UNIVERSITI TEKNOLOGI MARA

ASSIGNMENT 1

NUMERICAL SIMULATION OF BOUNDARY LAYER FLOW PAST ON EXPONENTIAL SHEET

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CONTENTS

INTRODUCTION

The Blasius's Solution for the laminar boundary layer on a flat plate can be solved by using analytical solution and Numerical Method (Weltry, J. R., Wicks, C. E., Wilson, R. E., & Rorrer, G. l., 2009). We have learnt on how to obtain general solution of differential equation with first order differential equation by recognizing them as separable, linear, exact, homogenous or perhaps Bernoulli equations. If the solution of ordinary differential equation of order two or higher we need to examine some of theory in linear equations (Zill & Wright, 2012). However there is a limitation when solving it analytically, hence we need to solve the higher order of differential equation by using numerical method. Applied numerical can get better approximation value by using iterative technique.

The purpose of this assignment is to apply the knowledge that we have learnt from the Blasius equation. Basically, the momentum equation can be transform to ODE by using similarity transformation, then we use the numerical method to solve the non linear ODE. In this assignment we were given task to find only one journal that has boundary layer equation for momentum. The article that has been chosen for this assignment is "Numerical simulation of boundary layer flow of nanofluid past an exponential stretching sheet" (Subhas Abel, M., & Rajeshsingh, N., 2015).

OBJECTIVE

The purpose of this report is :

1. to derive the momentum equation to non linear ODE by using similarity transformation.
2. to solve the non linear ODE by using numerical method which are Runge Kutta (RK4) and Runge Kutta Fehlberg (RKF)
3. to display the graph with different value of step size, h.
4. to show the value of output at point η by using Matlab which simultaneously write the result in excel and compare with actual solution.

METHODOLOGY

We consider a steady, incompressible, two dimensional boundary layer flow of viscous nanofluid past a flat sheet coinciding with the plane y=0 and being confined y>0. The basic steady of conversation of mass and momentum from article are given as below:

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (Continuity equation)

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2}$ (Momentum equation)

The boundary conditions are

$$v = 0, \qquad u = u_w(x) \qquad at\ y = 0$$

$$u = v = 0, \qquad as\ y \to \infty$$

$$u_w(x) = U_0\, e^{\frac{x}{L}}$$

Since we are only interested on similarity solution of the above boundary layer flow hence we need to introduce the following similarity transformation (dimensionless quantities).

$$\eta = y\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$\psi = \sqrt{2\upsilon L U_0}e^{\frac{x}{2L}}f(\eta)$$

Below are the derivation from momentum equation to the ODE form equation.

$$\boxed{\begin{aligned}\frac{\partial\eta}{\partial y} &= \frac{\partial}{\partial y}[y\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}] \\ &= \sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{\partial\eta}{\partial x} &= \frac{\partial}{\partial x}[y\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}] \\ &= \frac{y}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}\end{aligned}}$$

$$u = \frac{\partial \psi}{\partial y}$$

$$= \sqrt{2\upsilon L U_0} e^{\frac{x}{2L}} \frac{\partial}{\partial \eta}[f(\eta)] \frac{\partial \eta}{\partial y}$$

$$= \sqrt{2\upsilon L U_0} e^{\frac{x}{2L}} [f'(\eta)] \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}}$$

$$= U_0 e^{\frac{x}{L}} f'(\eta)$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$= -\sqrt{2\upsilon L U_0} \{ e^{\frac{x}{2L}} \frac{\partial}{\partial \eta}[f(\eta)] \frac{\partial \eta}{\partial x} + f(\eta) \frac{\partial}{\partial x}\left[e^{\frac{x}{2L}}\right] \}$$

$$= -\sqrt{2\upsilon L U_0} \{ e^{\frac{x}{2L}} f'(\eta) \frac{y}{2L} \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}} + f(\eta) \frac{1}{2L} e^{\frac{x}{2L}} \}$$

$$= -\sqrt{\frac{\upsilon U_0}{2L}} e^{\frac{x}{2L}} [\eta \, f'(\eta) + f(\eta)]$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}[U_0 e^{\frac{x}{L}} f'(\eta)]$$

$$= U_0 \{ e^{\frac{x}{L}} \frac{\partial}{\partial \eta}[f'(\eta)] \frac{\partial \eta}{\partial x} + f'(\eta) \frac{\partial}{\partial x}\left[e^{\frac{x}{L}}\right] \}$$

$$= U_0 \{ (e^{\frac{x}{L}} [f''(\eta)] \frac{y}{2L} \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}}) + f'(\eta) \left[\frac{1}{L} e^{\frac{x}{L}}\right] \}$$

$$= \frac{U_0 e^{\frac{x}{L}}}{L} \left[\frac{1}{2} \eta f''(\eta) + f'(\eta)\right]$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}[-\sqrt{\frac{\upsilon U_0}{2L}} e^{\frac{x}{2L}} [f(\eta) + \eta \, f'(\eta)]]$$

$$= -\sqrt{\frac{\upsilon U_0}{2L}} e^{\frac{x}{2L}} \{ \frac{\partial}{\partial y}[f(\eta) + \eta \, f'(\eta)] \}$$

$$= -\sqrt{\frac{\upsilon U_0}{2L}} e^{\frac{x}{2L}} \{ f'(\eta) \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}} + \eta f''(\eta) \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}} + f'(\eta) \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}} \}$$

$$= -\frac{U_0 e^{\frac{x}{L}}}{L} \left[\frac{1}{2} \eta f''(\eta) + f'(\eta)\right]$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta)+f'(\eta)\right]-\frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta)+f'(\eta)\right]$$
$$=0$$

It is proven that the continuity equation is satisfied. Then, use the governing equation of momentum takes the form of non-linear ordinary differential equations. Below are the derivation for non-linear ordinary equations.

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}[U_0 e^{\frac{x}{L}} f'(\eta)]$$
$$=U_0 e^{\frac{x}{L}}\frac{\partial}{\partial y}[f'(\eta)]$$
$$=U_0 e^{\frac{x}{L}} f''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$
$$=U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f''(\eta)$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}[U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f''(\eta)]$$
$$=U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}}\frac{\partial}{\partial y}[f''(\eta)]$$
$$=U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f'''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$
$$=\frac{{U_0}^2}{2\upsilon L}e^{\frac{2x}{L}} f''(\eta)$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}-\upsilon\frac{\partial^2 u}{\partial y^2}=0$$
$$U_0 e^{\frac{x}{L}} f'(\eta)\left\{\frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta)+f'(\eta)\right]\right\}+\left\{-\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}\,[\eta\, f'(\eta)+f(\eta)]\right\}U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f''(\eta)$$
$$-\upsilon\frac{{U_0}^2}{2\upsilon L}e^{\frac{2x}{L}} f'''(\eta)=0$$
$$\frac{{U_0}^2}{2L}e^{\frac{2x}{L}}[\eta f'(\eta)f''(\eta)+2[f'(\eta)]^2-\eta f'(\eta)f''(\eta)-f(\eta)f''(\eta)-\frac{\upsilon}{\upsilon}f'''(\eta)=0$$
$$f'''(\eta)+f(\eta)f''(\eta)-2[f'(\eta)]^2=0$$

Then, we can derived the formula by reducing the order to single ordinary differential equation with numerical method by using Runge Kutta Method and Runge Kutta Felhberg Method. Although the differential equation looks like linear however the presence of ff'' it shows that the differential equation is non-linear.

The differential equation is reduced to single ordinary differential equation as shown as below:

$$f' = p$$
$$f'' = p' = q$$
$$f''' = q' = -fq + 2p^2$$

The boundary conditions is set as below:

$$f(0)=0 \quad f'(0)=1 \text{ at } \eta=0$$

$$\lim_{\eta\to\infty} f'(\eta) = 0$$

We set the initial value problem which include initial value of η=a =0, final value of η=b =4 and the initial guess for q is -1.28437. The initial guess value of q is correct to 5 decimal places this is because to obtain a better approximation value of p=$f'(\eta)$. If $\lim_{\eta\to\infty} f'(\eta) = 0$ hence, the initial value of p is accurate. We know that when η is approaching to the final value of η, the value of p is approaching to 0. We use Runge Kutta Method and Runge Kutta Fehlberg Method in order to solve differential equation (Burden & Faires, 2010).

Runge Kutta Method:

$$y_i = a$$

$$k_1 = hf\,(t_i, y_i)$$

$$k_2 = hf\,(t_i + \frac{h}{2}, y_i + \frac{1}{2}k1)$$

$$k_3 = hf\,(t_i + \frac{h}{2}, y_i + \frac{1}{2}k2)$$

$$k_4 = hf\,(t_i + h, y_i + k3)$$

$$y_{i+1} = y_i + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)$$

Runge Kutta Fehlberg Method:

$$k_1 = hf\,(t_i, y_i)$$

$$k_2 = hf(\, t_i + \frac{h}{4}\,, y_i + \frac{1}{4}k_1\,)$$

$$k_3 = hf\,(\, t_i + \frac{3}{8}h, y_i + \frac{3}{32}k_1 + \frac{9}{32}\,k_2,)$$

$$k_4 = hf\,(\, t_i + \frac{12}{13}h\,, y_i + \frac{1932}{2197}\,k_1 - \frac{7200}{2197}\,k_2 + \frac{7296}{2197}\,k_3\,,)$$

$$k_5 = hf\,(t_i + h, y_i + \frac{439}{216}\,k_1 - 8k_2 + \frac{3680}{513}\,k_3 - \frac{845}{4104}\,k_4)$$

$$k_6 = hf\,(t_i + \frac{h}{2}\,, y_i - \frac{8}{27}\,k_1 + 2k_2 - \frac{3544}{2565}\,k_3 + \frac{1859}{4104}\,k_4 - \frac{11}{40}\,k_5)$$

CODING

<u>Runge Kutta (RK4)</u>

```
clc
%RK4
%declare variable
a=0;
b=4;
h=0.1;
N=(b-a)/h;
%higher order to ODE
g1=@(eta,f,p,q) p;
g2=@(eta,f,p,q) q;
g3=@(eta,f,p,q) -f*q+2*p^(2);
%initial value problem
eta(1)=a;
f(1)=0;
p(1)=1;
%initial guess
q(1)=-1.284367;
```

```
for i=1:N
    k1=g1(eta(i),f(i),p(i),q(i));
    m1=g2(eta(i),f(i),p(i),q(i));
    n1=g3(eta(i),f(i),p(i),q(i));

    k2=g1(eta(i)+h/2,f(i)+h*k1/2,p(i)+h*m1/2,q(i)+h*n1/2);
    m2=g2(eta(i)+h/2,f(i)+h*k1/2,p(i)+h*m1/2,q(i)+h*n1/2);
    n2=g3(eta(i)+h/2,f(i)+h*k1/2,p(i)+h*m1/2,q(i)+h*n1/2);

    k3=g1(eta(i)+h/2,f(i)+h*k2/2,p(i)+h*m2/2,q(i)+h*n2/2);
    m3=g2(eta(i)+h/2,f(i)+h*k2/2,p(i)+h*m2/2,q(i)+h*n2/2);
    n3=g3(eta(i)+h/2,f(i)+h*k2/2,p(i)+h*m2/2,q(i)+h*n2/2);

    k4=g1(eta(i)+h,f(i)+h*k3,p(i)+h*m3,q(i)+h*n3);
    m4=g2(eta(i)+h,f(i)+h*k3,p(i)+h*m3,q(i)+h*n3);
    n4=g3(eta(i)+h,f(i)+h*k3,p(i)+h*m3,q(i)+h*n3);

    f(i+1)=f(i)+h*(k1+2*k2+2*k3+k4)/6;
    p(i+1)=p(i)+h*(m1+2*m2+2*m3+m4)/6;
    q(i+1)=q(i)+h*(n1+2*n2+2*n3+n4)/6;
    eta(i+1)=eta(i)+h;

    %print output
    fprintf('Iteration=%2d\t eta(%d)=%.4f\t f(%d)=%.4f\t p(%d)=%.4f\t
q(%d)=%.4f\t \n',i,i,eta(i+1),i,f(i+1),i,p(i+1),i,q(i));

end

%plot graph
%since x-axis is actually an eta, so we need to plot the graph f,p,q with
%respect to eta
plot(eta,f, 'LineWidth', 1)
hold on
plot(eta,p, 'LineWidth', 1)
plot(eta,q, 'LineWidth', 1)
axis([0 4 -2 inf])
hold off
legend('f', 'p', 'q')
title('Runge Kutta')
xlabel('\eta')
ylabel('f,p,q')
```

Runge Kutta Fehlberg (RKF)

```
clc
%declare variable
a=0;
b=4;
h=0.1;
N=(b-a)/h;
%higher order to ODE
g1=@(eta,f,p,q) p;
g2=@(eta,f,p,q) q;
g3=@(eta,f,p,q) -f*q+2*p^(2);
```

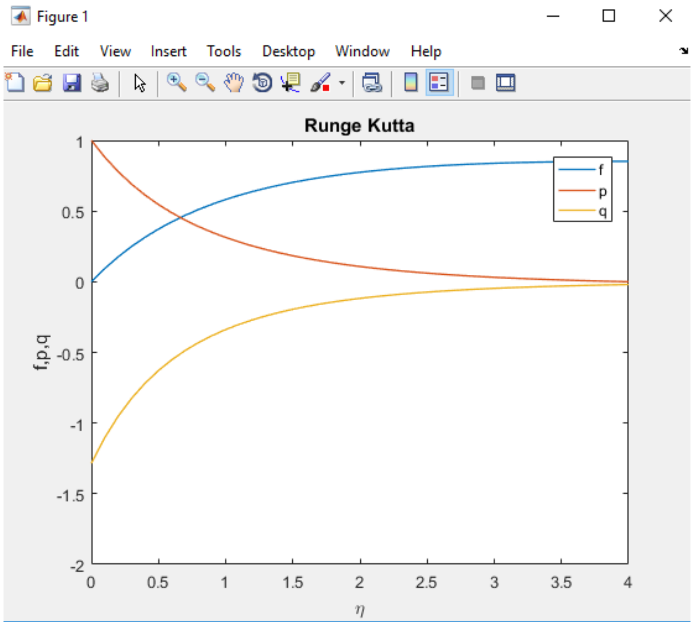
```
%initial value problem
eta(1)=a;
f(1)=0;
p(1)=1;
%initial guess
q(1)=-1.284367;
%RKF
for i=1:N
    k1=g1(eta(i),f(i),p(i),q(i));
    m1=g2(eta(i),f(i),p(i),q(i));
    n1=g3(eta(i),f(i),p(i),q(i));

    k2=g1(eta(i)+h/4,f(i)+h*k1/4,p(i)+h*m1/4,q(i)+h*n1/4);
    m2=g2(eta(i)+h/4,f(i)+h*k1/4,p(i)+h*m1/4,q(i)+h*n1/4);
    n2=g3(eta(i)+h/4,f(i)+h*k1/4,p(i)+h*m1/4,q(i)+h*n1/4);


k3=g1(eta(i)+3*h/8,f(i)+3*h*k1/32+9*h*k2/32,p(i)+3*h*m1/32+9*h*m2/32,q(i)+3*h
*n1/32+9*h*n2/32);

m3=g2(eta(i)+3*h/8,f(i)+3*h*k1/32+9*h*k2/32,p(i)+3*h*m1/32+9*h*m2/32,q(i)+3*h
*n1/32+9*h*n2/32);

n3=g3(eta(i)+3*h/8,f(i)+3*h*k1/32+9*h*k2/32,p(i)+3*h*m1/32+9*h*m2/32,q(i)+3*h
*n1/32+9*h*n2/32);

    k4=g1(eta(i)+12*h/13,f(i)+1932*h*k1/2197-
7200*h*k2/2197+7296*h*k3/2197,p(i)+1932*h*m1/2197-
7200*h*m2/2197+7296*h*m3/2197,q(i)+1932*h*n1/2197-
7200*h*n2/2197+7296*h*n3/2197);
    m4=g2(eta(i)+12*h/13,f(i)+1932*h*k1/2197-
7200*h*k2/2197+7296*h*k3/2197,p(i)+1932*h*m1/2197-
7200*h*m2/2197+7296*h*m3/2197,q(i)+1932*h*n1/2197-
7200*h*n2/2197+7296*h*n3/2197);
    n4=g3(eta(i)+12*h/13,f(i)+1932*h*k1/2197-
7200*h*k2/2197+7296*h*k3/2197,p(i)+1932*h*m1/2197-
7200*h*m2/2197+7296*h*m3/2197,q(i)+1932*h*n1/2197-
7200*h*n2/2197+7296*h*n3/2197);

    k5=g1(eta(i)+h,f(i)+439*h*k1/216-8*h*k2+3680*h*k3/513-
845*h*k4/4104,p(i)+439*h*m1/216-8*h*m2+3680*h*m3/513-
845*h*m4/4104,q(i)+439*h*n1/216-8*h*n2+3680*h*n3/513-845*h*n4/4104);
    m5=g2(eta(i)+h,f(i)+439*h*k1/216-8*h*k2+3680*h*k3/513-
845*h*k4/4104,p(i)+439*h*m1/216-8*h*m2+3680*h*m3/513-
845*h*m4/4104,q(i)+439*h*n1/216-8*h*n2+3680*h*n3/513-845*h*n4/4104);
    n5=g3(eta(i)+h,f(i)+439*h*k1/216-8*h*k2+3680*h*k3/513-
845*h*k4/4104,p(i)+439*h*m1/216-8*h*m2+3680*h*m3/513-
845*h*m4/4104,q(i)+439*h*n1/216-8*h*n2+3680*h*n3/513-845*h*n4/4104);

    k6=g1(eta(i)+h/2,f(i)-8*h*k1/27+2*h*k2-3554*h*k3/2565+1859*h*k4/4104-
11*h*k5/40,p(i)-8*h*m1/27+2*h*m2-3554*h*m3/2565+1859*h*m4/4104-
11*h*m5/40,q(i)-8*h*n1/27+2*h*n2-3554*h*n3/2565+1859*h*n4/4104-11*h*n5/40);
    m6=g2(eta(i)+h/2,f(i)-8*h*k1/27+2*h*k2-3554*h*k3/2565+1859*h*k4/4104-
11*h*k5/40,p(i)-8*h*m1/27+2*h*m2-3554*h*m3/2565+1859*h*m4/4104-
11*h*m5/40,q(i)-8*h*n1/27+2*h*n2-3554*h*n3/2565+1859*h*n4/4104-11*h*n5/40);
```

```
    n6=g3(eta(i)+h/2,f(i)-8*h*k1/27+2*h*k2-3554*h*k3/2565+1859*h*k4/4104-
11*h*k5/40,p(i)-8*h*m1/27+2*h*m2-3554*h*m3/2565+1859*h*m4/4104-
11*h*m5/40,q(i)-8*h*n1/27+2*h*n2-3554*h*n3/2565+1859*h*n4/4104-11*h*n5/40);

    f(i+1)=f(i)+h*(125*k1/216+7040*k3/2565+10985*k4/4101-k5)/5;
    p(i+1)=p(i)+h*(125*m1/216+7040*m3/2565+10985*m4/4101-m5)/5;
    q(i+1)=q(i)+h*(125*n1/216+7040*n3/2565+10985*n4/4101-n5)/5;
    eta(i+1)=eta(i)+h;

    %print output
    fprintf('Iteration=%2d\t eta(%d)=%.4f\t f(%d)=%.4f\t p(%d)=%.4f\t
q(%d)=%.4f\t \n',i,i,eta(i+1),i,f(i+1),i,p(i+1),i,q(i));

end

%plot graph
%since x-axis is actually an eta, so we need to plot the graph f,p,q with
%respect to eta
plot(eta,f, 'LineWidth', 1)
hold on
plot(eta,p, 'LineWidth', 1)
plot(eta,q, 'LineWidth', 1)
axis([0 4 -2 inf])
hold off
legend('f', 'p', 'q')
title('Runge Kutta Fehlberg')
xlabel('\eta')
ylabel('f,p,q')
```

RESULT AND DISCUSSION

<u>Runge Kutta (RK4)</u>

h=0.1

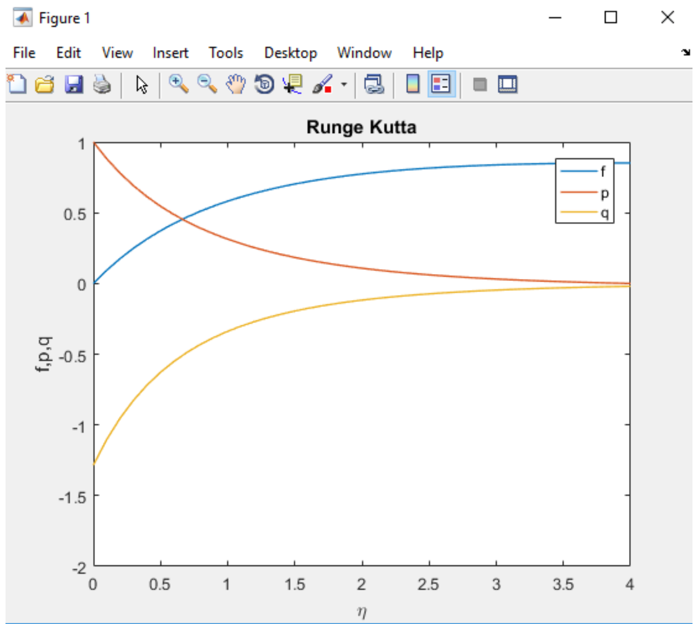


Graph f,p,q versus η with step size 0.1

Table 1. Value of f, p, q at certain points of η with step size 0.1 correct to 4 decimal places.

η	**f**	**p**	**q**
0	0	1	-1.2844
0.5	0.3730	0.5458	-0.6278
1	0.5818	0.3133	-0.3382
1.5	0.7031	0.1839	-0.1948
2	0.7745	0.1077	-0.1176
2.5	0.8157	0.0609	-0.0734
3	0.8382	0.0314	-0.0469
3.5	0.8488	0.0124	-0.0304
4	0.8517	0.0000	-0.0198

h=0.01

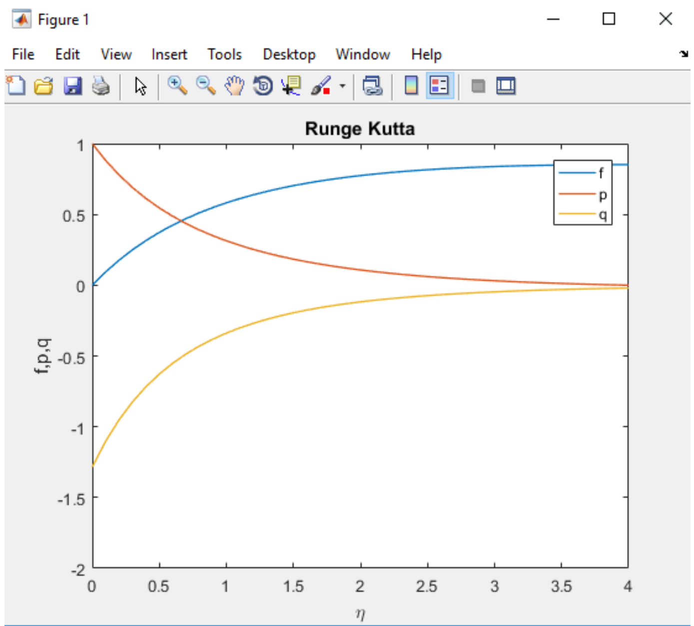


Graph f,p,q versus η with step size 0.01

Table 2. Value of f, p, q at certain points of η with step size 0.01 correct to 4 decimal places.

η	**f**	**p**	**q**
0	0	1	-1.2844
0.5	0.3730	0.5458	-0.6278
1	0.5818	0.3133	-0.3382
1.5	0.7031	0.1839	-0.1948
2	0.7745	0.1077	-0.1176
2.5	0.8157	0.0609	-0.0734
3	0.8382	0.0314	-0.0469
3.5	0.8488	0.0124	-0.0304
4	0.8517	0.0000	-0.0198

h=0.001



Graph f,p,q versus η with step size 0.001

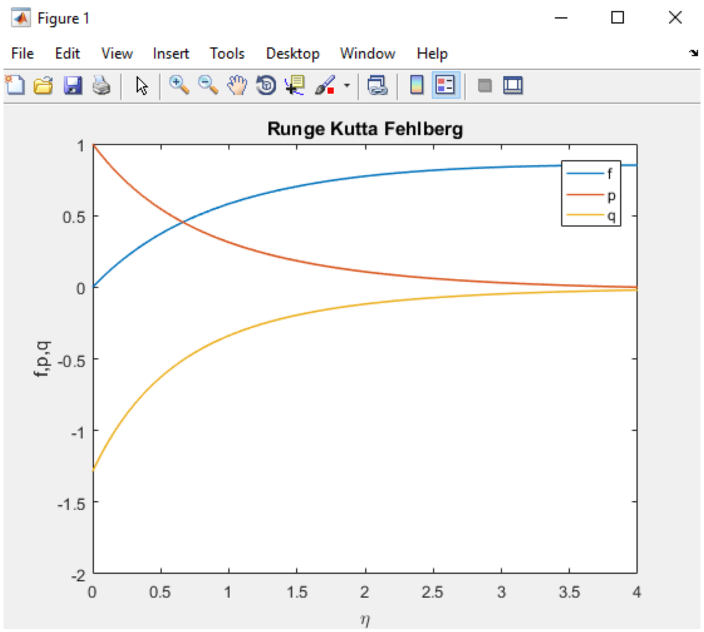
Table 3. Value of f, p, q at certain points of η with step size 0.001 correct to 4 decimal places.

η	**f**	**p**	**q**
0	0	1	-1.2844
0.5	0.3730	0.5458	-0.6278
1	0.5818	0.3133	-0.3382
1.5	0.7031	0.1839	-0.1948
2	0.7745	0.1077	-0.1176
2.5	0.8157	0.0609	-0.0734
3	0.8382	0.0314	-0.0469
3.5	0.8488	0.0124	-0.0304
4	0.8517	0.0000	-0.0198

When the step size is 0.1 it requires 40 iterations, while the step size for 0.01 it requires 400 iterations, lastly for the step size 0.001 the iterations is 4000. Therefore at certain points of η need to show the value of f, p, and q for test equation with different step size. As we can see that the value at certain points in the table are the same for each different value of step size. If we run the coding in Matlab we are able to see that after at point η=3.5 the value of p is approaching to 0.

Runge Kutta Fehlberg (RKF)

h=0.1



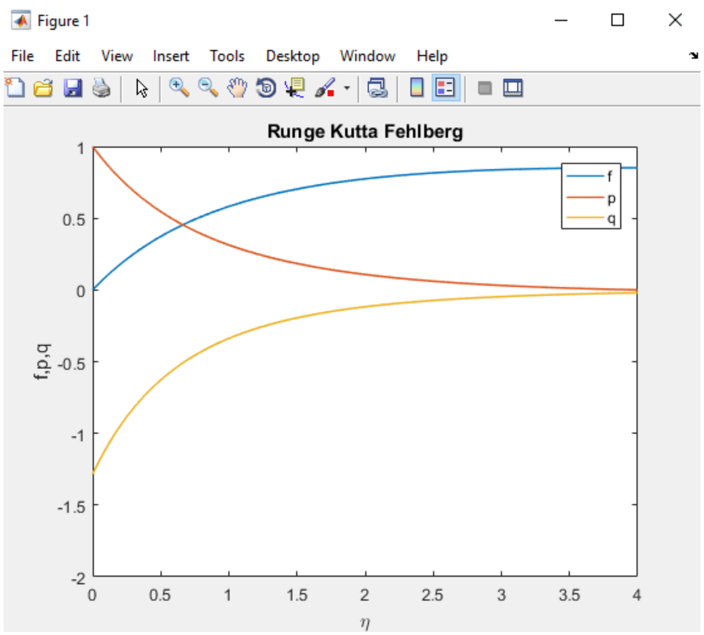
Graph f,p,q versus η with step size 0.1

Table 4. Value of f, p, q at certain points of η with step size 0.1 correct to 4 decimal places.

η	**f**	**p**	**q**
0	0	1	-1.2844
0.5	0.3731	0.5457	-0.6276
1	0.5819	0.3132	-0.3380
1.5	0.7033	0.1838	-0.1947
2	0.7745	0.1076	-0.1175

2.5	0.8157	0.0608	-0.0734
3	0.8382	0.0313	-0.0468
3.5	0.8488	0.0123	-0.0303
4	0.8517	0.0000	-0.0198

h=0.01



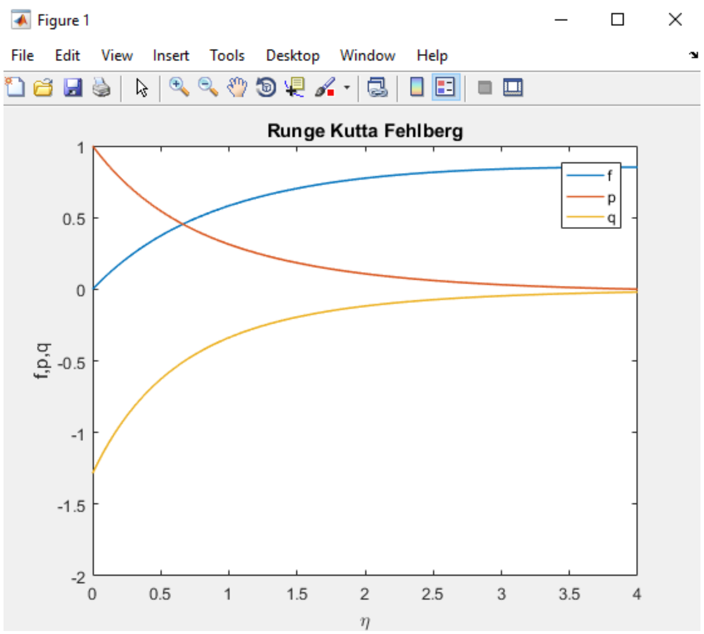
Graph f,p,q versus η with step size 0.01

Table 5. Value of f, p, q at certain points of η with step size 0.01 correct to 4 decimal places.

η	**f**	**p**	**q**
0	0	1	-1.2844
0.5	0.3731	0.5457	-0.6276
1	0.5819	0.3132	-0.3380
1.5	0.7033	0.1838	-0.1947
2	0.7745	0.1076	-0.1175
2.5	0.8157	0.0608	-0.0734
3	0.8382	0.0313	-0.0468

3.5	0.8488	0.0123	-0.0303
4	0.8517	0.0000	-0.0198

h=0.001



Graph f,p,q versus η with step size 0.001

Table 6. Value of f, p, q at certain points of η with step size 0.001 correct to 4 decimal places.

η	**f**	**p**	**q**
0	0	1	-1.2844
0.5	0.3731	0.5457	-0.6276
1	0.5819	0.3132	-0.3380
1.5	0.7033	0.1838	-0.1947
2	0.7745	0.1076	-0.1175
2.5	0.8157	0.0608	-0.0734
3	0.8382	0.0313	-0.0468
3.5	0.8488	0.0123	-0.0303
4	0.8517	0.0000	-0.0198

When the step size is 0.1 it requires 40 iterations, while the step size for 0.01 it requires 400 iterations, lastly for the step size 0.001 the iterations is 4000. Therefore at certain points of η need to show the value of f, p, and q for test equation with different step size. As we can see that the value at certain points in the table are the same for each different value of step size. By making comparison value that has obtained from the table above, we can see that the RK4 and RKF is slightly different for valuesof f,p and q at certain points of η. If we run the coding in Matlab we are able to see that after at point $\eta=3.5$ the value of p is approaching to 0.

CONCLUSION

In a conclusion, the derivation from momentum equation to non linear ODE is transformed by using similarity transformation. Then the non-linear ODE is solved by using numerical method which are are Runge Kutta (RK4) and Runge Kutta Fehlberg (RKF). In this assignment, the graph is shown for RK4 and RKF by displaying different values of step size which are 0.1, 0.01, 0.001. Lastly, the value of f, p, q represent $f(\eta)$, $f'(\eta)$, $f''(\eta)$ respectively. Therefore, the initial guess is satisfied since the $\lim_{n\to\infty} f'(\eta) = 0$

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