

Mining Billion-node Graphs: Patterns, Generators and Tools

Christos Faloutsos

CMU

THANK YOU!

- Prof. Lee Giles
- Louise Troxell



Our goal:

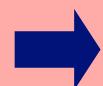
Open source system for mining huge graphs:

PEGASUS project (PEta GrAph mining System)

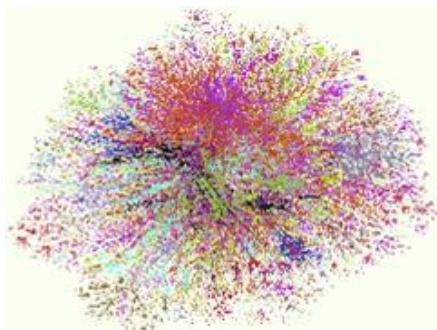
- www.cs.cmu.edu/~pegasus
- code and papers



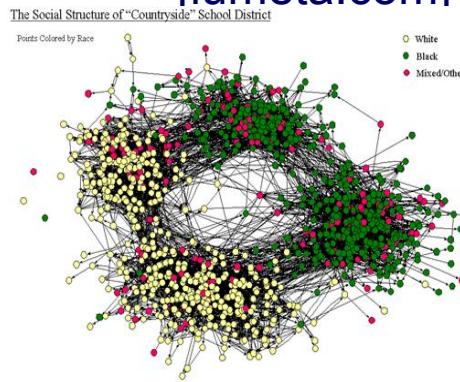
Outline

- 
- Introduction – Motivation
 - Problem#1: Patterns in graphs
 - Problem#2: Tools
 - Problem#3: Scalability
 - Conclusions

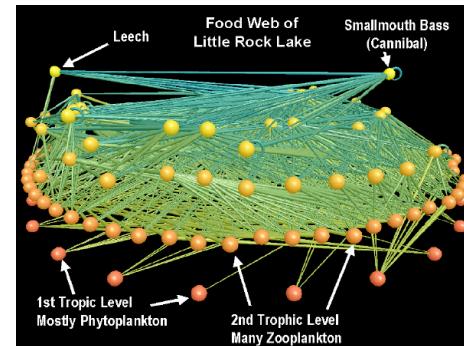
Graphs - why should we care?



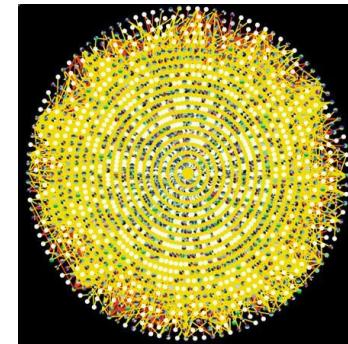
Internet Map
[lumeta.com]



Friendship Network
[Moody '01]



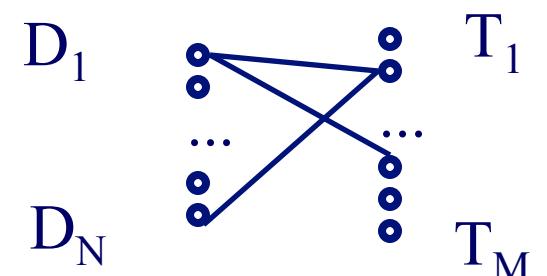
Food Web
[Martinez '91]



Protein Interactions
[genomebiology.com]

Graphs - why should we care?

- IR: bi-partite graphs (doc-terms)



- Citeseer: doc/authors/terms/...

- web: hyper-text graph

- ... and more:

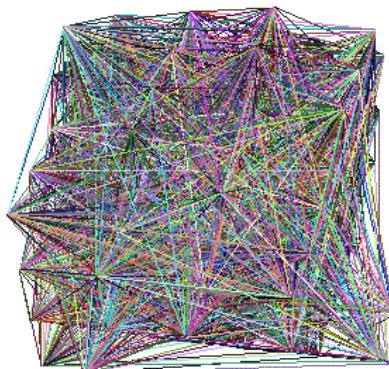
Graphs - why should we care?

- network of companies & board-of-directors members
- ‘viral’ marketing
- web-log (‘blog’) news propagation
- computer network security: email/IP traffic and anomaly detection
-

Outline

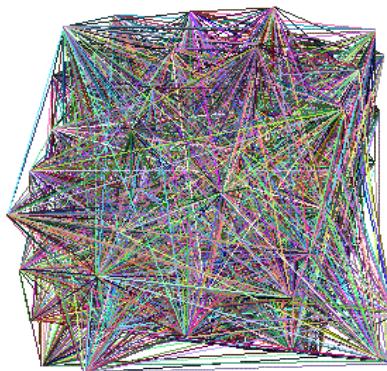
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 - Static graphs
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Problem #1 - network and graph mining



- How does the Internet look like?
- How does FaceBook look like?
- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?

Problem #1 - network and graph mining

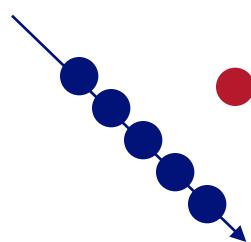
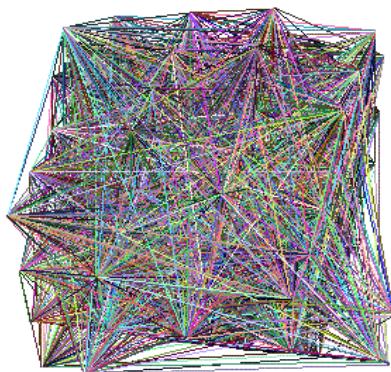


- •
-

- How does the Internet look like?
- How does FaceBook look like?
- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?
 - To spot **anomalies** (rarities), we have to discover **patterns**

Problem #1 - network and graph mining

- How does the Internet look like?
- How does FaceBook look like?
- What is ‘normal’/‘abnormal’?
- which patterns/laws hold?
 - To spot **anomalies** (rarities), we have to discover **patterns**
 - **Large** datasets reveal patterns/anomalies that may be invisible otherwise...



Graph mining

- Are real graphs random?

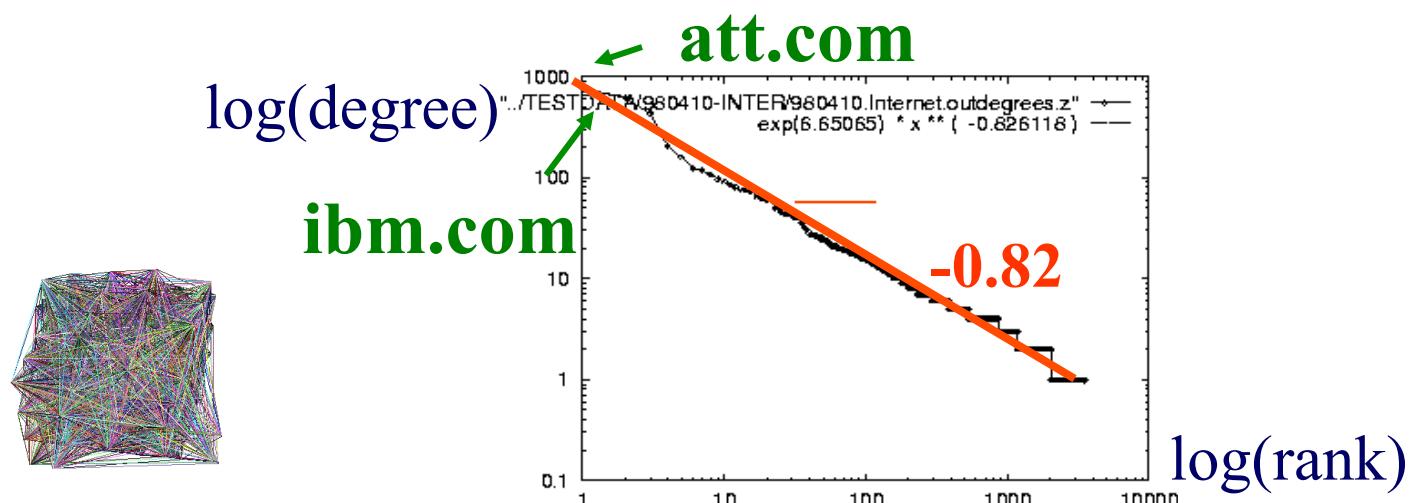
Laws and patterns

- Are real graphs random?
- A: NO!!
 - Diameter
 - in- and out- degree distributions
 - other (surprising) patterns
- So, let's look at the data

Solution# S.1

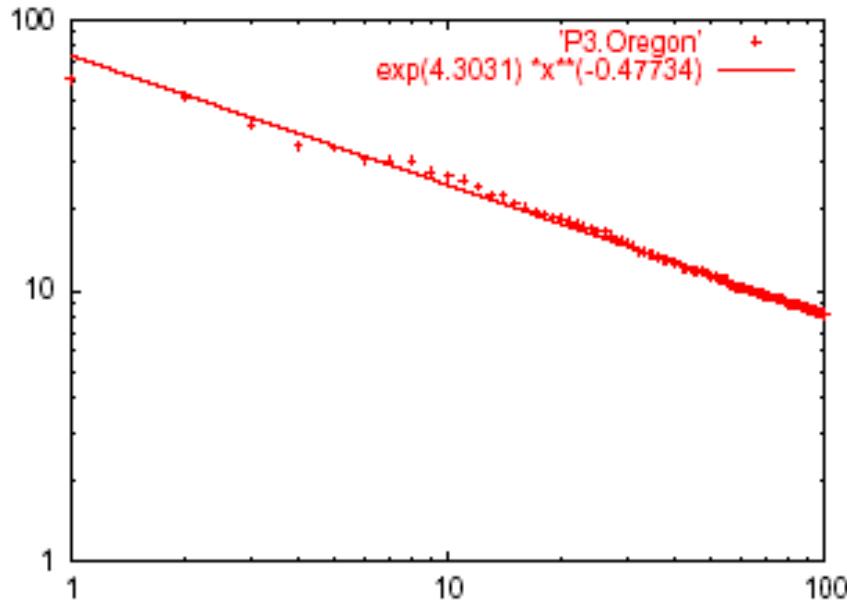
- Power law in the degree distribution
[SIGCOMM99]

internet domains



Solution# S.2: Eigen Exponent E

Eigenvalue



Exponent = slope

$$E = -0.48$$

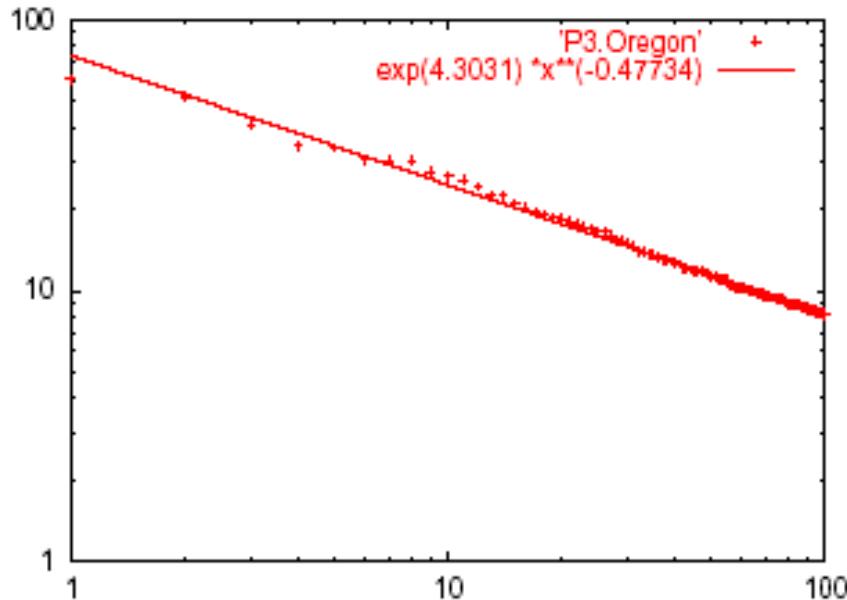
May 2001

Rank of decreasing eigenvalue

- A2: power law in the eigenvalues of the adjacency matrix

Solution# S.2: Eigen Exponent E

Eigenvalue



Exponent = slope

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May 2001

Rank of decreasing eigenvalue

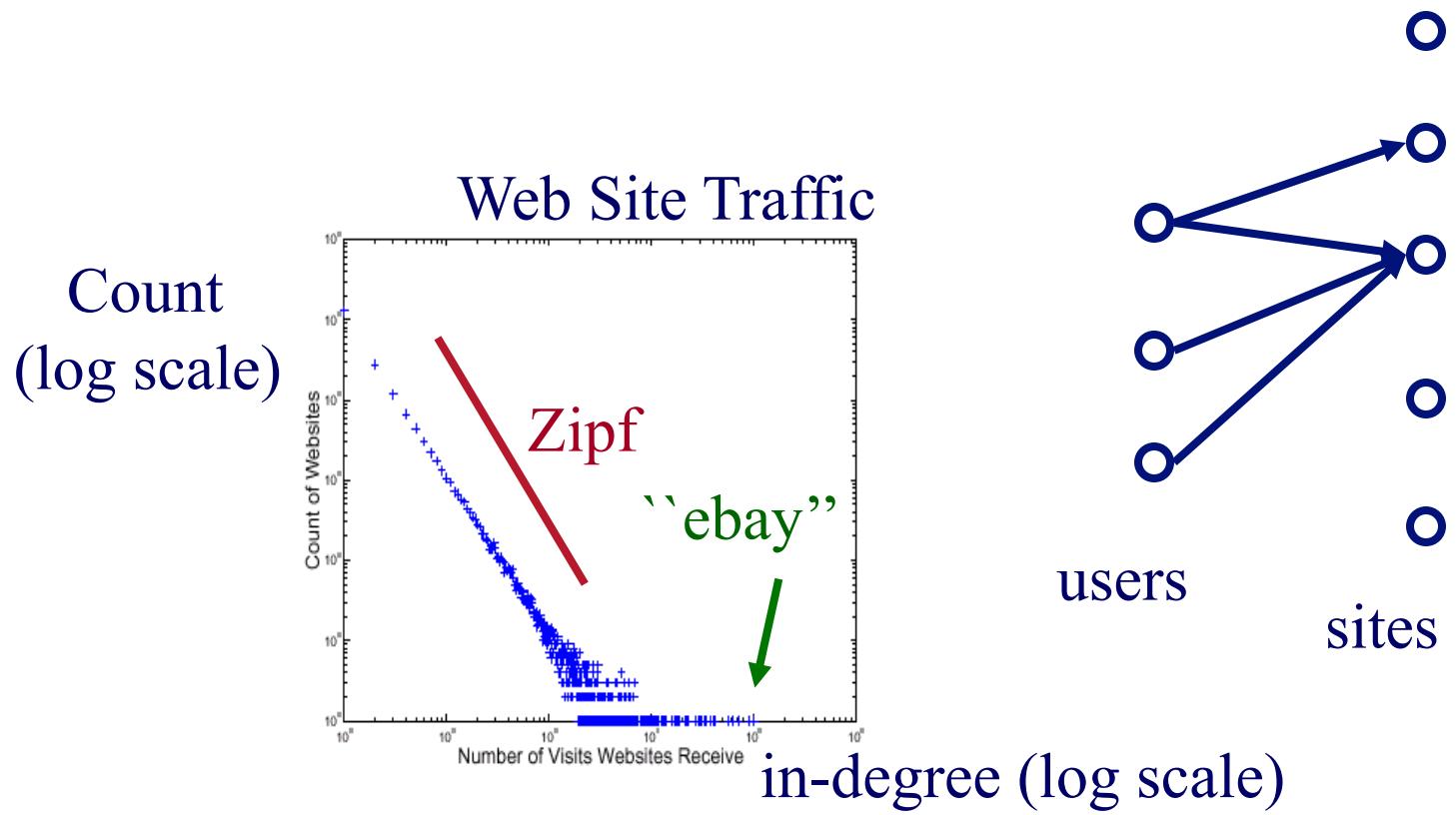
- [Mihail, Papadimitriou '02]: slope is $\frac{1}{2}$ of rank exponent

But:

How about graphs from other domains?

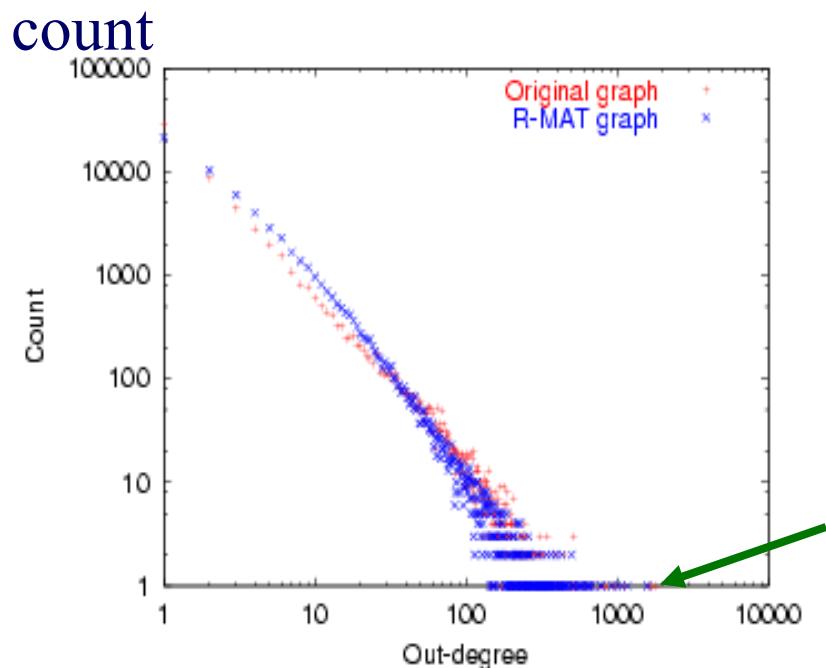
More power laws:

- web hit counts [w/ A. Montgomery]



epinions.com

- who-trusts-whom
[Richardson +
Domingos, KDD
2001]



(out) degree

And numerous more

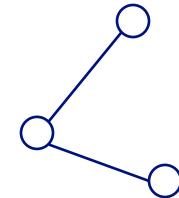
- # of sexual contacts
- Income [Pareto] – ‘80-20 distribution’
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs (‘mice and elephants’)
- Size of files of a user
- ...
- ‘Black swans’

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 - degree, diameter, eigen,
 - triangles
 - cliques
 - Weighted graphs
 - Time evolving graphs
- Problem#2: Tools

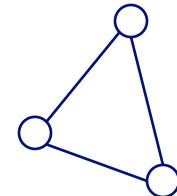


Solution# S.3: Triangle ‘Laws’



- Real social networks have a lot of triangles

Solution# S.3: Triangle ‘Laws’



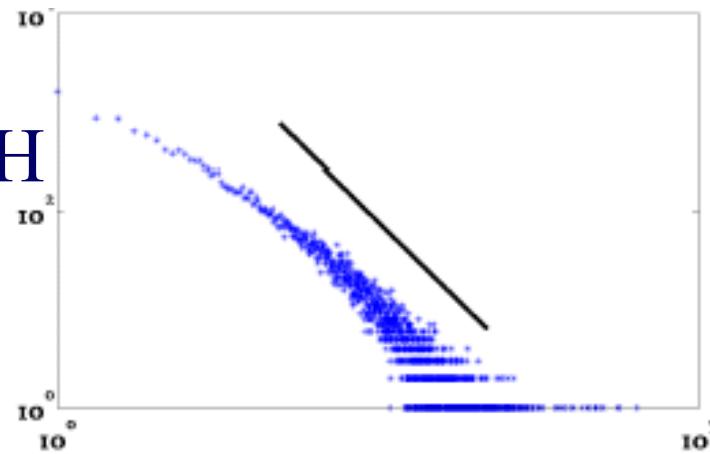
- Real social networks have a lot of triangles
 - Friends of friends are friends
- Any patterns?

Triangle Law: #S.3

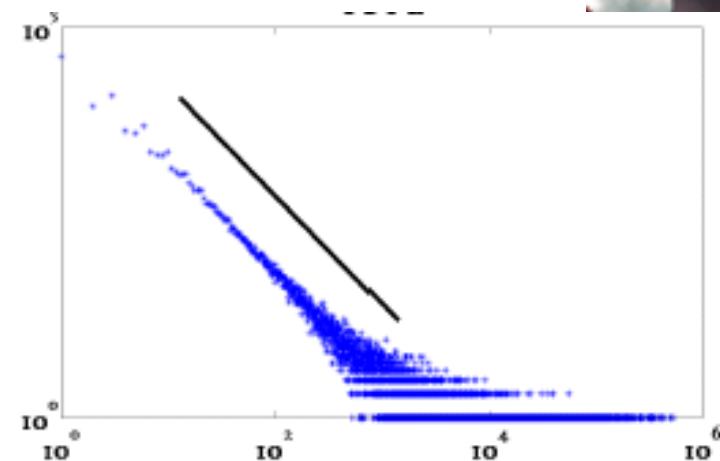
[Tsourakakis ICDM 2008]



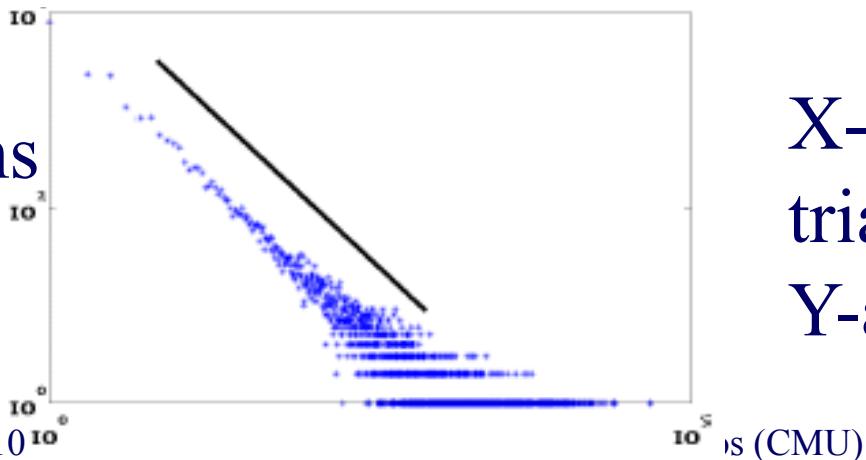
HEP-TH



ASN



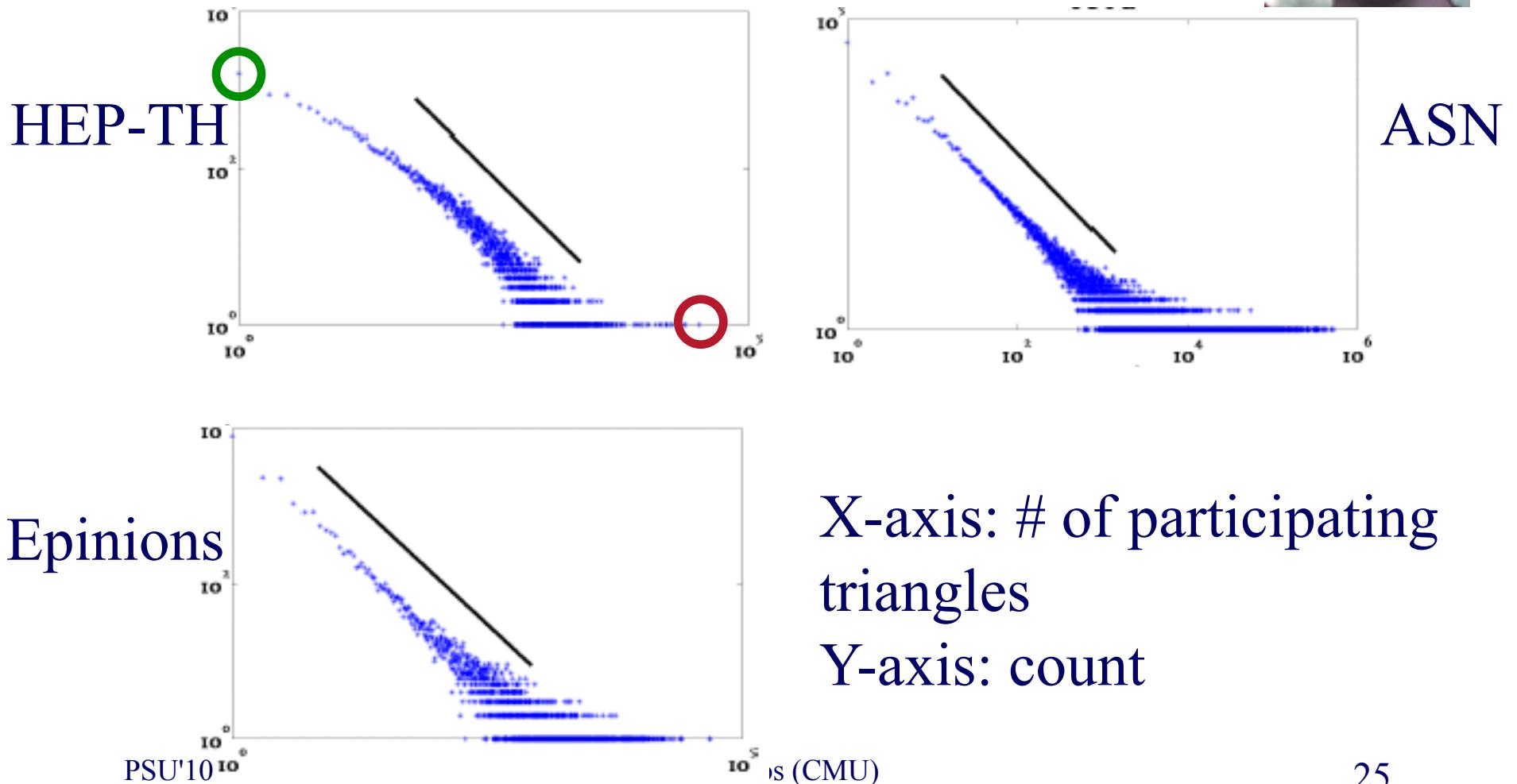
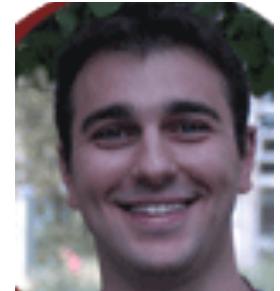
Epinions



X-axis: # of participating triangles
Y-axis: count



Triangle Law: #S.3 [Tsourakakis ICDM 2008]

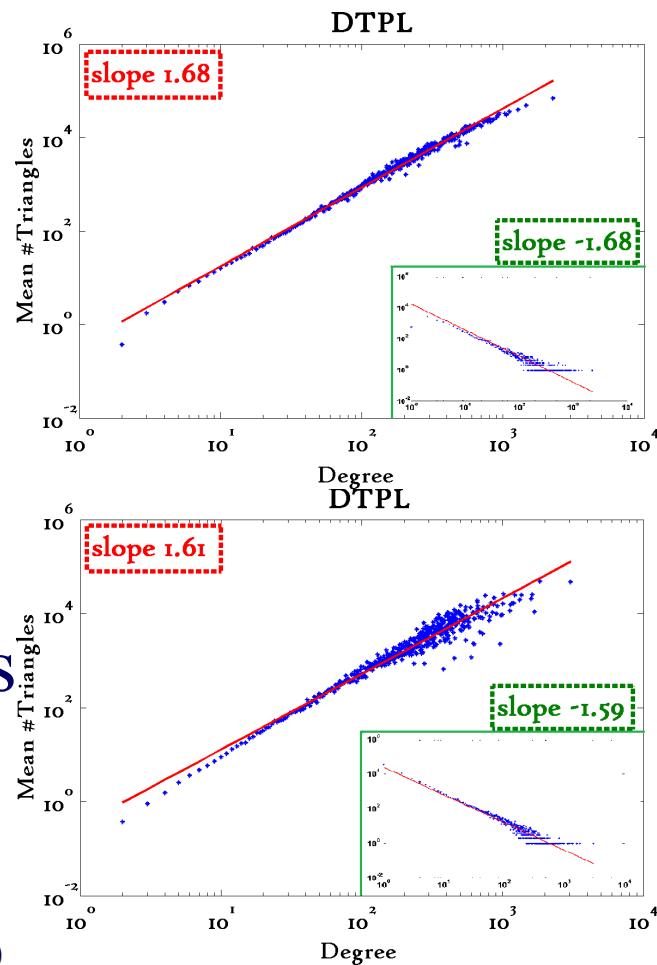




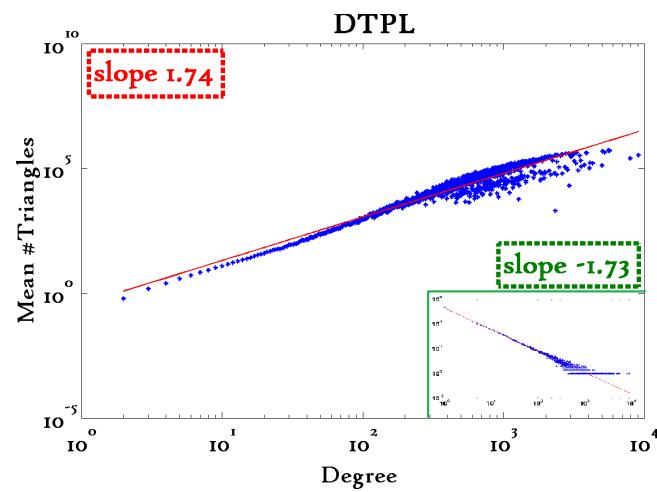
Triangle Law: #S.4

[Tsourakakis ICDM 2008]

Reuters



Epinions



PSU'10

X-axis: degree
Y-axis: mean # triangles
 n friends $\rightarrow \sim n^{1.6}$ triangles

Constantinos Daskalakis (CMU)



Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)

Q: Can we do that quickly?



Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)

Q: Can we do that quickly?

A: Yes!

#triangles = 1/6 Sum (λ_i^3)
(and, because of skewness, we only need
the top few eigenvalues!)

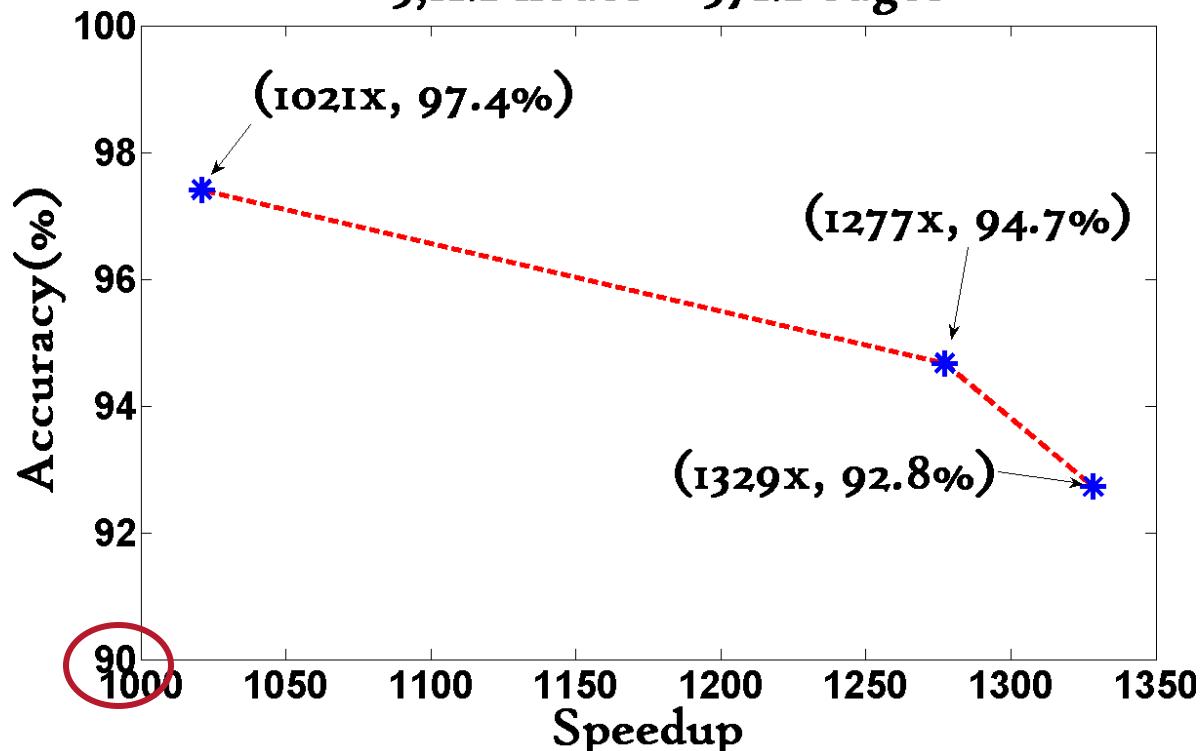


Triangle Law: Computations

[Tsourakakis ICDM 2008]

Wikipedia graph 2006-Nov-04

$\approx 3.1\text{M}$ nodes $\approx 37\text{M}$ edges



1000x+ speed-up, >90% accuracy

C. Faloutsos (CMU)



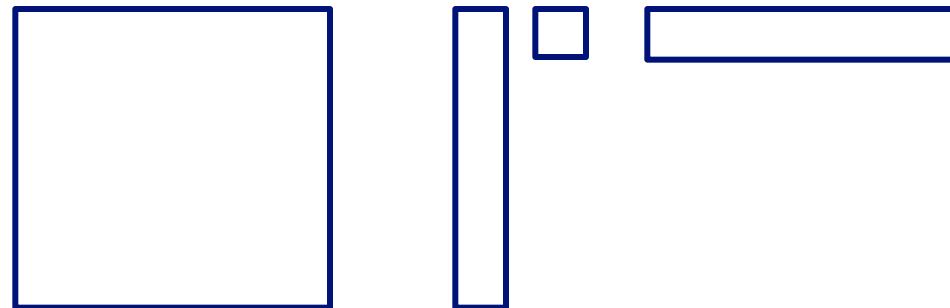
EigenSpokes

B. Aditya Prakash, Mukund Seshadri, Ashwin Sridharan, Sridhar Machiraju and Christos Faloutsos: *EigenSpokes: Surprising Patterns and Scalable Community Chipping in Large Graphs*, PAKDD 2010, Hyderabad, India, 21-24 June 2010.

EigenSpokes

- Eigenvectors of adjacency matrix
 - equivalent to singular vectors
(symmetric, undirected graph)

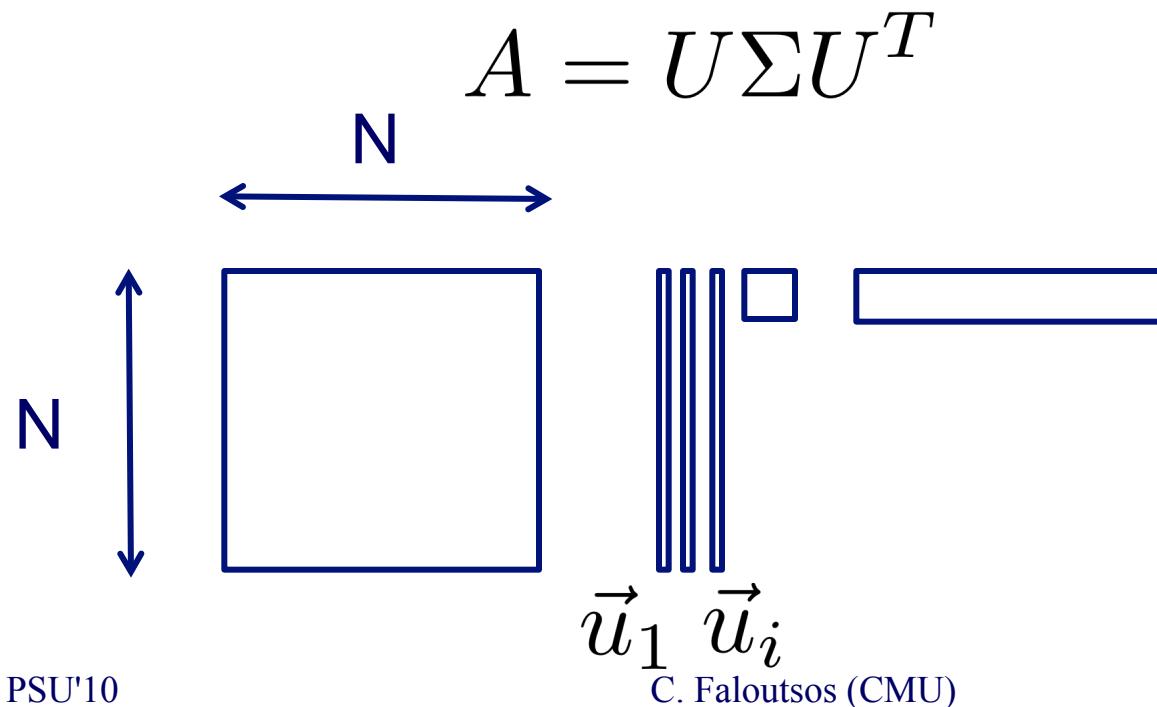
$$A = U\Sigma U^T$$





EigenSpokes

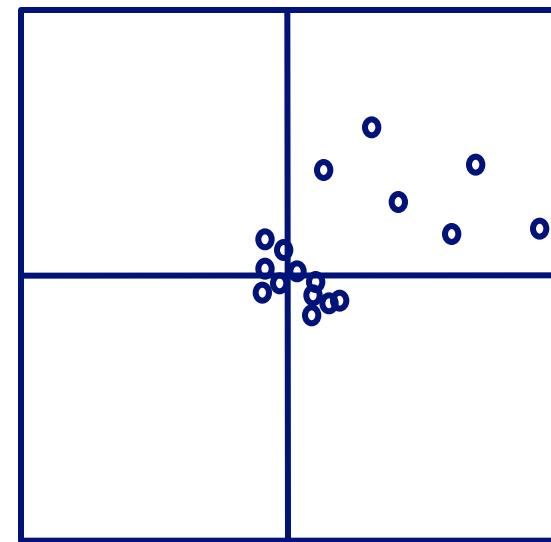
- Eigenvectors of adjacency matrix
 - equivalent to singular vectors
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EigenSpokes

- EE plot:
- Scatter plot of scores of u_1 vs u_2
- One would expect
 - Many points @ origin
 - A few scattered ~randomly

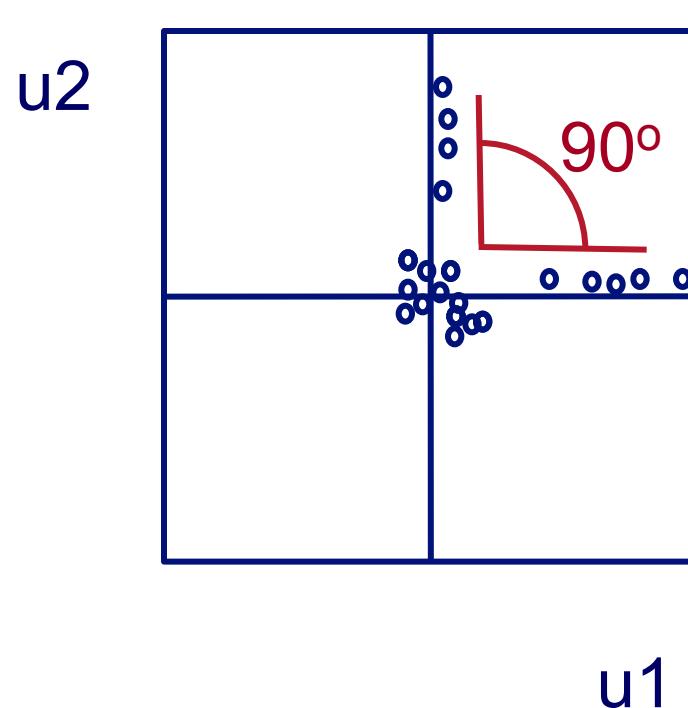
2nd Principal component u_2



1st Principal component

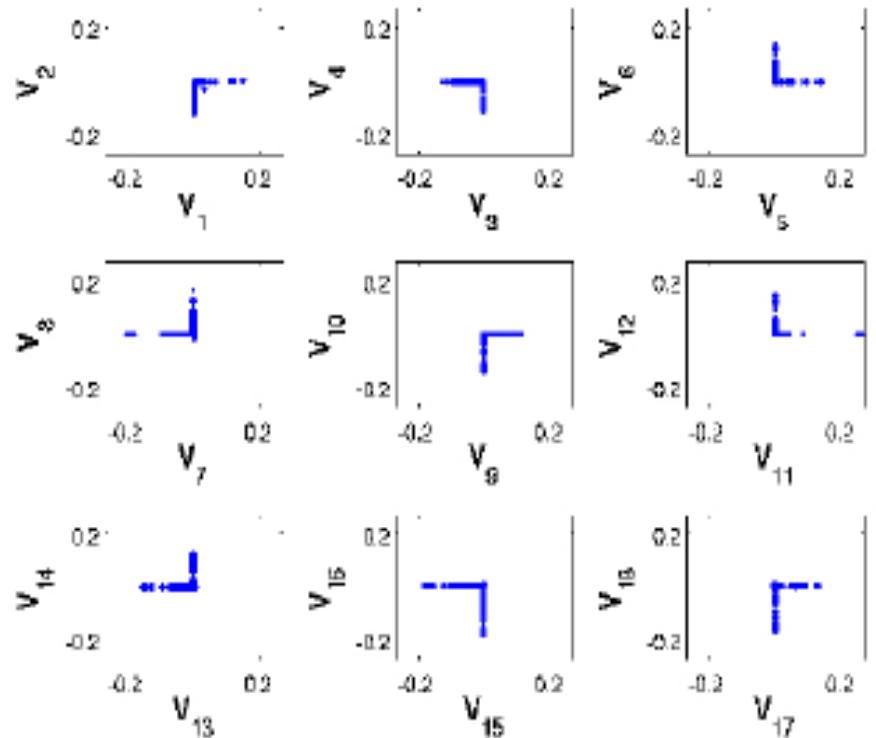
EigenSpokes

- EE plot:
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 - Many points @ origin
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EigenSpokes - pervasiveness

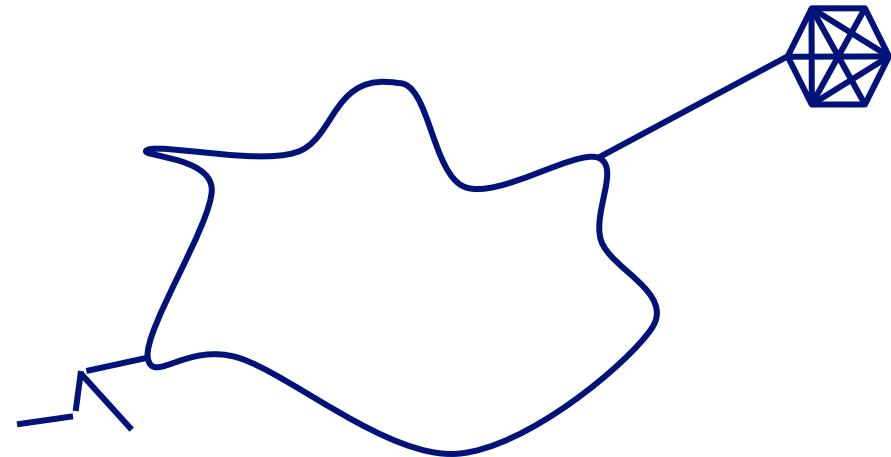
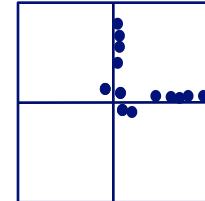
- Present in mobile social graph
 - across time and space



- Patent citation graph

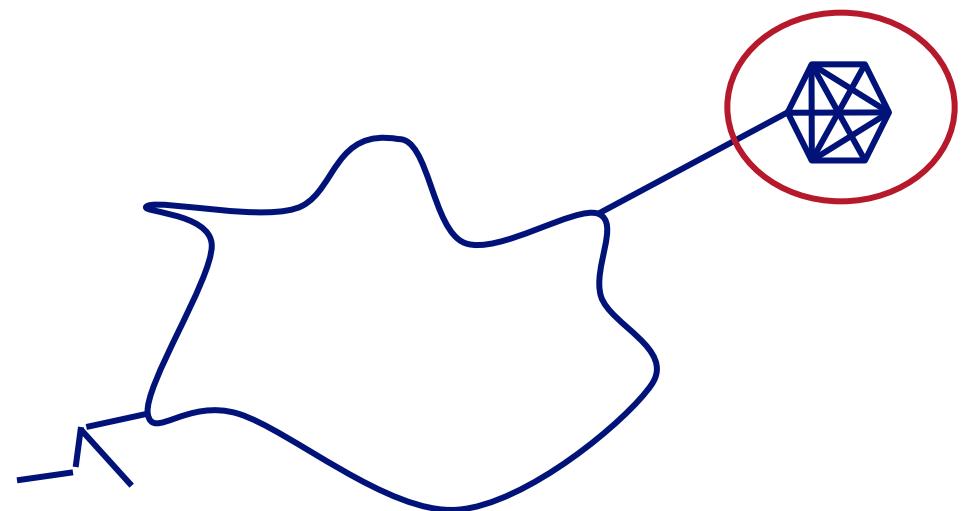
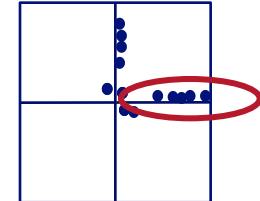
EigenSpokes - explanation

Near-cliques, or
near-bipartite-cores,
loosely connected



EigenSpokes - explanation

Near-cliques, or
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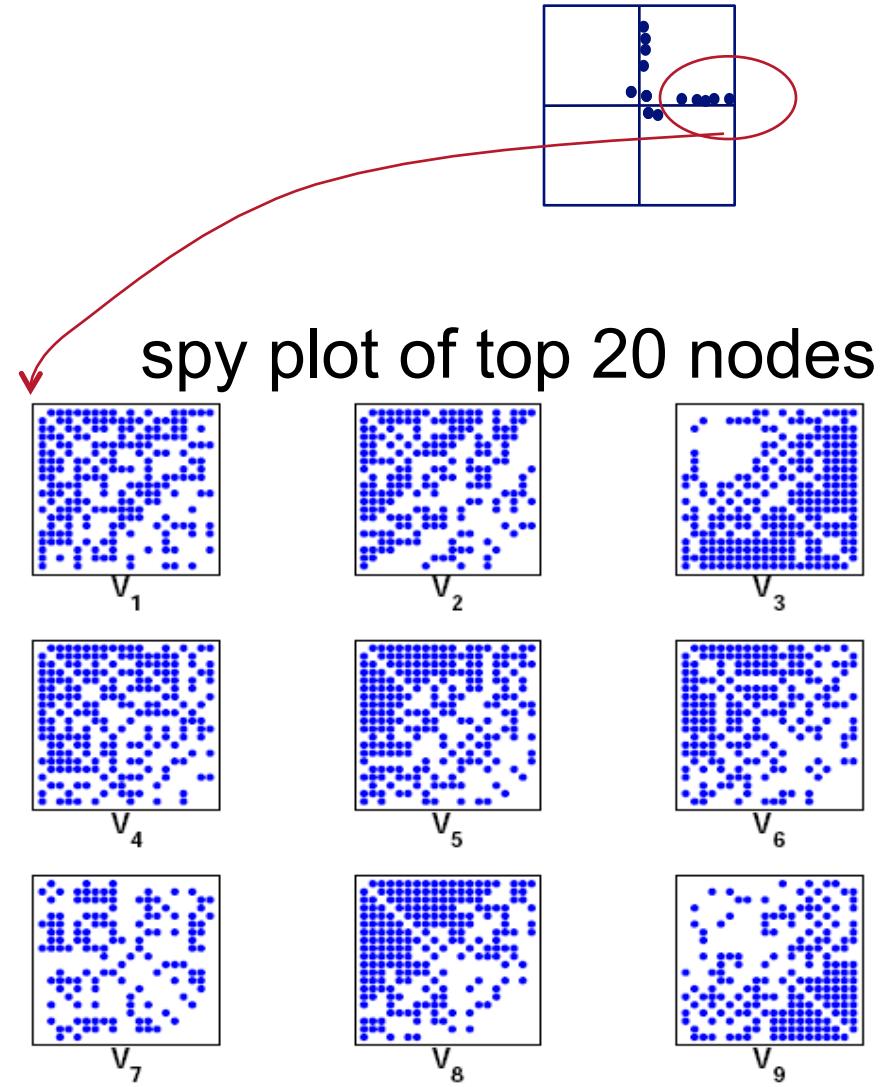


EigenSpokes - explanation

Near-cliques, or
near-bipartite-cores,
loosely connected

So what?

- Extract nodes with high *scores*
- high connectivity
- Good “communities”

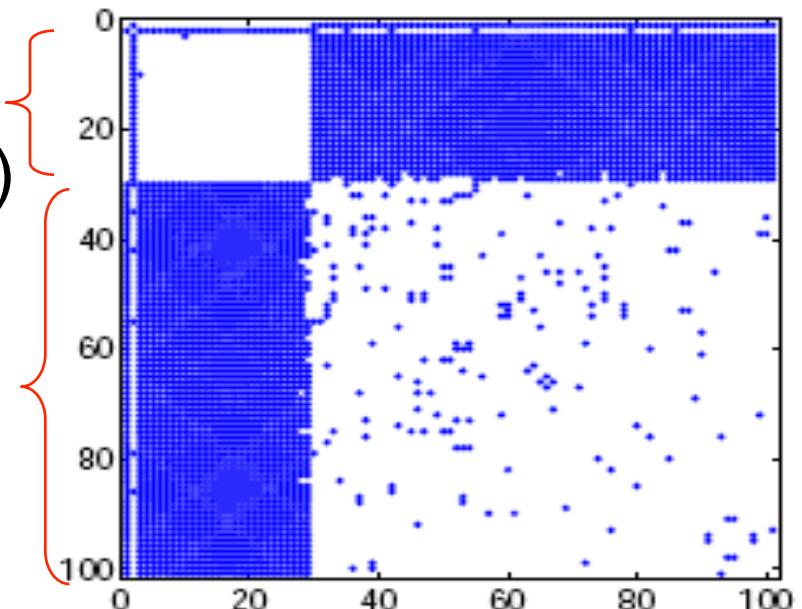
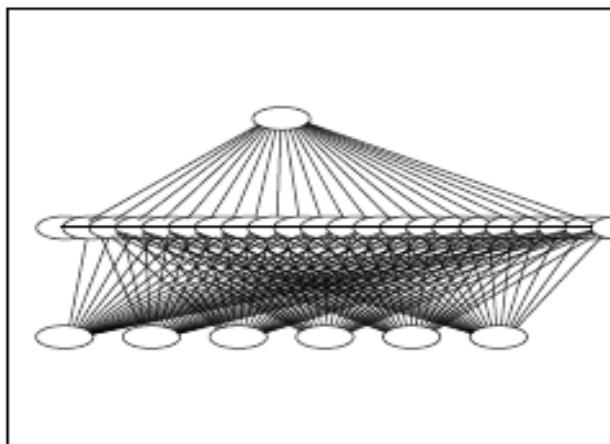


Bipartite Communities!

patents from
same inventor(s)

cut-and-paste
bibliography!

magnified bipartite community



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Observations on weighted graphs?

- A: yes - even more ‘laws’!



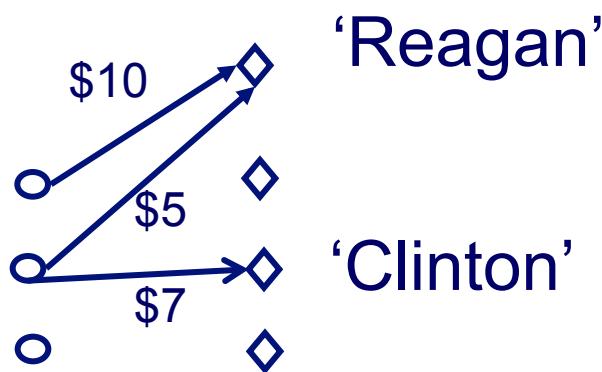
M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator.
SIG-KDD 2008

Observation W.1: Fortification

*Q: How do the weights
of nodes relate to degree?*

Observation W.1: Fortification

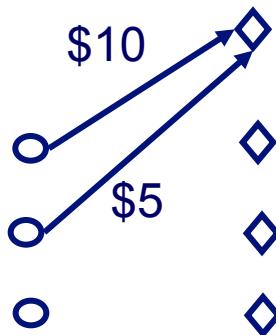
**More donors,
more \$?**



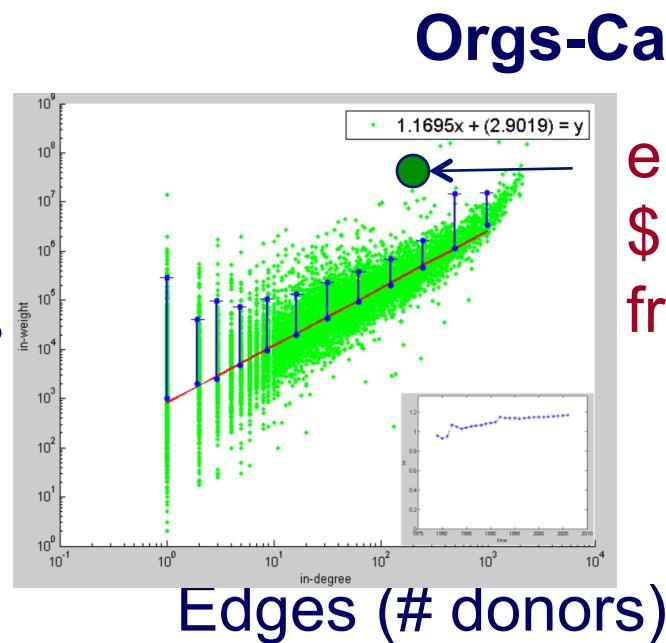
Observation W.1: fortification: Snapshot Power Law

- Weight: super-linear on in-degree
- exponent ‘iw’: $1.01 < iw < 1.26$

**More donors,
even more \$**



In-weights
(\$)



Edges (# donors)

e.g. John Kerry,
\$10M received,
from 1K donors

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Problem: Time evolution

- with Jure Leskovec (CMU -> Stanford)



- and Jon Kleinberg (Cornell – sabb. @ CMU)



T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
 - diameter $\sim O(\log N)$
 - diameter $\sim O(\log \log N)$
- What is happening in real data?

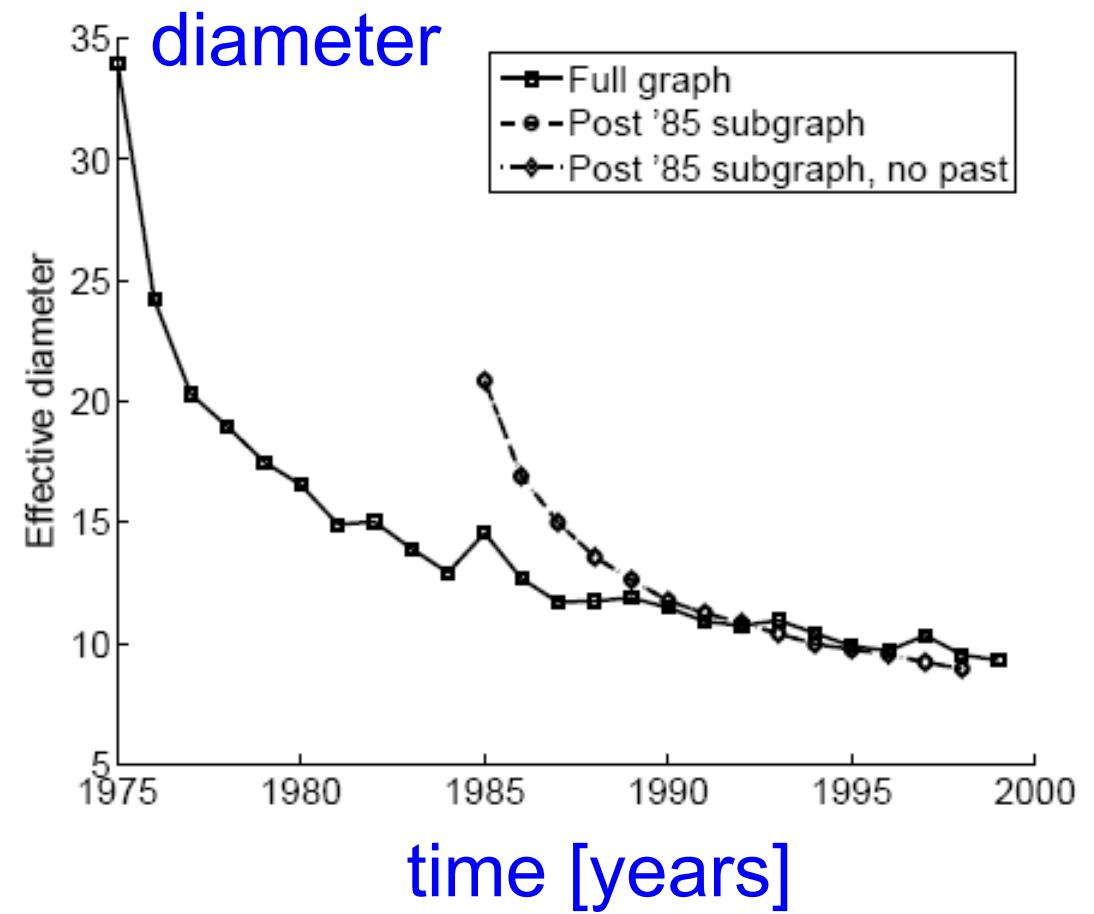


T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
 - diameter $\sim O(\log N)$
 - diameter $\sim O(\log \log N)$
- What is happening in real data?
- Diameter **shrinks** over time

T.1 Diameter – “Patents”

- Patent citation network
- 25 years of data
- @1999
 - 2.9 M nodes
 - 16.5 M edges



T.2 Temporal Evolution of the Graphs

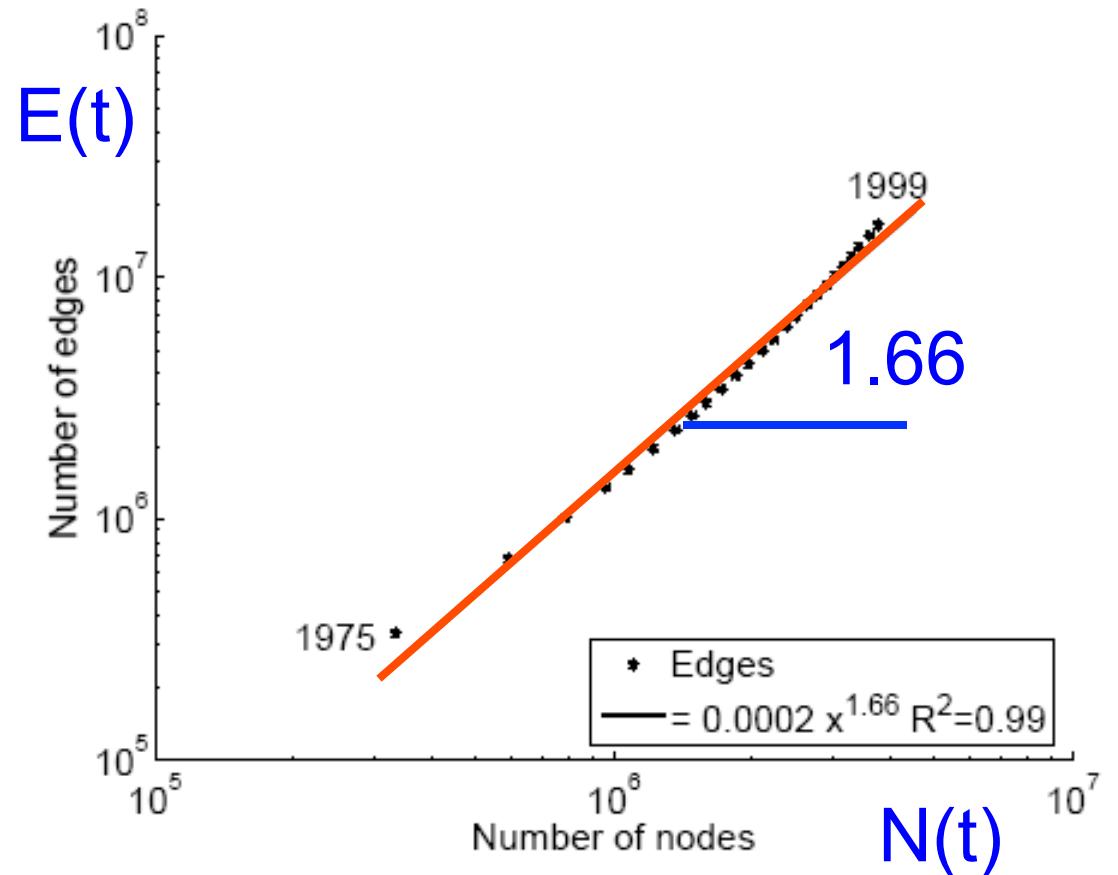
- $N(t)$... nodes at time t
- $E(t)$... edges at time t
- Suppose that
$$N(t+1) = 2 * N(t)$$
- Q: what is your guess for
$$E(t+1) =? 2 * E(t)$$

T.2 Temporal Evolution of the Graphs

- $N(t)$... nodes at time t
- $E(t)$... edges at time t
- Suppose that
$$N(t+1) = 2 * N(t)$$
- Q: what is your guess for
$$E(t+1) \approx ? \cdot 2 * E(t)$$
- A: over-doubled!
 - But obeying the “Densification Power Law”

T.2 Densification – Patent Citations

- Citations among patents granted
- @1999
 - 2.9 M nodes
 - 16.5 M edges
- Each year is a datapoint



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More on Time-evolving graphs

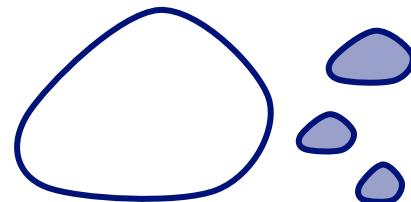
M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator.
SIG-KDD 2008

Observation T.3: NLCC behavior

Q: How do NLCC's emerge and join with the GCC?

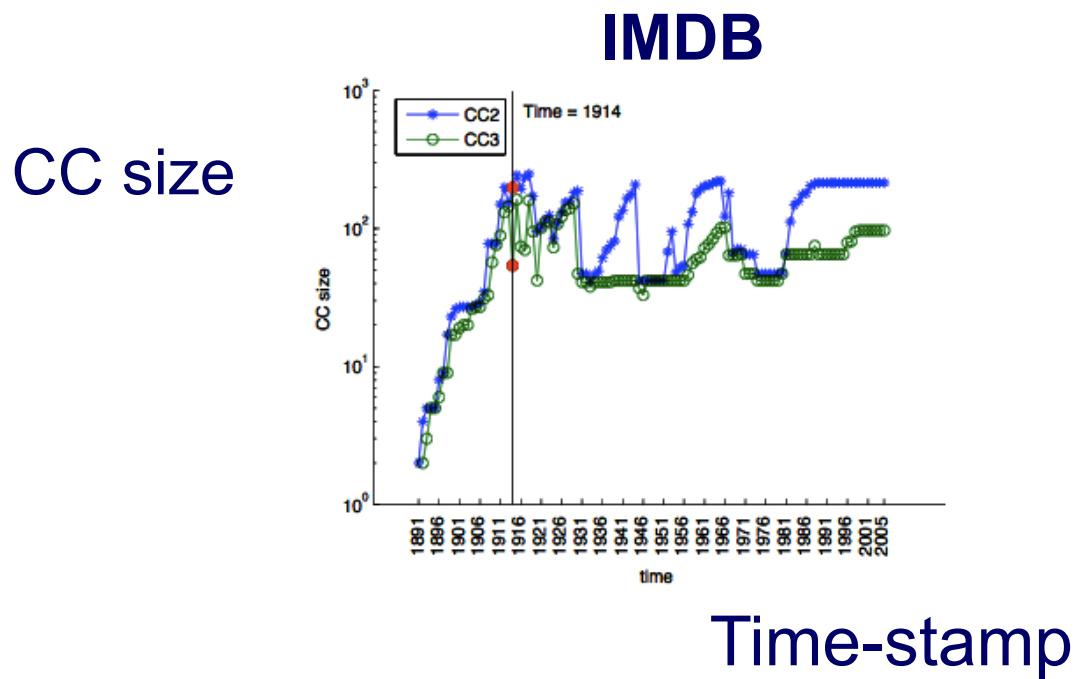
(‘‘NLCC’’ = non-largest conn. components)

- Do they continue to grow in size?
 - or do they shrink?
 - or stabilize?



Observation T.3: NLCC behavior

- After the gelling point, the GCC takes off, but NLCC's remain ~constant (actually, oscillate).

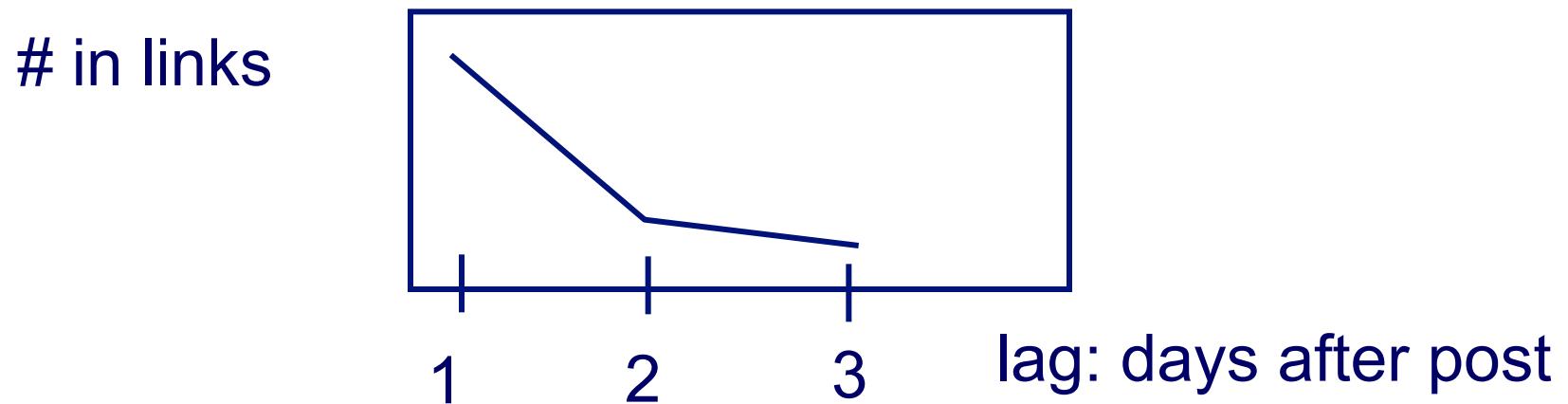


Timing for Blogs

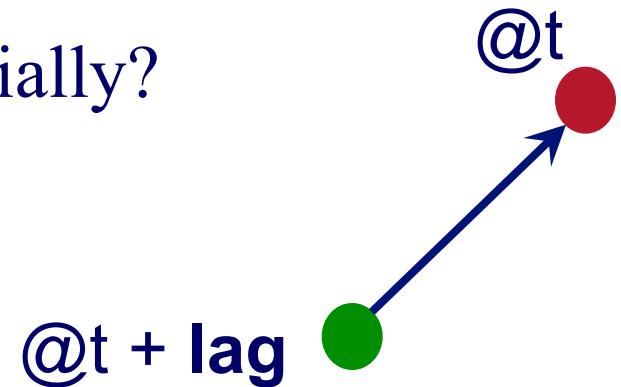
- with Mary McGlohon (CMU)
- Jure Leskovec (CMU->Stanford)
- Natalie Glance (now at Google)
- Mat Hurst (now at MSR)

[SDM'07]

T.4 : popularity over time

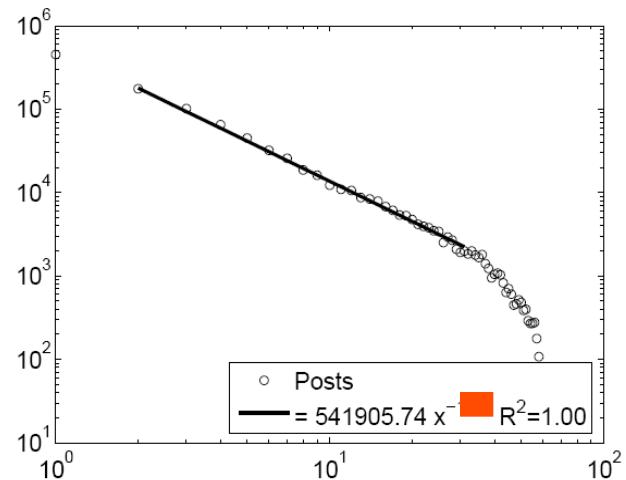


Post popularity drops-off – exponentially?



T.4 : popularity over time

in links
(log)

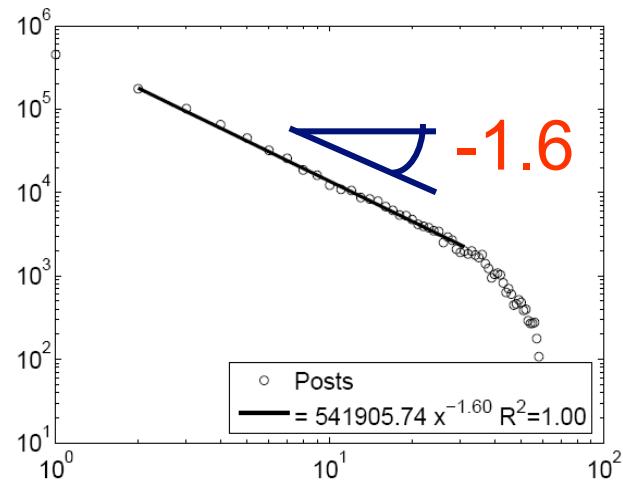


days after post
(log)

Post popularity drops-off – exponentially?
POWER LAW!
Exponent?

T.4 : popularity over time

in links
(log)



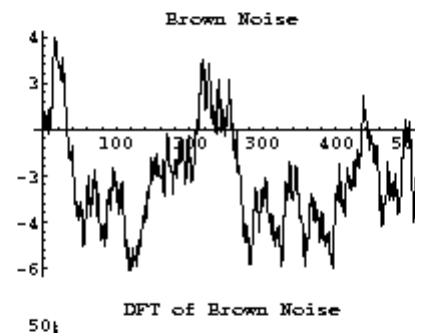
days after post
(log)

Post popularity drops-off – exponentially?

POWER LAW!

Exponent? -1.6

- close to -1.5: Barabasi's stack model
- and like the zero-crossings of a random walk



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 - CenterPiece Subgraphs; G-Ray
 - OddBall (anomaly detection)
 - PEGASUS
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CenterPiece Subgraphs

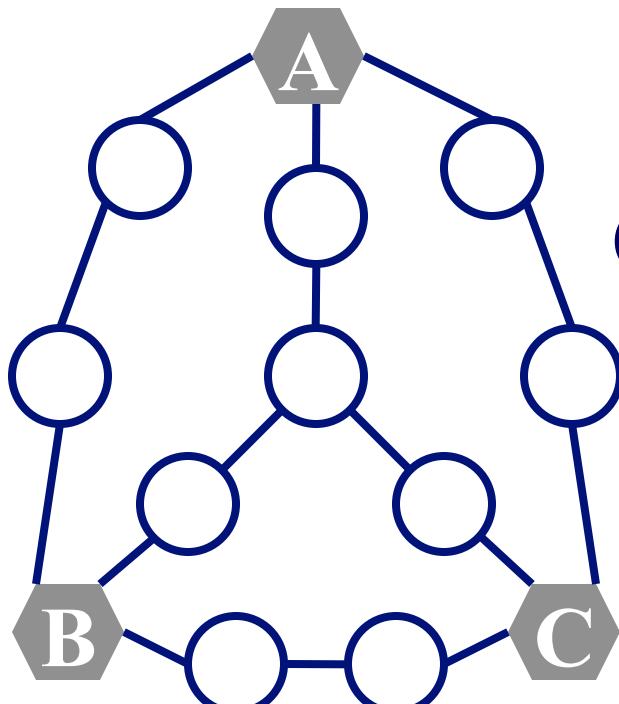
- Hanghang TONG et al,
KDD'06



Center-Piece Subgraph Discovery

[Tong+ KDD 06]

Input



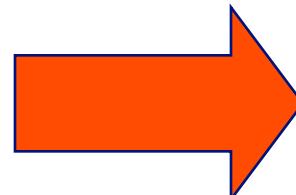
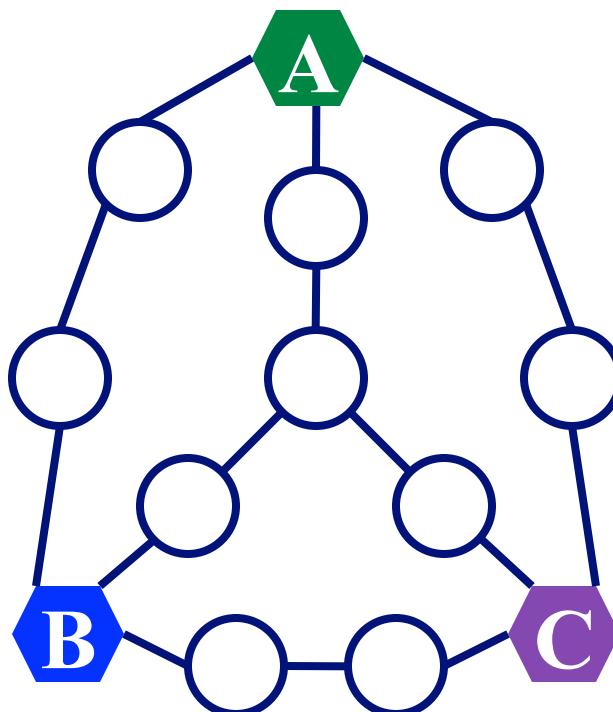
Original Graph

Q: Who is the most central node
wrt the black nodes?
(e.g., master-mind criminal, common
advisor/collaborator, etc)

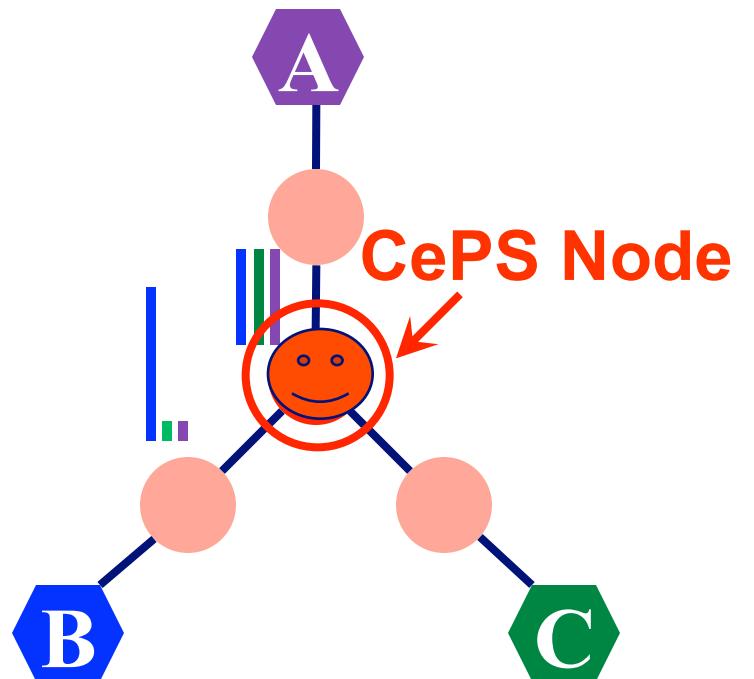
Center-Piece Subgraph Discovery

[Tong+ KDD 06]

Input: original graph



Output: CePS



Q: How to find hub for the query nodes?

A: Combine proximity scores (RWR)

CePS: Example (AND Query)



R. Agrawal



Jiawei Han

?



V. Vapnik

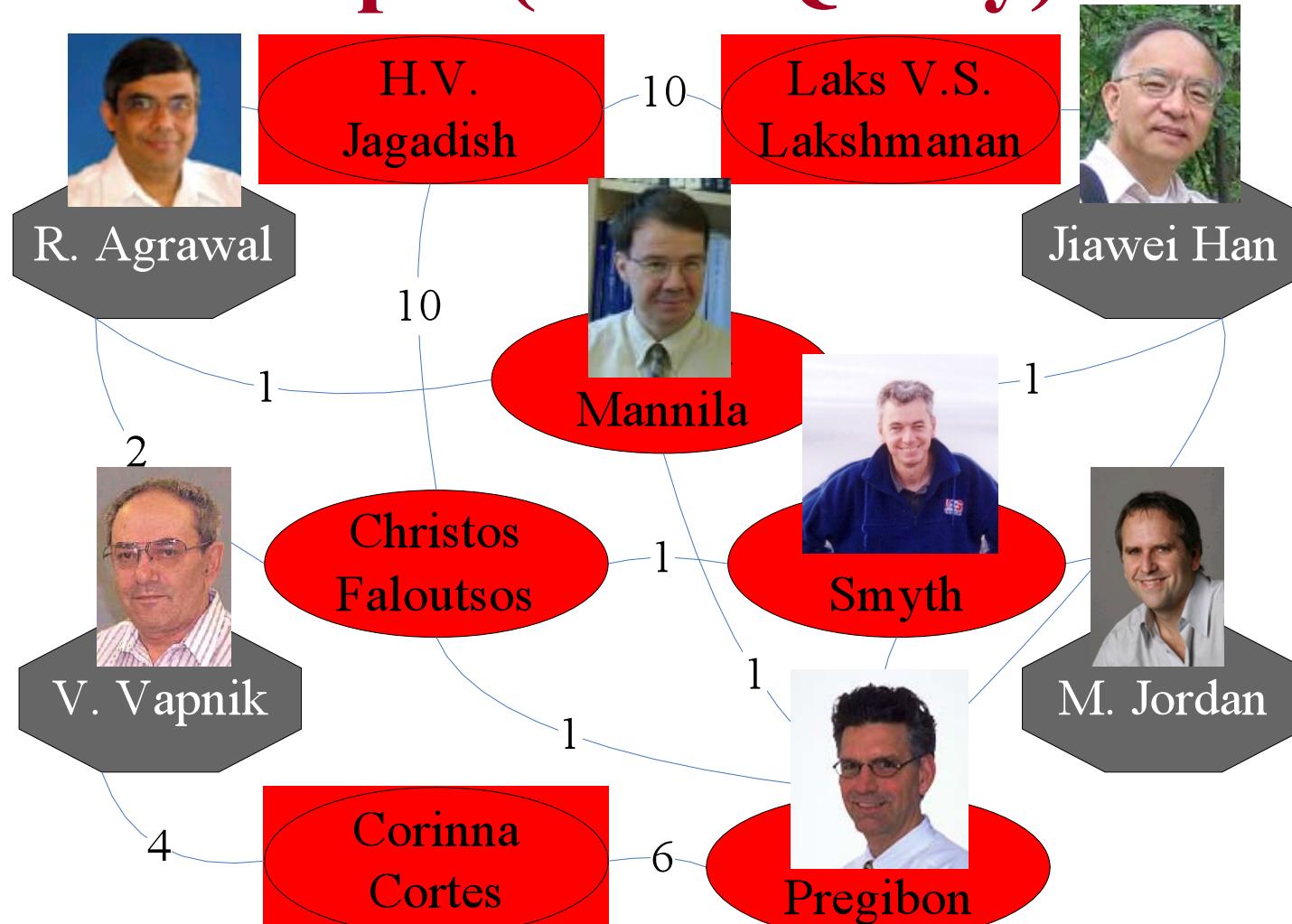


M. Jordan

DBLP co-authorship network:
-400,000 authors, 2,000,000 edges

Code at: <http://www.cs.cmu.edu/~htong/soft.htm>

CePS: Example (AND Query)



DBLP co-authorship network:
-400,000 authors, 2,000,000 edges

PSU'10

Code at: <http://www.cs.cmu.edu/~htong/soft.htm>

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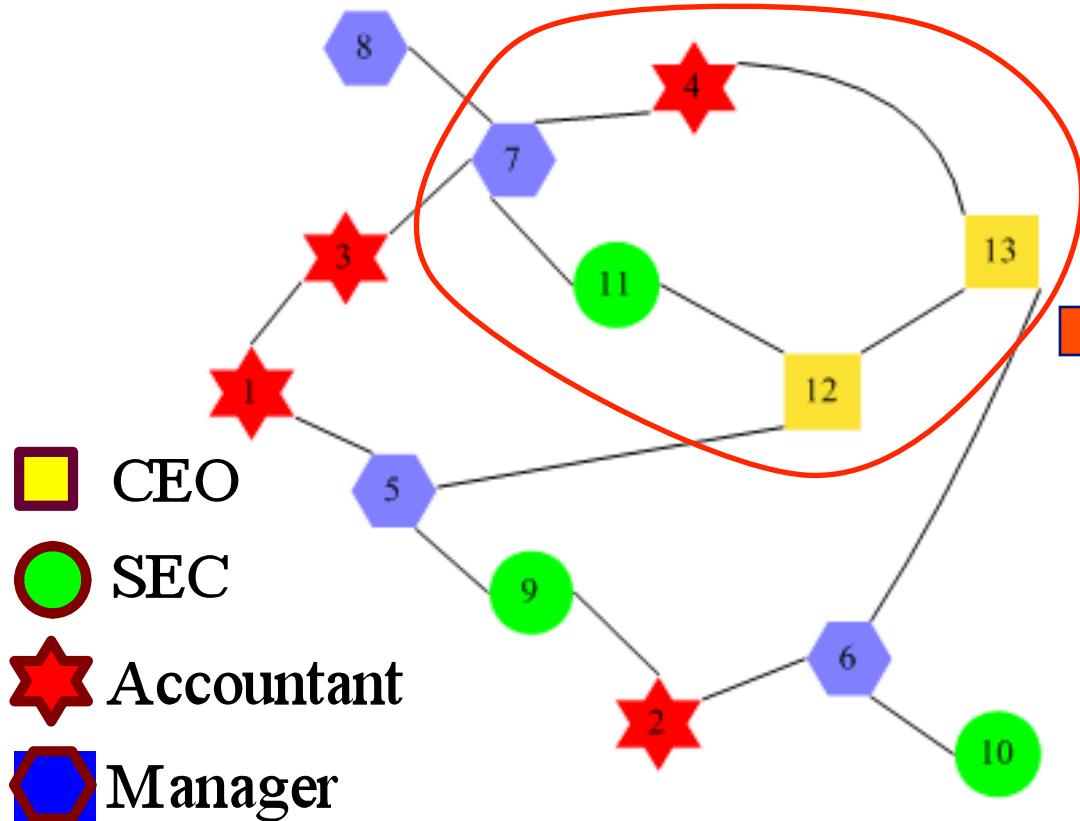
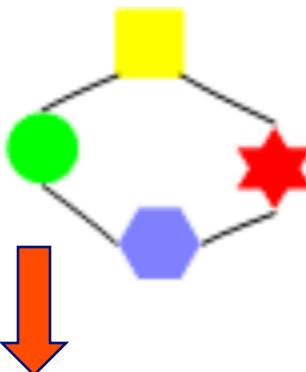
Graph X-Ray: Fast Best-Effort Pattern Matching in Large Attributed Graphs

Hanghang Tong, Brian Gallagher,
Christos Faloutsos, Tina Eliassi-Rad

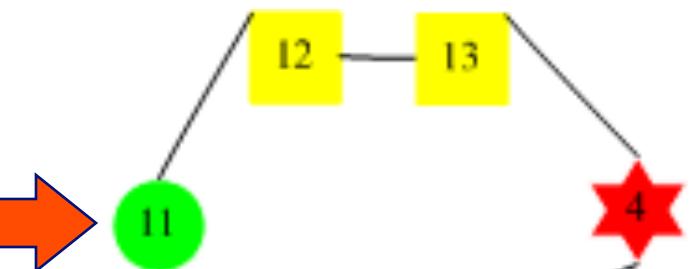
KDD'07

Input

Query Graph

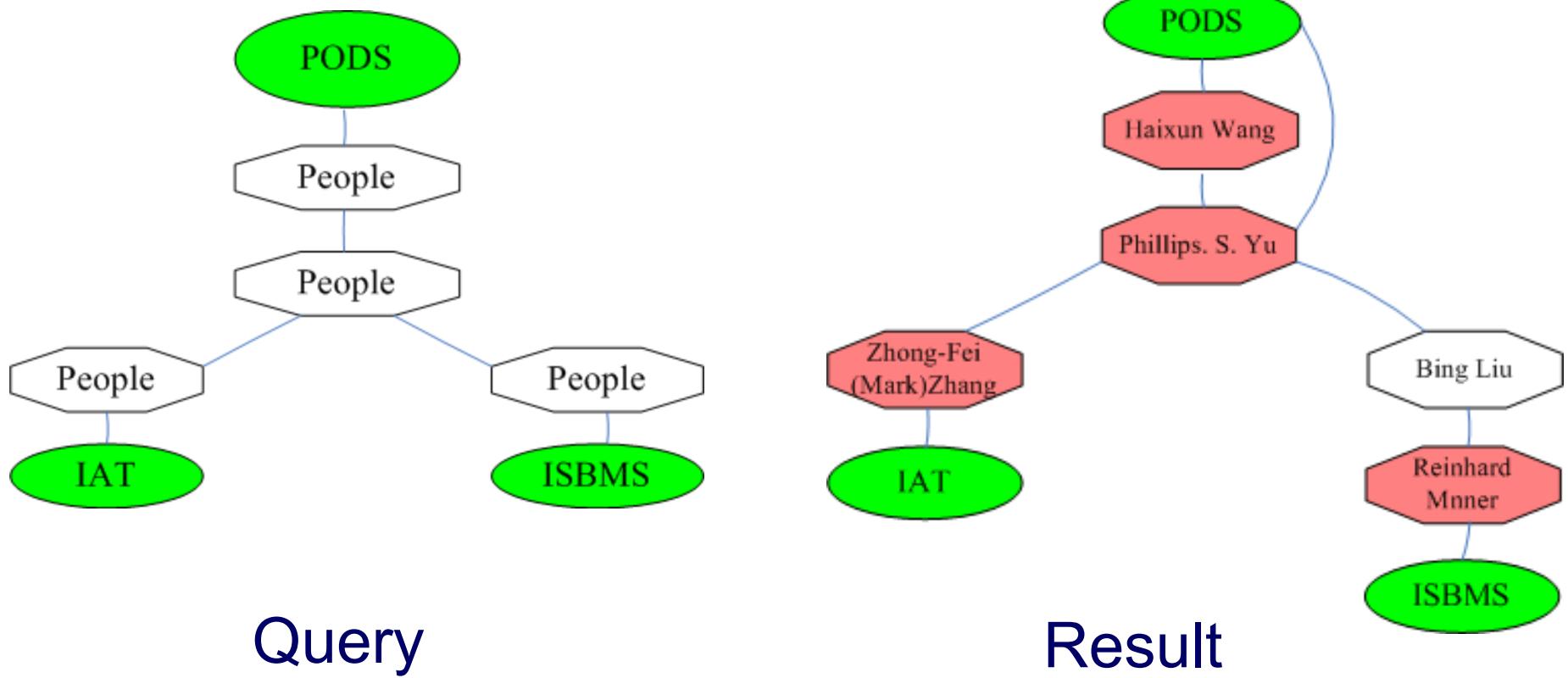


Output



Matching Subgraph

Effectiveness: star-query



Query

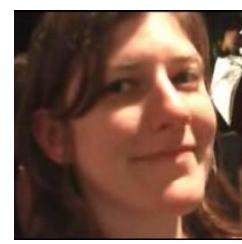
Result

Outline

- Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Tools
 - CenterPiece Subgraphs
 - OddBall (anomaly detection)
- Problem#3: Scalability - PEGASUS
- Conclusions



OddBall: Spotting Anomalies in Weighted Graphs



Leman Akoglu, Mary McGlohon, Christos
Faloutsos

*Carnegie Mellon University
School of Computer Science*

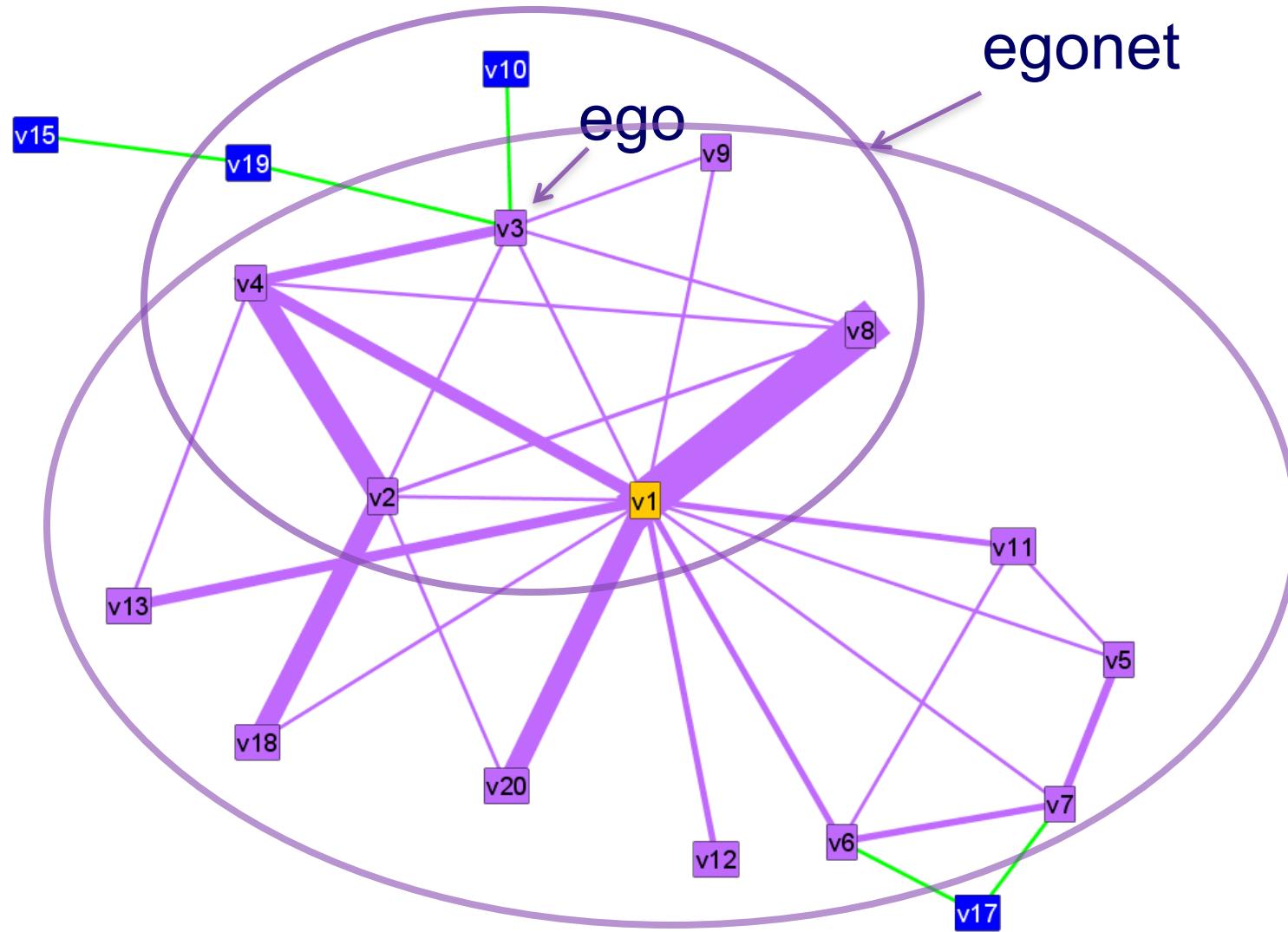
To appear in PAKDD 2010, Hyderabad, India

Main idea

For each node,

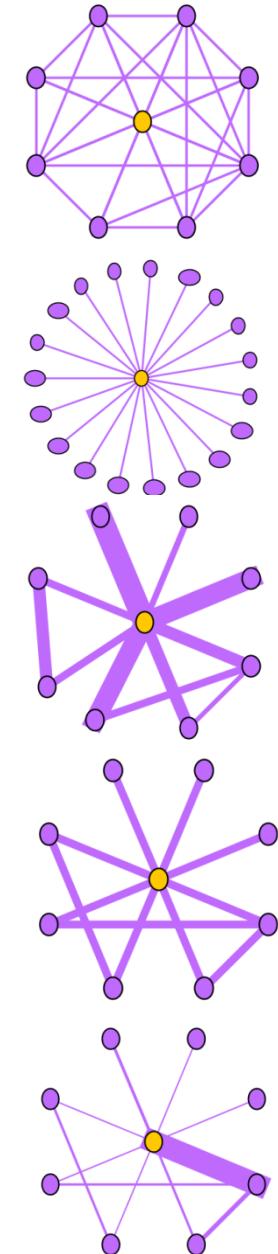
- extract ‘ego-net’ (=1-step-away neighbors)
- Extract features (#edges, total weight, etc etc)
- Compare with the rest of the population

What is an egonet?

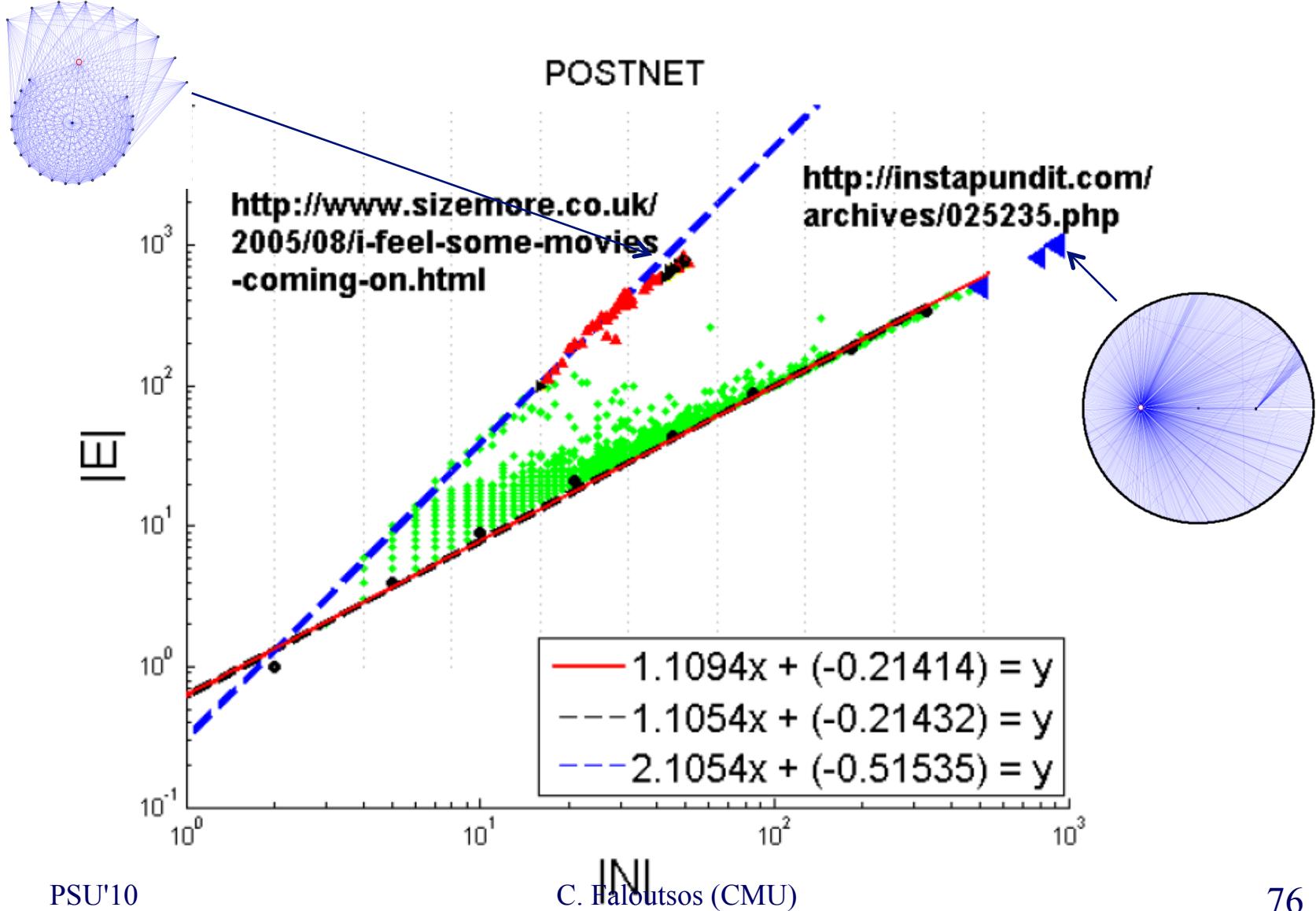


Selected Features

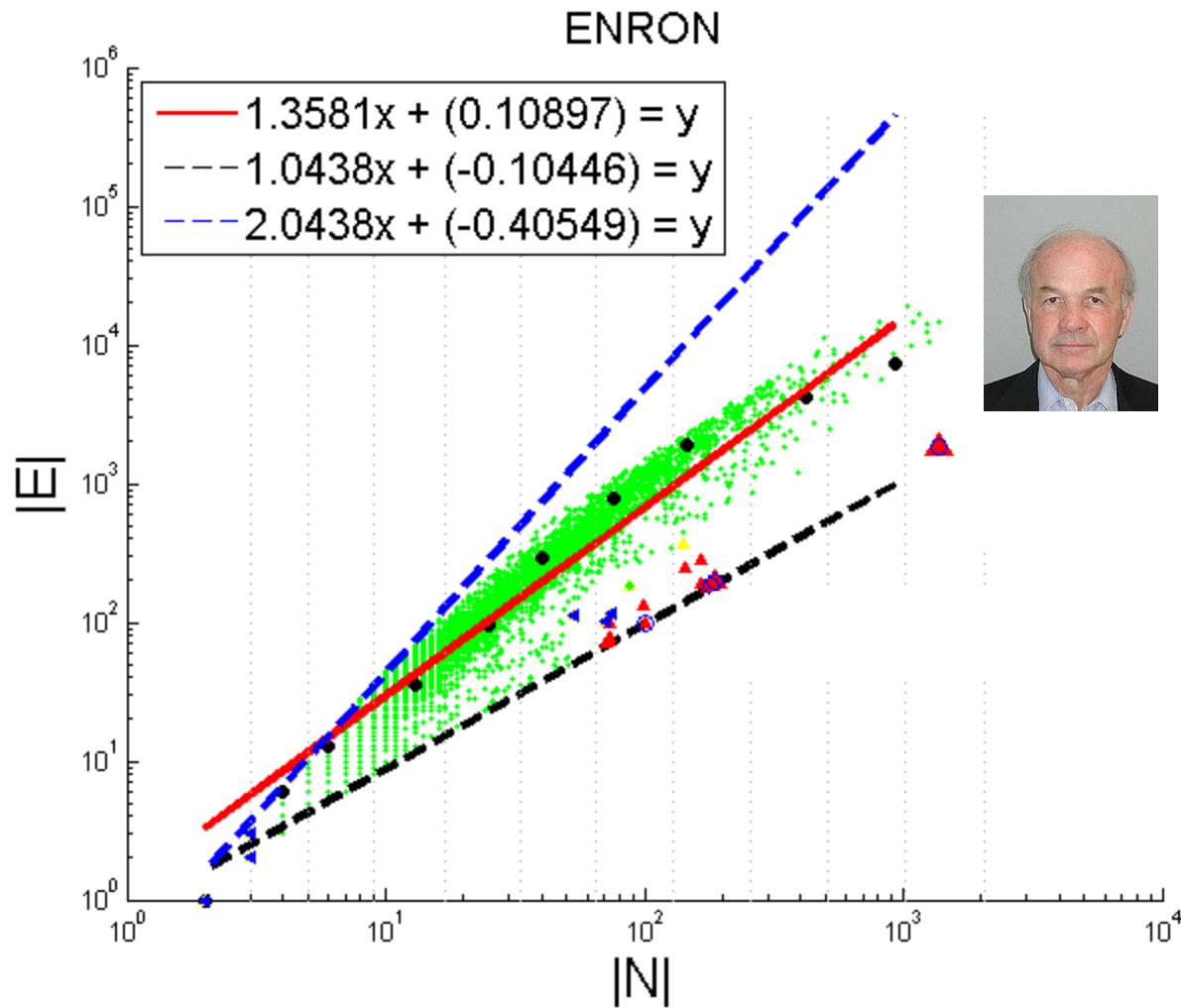
- N_i : number of neighbors (degree) of ego i
- E_i : number of edges in egonet i
- W_i : total weight of egonet i
- $\lambda_{w,i}$: principal eigenvalue of the **weighted** adjacency matrix of egonet I



Near-Clique/Star



Near-Clique/Star



Outline

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Outline – Algorithms & results

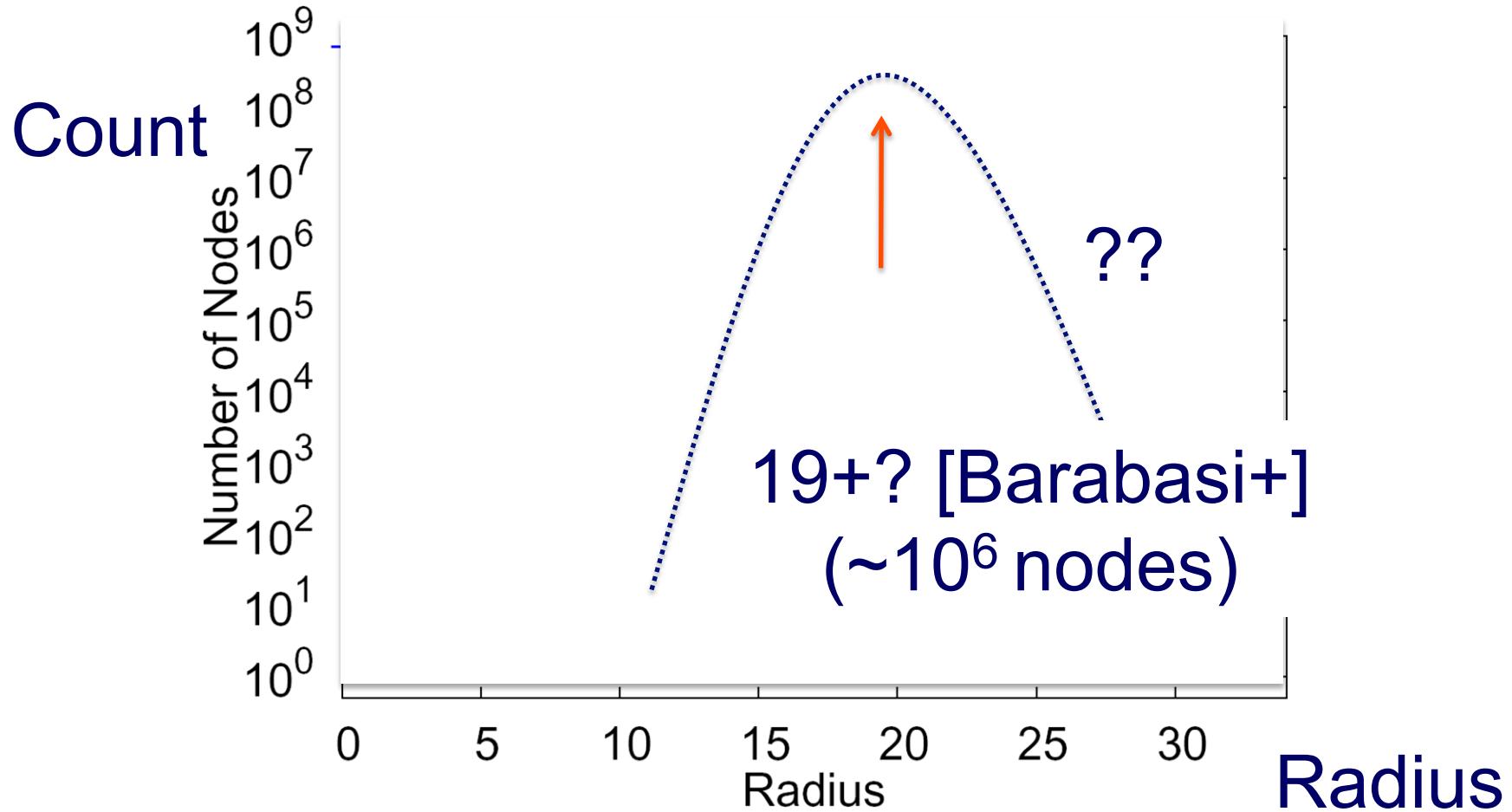


	Centralized	Hadoop /PEGASUS
Degree Distr.	old	old
Pagerank	old	old
Diameter/ANF	old	DONE
Conn. Comp	old	DONE
Triangles	DONE	
Visualization	STARTED	



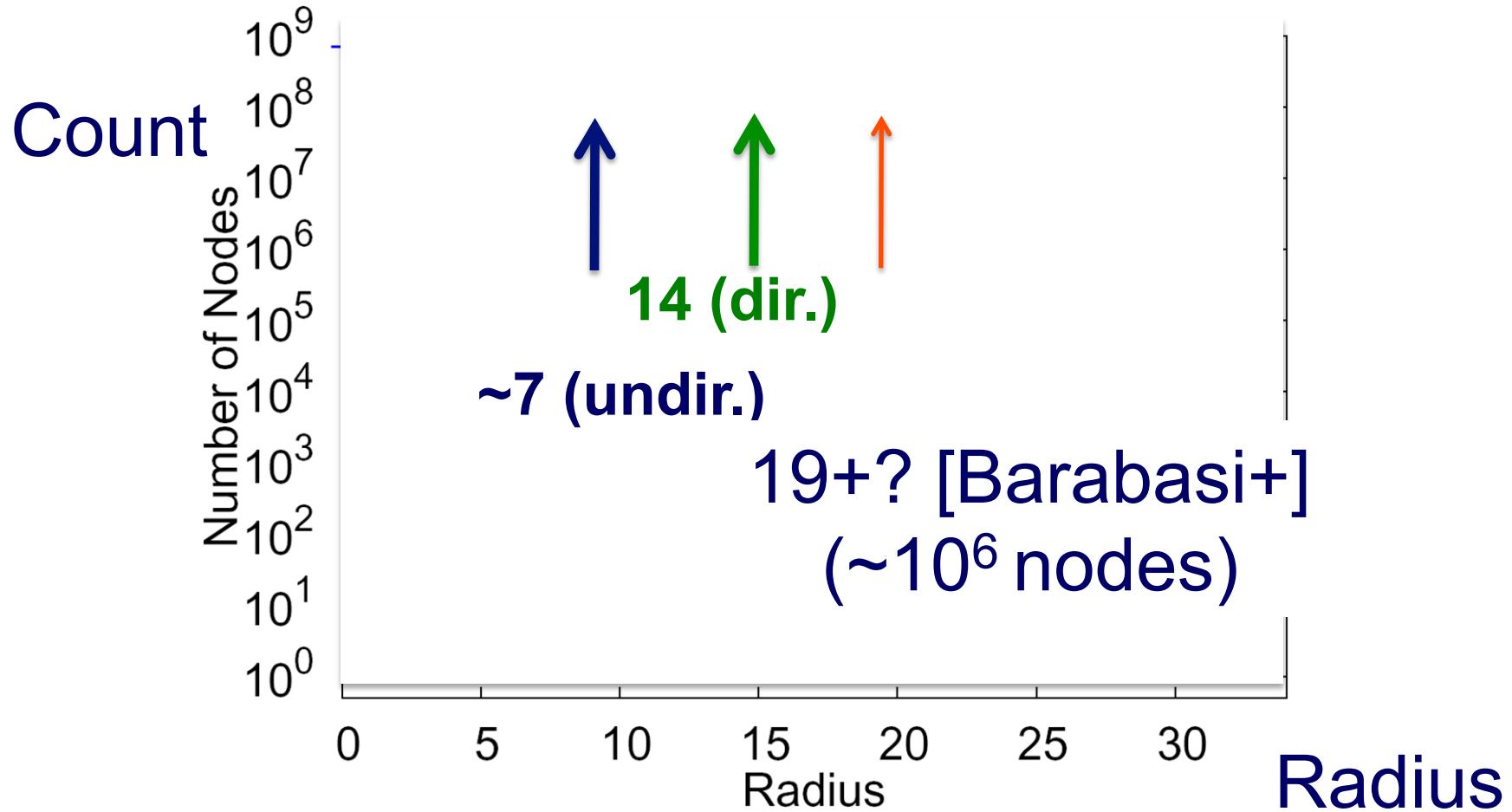
HADI for diameter estimation

- *Radius Plots for Mining Tera-byte Scale Graphs* U Kang, Charalampos Tsourakakis, Ana Paula Appel, Christos Faloutsos, Jure Leskovec, SDM'10
- Naively: diameter needs $O(N^{**2})$ space and up to $O(N^{**3})$ time – **prohibitive** ($N \sim 1B$)
- Our HADI: linear on E ($\sim 10B$)
 - Near-linear scalability wrt # machines
 - Several optimizations -> 5x faster



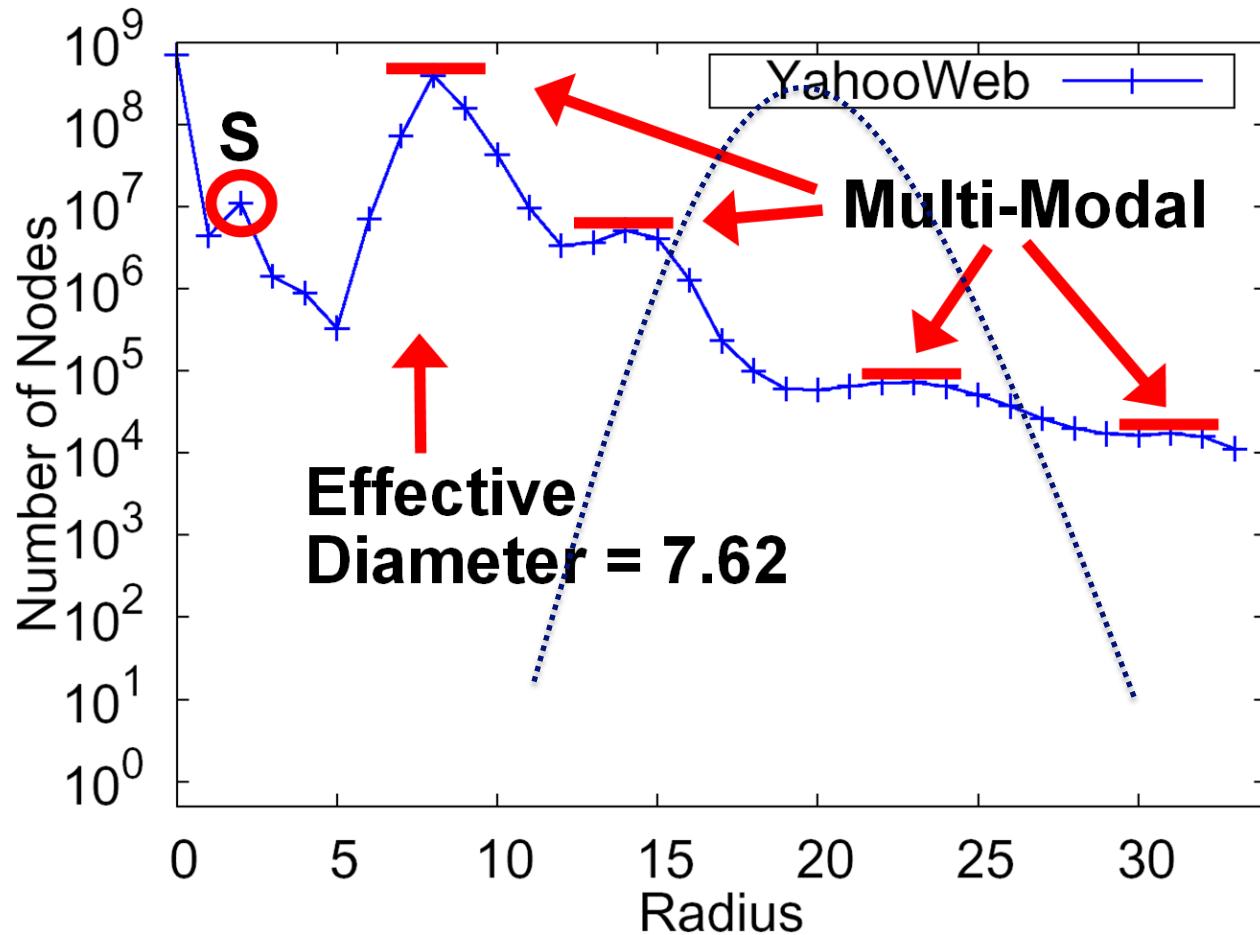
YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- Largest publicly available graph ever studied.



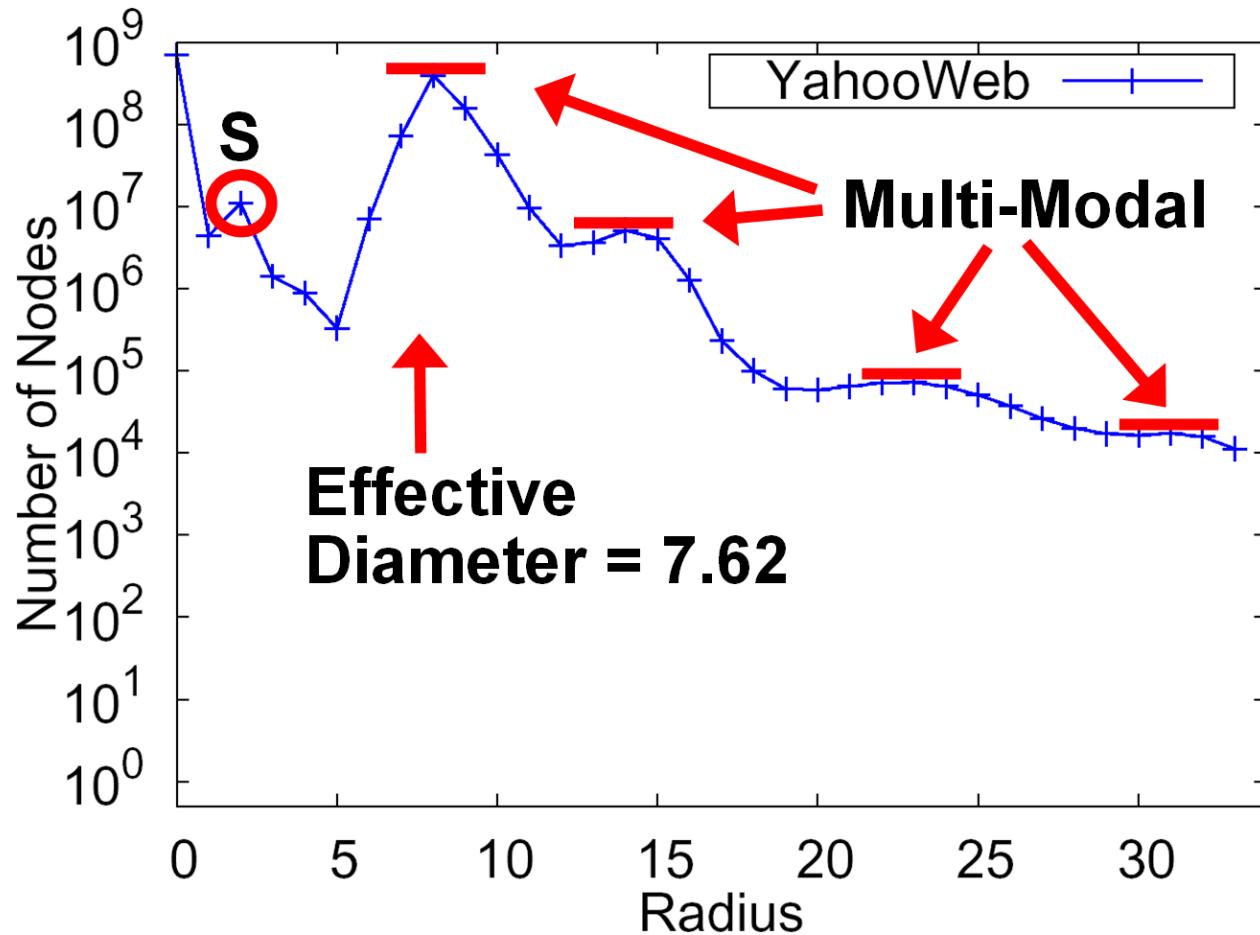
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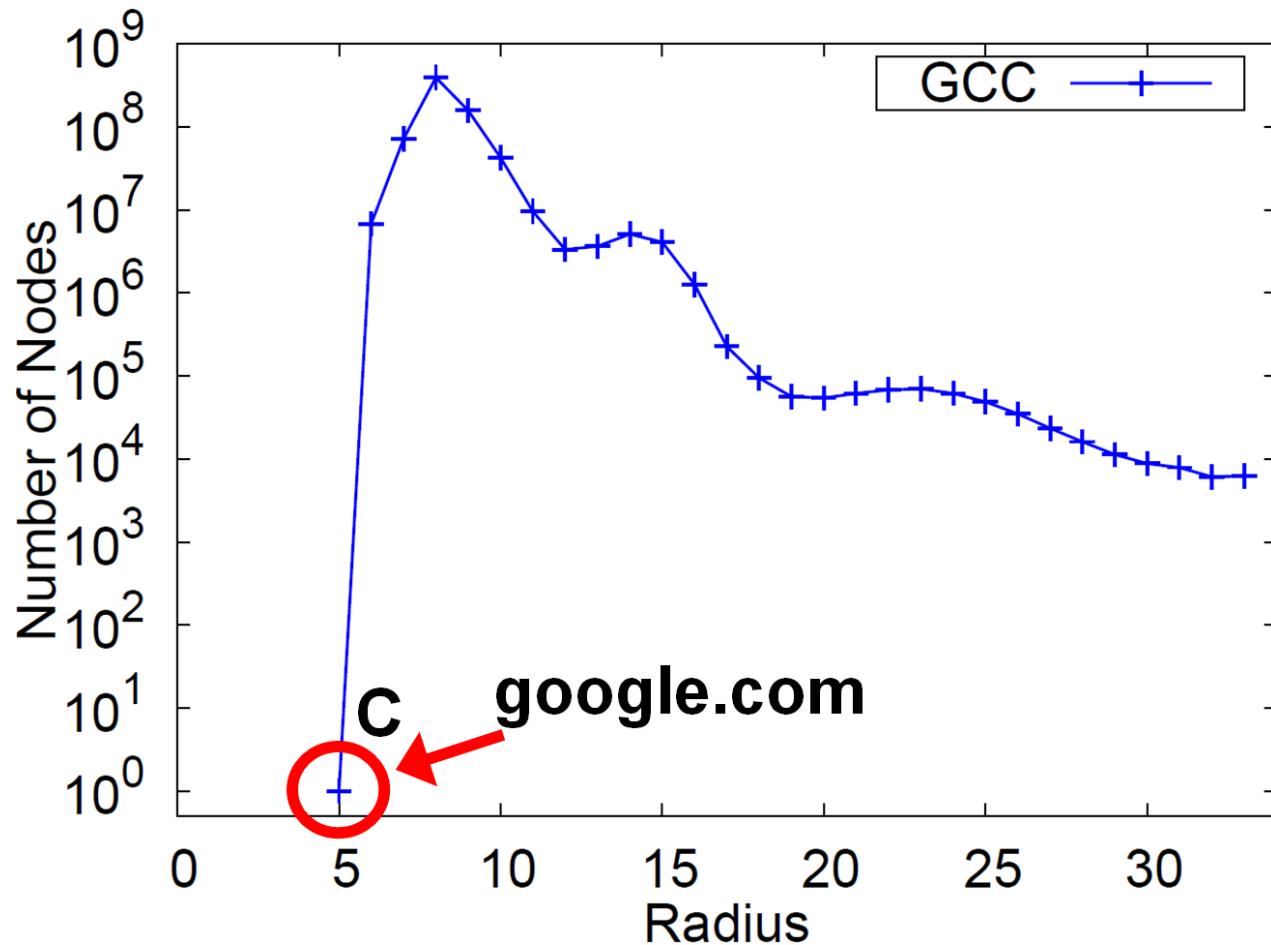
YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- effective diameter: surprisingly small.
- Multi-modality: probably mixture of cores .

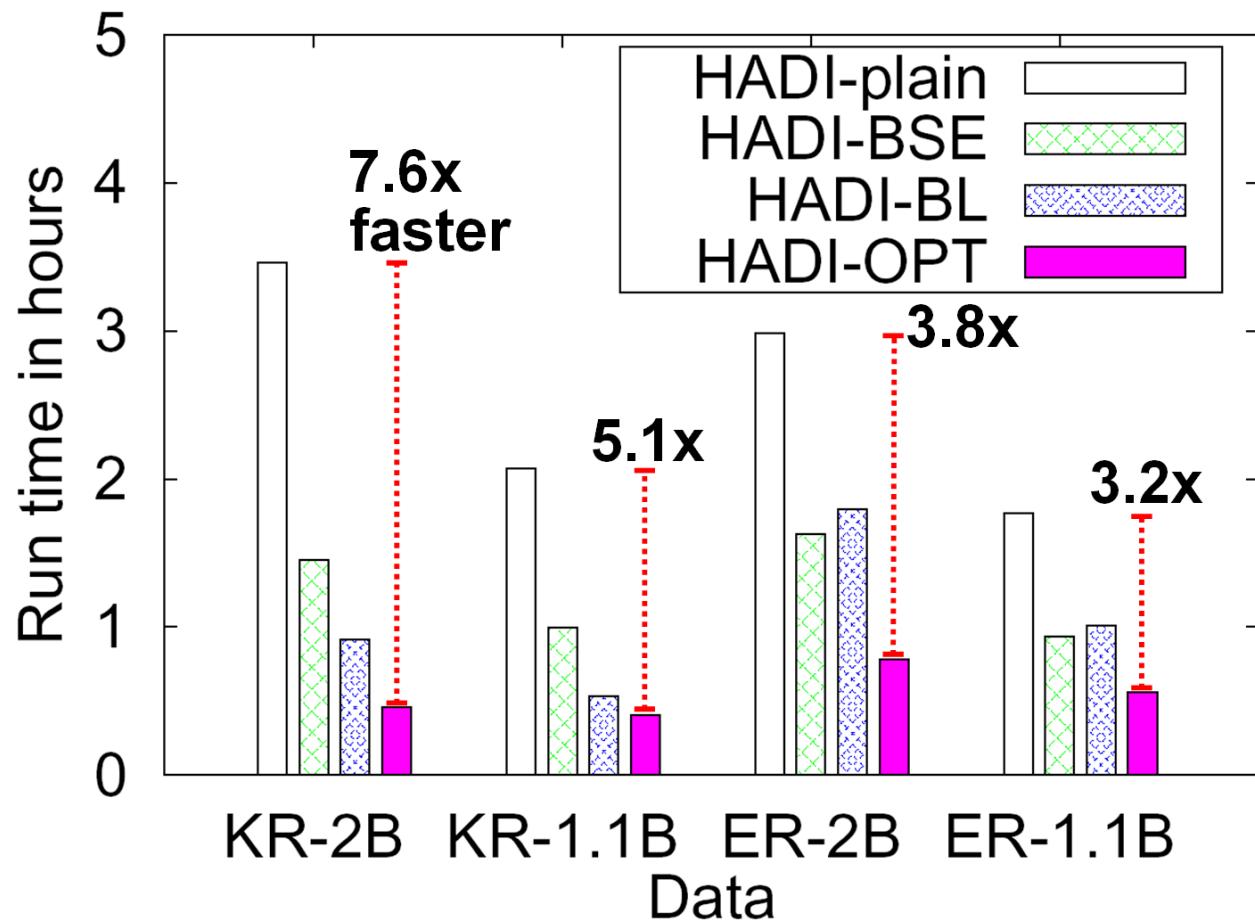


YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- effective diameter: surprisingly small.
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Radius Plot of **GCC** of YahooWeb.



Running time - Kronecker and Erdos-Renyi
Graphs with billions edges.

Outline – Algorithms & results

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Generalized Iterated Matrix Vector Multiplication (GIMV)

*PEGASUS: A Peta-Scale Graph Mining
System - Implementation and Observations.*

U Kang, Charalampos E. Tsourakakis,
and Christos Faloutsos.

([ICDM](#)) 2009, Miami, Florida, USA.
Best Application Paper (runner-up).

Generalized Iterated Matrix Vector Multiplication (GIMV)

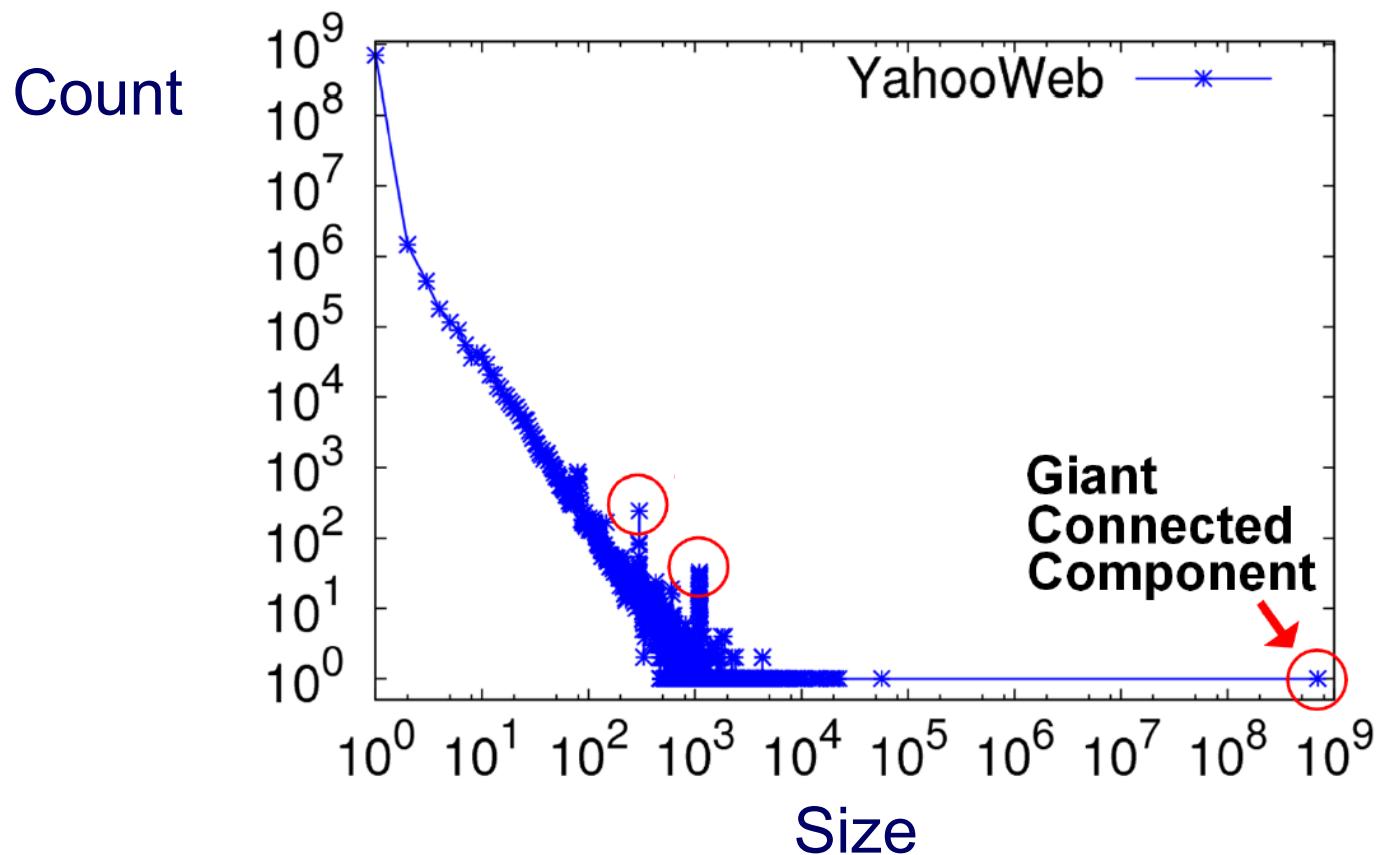
- PageRank
- proximity (RWR)
- Diameter
- Connected components
- (eigenvectors,
- Belief Prop.
- ...)



Matrix – vector
Multiplication
(iterated)

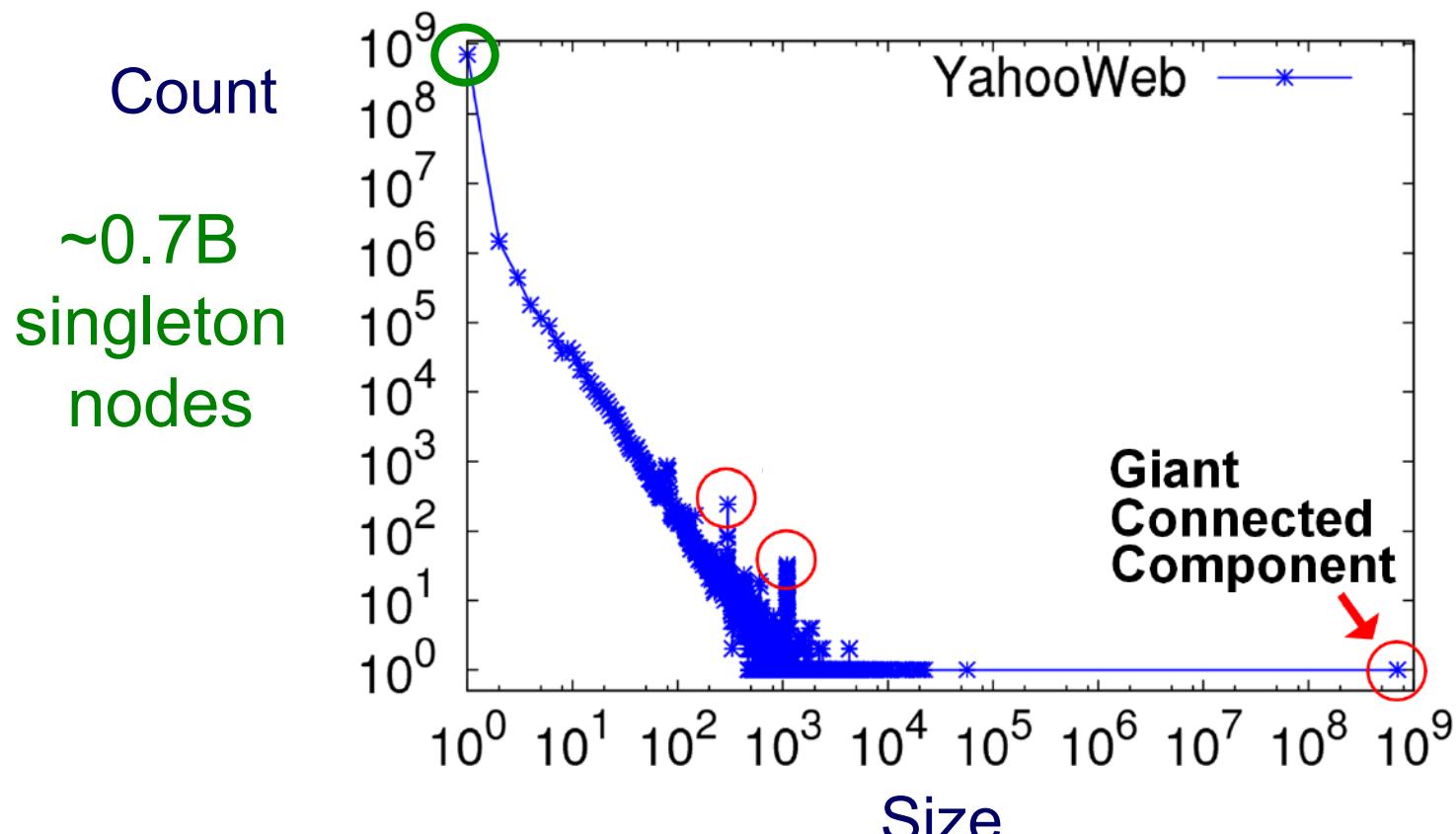
Example: GIM-V At Work

- Connected Components



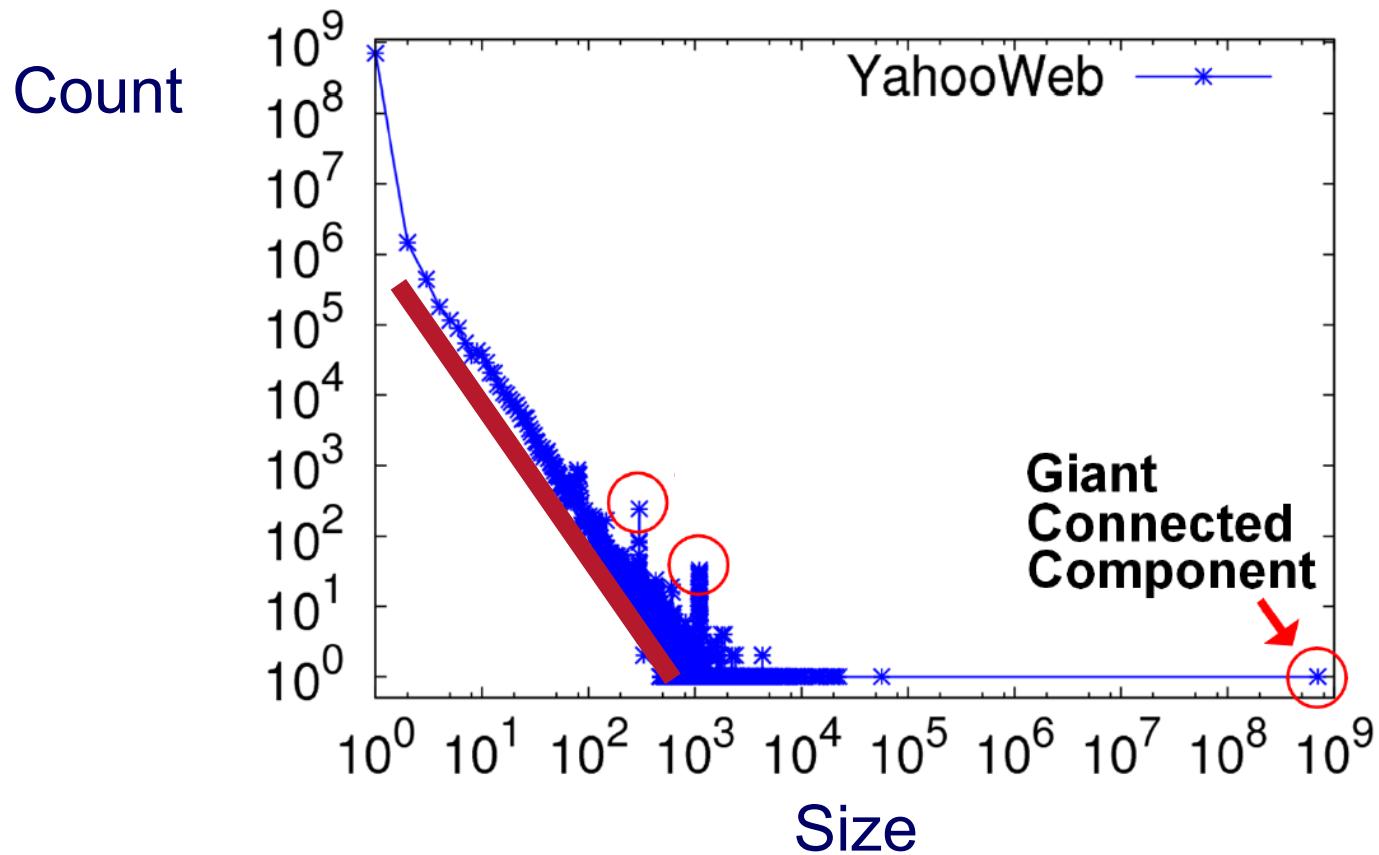
Example: GIM-V At Work

- Connected Components



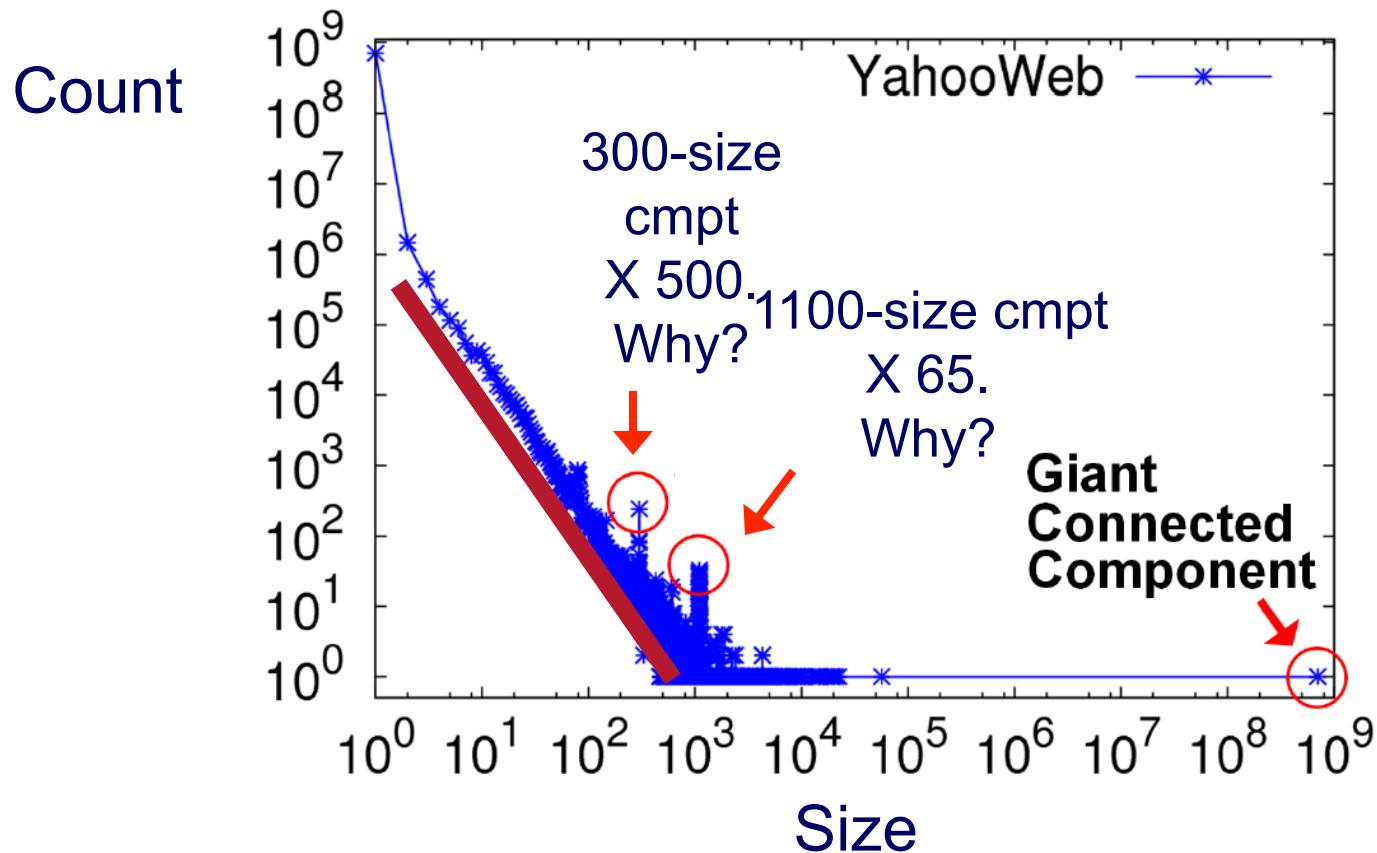
Example: GIM-V At Work

- Connected Components



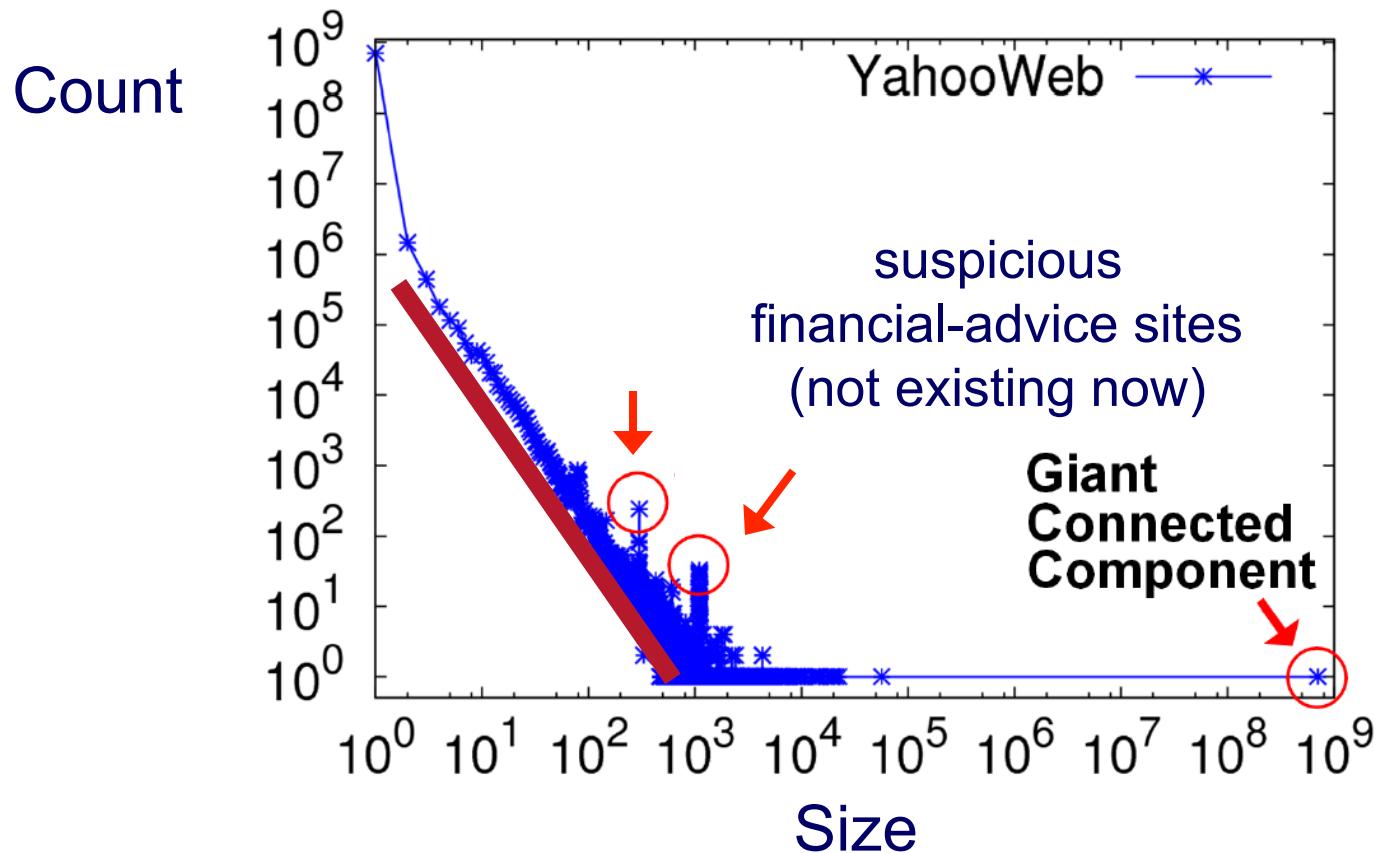
Example: GIM-V At Work

- Connected Components



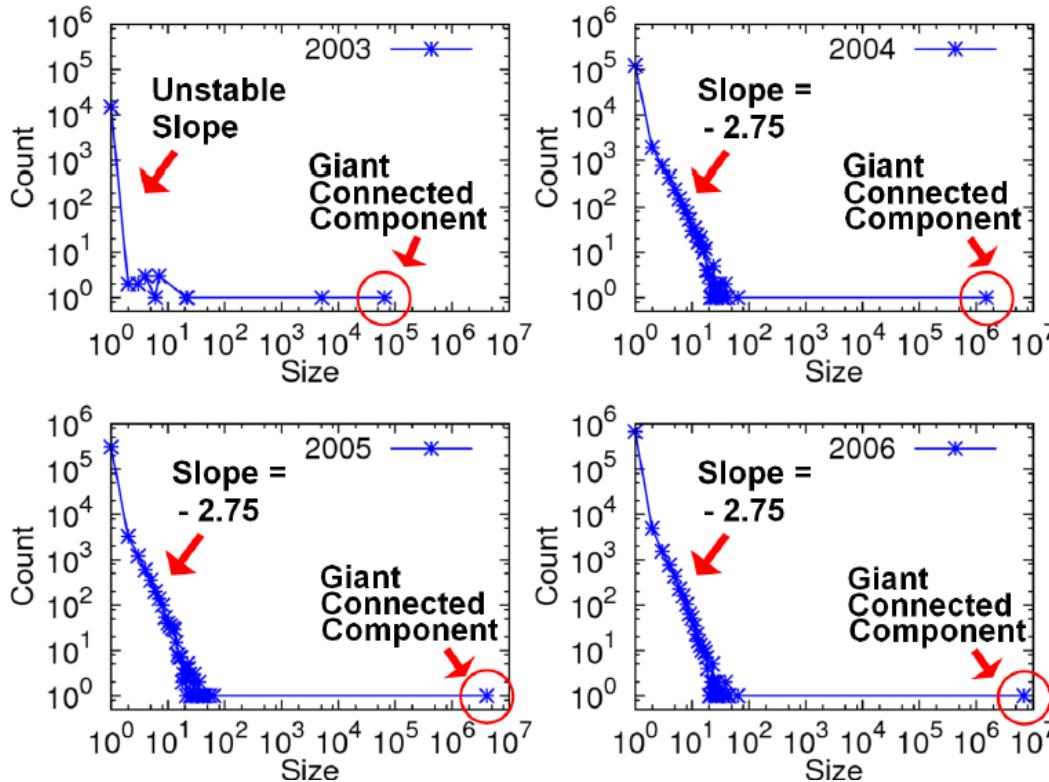
Example: GIM-V At Work

- Connected Components



GIM-V At Work

- Connected Components over Time
- LinkedIn: 7.5M nodes and 58M edges



Stable tail slope
after the gelling point

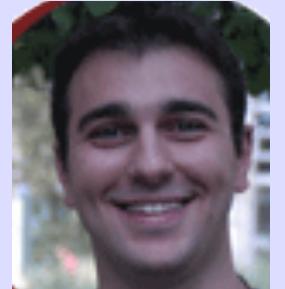
Outline – Algorithms & results

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Visualization	STARTED	



Mentioned already

Triangles : Computations [Tsourakakis ICDM 2008]



But: triangles are expensive to compute
(3-way join; several approx. algos)

Q: Can we do that quickly?

A: Yes!

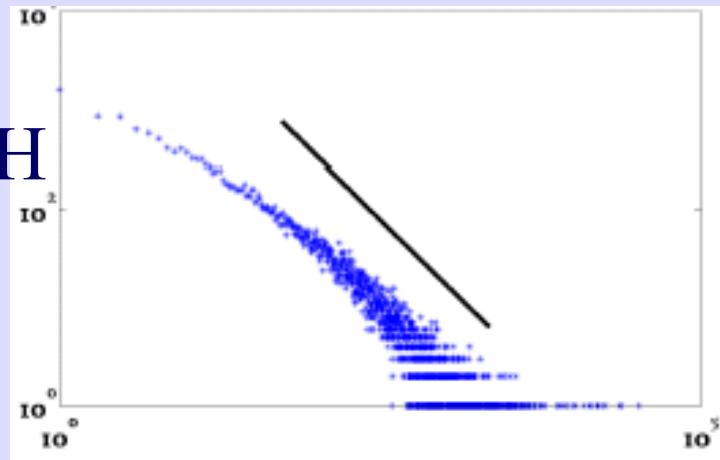
$$\# \text{triangles} = \frac{1}{6} \sum (\lambda_i^3)$$

(and, because of skewness, we only need
the top few eigenvalues!)

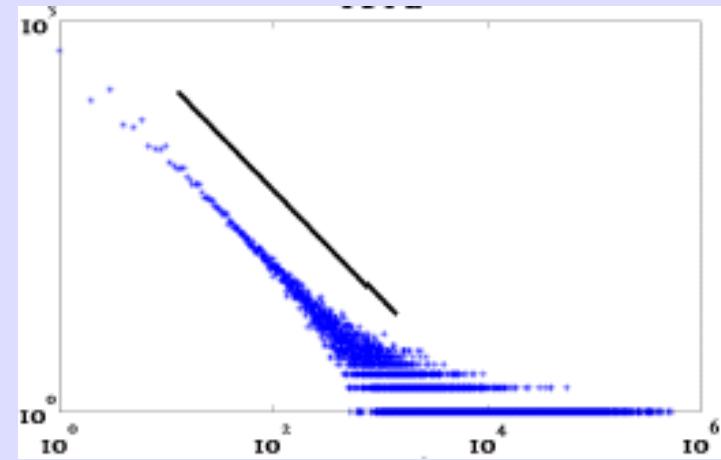
Mentioned already

Triangle Law: #1 [Tsourakakis ICDM 2008]

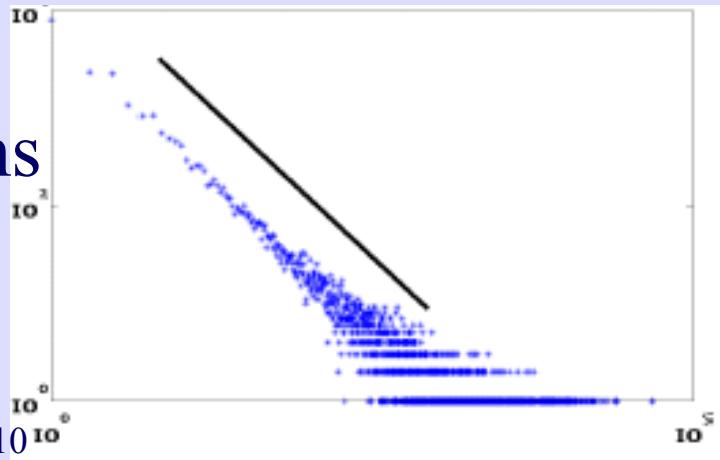
HEP-TH



ASN



Epinions



X-axis: # of Triangles
a node participates in
Y-axis: count of such nodes

PSU'10 vs (CMU)

Outline – Algorithms & results

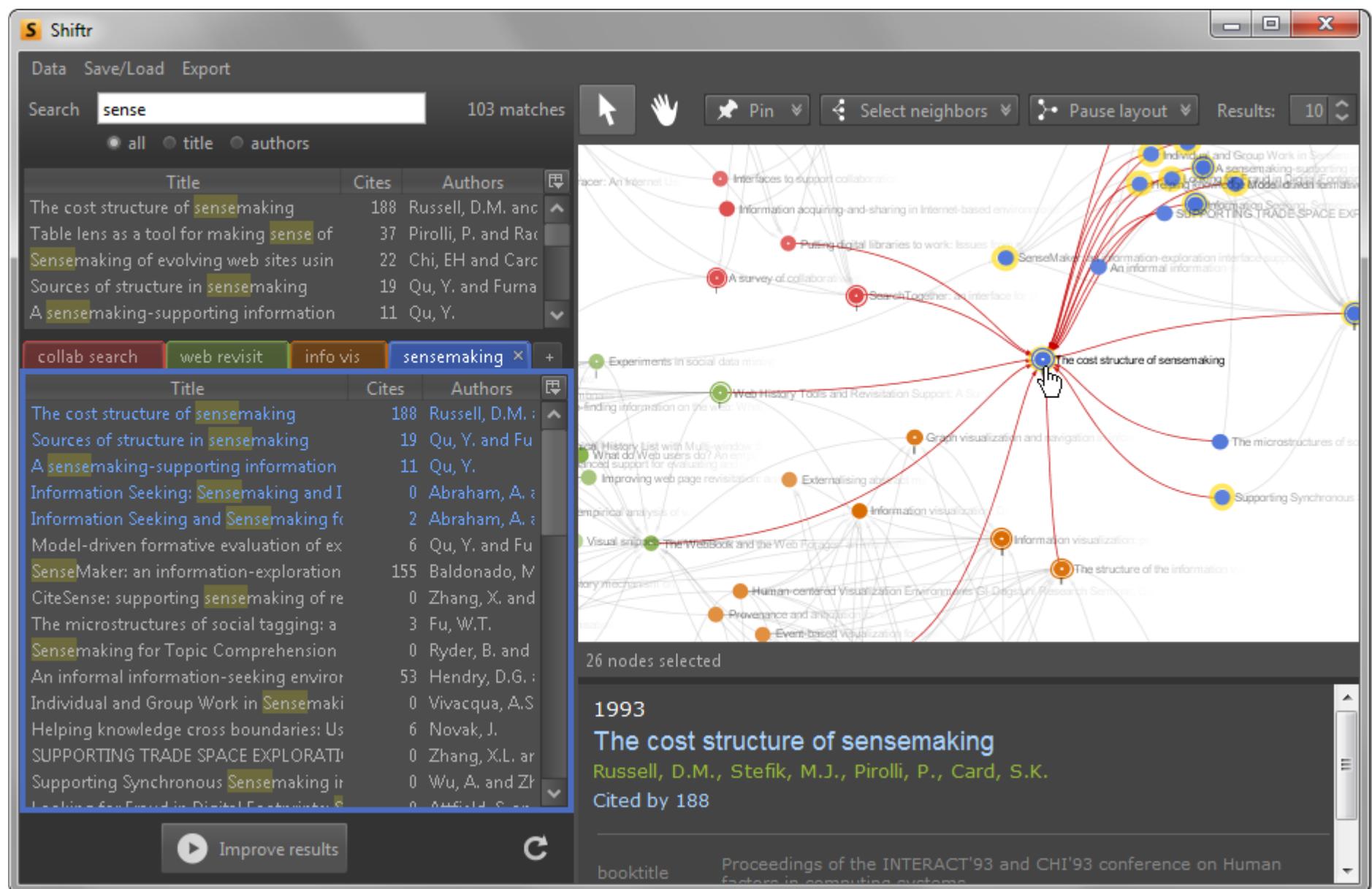
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Visualization: ShiftR

- *Supporting Ad Hoc Sensemaking:
Integrating Cognitive, HCI, and Data
Mining Approaches*
Aniket Kittur, **Duen Horng ('Polo') Chau**,
Christos Faloutsos, Jason I. Hong
Sensemaking Workshop at CHI 2009, April
4-5. Boston, MA, USA.



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- Introduction – Motivation
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- • (additional topics, skipped)
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Other topics - part#1 - tools

- Community detection – how many?
 - Cross-Associations [Chakrabarti +, KDD 2004]
- Time-evolving graphs
 - Tensors [Sun+, KDD'06],
 - [Kolda+ ICDM'05]
 - GraphScope [Sun+, KDD'07]
- Graph compression
 - CUR decomposition [Sun+ SDM'07]

Other topics - part#1 - tools

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 - Cross-Associations [Chakrabarti +, KDD 2004]
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Tensors

- Adjacency matrices, stacked (over time, and/or edge-type – ‘composite networks’)

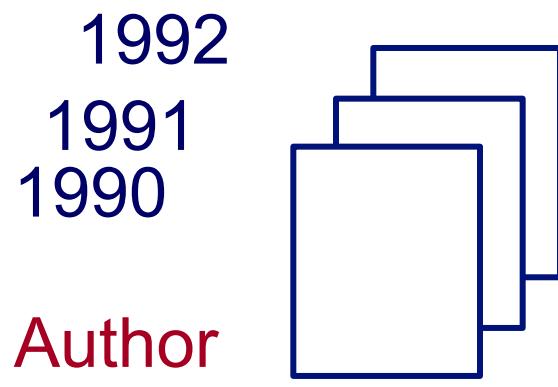
1990

Author



Tensors

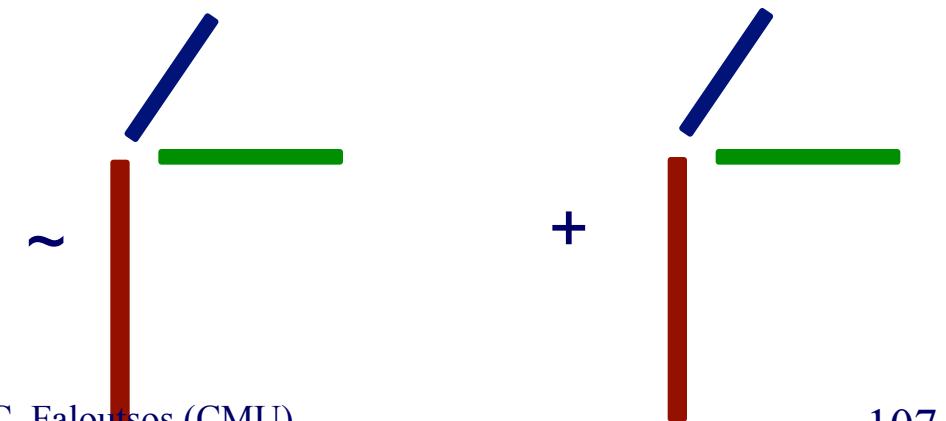
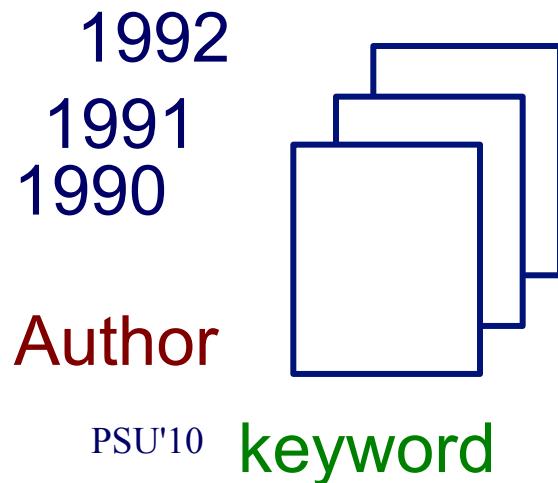
- Adjacency matrices, stacked (over time, and/or edge-type – ‘composite networks’)



Tensors

- Adjacency matrices, stacked (over time, and/or edge-type – ‘composite networks’)

PARAFAC tensor decomposition
(generalization of SVD)

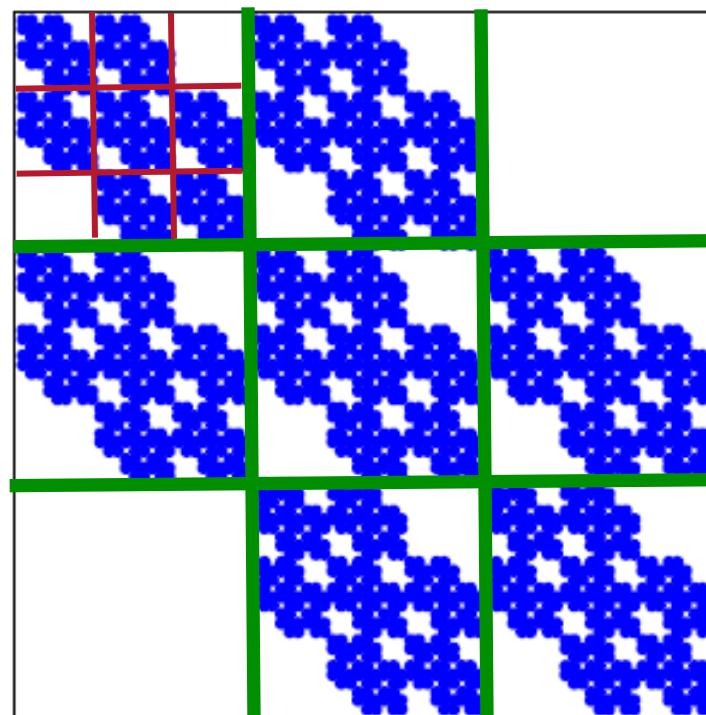


Other topics – part#2 - generators

- Kronecker [PKDD'05];
- Random Typing [Akoglu+, PKDD'09]

Kronecker Product – a Graph

- One of most realistic generators, with **provable** properties



Other topics - part#3 – virus propagation

- Epidemic threshold for SIS: depends **only** on first eigenvalue of adjacency matrix

$$\lambda_1$$

- [Chakrabarti+, TISSEC'07]

Other topics - part#3 – virus propagation

- Ditto for epidemic threshold for
 - SIR (mumps – lifetime immunity)
 - SEIR (incubation)
 - MSEIR (temp. immunity by birth)
 - S I1 I2 R (HIV)
- In all cases, the epid. threshold depends on

$$\lambda_1$$

- [B.A. Prakash ++, 2010]
- <http://arxiv.org/abs/1004.0060>

Other topics - part#3 – virus propagation

- Immunization policies [Tong+, under review]
- Drinking water sensor placement [KDD'07]

More info

Tutorial on graph mining: KDD'09
(w/ Gary Miller and C. Tsourakakis)

www.cs.cmu.edu/~christos/TALKS/09-KDD-tutorial/

Tutorial on tensors: SIGMOD'07
(w/ T. Kolda and J. Sun):

www.cs.cmu.edu/~christos/TALKS/SIGMOD-07-tutorial/

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OVERALL CONCLUSIONS – low level:

- Several new **patterns** (fortification, triangle-laws, conn. components, etc)
- New **tools**:
 - CenterPiece Subgraphs, G-Ray, anomaly detection (OddBall)
- **Scalability**: PEGASUS / hadoop

OVERALL CONCLUSIONS – high level

- Large datasets may reveal patterns/outliers that would be invisible otherwise
- Terrific opportunities
 - Large datasets, easily(*) available PLUS
 - s/w and h/w developments
- Promising collaborations between DB/Sys, AI/Stat, sociology, marketing, epidemiology, ++

References

- Leman Akoglu, Christos Faloutsos: *RTG: A Recursive Realistic Graph Generator Using Random Typing.* ECML/PKDD (1) 2009: 13-28
- Deepayan Chakrabarti, Christos Faloutsos: *Graph mining: Laws, generators, and algorithms.* ACM Comput. Surv. 38(1): (2006)

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- Deepayan Chakrabarti, Yang Wang, Chenxi Wang, Jure Leskovec, Christos Faloutsos: *Epidemic thresholds in real networks*. ACM Trans. Inf. Syst. Secur. 10(4): (2008)
- Deepayan Chakrabarti, Jure Leskovec, Christos Faloutsos, Samuel Madden, Carlos Guestrin, Michalis Faloutsos: *Information Survival Threshold in Sensor and P2P Networks*. INFOCOM 2007: 1316-1324

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Mining large graphs and streams using matrix and tensor tools. Tutorial, SIGMOD Conference 2007: 1174

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- T. G. Kolda and J. Sun. *Scalable Tensor Decompositions for Multi-aspect Data Mining*. In: ICDM 2008, pp. 363-372, December 2008.

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Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations, KDD 2005 (Best Research paper award).
- Jure Leskovec, Deepayan Chakrabarti, Jon M. Kleinberg, Christos Faloutsos: *Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication*. PKDD 2005: 133-145

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- B. Aditya Prakash, Deepayan Chakrabarti, Michalis Faloutsos, Nicholas Valler, Christos Faloutsos: *Got the Flu (or Mumps)? Check the Eigenvalue!* Apr 2010 arXiv:1004.0060v1

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- Jimeng Sun, Yinglian Xie, Hui Zhang, Christos Faloutsos. *Less is More: Compact Matrix Decomposition for Large Sparse Graphs*, SDM, Minneapolis, Minnesota, Apr 2007.
- Jimeng Sun, Spiros Papadimitriou, Philip S. Yu, and Christos Faloutsos, *GraphScope: Parameter-free Mining of Large Time-evolving Graphs* ACM SIGKDD Conference, San Jose, CA, August 2007

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- Jimeng Sun, Dacheng Tao, Christos Faloutsos: *Beyond streams and graphs: dynamic tensor analysis*. KDD 2006: 374-383

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- Hanghang Tong, Christos Faloutsos, and Jia-Yu Pan, *Fast Random Walk with Restart and Its Applications*, ICDM 2006, Hong Kong.
- Hanghang Tong, Christos Faloutsos, *Center-Piece Subgraphs: Problem Definition and Fast Solutions*, KDD 2006, Philadelphia, PA

References

- Hanghang Tong, Christos Faloutsos, Brian Gallagher, Tina Eliassi-Rad: Fast best -effort pattern matching in large attributed graphs. KDD 2007: 737-746

Project info

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Kang, U

Prakash,
Aditya

Tong,
Hanghang

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