Title of Book

Subtitle of Book

Author

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January 17, 2026

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PREFACE

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- You can add bullet points if needed
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Contents

V. MULTIPLICATION

59. 1. **Multiplication** is taking one number as many times as there are units in another; or,

2. Multiplication is a short method of adding numbers that are equal.

60. The number to be taken, is called the *Multiplicand*; the other number, the *Multiplier*; and the result obtained, the *Product*. The Multiplicand and Multiplier are together called *Factors* (makers), because they *make* the Product.

PROBLEM.–How many trees in 3 rows, each containing 42 trees?

SOLUTION.–Since 3 rows contain 3 *times* as many trees as *one* row, take 42 *three* times. This may be done by writing 42 three times, and then *adding*. This gives 126 trees for the whole number of trees.
Instead, however, of writing 42 *three* times, write it *once*; then placing under it the figure 3, the *number of times* it is to be taken, say, 3 times 2 are 6, and 3 times 1 are 12. This process is *Multiplication.*

OPERATION

First row,	42	trees
Second row,	42	trees
Third row,	42	trees
	126	trees

42	trees
3	
126	trees

PRINCIPLES.–1. *The multiplicand may be either concrete or abstract.*

2. *The multiplier must always be an abstract number.*

3. *The product is the same in kind as the multiplicand.*

4. *The product is the same, whichever factor is taken as the multiplier.*

5. *The partial products are the same in kind as the multiplicand.*

6. *The sum of the partial products is equal to the total product.*

MULTIPLICATION TABLE.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

61. Multiplication is divided into two cases:

1. *When the multiplier does not exceed 12.*

2. *When the multiplier exceeds 12.*

CASE I

62. When the multiplier does not exceed 12.

PROBLEM.–At the rate of 53 miles an hour, how far will a railroad car run in four hours?

SOLUTION.–Here say, 4 times 3 (units) are 12 (units); write the 2 in units' place, and carry the 1 (ten); then, 4 times 5 are 20, and 1 carried makes 21 (tens), and the work is complete.

OPERATION

53 miles.
4
212 miles

DEMONSTRATION.– The multiplier being written under the multiplicand for convenience, begin with units, so that if the product should contain tens, they may be carried to the tens; and so on for each successive order.

Since every figure of the multiplicand is multiplied, therefore, the *whole* multiplicand is multiplied.

Rule.–1. *Write the multiplicand, and place the multiplier under it, so that units of the same order shall stand in the same column, and draw a line beneath.*

2. *Begin with units; multiply each figure of the multiplicand by the multiplier, carrying as in Addition.*

PROOF.–Separate the multiplier into any two parts; multiply by these separately. The sum of the products must be equal to the first product.

EXAMPLES FOR PRACTICE

1. 195×3. *Ans.*
2. 3823×4. *Ans.*
3. 8765×5. *Ans.*
4. 98374×6. *Ans.*
5. 64382×7. *Ans.*
6. 58765×8. *Ans.*
7. 837941×9. *Ans.*
8. 645703×10. *Ans..*
9. 407649×11. *Ans.*
10. If 4 men can perform a certain piece of work in 15 days, how long will it require 1 man?

    SOLUTION.–One man must work four times as long as four men.

    $$4 \times 15 \text{ days} = 60 \text{ days}.$$

11. How many pages in a half-dozen books, each containing 336 pages?
12. How far can an ocean steamer travel in a week, at the rate of 245 miles a day?

13. What is the yearly expense of a cotton-mill, if $32053 are paid out every month?

14. A receives from his business an average of $45 a day. He pays three clerks $3; three, $9; and three, $12 a week; other expenses amount to $4 a day; what are his profits for one week?

CASE II

63. When the multiplier exceeds 12.

PROBLEM.–Multiply 246 by 235.

SOLUTION.– first Multiply by 5 (units), and place the first figure of the product, 1230, under the 5 (units). Then multiply by 3 (tens), and place the product, 738, under the 3 (tens). Lastly, multiply by 2 (hundreds), and place the first figure of the product, 492, under the 2 (hundreds). Then add these several products for the entire product.

OPERATION

246		
235		
1230	product by	5
738	product by	30
492	product by	200
57810	product by	235

DEMONSTRATION.–The 0 of the first product, 1230, is *units* (Art. 62). The 8 of the second product, 738, is *tens*, because 3 (tens) times 6 = 6 times 3 (tens) = 18 (tens); giving 8 (tens) to be written in the tens' column. The 2 of the third product, 492, is hundreds, because 2 (hundreds) times 6 = 6 times 2 (hundreds) = 12 (hundreds), giving 2 (hundreds) to be written in the hundreds' column. The right-hand figure of each product being in its proper column, the other figures will fall in their proper columns; and each line being the product of the multiplicand by a *part* of the multiplier, their sum will be the product by all the parts or the *whole* of the multiplier.

Rule.–1. *Write the multiplier under the multiplicand, placing figures of the same order in the same column, and draw a line beneath.*

2. *Multiply each figure of the multiplicand by each figure of the multiplier successively; first by the units' figure, then by the tens' figure, etc.; placing the right-hand figure of each product under that figure of the multiplier which produces it, then draw a line beneath.*

3. *Add the several partial products together; their sum will be the required product.*

METHODS OF PROOF.–1. Multiply the multiplier by the multiplicand; this product must be the same as the first product.

2. The same as when the multiplier does not exceed 12.

NOTE.–For proof by casting out the 9's, see Art. 105.

REMARK.– Although it is customary to use the figures of the multiplier in regular order beginning with units, it will give the same product to use them in any order, observing that *the right-hand figure of each partial product must be placed under the figure of the multiplier which produces it.*

OPERATION

246		
235		
738	product by	30
492	product by	200
1230	product by	5
57810	product by	235

EXAMPLES FOR PRACTICE

1. 7198×216. *Ans.*
2. 8862×189. *Ans.*
3. 7575×7575. *Ans.*
4. 15607×3094. *Ans.*
5. 93186×4455. *Ans.*
6. 135790×24680. *Ans.*
7. 3523725×2583. *Ans.*
8. 4687319×1987. *Ans.*
9. 9264397×9584. *Ans.*
10. 9507340×7071. *Ans.*
11. 1644405×7749. *Ans.*
12. 1389294×8900. *Ans.*
13. 2778588×9867. *Ans.*
14. 204265×562402. *Ans.*

PRACTICAL PROBLEMS

1. In a mile are 63360 inches: how many inches are there in the circumference of the earth at the equator if the distance be 25000 miles?
2. The flow of the Mississippi at Memphis is about 434000 cubic feet a second: required the weight of water passing that point in one day of 86400 seconds, if a cubic foot of water weigh 62 pounds?
3. John Sexton sold 25625 bushels of wheat, at \$1.20 a bushel, and received in payment 320 acres of land, valued at \$50 an acre; 60 head of horses, valued at \$65 a head; 10 town lots, worth \$150 each; and the remainder in money: how much money did he receive?
4. If light comes from the sun to the earth in 495 seconds, what is the distance from the earth to the sun, light moving 192500 miles a second?
5. If 3702754100 cubic feet of solid matter is deposited in the Gulf of Mexico by the Mississippi every year, what is the deposit for 6000 years?
6. The area of Missouri is 65350 square miles: how many acres are there in the State, allowing 640 acres to each square mile?
7. In the United States, at the close of 1878, there were 81841 miles of railroad: if the average cost of building be \$50000 a mile, what has been the total cost of building the railroads in this country?

8. The number of pounds of tobacco produced in this country in 1870 was 260000000. If this were manufactured into plugs one inch wide and six inches long, and four plugs weigh a pound, what would be the length in inches of the entire crop?

BUSINESS TERMS AND EXPLANATIONS

64. A **Bill** is an account of goods sold or delivered, services rendered, or work done. Usually the price or value is annexed to each article, and the date of purchase given.

It is customary to write the total amount off to the right, and not directly under the column of amounts added.

65. A **Receipt** is a written acknowledgment of payment. The common form consists in signing the name after the words "Received Payment" written at the foot of the bill.

1. Joseph Allen bought of Seth Ward, at Springfield, Ill., Jan. 2, 1879, 30 barrels of flour, at $3.60 a barrel; 48 barrels of mess pork, at $16.25 a barrel; 16 boxes of candles, at $3.50 a box; 23 barrels of molasses, at $28.75 a barrel; and 64 sacks of coffee, at $47.50 a sack. Place the purchases in bill form.

SOLUTION.

SPRINGFIED, ILI., *Jan.* 2, 1879.

JOSEPH ALLEN,

1879 Bought of SETH WARD.

Jan	2	To 30 bl. flour,	@ $3.60 a bl.	108	00		
„	2	„ 48 „ mess pork,	„ 16.25 „	780	00		
„	2	„ 16 Boxes candles,	„ 3.50 „ box	780	00		
„	2	„ 23 bl. molasses,	„ 28.75 „ bl.	661	25		
„	2	„ 64 sacks coffee,	„ 47.50 „ sack	3040	00		
						$4645	25

2.

1. At St. Louis, March 1, 1879, Chester Snyder bought of Thomas Glenn, 4 lb. of tea, at 40 ct.; 21 lb. of butter, at 21 ct.; 58 lb. of bacon, at 13 ct.; 16 lb. of lard, at 9 ct.; 30 lb. of cheese, at 12 ct.; 4 lb. of raisins, at 20 ct.; and 9 doz. of eggs, at 15 ct. Place these purchases in the form of a receipted bill.

66. A **Statement of Account** is a written form rendered to a customer, showing his debits and credits as they appear on the books. The following is an example:

CINCINNATI, *Feb.* 2, 1880.

JOHN SNITH,

1880 In Account *with* VAN ANTWERP, BRAGG & CO.

Jan	2	To 525 McGuffey's Revised First Readers, @ 16c.	84			
„	10	„ 50 Ray's New Higher Arithmetics, „ 75c.	37	50		
		Cr.			121	50
„	20	By Cash	20			
„	31	„ Merchandise	12	75	32	75
					$88	75

3. James Wilson & Co. bought of the Alleghany Coal Co., March 2, 1880, five hundred tons of coal, at $2.75 a ton, and sold the same Company during the month, as follows: March 3d, 14 barrels of flour, at $6.55 a barrel; March 10th, 6123 pounds of sugar, at 5 ct. a pound; they also paid them on account, on March 15th, cash, $687.50. Make out a statement of account in behalf of the Alleghany Coal Co. under date of April 1, 1880.

CONTRACTIONS IN MULTIPLICATION

CASE I

67. When the multiplier is a composite number.

A **Composite Number** is the product of two or more whole numbers, each greater than 1, called its *factors*. Thus, 10 is a composite number, whose factors are 2 and 5; and 30 is one whose factors are 2, 3, and 5.

PROBLEM.–At 7 cents a piece, what will 6 melons cost?

ANALYSIS.–Three times 2 times are 6 times. Hence, it is the same to take 2 times 7, and then take this product 3 times, as to take 6 times 7. The same may be shown of any other composite number.

OPERATION

7 cents, cost of 1 melon.
2
14 cents, cost of 2 melons.
3
42 cents, cost of 6 melons.

Rule.–*Separate the multiplier into two or more factors. Multiply first by one of the factors, then this product by another factor, and so on till each factor has been used as a multiplier. The last product will be the result required.*

EXAMPLES FOR PRACTICE

1. At the rate of 37 miles a day, how far will a man walk in 28 days?

2. Sound moves about 1130 feet per second: how far will it move in 54 seconds?

3. If an engine travel at an average speed of 25 miles an hour, how far can it travel in a week, or 168 hours?

CASE II

68. When the multiplier is 1 with ciphers annexed, as 10, 100, 1000, etc.

DEMONSTRATION.–By the principles of Notation (Art. 43), placing *one* cipher on the right of a number, changes the units into tens, the tens into hundreds, and so on, and, therefore, *multiplies the number by* 10.

Annexing *two* ciphers to a number changes the units into hundreds, the tens into thousands, and so on, and, therefore, *multiplies the number by* 100. Annexing *three* ciphers multiplies the number by 1000, etc.

Rule.–*Annex to the multiplicand as many ciphers as there are in the multiplier; the result will be the required product.*

EXAMPLES FOR PRACTICE

(a) Multiply 743 by 10. *Ans.*

(b) Multiply 375 by 100. *Ans.*

(c) Multiply 207 by 1000. *Ans.*

CASE III

69. When ciphers are on the right in one or both factors.

PROBLEM.–Find the product of 5400 by 130.

SOLUTION.–Find the product of 54 by 13, and then annex three ciphers; that is, as many as there are on the right in both the factors.

OPERATION

5400
130
162
54
702000

ANALYSIS.–Since 13 times 54 = 702, it follows that 13 times 54 hundreds (5400) = 702 hundreds (70200); and 130 times 5400 = 10 times 13 times 5400 = 10 times 70200 = 702000.

Rule.–*Multiply as if there were no ciphers on the right in the numbers; then annex to the product as many ciphers as there are on the right in both the factors.*

EXAMPLES FOR PRACTICE

1. 15460 × 3200. *Ans.*

2. 30700 × 5904000. *Ans.*

CASE IV

70. *When the multiplier is a little less or a little greater than 10, 100, 1000, etc.*

PROBLEM.–Multiply 3046 by 997.

ANALYSIS.–Since 997 is equal to 1000 diminished by 3, to multiply by it is the same as to multiply by 1000 (that is, to annex 3 ciphers) and by 3, and take the difference of the products; and the same can be shown in any similar case.

OPERATION

30046
997
3046000
9138
3036862

NOTE.–Where the number is a little *greater* than 10, 100, 1000, etc., the two products must be *added.*

Rule.–*Annex to the multiplicand as many ciphers as there are figures in the multiplier; multiply the multiplicand by the difference between the multiplier and 100, 1000, etc., and add or subtract the smaller result as the multiplier is greater or less than 100, 1000, etc.*

EXAMPLES FOR PRACTICE

1. 7023 × 99. *Ans.*

2. 16642 × 996. *Ans.*

3. 372051 × 1002. *Ans.*

CASE V

71. When one part taken as units, in the multiplier, is a factor of another part so taken.

PROBLEM.–Multiply 387295 by 216324.

SOLUTION.–Commence with the 3 of the multiplier, and obtain the first partial product, 1161885; then multiply this product by 8, which gives the product of the multiplicand by 24 at once (since 8 times 3 times any number make 24 times it). Set the right-hand figure under the right-hand figure 4 of the multiplier in use. Multiply the second partial product by 9, which gives the product of the multiplicand by 216 (since 9 times 24 times a number make 216 times that number). Set the right-hand figure of this partial product under the 6 of the multiplicand; and, finally, add to obtain the total product.

OPERATION

387295
216324
1161885
9295080
83655720
8378123580

Rule.–1. *Multiply the multiplicand by some figure or figures of the multiplier, which are a factor of one or more parts of the multiplier.*

2. *Multiply this partial product by a factor of some other figure or figures of the multiplier, and write the right-hand figure thus obtained under the right-hand figure of the multiplier thus used.*

3. *Continue thus until the entire multiplier is used, and then add the partial products.*

EXAMPLES FOR PRACTICE

(a) 38057×48618. *Ans.*

(b) 267388×14982. *Ans.*

(c) 481063×63721. *Ans.*

(d) 66917×849612. *Ans.*

(e) 102735×273162. *Ans.*

(f) 536712×729981. *Ans.*

Topical Outline.

MULTIPLICATION.

1. Definitions.
2. Terms. . . . . { 1. Multiplicand. 2. Multiplier. 3. Partial Product. 4. Product.
3. Sing.
4. Principles.
5. Operation. . . . . { 1. Writing Numbers. 2. Drawing Line Beneath. 3. Finding Partial Products. 4. Drawing a Line Beneath Partial Products. 5. Adding the Partial Products.
6. Rule.
7. Proof.
8. Applications.
9. Contractions.

MULTIPLICATION.—BILLS AND ACCOUNTS

Art. 65.

(2.)

ST. LOUIS, March 1, 1879.

CHESTER SNYDER,

1879. *Bought of* Thomas Glenn.

			$			
March	1	4 lb. tea @ 40 ct. a lb.,	1	60		
„	1	21 „ butter, @ 21 ct. a lb.	4	41		
„	1	58 „ bacon, „ 13 ct. „	7	54		
„	1	16 „ lard, „ 9 ct. „	1	44		
„	1	30 „ cheese, „ 12 ct. „	3	60		
„	1	4 „ raisins, „ 20 ct. „		80		
„	1	9 doz. eggs, „ 15 ct. a doz.,	1	35		
					$20	74

Received payment,

THOMAS GLENN.

Art. 66.

(3.)

Allegheny, April 1, 1880.

JAMES WILSON & CO.,

1880. *In Acc't with* ALLEGHENY COAL CO.

		DR.			$	
March	2	To 500 tons coal, @ $2.75 a ton,			1375	00
		CR.	$			
„	3	By 14 bbl. flour, @ $6.55 a bbl.,	91	70		
„	10	„ 6123 lb. sugar, @ 8 ct. a lb.,	489	84		
„	15	„ cash on acc't,	687	50	1269	04
		Balance due Allegheny Coal Co., ...			$105	96

CONTRACTIONS IN MULTIPLICATION

Case IV.

Art. 70.

(1.)

7023
99

702300
7023

695277

(2.)

16642
996

16642000
66568

16575432

(3.)

372051000
744102

372795102

Case V.

Art. 71.

(1.)

38057
48618

228342
685026
1826736

1850255226

(2.)

267388
14982

5347776
3743432
26204024

4006007016

(3.)

481063
63721

3367441
10102323
30306969

30653815423

(4.)

66917
849612

803004
6424032
5621028

56853486204

(5.)

102735
273162

308205
2773845
16643070

28063298070

(6.)

536712
729981

4830408
43473672
391263048

391789562472

ARITHMETICAL SIGNS

Art. 86.

(3.)
$$\left.\begin{array}{rl} 21 \div 3 \times 7 & = +49 \\ -1 \times 1 \div 1 \times 4 \div 2 & = -2 \\ 18 \div 3 \times 6 \div 4 & = +9 \\ 1 \times 4 \times 6 \div 8 & = +3 \end{array}\right\} = 59, \ \textit{Ans.}$$

(4.)
$$\left.\begin{array}{rl} 16 \times 4 \div 8 & = +8 \\ -7 + 48 \div 16 & = -4 \\ -3 - 28 \times 0 & = -3 \\ 24 \times 6 \div 48 & = +3 \\ -4 \times 9 \div 12 & = -3 \end{array}\right\} = 1, \ \textit{Ans.}$$

(5.) $\left.\begin{array}{ll} 16 \div 16 \times 96 \div 8 & = +12 \\ -7 - \ 5 + \ 3 & = \ -9 \end{array}\right\} = +3.$

$(27 \div 9) \div \ 3 - 1 \quad = \quad 0.$

$91 \div 13 \times \ 7 - 45 - 3 = \quad 1.$

Then, $3 \times 0 + 1 \times 9 = 9$, Ans.

CONTRACTIONS IN MULTIPLICATION AND DIVISION

Case I.

Art. 88.

(1.)

3)42200

$14066\frac{2}{3}$, *Ans.*

(2.)

656400

5

8)3282000

410250, *Ans.*

(3.)

6)1072400

$178733\frac{2}{6}$, *Ans.*

Case II.

Art. 89.

(1.)

4514020000

451402

3)4513568598

1504522866, *Ans.*

(2.)

281257000000

281257

281256718743

5

9)1406283593715

156253732635, *Ans.*

(3.)

6302240000000

630224000

9)6301609776000

700178864000

4

2800715456000, *Ans.*

Case III.

Art. 90.

(1.) 225) 300521761

4 4

9|00) 12020870|44

Quot. 1335652, *Ans.*

$244 \div 4 = 61$, *Rem.*

(2.) 43750) 1510337264

4 4

175000) 6041349056

4 4

7|00000) 241653|96224

Quot. = 34521, *Ans.*

$696224 \div 4 \div 4 = 43514$, *Rem.*

(3.)
$$\begin{array}{rr} 1406250) & 22500712361 \\ 8 & 8 \\ \hline 11250000) & 180005698888 \\ 8 & 8 \\ \hline 9|0000000) & 144004|5591104 \end{array}$$

Quot. = 16000, *Ans.*

$45591104 \div 8 \div 8 =$ 712361 = *Rem.*

(4.)
$$\begin{array}{rr} 20833\frac{1}{8}) & 620712480 \\ 3 & 3 \\ \hline 62500\ ) & 1862137440 \\ 4 & 4 \\ \hline 250000) & 7448549760 \\ 4 & 4 \\ \hline 1|000000) & 29794|199040 \end{array}$$

Quot. = 29794, *Ans.*

$199040 \div 4 \div 4 \div 3 = 4146\frac{2}{3}$, *Rem.*

(5.)
$$\begin{array}{rr} 2916\frac{2}{3}) & 742851692 \\ 3 & 3 \\ \hline 8750\ ) & 2228555076 \\ 8 & 8 \\ \hline 7|0000\ ) & 1782844|0608 \end{array}$$

Quot. = 254692, *Ans.*

$608 \div 8 \div 3 = 25\frac{1}{3}$, *Rem.*

Miscellaneous Exercises.

(1.) \$6 × 153 = \$918; \$918 ÷ 54 = \$17, *Ans.*

(2.) 217 × 35 + 25 = 7620, *Ans.*

(3.) 48794 ÷ 41 = 119, *Ans.*

(4.) 103 × 103 = 10609, *Ans.*

(5.) 53815 ÷ 375 = 143 and 190 rem.; 144 times 375 = 54000, this is 185 more than 53815, and since the next lower product is 190 less than 53815, 54000 is the nearest. *Ans.* 54000.

(6.) \$2675 ÷ 25 = \$107; \$107 × 19 = \$2033, *Ans.*

(7.) \$210 ÷ 15 = \$14, gain on each; \$75 + \$14 = \$89, *Ans.*

(8.) 391 mi.−139 mi. = 252 mi.; 11 hr. −4 hr. = 7 hr.; 252 mi. ÷ 7 = 36 miles, *Ans.*

(9.) 235 yd. −12 yd. = 223 yd.; \$5 × 235 = \$1175; \$7 × 223 = \$1561; \$1561 − \$1175 = \$386, *Ans.*

(10.) 135 bl. −83 bl. = 52 bl.; \$2 × 52 = \$104, *Ans.*

(11.) \$75 × 5 = \$375; \$68 × 12 = \$816; \$73 × 17 = \$1241; \$375 + \$816 = \$1191; \$1241 − \$1191 = \$50 gain, *Ans.*

Also \$1191 + \$118 = \$1309; \$1309 ÷ 17 = \$77, *Ans.*

(12.) \$240 − \$24 = \$216, whole cost; \$216 ÷ 3 = \$72, the cost of 1 piece; \$72 ÷ \$4 = 18. *Ans.* 18 yards.

(13.) 13 men -8 men $= 5$ men; 1 man can do it in 13 times 15 days, which is 195 days; and 5 men, in $\frac{1}{5}$ of 195 days, or 39 days, *Ans.*

(14.) 14 men $\div$ 7 men $= 21$ men; 1 man can do it in 14 times 24 days, that is, 336 days; and 21 men, in $\frac{1}{21}$ of 336 days, or 16 days, *Ans.*

(15.) For 1 day, the provisions will support 30 times 45 men, or 1350 men; for 50 days, one fiftieth of 1350 men $= 27$ men; 45 men -27 men $= 18$ men, *Ans.*

(16.) \$18 $\times$ 3 $=$ \$54; \$85 $+$ \$54 $=$ \$139, total value; \$139 $-$ \$41 $=$ \$98; 1 sheep cost one fourteenth of \$98, or \$7, *Ans.*

(17.) A sheep and a hog cost \$13; therefore, he will buy as many of each as \$13 is contained times in \$1482; 1482 $\div$ 13 $= 114$, *Ans.*

(18.) 1 horse and 2 oxen cost \$84; therefore, he bought as many horses as \$84 is contained times in \$1260; 1260 $\div$ 84 $= 15$; twice 15 equals 30. *Ans.* 15 horses and 30 oxen.

(19.) One seventh of 1050 ct. $= 150$ ct.; 150 $\div$ 25 $= 6$, of the 25-cent pieces. 1050 ct. -150 ct. $= 900$ ct.; 1 of each of the others would make 10 ct. $+$ 5 ct. $+$ 3 ct. $= 18$ ct.; 900 $\div$ 18 $= 50$, the number of each of the others. *Ans.* Of 25-cent pieces, 6; of the others, 50 each.

(20.) \$6300 $\div$ 140 $=$ \$45, gain per acre: \$210 $-$ \$45 $=$ \$165, the cost; \$5600 $\div$ 140 $=$ \$40, loss per acre; \$165 $-$ \$40 $=$ \$125, sold for per acre. *Ans.* \$165, cost; \$125, sold for.

Art. 134.

Case IV.

(1.) $\$\frac{37}{8} = \$37 \div 8 = \$4\frac{5}{8}$, *Ans.*

(2.) $\frac{137}{4}$ bu. $= 137$ bu. $\div\ 4 = 34\frac{1}{4}$ bu., *Ans.*

(3.) $\frac{785}{60}$ hr. $= 785$ hr. $\div\ 60 = 13\frac{1}{12}$ hr., *Ans.*

(4.) $\frac{1295}{37} = 1295 \div 37 = 35$, *Ans.*

(5.) $\frac{800}{9} = 800 \div 9 = 88\frac{8}{9}$, *Ans.*

(6.) $\frac{1162}{11} = 1162 \div 11 = 105\frac{7}{11}$, *Ans.*

(7.) $\frac{4260}{13} = 4260 \div 13 = 327\frac{9}{13}$, *Ans.*

(8.) $\frac{15780}{31} = 15780 \div 31 = 509\frac{1}{31}$, *Ans.*

Case V.

Art. 135.

(1.) $\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{7} = \frac{1}{7}$, *Ans.*

(2.) $\frac{\cancel{2}}{5} \times \frac{\cancel{4}}{\cancel{7}} \times \frac{\cancel{21}^{3}}{\cancel{8}_{\cancel{4}}} = \frac{8}{5}$, *Ans.*

(3.) $\dfrac{\cancel{4}}{\cancel{5}} \times \dfrac{\cancel{15}^{\,3}}{\cancel{16}^{\,4}} \times \dfrac{\cancel{8}^{\,2}}{\cancel{3}} = \dfrac{2}{1} = 2,$ *Ans.*

(4.) $\dfrac{1}{2} \times \dfrac{\cancel{4}}{\cancel{5}} \times \dfrac{\cancel{15}^{\,3}}{\cancel{4}} = \dfrac{3}{2} = 1\tfrac{1}{2},$ *Ans.*

(5.) $\dfrac{\cancel{3}}{\cancel{4}} \times \dfrac{\cancel{8}^{\,2}}{\cancel{9}^{\,3}} \times \dfrac{\cancel{4}}{\cancel{7}} \times \dfrac{\cancel{35}^{\,5}}{\cancel{4}} = \dfrac{10}{3} = 3\tfrac{1}{3},$ *Ans.*

(6.) $\dfrac{1}{\cancel{3}} \times \dfrac{\cancel{3}}{5} \times \dfrac{\cancel{6}^{\,\cancel{2}}}{\cancel{7}} \times \dfrac{3}{\cancel{4}^{\,\cancel{2}}} \times \dfrac{\cancel{14}^{\,\cancel{2}}}{\cancel{3}} = \tfrac{3}{5},$ *Ans.*

(7.) $\dfrac{\cancel{8}}{\cancel{11}} \times \dfrac{\cancel{3}}{\cancel{7}} \times \dfrac{\cancel{4}}{\cancel{19}} \times \dfrac{\cancel{77}^{\,7}}{\cancel{24}^{\,\cancel{8}}} \times \dfrac{\cancel{57}^{\,3}}{\cancel{8}^{\,2}} = \dfrac{3}{2} = 1\tfrac{1}{2},$ *Ans.*

(8.) $\dfrac{\cancel{12}^{\,\cancel{4}}}{\cancel{13}} \times \dfrac{\cancel{9}}{\cancel{16}^{\,4}} \times \dfrac{\cancel{7}}{\cancel{18}^{\,\cancel{2}}} \times \dfrac{\cancel{10}^{\,\cancel{5}}}{\cancel{21}^{\,\cancel{3}}} \times \dfrac{\cancel{39}^{\,3}}{\cancel{35}^{\,7}} = \tfrac{3}{28},$ *Ans.*

Case VI.

Art. 138.

(1.)

$1 \times 3 \times 5 = 15$
$2 \times 2 \times 5 = 20$
$3 \times 2 \times 3 = 18$
$2 \times 3 \times 5 = 30$ *Den.*

Ans. $\frac{15}{30}, \frac{20}{30}, \frac{18}{30}$.

(2.)

$1 \times 5 \times 6 = 30$
$1 \times 4 \times 6 = 24$
$1 \times 4 \times 5 = 20$
$4 \times 5 \times 6 = 120$ *Den.*

Ans. $\frac{30}{120}, \frac{24}{120}, \frac{20}{120}$.

(3.)

$2 \times 7 \times 8 = 112$
$3 \times 3 \times 8 = 72$
$5 \times 3 \times 7 = 105$
$3 \times 7 \times 8 = 168$ *Den.*

Ans. $\frac{112}{168}, \frac{72}{168}, \frac{105}{168}$.

(4.)

$1 \times 5 \times 6 \times 8 = 240$
$3 \times 2 \times 6 \times 8 = 288$
$5 \times 2 \times 5 \times 8 = 400$
$7 \times 2 \times 5 \times 6 = 420$
$2 \times 5 \times 6 \times 8 = 480$ *Den.*

Ans. $\frac{240}{480}, \frac{288}{480}, \frac{400}{480}, \frac{420}{480}$.

(5.) $\frac{1}{2} \times \frac{7}{2} = \frac{7}{4}$;
$\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$;

$2 \times 4 \times 5 = 40$
$7 \times 3 \times 5 = 105$
$2 \times 3 \times 4 = 24$
$3 \times 5 \times 5 = 60$ *Den.*
Ans. $\frac{40}{60}, \frac{105}{60}, \frac{24}{60}$.

(6.) $\frac{2}{3} \times \frac{6}{7} = \frac{4}{7}$; $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$;

$\frac{1}{2} \times \frac{4}{5} \times \frac{3}{7} \times \frac{21}{8} = \frac{9}{20}$.

$4 \times 3 \times 20 = 240$
$2 \times 7 \times 20 = 280$
$9 \times 3 \times 7 = 189$
$7 \times 3 \times 20 = 420$ *Den.*
Ans. $\frac{240}{420}, \frac{280}{420}, \frac{189}{420}$.

(1.) $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$.
$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$.
Ans. $\frac{4}{8}, \frac{6}{8}, \frac{5}{8}$.

(2.) $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$.
$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$.
Ans. $\frac{8}{12}, \frac{10}{12}, \frac{7}{12}$.

(3.) $\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$.
$\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$.
$\frac{9}{10} \times \frac{2}{2} = \frac{18}{20}$.
Ans. $\frac{15}{20}, \frac{16}{20}, \frac{18}{20}, \frac{11}{20}$.

CASE VII.

Art. 139.

(1.) The L. C. M. of 3, 4, and 6 = 12 :
$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$; $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$; $\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$. *Ans.* $\frac{4}{12}, \frac{9}{12}, \frac{10}{12}$.

(2.) The L. C. M. of 2, 5, 10, and 4 is 20 :
$\frac{1}{2} \times \frac{10}{10} = \frac{10}{20}$; $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$; $\frac{9}{10} \times \frac{2}{2} = \frac{18}{20}$; $\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$. *Ans.* $\frac{10}{20}, \frac{12}{20}, \frac{18}{20}, \frac{15}{20}$.

(3.) The L. C. M. of 7, 8, and 14 is 56 :
$\frac{3}{7} \times \frac{8}{8} = \frac{24}{56}$; $\frac{5}{8} \times \frac{7}{7} = \frac{35}{56}$; $\frac{11}{14} \times \frac{4}{4} = \frac{44}{56}$. *Ans.* $\frac{24}{56}, \frac{35}{56}, \frac{44}{56}$.

(4.) $2)\frac{6}{8} = \frac{3}{4}$; $3)\frac{9}{12} = \frac{3}{4}$; $5)\frac{15}{20} = \frac{3}{4}$. *Ans.* $\frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}$.

(5.) $3)\frac{6}{9} = \frac{2}{3}$; $3)\frac{9}{12} = \frac{3}{4}$; $4)\frac{12}{20} = \frac{3}{5}$;

L. C. M. of 3, 4, 5, and 10, is 60: $\frac{2}{3} \times \frac{20}{20} = \frac{40}{60}$;

$\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$; $\frac{3}{5} \times \frac{12}{12} = \frac{36}{60}$; $\frac{7}{10} \times \frac{6}{6} = \frac{42}{60}$. *Ans.* $\frac{40}{60}, \frac{45}{60}, \frac{36}{60}, \frac{42}{60}$.

(6.) $1\frac{3}{4} = \frac{7}{4}$; $3\frac{2}{3} = \frac{11}{3}$; $\frac{3}{\cancel{10}_{2}} \times \frac{\cancel{25}^{5}}{7} = \frac{15}{14}$;

L. C. M. of 4, 3, and 14, is 84; $\frac{7}{4} \times \frac{21}{21} = \frac{147}{84}$;

$\frac{11}{3} \times \frac{28}{28} = \frac{308}{84}$; $\frac{15}{14} \times \frac{6}{6} = \frac{90}{84}$. *Ans.* $\frac{147}{84}$, $\frac{308}{84}$, $\frac{90}{84}$.

ALLIGATION ALTERNATE

CASE I.

Art. 363.

(1.)

			Balance.			lb.		Balance		lb.		Balance		lb.
	25	3	17		2	19			2	4	6	17			17
	27	1		4		4		17			17		4	2	6
28	30	2			3	3 or,			3		3 or,			1	1
	32	4		1		1				3	3		1		1
	45	17		3		3					1	3			3
						Ans.					*Ans.*				*Ans.*

(2.)

			Balance.			lb.		Balance		lb.
	5	$\frac{7}{4}$		1		1	5			5
	$5\frac{1}{2}$	$\frac{5}{4}$	5			5		1		1
$6\frac{3}{4}$	6	$\frac{3}{4}$			5	5 or,			1	1
	7	$\frac{1}{4}$		7		7		5	3	8
	8	$\frac{5}{4}$	5		3	7	7			7
						Ans.				*Ans.*

(3.)

			Balance.			Gal.		Balance		Gal.
	84	3	9		1	10	7			7
	86	1		7		7		9	1	10
87	88	1			3	3 or,			1	1
	94	7		1		1	3			3
	96	9	3			3		1		1
						Ans.				*Ans.*

OPERATION

	Bulks per lb.	Diff	Bal.
$\frac{25}{421}$	$\frac{2}{21}$	$\frac{3487}{97251}$	723
	$\frac{4}{77}$	$\frac{723}{97251}$	3487

Explanation.—Since silver has sp. gr. $\frac{21}{2}$, *one lb.* silver has $\frac{2}{21}$ the bulk of one lb. water; since gold has sp. gr. $\frac{77}{4}$, *one lb.* gold has $\frac{4}{77}$ the bulk of one lb. water; the combination having sp. gr. $\frac{421}{25}$, *one lb.* of the combination must have $\frac{25}{421}$ the bulk of one lb. water. If we take the *whole* in silver, each pound will have $\frac{1}{\frac{3487}{97251}}$ bulk *too great*; if the whole be taken gold, each pound will have $\frac{723}{97251}$ bulk *too small*; hence, balance in ratio of 723 lb. silver to 3487 lb. gold, *Ans.*

Remark.—If the sp. gr. of a substance be 4, one pound of it will have a bulk equal to $\frac{1}{4}$ the bulk of a pound of water; if sp. gr. be $\frac{21}{2}$, a pound of the substance will have the bulk of $\frac{2}{21}$ of a pound of water. By thus *inverting* the numbers expressing the sp. gravities of different things, their bulks may be directly compared, just as we compare the *prices per pound* in other examples. When we compare *prices*, in a common example, we balance the losses against the gains, *calling* the units in the balancing, "*pounds*," though they may be transferred from a column of prices, named in "*cents.*"

Illustration.—Suppose, where the average price is $\frac{2}{9}$ ct. we find a *loss* of $\frac{714}{189}$ *cents on one kind*, and gain on *another kind* of a lb. $\frac{24}{189}$ *cents*, we take 714 of the latter kind of *pounds*, and 24 *pounds* of the former. The specific gravity came would read: We lose on a pound $\frac{714}{189}$ bulks, and gain on another pound $\frac{24}{189}$ *bulks*; hence balance by 714 of latter to 24 of former in *pounds.*

$\frac{2}{9}$	$\frac{2}{21}$	$\frac{24}{189}$	714
	$\frac{4}{1}$	$\frac{714}{189}$	24

(5.)

			Bal.	lb.
$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{40}$	1
	$\frac{9}{10}$	$\frac{1}{40}$	$\frac{1}{8}$	5

or,

Ans.

(6.)

			Bal.		Parts.
	24	2	2	1	3
22	20	2		1	1
	18	4	1		1

Ans.

CASE II.

Art. 364.

(1.)

			Bal.	
60	50	10	12	$\times 8 = 96$,
	72	12	10	$\times 8 = 80$.

Ans.

(2.)

			Bal.			
65	40	25	2			$2 \times 50 =$
	50	15		2		
	60	5			2	*Ans.*
	75	10	5	3	1	$4 + (5\times$
						$= 254$,

(3.)

			Bal.	
56	0	56	5	$\times\frac{29}{8} = \frac{145}{8}$ pt. $= 2$ gal. 1 qt. $\frac{1}{8}$ pr., *Ans.*
	91	35	8	$\times\frac{29}{8} = 29$ pt.

(4.)

gr.

$$\begin{array}{r|l|l|l|l} & 16 & 2 & 1.8 & \times\frac{81}{1.8} = 3 \\ 18 & & & & \text{pwt. } 9 \text{ gr.} \\ & 21.6 & 3.6 & 1 & \times\frac{81}{1.8} = \end{array}$$

1 pwt. 21 gr., *Ans.*

(5.)

pt.

$$\begin{array}{r|r|r|r|r|r|l} & 0 & \underline{60} & 3 & 3 & \frac{9}{5} & \frac{12}{5} & = \frac{21}{5} \\ 60 & 78 & 18 & & 10 & & 8 & \text{pt.}= \\ & 96 & 36 & 5 & & 3 & & 4\frac{1}{5} \text{ pt.} \end{array}$$

Ans.

(6.)

Part.		%		acidity
$12\frac{1}{2}$ pt.	×	0	=	0
$7\frac{1}{2}$ "	×	100	=	750
20				)750($37\frac{1}{2}$ % obtained.

	15			pt.	
		0	22.5	31	$\times\frac{7.5}{9} = 25\frac{5}{6}$ pt.
desired,	$22\frac{1}{2}$				$= 12\frac{1}{2}$
		100	77.5	9	$13\frac{1}{3}$ pt.

1 gal. 2 qt. $1\frac{1}{3}$ pt., *Ans.*

Explanation.—The combination is to *displace* water equal to *once* its weight, and, hence, 1 is the average. The lead, while in the water, displaces $\frac{1}{11}$ of its own weight; the copper displaces $\frac{1}{9}$ of its own weight; the cork, when wholly in water, displaces 4 times its own weight.

(7.)

OPERATION. (1st.)

	1	$\frac{10}{11}$	3			$\times\frac{352}{27} = \frac{352}{9}$	$\frac{352}{9}$ oz.,
1	$\frac{1}{9}$	$\frac{8}{9}$		3	$\frac{1}{2}$		*Ans.*
	4	3	$\frac{10}{11}$	$\frac{8}{9}$	$\frac{8}{54}$	$\frac{640}{54}$	

$12 - \frac{8}{54} = \frac{640}{54}$; $\frac{640}{54} \div \frac{10}{11} = \frac{352}{27}$.

Hence, the piece, say 1 oz., of lead *lacks* displacing $\frac{10}{11}$ of an oz.; 1 oz. of copper lacks displacing $\frac{8}{9}$; a cork oz. displaces *too much*, by 3 times its weight; hence, balancing, we take 3 oz. of lead for each $\frac{10}{11}$ oz. of cork, and 3 oz. of copper for each $\frac{8}{9}$ oz. of cork. But the conditions require only $\frac{1}{6}$ of 3 oz. of copper; hence, to balance that requires $\frac{1}{5}$ of $\frac{8}{9}$, or $\frac{8}{54}$ oz. cork. The conditions also require 12 oz. cork *in all*; therefore, $12 - \frac{8}{54}$, or $\frac{320}{27}$ oz. cork are yet required, and as this contains $\frac{10}{11}$, $\frac{352}{27}$ times, there must be, by first balancing, $\frac{352}{27} \times 3$, or $\frac{352}{9}$ oz. lead, which is 2 lb. $7\frac{1}{9}$ oz., *Ans.*

Explanation.—First, balancing in proportion to bulks, 40 of cork to 3 of lead, 32 of cork to 3 of copper. This makes their actual weights, as $40 \times \frac{1}{4}$ to 11×3, and $32 \times \frac{1}{4}$ to 9×3. Then the $\frac{1}{2}$ oz. copper requiring $\frac{8}{54}$ oz. cork, the remaining $\frac{640}{54}$ oz. cork require 2 lb. $7\frac{1}{9}$ oz. lead, *Ans.*

OPERATION. (2d.)

			proper bulks.			proper wrights.	
	$\frac{1}{4}$	$\frac{3}{4}$	40	32		10	8
1	9	8		3	=		27
	11	10	3			33	

$\frac{8}{54} + \frac{640}{54}$ $\frac{1}{2}$ $\frac{352}{9}$ oz.,

(8.)

120 ×	74	=	8880
150 ×	68	=	10200
130 ×	54	=	7020
400)			26100($65\frac{1}{4}$%

Since 60 is 120% of that desired, 60 ÷ 120, or 50% is the required average

	$65\frac{1}{4}$	$15\frac{1}{4}$	$10 \times 40 = 400$
50			
	40	10	$15\frac{1}{4} \times 40 = 610$

610 shares, *Ans.*

(9.)

		$				bbl.			
400	× 7.50	= 3000			7.06	.56	100	×20 = 2000
640	× 7.25	= 4640	req. av.	6.50				
960	× 6.75	= 6480			5.50	1	56	×20 = 1120
2000		)14120	(7.06					bbl., *Ans.*

average now.

Case III.

Art. 365.

(1.)

The given lbs. and prices make an average of $5\frac{1}{7}$ ct. Then,

6	3	3	1			$\frac{3}{4}$ lb., *Ans.*
	$5\frac{1}{7}$	$\frac{6}{7}$		7		
	7	1	3	6	$6 + 2\frac{1}{4} = 8\ \frac{1}{4}$ lb., *Ans.*	

The balancing requires 1 of the first for 3 of the third; and 7 of the second for 6 of the third. This gives the required 7; but as there are $16 - 13$, or 3 yet required, and as these must be taken in proportion, *as one* of the first to *three* of the third, we take $\frac{3}{4}$ of the first balance column and add it to the second; having $\frac{3}{4} + 7 + 8\frac{1}{4} = 16$.

(2.)

bbl.	pr.		$	
300	× 7.50	=	2250	
800	× 7.80	=	6240	
400	× 7.65	=	3060	
1500			)11550	($7.70, av. for 1500 bbl.

As there are to be 2000 bbl., there are yet 500 bbl. required, and their price must be $(2000 × 7.85 − 11550) ÷ 500 = $8.30, the av. Hence,

8.30	8	.30	2	× 100 =	200 bbl.	*Ans.*
	8.50	.20	3		300 bbl.	

(3.)

lb.		$		
14	× .30 =	4.20		
6	× .60 =	3.60		
40)		17.80	(44 $\frac{1}{2}$ ct., av.	

$28\frac{3}{4}$	25	$3\frac{3}{4}$	5	× 2 =	10 lb.	*Ans.*
	35	$6\frac{1}{4}$	3		6 lb.	

56 lb. − 40 lb. = 26 lb. desired.
$(56 × .40 − 17.80) ÷ 16 = 28\frac{3}{4}$ ct. av. for 16 lb.

(4.)

If the specific gravity of a body be $\frac{7}{1}$, it loses $\frac{1}{7}$ in water; so, copper loses $\frac{4}{31}$, and silver $\frac{2}{21}$, while the required loss of combination weight is $\frac{5}{43}$; hence,

$$\frac{5}{43}\left|\begin{array}{c|c|c}\frac{4}{31} & \frac{357}{27993} & 589\\ \frac{2}{21} & \frac{589}{27993} & 357\end{array}\right| \times \frac{12}{946} = \left\{\begin{array}{l} 7\frac{223}{473}\text{ oz.}\\ 4\frac{250}{473}\text{ oz.}\end{array}\right\} \textit{Ans.}$$

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(5.)

1 st. gr.

	15	3	6	2	64	2 pwt. 16 gr., *Ans.*
18	20	2		3	24	1 pwt., *Ans.*
	24	6		3	24	1 " *Ans.*
			9	5		$112 \div 14 = 8$.

Taking 3's and 5's to make 112 gr., we proceed thus to find other answers.

2d.	2	2	72	68	64	60	56	52	48
		3	6	15	24	33	42	51	60
	1	3	34	29	24	19	14	9	4
	3	5							

(6.)

	9	5	3	2	29	30	31	32	33	34	35	36	37	calves,
4	2	2		5	68	60	52	44	36	28	20	12	4	hogs,
	1	3	5		3	10	17	24	31	38	45	52	59	lambs.
			8	7										

Take 5ths of 8 and 5ths of 7 to make 100; *or whole* 8's and *whole* 7's to make 500.

(7.)

If a body have a specific gravity of 2, in water it displaces $\frac{1}{2}$ its own weight; if its sp. gr. be $\frac{4}{3}$, it displaces in like manner $\frac{3}{4}$; so the crown, sp. gr. $\frac{117}{8}$, displaced $\frac{8}{117}$, and thus with the two metals. Hence, the question is,—*If gold displace, in water* $\frac{4}{77}$ *of its own weight, and silver* $\frac{2}{21}$ *of its own weight, how should these be combined so as to displace* $\frac{8}{117}$ *of their weight?* The above balancing shows their actual weights should combine as 74 to 121; i.e., the gold should weigh $\frac{121}{195}$ of the *combined* weights. The whole weight being $17\frac{1}{2}$ lb., the weight of the gold must be $17\frac{1}{2}$ lb. $\times\frac{121}{195} = 10\frac{67}{78}$ lb., *Ans.*

$$\frac{8}{117}\left|\begin{array}{c|c|c}\frac{2}{21} & \frac{726}{27027} & 74\\ \frac{4}{77} & \frac{444}{27027} & 121\end{array}\right| \times \frac{17.5}{195} = \left\{\begin{array}{l} 6\frac{25}{39}\textit{ silver.}\\ 10\frac{67}{78}\textit{ gold.}\end{array}\right\} \textit{Ans.}$$

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INVOLUTION

Art. 370.

(1.) $(5)^2 = 5 \times 5 = 25$, *Ans.*

(2.) $(14)^3 = 14 \times 14 \times 14 = 2744$, *Ans.*

(3.) $(6)^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776$, *Ans.*

(4.) $(192)^2 = 192 \times 192 = 36864$, *Ans.*

(5.) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$, *Ans.*

(6.) $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{81}{625}$, *Ans.*

(7.) $\left(2\frac{1}{4}\right)^3 = \frac{9}{4} \times \frac{9}{4} \times \frac{9}{4} = \frac{729}{64} = 11\frac{25}{64}$, *Ans.*

(8.) $\left(\frac{7}{8}\right)^5 = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = \frac{16807}{32768}$, *Ans.*

(9.) $(.02)^3 = .02 \times .02 \times .02 = .000008$, *Ans.*

(10.) $(5)^4 = 5 \times 5 \times 5 \times 5 = 625$; $\therefore$ $(5^4)^2$, or $5^8 = 625 \times 625 = 390625$, *Ans.*

(11.) $(.046)^3 = .046 \times .046 \times .046 = .000097336$, *Ans.*

(12.) $\left(\frac{1}{9}\right)^7 = \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} = \frac{1}{4782969}$, *Ans.*

(13.) $(2056)^2 = 2056 \times 2056 = 4227136$, *Ans.*

(14.) $(7.62\frac{1}{2})^2 = 7.62\frac{1}{2} \times 7.62\frac{1}{2} = 58.1406\frac{1}{4}$, *Ans.*

Art. 371.

(1.) $19^2 = (10 + 9)^2 = 100 + 2(10 \times 9) + 81 = 361$, *Ans.*

(2.) $29^2 = (20 + 9)^2 = 400 + 2(20 \times 9) + 81 = 841$, *Ans.*

(3.) $4^2 = (1 + 3)^2 = 1 + 2(1 \times 3) + 9 = 16$, *Ans.*

(4.) $40^2 = (30 + 10)^2 = 900 + 2(30 \times 10) + 100 = 1600$, *Ans.*

(5.) $125^2 = (100 + 25)^2 = 10000 + 2(100 \times 25) + 625 = 15625$, *Ans.*

(6.) $59^2 = (50 + 9)^2 = 2500 + 2(50 \times 9) + 81 = 3481$, *Ans.*

Art. 372.

(1.) $19^3 = (10 + 9)^3 = 1000 + 3(100 \times 9) + 3(10 \times 81) + 729 = 6859$, *Ans.*

(2.) $29^3 = (20 + 9)^3 = 8000 + 3(400 \times 9) + 3(20 \times 81) + 729 = 24389$, *Ans.*

(3.) $4^3 = (1 + 3)^3 = 1 + 3(1 \times 3) + 3(1 \times 9) + 27 = 64$, *Ans.*

(4.) $40^3 = (20 + 20)^3 = 8000 + 3(400 \times 20) + 3(20 \times 400) + 8000 = 64000$, *Ans.*

(5.) $125^3 = (120 + 5)^3 = 1728000 + 3(14400 \times 5) + 3(120 \times 25) + 125 = 1953125$, *Ans.*

(6.) $216^3 = (200 + 16)^3 = 8000000 + 3(40000 \times 16) + 3(200 \times 256) + 4096 = 10077696$, *Ans.*

EVOLUTION

EXTRACTION OF THE SQUARE ROOT

Art. 375.

(1.)

$\dot{2}8\dot{0}\dot{9}$(53, *Ans.*

25

103 | 309

309

(2.)

$1\dot{4}\dot{4}\dot{4}$(38, *Ans.*

9

68 | 544

544

(3.)

$\dot{1}1\dot{8}8\dot{1}$(109, *Ans.*

1

209)1881

1881

(4.)

$\dot{1}8\dot{5}6\dot{4}0\dot{6}2\dot{5}$(13625, *Ans.*

1

23)85

69

266)1664

1596

2722) 6806

5444

27245) 136225

136225

(5.)

$8\dot{0}0\dot{1}2\dot{3}0\dot{4}$(8944.9−, *Ans.*

64

169 | 1601

1521

1784 | 8023

7136

17884 | 88704

71536

178889 | 1716800

1610001

6799

(6.)

$\dot{6}2\dot{0}3\dot{7}9\dot{4}$(2490.74, *Ans.*

4

44 | 220

176

489 | 4437

4401

49807 | 369400

348649

498144 | 20751

19926−

(7.)

$\dot{3}4\dot{4}\dot{7}3\dot{6}$(1856, *Ans.*

1

28 | 244

224

365 | 2047

1825

3706 | 22236

22236

(8.)

$\dot{5}\dot{7}\dot{6}\dot{0}\dot{0}(240$, *Ans.*

	4
44	176
	176
	00

(9.)

$1\dot{6}\dot{4}\dot{9}\dot{9}\dot{8}\dot{4}\dot{4}(4062$, *Ans.*

	1600
806	4998
	4836
8122	16244
	16244

(10.)

$4\dot{9}0\dot{9}8\dot{0}\dot{4}\dot{9}(7007$, *Ans.*

	49
14007	98049
	98049

(11.)

$\dot{7}3\dot{0}0\dot{5}(270.194$, *Ans.*

	4
47	330
	329
5401	10500
	5401
54029	509900
	486261
540384	2363900
	2161536

(12.)

$386^3 = 5\dot{7}5\dot{1}2\dot{4}5\dot{6}(7583.69$, *Ans.*

	49
45	851
	725
1508	12624
	12064
15163	56056
	45489
	10567
	9098
	1469
	1365

(13.)

$\dot{3}.0\dot{0}0\dot{0}0\dot{0}(1.7320508$, *Ans.*

	1
27	200
	189
343	1100
	1029
3462	7100
	6924
3464+	17600
	17320+
	279
	277

(14.)

$\dot{9}.8\dot{6}9\dot{6}0\dot{4}4\dot{0}1\dot{0}(3.1415926$, *Ans.*

	9
61	86
	61
624	2596
	2496
6281	10004
	6281
62825	372340
	314125
	58215
	56543
	1672
	1257
	415
	377

(15.) $\sqrt{.030625} = .175$; $\sqrt{40.96} = 6.4$; $\sqrt{.00000625} = .0025$; $.175 \times .0025 \times 6.4 = .0028$, *Ans.*

(16.) $126 \times 58 \times 604 = 4414032$, *Ans.*

(17.) $\sqrt{12.96} \times \sqrt{\frac{5}{6}} = \sqrt{10.8} = 3.2863$, *Ans.*

Art. 377.

(1.) $\sqrt{\frac{6}{7}} = \sqrt{\frac{42}{49}} = \frac{1}{7}\sqrt{42} = .92682+$, *Ans.*

(2.) $\sqrt{34\frac{5}{8}} = \sqrt{34.625} = 5.8843+$, *Ans.*

(3.) $\sqrt{\frac{4}{7}} = \sqrt{\frac{28}{49}} = \frac{1}{7}\sqrt{28}$ and *not* $\frac{1}{7}\sqrt{36}$; hence, $\frac{5}{7}$ more nearly than $\frac{6}{7}$, *Ans.*

(4.) $\sqrt{272.25} = 16.5$, *Ans.*

(5.) $\sqrt{6.40} = 2.5298+$, *Ans.*

(6.) $\frac{28}{57} \times \frac{392}{2527} \times \frac{35}{38} \times \frac{3}{1} = \frac{784}{361\times361} \times 35$; hence, sq. rt. $= \frac{28}{361}\sqrt{35} = 5.9160798 \times \frac{28}{361} = .45886+$, *Ans.*

(7.) $\sqrt{123.454321} \times .81 = 11.111 \times .9 = 9.9999$, *Ans.*

(8.) $\sqrt{1.728 \times 4.8 \times \frac{3}{7}} = \sqrt{1.44 \times 1.44 \times 4 \times \frac{3}{7}} = 1.2 \times 1.2 \times 2 \times \frac{1}{7}\sqrt{21} = \frac{2.88}{7}\sqrt{21}$, *Ans.*

EXTRACTION OF THE CUBE ROOT

Art. 380.

(1.)

$51\dot{2}(8$, *Ans.*
512

(2.) $1\dot{9}68\dot{3}(27$, *Ans.*

	8	
$4 \times 300 = 1200$	11683	
$2 \times 7 \times 30 = 420$		
$7 \times 7 = 49$		
1669		
	11683	

(3.) $\dot{7}30\dot{1}38\dot{4}(194$, *Ans.*

	1	
$1 \times 300 = 300$	6301	
$1 \times 9 \times 30 = 270$		
$9 \times 9 = 81$		
651	5859	
$361 \times 300 = 108300$	442384	
$19 \times 4 \times 30 = 2280$		
$4 \times 4 = 16$		
110596	442384	

(4.) $9\dot{4}81\dot{8}81\dot{6}(456$, *Ans.*

	64	
$16 \times 300 = 4800$	30818	
$4 \times 5 \times 30 = 600$		
$5 \times 5 = 25$		
5425	27125	
$2025 \times 300 = 607500$	3693816	
$45 \times 6 \times 30 = 8100$		
$6 \times 6 = 36$		
615636	3693816	

(5.)

$\dot{1}06\dot{7}46\dot{2}64\dot{8}$(1022, *Ans.*

	1	
$1 \times 300 = 300)$	67	
$100 \times 300 =$	30000	67462
$10 \times 2 \times 30 =$	600	
$2 \times 2 =$	4	
	30604	61208
$10404 \times 300 =$	3121200	6254648
$102 \times 2 \times 30 =$	6120	
$2 \times 2 =$	4	6254648
	3127324	

(6.)

$\dot{5}.08\dot{8}44\dot{8}$(1.72, *Ans.*

1	
300	4088
210	
49	3913
559	175448
86700	
1020	
4	
87724	175448

(7.)

$2\dot{2}18\dot{8}.04\dot{1}$(28.1, *Ans.*

8	
1200	14188
480	
64	13952
1744	236041
235200	
840	
1	
2360411	236041

(8.)

$3\dot{2}.65\dot{0}$(3.196154+, *Ans.*

27	
2700	5650
90	
1	2791
2791	2859000
288300	
8370	
81	2670759
296751	188241000
305283 00	
57420	
36	183514536
305857 56	4726464
	305858
	166788
	15292
	1380

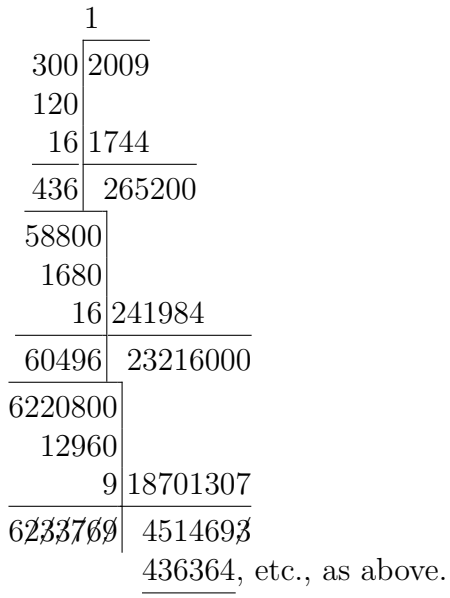
(9.)

$.00\dot{7}90\dot{0}$(.1991632+, *Ans.*

1	
300	6900
270	
81	5859
651	1041000
108300	
5130	
81	1021599
113511	19401000
11880300	
5970	
1	
11886271	11886271
	7514729
	713176
	38296
	3565
	264

(10.)

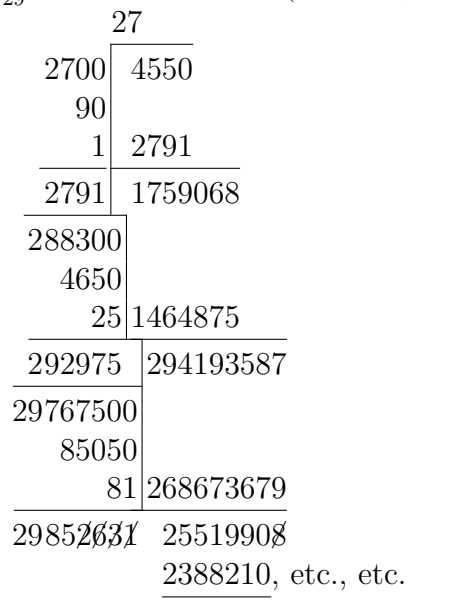
$\dot{3}.00\dot{9}20\dot{0}$(1.443724, *Ans.*



436364, etc., as above.

(11.)

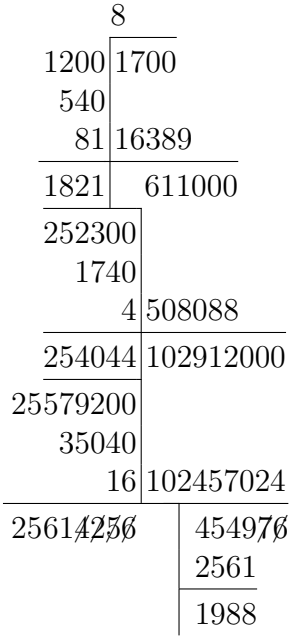
$\frac{23}{729}$ = .03$\dot{1}$55$\dot{0}$06$\dot{8}$587$\dot{}$(.315985, *Ans.*



2388210, etc., etc.

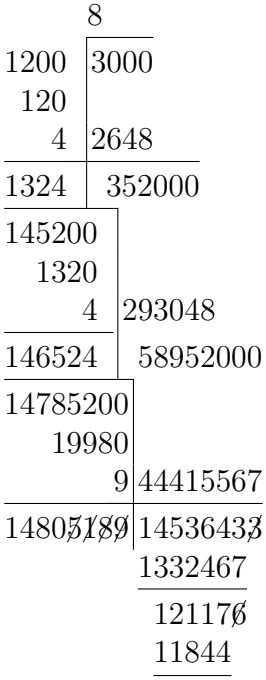
(12.)

$2\dot{5}$(2.924018, *Ans.*



(13.)

$1\dot{1}$(2.22398, *Ans.*



(14.)

$\frac{2}{3} = .66\dot{6}$(.87358, *Ans.*

512

19200 | 154666
1680 |
49 | 146503

20929 | 8163666

2270700 |
7830 |
9 | 6835817

~~2278539~~ | 1328049
113927

~~18877~~
1823

(15.)

$\frac{4}{15} = .26\dot{6}$(.64366, *Ans.*

216

10800 | 50666
720 |
16 | 46144

11536 | 4522667

1228800 |
5760 |
9 | 3703707

~~1234569~~ | 818960
74074

~~7822~~

(16.) $17\dot{1}.41632\dot{8}75\dot{}$(5.555, *Ans.*

125

$25 \times 300 = 7500$ | 4616
$5^2 \times 30 = 750$ |
$5^2 = 25$ |

8275 | 41375

5041328

$55^2 \times 300 = 907500$
$55 \times 5 \times 30 = 8250$
$5 \times 5 = 25$

915775 | 4578875

462453875

$555^2 \times 300 = 92407500$
$555 \times 5 \times 30 = 83250$
$5 \times 5 \times 25$

92490775 | 462453875

(17.) $\dot{7}01\dot{1}$ (19.1393267, *Ans.*

300		1
270		6011
81		5859
651		152000
	$19^2 \times 300 =$ 108300	
	$19 \times 30 =$ 570	
	1	
	108871	108871
		43129000
	$191^2 \times 300 =$ 10944300	
	$191 \times 3 \times 30 =$ 17190	
	9	
	10961499	32884497
		10244503000
	$1913^2 \times 300 =$ 1097870700	
	$1913 \times 30 \times 9 =$ 516510	
	$9 \times 9 =$ 81	
	1098387291	9885485619, etc., et.

(18.) $\sqrt[3]{\frac{48}{4394}} = \sqrt[3]{\frac{24}{2197}} = \frac{1}{13}\sqrt[3]{24} = \frac{1}{13}$ of $2.8844991 = .2218845$, *Ans.*

(19.) $\sqrt[3]{\frac{2}{3} \text{ of } \frac{4}{11}} = \sqrt[3]{.24242424+} = .6235319$, *Ans.*

EXTRACTION OF ANY ROOT

Art. 384.

	(1.)
0	15625(125, *Ans.*
1	1
1	*5625
1	44
*2	*1225
2	1225
22	
2	
*24	
5	
245	

(2.)

0	0		$6\dot{8}7\dot{1}9\dot{4}7\dot{6}7\dot{3}6\dot{}$(4096, *Ans.*
4	16		64
4	16		$*$ 4719476
4	32		4417929
8	$*$48		$*$ 302547736
4	10881		301547736
$*$12	490881		
09	10962		
1209	$*$501843		
9	73656		
1218	$*$50257956		
9			
$*$1227			
6			
12276			

(3.)

0	0	0	0	14348907(27, *Ans.*
2	4	8	16	*11148907
4	12	32	*80	11148907
6	24	*80	1592701	
8	*40	113243		
*10	4749			
107				

(4.)

0	0	151(5.325074, *Ans.*
5	25	125
5	50	∗ 26.00
10	*75	23.877
5	4.59	∗ 2.123000
∗15	79.59	1.691768
.3	4.68	∗ .431232000
15.3	∗84.27	424935125
.3	.3184	∗ .006296875
15.6	84.5884	595467
.3	.3188	34220
∗15.9	∗84.9072	
.02	.079825	
15.92	84.987025	
.02	.079850	
15.94	∗85.066875	
.02		
∗15.96		
.005		
15.965		
15.970		
∗15.975		

(5.) Proceeding by Art. 382, we have $\sqrt[4]{97.41}$ = sq. rt. of $\sqrt{97.41}$, which = $\sqrt{9.86965045}$ = 3.14150, or, 3.1416, *Ans.*

(6.) $\sqrt[4]{1.08} = \sqrt{\text{sq. rt. of } 1.08} = \sqrt{1.03923048} = 1.01943$, *Ans.*

(7.)

$\frac{5}{12} = .41\dot{6}$:

0	0	0	0	.4166(.83938, *Ans.*
8	64	512	4096	889866667
16	192	2048	204800000	227626024
24	384	5120000	220746881	9381865
32	64000	5315627	23729160	1944016
400	65209	5514908	24249351	
403	66427	571878	24775122	
406	67654	5799	2479283	
409	68890	58419	248105	
412		590	0	

(8.)

$35.2 = 32 \times 1.10$. Hence root $= 2\sqrt[5]{1.10}$

0	0	0	0	1.10	(1.01924
1	1	1	1	*.10	
2	3	4	*5	*.0489899499	
3	6	*10	5.10100501	*.0013207560	
4	*10.	10.100501	*5.20302005	109721	
*5.01	10.0501	10.201504	5.2965771	22354	
5.02	10.1003	*10.303010	5.390967+	21944	
5.03	*10.1506	10.395228	951	410	
5.04	10.2010	10.487854	5.4860		
*5.05	10.2464	92	&c.,		
&c.,	10.2918	10.579			

$1.01924 \times 2 = 2.03848$, *Ans.*

(8.) $\sqrt[12]{782757789696} = \sqrt[4]{\text{cube rt.}} = \sqrt[4]{9216} = \sqrt{96} = 9.79795897$, *Ans.*

(9.) $\sqrt{1367631} = \sqrt[3]{\text{cube rt.}} = \sqrt[3]{111} = 4.8058955$, *Ans.*

APPLICATIONS OF SQUARE AND CUBE ROOT

Art. 388.

(1.) $\sqrt{30^2 + 12^2} = \sqrt{1044} = 32.31+$ ft., *Ans.*

(2.) $\sqrt{10^2 \times 2} = 10\sqrt{2} = 14.142+$ ft., *Ans.*

(3.) $\sqrt{69^2 + 92^2} = \sqrt{13225} = 115$; and $(69 + 92) - 115 = 46$ rd., *Ans.*

(4.) $\sqrt{500^2 + 360^2} = \sqrt{379600} = 616+$ yd., *Ans.*