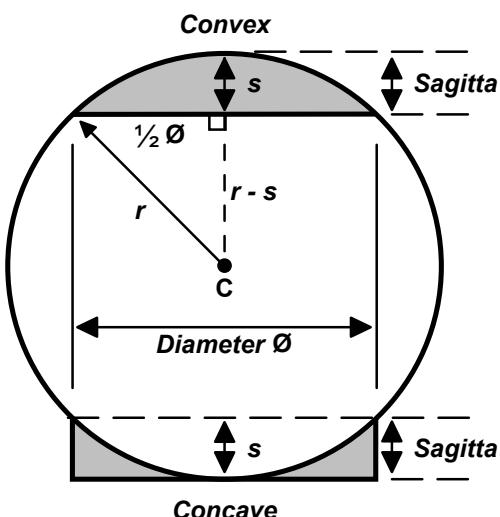


FIGURE 7:1 The sagittae for both convex and concave curves are shown. Using the figure, a right triangle can be formed using the radius r as the hypotenuse, $\frac{1}{2}\emptyset$ as one leg, and the quantity $(r - s)$ as the other leg. This is the basis of the sag formula.

If we assume that s and \emptyset will be relatively small compared to r , which is often the case for flatter curves and smaller diameters, an *approximation* of the sag s of a lens in millimeters can be found with

$$s = \frac{\left(\frac{1}{2}\emptyset\right)^2}{2r}$$

From our definition of surface power (Eq. 14), we can show that the radius r , in millimeters, is equal to

$$r = \frac{1000(n-1)}{F_s}$$

Therefore, we can substitute this quantity for r in the sag equation, allowing us to solve for sagitta using the surface power of the curve. The result is the *approximate sag formula*:

EQ. 36 $s = \frac{\left(\frac{1}{2}\emptyset\right)^2 F_s}{2000(n-1)}$

The sign (\pm) of the value is not important, since we are only concerned with the magnitude of the sag, and not the direction. This last formula shows a specific relationship between the *sagittal depth* and the surface power of a lens. If the refractive index is known, either value may be utilized for surfacing calculations. If the sagittae for both surfaces of a lens are known, it is also possible to determine the final thickness of the lens at a given diameter.

Remember that stronger surface powers produce shorter radii of curvature. Hence, for a given diameter, the sag is directly proportional to the surface power and will increase as the power of the surface increases.* Further, the sag will also increase as the diameter of the lens increases, as shown in Figure 7:2 for a *minus* lens.

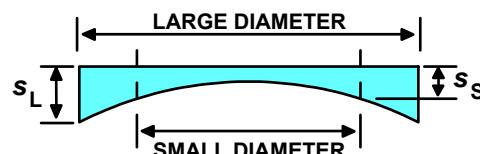


FIGURE 7:2 The sag s_S represents the sag at a smaller diameter, while s_L represents the sag at a larger diameter.

Example

A +8.50 D convex curve is ground on a lens material with a 1.500 index of refraction. What is the approximate sag value of the curve at a 70-mm diameter?

$$s = \frac{\left(\frac{1}{2}70\right)^2 8.50}{2000(1.500 - 1)}$$

* If the exact formula is used, it can be shown that the sag of a curve actually increases slightly faster than its power—especially for larger diameters.

$$s = \frac{10412.5}{1000}$$

$$s = 10.41$$

∴ Sagitta is 10.41 mm.

7.2 LENS THICKNESS

Now that we know how to calculate the sagitta of a surface, we need to consider the form of the entire lens. Most modern lenses are **meniscus** in form, having convex front curves and concave back curves. Recall that if the dioptric value of the front curve is greater than the value of the concave back curve (absolute values), the lens will be *positive* (plus) in power. Similarly, if the dioptric value of the back curve is greater than the value of the convex front curve (absolute values), the lens will be *negative* (minus) in power. Because these lenses have *two* surface curves, we need to consider the sag of both the front curve s_1 and the back curve s_2 for determining thickness. Fundamentally, the change in lens thickness is the addition of the two sagittae.

Generally, we are concerned with finding the *maximum* thickness of the lens. This will be the *center* thickness t_{CNTR} of plus lenses and the *edge* thickness t_{EDGE} of minus lenses. These lenses are often produced with a certain amount of *minimum* (or additional) thickness, as well. Therefore, in addition to the thickness of each curve, we also need to add additional *edge* thickness for plus lenses (the thinnest point of the lens) and additional *center* thickness for minus lenses (the thinnest point of the lens). Figure 7:3 depicts the factors affecting the final center thickness of a meniscus plus lens.

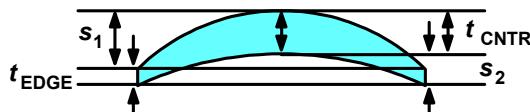


FIGURE 7:3 For a meniscus plus lens, the *center* thickness $t_{CNTR} = s_1 - s_2 + t_{EDGE}$.

To determine the final **center thickness** t_{CNTR} of a *plus* lens, use the formula:

$$\text{EQ. 37} \quad t_{CNTR} = s_1 - s_2 + t_{EDGE}$$

To determine the final **edge thickness** t_{EDGE} of a *minus* lens, use the formula:

$$\text{EQ. 38} \quad t_{EDGE} = s_2 - s_1 + t_{CNTR}$$

Example

A meniscus lens (convex front, concave back) has a front sag of 6.0 mm, a back sag of 2.0 mm, and an edge thickness of 1.0 mm. What is the center thickness and a plate height of the lens?

$$t_{CNTR} = 6.0 - 2.0 + 1.0$$

$$t_{CNTR} = 5.0$$

$$p = 6.0 + 1.0$$

$$p = 7.0$$

∴ Center thickness is 5.0 mm and the plate height is 7.0 mm.

When dealing with spectacle lenses of low-to-moderate power and reasonable diameter, however, we can further simplify the process by ignoring the surface curves and form of the lens altogether. This is simply an extension of our earlier sagitta approximation, which says that the sag of a curve will be directly proportional to its power. For a *thin lens* the surface powers of a lens (F_1 and F_2) must vary at the same rate to provide a given lens power according to the lensmaker's formula (Eq. 19):

$$F = F_1 + F_2$$

Consequently, the sags of each curve must also vary at the same rate. Therefore, the difference between the sags will remain constant as the surface powers change.*

To visualize this concept, consider the form of the lens as being *flat*, so that the lens power is produced by one surface curve with a single sagitta. The flat plus lens will have a convex front curve and a *plano* (flat) back curve, while the flat minus lens will have a *plano* front curve and a concave back curve. With our approximation, the difference between the sags of the front and back curves will remain constant.

At this point, we merely have to add the desired amount of minimum thickness to determine the final, maximum thickness of the lens. These simplified lenses are illustrated in Figure 7:4 and Figure 7:5.

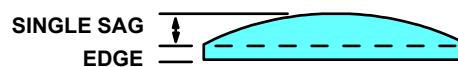


FIGURE 7:4 Flat, plano-convex *plus* lens.



FIGURE 7:5 Flat, plano-concave *minus* lens.

We can now substitute the focal power F of the lens—ignoring the (\pm) sign—in place of the surface power F_s in our simplified sag formula,

* This is consistent with our thin lens approximation, but will quickly lose accuracy for strong powers or large lens diameters.

$$s = \frac{\left(\frac{1}{2}\varnothing\right)^2 F}{2000(n-1)}$$

And, to determine the final, *maximum* thickness of the lens t_{MAX} , use the formula:

$$t_{MAX} = s + t_{MIN}$$

Which, after substituting for s , gives us

$$\text{EQ. 39} \quad t_{MAX} = \frac{\left(\frac{1}{2}\varnothing\right)^2 F}{2000(n-1)} + t_{MIN}$$

where t_{MIN} is the minimum thickness required for the lens.

When prism has been prescribed for the lens, a consideration should be made for the additional thickness produced by the prismatic effect. The effect that prism has on the over thickness of the lens will depend upon the power of the lens and the shape of the frame, as well as the amount and orientation of the prism. For instance, *base in* prism will typically make *plus* lens slightly *thicker* and *minus* lenses slightly *thinner*. Conversely, *base out* prism will typically make *plus* lens slightly *thinner* and *minus* lenses slightly *thicker*. The thickness difference produced by prism is discussed in Section 8.4.

Lens manufacturers often provide center thickness guidelines to ensure that these lenses will have enough thickness to provide acceptable flexural stability during processing. The thickness also has to be substantial enough to satisfy the FDA impact resistance requirements for safety.

Most minus lenses are either surfaced to, or supplied in finished form with, centers between 1.0 and 2.2 mm—depending upon the type of lens material and design. Plus lenses are typically provided with edges between 1.0 and 2.0 mm. Plus lenses intended for rimless frames may require thicker edges to allow for a groove to be cut. Both plus and minus lenses can be no thinner than 3.0 mm at the thinnest point when used in OSHA safety frames. High-powered plus lenses over +3.00 D can go down to 2.5 mm at the edge, though.

Up to this point, we have assumed two things: a lens power and a lens blank diameter. Obviously, the power should be known. If the diameter is unknown, a few more computations may be necessary. This will be discussed in more detail in Section 9.3. It is important to note that the center thickness of a *finished* plus lens is fixed with respect to the initial diameter of the lens blank. Once cast, plus lenses can only be *surfaced* to smaller diameters and thinner centers. When using finished plus lenses, the factory blank size should be utilized for determining the center thickness.

7.3 LENS FORMS AND BASE CURVES

When a manufacturer provides a lens blank with both the front and back curves finished to the desired power, the lens blank is referred to as a **finished lens**. When a blank is supplied with only one curve finished, the lens blank is referred to as a **semi-finished lens**. These blanks require the backside to be surfaced to the desired thickness and power, usually by a surfacing laboratory.

The relationship between the front and back surface curvatures of a lens is referred to as the **lens form**, or **lens profile**. It should now be obvious that a lens can be produced by many different lens forms, as long as the sum of the front and back surface powers remains constant—or at least nearly so (neglecting thickness for now). This concept is illustrated in Figure 7:6 and Figure 7:7 for +4.00 and -4.00 D lenses made using three different lens forms.

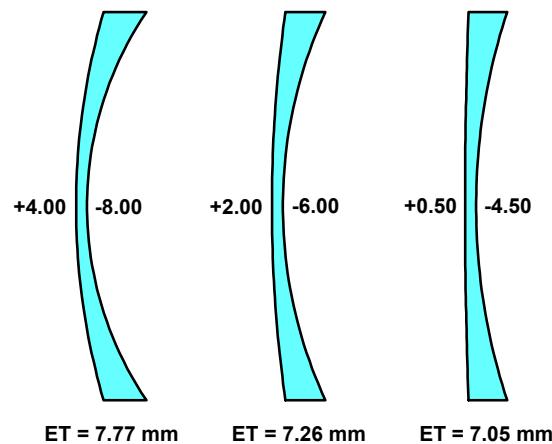


FIGURE 7:6 -4.00 D lens forms. Notice how the form of the lens affects the finished *edge thickness* in *minus* lenses. Steeper lens forms will have greater differences between the sagittal values of the front and back surfaces, thereby causing increases in the maximum edge thickness.

Figure 7:6 and Figure 7:7 show how the sum of the surface powers can remain relatively constant, even as the lens changes in form. However, for thicker plus lenses the back surface has to be compensated slightly for the gain in back vertex power. It is important to note that the maximum thickness—i.e. the edge thickness for minus lenses and center thickness for plus lenses—increases as the form of the lens becomes steeper, and vice versa as the form becomes flatter. Consequently, flatter lens forms are thinner for a given lens power.

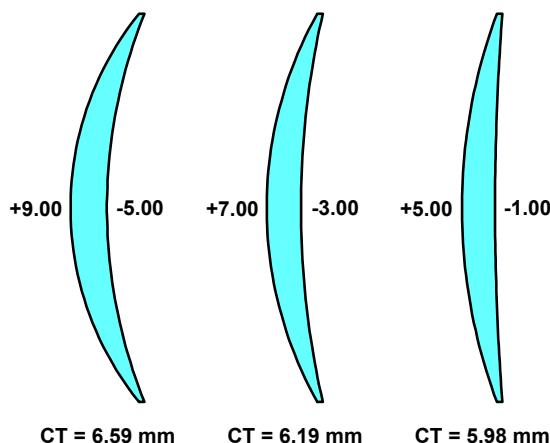


FIGURE 7:7 +4.00 D lens forms. In *plus* lenses, the *center thickness* will vary with the form of a lens. Again, steeper lens forms will have greater differences between the sagittal values of the front and back surfaces, thereby causing increases in the maximum center thickness.

Lens forms are often arbitrarily classified into either of two categories:

1. **Flat lenses:** These early lens forms, seldom employed today, include *plano-concave* and *plano-convex* lenses with one plane surface, as well as *bi-concave* and *bi-convex* lenses with two concave or two convex surfaces, respectively. Flat lenses are shown below in Figure 7:8.

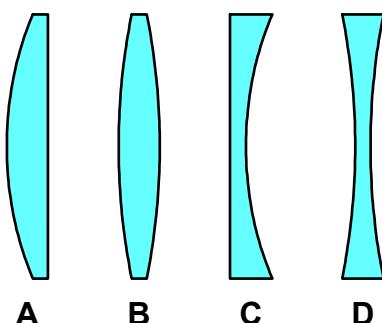


FIGURE 7:8 Flat lens forms. A) Plano-convex; B) bi-convex; C) plano-concave; and D) bi-concave lenses.

2. **Bent lenses:** These modern lenses are *meniscus* (or crescent-shaped) in form, having a convex front curve and a concave back curve. Bent plus and minus lenses are shown in Figure 7:9.

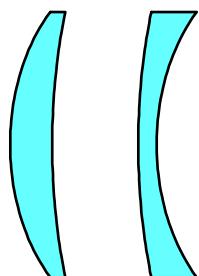


FIGURE 7:9 Bent (meniscus) lens forms.

The **base curve** of the lens, in diopters, is the surface curve that becomes the basis from which the remaining curves will be calculated. For a *semi-finished* lens, the base curve will be the factory-finished curve. For modern ophthalmic lenses, the base curve is typically the *front curve* of the lens blank, and is usually *convex*. There are very few exceptions to this (Brooks 371).

Manufacturers customarily produce an integral series of semi-finished lens blanks. Such a **base curve series** is a system of semi-finished lens blanks in several increments of base curve surface power, so that small prescription ranges may be grouped together upon common lens blanks. It is the selection of the *base curve* that determines the final *lens form* for a desired lens power. It will be shown later that the selection of the base curve is a primary consideration in spectacle lens design. Manufacturers make base curve selection charts available that provide the recommended ranges of final surfaced power for each base curve in the series.

Periodically, some practitioners may recommend matching a wearer's new base curves to his/her previous base curves for new eyewear. This is done because the base curve of a lens affects certain perceptual aspects of vision, such as *magnification* and *distortion*. By employing the same base curves when the wearer obtains new eyewear, such perceptual differences between the new lenses and the previous lenses are minimized. Some feel that this makes it easier for particularly sensitive wearers to 'adapt' to the new eyewear.

However, changes in the wearer's spectacle prescription will also create unavoidable perceptual differences. Moreover, the wearer will generally adjust to these perceptual differences within a week or so. If the same base curve is continually used as the wearer's prescription changes, which might necessitate a change in the 'best form' base curve, the peripheral optical performance of the lens may suffer as a consequence. When duplicating lenses of the same lens material, design, and power, matching base curves should not pose a problem—and is a recommended practice. However, unless the wearer has shown a previous sensitivity to base curve changes, you should use the manufacturer's recommended base curve when changing the Rx, or when using different lens materials and/or designs.

7.4 PLATE HEIGHT

The variation in thickness between different lens forms of the same power has already been described. Various lens forms will also produce significant differences in the **plate height**, or overall *bulge*, between lenses of the same power. The physical aspect of plate height is described in Figure 7:10. Essentially, the plate height is the height of a lens as measured from a flat plane, upon

which the lens rests, to a plane tangent to the apex of the front surface.

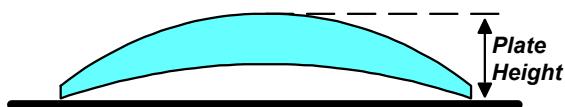


FIGURE 7:10 The *plate height* is the height of the lens, as measured perpendicularly from a flat plane to the apex of the front surface. The plate height can be found by simply adding the *center* thickness to the sag of the *back curve*, or by adding the *edge* thickness to the sag of the *front curve*.

Lens forms with flatter plate heights are more securely retained in frames, which is especially important with large or exotic frame shapes. It will be shown later that a reduction in plate height will also provide a reduction in the *magnification* associated with plus lenses, and the *minification* associated with minus lenses. This gives the wearer's eyes a more natural appearance through the lenses. In addition, flatter plate heights are also more cosmetically pleasing than steeper, bulbous ones.

The form of the lens for a given prescription can also have a significant effect upon both its cosmetic and optical properties. For instance, the weight, plate height, and thickness are provided for three different forms of two lens powers in Table 10. The plate height p of a lens can be found by adding the sag of the *back curve* to the *center* thickness of the lens, or by adding the sag of the *front curve* to the *edge* thickness:

$$\text{EQ. 40} \quad p = s_2 + t_{\text{CNTR}}$$

or,

$$\text{EQ. 41} \quad p = s_1 + t_{\text{EDGE}}$$

where s_1 is the sag of the front curve, s_2 is the sag of the back curve, t_{CNTR} is the center thickness, and t_{EDGE} is the edge thickness of the lens.

Table 10 and Table 11. All six of these lenses have been computed for a 1.500 index of refraction (CR-39) on a 70-mm blank size.

The plate height p of a lens can be found by adding the sag of the *back curve* to the *center* thickness of the lens, or by adding the sag of the *front curve* to the *edge* thickness:

$$\text{EQ. 40} \quad p = s_2 + t_{\text{CNTR}}$$

or,

$$\text{EQ. 41} \quad p = s_1 + t_{\text{EDGE}}$$

where s_1 is the sag of the front curve, s_2 is the sag of the back curve, t_{CNTR} is the center thickness, and t_{EDGE} is the edge thickness of the lens.

TABLE 10 +4.00 D lens

Front Curve (D)	Plate Height (mm)	Weight (g)	Edge Thick. (mm)	Center Thick. (mm)
9.00	13.4	20.4	1.0	6.6
7.00	10.2	18.8	1.0	6.2
5.00	6.9	17.9	1.0	6.0

TABLE 11 -4.00 D lens

Front Curve (D)	Plate Height (mm)	Weight (g)	Edge Thick. (mm)	Center Thick. (mm)
4.00	12.4	24.0	7.8	2.0
2.00	9.7	23.2	7.3	2.0
0.50	7.7	22.9	7.1	2.0

It should be apparent from the preceding tables that flattening a lens form will offer:

- Reduced plate height
- Reduced lens weight
- Reduced center thickness in plus lenses
- Reduced edge thickness in minus lenses

Although flatter lenses provide thinner and lighter lenses, there are many situations in which steeper lenses provide better vision for the wearer. Steeper base curves will often provide better vision when the person views through peripheral portions of the lens. Providing a correction *off-axis* that performs as well as the prescription *on-axis* is one of the primary motivations behind the selection of a particular lens form. This will be discussed further in Section 10. Table 12 shows the sag values for a range of surface power and lens diameter combinations.

TABLE 12 Sag values at various diameters

	Diameter (mm)					
	50	55	60	65	70	75
1.00 D	0.6	0.7	0.8	1.0	1.2	1.3
2.00 D	1.2	1.4	1.7	2.0	2.3	2.7
3.00 D	1.8	2.2	2.6	3.0	3.5	4.0
4.00 D	2.4	2.9	3.4	4.0	4.7	5.4
5.00 D	3.0	3.6	4.3	5.1	5.9	6.9
6.00 D	3.6	4.4	5.3	6.2	7.2	8.4
7.00 D	4.2	5.2	6.2	7.3	8.6	9.9
8.00 D	4.9	6.0	7.2	8.5	10.0	11.6
9.00 D	5.6	6.8	8.2	9.8	11.5	13.5
10.00 D	6.3	7.7	9.3	11.1	13.2	15.5

These values are based upon an arbitrary refractive index of 1.530. Tables like this can be used to determine the exact thickness of a lens. Notice how the values from the table increase non-uniformly (or non-linearly) as the surface power or diameter increases; the

sag of a curve actually increases faster than its surface power.

7.5 MEASURING SURFACE CURVATURE

Sag gauges and **lens measures** (or **lens clocks**) are both simple instruments that make use of this formula to calculate the surface power of a lens, based upon a sagittal measurement of its surface. Essentially, the *curvature* of the surface is found by measuring its sag at a given diameter. Then, by assuming an arbitrary index of refraction, the surface power can be determined. A refractive index of 1.530 is generally used for the calculation. A typical lens measure is shown in Figure 7:11. Sag gauges are similar devices that are generally larger. Some sag gauges may use a “bell” instead of three pins.

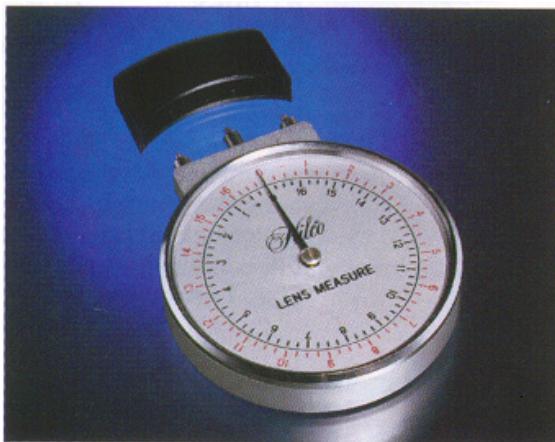


FIGURE 7:11 A typical lens measure.

A typical lens measure has two stationary outer pins and a moveable center pin. The outer pins serve as the *measuring aperture* of the device, allowing the center pin to measure the sagitta of a lens surface at a given diameter—as shown in Figure 7:12. The spring-loaded center pin, in turn, is geared to a pointer (like the hand of a clock), which indicates the dioptric value of the surface along a graduated scale encircling the face of the instrument (Jalie 61).

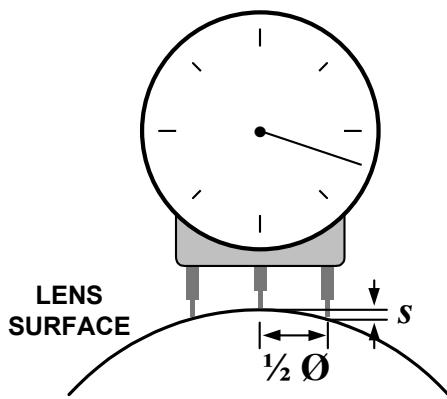


FIGURE 7:12 The device measures the sagitta s of a lens surface from the plane of its fixed diameter \emptyset , or measuring aperture.

Since the semi-diameter ($\frac{1}{2} \emptyset$) of a lens measure is relatively small, we can apply our simplified sag formula (Eq. 36) to relate the surface power F_s to the movement s of the center pin, which is the sag of the surface at the device’s diameter \emptyset :

$$s = \frac{\left(\frac{1}{2} \emptyset\right)^2 F_s}{2000(n-1)}$$

Now, if we assume that the separation of the outer pins of the lens measure is 20 mm ($\frac{1}{2} \emptyset = 10 \text{ mm}$) and then assign an arbitrary refractive index of 1.530, our formula simplifies to:

$$s = \frac{(10)^2 F_s}{2000(1.530-1)}$$

$$s = 0.094 \cdot F_s$$

Or,

$$F_s = \frac{s}{0.094}$$

This shows us that the surface power indicated by the lens measure is directly proportional to the movement s of the center pin of the device.

Example

A lens surface is measured with a lens measure and causes the center pin to rise a full 1.0 mm. What is its 1.530-based surface power?

$$F_s = \frac{1.0}{0.094}$$

$$F_s = 10.64$$

∴ 1.530-based surface power is 10.64 D.

Since surface power is directly proportional to the sagitta measured by the device, this lens measure will indicate 10.64 D of power for every 1.0 mm the pin is moved. If the pin is only moved a fourth of that distance—or 0.25 mm—the surface power indicated will be one-fourth of 10.64 D: $\frac{1}{4} \times 10.64 = 2.66 \text{ D}$.

It is important to note that the surface power indicated by the lens measure will only be accurate for lens materials with a 1.530 refractive index (or the index the device is calibrated to). For lens materials with a higher index of refraction, the surface power will actually be greater than the indicated power—and vice versa for materials with a lower index. It is quite simple, though,

to convert the measured surface power to the actual surface power, and this is discussed in Section 12.1.

Keep the following tips in mind when using these devices:

- The correct scale should be read when measuring concave versus convex surfaces. For instance, certain lens measures employ *black* letters on an outer scale for reading *convex* surfaces and *red* letters on an inner scale for reading *concave* surfaces. Other lens measures have two sets of numbers, going in opposite directions, and a sign (+ or -) to distinguish between convex and concave readings.
- Lens measures should be held perpendicularly to the lens surface when taking measurements. Tipping the pins with respect to the surface will produce inaccurate readings.
- Lens measures and sag gauges should be periodically checked for accuracy using either a perfectly flat surface or reference standard of known curvature.
- These are precision instruments, and should be handled delicately to avoid damaging the device, scratching the lens surface, or obtaining inaccurate results.

Lens measures can also be conveniently used to measure the amount of **slab-off** prism in a lens surface that has been *bi-centrally-ground* with two surfaces (see Section 12.4). **Bi-centric grinding** is generally done to correct vertical prism imbalance produced with multifocals during near vision, and is accomplished by grinding the same curvature twice—yet at different angles—on one surface of the lens. The tilt of one curve with respect to the other is what effectively produces the differential prismatic effect, or *slab-off*, between the distance and near portions of the lens. Because of the geometry of the lens measure and the relationship between surface tilt and prism, the amount of slab-off prism can be read directly from the lens measure (Jalie 154).

The amount of slab-off prism in a lens can be determined by taking a measurement of the curvature in the distance portion of the lens surface containing the slab-off, and comparing it to a measurement vertically straddled across the slab-off line. (The center pin of the lens measure should be positioned on the slab line.) The difference in curvature measurements between the two readings indicates the amount of slab-off prism present, in prism diopters. This is demonstrated in Figure 7:13.

For instance, consider a plastic lens with a slab-off on the rear surface. If the lens measure reads -4.00 D in the distance portion and -1.00 D across the slab line, the

amount of slab-off prism present is roughly 3^{Δ} (base up), or $4.00 - 1.00 = 3^{\Delta}$.

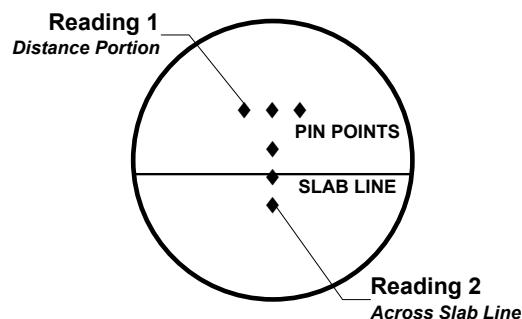


FIGURE 7:13 To determine the amount of slab-off with a lens measure, first take a reading in the distance portion of the lens surface containing the slab-off. Then take a reading across the slab-off line of the surface—ensuring that the center pin of the lens measure is positioned *on* the slab line. The difference between the two lens measure readings indicates the amount of slab-off prism present, in prism diopters.

8. Ophthalmic Prisms

Prism is often incorporated into spectacle lenses for several reasons: to compensate for anomalies of *binocular vision*, to decentre the *optical center*, to reduce thickness in *progressive lenses*, etcetera. We have already examined the refracting properties of *flat* prisms in detail. These prisms have plane sides, and do not change the vergence of incident light from a distant object. Most prisms utilized in conjunction with ophthalmic lenses, however, are *meniscus* in shape. These prisms have curved surfaces, and often possess focal power. This difference is demonstrated in Figure 8:1 (Wakefield 115).

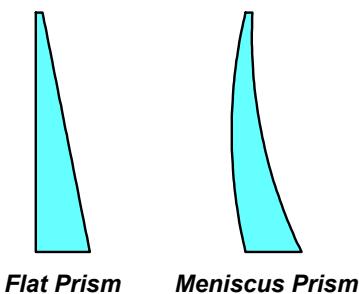


FIGURE 8:1 Flat and meniscus prisms.

This section will discuss the optics and applications of prisms incorporated into ophthalmic lenses.

8.1 BINOCULAR VISION

Prism is often incorporated into an optical prescription to treat disorders of binocular vision. Binocular vision refers to the ability to use the two eyes together when viewing an object. When the eyes are properly used together, the patient obtains better visual function and an improved sense of depth perception called **stereopsis**.

There are several *oculomotor imbalances* that can make comfortable binocular vision either difficult or impossible. When the visual axis of one eye converges—or tends to converge—*in* towards the visual axis of the other eye, the eyes exhibit an *eso* deviation. When the visual axis of one eye diverges—or tends to diverge—*out* from the visual axis of the other, the eyes exhibit an *exo* deviation. Both ‘*eso*’ and ‘*exo*’ deviations are illustrated in Figure 8:2.

Both *eso* and *exo* deviations designate a horizontal misalignment of the visual axes. There are also vertical deviations. When the visual axis of one eye rises—or tends to rise—*up* from the visual axis of the other, that eye exhibits a *hyper* (above) deviation. Conversely, the other eye with the lower visual axis exhibits a *hypo* deviation (below). Generally, we refer to the eye with the *hyper* deviation.

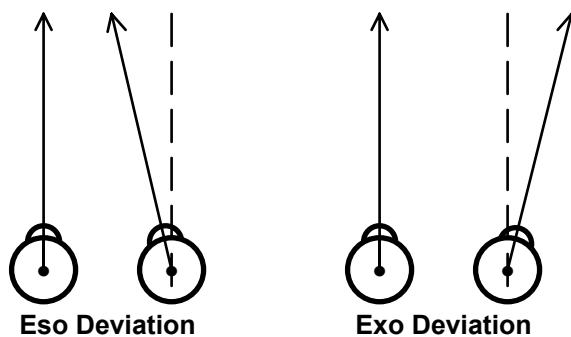


FIGURE 8:2 When the visual axis of one eye turns *in* relative to the other, the eyes exhibit an *eso* deviation. When the visual axis of one eye tends to turn *out* relative to the other, the eyes exhibit an *exo* deviation.

Some patients are unable to keep both eyes aligned and have a **heterotropia** (or **strabismus**). This is a *manifest* (obvious) deviation of the eyes. Two common types of heterotropia are **esotropia** (or *crossed eyes*) and **exotropia** (or *wall eyes*). Other patients are able to keep both eyes aligned, but only with excessive effort. This *latent* (concealed) deviation of the eyes is referred to as a **heterophoria**. Three types of heterophoria are: **esophoria**, which is when the eyes have a tendency to cross; **exophoria**, which is when the eyes have a tendency to diverge; and **vertical phoria**, which is when one eye has a tendency to view higher than the other. **Orthophoria** is the condition where no manifest or latent deviations exist between the two eyes.

Patients with an esophoria, exophoria, or vertical phoria problem can have symptoms such as eyestrain or headaches—especially when performing visually demanding tasks. These conditions also can result in double vision and discomfort associated with using the eyes.

In understanding prism prescriptions, it is important to keep in mind that the prism is usually prescribed in order to move the image of the world in the direction of the misalignment of the eyes. This makes it easier for the eyes to simultaneously view the object. For example, if the patient has an *eso* condition it means that the eyes cross or have a tendency to cross. To correct, or *relieve*, this condition the image seen by each eye should be moved towards the nose of the patient—thereby allowing the patient to more easily view the world with both eyes aligned on the image. Since images are displaced in the direction of the apex of the prism, in the case of the *eso* deviation the apex of the prism should be *in* or towards the nose and the base of the prism should be *out* or towards the edge of the face.

Similar reasoning leads to prescribing *base-in* prism for *exo* conditions in which the eyes tend to look outwards and the image is likewise displaced outwards. For vertical deviations, vertical prism is prescribed. For

example, if the right eye is misaligned such that it views higher than the left, then the patient has a *right hyper* deviation. Note that this is the same as saying the left eye views lower than the right, which is as a *left hypo* deviation. For this condition, prism is prescribed *base-down* in the right lens and/or *base-up* in the left lens. The prismatic relief of *eso* and *exo* deviations is illustrated in Figure 8:3.

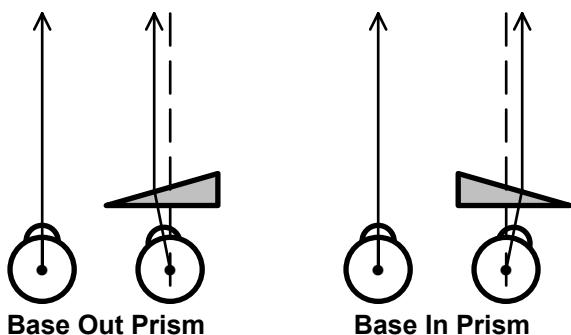


FIGURE 8:3 Prismatic relief of *eso* and *exo* deviations.

By convention, the ophthalmic industry specifies prism by the direction of the base. Therefore, for the *eso* condition discussed above, *base-out* prism is prescribed. Usually, but not always, the amount of prism will be split between both eyes. The net effect is fundamentally the same, but the increased thickness and magnification effects are divided more evenly between the two lenses.

The magnitude of horizontal deviations is usually greater than vertical deviations, the ability of the visual system to comfortably overcome horizontal deviations is also better. For these reasons, horizontal prism prescriptions are usually larger than vertical prescriptions. Related to this is the fact that patients are much less tolerant of unwanted vertical prism than unwanted horizontal prism.

8.2 OPTICAL CENTER

Earlier, we observed that the optical axis of a lens was an imaginary reference line passing through the centers of curvature of the front and back surfaces. Recall that the **vertices** (*V* and *V'*) are the positions on the lens where the optical axis intersects the front and back surfaces. Recall that the front and back surfaces are parallel at these points, as well (Section 4.2). We know that a ray of light striking a medium with *parallel* sides at an angle will leave the medium traveling at the same angle—although somewhat displaced. Consequently, if the lens is very thin, a ray striking the lens at one vertex will leave the lens from the other vertex without being deviated from its original path.

The vertices serve as the locations often marked as the **optical center** of the lens. The optical center of a lens is unique in that there is no refraction, or prismatic deviation, of the rays of light passing through it—or

along the optical axis. A ray crossing the optical axis at the optical center will exit the lens traveling in the same direction.

For *flat* lenses—such as bi-convex, bi-concave, plano-convex, and plano-concave lenses—the optical center lies within the actual lens. For *meniscus* lenses, however, the optical center may actually lie *outside* of the lens—along the optical axis. The vertex is still marked as the location of the optical center for practical purposes, though. Figure 8:4 illustrates how rays of light passing through the optical center *O* of an ‘infinitely thin’ lens emerge from the lens without being deviated. In reality, most lenses have at least some finite thickness. A ray of light passing through the optical center of a thick lens will be displaced slightly, but still exits the lens parallel to its original direction.

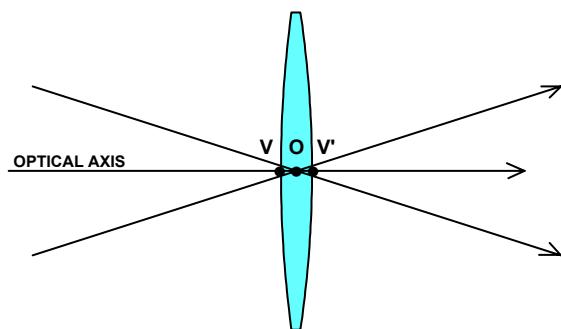


FIGURE 8:4 Optical center of a thin lens. Rays of light passing through the optical center of an infinitely ‘thin’ lens are not deviated from their original path. For thick lenses, rays passing through the optical center are slightly displaced but still exit the lens parallel to their original direction. The vertices *V* and *V'* are the locations on the lens surfaces intersected by the optical axis.

8.3 INDUCING PRISM

A prismatic effect can be produced by any one of the following three methods:

1. **Prism by grinding:** A prismatic effect can be produced in an ophthalmic lens by simply grinding a difference between the edge thicknesses across the *base-apex* meridian of the semi-finished lens blank. This essentially tilts the curves relative to each other, creating an *apical angle*. The required difference in thickness can be either calculated, or found with tables.

Grinding prism into the lens to move the optical center away from the geometric center of a semi-finished lens blank is routinely done by surfacing laboratories that utilize an *on-center blocking* technique.

2. **Prism by obliquity:** Tilting the optical axis of an ophthalmic lens away from the line of sight will also produce a slight prismatic effect, as perceived

by the wearer. This small amount of prism by *obliquity* will be discussed in Section 12.7. However, this method is not used to provide prescribed prism.

3. **Prism by decentration:** Ophthalmic lenses with power will produce a prismatic effect when the optical center is moved—or **decentered**—away from the line of sight. This results from the fact that the lens deviates light like an infinite series of prisms. This prismatic effect is identical in theory to prism produced by grinding, which produces prism by effectively moving the optical center of the lens. We will study this prismatic effect in more detail in the section below.

Recall that the *prism diopter* is defined as the number of *centimeters* of displacement over a given distance in *meters* (Section Prism). We can use the fact that parallel rays of light incident upon a lens will pass through its focal point to calculate the amount of prism produced at a given distance from the optical axis (or center). This means that a ray of light passing through the lens at a distance d from the optical axis, called the **decentration**, is prismatically deviated by the same distance over the focal length f' of the lens, as illustrated in Figure 8:5.

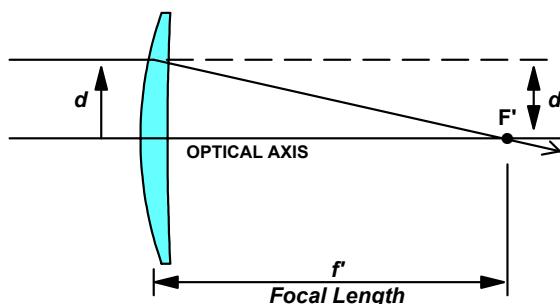


FIGURE 8:5 Derivation of Prentice's rule. A ray of light striking the lens at a distance d from the optical center has to be deviated that far at the secondary focal plane.

Therefore, the prismatic deviation Δ of the ray, in prism diopters, is equal to

$$\Delta = \frac{d}{f'}$$

when d is given in centimeters and f' is given in meters.

Substituting the quantity $1 / F$ for f' , we arrive at another useful formula often employed in ophthalmic optics, known as **Prentice's rule**, which should be committed to memory:

$$\text{EQ. 42 } \Delta = d \cdot F$$

Or, more simply, the prismatic effect Δ is equal to the product of the decentration d of the optical center away

from the line of sight multiplied by the focal power F of the lens.

As illustrated in Figure 8:6, *plus* lenses will have *positive* (+) values, and the base direction will be in the *same* direction of the decentration. *Minus* lenses will have *negative* (-) values, and the base direction will be in the *opposite* direction of the decentration.

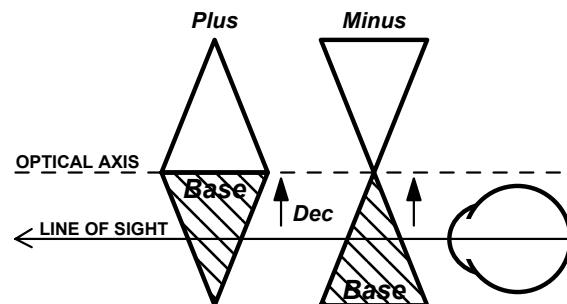


FIGURE 8:6 Prism base directions.

If a lens with cylinder is displaced along a principal meridian, the prismatic amount can be calculated by using the power in that meridian for the Prentice's rule calculation. If the lens is decentered along another meridian, then it becomes necessary to determine the amount of movement along the two principal meridians that would result in the same displacement of the lens along the non-principal meridian. This can be accomplished with vector analysis. The prismatic amount at that point on the lens is the sum of the two movements along the principal meridians. Fortunately, these calculations—which can be quite tedious—are easily performed by modern laboratory computer programs.

It should be kept in mind that Prentice's rule loses accuracy for both high-powered and extremely low-powered lenses.

Example

The optical center of +6.00 D lens is decentered *in* from the line of sight by 3 mm (0.3 cm). What is the prismatic effect?

$$\Delta = 0.3(6.00)$$

$$\Delta = 1.8$$

∴ Prismatic effect is 1.8^{Δ} base *in*.

8.4 PRISM THICKNESS

In Section 3.3, a prism was defined as a refracting element with non-parallel sides. Consequently, this means that anytime there is a difference in thickness between two given locations on a lens, there is also a prismatic effect produced between those two points. Conversely, a prismatic effect will produce a difference in thickness along the same meridian as the prism when measured at equal, but opposite distances, from the location of the prismatic effect. This meridian is referred to as the **base-apex line** of the prism. For instance, the thickness should be measured at the *top* and *bottom* of a lens exhibiting *vertical* prism.

With the assistance of Figure 8:7 and our familiar small angle approximations, we can see that the thickness difference t of a prism is roughly equal to

$$a = \frac{t}{\emptyset}$$

where a is the apical angle and \emptyset is the diameter of the prism, which is usually expressed in millimeters. Further, by combining the prism deviation equations (Eq. 9 and Eq. 10), we can also see that

$$a = \frac{\Delta}{100(n-1)}$$

where n is the index of the material and Δ is the prismatic deviation in prism diopters at the geometric center G of the blank.

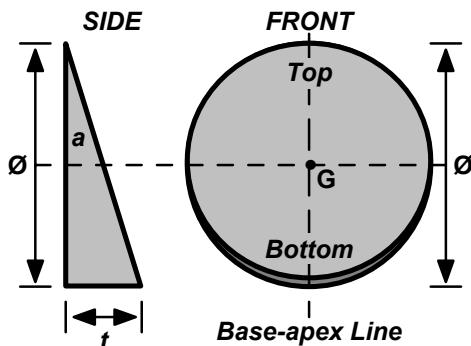


FIGURE 8:7 Thickness difference of a prism. Because this blank has base down (vertical) prism at the geometric center G , the thickness difference t is between the top and bottom of the lens blank (through the *base-apex line*).

After equating these two formulas for a , the thickness difference t in millimeters is given by the equation,

$$\text{EQ. 43} \quad t = \frac{\Delta \cdot \emptyset}{100(n-1)}$$

Similarly, if the edge thickness difference is known, the above equation can be solved for the amount of prism present at the geometric center.

When the optical center of a lens is decentered from the geometric center of the finished lens shape during the *edging* process, a prismatic effect occurs at the geometric center of the lens. This prism can be determined using Prentice's rule (Eq. 42), since the geometric center is now effectively *decentered* from the optical center. It is possible to calculate the *approximate* difference in edge thickness between the two edges of the lens along the meridian of decentration, which is the base-apex line of the prismatic effect (for spherical lenses). Although the sample principles apply, the calculation of prism and thickness for a spherocylindrical lens is more complex. The thickness difference Δt created by decentration (d) is illustrated in Figure 8:8 for a plus lens.

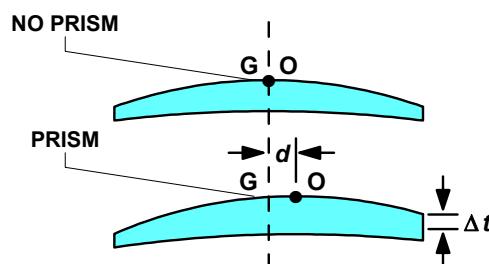


FIGURE 8:8 Because the optical center O of the bottom lens has been decentered from the geometric center G of the lens blank by a distance d , a thickness difference Δt is created between the left and right edges of the lens blank (through the *base-apex line*). The approximate amount of thickness difference can be determined if the prismatic effect at the geometric center G is known.

Example

A +5.00 D lens, made from high-index plastic ($n = 1.600$), is edged to a 45-mm lens diameter with 4 mm (0.4 cm) of decentration. What is the approximate edge thickness difference between the right and left sides along the meridian of decentration?

First, determine the prismatic effect, which is the effective decentration (or distance) of the geometric center relative to the optical center from:

$$\Delta = 0.4(5.00)$$

$$\Delta = 2.0$$

Then, determine the thickness difference along the prism base-apex line (i.e., meridian of decentration):

$$t = \frac{2.0(45)}{100(1.600-1)}$$

$$t = 1.5$$

∴ Thickness difference is 1.5 mm.

Example

A 50-mm lens without refractive power shows 2^Δ of base in prism at the geometric center. What is the edge thickness difference between the right and left sides?

$$t = \frac{2(50)}{100(1.500 - 1)}$$

$$t = 2.0$$

∴ Thickness difference is 2.0 mm.

Table 13 shows the thickness difference values for a range of prism and lens diameter combinations. These values are again based upon an arbitrary refractive index of 1.530. This table represents prisms with no thickness at the apex (*knife-edged*).

TABLE 13 Knife-edged prism thickness values

	Diameter (mm)					
	50	55	60	65	70	75
1.00 Δ	0.9	1.0	1.1	1.2	1.3	1.4
2.00 Δ	1.9	2.1	2.3	2.5	2.6	2.8
3.00 Δ	2.8	3.1	3.4	3.7	4.0	4.3
4.00 Δ	3.8	4.2	4.5	4.9	5.3	5.7
5.00 Δ	4.7	5.2	5.7	6.1	6.6	7.1
6.00 Δ	5.7	6.2	6.8	7.4	7.9	8.5
7.00 Δ	6.6	7.3	7.9	8.6	9.3	9.9
8.00 Δ	7.6	8.3	9.1	9.8	10.6	11.3
9.00 Δ	8.5	9.3	10.2	11.0	11.9	12.7
10.00 Δ	9.4	10.4	11.3	12.3	13.2	14.2

8.5 UNWANTED PRISM

Although any ophthalmic lens with power will produce a prismatic effect away from the optical center, an unprescribed prismatic effect is generally undesirable, and is referred to as **unwanted prism**. This is the primary reason behind decentering the optical center of the lens from the geometric center of the frame; to place it in front of the eye. The location on the lens where the desired prismatic effect, if any, should be verified is known as the **prism reference point**. In the absence of prescribed prism, the optical center of the lens should coincide with the prism reference point. *Unprescribed* prism at the prism reference point is generally unwanted.

Any net prismatic effect between the two eyes, or **prism imbalance**, can cause uncomfortable binocular vision, including head aches, eyestrain, etc. This occurs because the prism imbalance between the two lenses causes the images to separate. The more the images separate, the more difficult it becomes for the eyes to fuse them back together. In significant quantities,

diplopia—or double vision—can result. *Horizontal* prism imbalance in excess of $2/3^\Delta$, or *vertical* prism imbalance in excess of $1/3^\Delta$, is generally considered unacceptable.

For horizontal prisms, the net prism imbalance can be found by adding together *like* bases in the two lenses (e.g., base in and base in) and subtracting *unlike* bases. For *vertical* prisms the net prism imbalance can be found by adding *unlike* bases in the two lenses (e.g., base up and base down) and subtracting *like* bases.

Example

A pair of lenses has 2^Δ of base in prism in the right eye and 1^Δ of base out prism in the left eye. What is the net prism imbalance?

$$\Delta = 2 - 1 = 1$$

∴ Prism imbalance is 1^Δ .

The net imbalance is also base *in*, since the initial base in value was higher than the base out value.

8.6 PRISM-THINNING

There are certain situations, however, where it may be advantageous to produce a prismatic effect at the prism reference point. For example, grinding equal amounts of vertical prism (called **yoked prism**) in certain *progressive addition lenses* will help reduce their overall thickness. The geometry of a progressive lens produces a thickness difference between the top (distance zone) and bottom (near zone) edges of the lens blank. This requires a greater center thickness in order to provide the same minimum edge thickness in the near zone (at the bottom edge). Consequently, for plus-powered lenses—or lenses with a significant add power—a normal progressive lens blank will be thicker than conventional flat-top and single vision lens blanks of the same power.

This process of grinding vertical prism into both lenses, known as **prism-thinning** or **equi-thinning**, is acceptable, since the vertical prism is equal in each eye and no prismatic imbalance is created. Prism-thinning reduces the thickness differential between the upper and lower edges of the lens. It also reduces the overall thickness and weight of the lens—particularly in progressive lenses with high plus powers and/or add powers. This is illustrated in Figure 8.9. Base *down* prism is the most common base direction for prism-thinning, but base *up* prism may also be effective in some instances (Meister 21).

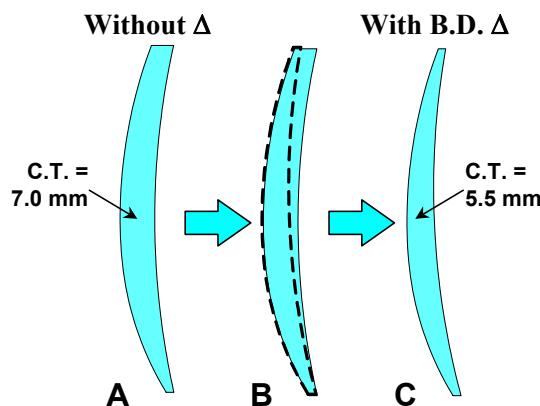


FIGURE 8:9 A progressive addition lens; +2.00 DS with a +3.00 D add power. A) This conventional progressive lens without prism-thinning has a center thickness of 7.0 mm. B) Base down prism can be ground into the lens to reduce the overall thickness. C) The same lens is only 5.5 mm thick at the center with prism-thinning. In addition, the top and bottom edge thicknesses are now relatively equal.

So, how much prism-thinning should be used? Ideally, the amount of prism-thinning used should be based upon the following factors:

- Distance power (in the vertical meridian)
- Add power
- Fitting cross height
- Fitting cross decentration
- Frame shape

For instance, lenses with higher plus powers in the distance portion require more prism-thinning. This is also true for progressive lenses with higher add powers. The fitting height needs to be considered to take into account the thickness difference produced by vertical decentration. A common rule-of-thumb formula provided in some progressive lens processing guides is:

$$\text{EQ. 44} \quad \text{Prism thinning} = 0.6 \times \text{Add}$$

This shows that a quantity of base down prism equal to roughly $2/3^{\text{rd}}$ of the add power should be used. This is often recommended when the power through the vertical meridian of the lens exceeds +1.50 D or so. This formula does not consider factors like the fitting height and the distance power, but still produces satisfactory results in most cases. For optimum results, sophisticated laboratory software can calculate the *exact* amount of prism-thinning required for a given frame and lens combination.

8.7 COMPOUNDING AND RESOLVING PRISMS

There are essentially two elements that are needed to specify a prismatic effect: the *magnitude* and the *orientation*. It is customary to specify the magnitude (or amount) of the prismatic effect in prism diopters ($^{\Delta}$),

and the orientation of the base of the prism. There are two popular systems of prism specification:

1. **Prescriber method:** This system, which is typically used by doctors who may incorporate prisms in their prescriptions, is based upon *rectangular coordinates*. Base directions, such as *base in*, *base out*, *base down*, and *base up*, are utilized in conjunction with the magnitude of prism required. For example, a prescription may call for 2^{Δ} *base down*. In order to attain an oblique prismatic effect, both horizontal and vertical directions are required (Brooks 7).
2. **Laboratory method:** This system, which is typically used by surfacing laboratories for fabrication purposes, is based upon *polar coordinates*. Using this system, the orientation of the base—or **prism axis**—is specified in conjunction with the desired amount of prism in that direction. The axis can be greater than or equal to 0 and less than 360° . Instead of specifying the desired prism as 2^{Δ} *base down*, it would be specified as 2^{Δ} at 270° . For an oblique prism, like 2^{Δ} *base up* and 2^{Δ} *base in*, the polar coordinate specification for a right lens would be 2.83^{Δ} at 45° . A modified style of this system uses only 180° , by noting the base direction as *down* for prism base axes between 180° and 360° .

A diagram demonstrating the notation of the two methods is provided in Figure 8:10. It is possible, using *vector analysis*, to convert between rectangular coordinates and polar coordinates. Since prism is often prescribed initially in a rectangular-coordinate format, it is usually necessary for the laboratory to convert to a polar-coordinate format. This can be accomplished by compounding the horizontal H_{Δ} and vertical V_{Δ} prism components into a single, **resultant prism** R_{Δ} . To determine the magnitude of the resultant prism, use the formula

$$\text{EQ. 45} \quad R_{\Delta} = \sqrt{H_{\Delta}^2 + V_{\Delta}^2}$$

To determine the prism axis, first calculate the initial reference angle θ of the prism—ignoring its sign (\pm). Next, determine the *quadrant* of the lens in which the base of the prism should be located using Figure 8:10. Finally, determine the actual axis of the prism by converting the reference angle θ into its correct quadrant using the following sign convention. To determine the initial reference angle θ , use the following formula (ignoring the sign of θ):

$$\text{EQ. 46} \quad \theta = \tan^{-1} \left(\frac{V_{\Delta}}{H_{\Delta}} \right)$$

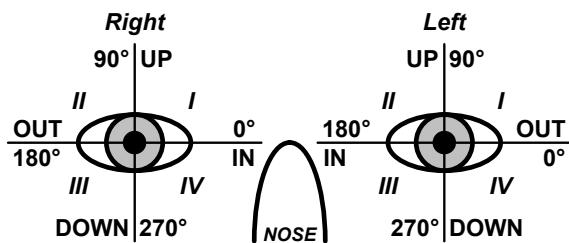


FIGURE 8:10 Rectangular and polar coordinates for the right and left eyes (looking at the wearer).

QUADRANT SIGN CONVENTION

I Quadrant: Base direction = θ

II Quadrant: Base direction = $180 - \theta$

III Quadrant: Base direction = $180 + \theta$

IV Quadrant: Base direction = $360 - \theta$

Example

A prescription written using the prescriber method calls for a right lens with 2^Δ base up and 2^Δ base in. What is this prism prescription in polar coordinates (using the laboratory method)?

$$R_\Delta = \sqrt{2^2 + 2^2}$$

$$R_\Delta = 2.83$$

$$\theta = \tan^{-1}\left(\frac{2}{2}\right)$$

$$\theta = 45^\circ$$

And, because the base of the prism is located *up* and *in* for the *right* eye, the orientation of the prism falls into the *first* quadrant. This means that the final axis of the base of the prism is equal to θ .

\therefore Prism is 2.83^Δ at 45° .

Sometimes it is necessary to *resolve* a single resultant prism into two separate vertical and horizontal components. This is the reverse of the previous procedure used to *compound* two horizontal and vertical components into a single resultant prism. Given a resultant prism R_Δ with an axis θ , the horizontal H_Δ and vertical V_Δ prism components can be found with these two equations:

$$\text{EQ. 47} \quad H_\Delta = R_\Delta \cdot \cos\theta$$

$$\text{EQ. 48} \quad V_\Delta = R_\Delta \cdot \sin\theta$$

The same sign convention from Figure 8:10 is used to determine the final directions of the horizontal and vertical components from the initial prism axis. The two examples used here to convert between polar

(laboratory) coordinates and rectangular (prescriber) coordinates are illustrated in Figure 8:11.

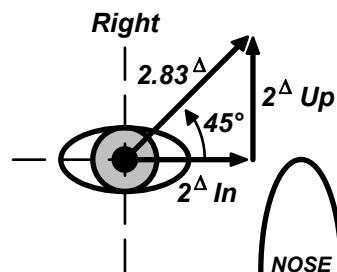


FIGURE 8:11 Converting prism coordinates for a *right* lens with 2^Δ *base in* and 2^Δ *base up*, or 2.83^Δ at 45° . For rectangular coordinates, *base in* always represents the *nasal* side of the lens, while *base out* represents the *temporal*.

When trying to resolve more than one resultant prism, or when trying to compound oblique prism components (i.e., not horizontal or vertical), the easiest approach is to simply resolve all of the prisms into horizontal and vertical components. Then add all of the horizontal prism components and, separately, all of the vertical prism components. If desired, the final horizontal and vertical components can also be compounded back into a single resultant prism.

Example

A prescription written using the laboratory method calls for a right lens with 2.83^Δ at 45° . What is this prism prescription in rectangular coordinates (using the prescriber method)?

$$H_\Delta = 2.83(\cos 45)$$

$$H_\Delta = 2.00$$

$$V_\Delta = 2.83(\sin 45)$$

$$V_\Delta = 2.00$$

Ignore the sign of these components; just use their positive values. We will determine their directions using our previous sign convention. Because the base of the resultant prism is located in the *first* quadrant of the right eye, the prism base directions should be *base in* for the horizontal component and *base up* for the vertical component. This yields

\therefore Prism is 2^Δ *base up* and 2^Δ *base in*.

Note that our result is completely consistent with the previous example. It is also interesting to note that many inexpensive scientific calculators have the capability to quickly convert between polar coordinates and rectangular coordinates.

With the use of a **focimeter**, which is the commonly used telescope-like device that measures both focal power and prism, the prismatic effect—if any—

produced at a given point through a lens can be quickly measured. The **target** of a manual focimeter is the cross-hairs image being observed through the test lens, which is superimposed upon a measuring scale consisting of concentric circles, called a **reticle**. The target and reticle of manual focimeters display both the magnitude and the orientation of any prismatic effects, as shown in Figure 8:12.

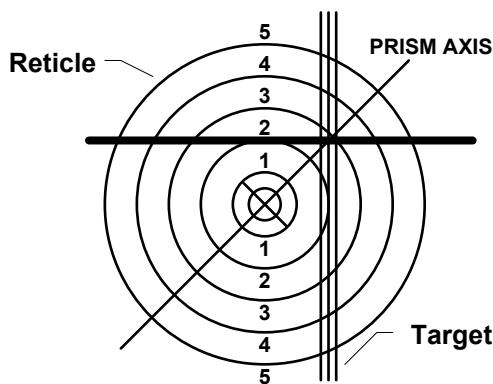


FIGURE 8:12 Prism in a focimeter. This prismatic effect can be described as 2.83^Δ at 45° using the polar coordinate, *laboratory* method. Or, if this was a right lens, 2^Δ base up and 2^Δ base in using the rectangular coordinate, *prescriber* method.

Example

You are given a semi-finished lens blank that is to be ground to a -4.00 D using on-center blocking. It is a *right* lens, and by grinding prism you wish to move the optical center *in* 4 mm (0.4 cm) from the geometric center and *down* 2 mm (0.2 cm). How much prism is required to do this (using the laboratory method)?

First, calculate the amount of vertical prism required to move a -4.00 D lens 0.2 cm *down*:

$$\Delta = 0.2(-4.00)$$

$$\Delta = -0.8$$

The negative (-) sign means that the prism base needs to be in the opposite direction of the downward movement. Hence, 0.8^Δ base *up*. Second, calculate the amount of horizontal prism required to move a -4.00 D lens 0.4 cm *in*:

$$\Delta = 0.4(-4.00)$$

$$\Delta = -1.6$$

The negative (-) sign once again indicates that the base needs to be in the opposite direction of the inward movement. Hence, 1.6^Δ base *out*.

Finally, the resultant prism and axis needs to be determined using the laboratory method. The magnitude of the prism is:

$$R_\Delta = \sqrt{1.6^2 + 0.8^2}$$

$$R_\Delta = 1.79$$

The prism base is up and out for the right eye. Using our sign convention, the prism axis lies in the *second* quadrant. This means that the final axis is equal to $180^\circ - \theta$.

$$\theta = \tan^{-1}\left(\frac{2}{4}\right)$$

$$\theta = 27^\circ$$

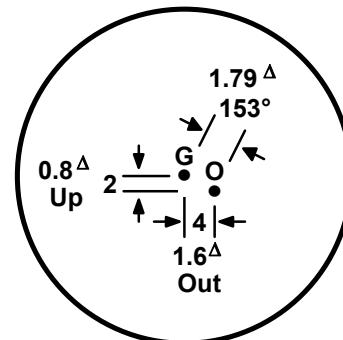
(Even if the angle had been negative, we still would have used the positive value in our system.) Finally, correcting for the *second* quadrant gives us:

$$\theta = 180^\circ - 27^\circ$$

$$\theta = 153^\circ$$

\therefore Prism required is 1.79^Δ at 153° .

These prism components, as well as the resultant movement of the optical center O from the geometric center G, are illustrated in Figure 8:13.



Right, Semi-Finished Lens Blank

FIGURE 8:13 To move the optical center of this -4.00 D lens 2 mm *down* and 4 mm *in*, 1.79^Δ at 153° is required. This is equivalent to 0.8^Δ base *up* and 1.6^Δ base *out*.

8.8 SPLITTING PRISMS

Except in some rare instances, prism is prescribed to induce *disjunctive* (or *vergence*) movements between the eyes. A *disjunctive* movement occurs when the eyes move in opposite directions. Prism that produces a *disjunctive* movement causes the images produced by the right and left lenses to separate, which—in turn—requires that the visual axes (or *lines of sight*) separate to maintain fixation on each image. Conversely, if the visual axes have a tendency to separate, because of an anomaly of binocular vision (i.e., tropia or phoria), prism can be prescribed to separate the two images and align each with its corresponding line of sight (i.e., right or left). This relieves the oculomotor stress created

while the eyes try to maintain fixation on an object when one eye has a tendency to deviate from the other. Prism prescribed in this manner actually allows the visual axes to move away from each other enough to maintain single binocular vision and/or relieve the oculomotor stress.

Base *in* prism in *each* eye or base *out* prism *each* eye will cause a horizontal disjunctive movement. Base *up* prism in one eye and base *down* prism in the other eye, or vice versa, will cause vertical disjunctive movements. On the other hand, base *down* prism in both eyes or base *up* prism in both eyes will cause *conjunctive* (or *version*) movements in which the visual axes move together in the same direction.

Large amounts of prism add appreciable thickness and weight to a lens. Moreover, because of certain imaging *aberrations* caused by prism, the optical performance of a lens with excess prism is also reduced. Consequently, if a refractionist prescribes a large amount of prism for only one eye, it is customary to split the prism equally between both lenses—unless otherwise specified.* This practice allows the thickness and weight to be shared evenly between both lenses, which improves the cosmetics of the eyewear. It also minimizes the aberrations caused by prism in each lens.

To split prescribed prism, follow the guidelines, below:

Horizontal prism: Split the amount of prism in half. The initial base direction (i.e., *out* or *in*) will be used for *both* eyes.

Vertical prism: Split the prism in half. The initial base direction (i.e., *up* or *down*) will be used for the eye it was originally prescribed for, and the opposite base direction will be used for the other eye.

Example

A prescription is written for 4^{Δ} base up and 2^{Δ} out down in the right eye lens. How would the prism be split equally between both lenses?

$$\text{Vertical } \Delta = 4 \div 2 = 2$$

$$\text{Horizontal } \Delta = 2 \div 2 = 1$$

The base direction for the horizontal prism is *out*, so both lenses will have base *out* prism. The base direction for the vertical prism is *up*, so the *right* eye will have base *up* prism and the *left* eye will have base *down* prism (opposite to the original eye).

∴ Prism required is 2^{Δ} base up and 1^{Δ} base out in the right eye, and 2^{Δ} base down and 1^{Δ} base out in the left eye.

* Certain conditions, such as *non-concomitant strabismus*, may necessitate different amounts of prism for each eye.

9. Spectacle Frames and Fitting

The general optics of spectacle lenses has already been discussed. These next sections will introduce the basics of spectacle frames, frame measurements, and the principles of spectacle lens fitting. For a more thorough discussion of these topics, you should review any of the dispensing references in the bibliography.

Ophthalmic lenses are held in place before the eyes by means of a **frame or mounting**, as described in Section 13. Finished eyewear includes both the lenses and the frame. The entire process of fitting and dispensing eyewear is beyond the scope of this workbook, however some of the key principles are presented.

9.1 SPECTACLE FRAMES

The primary purpose of the spectacle frame, or mounting, is to comfortably support and accurately position the spectacle lenses in front of the eyes. To accomplish this task, frames must be durable and properly fit. Figure 9:1 and Figure 9:2 show samples of frames. Key frame components can be seen in these pictures.

- **Rims/eyewires:** The parts of the frame front that surround and support the lenses, either partially (e.g. *rimless* frame) or entirely.
- **Bridge piece:** The part of the frame front that connects the two rims together, and is supported by the bridge of the nose.
- **Temples:** Hinged extensions connected to the frame front that extend to the ears for support, ending in **earpieces** that contour around the ears.

Like lenses, frames should be durable, lightweight/comfortable, and attractive. They must also retain their shape and adjustment, and be relatively hypoallergenic—since they are in contact with the skin. Frame materials are chosen based upon those criteria. Frames can be made from a variety of materials that are often categorized into two broad groups: **plastics** and **metals**.

Plastics include materials like *cellulose acetate* and *co-polyamide*. *Metal* frames are often constructed from various alloys, such as *Monel* (copper-nickel), that are designed to provide high tensile strength, corrosion resistance, and so on. Finer metal frames will often be plated with a layer of gold, as well. Combination frames are also available that include components made from both plastics and metal. Figure 9:1 and Figure 9:2 are pictures of frames made from plastics and metal (Obstfeld 78).



FIGURE 9:1 Plastics frame.



FIGURE 9:2 Metal frame.

Ideally, spectacle frames will only contact the wearer at three points. Collectively these points are known as the **fitting triangle**: the top of the right and left ears and the crest of the nose. These three contact points, as illustrated in Figure 9:3, bear the weight of the eyeglasses. The weight supported by each of the 3 points will depend upon many factors, including the mass of the frame, the type of bridge piece, the style of temples, the weight of the lenses, and how well the frame is fit (Stimson 200).

Frames are also chosen by the wearer on the basis of shape, color, and other aesthetic qualities. The shape and width of the frame plays an important role in the size, thickness, and positioning of the spectacle lenses with which it will be glazed.

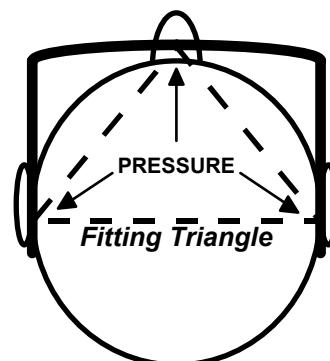


FIGURE 9:3 Spectacle frame fitting triangle.

The dimensions of the spectacle frame and lens are based upon a standardized system of measurement called the **Boxing System**, which is described in Table 1 and illustrated in Figure 9:4 and Figure 9:5 (ANSI 5).

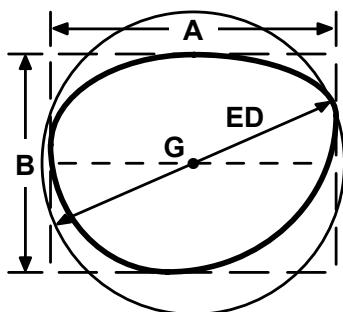


FIGURE 9:4 The Boxing System. The parameters are described in Table 11.

TABLE 14 Boxing System frame and lens dimensions

A	Eye size. The <i>horizontal</i> width between the two vertical lines tangent to the edges of the lens—or a box enclosing the lens.
B	The <i>vertical</i> height between the two horizontal lines tangent to the edges of the lens—or a box enclosing the lens.
G	The geometric center of the imaginary box enclosing the edges of the finished lens or aperture. ($\frac{1}{2}B$ and $\frac{1}{2}A$ measurements.)
DBC	Distance between centers. The separation between the geometric centers of the right and left lens apertures.
DBL	Bridge size. The minimum distance between the two lenses or apertures.
ED	Effective Diameter. Twice the longest radius from the geometric center of lens to the apex of the edge; the smallest circle that will completely enclose the lens.

The separation between the geometric centers of the lenses, or the lens apertures of the frame, is known as the **distance between centers** DBC. The DBC of a frame is illustrated in Figure 9:5. This distance is equal to the sum of the eye size (or, 2 times half of the eye size) plus the bridge size,

$$\text{EQ. 49} \quad \text{DBC} = A + \text{DBL}$$

where DBC is the distance between the geometric centers of the frame apertures, A is the eyesize (or A-measurement) of the frame, and DBL is the bridge size (or distance between lenses) is the bridge size in millimeters.

Most spectacle frames will have at least the eye size A, bridge size DBL, and temple length measurements marked upon the frame; usually on the back of the bridge piece or on the inside of a temple.

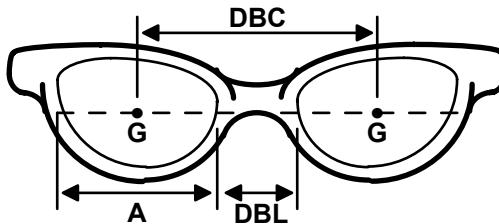


FIGURE 9:5 DBC measurement. Line DBC is the distance between the geometric centers of the right and left lenses, or lens apertures, and is equal to the sum of $A + \text{DBL}$.

9.2 LENS CENTRATION

Spectacle lenses intended for general use should be positioned in the frame to place the optical centers directly in front of the *lines of sight* (or *visual axes*) of the eyes. This prevents the wearer from encountering unwanted prism—caused by looking away from the optical centers—while staring straight ahead in **primary gaze**. If prism has been prescribed, the *prism reference points* should be directly in front of the lines of sight. The separation between the lines of sight, while fixating a distant object, is known as the **binocular interpupillary distance** (PD).

Although we often think of the interpupillary distance as the distance between the pupil centers, this is not entirely accurate. When fixating an object, the eye rotates so as to place the image upon the most sensitive area of the retina. This area, known as the **macula**, is offset temporally from the center of the back of the eye. Consequently, the *optical axis* of the eye and the pupil rotate out roughly 5° , to align the *visual axis* of the eye, which passes through the nodal point N to the macula M, with the object of fixation. This is illustrated in Figure 9:6 (Stimson 136).

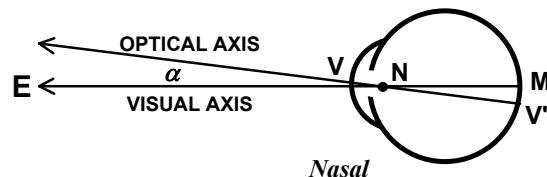


FIGURE 9:6 Top view of the right eye. The eye has to rotate out by the angle α , which is roughly 5° , to align the *macula* at point M with the object of fixation E.

The visual axis of the eye is not entirely coincident with the optical axis (which passes through the center of the optics of the eye). To measure the separation between the visual axes, corneal reflections can be used. These reflections can be measured with a penlight and a ruler, a **corneal reflection pupillometer**, or a similar device.

For precision measurements, the *monocular* interpupillary distances (PD_{EYE}) are often used since the face is seldom completely symmetrical. The monocular interpupillary distance is the distance from the center of

the bridge of the nose to the line of sight of each eye, individually. When separate monocular interpupillary distance measurements are not given, the monocular PD_{EYE} measurements are assumed to be half of the total PD measurement (PD_{EYE} = $\frac{1}{2}$ PD). Both the *binocular* and *monocular* interpupillary distance measurements are illustrated in Figure 9:7 (Obstfeld 223).

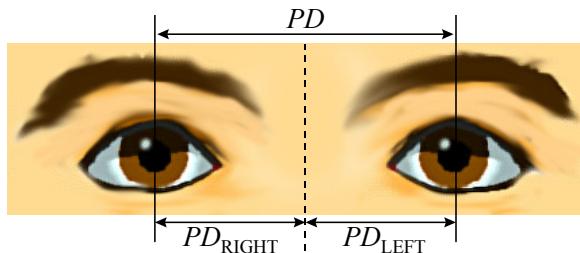


FIGURE 9:7 Monocular and binocular interpupillary distance measurements.

Generally, the DBC distance is wider than the interpupillary distance, which means that the optical center of each lens will have to be moved from the geometric center of the frame in order to position it in front of the line of sight. The process of moving the optical center of the lens from the geometric center of the lens aperture of the frame is known as **decentration**. This is illustrated in Figure 9:8.

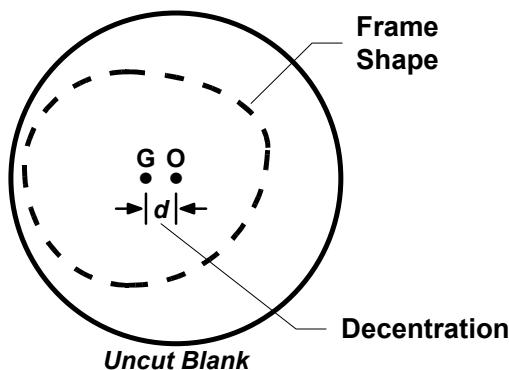


FIGURE 9:8 The decentration d is the movement of the optical center O of the lens blank from the geometric center G of the finished, edged lens.

For each eye, the decentration required to place the optical center in front of the line of sight of each eye is given by

$$\text{EQ. 50} \quad \text{dec}_{EYE} = \frac{\text{DBC}}{2} - \text{PD}_{EYE}$$

For *positive* (+) values of the decentration, the optical center should be decentered in (nasally) from the geometric center of the frame aperture. For *negative* (-) values, the optical center should be decentered out from the geometric center.

Example

A frame has an eye size of 54 mm, a bridge size of 16 mm, and is fit on a patient with a 64-mm PD. What is the required decentration (for each eye)?

$$\text{dec} = \frac{54 + 16}{2} - \frac{64}{2}$$

$$\text{dec} = 35 - 32$$

$$\text{dec} = 3$$

∴ Decentration is 3 mm per each eye.

Once the decentration of the lenses has been computed, it will be used to **layout**, or position, the optical centers of the lenses during the blocking process. This ensures that they will be positioned correctly within the frame, as illustrated in Figure 9:9. It is important to note that the **prism reference point**, which is the point on the lens that satisfies any prescribed prism requirements, is the actual point on the lens that is decentered during the layout process. In the absence of any prescribed prism, the prism reference point is coincident with the optical center of the lens. We will continue to use the term *optical center* for simplicity.

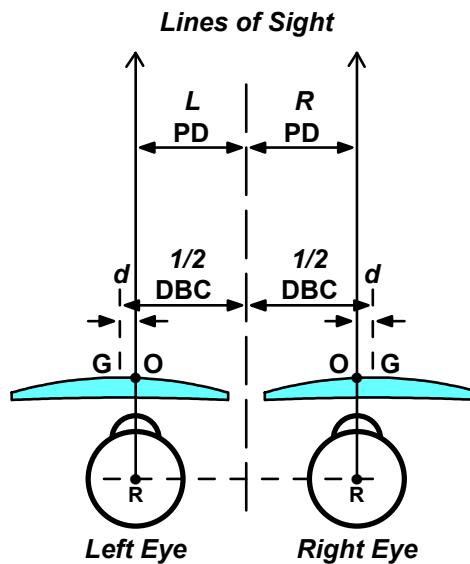


FIGURE 9:9 To avoid inducing unwanted prism, the optical center of the lens at point O is centered in front of the visual axis. It is decentered from the geometric center at point G, by the decentration d .

At this point, it is important to note that the term ‘decentration’ has two distinct meanings, depending upon the context (Brooks & Borish 543):

- Moving the optical center—or prism reference point—of a lens from the geometric center of the lens aperture of the frame.

- Moving the optical center away from the line of sight (Section 8.3), which induces a prismatic effect. This can be either wanted or unwanted.

So, what happens when spectacle lenses are *not* decentred properly? The effects can be readily understood if we refer back to Prentice's rule (Eq. 42). In Figure 9:10, two minus lenses have not been decentred *in* adequately. Consequently, the visual axes intersect the lenses *in* from the optical centers. At these points, the lenses produce a *base in* prismatic effect that causes the eyes to diverge out slightly in order to compensate for it.

For instance, a 2-mm centration error (*out*) per eye for a pair of -4.00 D lenses would produce an error of $0.2 (-4.00) = 0.8^\Delta$ base *in* for each eye. This induces a combined prism imbalance of 1.6^Δ base *in*.

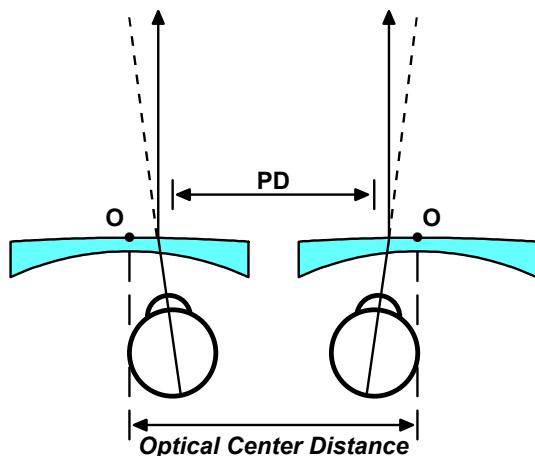


FIGURE 9:10 Incorrect lens centration results in unwanted prism. In the example above, these *minus* lenses have produced a *base in* prismatic effect as consequence of inadequate decentration. This *base in* prism causes the eyes to diverge.

Although the lines of sight are parallel with each other for distant vision, they *converge* together when fixating an object at near. The separation between the lines of sight (measured in the plane of the spectacle lenses) when they are converged to fixate a near object is referred to as the **near interpupillary distance NPD**. The near interpupillary distance, as the eyes converge to fixate an object at O, is illustrated in Figure 9:11. Using the similar triangles, we can show that the *monocular* near interpupillary distance NPD_{EYE} , in millimeters, is equal to

$$\text{EQ. 51} \quad NPD_{EYE} = \left(\frac{10w}{10w+h} \right) PD_{EYE}$$

where w is the patient's working distance in centimeters, PD_{EYE} is the monocular interpupillary distance in millimeters, and h is the distance in

millimeters from the center of rotation to the spectacle plane.

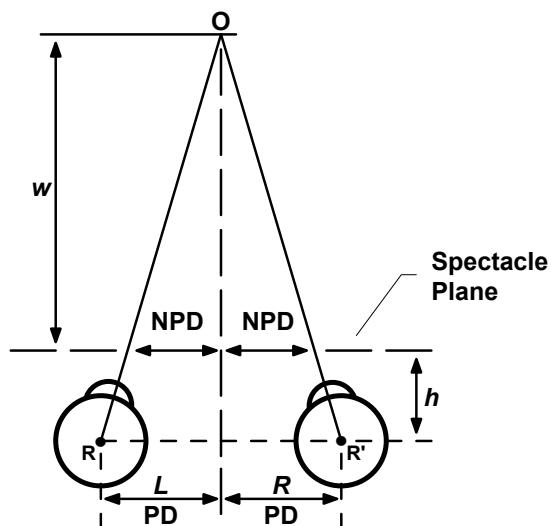


FIGURE 9:11 The *monocular* near interpupillary distance NPD for each eye, as the eyes converge to fixate an object at point O, can be found using similar triangles. NPD is proportional to PD, as w is proportional to $w + h$.

Table 15 below shows the calculated monocular NPD_{EYE} values for various monocular PD_{EYE} values and working distances. It is often simpler to use such a table, rather than to manually calculate the NPD.

TABLE 15 Monocular near interpupillary distances

PD (mm)	Working Distance (cm)					
	100	50	40	33	25	20
28	27.3	26.6	26.2	25.9	25.3	24.7
29	28.2	27.5	27.2	26.8	26.2	25.6
30	29.2	28.5	28.1	27.7	27.1	26.4
31	30.2	29.4	29.0	28.7	28.0	27.3
32	31.2	30.4	30.0	29.6	28.9	28.2
33	32.1	31.3	30.9	30.5	29.8	29.1
34	33.1	32.3	31.9	31.4	30.7	30.0
35	34.1	33.2	32.8	32.4	31.6	30.8

However, because of prismatic effects, the distance power of the lens will also affect the patient's near interpupillary distance. *Minus* lenses require slightly *less* decentration, and *plus* lenses require slightly *more*. This table has been calculated with a distance h of 27 mm. For longer values (i.e., longer vertex distances) the required decentration increases, and vice versa for shorter distances.

The difference between the PD_{EYE} and NPD_{EYE} for each lens is known as the **inset**, and is important when fabricating multifocals. Multifocal lenses have an additional segment for near viewing, which must be centered relative to the lines of sight as they converge for near vision. Lens inset is illustrated in Figure 9:12.

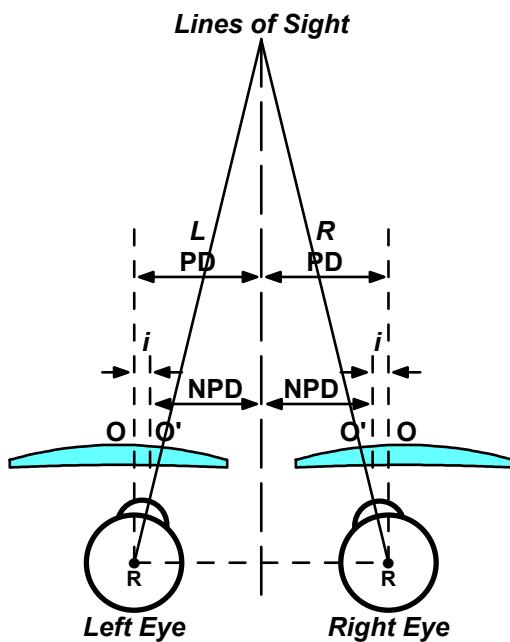


FIGURE 9:12 To position the segment in order to allow for the convergence of the eyes, the optical center of the segment O' is centered in front of the visual axis. It is decentred in from the distance optical center O, by the amount of inset i .

For each eye, the inset is simply equal to the difference between the monocular interpupillary distance PD_{EYE} and the monocular near interpupillary distance NPD_{EYE} , so that

$$\text{EQ. 52} \quad \text{inset}_{EYE} = PD_{EYE} - NPD_{EYE}$$

where PD_{EYE} and NPD_{EYE} are the *monocular* distance and near interpupillary distances for either the right or left eye.

For multifocals, a separate vertical measurement is also required to place the segment at the desired location below the lines of sight. The **segment height** is measured from a horizontal plane, tangent to the bottom of the lens, to the top edge of the segment.

The objective is to place the segment low enough as to not interfere with distance vision, yet high enough to allow the wearer to reach it comfortably when the lines of sight are lowered for near vision. For most bifocal lenses, the top edge of the segment should be placed at about the level of the lower eyelid when viewing straight ahead and level with the patient's eyes. For most trifocals, the top edge of the segment should be placed between the lower pupil edge and the lower limbus. The placements of flat-top bifocal and trifocal lenses are demonstrated in Figure 9:13.

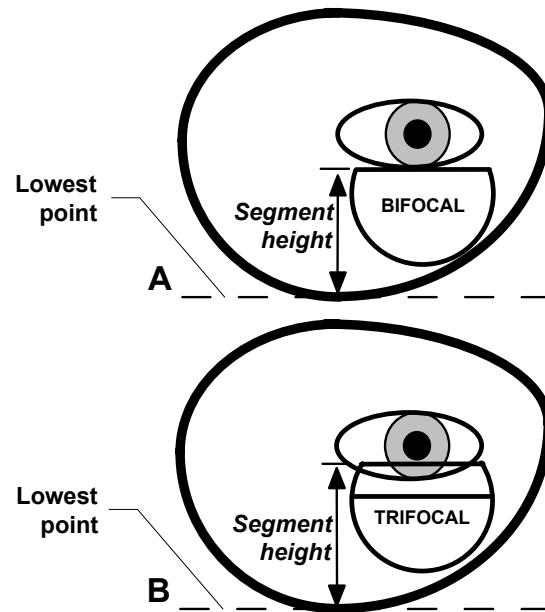


FIGURE 9:13 Placement of multifocal segments. A) Flat-top bifocals are generally placed at the lower lid or limbus. B) Trifocals are generally placed between the lower edge of the pupil and the lower limbus.

The segment height should always be measured from the bottom edge of the lens, or the lowest point in the rim of the frame. Since the bottom edge of the lens rests in the bevel of the frame, which can not be seen when looking straight at the wearer, it becomes necessary to consider this when making the measurement.

The vertical positioning of the segment within the frame aperture is determined by the **segment drop**, which is simply the vertical displacement of the segment (from the geometric center of the frame) required to achieve the desired segment height:

$$\text{EQ. 53} \quad \text{seg drop} = \frac{B}{2} - \text{seg height}$$

where B is the vertical depth of the frame. For *positive* (+) values of the segment drop, the segment will be lowered from the geometric center of the frame aperture. For *negative* (-) values, the segment will be raised from the geometric center.

The layout and positioning of a typical multifocal segment—including the decentration of the optical center O, the segment inset i , segment height s , and seg drop y —is summarized in Figure 9:14. Most of these measurements are made relative to the geometric center G of the finished lens (or the frame aperture).

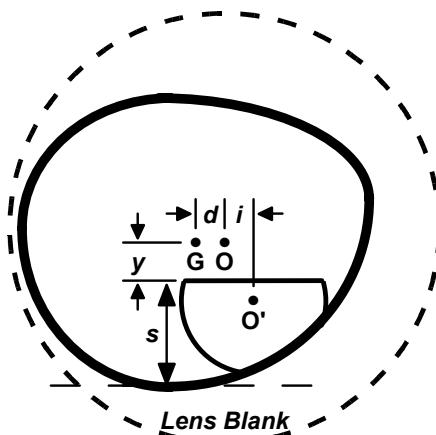


FIGURE 9:14 The distance s represents the *height* of the top edge of the segment from the bottom edge of the lens, or the lowest point inside the rim of the frame. The distance d represents the horizontal *decentration* of the distance optical center O from the geometric center G of the frame aperture. The distance i represents the horizontal *inset* of the segment optical center O' from the distance optical center O . The distance y represents the vertical *drop* of the top of the segment from either the geometric center of the frame aperture G , or the distance optical center O .

The placement of a progressive addition lens is determined by the location of its **fitting cross**, which is typically placed directly in front of the center of the pupil, as illustrated in Figure 9:15. This is also measured from the bottom edge of the lens.

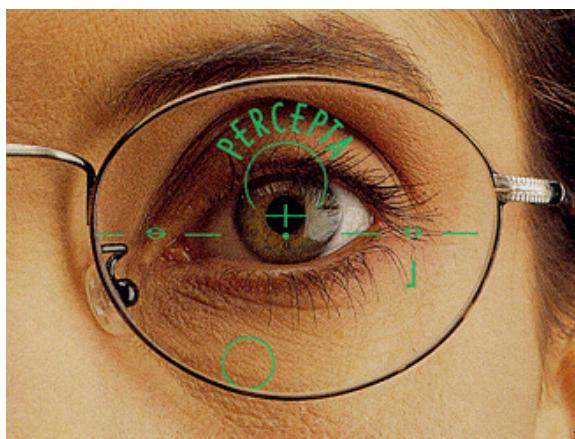


FIGURE 9:15 Fitting progressive addition lenses. The fitting cross should be centered directly in front of the pupil center with the eye in its primary gaze.

Because the zones of usable vision afforded by progressive addition lenses are often smaller than the zones of conventional multifocals, progressive lenses also require more careful fitting. Small errors in the placement of the fitting crosses can result in considerable losses in the field of view and wearer dissatisfaction.

Some additional fitting factors to consider include:

- Fit the frame to the patient before taking any measurements.
- Always take *monocular* interpupillary distance measurements. Most manufacturers recommend using the *distance* PD measurements, though some may suggest using the *near* PD measurements. If the *near* PD measurements are used, the insets of the lenses are added to them to arrive at the *distance* PD measurements. Most new progressive lens designs, however, use accurately computed variable sets, which supersede the need for this practice.
- Maintain a short vertex distance to maximize the fields of view through the near and intermediate zones. Eight to 14 mm is an acceptable range.
- Ensure that the frame has a short vertex distance, and at least 10 to 14° of *pantoscopic* tilt, to maximize the fields of view.
- Some face form wrap will also be beneficial.
- Measure the segment height to the center of the pupil as the patient's eye is in primary gaze.
- Select a frame with a minimum segment height of 22 mm for most lens designs.

For edging layout, the entire horizontal movement of the segment center from the geometric center of the frame aperture needs to be computed. The **total inset** of the segment of a lens is the sum of the decentration of the distance optical center (or the prism reference point) and the inset of the segment, so that $\text{total inset}_{EYE} = \text{dec}_{EYE} + \text{inset}_{EYE}$. This is the actual distance that the segment is moved from the geometric center G of the frame aperture:

$$\text{EQ. 54} \quad \text{total inset}_{EYE} = \text{dec}_{EYE} + \text{inset}_{EYE}$$

Total inset can also be expressed as,

$$\text{EQ. 55} \quad \text{total inset}_{EYE} = \frac{\text{DBC}}{2} - \text{NPD}_{EYE}$$

Example

You are given the following eyewear specifications; a 32-mm DPD for each eye (OU), a 30-mm NPD for each eye, a measured segment height of 18 mm, a 54-mm frame eye size, a 50-mm frame B measurement, and a 16-mm bridge. What are the measurements for lens layout?

$$\text{seg drop} = \frac{50}{2} - 18$$

$$\text{seg drop} = 7$$

$$\text{DBC} = 54 + 16$$

$$DBC = 70$$

$$\text{dec}_{R\&L} = \frac{70}{2} - 32$$

$$\text{dec}_{R\&L} = 3$$

$$\text{inset}_{R\&L} = 32 - 30$$

$$\text{inset}_{R\&L} = 2$$

$$\text{total inset}_{R\&L} = 35 - 30$$

$$\text{total inset}_{R\&L} = 5$$

∴ Final layout measurements are:

Segment drop: 7 mm down per eye.

Decentration: 3 mm *in* per eye.

Segment inset: 2 mm *in* per eye.

Total inset: 5 mm *in* per eye.

9.3 PANTOSCOPIC TILT

Other fitting considerations are needed to ensure maximum performance from the spectacle lenses. As with most fitting parameters, they are most critical for higher-powered lenses. The **vertex distance** v is the separation between the back vertex V' of the lens and the apex of the cornea. For high powers, differences between the *refracted* vertex distance and the *fitted* vertex distance may require a compensation of the lens power using the methods described in Section 12.2. The **stop distance** h is the separation between the back vertex V' of the lens and the **center of rotation** of the eye at point R , about which the eye turns. The stop distance is often utilized for lens design calculations. These fitting criteria are described in Figure 9:16.

At least some degree of *pantoscopic* tilt is usually desired for both optics and cosmetics. The bony orbits of the eyes actually have some degree of anatomical pantoscopic tilt—measured from the brow to the cheekbone. By ensuring that the tilt of the frame resembles this natural tilt, the field of view provided by the eyewear is maximized. For reasons discussed below, improper pantoscopic tilt can induce prescription errors (Stimson 328).

Angle θ is the **tilt** of the frame, as measured from a plane perpendicular to the lines of sight in primary gaze. When the bottom rims of the frame are inclined toward the wearer, the frame has **pantoscopic tilt**. When the bottom rims are tilted away from the wearer, the frame has **retroscopic tilt**. These vertical tilts are illustrated in Figure 9:17.

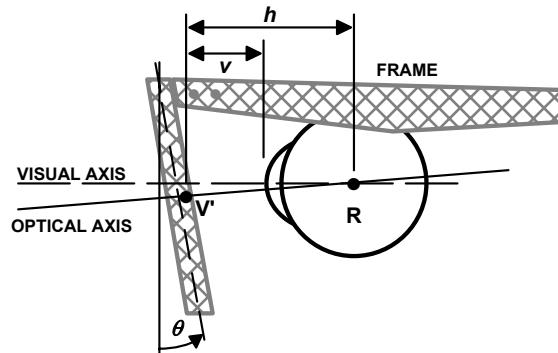


FIGURE 9:16 Lens positioning. The stop distance h is the distance from the back vertex V' of the lens to the center of rotation at point R . The vertex distance v is the distance from the back vertex V' of the lens to the apex of the cornea. Angle θ is the tilt of the frame front, as measured from a plane perpendicular to the visual axis in primary gaze.

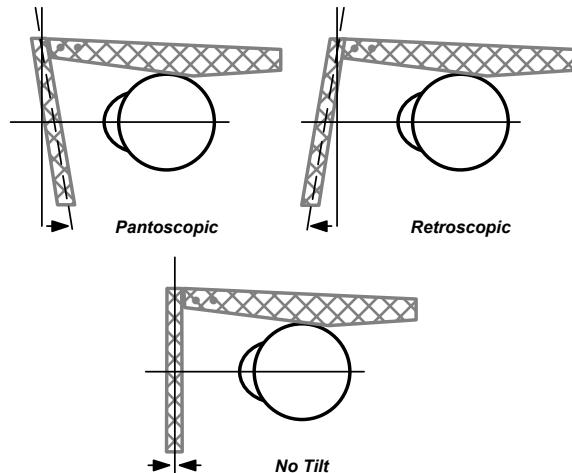


FIGURE 9:17 Vertical frame tilts.

Ideally, the optical axis of the lens should pass through the center of rotation R . This ensures that the lens is perpendicular to the eye, and that the visual axis of the eye is coincident with the optical axis of the lens. A lens usually provides the best vision if the optical center is located a small amount below the pupil of the eye, because we usually look slightly downward, and with a small amount of pantoscopic tilt. Some pantoscopic tilt provides a better facial fit, since the bony orbits of the skull wherein the eyes rest are slightly inclined, and also ensures that the line of sight is nearly perpendicular to the lens when viewing through the optical center.

The wrap of the entire frame around the face of the wearer is known as **face-form tilt**, or *wrap*. A slight amount of face-form tilt helps the frame contour better to the shape of the patient's face. This facial tilt is illustrated in Figure 9:18.

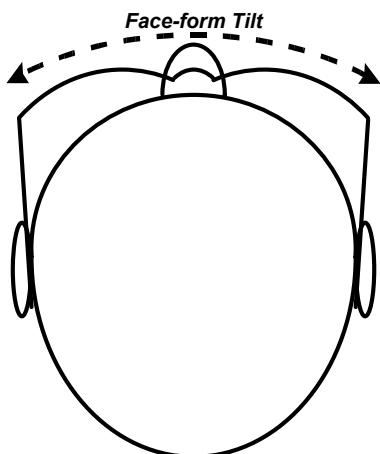


FIGURE 9:18 Face-form tilt.

Here is a simple rule of thumb: *There should be approximately 2° of pantoscopic tilt for every 1 mm that the pupil center is located above the optical center of the lens* (see Figure 9:16). Most commonly the optical center is best located 3 to 4 mm below the pupil with a pantoscopic tilt of 6 to 8°.

If the optical axis of the lens does not intersect the center of rotation of the eye, the tilted lens may induce an astigmatic error. This astigmatism, which is a departure from the desired prescription, causes the following two effects which are directly proportional to the power of the lens (Dowaliby 185):

1. **Increased sphere power.** The new sphere power F_{NEWSPH} is given for a thin lens by the formula:

$$\text{EQ. 56} \quad F_{\text{NEWSPH}} = \left(1 + \frac{\sin^2 \theta}{2n} \right) F_{\text{SPH}}$$

2. **Induced cylinder power** (with the same sign \pm as the sphere power) at axis 180°. The induced cylinder power C_{NEWCYL} is given for a thin lens by the formula:

$$\text{EQ. 57} \quad C_{\text{INDCYL}} = F_{\text{NEWSPH}} \cdot \tan^2 \theta$$

where θ is the angle of tilt, n is the refractive index of the lens, F_{SPH} is the original sphere power of the lens, F_{NEWSPH} is the induced sphere power, and C_{NEWCYL} is the induced cylinder power.

It is important to note that face form tilting can also introduce this astigmatic error. This is why face form should be minimal. In this case, the axis of the induced cylinder would be at 90°.

Example

The optical center of a lens is placed 5 mm below the line of sight. How much pantoscopic tilt is required to prevent an astigmatic error?

$$\theta = 2^\circ (5)$$

$$\theta = 10^\circ$$

\therefore Pantoscopic tilt required is 10°.

Example

A +4.00 D lens is given 15° of pantoscopic tilt (with the optical center at pupil level). The index of refraction is 1.500. What is the new sphere power and induced cylinder power?

$$F_{\text{NEWSPH}} = \left(1 + \frac{\sin^2 15^\circ}{2(1.500)} \right) 4.00$$

$$F_{\text{NEWSPH}} = \left(1 + \frac{0.067}{3} \right) 4.00$$

$$F_{\text{NEWSPH}} = (1.022)4.00$$

$$F_{\text{NEWSPH}} = +4.09$$

$$C_{\text{INDCYL}} = 4.09 \cdot \tan^2 15^\circ$$

$$C_{\text{INDCYL}} = 0.29$$

\therefore New sphere power is +4.09 D and the induced cylinder power is +0.29 D.

Therefore, the refractive power that the patient effectively receives is +4.09 +0.29 D \times 180.

9.4 MINIMUM BLANK SIZE

The dimensions of the patient's spectacle frame and the position of the lenses within that frame are required to determine the *minimum blank size* for a given pair of eyeglasses. The **minimum blank size** (abbreviated MBS) will be the smallest circular lens diameter required for a particular frame and lens combination. Essentially, this is the smallest circle that will completely enclose the edged lens—once the optical center O has been decentered from the geometric center G of the frame.

The minimum blank size is equal to twice the **minimum radius** r_{MBS} of the decentered lens, which is the distance from the optical center or prism reference point, to the farthest point along the edge of the finished lens shape. As shown in Figure 9:19, the diameter of the minimum blank size \varnothing_{MBS} of a single vision lens can be estimated with the following rule of thumb:

$$\text{EQ. 58} \quad \varnothing_{\text{MBS}} = \varnothing_{\text{ED}} + 2 \times \text{dec}$$

where dec is the required decentration (Eq. 50), and \varnothing_{ED} is the effective diameter of the frame. All values are typically given in millimeters.

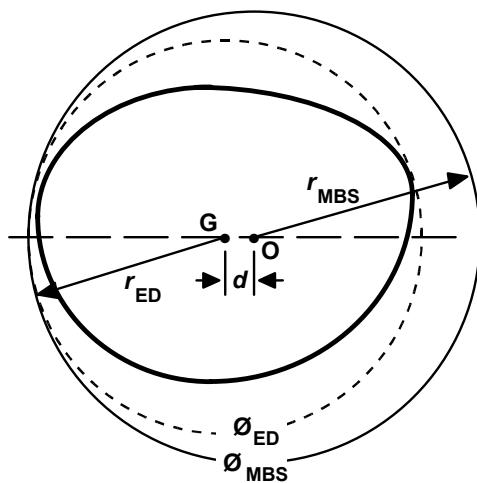


FIGURE 9:19 Minimum blank size calculation. The minimum radius r_{MBS} of the approximate minimum blank size is equal to the sum of the radius r_{ED} of the effective diameter $\Ø_{ED}$ plus the decentration d . Hence, the diameter $\Ø_{MBS}$ of the MBS is equal to twice the minimum radius, so that $\Ø_{MBS} = 2r_{MBS} = \Ø_{ED} + 2 \times d$.

The minimum blank size is necessary in order to calculate the required thickness of a lens (especially plus lenses) for surfacing, and to select the appropriate blank diameter for edging the lens into a given frame. This also allows the smallest possible lens blank to be used. For *minus* lenses, the MBS allows you to estimate the maximum *edge* thickness.

To see why it is advantageous to use the smallest blank size possible—particularly *plus* lenses—consider Figure 9:20, which depicts the center and edge thicknesses of two +4.00 D lenses that have been edged to the same frame (54-mm eyesize). One lens was made using a factory-finished (or *stock*) lens blank, which has an initial diameter of 75 mm. Once edged, the minimum edge thickness is 3.35 mm, which is quite thicker than desired in most cases. The other was made from a semi-finished lens blank that has been surfaced to its absolute minimum thickness. The minimum edge thickness of this lens once edged is only 1.0 mm. (This minimum edge thickness is quite common, but any desired edge thickness could be used.) The surfaced lens is comparable in thickness to a stock lens that is roughly 62-mm in diameter, which would be the minimum blank size for this particular frame. Note that the surfaced lens is roughly 32% thinner than the stock lens.

Example

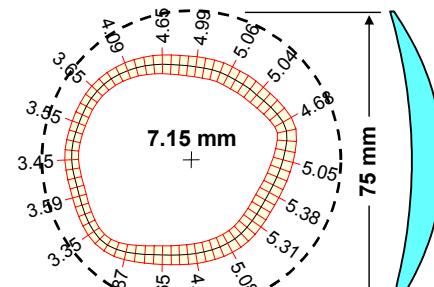
A certain frame requires a 3-mm decentration per each eye and has an effective diameter of 54 mm. What is the minimum blank size?

$$\Ø_{MBS} = 54 + 2(3)$$

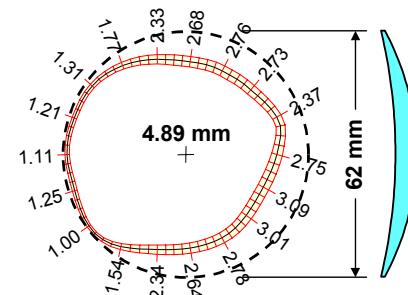
$$\Ø_{MBS} = 60$$

∴ Minimum blank size is 60 mm.

It should be noted, however, that this equation assumes, as is very often the case, that the lens is decentered *in* and that the farthest edge of the frame from the geometric center is *out* (i.e., temporal).



A. Stock +4.00 D lens



B. Surfaced +4.00 D lens

FIGURE 9:20 For plus lenses, the center thickness of the lens is dependent upon several factors, including the smallest possible blank size (MBS) for the frame. A) The center and minimum edge thickness of a stock lens is thicker than necessary for most edged plus lenses, unless the stock lens is about the same size as the MBS. B) Surfacing a plus lens produces the thinnest possible lens, and ensures that the minimum edge thickness is achieved.

It should be apparent that this rule of thumb method does *not* consider the frame shape or the angle of the effective diameter. Determining the exact minimum radius r_{MBS} of the lens will be more accurate, when possible. Plus-powered, spherocylindrical lenses (with a cylinder component) can further complicate the exact minimum blank diameter required, since the edge thickness varies around the perimeter of the lens. (Recall that the edge thickness serves as the limiting thickness factor for plus lenses). The axis of the cylinder—as it relates to the dimensions of the frame—is also an important factor, as well as any prescribed prism. Computers are now commonly employed to make exact calculations of lens thickness and blank size, and ensure that lenses are surfaced as thin as possible.

10. Single Vision Lens Design

This chapter describes single vision ophthalmic lens design, including *corrected curve*, *aspheric*, and *atomic* lenses. Earlier, it was suggested that a given lens power could be produced using any of a variety of lens forms (different base curves); as long as the combined power of the front and back surfaces summed to the desired power. However, proper selection of the base curve or lens design will result in better vision for the wearer. The fundamental advantage to using the proper base curve is that it results in better vision when the wearer looks through peripheral portions of the lens.

10.1 OPHTHALMIC LENS ABBERRATIONS

It was stated in Section 4.1 that Snell's law of refraction could be approximated to develop a simple equation for the power of a lens surface. This is true in a small area, immediately surrounding the optical axis of the lens, known as the **paraxial region**. Within this region, we can apply our small angle approximations to simplify the calculation of surface power. Within the paraxial region, Snell's law of refraction is given by this approximation:

$$\sin i \approx i$$

The focal power of an ophthalmic lens is based upon this simplified system of analysis, which is often referred to as **First-order Theory** or **Gaussian Optics**. However, this simplification quickly loses accuracy at further distances from the optical axis, or at oblique angles of view. Because the focal power of an ophthalmic lens is only entirely accurate within the paraxial region about the optical axis, the prescription *off-axis* (away from, or obliquely to, the optical axis) will perform differently than the prescription *on-axis* (in alignment with the optical axis). When prescribing, producing, and verifying the power of a spectacle lens, the lenses are typically measured *on-axis*. Therefore, the approximations developed for paraxial power—although not entirely precise—are quite accurate for practical purposes.

To understand how rays of light behave *off-axis*, a more accurate approximation for Snell's law can be utilized. If at least one additional term is added to our radian approximation of the angle of incidence, a more accurate evaluation becomes possible:

$$\sin i \approx i - \frac{i^3}{6}$$

This provides us with the basis of **Third-order Theory**. Third-order Theory shows us that light striking a lens obliquely is not necessarily focused to a single point in the plane of the secondary focal point of the lens. For ophthalmic lenses, an **optical aberration** occurs when

rays of light fail to come a point focus at the intended position of the *far point* M_R of the eye as it revolves about its center of rotation at point R. As the eye rotates about its center, the far point generates a spherical surface, which is called the **far-point sphere**. For any direction of gaze, the far-point sphere represents the ideal image position. The radius r_{FPS} of the far-point sphere is given by

$$\text{EQ. 59 } r_{FPS} = h - f'$$

where f' is the secondary focal length of the lens and h is the stop distance from the back vertex of the lens to the center of rotation R.

When viewing peripherally through a lens, we want the focus to fall on the far point sphere. However, it should be apparent that the base curve, or "bending" of the lens, will affect its distance from the eye and its tilt relative to the line of sight. These factors can significantly alter the power that the eye encounters, resulting in a power that deviates from that required for clear vision.

The design criteria for both plus and minus lenses are illustrated in Figure 10:1 and Figure 10:2. Like refractive errors, optical aberrations cause imperfect image formation and blurred vision. As the eye rotates, the stop distance h also describes a spherical surface called the **vertex sphere**. All measurements of off-axis power are referenced from the vertex sphere for consistency.

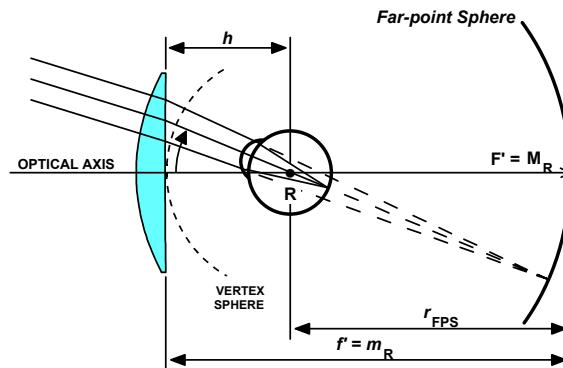


FIGURE 10:1 Lens design parameters for a *plus* lens. The far point M_R is rotated about the center of rotation R, creating a spherical surface called the **far-point sphere**. The radius of the far-point sphere $r_{FPS} = h - f'$, where h is the *stop distance* from the lens to the center of rotation.

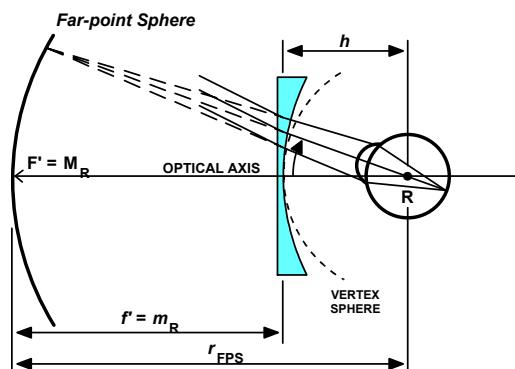


FIGURE 10:2 Lens design parameters for a *minus* lens. The same descriptors from plus lenses apply.

There are five unique **monochromatic aberrations**, first enumerated by Ludwig von Seidel in 1855: *spherical aberration*, *coma*, *oblique astigmatism*, *curvature of the field*, and *distortion*. They are referred to as *monochromatic*, which literally means ‘single color,’ since they are independent of color (Freeman 381).

The first two aberrations, **coma** and **spherical aberration**, result from the fact that the power of a lens effectively increases away from its optical axis (or center). Fortunately, for spectacle lenses these two aberrations are minimized because the small pupil of the eye limits the area of the lens that admits rays of light into the eye at any one time. Therefore, they are generally not a concern for ophthalmic lens designers. Spherical aberration, which affects object points along the optical axis, is illustrated in Figure 10:3. Coma is similar to spherical aberration but affects object points off the optical axis.

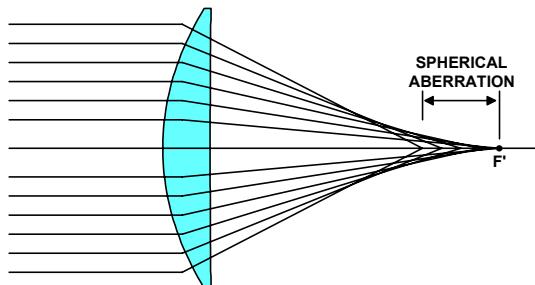


FIGURE 10:3 Spherical aberration. Rays of light refracted in the periphery of a lens come to a different focal point than central rays—producing *spherical aberration*. The paraxial approximation for lens power, which assumes a focus at point F' , loses accuracy for these peripheral rays.

Ophthalmic lens designers are typically concerned with only two of Seidel’s aberrations, which are often considered to have greater significance to vision. **Oblique astigmatism** is one of the principal lens aberrations that must be corrected for when designing spectacle lenses. This *astigmatic* focusing error, which is illustrated in Figure 10:4, results when rays of light

from an off-axis object strike the lens obliquely. Two focal lines are produced from a single object point, much like the regular astigmatism described in Section 5. The *dioptric difference* between these two foci, $F_T - F_S$, is the amount of the oblique astigmatism, or the **oblique astigmatic error** A . Oblique astigmatism is also referred to as **marginal** or **radial astigmatism**.

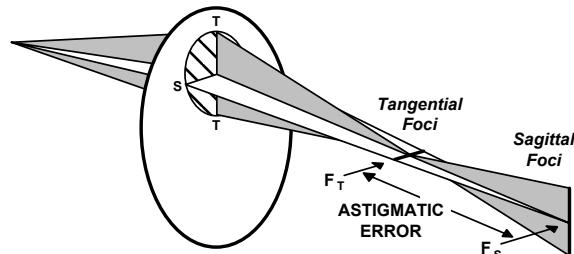


FIGURE 10:4 Oblique astigmatism. Rays of light from an object point strike the lens obliquely and are focused into two separate line foci, instead of a single point focus, when *oblique astigmatism* is present. The dioptric difference between these two foci, which are referred to as the *tangential* focus F_T and *sagittal* focus F_S , represents the amount of *astigmatic error*: $A = F_T - F_S$.

Rays of light striking the *tangential*, or *meridional*, plane of the lens come to a line focus at the **tangential focus** F_T . The resultant focal line is perpendicular to the actual tangential plane. Rays striking the *sagittal*, or *equatorial*, plane come to a line focus at the **sagittal focus** F_S . This focal line is perpendicular to the sagittal plane. Both of these planes are shown in Figure 10:5. The actual *tangential power error* P_T is equal to the difference between the desired back vertex power F_V and the tangential power F_T , or $P_T = F_V - F_T$. Similarly, the *sagittal power error* P_S is equal to the difference between the desired back vertex power the sagittal power F_S , or $P_S = F_V - F_S$. Therefore, the astigmatic error A is also given by, $A = P_T - P_S = F_T - F_S$.

Changing the base curve of a lens changes the angle with which the line of sight passes through the lens with peripheral viewing. This effectively changes the tilt encountered and therefore the power that is encountered.

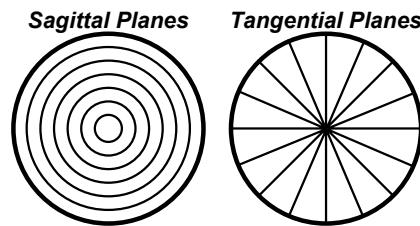


FIGURE 10:5 The *sagittal* (equatorial) and *tangential* (meridional) planes of a lens.

The next Seidel aberration and the second aberration of concern to ophthalmic lens designers is **curvature of the field**. When oblique astigmatism has been

completely corrected (by selecting the base curve which does so), the astigmatic focal lines become coincident and the lens produces a single point focus. However, instead of producing a flat image, a curved image is produced called **Petzval's surface**. Although a flat image surface is desired for many optical systems, such as cameras, the far-point sphere (or ideal image plane) of the eye is also curved. The radius r_{PS} of Petzval's surface is given by

$$\text{EQ. 60} \quad r_{PS} = -n \cdot f'$$

where n is the refractive index of the material and f' is the secondary focal length of the lens.

Unfortunately, Petzval's surface is *flatter* than the far-point sphere in most instances, which results in some residual focusing error as shown in Figure 10:6. The dioptric difference between Petzval's surface and the far-point sphere of the eye is known as the **power error** of the lens. This *spherical* focusing error usually reduces the power of the lens in the periphery (Grosvenor 362).

When an astigmatic error exists, the **mean power error** M is used to describe the dioptric difference between the far-point sphere and the *average* of the two astigmatic focal powers: $M = F_V - \frac{1}{2}(F_T + F_S)$. This is also equal to the average of the tangential and sagittal power errors, or $M = \frac{1}{2}(P_T + P_S)$.

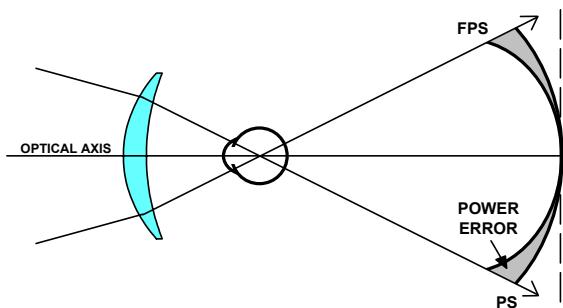


FIGURE 10:6 Curvature of the field. In the absence of oblique astigmatism (accomplished by selecting the base curve that eliminates oblique astigmatism), the tangential and sagittal foci coincide upon an image surface known as *Petzval's surface* (PS). The dioptric difference between the far-point sphere FPS of the eye and Petzval's surface PS for any given viewing angle is the amount of power error present through the lens.

In the presence of oblique astigmatism, there is no true Petzval's surface. As the base curve departs from the one that results in the Petzval's surface, the tangential and sagittal foci depart from Petzval's surface forming their own image shells, as shown in Figure 10:7. The tangential image shell deviates more rapidly from Petzval's surface than the sagittal image shell.

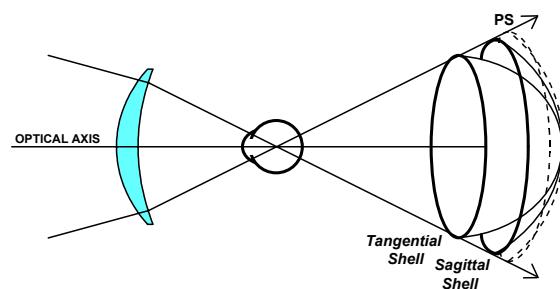


FIGURE 10:7 In the presence of oblique astigmatism, the tangential and sagittal image shells depart from Petzval's surface PS. The tangential shell deviates more rapidly.

Distortion is an aberration that affects not the focal quality of an image, but its size and shape (or its geometric reproduction). Just as the power of a lens effectively increases away from the optical axis of the lens, so does the magnification produced by the lens. For *plus*-powered lenses, excess magnification in the periphery of the lens causes *pincushion* distortion. For *minus*-powered lenses, excess minification causes *barrel* distortion. When there is no distortion present in the image, the image is said to be **orthoscopic**, as illustrated in Figure 10:8.

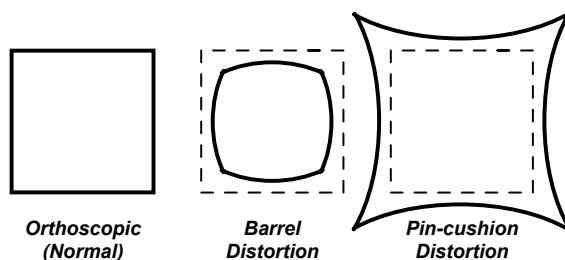


FIGURE 10:8 Distortion. *Orthoscopy* is a lack of distortion. *Minus* lenses produce *barrel* distortion. *Plus* lenses produce *pincushion* distortion.

Distortion causes objects to appear misshapen and curved—especially in higher lens powers. It cannot be eliminated using typical base curve ranges, and is generally not a consideration for lens design.

Oblique astigmatism and power error occur as the spectacle-wearer gazes away from the optical axis—or optical center—of the lens. Consequently, peripheral vision through a lens that suffers from an excess of either of these two aberrations is blurred, and the wearer experiences a limited field of clear vision. This is often the case with flatter lens forms, as we shall see later. This effect is illustrated in Figure 10:9 and Figure 10:10 for a 30° angle of view from the optical axis of the lens.

For our purposes, the **viewing angle** is the angle that the line of sight makes with the optical axis of the lens as the eye rotates. The eyes often make rotations of 30° or more—about its center of rotation R —from the optical axis to fixate objects within the wearer's field of view.

Historically, most design calculations have also been based upon a 30°-viewing angle (Davis 134).

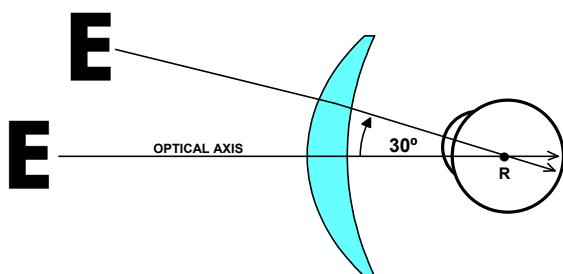


FIGURE 10:9 Clear peripheral vision at a 30° viewing angle with a *steeper* lens form.

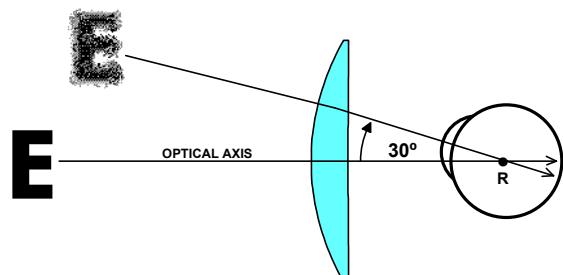


FIGURE 10:10 Blurred peripheral vision at a 30° viewing angle with a *flatter* lens form.

10.2 CORRECTED CURVE LENS DESIGN

The selection of the form of the lens (base curve) is a primary tool in the reduction of lens aberrations. Further, the final lens form for a given power is determined during base curve selection. It was demonstrated earlier that flatter lens forms produce thinner and lighter weight lenses. However, flatter lens forms can also suffer from significant lens aberrations in the periphery. Table 16 demonstrates the error from the desired prescription—for both a +4.00 and a -4.00 D lens when produced upon flatter base curves. Notice how the prescription error increases as the base curve becomes flatter.

TABLE 16 Off-axis performance

+4.00 D Lens		-4.00 D Lens	
Base Curve	Prescription	Base Curve	Prescription
	30° Off-axis		30° Off-axis
9.00	+3.90 -0.08	4.00	-3.90 -0.10
7.00	+4.31 -0.36	2.00	-4.01 -0.36
5.00	+4.90 -0.79	0.50	-4.12 -0.63

When a base curve is chosen to minimize lens aberrations, the resulting lens is referred to as a **corrected curve lens**. Since corrected curve lens forms will have the least amount of the most detrimental aberrations, they are also called **best form lenses**.

Most of the work developing corrected curve lenses began in the 1800s. In 1804, William Wollaston

developed a system of steeply bent lens forms that were free from oblique astigmatism. However, these lenses were much too steep for practical uses. In 1898, F. Ostwalt developed a system of lenses significantly flatter than Wollaston's that were also free from oblique astigmatism.

In 1909, Marius Tscherning demonstrated that there were two recommended *best form* base curves for each lens power. **Tscherning's ellipse**, which is depicted in Figure 10:11, is the locus of points that plot out the recommended base (front) curves for each focal power. Consequently, it can be shown that both Wollaston's and Ostwalt's lenses can be calculated using the same quadratic formula. Wollaston's branch represents the positive root and Ostwalt's represents the negative root. For near vision, the ellipse shifts *down* slightly. Therefore, lenses utilized for reading should be roughly 1 to 2 D flatter than those for distance.

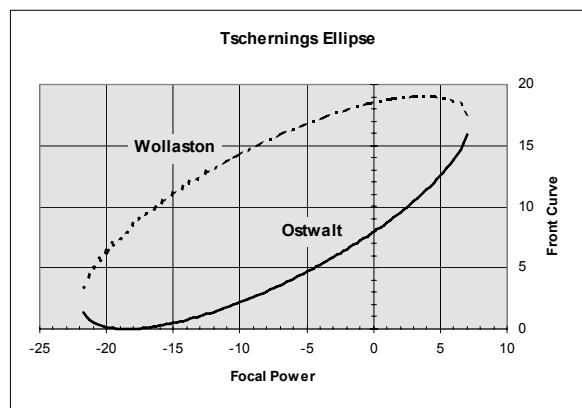


FIGURE 10:11 Tscherning's ellipse for an infinite object distance and a 27 mm stop distance. For near object distances, the ellipse shifts down slightly.

It is important to note that modern lens forms employ the flatter, *Ostwalt* branch of Tscherning's ellipse. Ideally, a separate front curve would be required for each individual lens power to minimize lens aberrations, as shown in Figure 10:11.

One of the first mass-produced series of corrected curve lenses was the Zeiss *Punktal* lens series, which was first released in 1913. These lenses utilized a separate base (front) curve for every prescription, which required a massive, costly inventory. In the 1920s, American Optical introduced their own *Tillyer* series of corrected curve lenses that was named after its designer, Edgar Tillyer. Tillyer grouped small ranges of prescriptions together upon common semi-finished blanks to keep costs and inventories down to a minimum. His series employed only 19 base curves. Soon, other

manufacturers followed with their own versions of corrected curve lens series (Fannin & Grosvenor 147).*

There is no definitive *best form* base curve. The base curve recommended by a particular manufacturer, for a given prescription range, will depend upon which lens aberration(s) that manufacturer is trying to eliminate or minimize. Therefore, different manufacturers may produce differing base curves for similar prescription ranges. Manufacturers may choose between various **design philosophies** (or **merit functions**) to determine which aberrations to minimize or eliminate. Unfortunately, it is not possible to eliminate all of these lens aberrations completely at once. Here are some of the most common design philosophies:

- **Eliminate oblique astigmatism:** To eliminate the oblique astigmatic error A , the tangential power P_T of the lens must be equal to the sagittal power, so that $P_T = P_S$ or $F_T = F_S$. Such a lens is referred to as a **point-focal** lens form.
- **Eliminate mean power error:** To eliminate the mean (or average) power error M , the average of the tangential and sagittal powers of the lens must be equal to the desired back vertex power, so that $\frac{1}{2}(F_T + F_S) = F_V$. Such a lens is referred to as a **Percival** lens form.
- **Eliminate tangential error:** To eliminate the tangential error P_T , the tangential power must be equal to the desired back vertex power, so that $F_T = F_V$. This design philosophy was originally employed in the 1960s for the AO *Tillyer Masterpiece* series (Davis 19).
- **Eliminate RMS error:** The **RMS**, or root-mean-square, **power error** is a scalar measure of both the astigmatic error (A) and mean power error (M). The RMS power error P is a useful predictor of blur and visual acuity that combines the two errors into a single, meaningful measure of defocus:

$$P = \sqrt{M^2 + \left(\frac{1}{2} A\right)^2}$$

Note that the astigmatic error A is weighted by a factor of $\frac{1}{2}$. Recall from Section 6.5, that a purely astigmatic error produces half as much blur as a spherical error of the same magnitude. We can also substitute the relationships that the astigmatic and

* It is also important to note that reducing the number of base curves utilized for a given range of lens powers also reduces the accuracy of the off-axis correction of the lens series. Recall that, ideally, each lens power requires its own front curve. However, the magnitude of the errors created by having a single base curve serve a limited range of powers is relatively small.

mean power errors have to the tangential and sagittal power errors (P_T and P_S) into the above equation to yield,

$$P = \sqrt{\left(\frac{P_T + P_S}{2}\right)^2 + \left(\frac{P_T - P_S}{2}\right)^2}$$

$$P = \sqrt{\frac{P_T^2 + P_S^2}{2}}$$

Consequently, in order to eliminate the RMS power error, the square root of the average of the squared power errors (P_T^2 and P_S^2) must be zero.

Note that you can only choose one of these merit functions at a time. Further, recall that you can not eliminate all errors simultaneously. Correcting any one aberration entirely still leaves a residual error in the others. For instance, consider Figure 10:12, which demonstrates the tradeoffs.

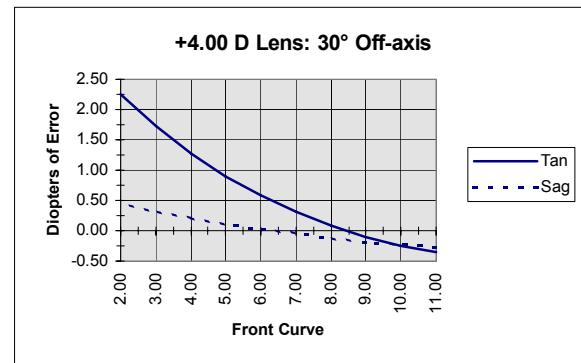


FIGURE 10:12 The off-axis performance for a +4.00 D lens at a 30° viewing angle. The *tangential* and *sagittal* power errors (P_T and P_S) have been plotted against a range of front curves.

This is a graph of the *tangential* and *sagittal* power errors (P_T and P_S) of a +4.00 D lens—at a 30° viewing angle—plotted against a range of front curves. Oblique astigmatism is eliminated with a front curve of 9.75 D, since the tangential and sagittal errors are equal for that curve. Note that there is still some residual power error. The mean power error is eliminated with a front curve of 8.00 D, since the average of the tangential and sagittal errors is equal to zero. Note that there is still some residual astigmatic error. The tangential error is eliminated with a front curve of 8.50 D, since the tangential error is zero with that front curve. There is still some residual astigmatic and mean power error, though. For this particular power, a well designed corrected curve lens will probably utilize a front curve between 8 to 10 D—depending upon the design philosophy and merit function of the manufacturer.

Recall that *best form*, or *corrected curve*, lenses use the optimum base curve for optical performance. It should now be obvious that the lens forms we would choose for good cosmetics (i.e., flatter lenses), provide poor optical performance in the periphery. This is the primary conflict that lens designers have to balance when designing ophthalmic lenses: *optical performance versus cosmetics*.

Table 17 shows typical base curve selection guidelines for producing best form lenses. Today, most base curve series have at most six or seven base curves. For lenses with cylinder power, the *spherical equivalent* should be used to determine the appropriate base curve. Above about +8.00 D, conventional base curves are no longer effective. (Note that Tscherning's ellipse does not pass this power range.)

TABLE 17 A typical base curve selection chart

Power Range (D)	Base Curve (D)
+8.00 to +4.75	10.00
+2.25 to +4.50	8.00
+2.00 to -2.00	6.00
-2.25 to -4.00	4.00
-4.25 to -7.00	2.50
-7.25 to -12.00	0.50

10.3 ASPHERIC LENS DESIGN

Fortunately, lens designers have another tool at their disposal when designing lenses: asphericity. As the name implies, an **aspheric surface** is a surface that departs from being perfectly spherical. Aspheric surfaces are rotationally-symmetrical surfaces that gradually vary in surface power from the center towards the edge, in a radial fashion. This change in surface power produces **surface astigmatism** that can counteract and neutralize the oblique astigmatism. Aspheric surfaces free lens designers from the constraints of *best form* lenses. Lenses can be made flatter, thinner, and lighter, while maintaining excellent optical performance.

Aspheric lenses were originally employed to provide acceptable vision in high-plus, *post-cataract* lenses, which generally exceed the +8.00 D limit of Tscherning's ellipse. We can see from the schematic eye (Table 6) that the crystalline lens is responsible for nearly a third of the refracting power of the eye. When the lens is removed from the eye as a result of a **cataract** (opacity of the crystalline lens), extremely high-plus spectacle lenses can be used to supplement the loss of refracting power.

Because of surgical advances and *intraocular lens implants*, such lenses are now all but obsolete. Today, aspheric surfaces are mainly used to allow lens designers to produce flatter, thinner lenses with the

superior optical performance of the steeper corrected curve, or best form, lenses.

To produce a three-dimensional aspheric surface, an aspheric curve is rotated about an **axis of symmetry**. The central curvature, or **vertex curvature**, of an aspheric surface will be nearly spherical. The vertex curvature of an aspheric surface will be the front curve value utilized for lens power and surfacing calculations. Away from the vertex curvature, the amount of surface astigmatism smoothly increases. The rate of increase in surface astigmatism depends upon the degree of asphericity. Figure 10:13 demonstrates the surface produced by rotating an ellipse about an axis of symmetry. Notice the changing radii of curvature in both the tangential and sagittal meridians (Jalie 516).

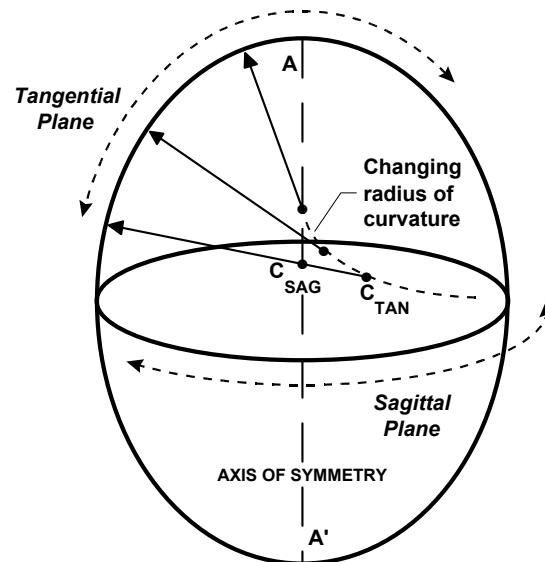


FIGURE 10:13 This elliptical curve has a radius of curvature that gradually changes away from the center. Once the ellipse has been rotated about the axis of symmetry AA', it produces a three-dimensional **conicoid** surface. The tangential radius of curvature, from point C_{TAN}, is longer than the sagittal radius of curvature, from point C_{SAG}. This results in both a change in surface power and astigmatism that is utilized to control lens aberrations.

Original aspheric designs utilized **conicoid surfaces**, produced by rotating a conic section about an axis of symmetry to produce a three-dimensional surface. The conic section could be any one of five from the family of conics, including the *circle*, *prolate ellipse*, *oblate ellipse*, *hyperbola*, and *parabola*. These conic sections are illustrated in Figure 10:14.

Conicoid surfaces can be described by the following formula (Jalie 517):

$$\text{EQ. 61 } z = f(x) = \frac{x^2 \cdot R_s}{1 + \sqrt{1 - p \cdot x^2 \cdot R_s^2}}$$

where z is the height of the surface, R_S is the vertex curvature, x is the semi-diameter of the lens, and p is a value that controls the amount/degree of asphericity (or eccentricity of the conic focus) for the surface.

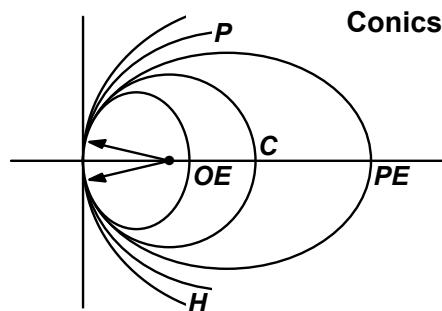


FIGURE 10:14 The family of conic sections includes the *circle* (p -value of 1), *oblate ellipse* (p -value > 1), *prolate ellipse* ($0 < p$ -value < 1), *parabola* (p -value = 0), and *hyperbola* (p -value < 0).

Modern aspheric lenses often employ *higher order* surfaces that allow for more complex shapes than the simple conic sections. These lenses can be described by *polynomial functions* of the form:

$$\text{EQ. 62} \quad z = f(x) = ax^2 + bx^4 + cx^6 + \dots$$

where z is the height of the surface; x is the semi-diameter of the lens; and a , b , and c are all coefficients that control the shape of the surface.

Surfaces described by this type of polynomial equation can be *deformed* conicoid surfaces or surfaces of even greater complexity. The first coefficient (a) in many of these polynomial equations describes the vertex curvature, and is either given by equation Eq. 61 or by

$$a = \frac{R_S}{2}$$

Once the first coefficient a has been determined, the final coefficients are then determined by '*ray tracing*' procedures that refine the off-axis performance of the lens for various viewing angles. Many designers chose to optimize the off-axis performance of a lens out to at least 30° to 35° .

Unfortunately, because of the fact that the curvature of an aspheric surface varies away from the center, normal measuring instruments such as the lens clock cannot measure the front curve value—or vertex curvature—of an aspheric lens accurately. The wider the spacing between the pins, or the wider the diameter of the sag gauge bell, the less accurate the instrument becomes for measuring aspheric surfaces. Moreover, because the asphericity of many of these surfaces varies only subtly away from the center of the lens, it is often equally difficult to detect asphericity using such instruments.

Most aspheric lenses are designed to allow use of a flatter, more cosmetically pleasing lens, while minimizing off-axis aberrations. Since flattening a lens introduces astigmatic and power errors, the peripheral curvature of the aspheric surface should change in a manner that neutralizes this effect. For instance, *plus* lenses with asphericity on the *front* surface require a *flattening* of curvature away from the center of the lens to reduce the effective gain in oblique power and astigmatic error. Asphericity on the *back* surface of a plus lens will require a *steepening* of curvature away from the center of the lens. The opposite holds true for *minus* lenses.

Aspheric lenses are generally more sensitive to the range of prescriptions that they have been optimized for. Consequently, aspheric lenses typically have more base curves available, in smaller increments of surface power. Proper base curve selection, as recommended by the manufacturer, is critical.

The further a lens form is flattened from its optimum, best form base (spherical) base curve, the more asphericity (or surface astigmatism) will be required to properly compensate for the off-axis optics.

We now know that flatter base curves produce thinner lenses (refer back to Section 7.2). It is interesting to note that the actual geometry of an aspheric surface also helps reduce lens thickness. This is a consequence of the fact that the sagitta of an aspheric surface differs from the sagitta of a spherical surface. Consider the comparison made in Figure 10:15. At a given diameter, the aspheric surface has a shallower sagitta than the spherical surface and therefore a reduced plate height. The thickness reduction is maximized when the surface with the highest surface power is made aspheric (Jalie 340).

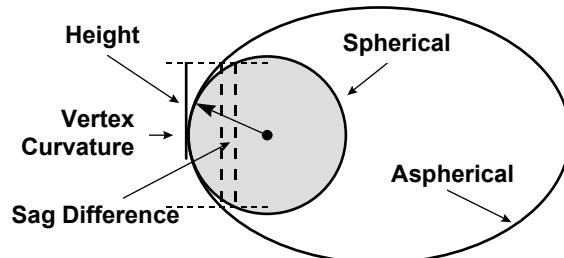


FIGURE 10:15 Difference in sags between spherical and aspherical convex curves. For a given diameter the aspheric surface is shallower than the spherical surface.

We can now compare the physical and optical properties of a steep *best form* lens, a flat lens with a spherical base curve, and a flat *aspheric* lens. Careful consideration of Table 18 shows that a best form lens (+4.00 D) can be produced having no astigmatic error and very little mean power error. A flat lens can also be made, which would be cosmetically superior, but would suffer significant astigmatic and mean power errors.

However, using an aspheric front surface, a lens can be produced that eliminates the astigmatic error while providing an even thinner center.

TABLE 18 +4.00 D lens design comparison

+4.00 D Lens Design Comparison*			
	Best Form Lens	Flat Lens	Aspheric Lens $p = -11.60$
Front Curve	9.75	4.25	4.25
Center Thickness	6.6	5.9	5.1
Weight (grams)	20.6	17.7	14.8
Plate Height	13.7	6.0	5.1
Astigmatic Error	0.00	0.99	0.00
Mean Power Error	-0.22	0.68	-0.23

* These are CR-39 lenses that have been computed with a 70-mm diameter, a 1-mm edge, and a 30° viewing angle.

Since the geometry of an aspheric surface can reduce the thickness of a lens, it becomes possible to use asphericity strictly for cosmetic purposes. By producing a highly aspherical surface with a rapid change in curvature towards the periphery of the lens, the sagitta of the surface can be made much shallower.

The aspheric surface of the **Hi-Drop** cataract lens, for instance, which is shown in Figure 10:16, rapidly drops between 3 to 4 diopters from the center of the lens to its edge to produce an extremely thin profile. This type of lens is called a **zonal aspheric**, because of its zones of changing surface power.

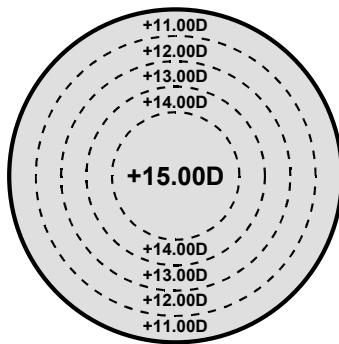


FIGURE 10:16 Decrease in surface power away from the center of a *Hi-Drop* lens.

When asphericity is ‘exaggerated’ in this fashion, the off-axis optics are generally poor—since the lens has been optimized solely for cosmetics, with little regard for optics. Another problem with making lenses too flat is that they do not provide enough clearance for the eyelashes.

In addition to aspheric lenses strictly optimized for optics and lenses strictly optimized for cosmetics, there are aspheric lenses available that combine the benefits of both. The surface of certain **continuous-surface** aspheric lenses is optimized out to a certain point across

for optics, and then drops off rapidly in curvature to the edge for cosmetics.

10.4 ATORIC LENSES

As described above, each focal power requires its own lens form to accurately correct for lens aberrations. Consequently, lenses with cylinder power are not entirely corrected using conventional lens designs. The lens designer may choose the optimum front curve based upon the sphere meridian, the cylinder meridian, or the average power (*spherical equivalent*) of the lens. For lenses with low cylinder power, the differences are generally negligible. For higher cylinder powers, though, the errors can be significant resulting in a reduced field of clear vision.

Advances in lens design have provided lens designers with the ability to produce surfaces even more complex than the rotationally-symmetric aspheric designs described earlier. By varying the degree of asphericity from one meridian of the lens to another, an **atomic surface** can be produced. This allows designers to optimize the lens for both the sphere and cylinder powers, by applying customized asphericity to each power. Just as ‘aspheric’ denotes a surface that departs from being completely spherical, ‘atomic’ denotes a surface that departs from being an exact circular toric. Figure 10:17 depicts a possible atomic surface.

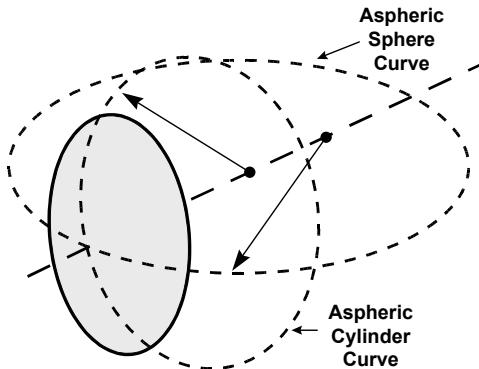


FIGURE 10:17 An *atomic* surface with unique amounts of asphericity individually applied to the sphere and cylinder meridians of the lens.

Rather than being rotationally-symmetrical—like an aspheric—atomic surfaces are symmetrical with respect to two planes of symmetry coincident with the principal meridians of the lens, like a toroidal surface. The curvatures in these two planes, which are also perpendicular to each other like a toroid, are not circular like a toroid, however. They can be conic in shape, or use of higher order polynomial curves.

Atoricity is an extension of aspheric technology, allowing lens designers to optimize for both the sphere and cylinder powers of a lens. This ensures a more

accurate refractive correction over wider fields of view for those with astigmatism. Let's look at the differences between best form (spherical base curve), aspheric, and atoric optimization strategies using an actual example. Figure 10:19 represents the relative asphericity of these three different lens designs for an astigmatic correction: +2.00 DS -1.00 DC × 090.

Figure 10:18 is a comparison between the error-free fields of view of a flattened lens with a spherical base curve, an aspheric lens, and an atoric lens—each with -2.00 D of cylinder power.

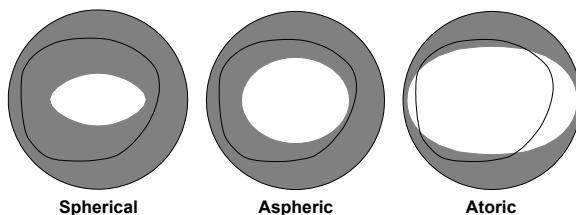
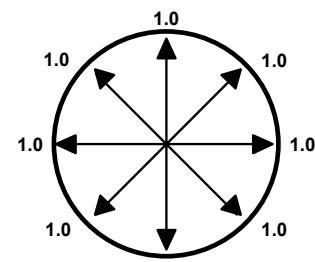


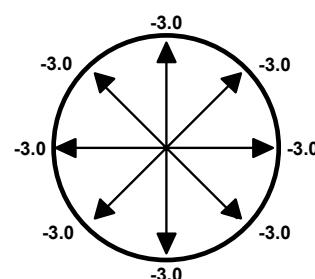
FIGURE 10:18 Atoric field of view comparison; +4.00 DS - 2.00 DC × 090 polycarbonate lenses with 4.85 D base curves. The first lens is a flattened spherical lens, the second lens is an aspheric, and the third lens is an atoric. The white area represents the field of error-free optics.

These values—which represent *conic p-values*—describe the relative departure of the surface from a perfect circle through each meridian, and can be thought of as the amount of asphericity present through that meridian. The further this value departs from 1.0, the more aspheric (or non-circular) the curvature of the surface is through that meridian.

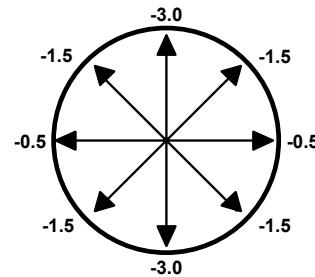
There is also a class of lens surfaces that has no particular symmetry. These surfaces are complex, *free-form* surfaces that can optimize for both the sphere and cylinder powers, as well as additional factors—like optical errors introduced while looking through the near zone of certain lenses. Currently, such surfaces have been employed mainly on the back surface of certain *progressive addition lenses*. In fact, these surfaces share the same complexity that progressive surfaces do (Section 11.3).



A) Spherical Surface



B) Aspheric Surface



C) Atoric Surface

FIGURE 10:19 Relative asphericity (a conic *p*-value) through each meridian of the lens for three different designs; +2.00 DS -2.00 DC × 090. A) The base curve of the *best form* lens is perfectly spherical in every meridian with a *p*-value of 1. B) The asphericity of an *aspheric* lens also remains the same in every meridian. A hyperboloidal *p*-value of -3 has been chosen to optimize for the sphere meridian. C) The relative asphericity of an *atomic* lens changes from meridian to meridian—optimizing for both the sphere and cylinder powers of the lens. The *p*-values vary from a minimum of -0.5—through the cyl meridian, to a maximum of -3.0—through the sphere meridian.

11. Multifocal Lens Design

It should be apparent that the primary purpose of a multifocal lens is to supplement—or replace, if necessary—the loss of accommodation that was once provided by the eye to bring near objects into focus. This loss of accommodation is usually age-related, in which case the condition is called *presbyopia*. However, multifocals are also sometimes used in younger patients for accommodative or binocular vision disorders, such as *accommodative esotropia*. Multifocal lenses include **bifocals** (with two focal powers), **trifocals** (with three focal powers), and **progressive addition lenses** (with a continuously varying focal power from distance to near).

11.1 CONVENTIONAL MULTIFOCALS

Recall that light rays from an object located at the primary focal point will be rendered parallel after refraction through the lens. That is, an object at the primary focal point is conjugate with an image at optical infinity (∞). Figure 11:1 demonstrates how a bifocal lens can completely replace the need for accommodation, if necessary, by rendering diverging rays of light from near objects completely parallel as they pass through the *segment*. This produces an object located at infinity for the distance—or *major*—portion of the lens, which then brings it to a focus at the secondary focal point F' (as well as the far point of the eye).

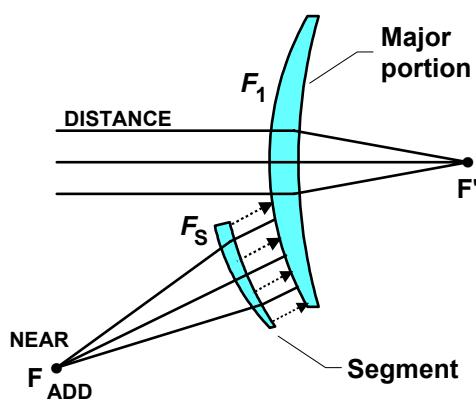


FIGURE 11:1 Bifocal correction. The diverging rays of light from a near object, positioned at the primary focal length of the lens through the bifocal *segment*, are rendered parallel and hence properly focused on the retina after refraction through the *major* portion of the lens. For this plastic lens, the add power is produced by a difference in the surface powers of the major portion and the segment.

For presbyopes that still possess a sufficient amount of accommodation, the add power of the segment is prescribed in order to supplement the remaining accommodation, not necessarily to replace it. Recall from Section 6.4 that the add power of a spectacle Rx is

prescribed by considering the wearer’s *amplitude of accommodation*—i.e., how much remains.

Bifocal segments act like small lenses fastened to the *major* (distance) portion of the lens. The segment surface of a plastic bifocal is cast in the mold. The curvature of the segment will be steeper than the curvature of the major portion by the amount needed to produce a difference in surface powers equal to the add power, as illustrated in Figure 11:1. Therefore, the add power is given by $Add = F_S - F_1$, where F_1 and F_S are the surface powers of the major portion and segment, respectively. For instance, a +6.00 D surface power in the major portion combined with a +8.00 D surface power in the segment will yield an add power of +8.00 - 6.00 = +2.00 D.

Measuring the add power of multifocal lenses is discussed in Section 4.5. For glass lenses, a segment made from a higher-index glass lens material is *fused* into the major portion of the lens. The gain in surface powers, caused by the increased refractive index of the segment at the interfaces between air and the segment front and between the segment and major portion, produces the desired add power. Both types of lenses (glass and plastic) are shown in Figure 11:2.

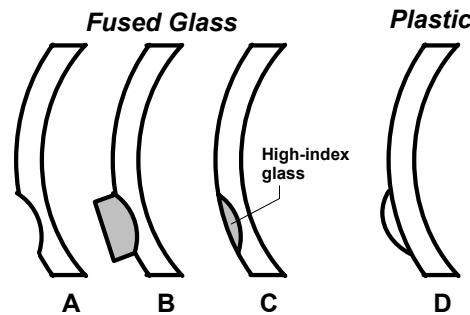


FIGURE 11:2 A) Glass major portion with a *countersink curve* ground into it; B) Major portion with a *high-index glass segment button* fused into the countersink curve; and C) Completed bifocal lens after grinding and polishing the excess button. D) Plastic, *one-piece* bifocal created by molding a steeper curvature in the segment area of the lens.

Trifocals add a second segment, as shown in Figure 11:3, directly above the bifocal segment, for intermediate vision. The **range of vision** through the lens, which is how close or how far away an object can be held while remaining in focus through the various zones of the lens, decreases as the add power increases with conventional bifocals. The second, upper segment of a trifocal provides an **intermediate power** that effectively increases the range of vision through the lens, by providing a weaker power for distances that fall in between the distance and near corrections. This is often called **mid-range vision**.

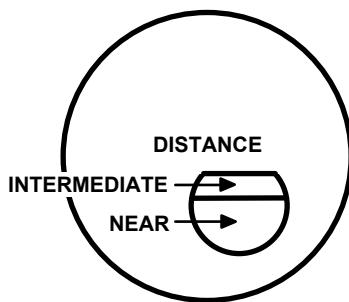


FIGURE 11:3 The trifocal.

Mid-range vision generally covers the distance range between 40 cm and 200 cm from the eye, with a distance of 63 cm serving as a convenient reference standard. The ranges of vision for both bifocal and trifocal lenses have been illustrated in Figure 11:4. A loss of intermediate vision occurs with higher add powers for two reasons:

- 1) As the add power increases the range of clear vision through the segment decreases, since objects beyond the primary focal length of the bifocal become increasingly out-of-focus. This focal length distance is given by $1 \div \text{add power}$. For a +2.50 D add, for instance, objects beyond 40 cm (0.4 m) will be blurred ($1 / 2.50 = 0.4$).
- 2) Higher add powers generally indicate lower *amplitudes of accommodation*, which means that the eye can not bring near objects into focus as readily through the distance portion of the lens. Higher reserves of accommodation, on the other hand, allow a person to increase the plus power of the eye to some extent using accommodation. This allows objects nearer than optical infinity to be brought into focus somewhat.

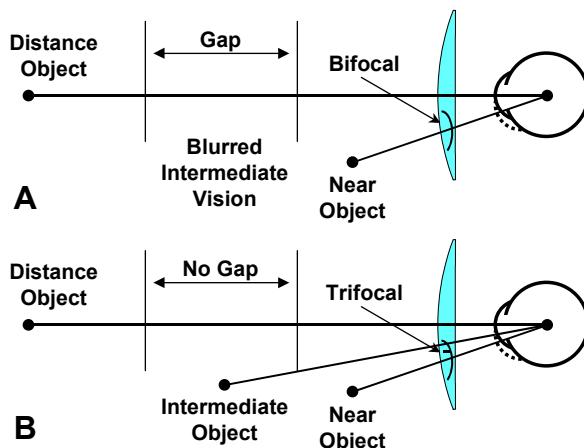


FIGURE 11:4 A) The intermediate vision of a bifocal becomes blurred as the add power increases, leaving the wearer with a gap in the range of vision afforded by the lenses. B) Trifocal lenses have an additional segment that provides intermediate vision for mid-range distances.

The intermediate power of most plastic trifocals is approximately 50% of the add power. For glass lenses, the intermediate power may vary slightly, depending upon the high-index glass materials utilized for the segments. In some cases, special intermediate powers may be ordered to accommodate unusual occupational or recreational working distances. In general, though, trifocals are only a small fraction of the multifocal market.

Example

To understand why a trifocal is necessary, consider a presbyope with 0.50 D of accommodation. To read at 40 cm, this presbyope would likely receive a +2.25 D add power. The maximum linear extent to which the presbyope can read through his/her segment with this add power is about $1 / 2.25 = 0.44$ m (44 cm). Past 44 cm, objects blur through the segment. Through the distance portion, our presbyope can see clearly from optical infinity to about $1 / 0.50 = 2.0$ m (200 cm). The gap in the range of vision between 44 and 200 centimeters represents blurred mid-range vision, which can be recovered with a trifocal lens.

11.2 MULTIFOCAL OPTICAL PROPERTIES

A multifocal segment behaves optically like a small lens mounted to the major distance portion. The multifocal segment has its own optical center and axis, as illustrated in Figure 11:5. Recall from Section 4.2 that the optical axis of a lens is the reference axis joining the centers of curvature of the front and back surfaces. This also holds true for the multifocal segment when considered as an individual lens. Of course, however, the total optical power at that portion of the lens also includes the refractive effects of the distance lens. This is why the power of the segment, by itself, is called an *add*—it is in addition to the distance power of the lens.

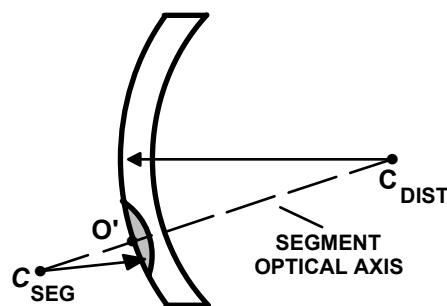


FIGURE 11:5 The optical axis of a multifocal segment is the line connecting the centers of curvature of the segment and the major portion containing the segment (C_{SEG} and C_{DIST}). The segment optical center O' is the vertex of the segment intersected by the optical axis of the segment.

Bifocal and trifocal segments are available in a variety of sizes and shapes. Some common bifocal styles, along with the optical center and size of each segment are

shown in Figure 11:6. Many trifocal segments use similar shapes. In addition to these conventional styles, there are dozens of other styles available to meet various occupational and recreational needs.

It is also important to note that in most situations, you can not actually locate the segment optical center of a multifocal lens using a focimeter. Most focimeters can only be used to measure the *combined* prismatic effects of both the major (distance) portion *and* the segment. When the optical center of a multifocal segment has been located in this fashion, it is actually the **resultant optical center**. This is the location where the net prismatic effects of the major portion and the segment total zero.

The add power of a multifocal is the difference between the *front vertex* powers of the segment and the distance/major portion of the lens. To verify the front vertex power, both the segment and the distance portion should be measured with the *front* surface (containing the segment) against the lens stop of the focimeter. This is especially critical for *plus* lenses, where the center thickness and curvature of the lens can produce significant differences between front and back vertex power measurements. A measurement is first taken in the major portion, and then a second measurement is taken within the segment; the difference yielding the *add power*.

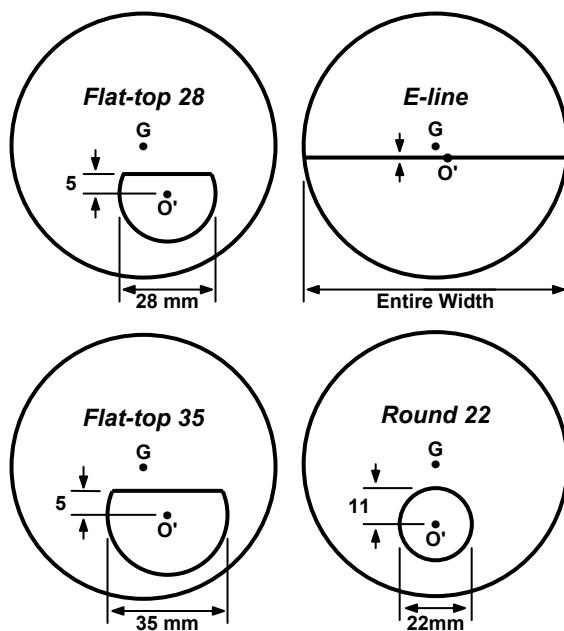


FIGURE 11:6 Some common bifocal sizes and shapes. Point G is the geometric center of the lens blank and point O' is the optical center of the segment.

One phenomenon produced by conventional multifocals is a prismatic effect called differential image displacement, or **image jump**. This is an apparent displacement of an object as the eye crosses the top

edge of the segment, as shown in Figure 11:8. Another consequence of this effect is a *blind area*, or **scotoma**, in which objects temporarily appear to collapse—or partially disappear (Rubin 272).

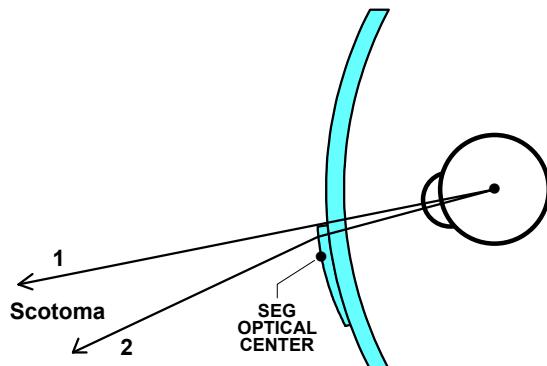


FIGURE 11:7 Differential image displacement and near scotoma. Ray 1 is passing through the major portion, immediately above the segment. Ray 2 is passing through the segment, above the optical center. Because of the base *down* prismatic effect of the segment, ray 2 is deflected downwards. This causes the image to *jump*, and results in a *blind area* between rays 1 and 2.

These two optical phenomena are consequences of the fact that a multifocal segment acts like a small lens, and produces its own prismatic effect separate from that of the distance portion. In Figure 11:8, the scotoma from the wearer's perspective has been illustrated. Note that a portion of the letter is missing as the wearer crosses into the bifocal line.

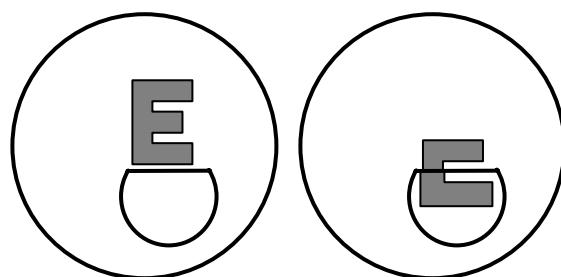


FIGURE 11:8 Differential image displacement and near scotoma. As the flat-top bifocal is moved over the object, the image of the lower portion of the object is displaced up (and magnified) from the prismatic effect of the segment. This causes objects to appear to *jump* as the wearer moves into the segment portion of the lens. Although this hasn't been illustrated here, the wearer actually perceives a double image of the object as the pupil crosses the edge of the segment. This occurs because light rays passing through both the segment and major portion of the lens enter into the pupil as it crosses segment border.

The image jump Δ_{JUMP} , in prism diopters, can be found using the add power F_{ADD} of our small segment lens and Prentice's rule:

$$\text{EQ. 63} \quad \Delta_{JUMP} = d \cdot F_{ADD}$$

where d is the distance from the top edge (or border) to the optical center O' of the segment in centimeters.

Example

A flat-top 28-mm bifocal has an add power of 2.50 D and a segment optical center 5 mm (0.5 cm) below the top edge. How much image jump is produced?

$$\Delta_{JUMP} = 0.5(2.50)$$

$$\Delta_{JUMP} = 1.25$$

\therefore Image jump is 1.25^Δ .

It should now be apparent that the farther the segment optical center of a multifocal is from the top of the segment, the more pronounced these effects become and vice versa. A multifocal lens like the E-line style (full width segment), which has its segment optical center located on the segment border, does not produce any image jump.

11.3 PROGRESSIVE ADDITION LENSES

The concept of a **progressive addition lens (PAL)** has been around since Owen Aves first patented such a lens in 1907. Although early progressive lenses were rather crude in design, they have consistently improved in both performance and sales over the past few decades. Conventional progressive addition lenses are one-piece lenses that vary gradually in front surface curvature from a minimum value in the upper, *distance* portion, to a maximum value in the lower, *near* portion (Wakefield 107).

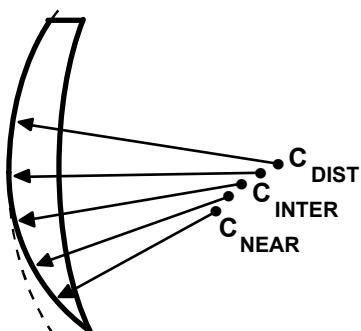


FIGURE 11:9 Cross-sectional representation of a progressive lens surface. The shorter radius of curvature in the near portion at point C_{NEAR} provides a stronger surface power than the longer radius of the distance portion at point C_{DIST} . In between these two points, the radii vary gradually to provide a smooth power change.

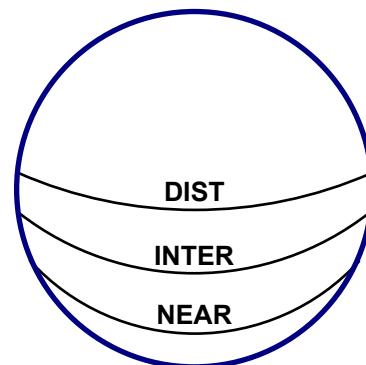


FIGURE 11:10 The change in curvature down the front surface of a progressive addition lens.

Figure 11:9 and Figure 11:10 show the gradual increase in curvature and surface power towards the lower, near portion. The result is a smooth, continuous increase in surface power that provides the necessary add power, without any visible lines of demarcation or abrupt disturbances of vision.

A typical, general-purpose progressive lens is often described as having three distinct zones of vision:

1. **Distance.** A stabilized zone located in the upper portion of the lens, which provides the necessary distance correction.
2. **Near.** A stabilized zone in the lower portion of the lens, which provides the required near addition (or add power).
3. **Intermediate.** A ‘corridor’ in the central portion of the lens connects these two zones; increasing progressively in plus power from the distance to near for mid-range vision.

These three zones of vision blend together seamlessly, providing the wearer with a continuous range of vision from near to far, as illustrated in Figure 11:11.

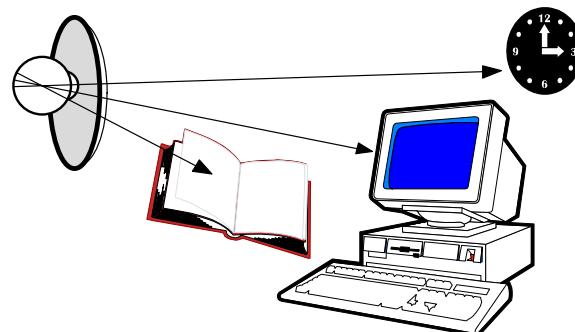


FIGURE 11:11 The three ranges of vision through a progressive addition lens.

Several methods exist for evaluating the optics of progressive addition lenses. One particularly convenient method of describing progressive lens surfaces is with **contour plots**. Contour plots are similar to

topographical maps, and use lines connecting points of equal power on the lens surface. Each contour line/shade represents an increasing level of power at a given interval, typically 0.50 D.

The gradual increase in power provided by a progressive lens surface can be described by either a **mean power plot** or a **power profile plot**. Both quantify the change in power, either by mapping the zones of increasing power, or by plotting it as the power changes along the progressive corridor (or **umbilical line**). The lens depicted in Figure 11:12 has a +2.00 D add power.

Contour plots can be a useful tool for analyzing and comparing the optics of progressive addition lenses.

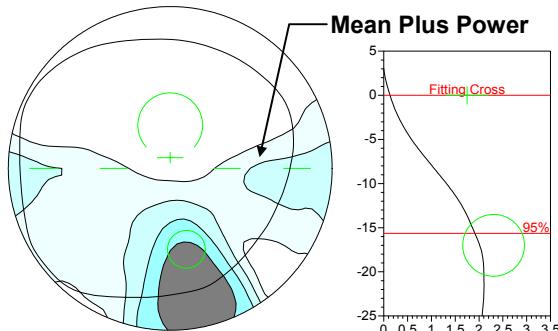


FIGURE 11:12 A contour plot is similar to a topographical map. This mean power plot shows the gradual increase in plus power towards the near portion of the lens. Each contour level represents 0.50 D of additional plus power. The four contour levels of this lens confirm the +2.00 D add power.

Progressive addition lenses offer the following advantages over conventional multifocals:

- No visible segments or lines of demarcation—provides more cosmetically appealing lenses with continuous vision, free from visually distracting borders;
- Clear vision at all distances—provides vision that more closely resembles the lost accommodation of the eyes; and
- No unwanted differential image displacement—or *jump*—ensures that there are no abrupt disturbances of vision.

There are an infinite number of possible ways to design a PAL. Every progressive lens design requires a globally smooth surface that provides a gradual transition in curvature from the distance portion down into the near portion. Further, this gradual blending of curvature means that the add power is gradually changing across a large area of the lens surface, not just at the bottom (see Figure 11:12).

Unfortunately, this change in curvature results in an inevitable consequence: **unwanted surface**

astigmatism. Surface astigmatism produces an unwanted **astigmatic error** (or **cylinder error**) that can, in sufficient quantities, blur vision and limit the wearer's field of clear vision. Therefore, this astigmatic error essentially serves as a boundary for the various zones on the progressive lens surface. By design, the areas of unwanted astigmatism are located in the lower quadrants, lateral to the corridor and the near area, where they will be least noticeable or bothersome to the wearer.

The unwanted astigmatism, which is a consequence of the lens design, is influenced by:

- **Add power.** The amount of astigmatism will be proportional to the add power of the lens. A +2.00 D add, for example, will generally produce twice as much astigmatic error as a +1.00 D add.
- **Length of the progressive corridor.** The area of the lens which generally connects the distance area and the near area and which contains the power progression is referred to as the **corridor**. Shorter corridors produce more rapid power changes along the corridor and higher levels of astigmatism. This reduces the eye movement required to reach the near zone. Longer corridors provide more gradual power changes and lower levels of unwanted astigmatism, but increase the eye movement required to reach the near zone of the lens.
- **Width of the distance and near zones.** Wider distance and near zones, which have the advantages of wider fields of clear vision, confine the astigmatism to smaller regions of the lens surface, but produce higher magnitudes of unwanted astigmatism. Narrower distance and near zones have the opposite effects.

A well-designed progressive lens will reduce the amount of astigmatic error to its mathematical limits for a given design. During the design and optimization process, various parameters are adjusted to control and manipulate the distribution and magnitude of this astigmatic error across the progressive lens surface. The width of the near and distance zones, and the length of the progressive corridor, are the chief parameters that are altered, as shown in Figure 11:13.

In order for the lens to have a usable progressive corridor and near zone, the unwanted astigmatism produced by the change in curvature along the umbilical line needs to be minimized. To achieve this, any horizontal section of the progressive corridor and near zone should have nearly the same surface power as the vertical section at that point. This will ensure that the area appears nearly spherical to the eye. Away from the central umbilic, the curvature is adjusted to produce a

smooth, continuous surface from the distance zone down to the near zone (Smith & Atchison 147).

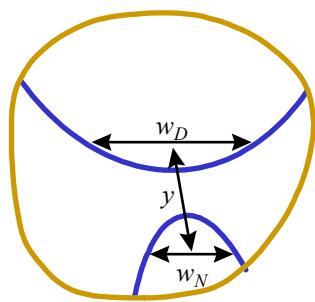


FIGURE 11:13 Line w_D is the width of the distance zone at a specified height; w_N is the width of the near zone at a specific depth; and y is the length of the progressive corridor (or *umbilical line*) connecting them.

To better understand how the length of the progressive corridor and the add power can affect the rate of change (and hence the magnitude of the astigmatic error), the approximate change in add power—per millimeter—is given by (Jalie 20):

$$\text{EQ. 64} \quad \Delta F = \frac{F_{\text{ADD}}}{y}$$

where ΔF is the increase in power for every 1 millimeter in diopters, F_{ADD} is the add power of the lens, and y is the length of the corridor in millimeters.

Consequently, the power of a progressive lens surface changes more rapidly down the progressive corridor as the add power increases, or as the length of the progressive corridor decreases. The magnitude, distribution, and rate of change (or *gradient*) of resulting astigmatism are all performance factors that can affect the wearer's acceptance of the lens.

Example

A certain PAL has a +2.00 D add, and a corridor length of 17 mm. What is the approximate increase in add power for every one millimeter?

$$\Delta F = \frac{2.00}{17}$$

$$\Delta F = 0.118$$

∴ Power changes 0.12 D every 1 mm.

Progressive lenses are often arbitrarily classified into two broad categories, or **design philosophies**, by the relative magnitude, distribution, and gradients of their surface astigmatism. (The *gradient* is the rate of change of power and unwanted astigmatism.) These various characteristics describe the *relative hardness* of the design. Lenses within each category show broad

similarities in the magnitude, distribution, and gradient of their surface astigmatism (or *hardness*):

- **Harder Designs.** A *harder* PAL design concentrates the astigmatic error into smaller areas of the lens surface, thereby expanding the areas of perfectly clear vision at the expense of higher levels of blur and distortion. Consequently, harder PAL designs generally exhibit four characteristics when compared to softer designs: wider distance zones; wider near zones; shorter narrower corridors; and higher, more rapidly increasing levels of astigmatic error (i.e., *higher* gradients).

The astigmatic error produced by a *harder* progressive lens design is demonstrated with the **surface astigmatism plot** in Figure 11:14. This contour plot is in 0.50 D levels of cylinder. A power profile plot next to it illustrates the shorter corridor length.

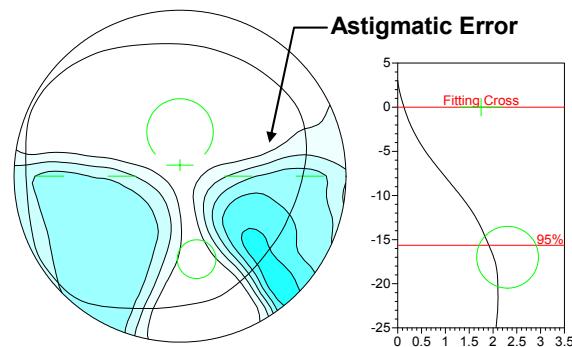


FIGURE 11:14 Relatively 'hard' PAL design.

- **Softer Designs.** A *softer* PAL design spreads the astigmatic error across larger areas of the lens surface, thereby reducing the overall magnitude of blur at the expense of narrowing the zones of perfectly clear vision. The astigmatic error may even encroach well into the distance zone. Consequently, softer PALs generally exhibit four characteristics when compared to harder designs: narrower distance zones; narrower near zones; longer, wider progressive corridors; and lesser, more slowly increasing levels of astigmatic error (i.e., *lower* gradients). See Figure 11:15.

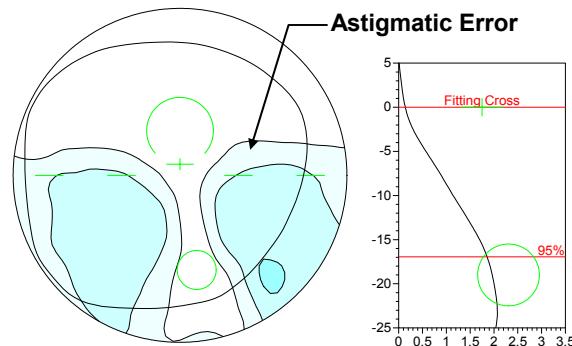


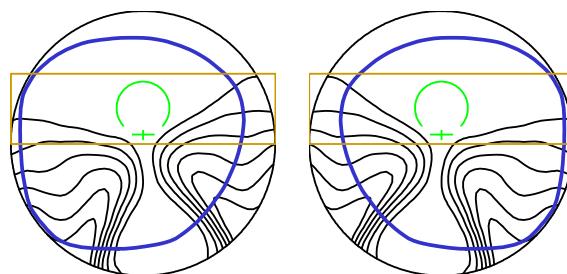
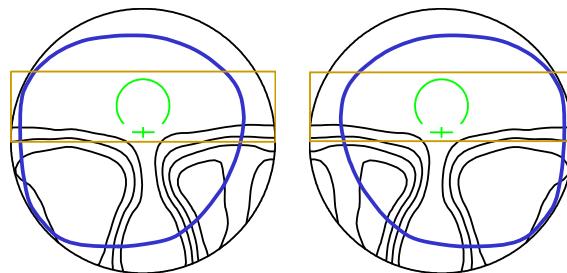
FIGURE 11:15 Relatively ‘soft’ PAL design.

In general, harder PAL designs will provide wider fields of view, and will require less head and eye movement, at the expense of more swim and blur. Softer PAL designs will provide reduced levels of astigmatism and swim, while limiting the size of the zones of clear vision and requiring more head and eye movement. Newer design PALs are seldom absolutely ‘hard’ or absolutely ‘soft.’ Unfortunately, such terms do not satisfactorily describe newer lenses. Many of the recent PAL designs incorporate a balance between the two design philosophies.

Some manufacturers now choose to vary the specific progressive lens design used for each add power and/or base curve throughout the entire lens series:

- **Multi-designs.** Some PAL manufacturers vary the surface design of each lens in a progressive lens series based upon the add power. These *multi-design* (or ‘add-specified’) lenses are designed to consider the effects of increasing astigmatic errors, as well as the change in the wearer’s visual needs as his/her presbyopia advances. Since these effects accompany an increase in add power, the lens design can be modified accordingly for each add.
- **Design by Prescription Designs.** One manufacturer has recently introduced a *Design by Prescription™* progressive lens series. These lenses vary the design of each base curve and add power combination based upon both the add power *and* the distance prescription. This philosophy considers not only advances in presbyopia, but also optical differences between various distance refractive errors.

Early progressive lenses were *symmetrically* designed so that the right and left lenses were identical. To achieve the desired inset for the near zone, the lens blanks were rotated in opposite directions by 9 or 10° to create right and left lenses. The principal drawback to this was the disruption of binocular vision as the wearer gazed laterally across the lens, since the astigmatism differed between the nasal and temporal sides of the distance zone. Most newer lens designs, however, are *asymmetrically* designed with separate right and left lenses. The amount of astigmatic error on either side of the progressive corridor can now be adjusted independently. Compare the distance zones in the early *symmetrical* right and left designs in Figure 11:16 with the *asymmetrical* designs in Figure 11:17.

**FIGURE 11:16** Early *symmetrical* PAL design in which the distortion rises into the nasal distance zone on both lenses after they have been rotated. When the right and left eyes look to the left, for instance, the right eye encounters blur while the left eye does not.**FIGURE 11:17** Newer *asymmetrical* PAL design.

Because of their change in curvature, all progressive lenses are inherently *aspheric* (or non-spherical). Today, this term is used quite loosely and should probably be qualified with some additional details. For example, some manufacturers use base curves that are flatter than conventional *best form* base curves for their PAL designs. Asphericity is applied in the distance portion of the lens to compensate for this; this is the same concept utilized for single vision aspheric lenses. Others may refer to their designs as aspheric if the distribution of unwanted surface astigmatism encroaches significantly into the upper half of the lens. Still others may call any PAL design aspheric by using the word in its strictest sense.

In addition to the astigmatic error inherent in a progressive lens, the change in power and magnification produced by the corridor and near zone of the lens results in **skew distortion**. Objects, like straight lines, may appear curved—or skewed—when viewed through the lateral areas in the lower portion of the lens. The *more curved* a vertical line appears, the *less orthoscopic* the lens is through that particular zone. This is illustrated in Figure 11:18 (Jalie 23).

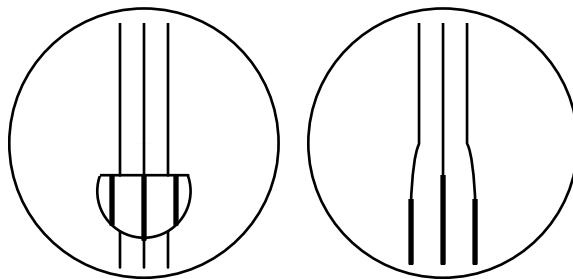


FIGURE 11:18 The change in power and magnification through the progressive corridor and near zone produces skew distortion, when compared to conventional multifocals.

TABLE 19 ISO/ANSI PAL markings

Reference Markings	Abbr	Type
Prism reference point Surfacing layout Prism verification	PRP	Remov.
Distance reference point Distance power verification	DRP	Remov.
Fitting cross Fitting reference Finishing layout	FC	Remov.
Near reference point Add power verification	NRP	Remov.
Alignment reference mark Axis alignment Re-marking lenses Identification of lens type or manufacturer	ARM	Perm.
Add power Identification of add power	ADD	Perm.
Logo Identification of lens type or manufacturer	LOGO	Perm.

Progressive addition lenses are supplied with two types of markings for layout, power verification, dispensing, and identification purposes. *Removable* markings, which are inked on, identify the layout, verification, and dispensing points of the lens. *Permanent* markings, which are engraved upon the surface, provide the identification and add power of the lens, as well as locator marks to reapply the ink markings if necessary. The standardized locations of these markings are shown in Figure 11:19 and Figure 11:20. Table 19 above provides some additional information about each marking, and its application (ANSI 23).

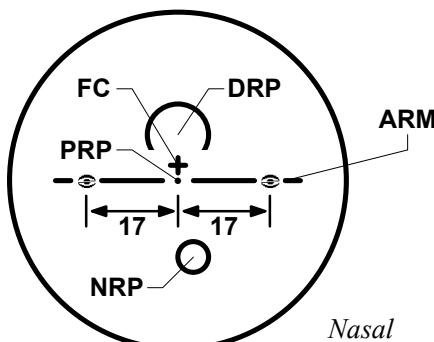


FIGURE 11:19 Temporary ink markings.

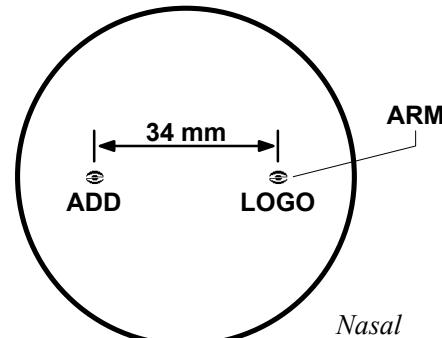


FIGURE 11:20 Permanent engravings.

11.4 BLENDED BIFOCAL LENSES

There is another type of bifocal lens that falls under the ‘invisible multifocal’ umbrella. The **blended bifocal** is a one-piece, round-style bifocal that has had the border between the segment and the major portion literally blended away. These multifocals have no visible line of demarcation (or apparent segment border) as a result. Unfortunately, an annular zone surrounding the segment, with a high degree of surface astigmatism, is created from the blending process. This astigmatic error causes blurred vision through the zone, which increases as the add power increases. Further, the level of surface astigmatism also becomes greater as the width of the blending zone is made narrower.

It is important to note that blended bifocals have no progressive power characteristics. They also suffer from image jump like conventional multifocals. A typical blended bifocal, the **EZ 2 Vue lens**, is shown in Figure 11:21.

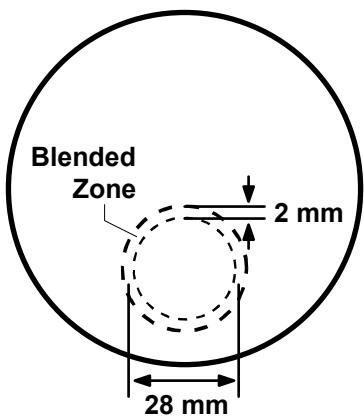


FIGURE 11:21 The *EZ 2 Vue* lens. Notice the annular zone around the segment, where the border between the segment and the major portion has been blended out. This is an area of high surface astigmatism and blur, whose magnitude increases as the add power increases. The surface astigmatism also increases as the width of the blending zone becomes narrower.

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12. Assorted Topics

Now that we have covered the basics of ophthalmic lenses, and their application to the eye, we can consider some additional, ‘miscellaneous’ concepts. These next sections extend many of the principles that we have examined previously. Various topics like lap tools, vertex distance compensation, vertical imbalance, and magnification will be presented.

12.1 LAP TOOLS AND THE CURVE VARIATION FACTOR

To produce a curved refracting surface upon a spectacle lens material, the surface can be *cast* to the desired curvature when the material is in its liquid monomer state, or it can be *ground* to the desired curvature using any one of various milling machines. Once the surface curve of an ophthalmic lens has been ground, the surface is fined and polished using abrasives against a hard tool, called a **lap tool**, that corresponds to the desired curvature of the lens. For instance, a -6.00 D, *concave* surface curve is produced using a +6.00 D, *convex* lap tool as shown in Figure 12:1. Lap tools are available in an assortment of surface powers that allow the laboratory to produce a wide range of back curves.

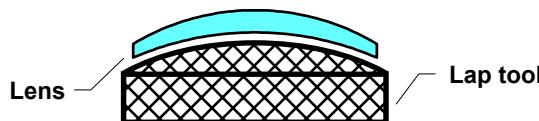


FIGURE 12:1 Lap tool and lens.

Keep in mind that a given lap tool, with its single radius of curvature, will produce different surface powers on lens materials with different indices of refraction. Currently, there are dozens of different lens materials available with a variety of refractive indices. For convenience the powers of these tools, as well as the instruments designed to measure them, are frequently referenced to a single, standard index of refraction. In the United States, this standard **tooling index** is 1.530. Manufacturers may also use this tooling index when describing the surface powers of their lens blanks. This new, *1.530-based* curve is known as the **true curve**. Here are some other general points to remember about lap tools:

- The degree of accuracy for producing surface powers using a set of lap tools can be no better than $\frac{1}{2}$ of the increment between each lap tool. For instance, the accuracy of a set of lap tools that come in 0.10 D increments is +/-0.05 D.
- As the refractive index of a material *increases*, the increment of the lap tools effectively *decreases*. For instance, a 0.10 D change in a lap tool (referenced to a 1.530 refractive index) will produce a 0.09 D change in surface power on a lens material with a

1.500 refractive index, but only a 0.12 D change in surface power on a lens material with a 1.66 refractive index.

- Since placing abrasive pads (with a given thickness) on the lap tool effectively changes its radius of curvature, a slight *pad compensation* is sometimes necessary for steeper curves. This can be done by simply adding the thickness of the pad to the radius of curvature of the lap tool to determine the effective radius of curvature of the pad/lap tool combination.

We should emphasize the point that two curves with the same curvature can have two different surface powers—depending upon the refractive index of the material (Eq. 14). A given lap tool can be used to grind its curvature onto a wide variety of lens materials and, consequently, can produce a wide variety of surface powers. Lap tools are labeled with a surface power corresponding to a refractive index of 1.530 for simplicity and consistency.

It is possible to quickly determine the actual surface power of a curve based upon the *1.530-based* value. This might be necessary if a lens surface is measured by a lens measure, which is based upon a 1.530 index. It might also be appropriate if you wish to determine the lap tool required to grind a certain surface power on a lens, which will generally have an index that differs from the 1.530 tooling index. The actual power F_{ACT} of a lens surface is given by

$$F_{ACT} = \frac{n_{ACT} - 1}{r}$$

where n_{ACT} is the refractive index of the lens material and r is the radius of curvature. The reference power F_{REF} of a lens surface is given by

$$F_{REF} = \frac{n_{REF} - 1}{r}$$

where n_{REF} is the reference refractive index (often 1.530). Equating both formulas for r gives us

$$\frac{F_{ACT}}{F_{REF}} = \frac{n_{ACT} - 1}{n_{REF} - 1}$$

Finally, we are able to solve for the actual surface power F_{ACT} —without knowing the curvature—by using the **curve variation factor**:

$$\text{EQ. 65} \quad F_{ACT} = \left(\frac{n_{ACT} - 1}{n_{REF} - 1} \right) F_{REF}$$

This formula allows us to calculate the actual surface power of a lens, based upon the refractive index n_{ACT} of the lens material, if the measured or reference surface power is known, relative to a particular reference index

n_{REF} . This curve variation factor can also be used to compare different materials to each other, as well.

Example

A lens measure—based upon a 1.530 tooling index—is used on a high-index glass lens with an index of 1.700 and shows a reading of +4.50 D. What is the actual surface power?

$$F_{\text{ACT}} = \left(\frac{1.700 - 1}{1.530 - 1} \right) 4.50$$

$$F_{\text{ACT}} = 1.321(4.50)$$

$$F_{\text{ACT}} = 5.94$$

∴ Surface power is +5.94 D.

12.2 EFFECTIVE POWER

This section will explore two closely related concepts: *effective power* and *vertex distance compensation*. We have previously pointed out that the focal power of a lens is based upon the position of the reference plane from which the focal points are measured. In our discussion of thick lenses (Section 4.4), for instance, we referred to the *back vertex power* of a lens as the reciprocal of the distance from the back vertex of the lens surface to the secondary focal point.

Consider the *thin lens* in Figure 12:2. The vergence of light at the plane of the lens is +5.00 D. At 10 cm from the lens, however, the vergence is +10.00 D! At 15 cm from the lens, which is 5 cm from the secondary focal point F' , the vergence is +20.00 D. The **effective power** of the lens is simply the image vergence of light at some chosen reference plane.

If the reference plane is kept stationary while the lens is moved, the same effect is produced: a change in the effective power of the lens relative to the intended reference plane. Therefore, the position of the lens is critical in order to maintain the correspondence between the secondary focal point and the intended focal plane of a lens. If the lens is moved away from its intended position—or reference plane—it creates a change in power (or vergence) relative to the intended location of the lens. If the lens is moved to the left or right, for instance, the image created is also shifted to left or right by the same distance.

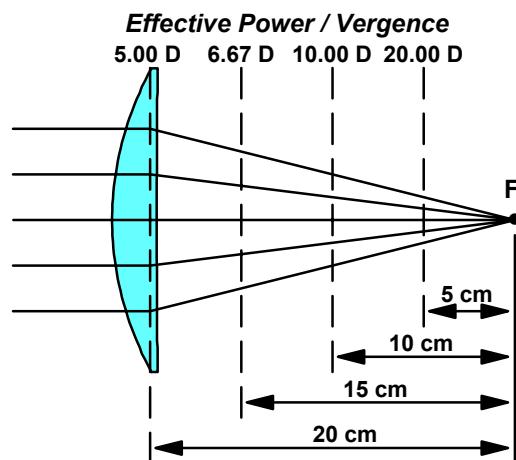


FIGURE 12:2 The effective power of the +5.00 D lens is based upon the distance of the lens from various reference planes. For instance, at a reference plane 5 cm from the lens (or 15 cm from the secondary focal point F'), the image vergence becomes 6.67 D. At 15 cm from the lens (or 5 cm from F'), the vergence becomes +20.00 D.

Ideally, the secondary focal point F' of a lens should fall on the *intended focal plane* of the optical system when the lens is at its intended position (or reference plane). For an image to be formed at this plane from an object at infinity, the secondary focal length f' of the lens—when placed at the intended position—should be equal in length to the distance between the intended focal plane and the intended position of the lens. Figure 12:3 and Figure 12:4 demonstrate the shift of the secondary focal points F' of a plus lens and a minus, as the lenses are moved by a distance d to the left of the intended positions.

The *effective power*, or vergence at the reference plane, in both situations is simply equal to the reciprocal of the **effective focal length** f_E . This is the distance from the actual location of the *secondary focal point* F' of the lens to the intended lens position, or reference plane.

In these examples, the *plus* lens has been shifted away from the focal plane, effectively *increasing* the power of the lens relative to the intended position of the lens. Moving the *minus* lens in the same direction, however, shifts the lens towards the focal plane, effectively *decreasing* the power of the lens relative to the intended position of the lens. After a given shift d in lens position, the *effective* focal length f_E is equal to $f_E = f' \pm d$. All distances should preferably be in meters, though millimeters will also work if each variable uses the same unit of measure.

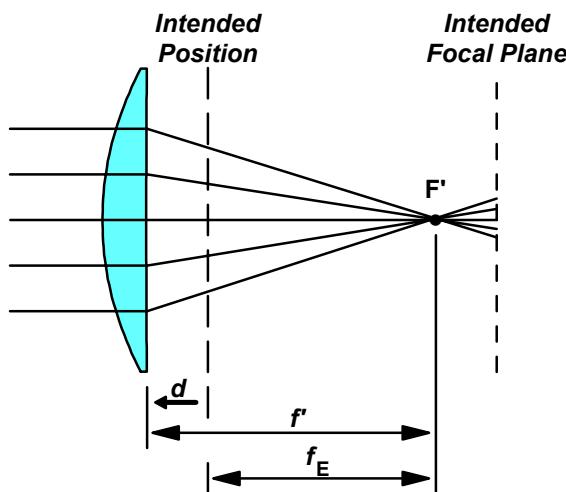


FIGURE 12:3 This *plus* lens has been moved to the left from its intended position by a distance d ; effectively increasing its power. The *effective* focal length f_E , at the reference plane, is now $f_E = f' - d$. This is *shorter* than the original focal length f' .

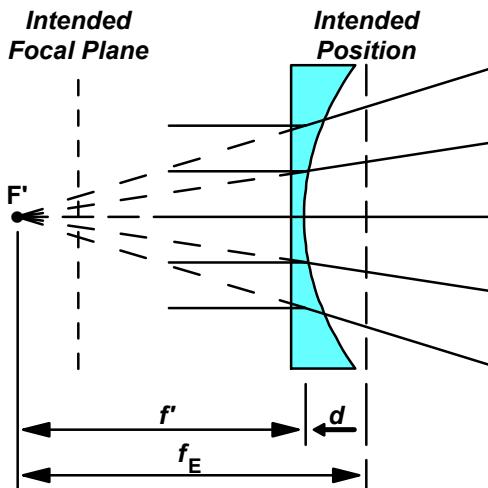


FIGURE 12:4 This *minus* lens has been moved to the left from its intended position by a distance d ; effectively reducing its power. The *effective* focal length f_E , at the reference plane, is now $f_E = f' - d$ (f' is negative). This is *longer* than the original focal length f' .

In order to compensate for this shift, the secondary focal length must be adjusted by the same distance d that the lens was moved, as illustrated in Figure 12:5 and Figure 12:6. This will require a modification to the actual focal power of the lens. To find the new, **compensated power** F_C required to offset the change in *effective power* of the original lens—with a focal power of F —we can use Figure 12:5 and Figure 12:6. First consider that:

$$F_C = \frac{1}{f_C}$$

where f_C is the **compensated focal length** in meters.

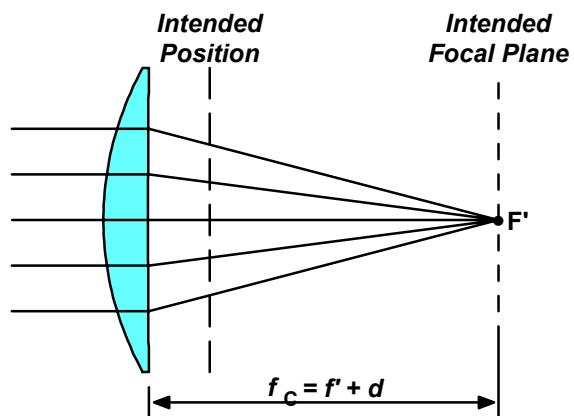


FIGURE 12:5 For this *plus* lens the *compensated* focal length f_C is *longer* than the original focal length f' . The compensated focal length is equal to $f_C = f' + d$.

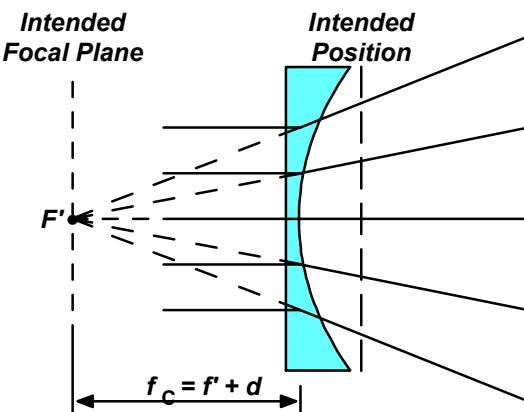


FIGURE 12:6 For this *minus* lens the *compensated* focal length f_C is *shorter* than the original focal length f' . The compensated focal length is equal to $f_C = f' + d$.

From the diagrams, we can see that $f_C = f' \pm d$. Remember that the secondary focal length f' of a lens is equal to the reciprocal of its focal power, $1 / F$. Substituting for both f' and f_C , gives us

$$F_C = \frac{1}{\frac{1}{F} \pm d}$$

Clearing the fraction from the denominator gives us the **compensated power formula**:

$$\text{EQ. 66} \quad F_C = \frac{F}{1 \pm d \cdot F}$$

where d is the movement in meters.

The following sign convention should be used for d :

- Lens movement to the *left*, or *away* from the eye, represents a *positive* (+) value for the distance d .
- Lens movement to the *right*, or *towards* the eye, represents a *negative* (-) value.

For ophthalmic lenses, the intended position of the lens, relative to the *corneal apex* of the eye, is determined during the eye examination. Lenses used during an examination to refract the eye are generally positioned at approximately 13 to 14 mm from the apex of the cornea. The distance of the back vertex V' of the lens to the apex of the cornea is known as the **vertex distance**. When the spectacle lens is placed at the refracted vertex distance, the secondary focal point should coincide with the intended focal plane of the eye. Recall from Section 6.2 that the intended focal plane of the eye is called the **far point (M_R)**.

We have discussed the actual compensated power required to offset the change in effective power. In addition, it is possible to determine the approximate amount of power ΔF for a given movement if we expand our previous formula (using a binomial expansion) and drop ‘higher order’ terms. This approximation gives us:

$$\text{EQ. 67} \quad \Delta F = F^2 \cdot d$$

where ΔF is the change in power, F is the original focal power, and d is the lens movement in meters.

This formula can be used to both predict the *approximate* change in effective power, and to determine the amount of power change needed to compensate for it (compensated power).

The concept of compensated power applies if the spectacle lens is not fitted at the vertex distance. A change in effective power will result unless the focal power of the lens is adjusted accordingly. This process is known as **vertex distance compensation**. The same formula (Eq. 66) is used. The shift d in this situation refers to the difference between the *refracted* vertex distance and the *fitted* vertex distance. Recall that the *refracted* vertex distance is the vertex distance used by the prescriber while determining the initial spectacle correction. The *fitted* vertex distance is the vertex distance of the actual spectacle frame and lenses (or the finished eyewear).

Figure 12:7 outlines the steps involved in vertex distance compensation for a *plus* lens that has been moved away from the eye. Of course, the steps are automatically performed for you when the formula is used. You should pay careful attention to your sign convention (for d), until you develop an intuition about the effects of vertex distance. After that, you should be able to tell whether or not you used the wrong sign (\pm) for d .

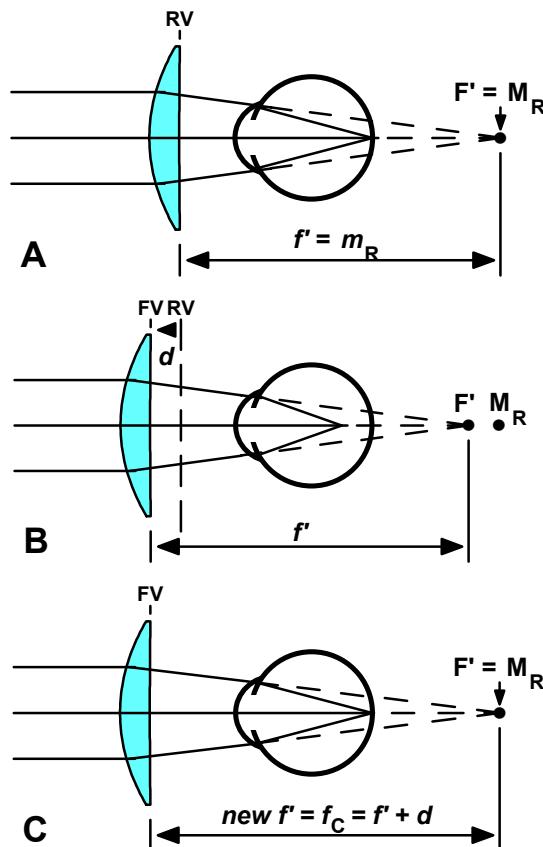


FIGURE 12:7 Vertex distance compensation. A) The secondary focal point F' of this *plus* lens has been prescribed to coincide with the far point M_R at the refracted vertex distance (RV). B) The lens has been moved forward by a distance d from the refracted vertex distance to the new, fitted vertex distance (FV). The far point M_R no longer coincides with the secondary focal point F' . C) The original lens has been replaced with a *weaker* plus lens to compensate for the change in effective power. The secondary focal length of the new lens is equal to the compensated focal length, or $\text{new } f' = f'_c = f' + d$. The new secondary focal point F' now coincides with M_R again.

Here are some additional considerations regarding effective power compensation:

- **Low-powered lenses:** The effective power for lenses with little or no power will not need compensation. It only becomes necessary to compensate for effective power when fitting spectacle lenses over ± 6.00 D, or when fitting contact lenses over ± 4.00 D (because of the significant distance from the spectacle plane). The prescriber may note the refracted vertex distance on the prescription if compensation may be necessary. If it is not noted, then 13 to 14 mm is a good assumption.

To understand why compensation is only critical for high-powered lenses, consider moving a 10.00 D lens—with a focal length of 100 mm—a distance of 5 mm. This distance represents 5% of the focal

length. Now consider moving a 1.00 D lens, with a focal length of 1000 mm, the same distance of 5 mm. This distance only represents 0.5% of the focal length.

- **Plus lenses:** Increasing the vertex distance of a plus lens will increase the effective power of the lens. To compensate for this, a weaker lens should be ordered. Alternately, a decrease in the vertex distance will require a stronger lens.
- **Minus lenses:** Increasing the vertex distance of a minus will decrease the effective power of the lens. A stronger lens should be ordered to compensate for this. Alternately, a decrease in the vertex distance will require a weaker lens.
- **Sphero-cylindrical lenses:** Each principal meridian of a lens with cylinder power should be calculated independently. Do NOT compensate the actual cylinder power! The new nominal cylinder power will be the difference between the two compensated principal meridians.

Example

A +10.00 D lens is moved away from the eye by 5 mm (0.005 m). What is the change in power?

$$\Delta F = 10.00^2(0.005)$$

$$\Delta F = 0.50$$

∴ Change in power is 0.50 D.

Because this is a *plus* lens moved *away* from the eye, the change is an *increase* in *plus* power. It is interesting to note that the *exact* value is closer to 0.53 D. This difference is quite negligible for our purposes. To compensate for the change in vertex distance using the approximate formula, simply subtract 0.50 D from the initial power (since it is effectively increased) so that $+10.00 - 0.50 = +9.50$.

Example

A prescription calling for a -6.50 DS -1.50 DC × 090 lens is prescribed at a 14-mm vertex distance, but fit at an 11-mm vertex distance. In this case, the lens is being moved to the *right* towards the eye (remember the sign convention). This represents a negative (-) movement of 3 mm (0.003 m). What compensated power should be ordered?

First, calculate both principal meridians individually. For the meridian F_{CSPH} containing the *sphere* power use -6.50. For the meridian F_{CCYL} containing the *cylinder* power use $-6.50 + (-1.50) = -8.00$.

$$F_{CSPH} = \frac{-6.50}{1 - 0.003(-6.50)}$$

$$F_{CSPH} = \frac{-6.500}{1.0195}$$

$$F_{CSPH} = -6.38$$

$$F_{CCYL} = \frac{-8.00}{1 - 0.003(-8.00)}$$

$$F_{CCYL} = \frac{-8.000}{1.024}$$

$$F_{CCYL} = -7.81$$

The new sphere power F_{CSPH} is -6.38 D. The new cylinder power C_C is the difference between the two compensated principal meridians ($F_{CCYL} - F_{CSPH}$):

$$C_C = -7.81 - (-6.38)$$

$$C_C = -1.43$$

∴ Compensated prescription is:

$$-6.38 \text{ DS } -1.43 \text{ DC } \times 090.$$

12.3 NEAR VISION EFFECTIVITY

Earlier, we examined the conjugate foci formula (Eq. 21) for calculating the image vergence produced by a *thin* lens for a given object vergence. Thick lenses complicate the use of our conjugate foci formula, somewhat. Instead of simply adding the incident object vergence to the vertex power F_V of the lens, we must add the entering object vergence L to the surface power F_1 of the front curve of the lens, or $F_1 + L$. This combined value should be substituted back into the back vertex power formula. Hence, the final image vergence L' is given by

$$\text{EQ. 68} \quad L' = \frac{F_1 + L}{1 - \frac{t}{n}(F_1 + L)} + F_2$$

where L is the object vergence and L' is the image vergence in diopters.

Example

A wave front with -3 D of divergence (from an object at 33 cm)—strikes a +4.00 D thick lens. The lens has a front curve of +8.00 D, a back curve of -4.17 D, a center thickness of 4 mm (0.004 m), and a refractive index of 1.500. What is the image vergence exiting the lens?

$$L' = \frac{8 + (-3)}{1 - \frac{0.004}{1.500}[8 + (-3)]} + (-4.17)$$

$$L' = \frac{5}{0.9867} + (-4.17)$$

$$L' = 5.07 + (-4.17)$$

$$L' = 0.90$$

∴ Final image vergence is +0.90 D.

Now consider the thin lens result using the conjugate foci formula (Eq. 21):

$$L' = F + L$$

$$L' = +4.00 + (-3) = 1$$

The difference between the *exact* image vergence and the *approximate* image vergence is referred to as the **near vision effectivity error**. In our example, the near vision effectivity error is $0.90 - 1.00 = -0.10$ D.

The clinical significance of this error is that the form and thickness of a *thick* lens will affect the actual image vergence produced for a given object distance. Although two lenses may have the same back vertex power (for an object at optical infinity), they will produce slightly different image vergences for objects at *near* if their lens forms are different. Consequently, a reading prescription determined using *trial* lenses of one form and thickness may differ slightly in performance once the actual lenses are produced using another form.

12.4 VERTICAL IMBALANCE AT NEAR

For single vision lenses, the wearer does not necessarily have to look away from the optical centers of the lenses. For multifocal lenses, however, the wearer is required to depress the lines of sight down into the segments or near zones of the lenses. For lenses with unequal powers—for the correction of *anisometropia*—this can produce certain undesirable side effects. **Anisometropia** is a difference in the refractive errors between the two eyes (e.g. OD -4.00 DS and OS -1.00 DS). The most notable consequence of anisometropia is *vertical prismatic imbalance at near*. When the eyes lower to read something through the segments of multifocal lenses with differing focal powers, unequal vertical prismatic effects are encountered.

The **reading level** of a multifocal is the position through the segment or near zone of a multifocal that the wearer will most likely look through for near vision. For a flat-top bifocal, the reading level is typically about 5 mm below the top of the segment. The differences in prismatic effects between two lenses are illustrated with the contour plots below in Figure 12:8. Notice that at the segment reading levels, 15 mm below the optical centers of the lenses, the +1.00 D lens produces 1.50^Δ of

prism, while the +2.00 D lens produces twice that with 3.00^Δ of prism.

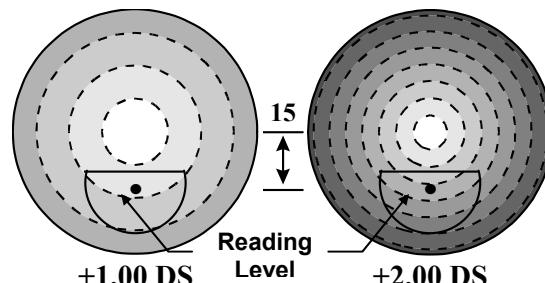


FIGURE 12:8 Differential prismatic effects at near. Each shaded contour represents 1^Δ of prism. Notice that the +1.00 DS lens has 1.50^Δ (base up) at the reading level, while the +2.00 DS lens has 3.00^Δ (base up) at the reading level. The total vertical imbalance is $3.00 - 1.50 = 1.50^\Delta$ (base up in the +2.00 DS lens).

There are several methods available to compensate for the vertical prism imbalance produced at near. The most common method involves grinding a second distance curve on the lower half of one of the lenses. The second curve is ground with *base up* prism in the *lower* portion of the lens only; enough to compensate for the vertical prism imbalance produced between the two lenses at near. Essentially, the two curves are tilted with respect to each other to produce the differential prismatic effect. This process is referred to as **bi-centric grinding**, since it effectively produces two distance optical centers. For plastic lenses, the bi-centric grinding is done on the back surface, as illustrated in Figure 12:9.

The finished lens is often referred to as a **slab-off** lens. The lens with the least *plus* or most *minus* power in the *vertical* meridian will be the lens used for the slab-off. For glass lenses, the bi-centric grinding is done on the front surface

There are also pre-made plastic lenses available that have been *molded* with bi-centric curves. These lenses typically have a *base down* prismatic effect cast onto the near portion of the *front* of the lens. Consequently, such a lens is generally referred to as **reverse slab-off** lens. The *reverse slab-off* lens will be the lens with the most *plus* or least *minus* power through the *vertical* meridian.

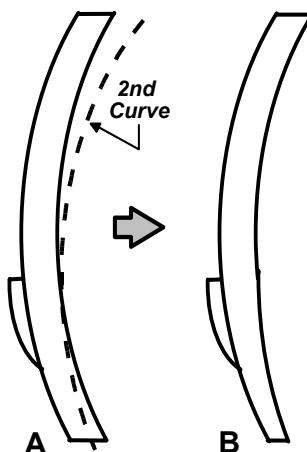


FIGURE 12:9 A) A conventional bifocal lens blank is ground with a second distance curve, containing the desired amount of prism, onto the lower half of the back surface. B) The final slab-off lens blank is now *bi-centric* and has two back curves.

The process of bi-centric grinding, or molding, produces a definite line across the entire lens. This line represents the junction between the two distance curves and should follow the top edge of the bifocal segment—as shown in Figure 12:10. Once the wearer crosses this *slab* line, the prismatic effect of the lower curve is encountered.

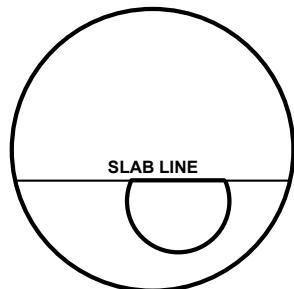


FIGURE 12:10 The line produced by bi-centric grinding.

The fastest way to calculate the vertical prism imbalance Δ_{VERT} at near is to simply determine the difference in power F_{DIF90} between the two vertical meridians of the lenses, and then multiply this difference by the reading level distance y , in centimeters (Prentice's rule, Eq. 42):

$$\text{EQ. 69} \quad \Delta_{VERT} = y \cdot F_{DIF90}$$

where Δ_{VERT} is the vertical prism imbalance, F_{DIF90} is the difference in power between the two vertical meridians, and y is the reading level distance from the optical centers in centimeters.

It is also important to note that the power through the vertical meridian of lenses with cylinder should be used to determine the net imbalance. When the cylinder has an oblique axis, use the sine-squared method to

determine the power through the vertical meridian (Eq. 31).*

Example

You are given a prescription for a right (OD) lens of +2.00 DS -1.00 DC \times 180 and a left (OS) lens of +3.50 DS -2.00 DC \times 060. The lenses will have a reading level of 10 mm (1.0 cm). What amount of slab-off prism would you order to compensate for the vertical imbalance at near?

First determine the power through the vertical meridians. For the *right* lens, the power F_{R90} through the vertical meridian is simply equal to combined sphere and cylinder power since the axis is at 180°, so that $F_{R90} = F_{SPH} + C$:

$$F_{R90} = +2.00 + (-1.00)$$

$$F_{R90} = +1.00$$

For the *left* lens, the power F_{L90} through the vertical meridian can be found with our sine-squared formula—once we determine the axis θ between the vertical meridian and the cylinder axis:

$$\theta = 90 - 60 = 30^\circ$$

$$F_{L90} = 3.50 + \sin^2 30(-2.00)$$

$$F_{L90} = +3.50 + (-0.50)$$

$$F_{L90} = +3.00$$

These powers have been confirmed on the optical crosses in Figure 12:11.

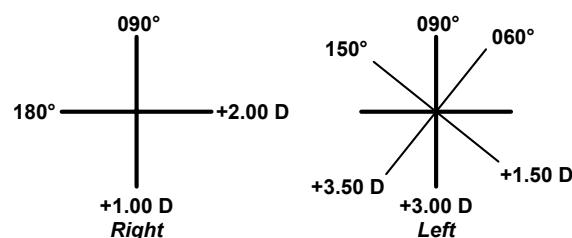


FIGURE 12:11 Optical cross diagrams for the right and left eyes. The power through the vertical meridian F_{R90} of the *right* lens is +1.00 D, while the power through the vertical meridian F_{L90} of the *left* lens is +3.00 D. The difference in power between the two vertical meridians is $F_{R90} - F_{L90} = +2.00$ D.

Now, calculate the prism imbalance at near:

* Although there are trigonometric methods that are more accurate, we will use our sine-squared approximation here. Since the final prism values are often rounded to the nearest $\frac{1}{2}^\Delta$, our approximation provides an acceptable level of accuracy.

$$\Delta_{VERT} = 1.0(+2.00)$$

$$\Delta_{VERT} = 2.00$$

\therefore Vertical imbalance is 2.00^Δ .

The slab-off lens will be the lens with the least plus or most minus. In this instance, the right lens—which has the least plus power through the vertical meridian—would be the lens to use for the bi-centric grinding (slab-off) operation.

Another way to look at this is to consider the residual (net) prismatic imbalance. The net prism imbalance is actually 2.00^Δ base up in the left lens, which had the most plus power to begin with. Balancing this prism effect would require adding base up prism to the near portion of the right lens as well. Conversely, if a reverse slab-off lens were used, base down prism would be added to near portion of the left lens to neutralize the residual base up effect.

Although bi-centric grinding is the most popular way to minimize vertical imbalance at near, it is certainly not the only way. The following methods can also be employed to reduce or eliminate vertical imbalance (Brooks & Borish 459):

- **Two pairs of eyewear.** Using separate glasses for distance vision and near vision precludes the need for additional vertical imbalance correction.
- **Contact lenses.** Since contact lenses follow the rotation of the eyes, prism—and prismatic imbalance—is generally not encountered.
- **Prism and R-compensated segments.** Certain glass multifocals can be ground to produce prism compensation directly in the reading segment.
- **Dissimilar segments.** If two multifocals are positioned with their segment optical centers at different vertical locations, they will produce a vertical prism imbalance at near. This prism imbalance, which varies as a function of the add power and the vertical separation between the segment optical centers, can be used to help neutralize the vertical prism imbalance induced by the major portion of the lenses.
- **Fresnel press-on prisms.** Fresnel press-on prisms, which are thin plastic sheets with optical qualities, provide an inexpensive—yet relatively temporary—solution for a vertical imbalance correction similar in nature to a slab-off.
- **Optical center placement.** Lowering the optical centers of the lenses brings them closer to the near points, and reduces the reading level distance. Although this method reduces the vertical imbalance at near by shortening the drop from the

distance optical centers, it does introduce some unwanted vertical imbalance during distance vision.

Vertical imbalance compensation is not usually considered until the prism imbalance exceeds 1.50 to 2.00^Δ . Even then, it may not be necessary since many wearers with vertical imbalance may adapt to it over time or may not possess normal binocular vision. Wearers who have recently had an appreciable change in the vision of one eye are the most likely candidates for vertical imbalance correction, including refractive and cataract surgery patients. If the wearer exhibits symptoms such as *asthenopia* (i.e., eye fatigue) or *diplopia* (i.e., double vision) during near vision, a vertical imbalance correction should be considered.

12.5 SPECTACLE MAGNIFICATION

In the previous sections, we have discussed the ability of a lens to change the vergence of light passing through it. A secondary effect produced by lenses is the apparent increase or decrease in the perceived size of the object. This is referred to as **magnification**, when the image appears *larger* than the object, or **minification**, when the image appears *smaller* than the object. We will use the term ‘magnification’ to refer to *any* generic change in the apparent size of an object. Magnification is often desired for instruments such as binoculars. Figure 12:12 and Figure 12:13 show the magnification and minification produced by plus- and minus-powered lenses, respectively (Wakefield 59).

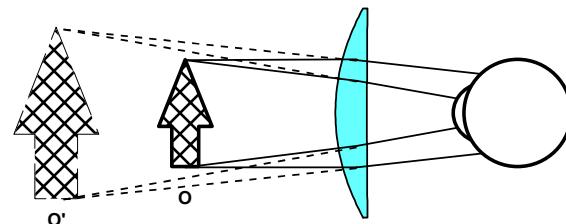


FIGURE 12:12 Magnification by a plus lens. The original object O is magnified by a plus-powered lens, producing a larger image O'.

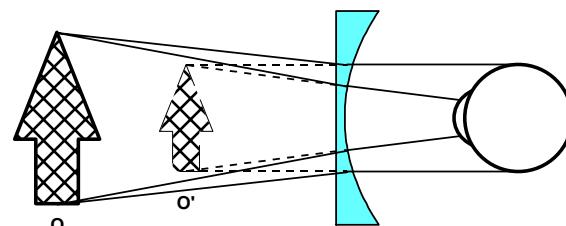


FIGURE 12:13 Minification by a minus lens. The original object O is minified by a minus-powered lens, producing a smaller image O'.

For most spectacle-wearers, magnification (or minification) is generally an inconsequential side effect of their lenses. It is typically only problematic when the

wearer experiences a significant change in magnification from his or her habitual lenses, or when the magnification is significantly different between the right and left eyes due to *anisometropia*. For instance, a change in the wearer's prescription can produce a noticeable change in the magnification produced by his or her new lenses.

Spectacle magnification is typically defined as the amount of *angular magnification* produced by the lens. The angular magnification M is the ratio of the angle θ' subtended by the *image* produced by the lens at the nodal point of the eye N, compared to angle θ subtended by the original *object*, so that

$$\text{EQ. 70} \quad M = \frac{\theta'}{\theta}$$

We are going to make two approximations at this point: We will assume that the lens is thin, and that angles θ and θ' are small. Using these two approximations, we can show that the spectacle magnification M of a *thin lens* is given by*

$$\text{EQ. 71} \quad M = \frac{1}{1 - h \cdot F}$$

where h is the vertex distance from the back of the lens to the nodal point N of the eye in meters.**

A value of M equal to *unity* (or 1) refers to a zero change in image size. Values *above* unity represent *magnification*, and values *below* unity represent *minification*. Hence, *minus-powered* lenses *minify* objects, and *plus-powered* lenses *magnify* objects. To convert this value into a *percentage* of change in spectacle magnification, use:

$$\text{EQ. 72} \quad M\% = 100(M - 1)$$

For thick lenses, a slightly more complex formula is required to take into consideration both the *power* and the *form* of the lens:

$$\text{EQ. 73} \quad M = \left(\frac{1}{1 - h \cdot F_V} \right) \left(\frac{1}{1 - \frac{t}{n} F_1} \right)$$

* The complete derivation for this formula can be found in Appendix B.

** The *entrance pupil* of the eye can also be used in place of the nodal point, and is often preferred. The entrance pupil is located roughly 3 mm behind the apex of the cornea.

where h is the distance to the nodal point of the eye in meters, F_V is the back vertex power of the lens, t is the center thickness in meters, n is the refractive index, and F_1 is the front curve.

The first factor of the equation is our thin lens formula for magnification, based upon the power and nodal point distance alone. This is known as the **power factor**. The second factor of the equation takes into account the form of the lens, including the thickness, refractive index, and front curve. This is known as the **shape factor**.

If we consider both the shape and power factors, we can make some general statements about spectacle magnification. For instance, consider the following points below:

- **Shape factor:** Increasing the front curve or the center thickness will *increase* the spectacle magnification (or reduce minification).
- **Power factor:** Increasing the vertex distance—which in turn increases the nodal point distance—will increase the *magnification of plus lenses* and the *minification of minus lenses*.

Based upon our *reduced eye* model, we can assume that the nodal point typically lies approximately 6 mm behind the apex of the *cornea*. When the exact distance from the back vertex V' of the lens to the nodal point N of the eye is unknown, consider adding this 6 mm value to the *vertex distance* measurement (which will be discussed in Section 7) to determine the nodal point distance h . For instance, a vertex distance of 13.5 produces a nodal point distance of $13.5 + 6 = 19.5$ mm.

Example

A -3.00 D lens is worn 15 mm from the nodal point of the eye. It has a 4.00 D front curve, a 2-mm center thickness, and a 1.500 refractive index. What is the percentage of spectacle magnification?

$$M = \left(\frac{1}{1 - 0.015(-3)} \right) \left(\frac{1}{1 - \frac{0.002}{1.500}(4)} \right)$$

$$M = (0.9569)(1.0054)$$

$$M = 0.962$$

$$M\% = 100(0.962 - 1)$$

$$M\% = -3.8$$

∴ Magnification is -3.8% (*smaller*).

Example

A +5.00 D *thin* lens is worn 15 mm from the nodal point of the eye. What is the percentage of spectacle magnification?

$$M = \frac{1}{1 - 0.015(5.00)}$$

$$M = 1.081$$

$$M\% = 8.1$$

∴ Magnification is 8.1% (*larger*).

There are certain situations where the difference in magnification effects between a pair of lenses can be problematic. A condition known as *spectacle-induced aniseikonia* may occur after the correction of *anisometropia* with spectacle lenses. Recall that anisometropia is a difference in the refractive errors between the two eyes (e.g. OD -4.00 DS and OS -1.00 DS). **Aniseikonia** is the relative difference in size or shape between the ocular images of the two eyes. Since spectacle lenses of differing power produce differing magnification effects, the eyes are presented with disparate images of dissimilar sizes or shapes. If a sufficient amount of aniseikonia exists, binocular fusion may become difficult or impossible.

Figure 12:14 and Figure 12:15 are graphs of the spectacle magnification produced by shape and power factors for a range of center thicknesses, front curves, back vertex powers, and vertex distances. To determine the total spectacle magnification using the graphs, simply add together the percentage of shape magnification to the percentage of power magnification. Tables like these, as well as the formulas described earlier, can also be used to design iseikonic lenses.

Aniseikonia can be reduced by decreasing the magnification difference between the two lenses. A lens designed to minimize this magnification difference, and the ensuing aniseikonia, is referred to as an **iseikonic lens**. An iseikonic lens can be created by adjusting parameters like the center thickness and base curve of a normal spectacle lens.

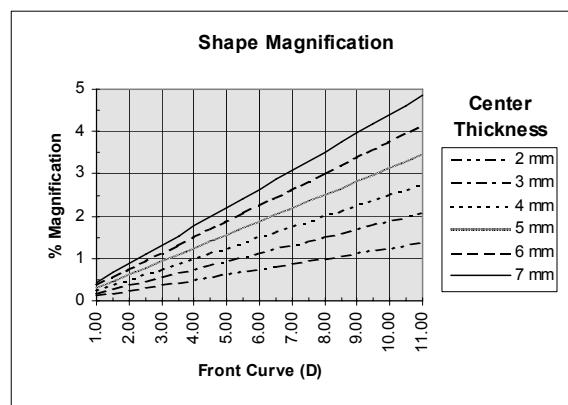


FIGURE 12:14 Magnification by shape for a CR-39 lens.

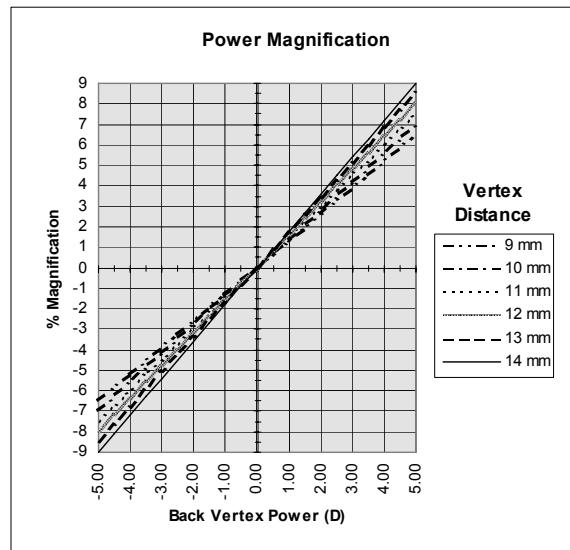


FIGURE 12:15 Magnification by power.

For instance, to decrease the magnification difference between a pair of +4.00 D and a +2.00 D lenses, an iseikonic lens can be designed by making the following modifications to the lenses:

1. Use a *flatter* (preferably aspheric) base curve for the +4.00 lens. This will *reduce* the shape magnification. Similarly, a steeper base curve can be used for the +2.00 lens to *increase* magnification.
2. *Reduce* the center thickness of the +4.00 lens as much as possible to *reduce* shape magnification. Thickness can be added to the +2.00 lens to *increase* magnification.
3. Ensure that the +4.00 lens sits as *close* to the eye as possible to *reduce* power magnification. This can be done by shortening the vertex distance of the frame or by moving the bevel of the lens towards the front surface.

Many spectacle wearers with appreciable levels of anisometropia seem to function without a correction for

aniseikonia. Many people can tolerate up to a 3% difference in magnification between the two eyes, which is roughly equivalent to 2.00 D of anisometropia. Wearers with a significant amount of aniseikonia may not have normal binocular vision. Another reason for why aniseikonia may not be a widespread problem is that higher levels of refractive error are often indicative of *axial* ametropia (i.e., length of the eyeball is too long or too short). **Knapp's law** states that a spectacle lens placed at the primary focal point (roughly 14 mm) of an eye with axial ametropia will produce a retinal image size equal to that of an emmetropic eye. Hence, spectacle lenses will actually minimize the magnification difference between the retinal image sizes in this case. However, if the wearer has anisometropia and exhibits symptoms of aniseikonia, you should consider minimizing the magnification difference through iseikonic lens design. These symptoms may include *asthenopia* (i.e., eye fatigue), impaired binocular vision, spatial distortion (e.g., slanting floors), or the report of more comfortable vision when using only one eye (Fannin & Grosvenor 313).

12.6 FIELD OF VIEW

The magnification (or minification) produced by a spectacle lens is directly related to the field of view provided by a lens of a given size. For instance, a *plus* lens magnifies everything seen through the lens. This effectively *shrinks* the overall field of view seen through the lens. A *minus* lens, though, minifies everything seen through the lens. This effectively *expands* the overall field of view seen through the lens. Imagine looking through a telescope the correct way. Everything looks larger, but at the same time the field of vision becomes narrower. Flip the telescope around, and everything looks smaller. Now the field of vision becomes wider.

When discussing the 'field of view' of a lens, we should distinguish between the *field of fixation* and the *peripheral field of view* as shown in Figure 12:16. The **field of fixation** is the field of view for the rotating eye. This describes the extent to which our lines of sight are able to fixate objects through the lens. The field of fixation is subtended at the center of rotation R of the eye. The **peripheral field of view** is the field of view for the steadily fixating eye. This represents the *total* extent of our vision through the lens, including our peripheral vision. The peripheral field of view is subtended at the entrance pupil P of the eye.

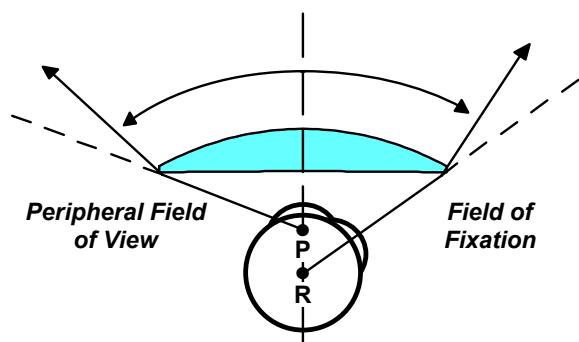


FIGURE 12:16 Fields of view through a spectacle lens. The *peripheral field of view* is subtended by the entrance pupil P of the eye. The *field of fixation* is subtended at the center of rotation R of the eye.

The change in the fields of fixation caused by plano-, plus-, and minus-powered lenses of the same diameter are illustrated in Figure 12:17. Notice how the *plus* lens has a *reduced* field of view, while the *minus* lens has an *expanded* field of view, compared to the unaltered field of view afforded by the *plano* lens with no power.

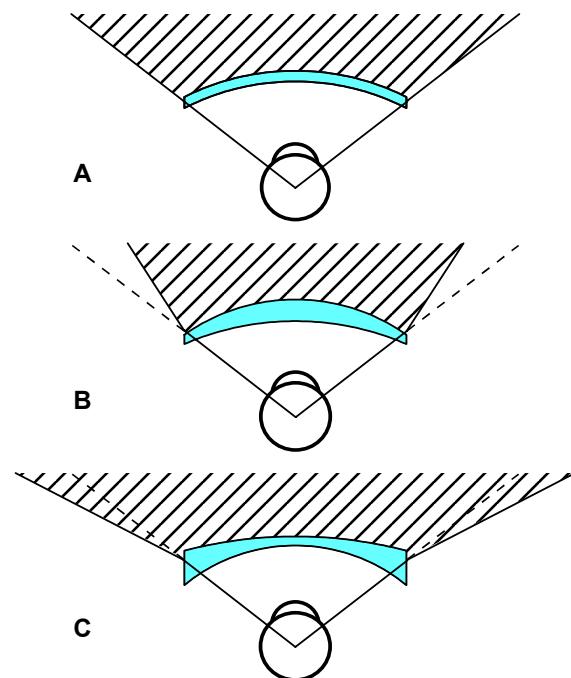


FIGURE 12:17 Fields of fixation for various lenses of the same diameter. A) Normal field of fixation provided by a *plano* lens for an emmetropic eye; B) Reduced field of fixation provided by a *plus* lens for a hyperopic eye; and C) Expanded field of fixation provided by a *minus* lens for a myopic eye.

In addition to the changes in the fields of view, spectacle magnification alters the amount of ocular rotation required to fixate objects. For instance, a *plus* lens (which reduces the field of view) requires *more* ocular rotation to see an object in the periphery than a *minus* lens (which expands the field of view).

12.7 PRISM BY OBLIQUITY

We made reference in earlier sections (i.e., Section 3.3 and Section 8.3) to the fact that the angle of incidence will affect the amount of prismatic deviation produced by lenses and prisms. When light strikes an ophthalmic lens at an angle θ with the optical axis, the amount of prismatic deviation Δ produced in prism diopters is approximately equal to

$$\text{EQ. 74} \quad \Delta = 100 \tan \theta \cdot \frac{t}{n} F_1$$

where θ is the angle of incidence with the optical axis, t is the center thickness of the lens in meters, F_1 is the front curve, and n is the index of refraction.

This prismatic deviation, produced by the *obliquity* of the incident rays of light, is quite small and, consequently, it is often ignored for practical applications. For instance, it is not used to provide (or induce) prescribed prism. For thin lenses at small angles with—and distances from—the optical axis (or center), the formulas described earlier will suffice. For more exact ray tracing procedures, however, the prismatic effects produced by obliquity should be considered. The total prismatic deviation is approximately equal to the combination of the prismatic effects produced by *obliquity* and the prismatic effects produced by *decentration*—if the deviation of the incident ray is being measured at some distance from the optical axis (or center).

13. Lens Materials and Treatments

Ophthalmic lenses can be made from various mineral or organic materials. These **lens materials** must have the following characteristics to be suitable for spectacle lenses:

- Nearly transparent to all visible wavelengths (free from color, unless otherwise specified)
- Free from defects, like bubbles and inclusions
- Homogeneity (uniform in both physical and chemical composition)
- Chemically and physically stable
- Durable and scratch-resistant
- Safe and impact-resistant
- Lightweight
- A high index of refraction and low dispersion are desired

13.1 PERFORMANCE CHARACTERISTICS

A list of common lens materials, and their pertinent characteristics, is provided in Table 20.

TABLE 20 Lens material characteristics

Material	Index	Abbé	Density
CR-39®	1.499	58	1.32
Crown Glass	1.523	58	2.54
Spectralite®	1.537	47	1.21
Polycarbonate	1.586	30	1.20
RLX Lite®	1.555	36	1.24
Ormex®	1.558	37	1.23
Finalite™	1.600	42	1.22
1.60 MR-6	1.597	36	1.34
1.66 MR-7	1.660	32	1.35
1.6 Index Glass	1.601	40	2.62
1.7 Index Glass	1.701	30	2.93
1.8 Index Glass	1.805	25	3.37

The refractive index of media was discussed earlier in Section 2.4. The chromatic dispersion produced by a lens material is rated in terms of its **Abbé value**. Lens materials with *higher* Abbé values produce *less* chromatic dispersion.

The weight of a material is controlled by its **density**, which is the mass of the material per unit volume (generally measured in grams per cubic centimeter). The *lower* the density, the *lighter* the material will be for a given quantity. This can significantly affect the comfort of the lenses on the face. Another term frequently used is **specific gravity**, which is the ratio of the mass of a material or liquid compared to the mass of an equal volume of water (at 4°C). The term *density*, when measured in g/cm³, is synonymous with *specific gravity* since a gram is equal to one cubic centimeter of water.

13.2 REVIEW OF LENS MATERIALS

Spectacles (or eyeglasses) have been around for some time; it is believed that lenses were first utilized for spectacles in the late 13th century. Originally, these spectacle lenses were ground from relatively expensive *quartz crystal* (Bruneni 2).

Today, quartz crystal has been abandoned in favor of more modern, inexpensive materials. Modern lenses typically belong to either of two broad classes of lens materials: **organic materials** (plastics) and **mineral materials** (glass). Organic materials (plastics) can be further classified by whether they are **thermosetting**, or **thermoplastic** materials.

Thermosetting materials begin as a liquid, monomer resin that is eventually polymerized into a solid polymer material. A common manufacturing process for producing lenses made from thermosetting materials is called **casting**. This process involves first mixing either a *chemical* or *photo initiator* into the monomer. Two glass molds, held together by a gasket, are then filled with the liquid monomer as illustrated in Figure 13:1. The mold/monomer composite is then cured in either an oven (for chemical initiation) or under an ultraviolet light source (for photo initiation). During the cure cycle, polymerization occurs and the molecules of the liquid monomer ‘cross-link’ tightly together—hardening the monomer into a polymer lens. The finished lens blank has roughly the same surface curvatures as the molds—although some shrinkage does occur (Fannin & Grosvenor 10).

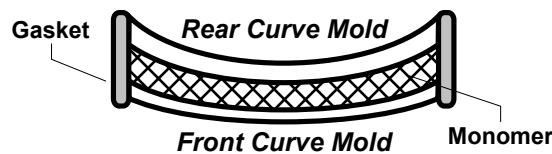


FIGURE 13:1 Lens casting. Monomer resin is poured between two glass molds that are held together by a gasket. The monomer is then cured until it polymerizes into a solid polymer lens material. It is interesting to note that the lens material shrinks somewhat during polymerization. The curvature of the molds will depend upon both the desired surface powers of the lens, and the amount of material shrinkage that occurs during the cure cycle.

Various ingredients are often added to the initial monomer resin to improve various physical and/or optical properties. These additives may be used to reduce the yellowness of the material, protect the material from ultraviolet degradation and weathering, increase the refractive index, help the lens material release from the mold, etc. Once cast and polymerized, thermosetting materials can not be melted back down to liquid, monomer resins.

The majority of spectacle lenses are made from *allyl diglycol carbonate*, a thermosetting plastic lens material referred to as **CR-39** (Columbia Resins). CR-39 was originally invented in the 1940s for military applications as a bonding agent. Although it didn't become popular as a spectacle lens material until the late 1970s, it is presently considered to be the 'standard' by which other materials are compared. It is lightweight, impact-resistant, and can be tinted to various shades with chemicals.

Thermoplastic materials are not cross-linked like thermosets, and can be melted and cooled back and forth from liquid to solid states. A common manufacturing process for producing lenses made from thermoplastic materials is called **injection-molding**. This process involves first heating and melting down thermoplastic pellets, and then injecting the melted resin between two metal molds under controlled pressure. The melted plastic is then allowed to cool, and hardens into a completed lens blank.

The most common thermoplastic lens material is **polycarbonate**, which was originally invented by General Electric in the 1950s under the trade name *Lexan*. Polycarbonate is known for its exceptional impact resistance. It is also quite thin and light-weight, compared to many other materials. Another thermoplastic material, an acrylic called *polymethyl methacrylate*, was also used for spectacle lenses for a number of years in Europe. This material is now used primarily for certain rigid contact lenses.

The majority of mineral lenses are made from a variety of glass known as **crown glass**. Crown glass, which dominated the industry for centuries, is comprised chiefly of *silica*, *soda*, and other miscellaneous ingredients. Rough glass lens blanks are produced using a **continuous flow** process. These rough blanks will have the front and back surfaces ground to the appropriate curvatures by the lens manufacturer. Fused bifocal segments can also be added. Optically, crown glass is very similar to CR-39 plastic. It also offers much greater scratch-resistance. However, glass materials are significantly heavier than the plastics because of their higher densities, and are much less impact-resistant.

13.3 HIGH-INDEX LENS MATERIALS

In the United States, when a lens material has an index of refraction greater than that of the standard tooling index of 1.530, the material is generally referred to as a **high-index** lens material. For instance, glass lens materials can have certain metallic oxides, like *flint* or *titanium*, added to their initial batches to increase their index of refraction. There are also thermosetting plastics available, like *polyurethane*, with higher indices of

refraction. Polycarbonate is another high-index lens material.

High-index materials produce thinner spectacle lenses, usually at the expense of lower Abbé values. To understand how a high-index lens produces a thinner profile, imagine two identical prisms: one with a refractive index of 1.500 and the other with an index of 1.700. Because of the higher index of refraction, the 1.700-prism deviates light more than the 1.500-prism as illustrated in Figure 13:2.

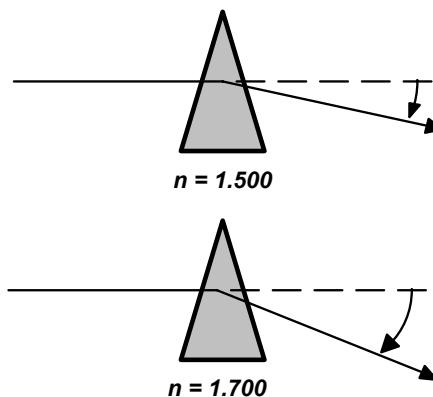


FIGURE 13:2 As the index of refraction increases, light is refracted to a greater extent.

Conversely, to produce the same deviation as the 1.500-prism, the 1.700-prism would have to be made with a thinner base. The same holds true for 'high-index' spectacle lenses. Lens curves can be made shallower, while still providing the same refractive power. Consider a comparison between two -4.00 D lenses, one with a refractive index of 1.500 and the other with an index of 1.700. These lenses are shown in Figure 13:3. Notice that the 1.700 high-index lens is 25% thinner than the 1.500 lens.

Further, high-index plastics are typically lighter in weight than conventional CR-39. This is a result of the fact that high-index lenses produce thinner lenses with less mass. These lens materials often have lower densities than CR-39, as well. High-index glass materials, on the other hand, are not necessarily lighter in weight than conventional crown glass (especially in low to moderate powers). Although these high-index glass materials have less mass, they often have noticeably higher densities. Therefore, the reduction in weight is less significant. Sometimes spectacle lenses made from these glasses can even weigh more than crown glass lenses.

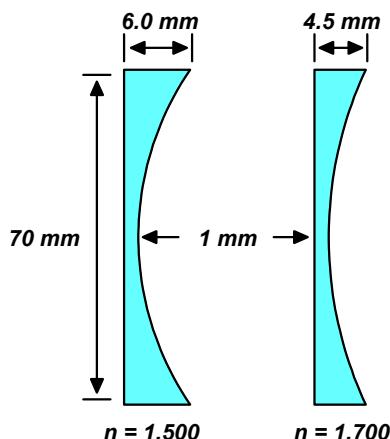


FIGURE 13:3 A -4.00 D lens comparison.

13.4 IMPACT RESISTANCE

The use of glass has declined over the last few decades, since the advent of larger spectacle frames and the FDA's 1972 impact-resistance regulation—which made the use of impact-resistant lens materials a requirement. The FDA now requires that every lens dispensed in the United States be capable of passing a ‘referee’ drop-ball test (Optical Industry Assoc. 2).

This test involves dropping a 15.9 mm (5/8”), 16 g (0.56 oz) steel ball upon the lens from a distance of 127 cm (50”). This impact is equivalent to about 0.2 joules of energy. To survive this test without fracturing, most glass lenses must be *tempered* to increase their impact resistance, and then tested individually. **Tempering** places the outer surface of the lens in a state of compression. Two methods of tempering are currently in use for spectacle lenses:

- **Thermal tempering:** This process involves heating the lens to its softening point, and then rapidly cooling the lens with blasts of air. The outer surfaces of the lens cool quickly, while the inner core of the lens cools more slowly. This places the outer surfaces of the lens in a state of compression, and the inner core in a state of tension. This process takes only a few minutes. The resulting strain produced within the lens makes the material **birefringent** (or doubly refracting). This birefringence can be evaluated using crossed *polaroid* filters.
- **Chemical tempering:** This process involves exchanging larger ions from a heated salt bath for smaller ions from the surfaces of the lens. These larger ions place the surfaces in a state of compression. For instance, *potassium* ions from the salt bath might be exchanged with smaller *sodium* ions from the lens surface (for crown glass lenses). This process takes between four to sixteen hours,

depending upon the method. It is also done at lower temperatures than thermal tempering.

Tempering increases impact resistance by placing the outer surfaces of the lens into a state of compression. Lens materials have a greater resistance to compressive stress than tension. When a projectile strikes the surface of a tempered lens, the opposite surface is put under tension as the lens flexes. This serves to neutralize the compressive stress already created in the surface by tempering—as opposed to breaking the lens under the additional strain.

Two particularly common types of lens fractures are described below (Fannin & Grosvenor 16):

1. The first breakage (Figure 13:4) is a fracture of *rear surface origin*, caused by a compound flexure of the lens that sets up a tension stress at the rear surface resulting in a tear at the back surface. This occurs primarily when a *minus-powered* (thinner at the center) lens is hit by a missile of moderate mass and velocity.

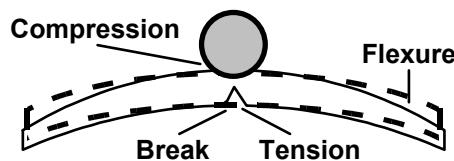


FIGURE 13:4 Lens fracture of *rear surface origin*.

2. The second breakage (Figure 13:5) is a fracture of *front surface origin*, caused by simple elastic denting of the front surface and a tear that propagates to the back. This occurs primarily when a small, lightweight, and high-velocity missile hits the lens.

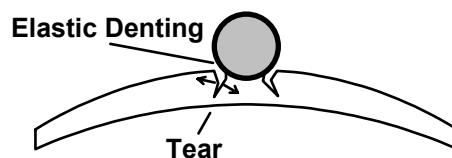


FIGURE 13:5 Lens fracture of *front surface origin*.

Most plastic lenses are inherently impact resistant, and are not individually tempered. These lenses are usually sample-tested at the manufacturing level to ensure that each batch is exhibiting a minimum of impact resistance. In certain situations, however, an individual lens may need to be tested if it has been ground below a certain center thickness, or if it has had additional coatings applied to it.

13.5 TRANSMITTANCE AND ABSORPTION

In Section 3, we discussed the interaction of lens materials with the light incident upon them. Recall that light may be *reflected* by a material, *transmitted* by a material, and/or *absorbed* by a material. We looked

closely at the reflectance of lens materials, and learned that a typical ophthalmic lens reflects 8% or more of incident light. So, what happens to the rest of the light striking the lens?

The light passing through the lens may be either completely transmitted if the lens is perfectly clear, or absorbed to some extent if the lens has filtering properties. The fraction of incident light transmitted through a lens is referred to as its **transmittance** τ .

The transmittance of a lens may vary from wavelength to wavelength, especially for *tinted* lenses with color. The transmittance spectra of most clear materials will be quite similar—particularly within the visible spectrum. For instance, high-index lenses will closely resemble the spectrum for CR-39, with two significant differences. First, with the use of Fresnel's formula (Eq. 7) you can see that high-index lenses reflect slightly more light than either crown glass or CR-39, thereby transmitting less light. Second, high-index lens materials generally absorb more ultraviolet radiation than CR-39. A graph showing the percent of transmittance for each wavelength is shown in Figure 13:6 for a clear polycarbonate lens. Such a graph is referred to as the *spectral transmittance* of the lens. It is important to note that, since this lens has no color, the entire visible spectrum is transmitted evenly.

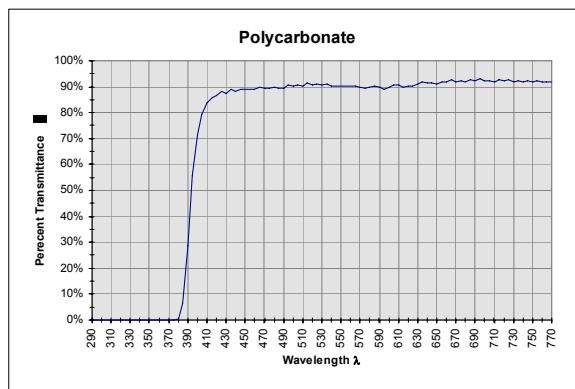


FIGURE 13:6 Transmittance of a *clear* polycarbonate lens. Note that the lens transmits *no* ultraviolet radiation (below 380 nm). Most high-index plastic lens materials absorb 100% of UVB radiation and 98% or more of UVA.

Absorption refers to the loss of light as it passes through a lens material. Lenses that absorb light evenly across the visible spectrum are referred to as **neutral filters**, and will be *gray* in appearance. Lenses that selectively absorb light across the visible spectrum are referred to as **selective filters**. For instance, a lens that selectively absorbs *blue* light may appear *amber* or *yellow* in hue since it transmits wavelengths other than blue. The principal use of absorptive filters is for protection against glare (sunglasses). Both neutral and selective filters are shown in Figure 13:7 and Figure 13:8.

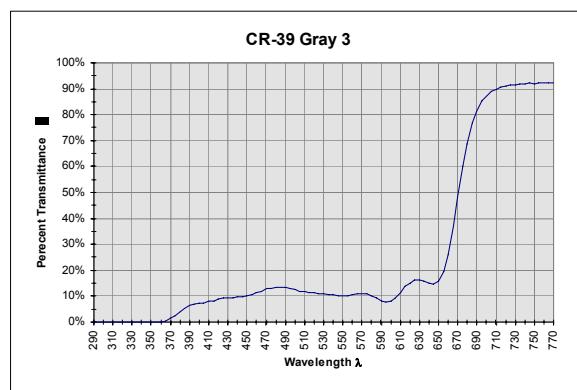


FIGURE 13:7 Transmittance of a *neutral gray* CR-39 lens. Note that the lens attenuates light quite evenly over the majority of the visible spectrum (between 380 to 760 nm).

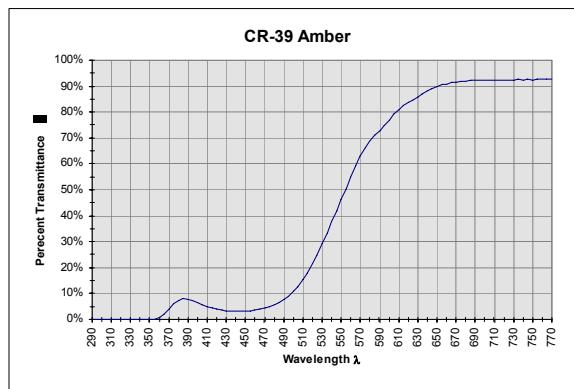


FIGURE 13:8 Transmittance of a *selective amber* CR-39 lens. Note that the lens significantly attenuates blue light, while transmitting a considerable amount of the yellow and red end of the spectrum—giving it an amber hue.

It is interesting to note that tinted glass (mineral) lenses are made at the factory level by adding metallic oxides into the raw material. This disperses the tint throughout the mass of the lens blank. Consequently, thicker mineral lenses will appear darker than thinner ones, since increasingly more light is absorbed as it passes through the thickness of the lens blank. This is a consequence of **Lambert's law**, which states that light passing through a given thickness of a homogenous material is absorbed by the same percentage—no matter what the original intensity (Brooks & Borish 359).

To use Lambert's law, we need to know the transmittance of the lens material at some arbitrary thickness. This thickness will become the *layer* and the transmittance is the *transmission factor* (q) for that layer. Each additional 'layer' of the material reduces the remaining light from the previous layer by its transmission factor. For instance, if a 2-mm layer of a given lens material transmits 50% of the incident light, 4 mm of this lens material—which is 2 layers at 2 mm—will transmit $50\% \times 50\% = 25\%$ of the original light intensity. The final transmittance τ of a lens, after

passing through N layers, can be mathematically expressed by:

$$\text{EQ. 75} \quad \tau = 100 \cdot I_O \cdot q^N$$

where τ is the final transmittance of the lens, I_O is the original intensity of the incident light, q is the transmission factor for a layer of the material at a given thickness, and N is the number of layers. N is given by the thickness of the lens divided by the thickness of the ‘layer,’ which is generally chosen to be 2 mm for spectacle lenses.

Plastic (organic) lenses, on the other hand, are tinted by dyeing the surface of the lens in a bath of organic dye. The depth of the tint is a mere fraction of the overall lens thickness. Therefore, lens thickness has no significant effect on the tint appearance of plastic lenses.

There are also lens materials available, in both glass and plastic, that are sensitive to sunlight. These lens materials are referred to as **photochromic filters**, and automatically darken when exposed to sunlight (or ultraviolet radiation). Figure 13:9 demonstrates both the faded and darkened states of a common glass photochromic lens, Corning’s PhotoGray Extra®.

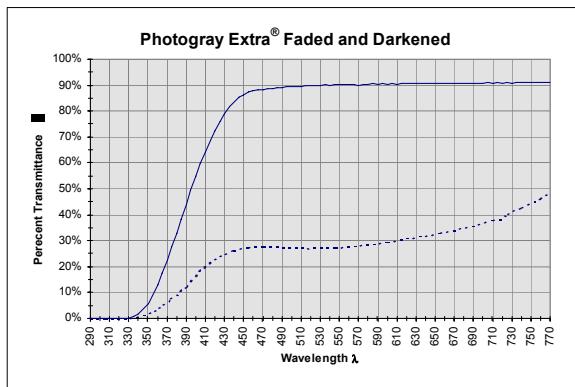


FIGURE 13:9 Transmittance of a *photochromic* glass lens in both its faded (solid line) and darkened (dashed) states.

Table 21, below, shows the *luminous transmittance* of several photochromic lens materials in both their *bleached* (faded) and *activated* (darkened) states. The **luminous transmittance** of a lens is its spectral transmittance weighted by the *photopic* (or daytime vision) sensitivity of the human eye. This is a more meaningful measure of lens transmittance, since the sensitivity of the human eye varies between the colors of the visible spectrum, as shown in Figure 13:10. For instance, blue light at 470 nm—with a 10% efficiency—needs to be 10 times as intense to evoke the same sensation of brightness as yellow-green light at 560 nm (with a 100% efficiency).

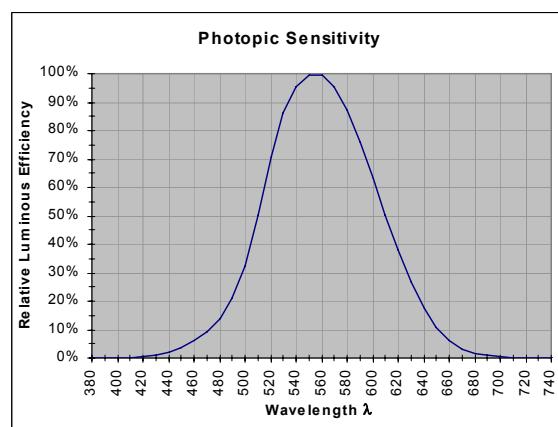


FIGURE 13:10 The sensitivity of the human eye varies from wavelength to wavelength, with its peak sensitivity for photopic (daytime) vision lying at roughly 555 nm.

TABLE 21 Transmittance of popular photochromics*

Glass Materials (mineral)	Faded % τ	Dark % τ
PhotoGray Extra®	85%	22%
PhotoBrown Extra®	85%	22%
PhotoSun II®	40%	12%
PhotoGray Extra® 16™	83%	22%
PhotoGray® Thin & Dark™	86%	16%
Plastic Materials (organic)	Faded % τ	Dark % τ
Transitions® Plus	84%	28%
Transitions® III Gray	87%	22%
Transitions® III Brown	87%	22%
Transitions® XTRActive™	75%	11%
Spectralite® Transitions® III	87%	22%
Seiko Changers™	89%	30%

Glass photochromics work by the action of *silver halide crystals* distributed throughout the lens blank. Upon exposure to sunlight, these crystals break apart into free silver particles. The silver particles then cluster together, forming silver colloids that absorb visible light—causing the lens to darken in color. Upon removal from sunlight, the process reverses itself—causing the lens to fade in color. The photochromic performance of glass lenses, which lasts indefinitely, is affected by several variables:

1. **Method of tempering.** Thermally tempered lenses will generally have a lower transmittance in both their faded and darkened states than chemically tempered lenses.
2. **Lens thickness.** As the thickness of the lens increases the transmittance decreases in its

* These measurements were conducted within a temperature range of 72 to 78°F (22 to 26°C).

darkened state, because of the additional level of silver halide crystals present throughout the greater mass of the lens.

3. **Ambient temperature.** Photochromic lenses are temperature sensitive, and will darken more in colder temperatures than in warmer.
4. **Ultraviolet radiation.** Photochromic lenses are activated by ultraviolet radiation and short wavelength visible light. Therefore, the lenses do not darken as much when some of the ultraviolet radiation is blocked—such as by an automobile windshield.

The most common plastic photochromics work by the action of a thin organic layer of *indolino spironaphthoxazine* molecules that are *imbibed* (or impregnated) into the front surface of the lens blank. These molecules undergo a chemical reaction when exposed to sunlight, which causes a change in their structure. This new configuration absorbs visible light. As with the glass photochromics, the process reverses itself once the sunlight is removed. Most of the plastic photochromics, however, are not affected by lens thickness—since only a thin layer of the front surface actually contains the photochromic molecules. There are some plastic materials that contain the photochromic molecules throughout the lens, and their performance *is* affected by lens thickness.

The photochromic performance of plastic materials is also affected by the ambient temperature and the amount of ultraviolet radiation. In addition, the photochromic performance of these materials gradually deteriorates over time, through a process called *photo-oxidation*.

For certain occupational or recreational visual needs, a selective filter may be appropriate. For instance, yellow and amber tints are often desired for their high-contrast qualities. Since the atmosphere scatters more blue light, lenses that absorb blue light are going to attenuate more of the veiling glare scattered by the atmosphere—thereby increasing contrast slightly. A typical yellow CR-39 lens is shown in Figure 13:11. In addition to neutral gray tints, many people prefer brown tints—particularly in Europe. A typical brown CR-39 lens is shown in Figure 13:12.

Certain occupational tasks may require specific eye protection, as well. For instance, specific filters can be obtained for use in conjunction with lasers, welding devices, x-ray machines, furnaces, etc. The transmittance of a *didymium* lens, sometimes used by glass blowers, is shown in Figure 13:13.

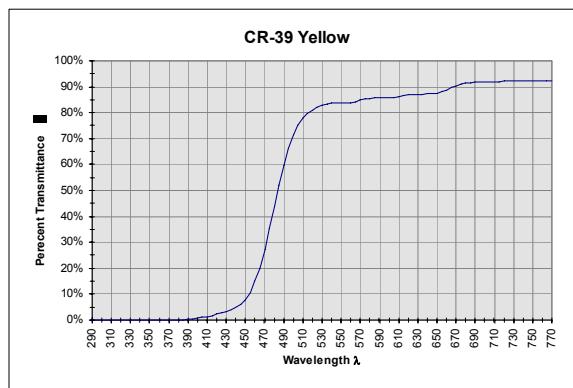


FIGURE 13:11 Transmittance of a yellow CR-39 lens. Note that, like amber lenses, the lens significantly attenuates blue light.

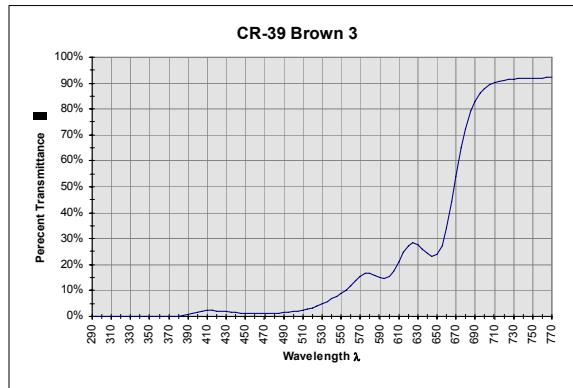


FIGURE 13:12 Transmittance of a brown CR-39 lens. This color can also be used for glare and sun protection.

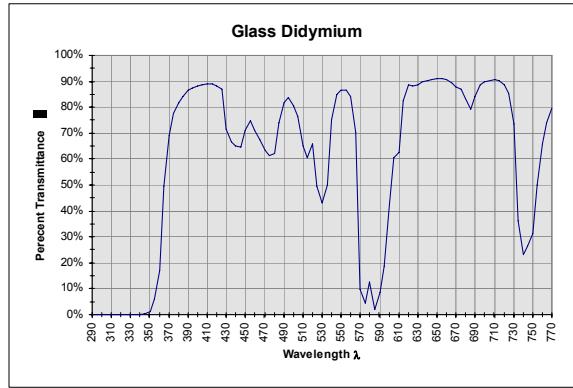


FIGURE 13:13 Transmittance of a didymium glass lens. Note the sharp drop in transmittance (increase in absorption) between the 550 to 600 nm range; this is the ‘sodium flare’ region.

Popular glass tint hues are typically available in several different transmittance options from the factory (e.g., gray 1, 2, and 3). Typically, as the filter’s number or letter designation *increases* the luminous transmittance of the lens *decreases*. Plastic lenses can be dyed to a reach a wide range of transmittance values.

Sometimes, tints with a relatively high transmittance are recommended for purely cosmetic reasons. Light

'fashion tints' can help accent the wearer's eye color, add color to the cheeks, or make the lenses appear more natural looking (less 'glassy'). Some have also advocated the use of light tints for reducing the discomfort associated with harsh lighting, glare from computer screens, and similar visually uncomfortable situations. Two such light filters, a blue and a pink lens, appear in Figure 13:14.

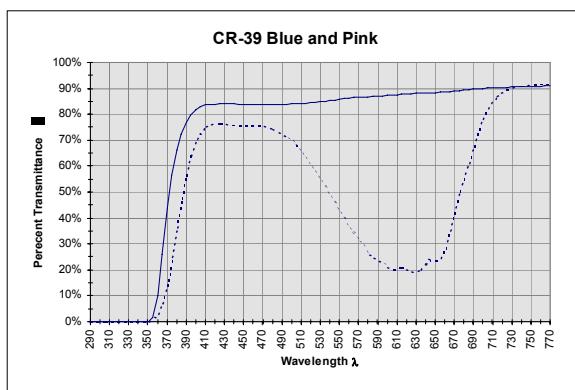


FIGURE 13:14 Transmittance of both *blue* and *pink* CR-39 lenses (light fashion tints).

Another popular filter for glare attenuation is a **polarized** filter. Although the electrical and magnetic fields of an electromagnetic radiation wave train remain perpendicular to their direction of propagation as they move through space, they don't necessarily start out vibrating in any particular orientation about this path. Light waves from many sources, like the sun and incandescent lamps, can vibrate in every possible orientation as they travel along their path. (Imagine all of the possible orientations on a protractor.) When the vibration of light waves is confined to a single plane (i.e., horizontal, vertical, or some plane in between), the light is referred to as **polarized**. Figure 13:15 depicts the difference between non-polarized and *linearly* (or completely) polarized light from a front view of an imaginary wave train.

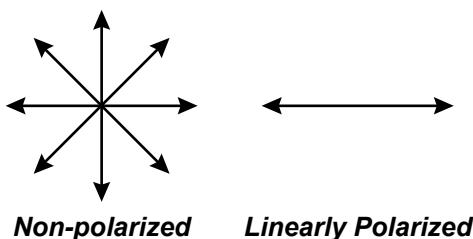


FIGURE 13:15 Light waves from many sources are not polarized, and vibrate in every possible direction about their direction of propagation (or travel). When light waves only vibrate in one plane, they are *linearly* polarized. The linearly polarized light above is *horizontally* polarized.

Light waves reflected off many dielectric surfaces, such as water, asphalt, and glass, become partially or completely polarized parallel to the surface. (Light

waves incident on the surface at *Brewster's angle* are linearly, or completely, polarized.* Glare specularly reflected off a horizontal surface, for instance, is horizontally polarized. To reduce this reflected glare, a **polarized filter** can be employed that transmits only the vertical components of incident light—absorbing the horizontally polarized veiling glare, which has been reflected off the surface. The action of a polarized filter on non-polarized light waves is depicted in Figure 13:16.

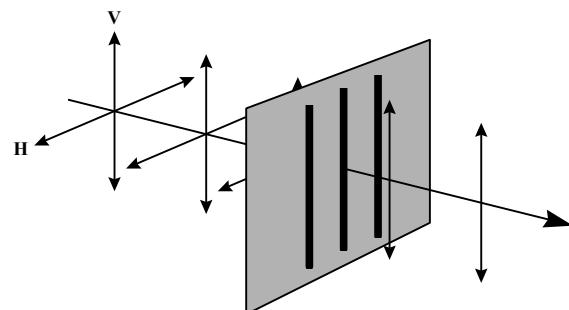


FIGURE 13:16 The molecules in a polarized filter only allow light vibrating in a single plane to pass through—thereby acting like imaginary 'slits.' Light vibrating perpendicularly to these slits is absorbed.

Polarized filters are generally made by stretching a thin sheet of *polyvinyl alcohol* and soaking it with iodine. These filters are particularly useful for people engaged in water, snow, or driving activities. It is important to note that polarized filters selectively attenuate certain forms of glare (horizontally reflected glare), not just the overall amount of light passing through the lens like the conventional filters previously discussed.

Recall from Section 2.1 that **ultraviolet radiation** is a form of electromagnetic radiation immediately adjacent to the blue/violet end of the visible spectrum (from 200 to 380 nm). Although invisible to the human eye, ultraviolet radiation does contain more energy than visible radiation. It is now generally accepted that lenses used for sun protection should attenuate nearly all ultraviolet (UV) radiation. Over the years, it has been determined that UV radiation contributes to the development of various ocular disorders, such as *photokeratitis* (or snow blindness), *cortical cataract*, *pinguecula*, and *pterygium*. It also might be as a contributing factor for *age-related macular degeneration*.

* Brewster's angle β is given by

$$\beta = \tan^{-1} n$$

where n is the index of refraction. For instance, β is 53° for water and 57° for glass.

13.6 LENS COATINGS

In addition to lens tints and treatments, a variety of coatings are often applied to spectacle lenses to enhance either their optical and/or mechanical performance. *Anti-reflective coatings* can be applied to glass and plastic lenses to minimize surface reflections. *Scratch-resistant coatings* can be applied to improve the abrasion resistance of plastic lenses.

Each surface of a spectacle lens acts as a curved mirror, and reflects a fraction of the incident light. The reflected image of an object is often referred to as a **ghost image**. While the size and clarity of the reflected ghost images vary with the power and form of the spectacle lens, the brightness (or intensity) of the ghost images increases with the refractive index of the lens material, as discussed in Section 3.2. The reflectances of some common lens materials are shown in Table 22.

TABLE 22 Reflectances of common lens materials

Lens Material	Refractive Index	Reflectance (ρ) 1 Side	Reflectance (ρ) 2 Sides
CR-39 Plastic	1.499	4.0%	7.7%
Crown Glass	1.523	4.3%	8.3%
Spectralite	1.537	4.5%	8.6%
Polycarbonate	1.586	5.1%	9.8%
1.60 Plastic	1.600	5.3%	10.1%
1.66 Plastic	1.660	6.2%	11.7%

Specular surface reflections and ghost images produce visual “noise,” which degrades retinal image quality without contributing useful visual information. They reduce visual performance and comfort via two principal phenomena: visual disturbance caused by ghost images and reduced visual discrimination caused by veiling glare.

There are five unique specular reflections that the wearer may notice from spectacle lenses, as shown in Figure 13:17. These reflected ghost images could become visually disturbing to the wearer when the following conditions are met:

1. The reflected ghost image is bright (or intense) enough to stand out against the background.
2. The vergence (or power) of the reflected ghost image is similar to the focal power provided by the spectacle lens, or can be made similar through accommodation.
3. The reflected ghost image lies close to, but not necessarily in, the wearer’s line of sight.

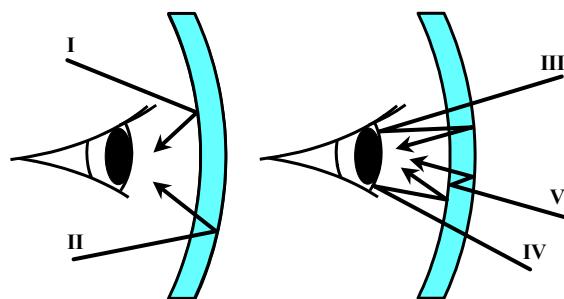


FIGURE 13:17 The Specular lens reflections from front (III – V) and rear (I & II) light sources.

These ghost images can also serve as sources of glare within the visual field. Moreover, when the reflected glare source is large or defocused, it can produce a *veiling glare* over a large portion of the visual field. Since this reflected glare is added to the brightness of both the object of interest and its background, the difference in brightness between them remains constant. However, since the background brightness, which is the denominator of the contrast expression above, still increases, the contrast of the retinal image decreases. Images with lower contrast are more difficult to resolve than images with higher contrast, which effectively reduces visual acuity. Hence, glare from lens reflections serves to reduce both contrast sensitivity and visual acuity.

Anti-reflective coatings consist of one or more thin layers (or *films*) of various inorganic oxides such as magnesium fluoride, titanium dioxide, and silicon dioxide. These films are often applied by vacuum deposition, and utilize the principles of **constructive** and **destructive light interference**. These principles are illustrated Figure 13:18.

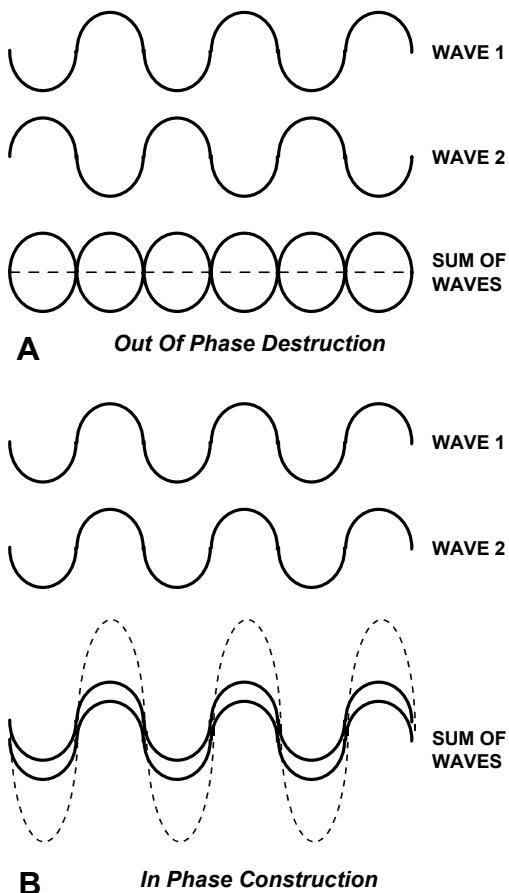


FIGURE 13:18 Light interference produced by thin films. A) Light waves ‘out of phase,’ so that their crests and troughs do *not* coincide, exhibit *destructive* interference and cancel out each other. (The dotted lines are the resultant wave.) B) Light waves ‘in phase,’ so that their waves *do* coincide, exhibit *constructive* interference and reinforce each other.

Anti-reflection coatings on glass lenses generally employ a single layer of magnesium fluoride. This coating allows for maximum interference in the yellow-green band of the visible spectrum. Red and blue light is not completely canceled, so these lenses produce a slight purple surface reflection (or *reflex color*). Modern AR coatings on plastic lenses often employ five or more layers, which alternate between lower and higher indices of refraction. These multi-layer (or *broadband*) coatings are able to cancel reflected light over a wider band of colors.

The application of an anti-reflection (AR) coating can increase the transmittance of a lens up to nearly 100%, while virtually eliminating visible reflections from the surface of the lens. This can be thought of as a two step process: The reflections are almost completely canceled by *destructive* interference, while the light passing through the lens is reinforced to almost 99% or more transmittance by *constructive* interference. These optical interference effects are produced by the interaction of reflections between the various interfaces

of the AR-coated lens (i.e., the interfaces between air, the AR layers, and the lens substrate). For comparison, the transmittance spectra of an AR-coated and a non-coated CR-39 lens are provided in Figure 13:19.

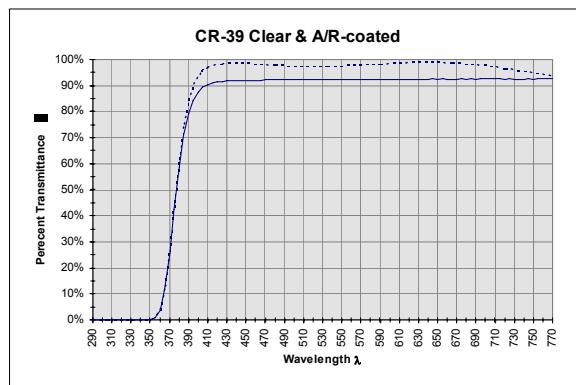


FIGURE 13:19 Transmittance of both a *clear* (solid line) and an *AR-coated* (dashed) CR-39 lens. Note that the lens transmits little ultraviolet radiation (below 380 nm), and evenly transmits the visible spectrum (between 380 to 760 nm). With the application of an anti-reflection coating, the transmittance of the lens material approaches 100%.

Scratch-resistant coatings are often applied to plastic lenses to increase the abrasion resistance of the lens surface, which is generally quite soft. Many scratch-resistant (SR) coatings are made from chemical resins containing compounds, such as *polysiloxane*, which contain silica (like glass) and organic polymers. SR coatings often have to balance tintability with scratch resistance—i.e., softer coatings tint better but scratch more. Modern SR coatings are now available that are *extremely* durable. Conventional SR coatings are often 1.5 to 3 × as scratch-resistant as uncoated CR-39, while many ‘ultra-tough’ coatings are 3 to 6 × as scratch-resistant. These ultra-tough type coatings are often non-tintable. SR coatings can be applied at the manufacturing, laboratory, or in-office level. The quality of these coatings may vary.

Scratch-resistant and anti-reflective coatings can be quite brittle and glass-like in nature, which can reduce impact resistance. Consequently, lenses with these coatings may be more susceptible to fracturing upon impact. Furthermore, some of the newer high-index materials in use today are often ground to a 1.5-mm—or even 1.0-mm—center thickness. Many of these lens materials are robust enough to pass the drop ball test without any additional treatments. Other materials may have a special **primer coating** applied to them to help absorb and dissipate some of the impact energy.

The base material (e.g., CR-39) upon which these lens treatments and coatings are applied is referred to as the **lens substrate**. For optimum optical and mechanical performance, lens treatments and coatings should be chemically engineered for compatibility with the

substrate, as well as with each other. Adhesion, thermal expansion, and other physical properties must be optimized to create an integrated system, as depicted in Figure 13:20.

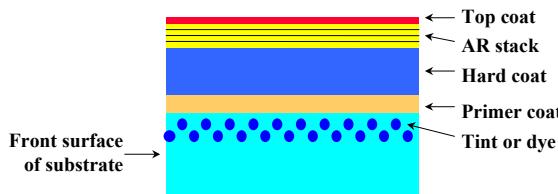


FIGURE 13:20 For optimum mechanical and optical performance, each lens treatment and coating should be chemically engineered for compatibility to create an integrated system.

13.7 CHROMATIC ABERRATION

Lens design generally addresses the monochromatic aberrations, like oblique astigmatism, which are independent of color. **Chromatic aberration** is a result of the inherent dispersive properties of a lens material.

Recall from Section 2.4 that the refractive index of a lens material actually varies as a function of color (or wavelength). The **Abbé value** of a lens material is a measure of the **refractive efficiency** of the material; or how efficiently the material bends light without breaking it up into its component colors (dispersion). Put another way, the Abbé value provides us with a comparison between the *mean refractivity* ($n_D - 1$) of a lens material and its *mean dispersion* ($n_F - n_C$):*

$$\text{EQ. 76} \quad v = \frac{n_D - 1}{n_F - n_C}$$

where v is the Abbé value of the material, n_D is the refractive index of the material at 587.56 nm (yellow light), n_F is the refractive index at 486.13 nm (blue light), and n_C is the refractive index at 656.28 nm (red light).

The *mean refractivity* is related to the ‘quoted’ index of refraction of the lens material, which is calculated using a wavelength very near to the center of the visible spectrum. In addition, the chosen wavelength generally lies close to the peak photopic sensitivity of the human eye, which is related to the *maximum luminous efficiency* of the eye. This *reference index*, which is used to calculate mean refractivity, is based upon the helium d line in the United States. (See Section 2.4). The *mean dispersion* is the difference in refractive

indices between the blue and red ends of the spectrum at specified wavelengths.

Chromatic aberration is a measure of the color dispersion produced by optical materials. Materials with lower Abbé values are more susceptible to chromatic aberration, since they disperse light more. White light refracted through a lens or prism with significant chromatic aberration will be spread into its component colors, producing multiple images (one for each color) as shown in Figure 13:21. When a lens with power disperses light, each color can create its own focal length and image size.

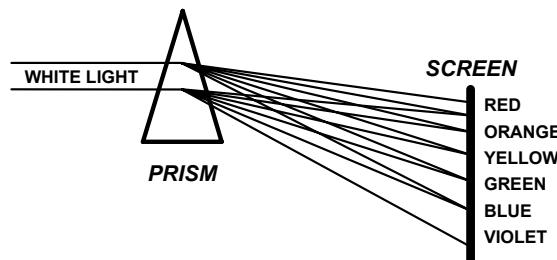


FIGURE 13:21 Chromatic dispersion of white light into its component colors by a prism.

Spectacle lenses can suffer from both *monochromatic* and *chromatic* aberrations. Like the *monochromatic* aberrations, *chromatic aberration* also increases towards the periphery, as shown in Figure 13:22.

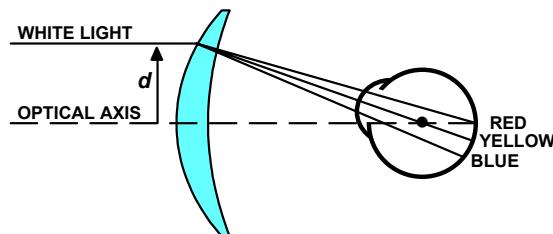


FIGURE 13:22 Chromatic aberration will vary directly with the amount of prism Δ at a given point through the lens using Prentice’s rule (Eq. 42), and is inversely proportional to Abbé value of the lens material.

The amount of *lateral chromatic aberration* produced by a lens is given by (Davis 20):

$$\text{EQ. 77} \quad \delta = \frac{\Delta}{v}$$

where δ is the amount of chromatic aberration in prism diopters (^), Δ is the amount of prism at a given point through the lens, and v is the Abbé value of the lens material.

Example

A person looks 10 mm (1 cm) away from the optical center of a +5.00 D polycarbonate lens, which has an

* The Abbé value is actually equal to the reciprocal of the *dispersive power* of a lens material, which is the ratio of mean dispersion to mean refractivity.

Abbé value of 30. How much chromatic aberration does the wearer experience?

$$\delta = \frac{1(5.00)}{30}$$

$$\delta = 0.17$$

∴ Chromatic aberration is 0.17^Δ .

The calculated amount of chromatic aberration is the prismatic difference between the red and blue portions of the spectrum. It is important to consider the fact that the Abbé value of both mineral and organic lens materials generally *decreases* as the refractive index of the material *increases*. Consequently, higher-index materials have more chromatic aberration for the same power. Notice the decrease in Abbé values as the refractive indices increase in Figure 13:23.

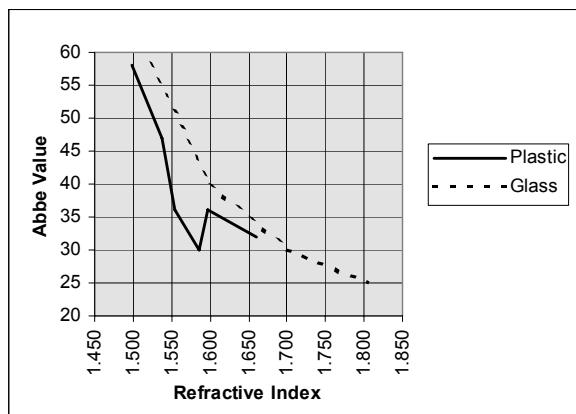


FIGURE 13:23 A comparison between the Abbé value and refractive index of representative glass and plastic lenses. Notice how the Abbé value decreases as the index of refraction increases for both materials.

The chromatic aberration experienced by the wearer is directly proportional to the prism produced at a given point through the lens, and inversely proportional to the Abbé value of the lens material. Chromatic aberration can result in both blurred vision and a fringe of color around objects viewed through the periphery. Fortunately, this aberration is only a concern in high powers, or in materials with low Abbé values.

For precision optical instruments, **achromatic doublets** are often used to minimize chromatic aberration. Doublets are constructed by combining two lenses made from glasses of differing refractive index and Abbé values. For spectacle lenses, chromatic aberration can be minimized with the judicious selection of lens materials (i.e., low Abbé value). Refer back to Table 20 a description of common lens materials and their Abbé values.

Since modern ‘thin & light’ lens materials often have lower Abbé values, proper lens design is critical.

Whenever possible, the *best form* base curve recommended by the manufacturer should be utilized. If the blur caused by *monochromatic* aberrations is minimized with the proper selection of base curves, or with a well-designed aspheric surface, the wearer’s sensitivity to the *chromatic* blur may be kept below his or her threshold of tolerance. This can reduce non-adapt cases (Davis 141).

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RLX Lite is a registered trademark of Signet Armorlite, San Marcos, California.

PhotoGray Extra, PhotoBrown Extra, and PhotoSun II are registered trademarks; and, 16 and Thin & Dark are trademarks of Corning, Inc, Corning, New York.

Transitions is a registered trademark and XTRActive is a trademark of Transitions Optical, Inc., Pinellas Park, Florida.

Ormex is a registered trademark of Essilor International.

Changers is a trademark of Seiko Optical.

CR-39 is a registered trademark of PPG Industries, Pittsburgh Pennsylvania.

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APPENDIX A: BASIC OPTICAL MATHEMATICS

METRIC CONVERSION

Optical problems typically utilize the metric system of measurement. Here are some common conversions:

- 1 kilometer (km) = 1,000 meters = 100,000 centimeters = 1,000,000 millimeters
- 1 meter (m) = 0.001 kilometers = 100 centimeters = 1,000 millimeters
- 1 centimeter (cm) = 0.01 meters = 10 millimeters
- 1 millimeter (mm) = 0.1 centimeters = 0.001 meters
- 1 micrometer (μm) = 0.001 millimeters = 0.000001 meters
- 1 nanometer (nm) = 0.000001 millimeters = 0.000000001 meters
- 1 angstrom (\AA) = 0.0000001 millimeters = 0.0000000001 meters
- 1 inch = 2.54 centimeters
- 1 centimeter = 0.3937 inches

For instance,

$$5,000 \text{ \AA} = 500 \text{ nm} = 0.500 \mu\text{m} = 0.000500 \text{ mm} = 0.0000500 \text{ cm} = 0.000000500 \text{ m}$$

For extremely large or small numbers, **scientific notation** is often used. A number expressed in scientific notation is written as a number between 1 and 10 multiplied by a power of 10 (i.e. $x \times 10^y$). Remember that negative powers of 10 denote fractions (i.e. $10^{-y} = 1/10^y$). For instance,

$$1 \times 10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

$$1 \times 10^{-3} = 1 \times 1 / (10 \times 10 \times 10) = 1 \times 1/1000 = 0.001$$

$$0.00006782 = 6.782 \times 10^{-5}$$

$$67820000 = 6.782 \times 10^7$$

RECIPROCALS

Reciprocal relationships are used extensively in optics. The **reciprocal**, or **inverse**, of a number can be obtained by dividing the number into 1. If a number (y) is equal to the reciprocal of another number (x), then

$$y = \frac{1}{x} = x^{-1} \quad \text{and} \quad x = \frac{1}{y} = y^{-1}$$

If y is the reciprocal of x , then x is also the reciprocal of y . Further, as y increases, x decreases. This means that when x becomes infinitely large ($x \rightarrow \infty$), y becomes infinitely small ($y \rightarrow 0$), and vice versa. The \rightarrow symbol means “approaches.” If the **absolute value** (or magnitude) of the denominator is less than 1, the absolute value of the reciprocal will be greater than 1 (or $>$ unity). Similarly, if the absolute value of the denominator is greater than 1, the absolute value of the reciprocal will be less than 1 (or $<$ unity). Negative numbers will still have negative reciprocals. Remember that the denominator can not equal zero, or the result is undefined and cannot be solved.

NUMERIC RELATIONSHIPS

In this section we will briefly identify and describe three common numeric relationships employed throughout this workbook. This workbook often speaks of *proportional* and *inversely proportional* relationships. When one quantity varies at the exact same rate that a second quantity does, we say that the quantity *varies directly as*, or is *directly proportional to*, the second quantity. If k is some constant, nonzero value, y will vary directly as x , so that

$$y = k \cdot x$$

If the value of x doubles, for example, y will also double. The value of x will always be a multiple of y .

As a result, an increase in x will result in a similar increase in y . When one quantity varies inversely with a second quantity (e.g. the reciprocal), we say that the quantity *varies inversely as*, or is *inversely proportional to*, the second quantity. Once again, if k is some constant value, y will vary inversely as x , so that

$$y = \frac{k}{x}$$

Another relationship, the **ratio**, is a simple method of comparing numbers or variables to each other. A ratio is basically the fraction of one number to the other, and is usually written in either one of two forms: $y : x$ or y / x .

GEOMETRY

Optics makes so much use of geometrical constructions, that it has an entire branch called *geometrical optics*. A concise review of the more fundamental concepts of geometry is provided here. Consider the circle in Figure I. Point C serves as the center of this circle. The line segment CR begins at the center and terminates at the circle. This line segment is referred to as the **radius** of the circle. The line segment DD' passes through the center (across the entire circle) and has its endpoints located on the circle. This line segment is referred to as a **diameter** of the circle. The diameter of a circle is equal to twice its radius. The line TT' contacts the circle at a single point (R). We say that this line is **tangent** to the circle at that single point. Lines tangent to a circle are perpendicular to the radius or diameter ending at that point. Therefore, line TT' is perpendicular to line CR.

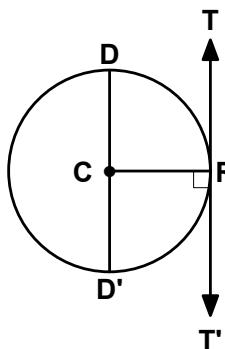


FIGURE I The circle, radius, and diameter.

Two triangles that have the exact same shape (i.e., angles), but are different in size, are referred to as **similar triangles**. Similar triangles will have equal corresponding angles and corresponding sides that are proportional in size. For instance, we know that the two triangles in Figure II are similar because they share a common angle (A and A'), and they both have a 90° angle (C and C'). We can infer from this that angles B and B' are also equal. Therefore, the sides of these two triangles are also proportional to each other.

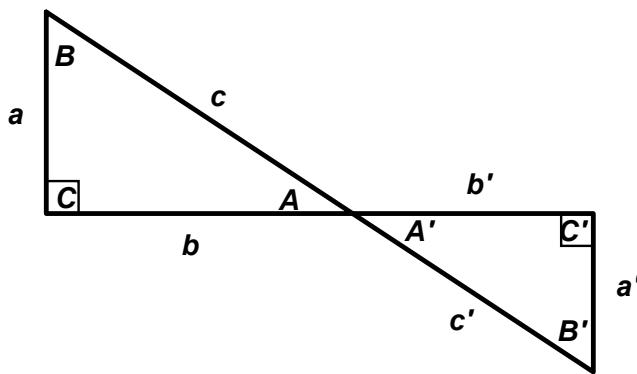


FIGURE II Similar triangles.

This type of proportional relationship can be expressed as

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

Also,

$$\frac{\angle A}{\angle A'} = \frac{\angle B}{\angle B'} = \frac{\angle C}{\angle C'}$$

Triangles have some useful other properties, as well. Using the triangle illustrated in Figure III:

- The sum of the interior angles of a triangle add up to 180° :

$$\angle A + \angle B + \angle C = 180$$
- The exterior angle of a triangle is equal to the sum of the opposite (or remote) interior angles:

$$\angle D = \angle A + \angle B$$

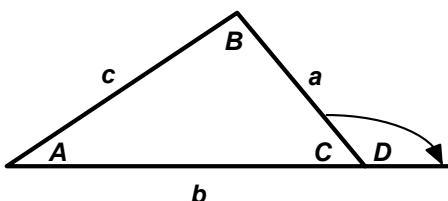


FIGURE III Interior and exterior angles.

Lastly, we will consider some additional concepts from geometry related to parallel lines and the angles created by them. In Figure IV, the lines AA' and BB' are parallel to each other. A third line, CC', is a transversal intersecting the other two. The following relationships can be established between the angles created by these intersections:

$$\angle 1 = \angle 3 = \angle 1' = \angle 3'$$

$$\angle 2 = \angle 4 = \angle 2' = \angle 4'$$

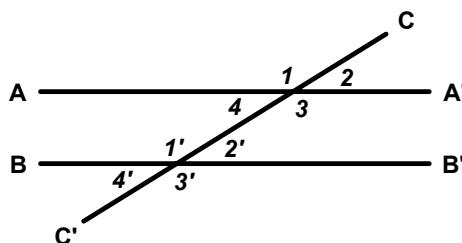


FIGURE IV The corresponding angles of two parallel lines and a transversal.

TRIGONOMETRY

Several relationships can be established when one angle of a triangle is equal to 90° . This special triangle is referred to as a **right triangle**, and is shown in Figure V.

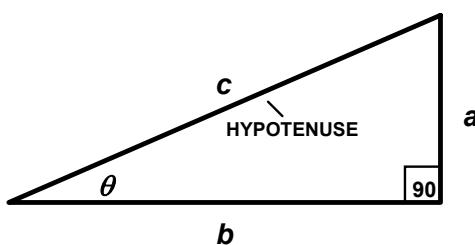


FIGURE V A *right* triangle.

One such relationship, the **Pythagorean theorem**, states that for a right triangle the sum of the squares of the legs is equal to the square of the hypotenuse (the longest side). This can be expressed as

$$a^2 + b^2 = c^2$$

Optical mathematics frequently involves trigonometric functions, as well. For a given angle, a predetermined ratio exists between certain sides of a right triangle. These ratios can be found using a table of trigonometry functions or a scientific calculator. There are three common trigonometric functions: the **sine** (abbreviated sin), the **tangent**

(abbreviated tan), and the **cosine** (abbreviated cos). For the angle (θ) in the triangle above, these trigonometric relationships can be expressed as:

$$\sin \theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cos \theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

In addition to these trigonometric functions, there are important trigonometric identities like:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

We can also relate trigonometric functions to convert between polar and rectangular coordinates. Rectangular coordinates use a horizontal directed distance x and vertical directed distance y to specify the location of a point P at (x, y) . These directed distances can also represent the magnitude and direction of a vector. Polar coordinates use a single directed distance r and the directed angle θ from the x -axis to specify the location of a point P at (r, θ) . The directed distances of either system are from the origin O at $(0,0)$.

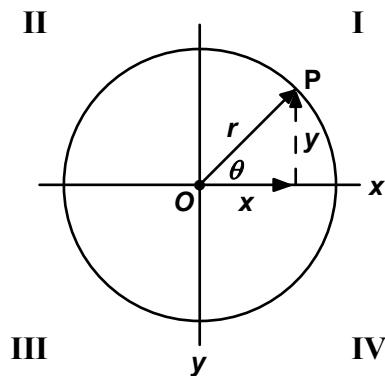


FIGURE VI Polar and rectangular coordinates.

To convert back and forth from polar to rectangular coordinate form, consider Figure VI. The rectangular coordinates (x, y) are related to the polar coordinates (r, θ) by:

$$x = r \cdot \cos \theta$$

$$y = r \cdot \sin \theta$$

Moreover, the polar coordinates $(r$ and θ) are related to the rectangular coordinates $(x$ and y) by:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

When solving for the angle θ , it is important to note which *quadrant* (i.e., I, II, III, or IV) the point P falls in. For instance, angle 30° has the same tangent that angle 210° has. In many cases, the simplest way to deal with this is to first determine the *arctangent* (\tan^{-1}) of y / x , and then apply the following rules for the quadrant that the point P falls in—ignoring the sign (\pm) of the originally calculated angle θ so that it is always positive:

Quadrant I: Actual θ is equal to calculated θ

Quadrant II: Actual θ is equal to $180^\circ - \text{calculated } \theta$

Quadrant III: Actual θ is equal to $180^\circ + \text{calculated } \theta$

Quadrant IV: Actual θ is equal to $360^\circ - \text{calculated } \theta$

Remember to ignore the sign of the original angle θ after you have calculated it; simply treat it as positive (+). Following the quadrant rules will ensure that the final angle adheres to the appropriate convention.

RADIANS

We are all familiar with the *degree* system of angular measurement. A central angle of a circle that subtends an arc equal to $1/360$ of the circumference of that circle is equal to 1° . Therefore, it takes 360° to make a complete revolution around a circle. Another method of angular measurement uses the *radian* system. The angle θ , in **radians**, is equal to the length of the arc a subtended by that central angle, divided by the radius of the circle r :

$$\theta = \frac{a}{r}$$

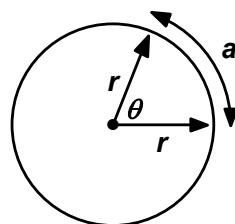


FIGURE VII Radian measure.

When the length of arc a is equal to the radius r , angle θ is equal to 1 radian (abbreviated rad). Because the circumference of a circle is equal to $2\pi r$, an angle of 360° is equal to 2π rad. This means that $1 \text{ rad} \approx 57.3^\circ$. Conversely, $1^\circ \approx 0.0175 \text{ rad}$.

The \approx symbol means “approximately equal to.” Therefore, the *sine* of an angle is approximately equal to the angle itself (in radians) for small angles. This also holds true for the *tangent* of a small angle.

SMALL ANGLE APPROXIMATIONS

Periodically, we will employ ‘small angle approximations’ to simplify some of the formulas that we will encounter. This is a way to approximate a trigonometric function. For instance, the sine of an angle (θ) can be expressed as

$$\sin \theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \dots$$

where θ is the angle in radians.

For small angles, this reduces to

$$\sin \theta \approx \theta$$

Small angle approximations also make it possible to find the sine or tangent of a small angle without the use of a scientific calculator or table of trigonometric functions. Since we know that there are approximately 0.0175 rad in every 1° , we can multiply the angle θ by this amount to determine its value in radians. And for small angles, this value is roughly equal to both the sine and the tangent of angle θ . For instance, consider the approximate radian value of a 15° angle:

$$\theta_{RAD} = 0.0175\theta_{DEG}$$

$$\theta_{RAD} = 0.0175(15^\circ)$$

$$\theta_{RAD} = 0.2625 \text{ rad}$$

The *sine* of a 15° angle is 0.2588 and the *tangent* of a 15° angle is 0.2679. These are both quite close to our radian value for 15° . Note that in general the $\sin \theta < \theta_{\text{RAD}} < \tan \theta$.

BINOMIAL EXPANSION

Another approximation that we will make frequent use of involves a binomial (having two terms) expansion. When one term of a binomial is equal to 1, and the other term x is some value that is less than 1, a binomial can be represented with the following series

$$(1 \pm x)^n = 1 \pm n \cdot x + \frac{n(n-1)x^2}{2} \pm \frac{n(n-1)(n-2)x^3}{6} + \dots$$

When x is much less than 1 ($x \ll 1$), we can drop off the ‘higher order’ terms and reduce this series to

$$(1 \pm x)^n \approx 1 \pm n \cdot x$$

Although the application of this approximation may not be obvious at first, some formulas that involve square-roots ($n = \frac{1}{2}$) or reciprocals ($n = -1$) can be made more manageable. For instance, consider this square-root function:

$$y = \sqrt{1+x}$$

This can be rewritten algebraically as

$$y = (1+x)^{\frac{1}{2}}$$

Now, after using our binomial expansion and dropping higher order terms, we have:

$$y \approx 1 + \frac{1}{2}x$$

An approximation of this form is easier to work with in some situations.

APPENDIX B: DERIVATIONS

CONJUGATE FOCI FORMULA

To derive the *conjugate foci formula* (Eq. 13) for a lens surface, we will make use of Figure I. Figure I uses ‘all positive’ measurements; meaning that object vergence, surface power, and image vergence are all positive values. Recall that for small angles, the sine and tangent of an angle are nearly equal to the angle itself, when it is expressed in radians. Applying a small angle approximation ($\sin i \approx i$) for Snell’s law of refraction (Eq. 5) gives us:

$$n \cdot \sin i = n' \cdot \sin i'$$

$$n' \cdot i' \approx n \cdot i$$

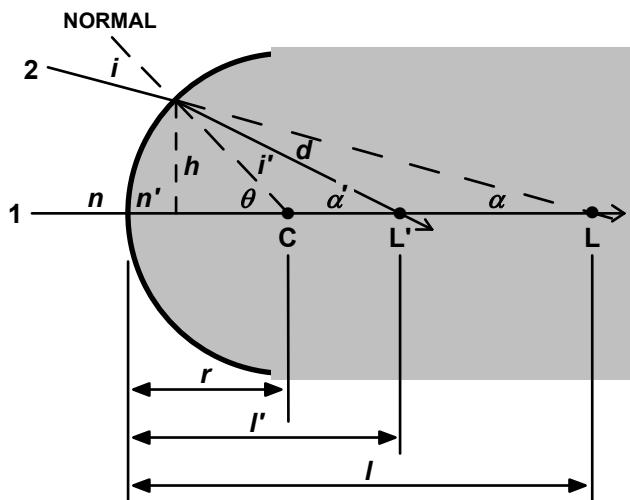


FIGURE I Conjugate foci formula derivation.

Rays 1 and 2 in Figure I are converging to form a *virtual* object at point L. After refraction by the lens surface, they intersect each other at the *real* image point L'. We are trying to develop a relationship between the reduced object distance (or vergence), the image distance (or vergence), and the refractive power of the surface. Ray 1 strikes the surface perpendicularly (normal), and is not refracted. Ray 2 strikes the surface at the height h ; the normal to the surface at this point has been drawn through the center of curvature at C. The angle of incidence i of the ray 2 is equal to the difference between angle θ and angle α (because of our remote interior relationship, $\theta = i + \alpha$), or

$$i = \theta - \alpha$$

The angle of refraction i' is equal to the difference between angle θ and angle α' (because of our remote interior relationship, $\theta = i' + \alpha'$), or

$$i' = \theta - \alpha'$$

Angle i is also related to the angle of refraction i' , by our approximation of Snell’s law,

$$n \cdot i \approx n' \cdot i'$$

After substituting for angles i and i' , we can show that

$$n(\theta - \alpha) = n'(\theta - \alpha')$$

Now we can make some small angle approximations for the angles α , α' , and θ . From Figure I, we know that:

$$\alpha \approx \frac{h}{l}$$

And,

$$\alpha' \approx \frac{h}{l'}$$

Finally,

$$\theta \approx \frac{h}{r}$$

where h is the height of the ray, l is the object distance, l' is the image distance, and r is the radius of curvature of the surface in meters.

Now let's substitute these back into our earlier relationship:

$$n\left(\frac{h}{r} - \frac{h}{l}\right) = n'\left(\frac{h}{r} - \frac{h}{l'}\right)$$

We can now factor out and cancel the height h , which is common to each term. This means that the heights of the rays no longer matter (at least for small angles). We can then distribute n and n' , and rearrange the terms to give us the *conjugate foci formula* (Rubin 70):

$$\frac{n'}{l'} = \frac{n' - n}{r} + \frac{n}{l}$$

where n is the refractive index to the *left* of the surface (on the *object* side), n' is the refractive index to the *right*, r is the radius of curvature, l is the object distance, and l' is the image distance in meters.

SPECTACLE MAGNIFICATION FORMULA

To derive the *spectacle magnification formula* (Eq. 71) for a thin lens, we will make use of Figure II. Recall that angular magnification M is the ratio of the angle θ' subtended by the *image* produced by the lens, at the nodal point of the eye N , compared to angle θ subtended by the original *object* (Eq. 70), so that (Rubin 207):

$$M = \frac{\theta'}{\theta}$$

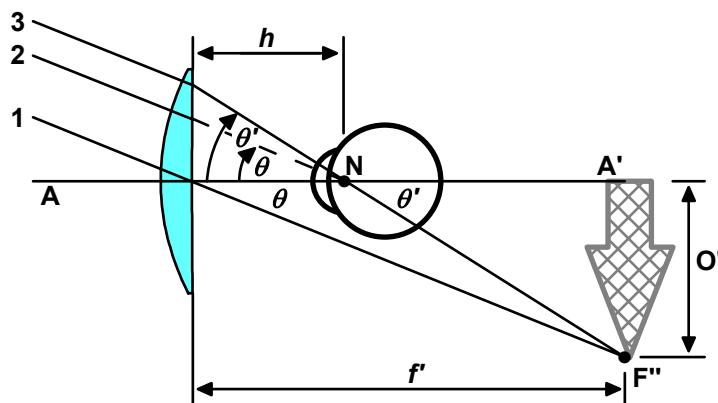


FIGURE II Thin lens magnification.

Rays 1, 2, and 3 are parallel rays of light from an object at optical infinity (∞). All three rays eventually intersect in the *secondary focal plane* of the lens at point F'' . Ray 1 passes without deviation through the optical center of the lens—which, we are assuming, is infinitely *thin*. Ray 2 from the object has been extended through the nodal point N to demonstrate the angle θ that the *object* would have subtended at the nodal point, if no lens had been in place. This angle (θ) is also the same angle the angle that ray 1 creates as it crosses the optical axis AA' . Recall that rays of light passing through the nodal point of the eye are not deviated. Ray 3 from the object is refracted to point F'' and passes through the nodal point N along the way. Ray 3 creates angle θ' , which is the angle the *image* subtends at the nodal point N .

Using the construction in Figure II and small angle approximations for θ' and θ , we can see that angle θ is given by

$$\theta \approx \frac{O'}{f'}$$

Further, angle θ' is given by

$$\theta' \approx \frac{O'}{f' - h}$$

Recall that the angular magnification is M defined as

$$M = \frac{\theta'}{\theta}$$

After substituting our new relationships for angle θ and angle θ' , the magnification M is given by

$$M = \frac{\frac{O'}{f' - h}}{\frac{O'}{f'}} = \frac{f'}{f' - h}$$

Substituting the focal power relationship (i.e., the quantity $1 / F$) for f' , and clearing out the fractions, gives us the formula for the *spectacle magnification formula* for a thin lens:

$$M = \frac{1}{1 - h \cdot F}$$

where h is the distance from the nodal point to the lens in meters and F is the focal power.

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