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Heat and Mass Transfer in Magnetohydrodynamics (MHD) Flow over an Exponentially Stretching Sheet in a Thermally Stratified Medium

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Abstract. This analysis is on a steady two-dimensional laminar MHD boundary layer flow of an incompressible viscous fluid over an exponential stretching sheet with the presence of thermal stratification and suction. The governing partial differential equations of continuity, momentum, energy, and concentration are transformed into nonlinear ordinary differential equations by using suitable similarity variables. The nonlinear differential equations are then solved numerically by using Runge-Kutta-Fehlberg method along with shooting technique. The effects of various material parameters on the velocity, temperature and concentration profiles are presented graphically and discussed. The results show that the velocity, temperature, and concentration profiles decrease with increasing suction parameter. Higher magnetic parameter reduces the velocity but increases both temperature and concentration. An increase in Prandtl number and thermal stratification parameter reduce the fluid temperature.The concentration boundary layer thickness decreases with increasing Schmidt number.

Keywords: Boundary layer flow, Exponentially stretching sheet, Magnetohydrodynamics, Suction, Thermally stratified medium.

INTRODUCTION

Studies of heat and mass transfer over a stretching sheet are important for industrial manufacturing processes such as cooling of metallic sheets in a cooling bath and paper production [1]. The quality of the final products produced are affected by the simultaneous heating or cooling and the kinematics of stretching. [2]. The study of boundary layer flow over a stretching surface moving with a constant velocity was initiated by [3]. The study was furthered by [4] using a linearly stretching surface. Researchers such as [5] and [6] studied a linearly stretching surface by considering flow with different physical situations. The velocity of the stretching sheet is not necessarily a linear [5]. Flow with general quadratic stretching sheet was studied by [7]. Researchers such as [8], [9], and [10] studied boundary layer flow problems due to an exponentially stretching sheet.

Suction and temperature parameter control the quantity and direction of the heat flow [11]. Suction increases the skin friction coefficient which opposes the velocity while injection does the opposite [12]. Thermally oil recovery is an example of suction or injection processes [13].

A thermally stratified medium is layering of bodies based on temperature difference. A thermally stratified medium is important in industrial applications for heat suction and energy storage process. A number of researchers like [14], [15], and [16] considered boundary layer flow that involved thermal stratification.

In manufacturing of metallic material, unwanted elements are removed from the metal thoroughly in the molten state. The rate of cooling determines the quality of the final products which can be achieved by passing the strips

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through an electrically conducting fluid with applied magnetic field [17]. MHD is applied in a process of cooling nuclear reactors by using liquid sodium [18]. Applied magnetic field is important in controlling the heat and mass transfer in fluid flow [19]. MHD boundary layer flow over a stretching sheet in an electrically conducting fluid was studied by [20] where a uniform transverse magnetic field was applied to the fluid flow. Researchers such as [21], [22], [23], and [24] studied various physical situations that concerned MHD flow over a stretching surface. Motivated by the importance of heat and mass transfer in above discussions, this study will investigate heat and mass transfer effect on the boundary layer flow over a stretching sheet with the presence thermal stratification and suction.

MATHEMATICAL MODEL

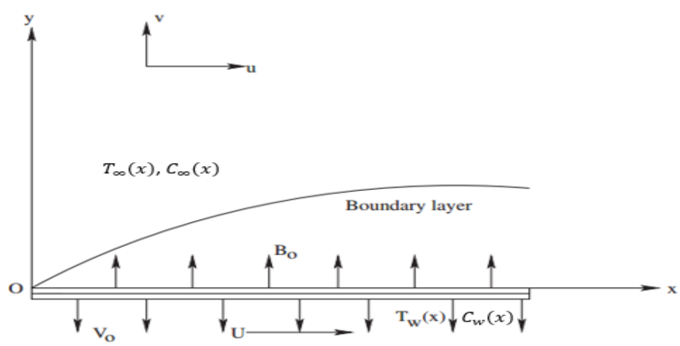

Figure 1: Sketch of the physical problem

Figure 1 shows the flow of an incompressible viscous electrically conducting fluid past a flat heated sheet on the plane $y = 0$. The flow is set to $y > 0$ and the sheet is stretched exponentially with two equal forces opposite to each other along the x-axis to keep the origin fixed. A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied transversely to the sheet where B_0 is a constant. The sheet is of concentration $C_w(x)$ and temperature $T_w(x)$ which is placed in a thermally stratified medium with variable ambient temperature $T_\infty(x)$. Temperature of the wall is considered to be greater than temperature of the ambient surrounding. It is also assumed that $T_w(x) = T_0 + be^{\frac{x}{2L}}$, $T_\infty(x) = T_0 + ce^{\frac{x}{2L}}$, $C_w(x) = C_\infty + C_0 e^{\frac{x}{2L}}$ where T_0 is the reference temperature and C_0 is the reference concentration, C_∞ is the ambient concentration, $b > 0$ and $c \geq 0$ are constants. The governing equations consist of continuity, momentum, energy and concentration, following [16] and [10] are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial \mathrm{y}} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial \mathrm{y}} = \nu\frac{\partial^2 u}{\partial \mathrm{y}^2} - \frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial \mathrm{y}} = \frac{\kappa}{\rho \mathrm{c_p}}\frac{\partial^2 T}{\partial \mathrm{y}^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial \mathrm{y}} = \mathrm{D}\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v are components of velocity in x and y directions respectively, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the fluid density, μ is the dynamic viscosity, c_p is the specific heat at constant pressure and κ is the thermal conductivity of the fluid, σ is the electrical conductivity, T is the temperature of the fluid, D is the coefficient of the mass diffusivity, and C is the concentration of the fluid.

Boundary Conditions

The governing equations consist equations (1), (2), (3) and (4) are subjected to boundary conditions as follows:

$$u = U, v = -V(x), T = T_w(x), C = C_w(x) \text{ at } y = 0 \tag{5}$$

$$u \to 0, T \to T_\infty(x), C \to C_\infty(x) \text{ as } y \to \infty \tag{6}$$

where the stretching velocity $U = U_0 e^{\frac{x}{L}}$, U_0 is the reference velocity, $V(x) > 0$ and $V(x) < 0$ is the velocity of suction and blowing respectively. In addition, a special velocity of $V(x) = V_0 e^{\frac{x}{2L}}$ at the wall is considered, V_0 is a constant of the initial strength of suction.

Method Of Solution

The suitable similarity transformations are as follows:

$$\eta(x, y) = \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y \tag{7}$$

$$\psi(x, y) = \sqrt{2\nu U_0 L} e^{\frac{x}{2L}} f(\eta) \tag{8}$$

$$\frac{T - T_\infty(x)}{T_w(x) - T_0} = \theta(\eta) \tag{9}$$

$$\frac{C - C_\infty(x)}{C_w(x) - C_\infty(x)} = \phi(\eta) \tag{10}$$

where η is the similarity variable, $\psi(x, y)$ is the stream function where $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, and L is the reference length. $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ are the dimensionless stream function, temperature, and concentration respectively. u and v are as follows:

$$u = \frac{\partial\psi}{\partial y} = U_0 e^{\frac{x}{L}} f'(\eta) \tag{11}$$

$$v = -\frac{\partial\psi}{\partial x} = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \left[f(\eta) + \eta f'(\eta)\right] \tag{12}$$

The partial differential equation (1) is automatically satisfied while equations (2) to (4) are transformed into the nonlinear ordinary differential equations (13) to (15):

$$f''' + ff'' - 2f'^2 - Mf' = 0 \tag{13}$$

$$\theta'' + \Pr\left[f\theta' - f'\theta - (St)f'\right] = 0 \tag{14}$$

$$\phi''+Sc\left[f\phi'-f'\phi\right]=0 \tag{15}$$

where $M=\frac{2L\sigma {B_0}^2}{\rho U_0}$ is the magnetic parameter, Pr= $\frac{\mu c_p}{\kappa}$ is the Prandtl number, $Sc=\frac{\nu}{D}$ is the Schmidt number and $St=\frac{c}{b}$ is the thermal stratification parameter. The new transformed boundary conditions are:

$$f=S, f'=1, \theta=1-St, \phi=1 \text{ at } \eta=0 \tag{16}$$

$$f'\to 0, \theta\to 0, \phi\to 0 \text{ as } \eta\to\infty \tag{17}$$

where $S=\frac{V_0}{\sqrt{\frac{\nu U_0}{2L}}}$ is the suction or blowing parameter. $S>0$ is the suction parameter while $S<0$ is the blowing parameter.

Equations (13) to (15) together with the corresponding boundary conditions (16) and (17) are solved numerically by Runge-Kutta Fehlberg method along with shooting technique. The local skin friction coefficient C_f, local Nusselt number Nu_x and Sherwood number Sh_x are defined as:

$$C_f=\frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U^2}{2}} \tag{18}$$

$$Nu_x=\frac{-x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w(x)-T_\infty(x)} \tag{19}$$

$$Sh_x=\frac{-x\left(\frac{\partial C}{\partial y}\right)_{y=0}}{C_w(x)-C_\infty(x)} \tag{20}$$

Using $(7)-(12)$ to obtain:

$$f''(0)=\frac{C_f\sqrt{\mathrm{Re}_x}}{\sqrt{\frac{2x}{L}}} \tag{21}$$

$$-\theta'(0)=\frac{(1-St)Nu_x}{\sqrt{\mathrm{Re}_x}\sqrt{\frac{x}{2L}}} \tag{22}$$

$$-\phi'(0)=\frac{Sh_x}{\sqrt{\mathrm{Re}_x}\sqrt{\frac{x}{2L}}} \tag{23}$$

where the local Reynolds number, $Re_x=\frac{xU}{\nu}$.

RESULTS AND DISCUSSION

The values of local Nusselt number $-\theta'(0)$ for several values of Pr and M parameters are presented in Table 1. The results obtained are compared with [8], [9], and [25]. The results show an excellent agreement with the existing studies.

TABLE 1: Comparison of values of local Nusselt number $[-\boldsymbol{\theta}'(\mathbf{0})]$ for $\boldsymbol{S=0}, \boldsymbol{St=0}, \boldsymbol{Sc=0}$ to the previously published data

Pr	M	[8]	[9]	[25]	Present study
1	0	0.9548	0.9548	0.9547	0.9548
2	0	1.4714	1.4715	1.4714	1.4715
3	0	1.8691	1.8691	1.8691	1.8691
1	1	-	0.8611	-	0.8612

The effects of Prandtl number, Schmidt number, suction parameter, thermal stratification parameter and magnetic parameter on the velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ profiles are shown graphically. Figure 2 shows an increase in magnetic field reduces the momentum boundary layer thickness thus reducing the rate of transport due to the Lorentz force generated. The higher magnetic parameter produce bigger Lorentz force which adds more resistance to the fluid flow. Figures 3 and 4 show that both temperature and concentration increase with an increase in magnetic parameter. The fluid gets warmer with the presence of magnetic field. Through the velocity distribution, it indirectly affects the temperature as well as the concentration of the fluid.

Figure 5 shows an increase in Prandtl number decreases the temperature profiles. The Prandtl number is equivalent to the ratio of momentum diffusivity to thermal diffusivity. The fluid with greater Prandtl number has lower thermal conductivity and thinner boundary layer structure resulting in the heat being diffused away from the surface more slowly.

Figure 6 depicts that the temperature profiles decreases due to the increasing thermal stratification parameter. An increase in thermal stratification parameter resulted with a decrease in the surface temperature and an increase in the free-stream temperature. The temperature difference between the surface and the fluid is thus reduced.

Figure 7 shows as Schmidt number increases, the concentration profiles decreases as well as the concentration boundary layer thickness. Schmidt number is equivalent to the ratio of momentum diffusivity to mass diffusivity. Increase in Schmidt number corresponds to the decrease in mass diffusion. The mass flux from the sheet decreases.

Figure 8 displays that the velocity profiles decreases when suction parameter increases. The imposition of the wall fluid suction increases the drag force, the fluid experiences resistance to its flow and the velocity decreases. Figures 9 and 10 illustrate that the temperature and concentration profiles decrease as suction parameter increases.

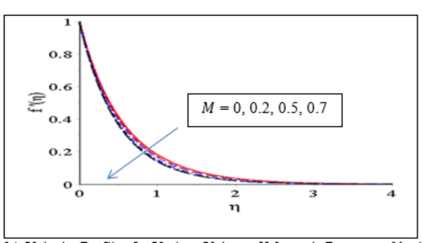

FIGURE 2: Velocity profiles for M with $S=1, St=0.2$, $Pr=0.7, Sc=0.6$

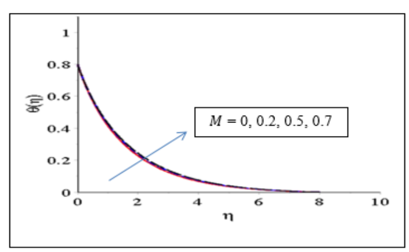

FIGURE 3: Temperature profiles for M with $S=1$, $St=0.2, Pr=0.7,\ Sc=0.6$

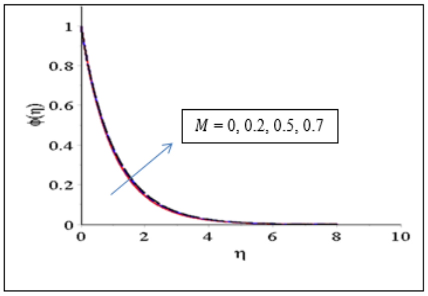

FIGURE 4: Concentration profiles for M with $S = 1, St = 0.2, Pr = 0.7, Sc = 0.6$

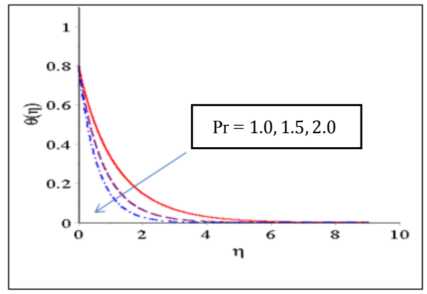

FIGURE 5: Temperature profiles for Pr with $S = 1$, $M = 0.5, St = 0.2, \quad Sc = 0.6$

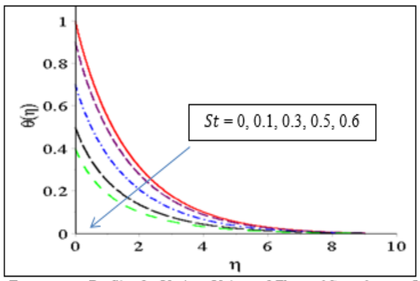

FIGURE 6: Temperature profiles for St with $S = 1$, $M = 0.5, \; Pr = Sc = 0.6$

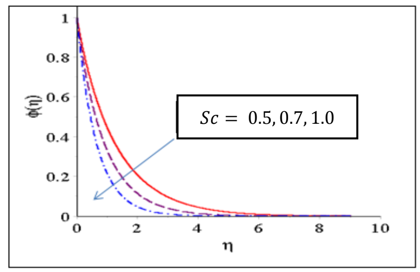

FIGURE 7: Concentration profiles for Sc with $S = 1$, $M = 0.5, St = 0.2, Pr = 0.7$

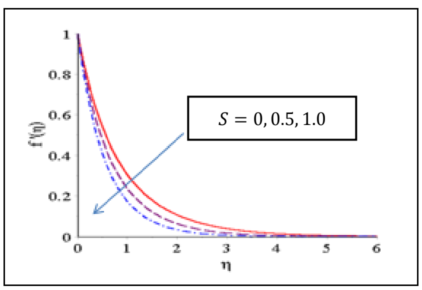

FIGURE 8: Velocity profiles for S with $M = 0.1$, $St = 0.2, Pr = 0.7, Sc = 0.6$

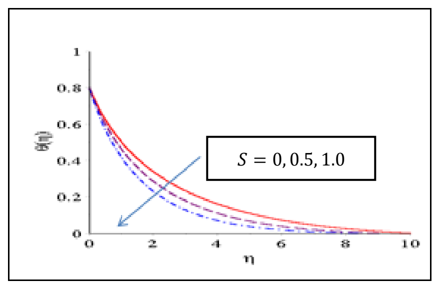

FIGURE 9: Temperature profiles for S with $M = 0.1$, $St = 0.2, Pr = 0.7, Sc = 0.6$

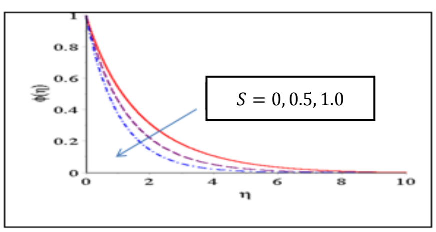

FIGURE 10: Concentration profiles for S with $M = 0.1, St = 0.2, Pr = 0.7, Sc = 0.6$

CONCLUSIONS

This study provides the numerical solutions of heat and mass transfer in MHD flow over an exponentially stretching sheet with the presence of thermal stratification and suction. The results indicated that the presence of magnetic parameter reduces the fluid velocity. Higher Prandtl number reduces the temperature of the fluid. The velocity, temperature, and concentration of the fluid decrease with an increase in suction parameter. An increase in thermal stratification parameter and Schmidt number decrease the temperature and concentration respectively.

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