UNIVERSITI TEKNOLOGI MARA

ASSIGNMENT 2

Heat and Mass Transfer in Magnetohydrodynamics (MHD) Flow over an Exponentially Stretching Sheet in a Thermally Stratified Medium

LECTURER'S NAME : ASSOCIATE PROFESSOR DR. SERIPAH AWANG KECHIL

NAME : NUR IMAN NAZIRAH BINTI NASIR (2018469592)

CONTENTS

INTRODUCTION

The Blasius's Solution for the laminar boundary layer on a flat plate can be solved by using analytical solution and Numerical Method (Weltry, J. R., Wicks, C. E., Wilson, R. E., & Rorrer, G. l., 2009). We have learrnt on how to obtain general solution of differential equation with first order differential equation by recognizing them as separable, linear, exact, homogenous or perhaps Bernoulli equations. If the solution of ordinary differential equation of order two or higher we need to examine some of theory in linear equations (Zill & Wright, 2012). However there is a limitation when solving it analytically, hence we need to solve the higher order of differential equation by using numerical method. Applied numerical can get better approximation value by using iterative technique.

The purpose of this assignment is to apply the knowledge that we have learnt from the Blasius equation. Basically, the momentum,energy and concentration equation can be transform to ODE by using similarity transformation, then we use the numerical method to solve the non linear ODE. In this assignment we were given task to find only one journal that has boundary layer equation for momentum. The article that has been chosen for this assignment is "Heat and mass transfer magnetohydrodynamics (MHD) flow over an exponentially stretching sheet in a thermally stratified" (Faudzi,et.al,,2018) . Nanofluids have many applications in industry such as coolants, lubricants, heat exchanges, micro-channel heat sinks. It is known that nanofluids can tremendously enhance the heat transfer characteristics of the original (base) fluid. Heat and mass transfer, fluid dynamic and also boundary layer theory are an application that important to science and engineering area (Welty,et al.,2009). Heat and mass transfer are kinetic processes that possibly will take place and be researched individually or together. Boundary layer in fluid mechanics, which the thin layer of a flowing liquid or gas in touching base with a surface for instance that of a flying machine wing or of the inner of a pipe. Shearing stress is exposed to the fluid in the boundary layer.

Similarity equation need to perform a similarity transformation for governing the Partial Differential Equation. Partial differential equation can be used as a topic in solving the science engineering applications. The continuity, the momentum, and the energy equations is one of the example of PDE system. Usually the system of partial differential equations (PDE) is commonly problematic to solve. These equations can be relegated to two or more non-linear ODEs by using similarity transformation. The momentum equation and continuity equation are relegated to a single non-linear ODE and energy equation is relegated to another ODE. The incompressible boundary layer analysis, the influence of compressibility on the whole velocity and temperature field should be take into account. In consequence, the system of equations in compressible boundary layer is a more complex PDE system, combined of the continuity equation, the momentum equation, the energy equation and an equation of state.

The solutions of this ordinary differential equation set are usually nondimensionalized velocities and temperature. Usually after we transforming the similarity transformation, we obtained the non-linear differential equations. We have learnt that the ordinary differential equation can be solve analytically and numerically. However there is a limitation when solving it analytically, hence we need to solve the higher order of non linear differential equation by using numerical method. Applied numerical can get better approximation by using iterative technique. It is impossible to solve the complex form differential equation by using analytical method since the analytical solution can only obtained in a general solution, this is why we need to solve it numerically. Numerical method is a very powerful method since it can solve the complex term by showing the numerical value at a certain points. Therefore, the simplified ODE set creates it potential to get the solution from the present solutions of the incompressible analysis and correspondingly trim down the computing time in the numerical analysis.

In this project, I have chosen the journal title Heat and mass transfer magnetohydrodynamics (MHD) flow over an exponentially stretching sheet in a thermally stratified" written by (Faudzi,et.al,,2018). The purpose of this project is to have a better understanding for this course.

OBJECTIVE

The purpose of this report is :

i. To investigate the researcher paper on how they solve the problem and observing the physical model that they have shown.
ii. To have a better understanding on what is the application of this course.
iii. To apply the similarity of transformation for governing Partial Differential Equation

iv. To solve the Partial Differential Equation by using numerical method which reducing the higher order of differential equation.

v. To solve the ODE equation by using numerical method which is Runge Kutta Method with Matlab software.

METHODOLOGY

We consider a steady, incompressible, two dimensional boundary layer flow of viscous nanofluid past a flat sheet coinciding with the plane y=0 and being confined y>0. The basic steady of conversation of mass and momentum from article are given as below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \text{ (Continuity equation)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-v\frac{\partial^2 u}{\partial y^2}+\sigma\frac{B^2 u}{\rho}=0 \text{ (Momentum equation)}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}-\frac{\kappa}{\rho C_P}\frac{\partial^2 T}{\partial y^2}=0 \text{ (Energy equation)}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}-D\frac{\partial^2 C}{\partial y^2}=0 \text{ (Concentration equation)}$$

The boundary conditions are

$$u=U, v=-V(x), T=T_w(x), C=C_w(x) \text{ at } y=0$$
$$u\to 0, T\to T_\infty(x), C\to C_\infty(x) \text{ as } y\to\infty$$

Since we are only interested on similarity solution of the above boundary layer flow hence we need to introduce the following similarity transformation (dimensionless quantities).

$$\eta = y\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$\psi=\sqrt{2\upsilon L U_0}e^{\frac{x}{2L}}f(\eta)\ ,\ \theta(\eta)=\frac{T-T_\infty(x)}{T_w(x)-T_0}\ ,\ \phi(\eta)=\frac{C-C_\infty(x)}{C_w(x)-C_\infty(x)}$$

Below are the derivation from momentum equation to the ODE form equation.

$$\frac{\partial\eta}{\partial y} = \frac{\partial}{\partial y}[y\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}] = \sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$\frac{\partial\eta}{\partial x} = \frac{\partial}{\partial x}[y\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}] = \frac{y}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$\begin{aligned} u &= \frac{\partial\psi}{\partial y} \\ &= \sqrt{2\upsilon L U_0}e^{\frac{x}{2L}}\frac{\partial}{\partial\eta}[f(\eta)]\frac{\partial\eta}{\partial y} \\ &= \sqrt{2\upsilon L U_0}e^{\frac{x}{2L}}[f'(\eta)]\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}} \\ &= U_0 e^{\frac{x}{L}}f'(\eta) \end{aligned}$$

$$\begin{aligned} v &= -\frac{\partial\psi}{\partial x} \\ &= -\sqrt{2\upsilon L U_0}\{e^{\frac{x}{2L}}\frac{\partial}{\partial\eta}[f(\eta)]\frac{\partial\eta}{\partial x} + f(\eta)\frac{\partial}{\partial x}\left[e^{\frac{x}{2L}}\right]\} \\ &= -\sqrt{2\upsilon L U_0}\{e^{\frac{x}{2L}}f'(\eta)\frac{y}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}} + f(\eta)\frac{1}{2L}e^{\frac{x}{2L}}\} \\ &= -\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}[\eta f'(\eta) + f(\eta)] \end{aligned}$$

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial}{\partial x}[U_0 e^{\frac{x}{L}}f'(\eta)] \\ &= U_0\{e^{\frac{x}{L}}\frac{\partial}{\partial\eta}[f'(\eta)]\frac{\partial\eta}{\partial x} + f'(\eta)\frac{\partial}{\partial x}\left[e^{\frac{x}{L}}\right]\} \\ &= U_0\{(e^{\frac{x}{L}}[f''(\eta)]\frac{y}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}) + f'(\eta)\left[\frac{1}{L}e^{\frac{x}{L}}\right]\} \\ &= \frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta) + f'(\eta)\right] \end{aligned}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}[-\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}[f(\eta) + \eta\, f'(\eta)]]$$

$$= -\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}\{\frac{\partial}{\partial y}[f(\eta) + \eta\, f'(\eta)]\}$$

$$= -\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}\{f'(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}} + \eta f''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}} + f'(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}\}$$

$$= -\frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta) + f'(\eta)\right]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta) + f'(\eta)\right] - \frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta) + f'(\eta)\right]$$

$$= 0$$

It is proven that the continuity equation is satisfied. Then, use the governing equation of momentum takes the form of non-linear ordinary differential equations. Below are the derivation for non-linear ordinary equations.

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}[U_0 e^{\frac{x}{L}} f'(\eta)]$$

$$= U_0 e^{\frac{x}{L}}\frac{\partial}{\partial y}[f'(\eta)]$$

$$= U_0 e^{\frac{x}{L}} f''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$= U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f''(\eta)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}[U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f''(\eta)]$$

$$= U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}}\frac{\partial}{\partial y}[f''(\eta)]$$

$$= U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}} f'''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$= \frac{{U_0}^2}{2\upsilon L}e^{\frac{2x}{L}} f''(\eta)$$

From Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-v\frac{\partial^2 u}{\partial y^2}+\sigma\frac{B^2 u}{\rho}=0$$

$$U_0 e^{\frac{x}{L}}f'(\eta)\left\{\frac{U_0 e^{\frac{x}{L}}}{L}\left[\frac{1}{2}\eta f''(\eta)+f'(\eta)\right]\right\}+\left\{-\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}\left[\eta\, f'(\eta)+f(\eta)\right]\right\}U_0\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{3x}{2L}}f''(\eta)$$

$$-\upsilon\frac{{U_0}^2}{2\upsilon L}e^{\frac{2x}{L}}f'''(\eta)+\sigma\frac{{B_0}^2 e^{\frac{x}{L}}}{\rho}U_0 e^{\frac{x}{L}}f'(\eta)=0$$

$$\frac{{U_0}^2}{2L}e^{\frac{2x}{L}}[\eta f'(\eta)f''(\eta)+2[f'(\eta)]^2-\eta f'(\eta)f''(\eta)-f(\eta)f''(\eta)-\frac{\upsilon}{\upsilon}f'''(\eta)+\frac{2LB_0{}^2\sigma}{\rho U_0}=0$$

$$f'''(\eta)+f(\eta)f''(\eta)-2[f'(\eta)]^2-Mf'(\eta)=0$$

Then, use the governing equation of heat takes the form of non-linear ordinary differential equations. Below are the derivation for non-linear ordinary equations.

$$\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}[(T_w(x)-T_0)\theta(\eta)+T_\infty(x)]$$

$$=\frac{\partial}{\partial x}[\left(T_0+be^{\frac{x}{2L}}-T_0\right)\theta(\eta)+T_0+ce^{\frac{x}{2L}}]$$

$$=\theta(\eta)\left[\frac{1}{2L}be^{\frac{x}{2L}}\right]+be^{\frac{x}{2L}}\theta'(\eta)\frac{y}{2L}\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}+\frac{c}{2L}e^{\frac{x}{2L}}$$

$$=\frac{1}{2L}e^{\frac{x}{2L}}[b\theta(\eta)+b\eta\theta'(\eta)+c]$$

$$=\frac{b}{2L}e^{\frac{x}{2L}}[\theta(\eta)+\eta\theta'(\eta)+\frac{c}{b}]$$

$$\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}[(T_w(x)-T_0)\theta(\eta)+T_\infty(x)]$$

$$=\frac{\partial}{\partial y}[\left(T_0+be^{\frac{x}{2L}}-T_0\right)\theta(\eta)+T_0+ce^{\frac{x}{2L}}]$$

$$=be^{\frac{x}{2L}}\theta'(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}$$

$$=be^{\frac{x}{L}}\theta'(\eta)\sqrt{\frac{U_0}{2\upsilon L}}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}[be^{\frac{x}{L}}\theta'(\eta)\sqrt{\frac{U_0}{2\upsilon L}}]$$
$$= be^{\frac{x}{2L}}\sqrt{\frac{U_0}{2\upsilon L}}[\theta''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}]$$
$$= be^{\frac{x}{2L}}\sqrt{\frac{U_0}{2\upsilon L}}[\theta''(\eta)\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}]$$
$$= be^{\frac{3x}{2L}}\theta''(\eta)\frac{U_0}{2\upsilon L}$$

From Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} = 0$$
$$U_0 e^{\frac{x}{L}}f'(\eta)\left[\frac{b}{2L}e^{\frac{x}{2L}}\left(\theta(\eta) + \eta\theta'(\eta) + \frac{c}{b}\right)\right] - \sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}[\eta f'(\eta) + f(\eta)](be^{\frac{x}{L}}\theta'(\eta)\sqrt{\frac{U_0}{2\upsilon L}})$$
$$- \frac{k}{\rho C_P}be^{\frac{3x}{2L}}\theta''(\eta)\frac{U_0}{2\upsilon L} = 0$$
$$be^{\frac{3x}{2L}}\frac{U_0}{2L}\{f'(\eta)\left(\theta(\eta) + \eta\theta'(\eta) + \frac{c}{b}\right) - \theta'(\eta)[\eta f'(\eta) + f(\eta)] - \frac{k}{\upsilon\rho C_P}\theta''(\eta) = 0$$
$$\theta''(\eta) + \frac{\upsilon\rho C_P}{k}\{f(\eta)\theta'(\eta) - f'(\eta)\theta(\eta) - \frac{c}{b}f'(\eta)\} = 0$$
$$\theta''(\eta) + \frac{\mu C_P}{k}\{f(\eta)\theta'(\eta) - f'(\eta)\theta(\eta) - \frac{c}{b}f'(\eta)\} = 0$$
$$\theta''(\eta) + \mathrm{Pr}\{f(\eta)\theta'(\eta) - f'(\eta)\theta(\eta) - Stf'(\eta)\} = 0$$

Below are the derivation that will be using for deriving the Concentration equation to non linear ODE.

$$\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}[C_\infty + C_0 e^{\frac{x}{2L}} \phi(\eta)]$$
$$= \phi(\eta) \frac{\partial}{\partial x}\left[C_0 e^{\frac{x}{2L}}\right] + C_0 e^{\frac{x}{2L}} \frac{\partial}{\partial x} \phi(\eta)$$
$$= \phi(\eta) \frac{1}{2L}\left[C_0 e^{\frac{x}{2L}}\right] + C_0 e^{\frac{x}{2L}} \phi'(\eta) \frac{y}{2L} \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}}$$
$$= \frac{1}{2L} C_0 e^{\frac{x}{2L}} [\phi(\eta) + \eta\phi'(\eta)]$$

$$\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}[C_\infty + C_0 e^{\frac{x}{2L}} \phi(\eta)]$$
$$= \frac{\partial}{\partial y}\left[C_0 e^{\frac{x}{2L}} \phi(\eta)\right]$$
$$= C_0 e^{\frac{x}{2L}} \frac{\partial}{\partial y}[\phi(\eta)]$$
$$= C_0 e^{\frac{x}{2L}} \phi'(\eta) \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}}$$
$$= C_0 e^{\frac{x}{L}} \phi'(\eta) \sqrt{\frac{U_0}{2\upsilon L}}$$

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y}[C_0 e^{\frac{x}{L}} \phi'(\eta) \sqrt{\frac{U_0}{2\upsilon L}}]$$
$$= C_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\upsilon L}} \frac{\partial}{\partial y}[\phi'(\eta)]$$
$$= C_0 e^{\frac{x}{2L}} \sqrt{\frac{U_0}{2\upsilon L}} [\phi''(\eta) \sqrt{\frac{U_0}{2\upsilon L}} e^{\frac{x}{2L}}]$$
$$= C_0 e^{\frac{3x}{2L}} \phi''(\eta) \frac{U_0}{2\upsilon L}$$

From concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}-D\frac{\partial^2 C}{\partial y^2}=0$$

$$U_0e^{\frac{x}{L}}f'(\eta)\left\{\frac{C_0e^{\frac{x}{L}}}{2L}[\phi(\eta)+\eta\phi'(\eta)]\right\}+\left\{-\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}[\eta\, f'(\eta)+f(\eta)]\right\}C_0e^{\frac{x}{L}}\phi'(\eta)\sqrt{\frac{U_0}{2\upsilon L}}$$

$$-DC_0e^{\frac{3x}{2L}}\phi''(\eta)\frac{U_0}{2\upsilon L}=0$$

$$\frac{C_0U_0}{2L}e^{\frac{3x}{L}}[f'(\eta)\big(\phi(\eta)+\eta\phi'(\eta)\big)-\phi'(\eta)\,[\eta\, f'(\eta)+f(\eta)]-\frac{D}{\upsilon}\phi''(\eta)=0$$

$$\phi''(\eta)+\frac{v}{D}\{f'(\eta)\phi(\eta)+f(\eta)\phi'(\eta)\}=0$$

$$\phi''(\eta)+Sc\{f'(\eta)\phi(\eta)+f(\eta)\phi'(\eta)\}=0$$

BCS

$$u=U$$
$$U_0e^{\frac{x}{L}}f'(\eta)=U_0e^{\frac{x}{L}}$$
$$f'(\eta)=1$$
$$f'(0)=1$$

$$C=C_w(x)$$
$$C_\infty+C_0e^{\frac{x}{2L}}\phi(\eta)=C_\infty+C_0e^{\frac{x}{2L}}$$
$$\phi(\eta)=1$$
$$\phi(0)=1$$

$$v=-V(x)$$
$$-\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x}{2L}}[\eta\, f'(\eta)+f(\eta)]=-V_0e^{\frac{x}{2L}}$$
$$f(\eta)=\frac{V_0}{\sqrt{\frac{\upsilon U_0}{2L}}}$$
$$f(0)=S$$

$$
\begin{aligned}
T &= -T_w(x) \\
[(T_0 + be^{\frac{x}{2L}} - T_0)\theta(\eta) + T_0 + ce^{\frac{x}{2L}} &= T_0 + be^{\frac{x}{2L}} \\
be^{\frac{x}{2L}}\theta(\eta) &= be^{\frac{x}{2L}} - \ ce^{\frac{x}{2L}} \\
\theta(\eta) &= 1 - \frac{c}{b} \\
\theta(0) &= 1 - St
\end{aligned}
$$

$$
\begin{aligned}
u &\to 0 \\
U_0 e^{\frac{x}{L}} f'(\eta) &\to 0 \\
f'(\eta) &\to 0 \\
f'(\infty) &\to 0
\end{aligned}
$$

$$
\begin{aligned}
T &\to T_\infty(x) \\
(T_w(x) - T_0)\theta(\eta) + T_\infty(x) &\to T_\infty(x) \\
(T_w(x) - T_0)\theta(\eta) &\to 0 \\
\theta(\eta) &\to 0 \\
\theta(\infty) &\to 0
\end{aligned}
$$

$$
\begin{aligned}
C &\to C_\infty(x) \\
C_\infty + C_0 e^{\frac{x}{2L}}\phi(\eta) &\to C_\infty + C_0 e^{\frac{x}{2L}}\phi(\eta) - C_0 e^{\frac{x}{2L}}\phi(\eta) \\
C_0 e^{\frac{x}{2L}}\phi(\eta) &\to 0 \\
\phi(\infty) &\to 0
\end{aligned}
$$

Since we have done the governing of PDE, then we solve the non-linear differential equation by using numerical method. So we need to perform Runge Kutta Method. Therefore the following system is constructed in the matrix form so that it is easier to make the coding in a Matlab.

$$
\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \\ x_6' \\ x_7' \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ -x_1x_3 + 2x_3^2 + Mx_2 \\ x_5 \\ -Pr(x_1x_5 - x_2x_4 - Stx_2) \\ x_7 \\ -Sc(x_1x_7 - x_2x_6) \end{pmatrix}
$$

The initial conditions is shown as follows:

$$
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} S \\ 1 \\ u_1 \\ 1 - St \\ u_2 \\ 1 \\ u_3 \end{pmatrix}
$$, where u_1 ,u_2, u_3 are initial guess.

RESULT AND DISCUSSION

Matlab bvp4c is an easier method for solving the non linear differential equation in Matlab instead of writing the long code of Runge Kutta method. The coding that has been written can refer to the Apendix section. It is very important to us to study the programming language so that we can reduce the time of soving complex problem. Nowadays, all problem need to be solve by using tools such as Matlab, Maple and Phyton since all the problem are very complex.

In this section, the results are obtained as below. The output of the graph and also numerical value are shown and will be discuss briefly. The numerical results are written in the excel file from the written coding in Matlab. MATLAB can export the numerical values to Excel. The command of xlswrite in MATLAB is very useful to the user so that they can easily observe the numerical value, since MATLAB has limitation of showing large data of numerical value in command window. Hence we need to use the the graphs shown respective to different value of M which are 0.1,0.2, and 0.4.

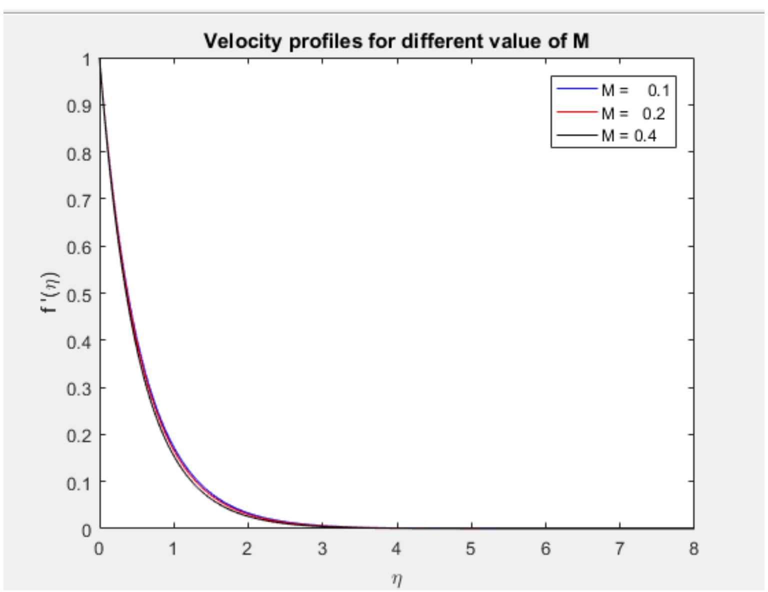

Figure 1. $f'(\eta)$ $versus$ η

In Figure 1. the graph is approaching to the value of 0 as η increases. This graph is satisfied to the initial condition $f'(\infty) = 0$. The initial guess of $f'(0) = -1.88753$.

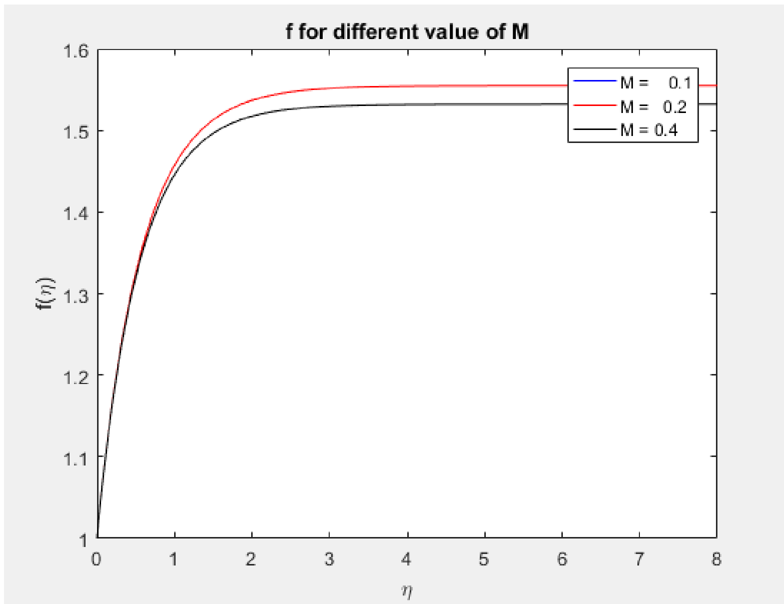

Figure 2. $\boldsymbol{f(\eta)\ versus\ \eta}$

In Figure 2. the graph for different values is shown as above. As the η is increasing the value $f(\eta)$ is the same starting at some point.

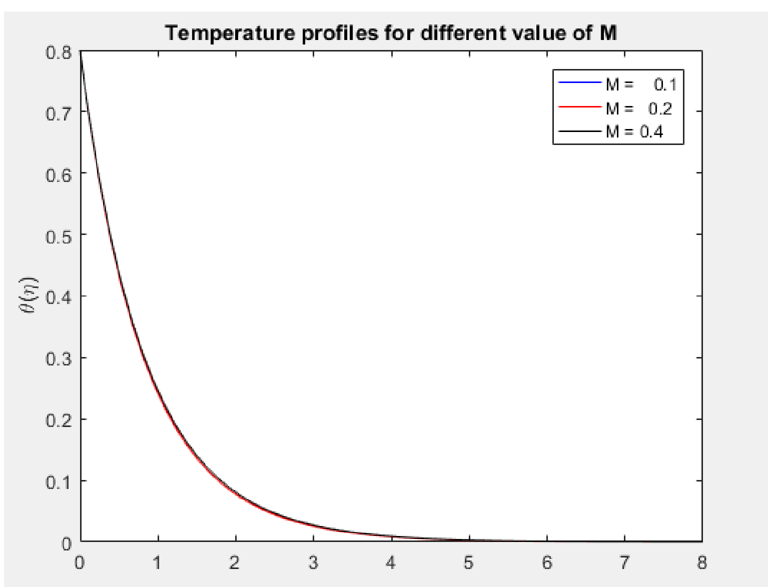

Figure 3. $\boldsymbol{\theta(\eta)\ versus\ \eta}$

In Figure 3. as η increases the value of $\theta(\eta)$ will approaching to 0. This graph is satisfied to the initial condition which is $\theta(\infty) = 0$. The initial guess of $\theta'(0) = -1.01045$.

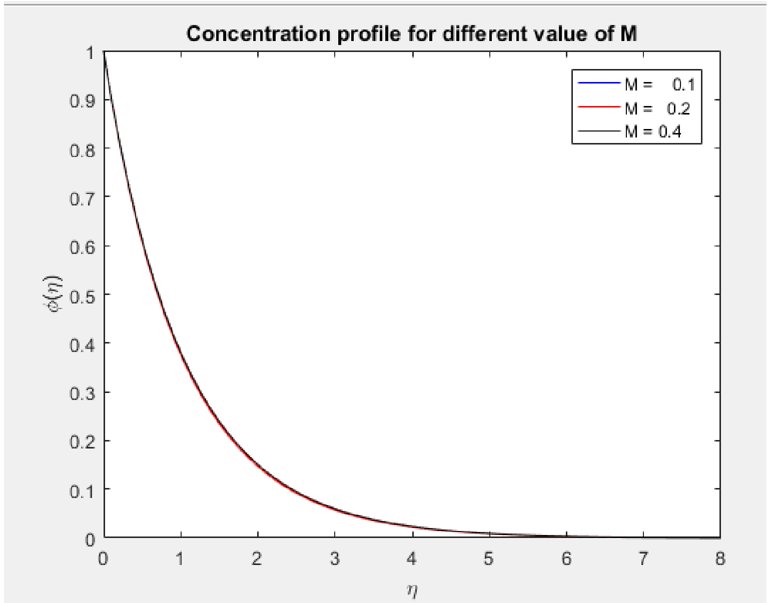

Figure 4. $\boldsymbol{\phi(\eta)\ versus\ \eta}$

In Figure 4. as η increases the value of $\theta(\eta)$ will approaching to 0. This graph is satisfied to the initial condition which is $\phi(\infty) = 0$. The initial guess of $\phi'(0) = -1.03103$.

The numerical values of f,f',f'',θ,θ', ϕ, $\phi'$is shown as below at certain points of η. Note that all the numerical results has been display in section Appendix.

Table 1. The numerical values of $\boldsymbol{f}$,$\boldsymbol{f'}$,$\boldsymbol{f''}$,$\boldsymbol{\theta}$ and $\boldsymbol{\theta'}$ when the value of M=0.1

η	f	f'	f''	θ	θ'	ϕ	ϕ'
0	1	1	-1.88753	0.8	-1.01045	1	-1.03103
0.8	1.422278	0.24208	-0.41126	0.302357	-0.35761	0.452784	-0.43692
1.2	1.492705	0.123617	-0.20584	0.189187	-0.22	0.308535	-0.29441
1.6	1.528846	0.063756	-0.10502	0.119163	-0.13693	0.210881	-0.20016
2.4	1.557242	0.017197	-0.02805	0.047878	-0.05421	0.098815	-0.0936
3.2	1.564934	0.004687	-0.00758	0.019419	-0.02185	0.046252	-0.04401
4.8	1.567639	0.000383	-0.00056	0.003186	-0.00365	0.009867	-0.00976
5.6	1.567825	0.000136	-0.00015	0.001253	-0.0015	0.004378	-0.0046

6.4	1.567902	7.01E-05	-3.8E-05	0.000458	-0.00062	0.001792	-0.00217
8	1.567992	5.14E-05	0	0	-0.0001	0	-0.00048

CONCLUSION

In a conclusion, the derivation from momentum equation, energy equation and concentration equation to non linear ODE is transformed by using similarity transformation. In this assignment, the graph is shown is the same as the journal. Therefore, the initial guess is satisfied since the $\lim_{n\to\infty} f'(\eta) = 0,\ \lim_{n\to\infty} \theta(\eta) = 0\ and\ \lim_{n\to\infty} \phi(\eta) = 0.$

REFERENCES

Welty, J. R., Wicks, C. E., Rorrer, G., & Wilson, R. E. (2009). *Fundamentals of momentum, heat, and mass transfer*. John Wiley & Sons.

Zill, D. G., & Wright, W. S. (2012). *Differential equations with boundary-value problems*. Cengage Learning.

Faudzi, M. A. M., Aziz, A. S. A., & Ali, Z. M. (2018, June). *Heat and mass transfer in magnetohydrodynamics (MHD) flow over an exponentially stretching sheet in a thermally stratified medium*. In AIP Conference Proceedings (Vol. 1974, No. 1, p. 020007). AIP Publishing.

APPENDIX

CODING

```
function MHD
format long g
global M Pr Sc St s
M=0.1;
Pr=0.7;
Sc=0.6;
St=0.2;
s=1;
 %beta
%==========INITIAL CONDITION=======
solinit = bvpinit(linspace(0,8,16),@MHD_init);
%==========SOLVE USING BVP4C=======
```

```
options = bvpset('stats','on','RelTol',1e-10);
sol = bvp4c(@MHD_ode,@MHD_bc,solinit,options);
x=linspace(0,8,16);
y=deval(sol,x);
%Result will be display at excel file, where x,f,f',f'',theta,theta'
xlswrite('ProjectFluid2019.xlsx',[(sol.x).',(sol.y(1,:)).',(sol.y(2,:)).',(so
l.y(3,:)).',(sol.y(4,:)).',(sol.y(5,:)).',(sol.y(6,:)).',(sol.y(7,:)).']);
%==========OUTPUT==================
figure(1)
lines={'blue','red','black'};
plot(sol.x,sol.y(2,:),lines{1})
xlabel('\eta')
ylabel('f ''(\eta)')
title('Velocity profiles for different value of M')
hold on
fprintf('\nFirst solution:\n');
fprintf('f"(0)=%0.8f\n',y(3));
hold on
for i=2:3
  M = M*2;
  sol = bvp4c(@MHD_ode,@MHD_bc,solinit,options);
  %fprintf('For M = %5i, A = %4.2f.\n',M,sol.parameters);
  plot(sol.x,sol.y(2,:),lines{i});
  drawnow
end
legend('M =    0.1','M =   0.2','M = 0.4');
hold off

figure(2)
lines={'blue','red','black'};
plot(sol.x,sol.y(4,:),lines{1})
xlabel('\eta')
ylabel('\theta(\eta)')
title('Temperature profiles for different value of M')
hold on
fprintf('\nFirst solution:\n');
fprintf('\theta''(0)=%0.8f\n',y(5));
hold on
M=0.1;
for i=2:3
  M = M*2;
  sol = bvp4c(@MHD_ode,@MHD_bc,solinit,options);
  %fprintf('For M = %5i, A = %4.2f.\n',M,sol.parameters);
  plot(sol.x,sol.y(4,:),lines{i});
  drawnow
end
legend('M =    0.1','M =   0.2','M = 0.4');
hold off

figure(3)
lines={'blue','red','black'};
plot(sol.x,sol.y(1,:),lines{1})
xlabel('\eta')
ylabel('f(\eta)')
legend('M=1')
title('f for different value of M')
```

```
hold on
fprintf('\nFirst solution:\n');
fprintf('\theta''(0)=%0.8f\n',y(2));
hold on
M=0.1;
for i=2:3
  M = M*2;
  sol = bvp4c(@MHD_ode,@MHD_bc,solinit,options);
  %fprintf('For M = %5i, A = %4.2f.\n',M,sol.parameters);
  plot(sol.x,sol.y(1,:),lines{i});
  drawnow
end
legend('M =    0.1','M =   0.2','M = 0.4');
hold off

figure(4)
lines={'blue','red','black'};
plot(sol.x,sol.y(6,:),lines{1})
xlabel('\eta')
ylabel('\phi(\eta)')
legend('M=1')
title('Concentration profile for different value of M')
hold on
fprintf('\nFirst solution:\n');
fprintf('\tphi(0)=%0.8f\n',y(6));
hold on
M=0.1;
for i=2:3
  M = M*2;
  sol = bvp4c(@MHD_ode,@MHD_bc,solinit,options);
  %fprintf('For M = %5i, A = %4.2f.\n',M,sol.parameters);
  plot(sol.x,sol.y(6,:),lines{i});
  drawnow
end
legend('M =    0.1','M =   0.2','M = 0.4');
hold off
%============INPUT ODEs===================
function dydx = MHD_ode(x,y,M,Pr,Sc,St)
global  M Pr Sc St
dydx = [ y(2)
         y(3)
         -y(1)*y(3)+2*y(2)^2+M*y(2)
         y(5)
         -Pr*(y(1)*y(5)-y(2)*y(4)-St*y(2))
         y(7)
         Sc*(-y(1)*y(7)+y(2)*y(6))];
end
%===========INPUT BCs=====================
function res = MHD_bc(ya,yb,St,Sc,s)
global St Sc s
res = [ya(1)-s
       ya(2)-1
       yb(3)
       ya(4)-1+St
       yb(4)
       ya(6)-1
       yb(6)
```

```
        ];end
%===========INPUT INITIAL VALUEs============
function v = MHD_init(x,St,Sc,s)
global St Sc s
v = [0
0
0
0
0
0
0];end end
```

NUMERICAL VALUE

η	f	f'	f''	θ	θ'	ϕ	ϕ'
0	1	1	-1.88753	0.8	-1.01045	1	-1.03103
0.014815	1.01461	0.972469	-1.82952	0.785184	-0.98985	0.984858	-1.01318
0.02963	1.028818	0.945782	-1.77358	0.770669	-0.96974	0.969978	-0.99572
0.044444	1.042637	0.919909	-1.71961	0.756448	-0.95011	0.955353	-0.97865
0.059259	1.056078	0.894821	-1.66755	0.742515	-0.93094	0.940979	-0.96195
0.074074	1.069154	0.870491	-1.61731	0.728863	-0.91222	0.926849	-0.94562
0.088889	1.081874	0.846892	-1.56881	0.715484	-0.89394	0.912959	-0.92963
0.103704	1.094251	0.823999	-1.52198	0.702374	-0.87608	0.899303	-0.91399
0.118519	1.106293	0.801788	-1.47677	0.689524	-0.85863	0.885876	-0.89868
0.133333	1.11801	0.780236	-1.43309	0.676931	-0.84158	0.872674	-0.88369
0.148148	1.129414	0.759319	-1.39089	0.664587	-0.82492	0.859691	-0.86901
0.162963	1.140512	0.739017	-1.35011	0.652487	-0.80864	0.846924	-0.85463
0.177778	1.151314	0.719309	-1.31069	0.640625	-0.79272	0.834368	-0.84055
0.192593	1.161828	0.700176	-1.27258	0.628997	-0.77716	0.822018	-0.82675
0.207407	1.172062	0.681597	-1.23574	0.617597	-0.76195	0.80987	-0.81324
0.222222	1.182026	0.663555	-1.2001	0.606419	-0.74708	0.79792	-0.79999
0.237037	1.191726	0.646033	-1.16563	0.595459	-0.73253	0.786165	-0.78701
0.251852	1.20117	0.629012	-1.13228	0.584713	-0.71831	0.7746	-0.77429
0.266667	1.210366	0.612478	-1.10001	0.574175	-0.7044	0.763222	-0.76181
0.296296	1.228039	0.580806	-1.03854	0.553705	-0.67748	0.741011	-0.73759
0.325926	1.244801	0.550897	-0.98092	0.534016	-0.65171	0.719504	-0.71429
0.355556	1.260701	0.522642	-0.92686	0.515075	-0.62703	0.698674	-0.69187
0.385185	1.275788	0.495939	-0.87612	0.496849	-0.6034	0.678496	-0.67028
0.414815	1.290105	0.470694	-0.82846	0.479308	-0.58075	0.658946	-0.64948
0.444444	1.303694	0.446817	-0.78367	0.462425	-0.55904	0.640001	-0.62945
0.474074	1.316596	0.424228	-0.74154	0.446171	-0.53822	0.621639	-0.61013
0.503704	1.328846	0.402849	-0.70191	0.430522	-0.51826	0.603838	-0.5915
0.533333	1.340479	0.38261	-0.6646	0.415452	-0.49911	0.586581	-0.57352
0.562963	1.351529	0.363444	-0.62945	0.400937	-0.48073	0.569846	-0.55617

0.592593	1.362027	0.345289	-0.59633	0.386957	-0.46308	0.553616	-0.53942
0.622222	1.372	0.328088	-0.5651	0.373489	-0.44614	0.537874	-0.52324
0.651852	1.381478	0.311784	-0.53564	0.360512	-0.42988	0.522604	-0.50761
0.681481	1.390485	0.296329	-0.50784	0.348008	-0.41425	0.507788	-0.49251
0.711111	1.399046	0.281675	-0.4816	0.335958	-0.39924	0.493413	-0.47791
0.740741	1.407184	0.267776	-0.45681	0.324344	-0.38481	0.479463	-0.46379
0.77037	1.414921	0.254591	-0.43339	0.313149	-0.37094	0.465925	-0.45013
0.8	1.422278	0.24208	-0.41126	0.302357	-0.35761	0.452784	-0.43692
0.82963	1.429273	0.230208	-0.39033	0.291952	-0.34479	0.440029	-0.42413
0.859259	1.435926	0.218939	-0.37053	0.28192	-0.33246	0.427646	-0.41176
0.888889	1.442253	0.20824	-0.3518	0.272246	-0.32061	0.415625	-0.39978
0.933333	1.451169	0.193196	-0.32556	0.258377	-0.30366	0.398243	-0.38251
0.977778	1.459442	0.179271	-0.30139	0.24524	-0.28766	0.381612	-0.36605
1.022222	1.46712	0.166378	-0.2791	0.232794	-0.27255	0.365694	-0.35036
1.066667	1.474246	0.154436	-0.25854	0.221001	-0.25828	0.350458	-0.33538
1.111111	1.48086	0.143373	-0.23956	0.209825	-0.24479	0.335872	-0.32109
1.155556	1.487002	0.133121	-0.22204	0.199231	-0.23205	0.321907	-0.30744
1.2	1.492705	0.123617	-0.20584	0.189187	-0.22	0.308535	-0.29441
1.244444	1.498	0.114805	-0.19087	0.179665	-0.20861	0.295729	-0.28196
1.288889	1.502919	0.106634	-0.17703	0.170635	-0.19783	0.283464	-0.27006
1.333333	1.507488	0.099054	-0.16423	0.162071	-0.18764	0.271715	-0.25869
1.377778	1.511732	0.092022	-0.15237	0.153948	-0.17799	0.260461	-0.24782
1.422222	1.515675	0.085497	-0.1414	0.146242	-0.16886	0.24968	-0.23743
1.466667	1.519338	0.079441	-0.13124	0.138931	-0.16022	0.23935	-0.22748
1.511111	1.522743	0.07382	-0.12183	0.131994	-0.15203	0.229453	-0.21797
1.555556	1.525906	0.068601	-0.11311	0.125411	-0.14428	0.219969	-0.20887
1.6	1.528846	0.063756	-0.10502	0.119163	-0.13693	0.210881	-0.20016
1.644444	1.531579	0.059257	-0.09753	0.113233	-0.12998	0.202171	-0.19183
1.711111	1.53532	0.053102	-0.0873	0.104898	-0.12022	0.189782	-0.17999
1.777778	1.538673	0.047593	-0.07816	0.097187	-0.11122	0.178156	-0.1689
1.822222	1.540713	0.044244	-0.07261	0.09237	-0.10561	0.170807	-0.16189
1.866667	1.542609	0.041133	-0.06746	0.087795	-0.10028	0.163762	-0.15519
1.955556	1.546012	0.035556	-0.05825	0.079326	-0.09045	0.150534	-0.14261
2.044444	1.548953	0.03074	-0.05031	0.071686	-0.08161	0.138378	-0.13107
2.133333	1.551496	0.02658	-0.04346	0.064792	-0.07365	0.127206	-0.12047
2.222222	1.553695	0.022986	-0.03755	0.058569	-0.06648	0.116936	-0.11074
2.311111	1.555597	0.019881	-0.03245	0.052951	-0.06003	0.107494	-0.10181
2.4	1.557242	0.017197	-0.02805	0.047878	-0.05421	0.098815	-0.0936
2.488889	1.558665	0.014877	-0.02424	0.043296	-0.04897	0.090835	-0.08606
2.577778	1.559896	0.012871	-0.02096	0.039156	-0.04425	0.083497	-0.07913
2.666667	1.560961	0.011137	-0.01812	0.035416	-0.03998	0.07675	-0.07276

2.755556	1.561883	0.009638	-0.01567	0.032036	-0.03614	0.070546	-0.06691
2.844444	1.562681	0.008342	-0.01355	0.02898	-0.03267	0.064841	-0.06153
2.933333	1.563371	0.007221	-0.01172	0.026218	-0.02954	0.059595	-0.05658
3.022222	1.563969	0.006251	-0.01013	0.02372	-0.02671	0.05477	-0.05204
3.111111	1.564486	0.005413	-0.00877	0.021462	-0.02415	0.050333	-0.04786
3.2	1.564934	0.004687	-0.00758	0.019419	-0.02185	0.046252	-0.04401
3.288889	1.565322	0.00406	-0.00656	0.017571	-0.01976	0.042499	-0.04048
3.377778	1.565659	0.003517	-0.00567	0.015899	-0.01788	0.039048	-0.03723
3.555556	1.566202	0.002642	-0.00424	0.013018	-0.01464	0.032954	-0.03149
3.733333	1.566611	0.001987	-0.00317	0.010658	-0.01199	0.027799	-0.02664
3.911111	1.566919	0.001498	-0.00238	0.008725	-0.00983	0.023438	-0.02253
4.088889	1.567151	0.001131	-0.00178	0.007141	-0.00806	0.01975	-0.01906
4.266667	1.567326	0.000857	-0.00133	0.005842	-0.00661	0.016629	-0.01613
4.444444	1.567459	0.000652	-0.00099	0.004776	-0.00542	0.013989	-0.01364
4.622222	1.567561	0.000498	-0.00074	0.003903	-0.00444	0.011756	-0.01154
4.8	1.567639	0.000383	-0.00056	0.003186	-0.00365	0.009867	-0.00976
4.977778	1.567699	0.000298	-0.00042	0.002598	-0.00299	0.008268	-0.00826
5.155556	1.567746	0.000234	-0.00031	0.002115	-0.00246	0.006916	-0.00699
5.333333	1.567783	0.000186	-0.00023	0.001718	-0.00202	0.005772	-0.00591
5.6	1.567825	0.000136	-0.00015	0.001253	-0.0015	0.004378	-0.0046
5.866667	1.567857	0.000104	-9.6E-05	0.000907	-0.00112	0.003293	-0.00358
6.133333	1.567882	8.32E-05	-6.1E-05	0.00065	-0.00083	0.002449	-0.00279
6.4	1.567902	7.01E-05	-3.8E-05	0.000458	-0.00062	0.001792	-0.00217
6.666667	1.56792	6.2E-05	-2.4E-05	0.000316	-0.00046	0.001281	-0.00169
7.066667	1.567943	5.53E-05	-1.1E-05	0.000168	-0.00029	0.000719	-0.00116
7.533333	1.567968	5.21E-05	-3.5E-06	6.23E-05	-0.00017	0.000282	-0.00075
8	1.567992	5.14E-05	0	0	-0.0001	0	-0.00048