

# Title of Book

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January 17, 2026

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# PREFACE

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## V. MULTIPLICATION

59. 1. **Multiplication** is taking one number as many times as there are units in another; or,  
 2. Multiplication is a short method of adding numbers that are equal.
60. The number to be taken, is called the *Multiplicand*; the other number, the *Multiplier*; and the result obtained, the *Product*. The Multiplicand and Multiplier are together called *Factors* (makers), because they *make* the Product.

**PROBLEM.**—How many trees in 3 rows, each containing 42 trees?

**SOLUTION.**—Since 3 rows contain 3 *times* as many trees as *one* row, take 42 *three* times. This may be done by writing 42 three times, and then *adding*. This gives 126 trees for the whole number of trees.

Instead, however, of writing 42 *three* times, write it *once*; then placing under it the figure 3, the *number of times* it is to be taken, say, 3 times 2 are 6, and 3 times 1 are 12. This process is *Multiplication*.

### OPERATION

First row,	42	trees
Second row,	42	trees
Third row,	42	trees
	<hr/>	
	126	trees
	42	trees
	3	
	<hr/>	
	126	trees

**PRINCIPLES.**—1. *The multiplicand may be either concrete or abstract.*

2. *The multiplier must always be an abstract number.*

3. *The product is the same in kind as the multiplicand.*

4. *The product is the same, whichever factor is taken as the multiplier.*

5. *The partial products are the same in kind as the multiplicand.*

6. *The sum of the partial products is equal to the total product.*

MULTIPLICATION TABLE.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

61. Multiplication is divided into two cases:

1. *When the multiplier does not exceed 12.*
2. *When the multiplier exceeds 12.*

## CASE I

## 62. When the multiplier does not exceed 12.

**PROBLEM.**—At the rate of 53 miles an hour, how far will a railroad car run in four hours?

**SOLUTION.**—Here say, 4 times 3 (units) are 12 (units); write the 2 in units' place, and carry the 1 (ten); then, 4 times 5 are 20, and 1 carried makes 21 (tens), and the work is complete.

**OPERATION**

$$\begin{array}{r} 53 \text{ miles.} \\ \quad 4 \\ \hline 212 \text{ miles} \end{array}$$

**DEMONSTRATION.**—The multiplier being written under the multiplicand for convenience, begin with units, so that if the product should contain tens, they may be carried to the tens; and so on for each successive order.

Since every figure of the multiplicand is multiplied, therefore, the *whole* multiplicand is multiplied.

**Rule.—1.** Write the multiplicand, and place the multiplier under it, so that units of the same order shall stand in the same column, and draw a line beneath.

**2.** Begin with units; multiply each figure of the multiplicand by the multiplier, carrying as in Addition.

**PROOF.**—Separate the multiplier into any two parts; multiply by these separately. The sum of the products must be equal to the first product.

## EXAMPLES FOR PRACTICE

1.  $195 \times 3$ . *Ans.*
2.  $3823 \times 4$ . *Ans.*
3.  $8765 \times 5$ . *Ans.*
4.  $98374 \times 6$ . *Ans.*
5.  $64382 \times 7$ . *Ans.*
6.  $58765 \times 8$ . *Ans.*
7.  $837941 \times 9$ . *Ans.*
8.  $645703 \times 10$ . *Ans.*
9.  $407649 \times 11$ . *Ans.*
10. If 4 men can perform a certain piece of work in 15 days, how long will it require 1 man?  
**SOLUTION.**—One man must work four times as long as four men.

$$4 \times 15 \text{ days} = 60 \text{ days.}$$

11. How many pages in a half-dozen books, each containing 336 pages?
12. How far can an ocean steamer travel in a week, at the rate of 245 miles a day?

13. What is the yearly expense of a cotton-mill, if \$32053 are paid out every month?
14. A receives from his business an average of \$45 a day. He pays three clerks \$3; three, \$9; and three, \$12 a week; other expenses amount to \$4 a day; what are his profits for one week?

## CASE II

### 63. When the multiplier exceeds 12.

**PROBLEM.**—Multiply 246 by 235.

**SOLUTION.**—first Multiply by 5 (units), and place the first figure of the product, 1230, under the 5 (units). Then multiply by 3 (tens), and place the product, 738, under the 3 (tens). Lastly, multiply by 2 (hundreds), and place the first figure of the product, 492, under the 2 (hundreds). Then add these several products for the entire product.

OPERATION			
246			
<u>235</u>			
1230	product by	5	
738	product by	30	
<u>492</u>	product by	<u>200</u>	
57810	product by	235	

**DEMONSTRATION.**—The 0 of the first product, 1230, is *units* (Art. 62). The 8 of the second product, 738, is *tens*, because 3 (tens) times 6 = 6 times 3 (tens) = 18 (tens); giving 8 (tens) to be written in the tens' column. The 2 of the third product, 492, is *hundreds*, because 2 (hundreds) times 6 = 6 times 2 (hundreds) = 12 (hundreds), giving 2 (hundreds) to be written in the hundreds' column. The right-hand figure of each product being in its proper column, the other figures will fall in their proper columns; and each line being the product of the multiplicand by a *part* of the multiplier, their sum will be the product by all the parts or the *whole* of the multiplier.

**Rule.—1.** Write the multiplier under the multiplicand, placing figures of the same order in the same column, and draw a line beneath.

**2.** Multiply each figure of the multiplicand by each figure of the multiplier successively; first by the units' figure, then by the tens' figure, etc.; placing the right-hand figure of each product under that figure of the multiplier which produces it, then draw a line beneath.

**3.** Add the several partial products together; their sum will be the required product.

**METHODS OF PROOF.—1.** Multiply the multiplier by the multiplicand; this product must be the same as the first product.

2. The same as when the multiplier does not exceed 12.

**NOTE.**—For proof by casting out the 9's, see Art. 105.

**REMARK.**—Although it is customary to use the figures of the multiplier in regular order beginning with units, it will give the same product to use them in any order, observing that *the right-hand figure of each partial product must be placed under the figure of the multiplier which produces it.*

OPERATION			
246			
<u>235</u>			
738	product by	30	
492	product by	<u>200</u>	
<u>1230</u>	product by	<u>5</u>	
57810	product by	235	

## EXAMPLES FOR PRACTICE

1.  $7198 \times 216$ . *Ans.*
2.  $8862 \times 189$ . *Ans.*
3.  $7575 \times 7575$ . *Ans.*
4.  $15607 \times 3094$ . *Ans.*
5.  $93186 \times 4455$ . *Ans.*
6.  $135790 \times 24680$ . *Ans.*
7.  $3523725 \times 2583$ . *Ans.*
8.  $4687319 \times 1987$ . *Ans.*
9.  $9264397 \times 9584$ . *Ans.*
10.  $9507340 \times 7071$ . *Ans.*
11.  $1644405 \times 7749$ . *Ans.*
12.  $1389294 \times 8900$ . *Ans.*
13.  $2778588 \times 9867$ . *Ans.*
14.  $204265 \times 562402$ . *Ans.*

## PRACTICAL PROBLEMS

1. In a mile are 63360 inches: how many inches are there in the circumference of the earth at the equator if the distance be 25000 miles?
2. The flow of the Mississippi at Memphis is about 434000 cubic feet a second: required the weight of water passing that point in one day of 86400 seconds, if a cubic foot of water weigh 62 pounds?
3. John Sexton sold 25625 bushels of wheat, at \$1.20 a bushel, and received in payment 320 acres of land, valued at \$50 an acre; 60 head of horses, valued at \$65 a head; 10 town lots, worth \$150 each; and the remainder in money: how much money did he receive?
4. If light comes from the sun to the earth in 495 seconds, what is the distance from the earth to the sun, light moving 192500 miles a second?
5. If 3702754100 cubic feet of solid matter is deposited in the Gulf of Mexico by the Mississippi every year, what is the deposit for 6000 years?
6. The area of Missouri is 65350 square miles: how many acres are there in the State, allowing 640 acres to each square mile?
7. In the United States, at the close of 1878, there were 81841 miles of railroad: if the average cost of building be \$50000 a mile, what has been the total cost of building the railroads in this country?

8. The number of pounds of tobacco produced in this country in 1870 was 260000000. If this were manufactured into plugs one inch wide and six inches long, and four plugs weigh a pound, what would be the length in inches of the entire crop?

## BUSINESS TERMS AND EXPLANATIONS

- 64.** A **Bill** is an account of goods sold or delivered, services rendered, or work done. Usually the price or value is annexed to each article, and the date of purchase given.

It is customary to write the total amount off to the right, and not directly under the column of amounts added.

- 65.** A **Receipt** is a written acknowledgment of payment. The common form consists in signing the name after the words "Received Payment" written at the foot of the bill.

1. Joseph Allen bought of Seth Ward, at Springfield, Ill., Jan. 2, 1879, 30 barrels of flour, at \$3.60 a barrel; 48 barrels of mess pork, at \$16.25 a barrel; 16 boxes of candles, at \$3.50 a box; 23 barrels of molasses, at \$28.75 a barrel; and 64 sacks of coffee, at \$47.50 a sack. Place the purchases in bill form.

### SOLUTION.

SPRINGFIELD, ILL., *Jan.* 2, 1879.

JOSEPH ALLEN,

1879

Bought of SETH WARD.

Jan	2	To 30 bl. flour,	@ \$3.60 a bl.	108	00		
"	2	" 48 " mess pork,	" 16.25 "	780	00		
"	2	" 16 Boxes candles,	" 3.50 " box	780	00		
"	2	" 23 bl. molasses,	" 28.75 " bl.	661	25		
"	2	" 64 sacks coffee,	" 47.50 " sack	3040	00		
						\$4645	25

2.

1. At St. Louis, March 1, 1879, Chester Snyder bought of Thomas Glenn, 4 lb. of tea, at 40 ct.; 21 lb. of butter, at 21 ct.; 58 lb. of bacon, at 13 ct.; 16 lb. of lard, at 9 ct.; 30 lb. of cheese, at 12 ct.; 4 lb. of raisins, at 20 ct.; and 9 doz. of eggs, at 15 ct. Place these purchases in the form of a receipted bill.

- 66.** A **Statement of Account** is a written form rendered to a customer, showing his debits and credits as they appear on the books. The following is an example:

CINCINNATI, *Feb.* 2, 1880.

JOHN SMITH,

1880

In Account *with* VAN ANTWERP, BRAGG & CO.

Jan	2	To 525 McGuffey's Revised First Readers, @ 16c.	84			
„	10	„ 50 Ray's New Higher Arithmetics, „ 75c.	37	50		
		Cr.			121	50
„	20	By Cash	20			
„	31	„ Merchandise	12	75	32	75
					\$88	75

3. James Wilson & Co. bought of the Alleghany Coal Co., March 2, 1880, five hundred tons of coal, at \$2.75 a ton, and sold the same Company during the month, as follows: March 3d, 14 barrels of flour, at \$6.55 a barrel; March 10th, 6123 pounds of sugar, at 5 ct. a pound; they also paid them on account, on March 15th, cash, \$687.50. Make out a statement of account in behalf of the Alleghany Coal Co. under date of April 1, 1880.

## CONTRACTIONS IN MULTIPLICATION

### CASE I

#### 67. When the multiplier is a composite number.

A **Composite Number** is the product of two or more whole numbers, each greater than 1, called its *factors*. Thus, 10 is a composite number, whose factors are 2 and 5; and 30 is one whose factors are 2, 3, and 5.

**PROBLEM.**—At 7 cents a piece, what will 6 melons cost?

**ANALYSIS.**—Three times 2 times are 6 times. Hence, it is the same to take 2 times 7, and then take this product 3 times, as to take 6 times 7. The same may be shown of any other composite number.

#### OPERATION

7	cents, cost of 1 melon.
2	
14	cents, cost of 2 melons.
3	
42	cents, cost of 6 melons.

**Rule.**—*Separate the multiplier into two or more factors. Multiply first by one of the factors, then this product by another factor, and so on till each factor has been used as a multiplier. The last product will be the result required.*

## EXAMPLES FOR PRACTICE

- At the rate of 37 miles a day, how far will a man walk in 28 days?
- Sound moves about 1130 feet per second: how far will it move in 54 seconds?
- If an engine travel at an average speed of 25 miles an hour, how far can it travel in a week, or 168 hours?

## CASE II

68. When the multiplier is 1 with ciphers annexed, as 10, 100, 1000, etc.

**DEMONSTRATION.**—By the principles of Notation (Art. 43), placing *one* cipher on the right of a number, changes the units into tens, the tens into hundreds, and so on, and, therefore, *multiplies the number by 10*.

Annexing *two* ciphers to a number changes the units into hundreds, the tens into thousands, and so on, and, therefore, *multiplies the number by 100*. Annexing *three* ciphers multiplies the number by 1000, etc.

**Rule.**—*Annex to the multiplicand as many ciphers as there are in the multiplier; the result will be the required product.*

## EXAMPLES FOR PRACTICE

- |                           |             |
|---------------------------|-------------|
| (a) Multiply 743 by 10.   | <i>Ans.</i> |
| (b) Multiply 375 by 100.  | <i>Ans.</i> |
| (c) Multiply 207 by 1000. | <i>Ans.</i> |

## CASE III

69. When ciphers are on the right in one or both factors.

**PROBLEM.**—Find the product of 5400 by 130.

**SOLUTION.**—Find the product of 54 by 13, and then annex three ciphers; that is, as many as there are on the right in both the factors.

**OPERATION**

$$\begin{array}{r}
 5400 \\
 130 \\
 \hline
 162 \\
 54 \\
 \hline
 702000
 \end{array}$$

**ANALYSIS.**—Since 13 times 54 = 702, it follows that 13 times 54 hundreds (5400) = 702 hundreds (70200); and 130 times 5400 = 10 times 13 times 5400 = 10 times 70200 = 702000.

**Rule.**—*Multiply as if there were no ciphers on the right in the numbers; then annex to the product as many ciphers as there are on the right in both the factors.*

## EXAMPLES FOR PRACTICE

- |                             |             |
|-----------------------------|-------------|
| 1. $15460 \times 3200$ .    | <i>Ans.</i> |
| 2. $30700 \times 5904000$ . | <i>Ans.</i> |

## CASE IV

70. When the multiplier is a little less or a little greater than 10, 100, 1000, etc.

**PROBLEM.**—Multiply 3046 by 997.

**ANALYSIS.**—Since 997 is equal to 1000 diminished by 3, to multiply by it is the same as to multiply by 1000 (that is, to annex 3 ciphers) and by 3, and take the difference of the products; and the same can be shown in any similar case.

**OPERATION**

$$\begin{array}{r} 30046 \\ 997 \\ \hline 3046000 \\ 9138 \\ \hline 3036862 \end{array}$$

**NOTE.**—Where the number is a little *greater* than 10, 100, 1000, etc., the two products must be *added*.

**Rule.**—*Annex to the multiplicand as many ciphers as there are figures in the multiplier; multiply the multiplicand by the difference between the multiplier and 100, 1000, etc., and add or subtract the smaller result as the multiplier is greater or less than 100, 1000, etc.*

## EXAMPLES FOR PRACTICE

1.  $7023 \times 99$ . *Ans.*

2.  $16642 \times 996$ . *Ans.*

3.  $372051 \times 1002$ . *Ans.*

## CASE V

**71. When one part taken as units, in the multiplier, is a factor of another part so taken.**

**PROBLEM.**—Multiply 387295 by 216324.

**SOLUTION.**—Commence with the 3 of the multiplier, and obtain the first partial product, 1161885; then multiply this product by 8, which gives the product of the multiplicand by 24 at once (since 8 times 3 times any number make 24 times it). Set the right-hand figure under the right-hand figure 4 of the multiplier in use. Multiply the second partial product by 9, which gives the product of the multiplicand by 216 (since 9 times 24 times a number make 216 times that number). Set the right-hand figure of this partial product under the 6 of the multiplicand; and, finally, add to obtain the total product.

**OPERATION**

$$\begin{array}{r} 387295 \\ 216324 \\ \hline 1161885 \\ 9295080 \\ 83655720 \\ \hline 8378123580 \end{array}$$

**Rule.**—**1.** *Multiply the multiplicand by some figure or figures of the multiplier, which are a factor of one or more parts of the multiplier.*

**2.** *Multiply this partial product by a factor of some other figure or figures of the multiplier, and write the right-hand figure thus obtained under the right-hand figure of the multiplier thus used.*

**3.** *Continue thus until the entire multiplier is used, and then add the partial products.*

## EXAMPLES FOR PRACTICE

- |                              |             |
|------------------------------|-------------|
| (a) $38057 \times 48618$ .   | <i>Ans.</i> |
| (b) $267388 \times 14982$ .  | <i>Ans.</i> |
| (c) $481063 \times 63721$ .  | <i>Ans.</i> |
| (d) $66917 \times 849612$ .  | <i>Ans.</i> |
| (e) $102735 \times 273162$ . | <i>Ans.</i> |
| (f) $536712 \times 729981$ . | <i>Ans.</i> |

---

**Topical Outline.**
**MULTIPLICATION.**

1. Definitions.

2. Terms. . . . .  $\left\{ \begin{array}{l} 1. \text{ Multiplicand.} \\ 2. \text{ Multiplier.} \\ 3. \text{ Partial Product.} \\ 4. \text{ Product.} \end{array} \right.$

3. Sing.

4. Principles.

5. Operation. . . . .  $\left\{ \begin{array}{l} 1. \text{ Writing Numbers.} \\ 2. \text{ Drawing Line Beneath.} \\ 3. \text{ Finding Partial Products.} \\ 4. \text{ Drawing a Line Beneath Partial Products.} \\ 5. \text{ Adding the Partial Products.} \end{array} \right.$

6. Rule.

7. Proof.

8. Applications.

9. Contractions.

**MULTIPLICATION.—BILLS AND ACCOUNTS****Art. 65.**

(2.)

ST. LOUIS, March 1, 1879.

CHESTER SNYDER,

1879.

*Bought of* Thomas Glenn.

			\$			
March	1	4 lb. tea @ 40 ct. a lb.,	1	60		
„	1	21 „ butter, @ 21 ct. a lb.	4	41		
„	1	58 „ bacon, „ 13 ct. „	7	54		
„	1	16 „ lard, „ 9 ct. „	1	44		
„	1	30 „ cheese, „ 12 ct. „	3	60		
„	1	4 „ raisins, „ 20 ct. „		80		
„	1	9 doz. eggs, „ 15 ct. a doz.,	1	35		
					\$20	74

*Received payment,**THOMAS GLENN.***Art. 66.**

(3.)

Allegheny, April 1, 1880.

JAMES WILSON &amp; CO.,

1880.

*In Acc't with* ALLEGHENY COAL CO.

		DR.			\$	
March	2	To 500 tons coal, @ \$2.75 a ton,			1375	00
		CR.	\$			
„	3	By 14 bbl. flour, @ \$6.55 a bbl.,	91	70		
„	10	„ 6123 lb. sugar, @ 8 ct. a lb.,	489	84		
„	15	„ cash on acc't,	687	50	1269	04
		Balance due Allegheny Coal Co., .....			\$105	96

## CONTRACTIONS IN MULTIPLICATION

## Case IV.

## Art. 70.

$$\begin{array}{r}
 (1.) \\
 7023 \\
 \underline{99} \\
 702300 \\
 7023 \\
 \hline
 695277
 \end{array}$$

$$\begin{array}{r}
 (2.) \\
 16642 \\
 \underline{996} \\
 16642000 \\
 66568 \\
 \hline
 16575432
 \end{array}$$

$$\begin{array}{r}
 (3.) \\
 372051000 \\
 \underline{744102} \\
 372795102
 \end{array}$$

## Case V.

## Art. 71.

$$\begin{array}{r}
 (1.) \\
 38057 \\
 \underline{48618} \\
 228342 \\
 685026 \\
 1826736 \\
 \hline
 1850255226
 \end{array}$$

$$\begin{array}{r}
 (2.) \\
 267388 \\
 \underline{14982} \\
 5347776 \\
 3743432 \\
 26204024 \\
 \hline
 4006007016
 \end{array}$$

$$\begin{array}{r}
 (3.) \\
 481063 \\
 \underline{63721} \\
 3367441 \\
 10102323 \\
 30306969 \\
 \hline
 30653815423
 \end{array}$$

$$\begin{array}{r}
 (4.) \\
 66917 \\
 \underline{849612} \\
 803004 \\
 6424032 \\
 5621028 \\
 \hline
 56853486204
 \end{array}$$

$$\begin{array}{r}
 (5.) \\
 102735 \\
 \underline{273162} \\
 308205 \\
 2773845 \\
 16643070 \\
 \hline
 28063298070
 \end{array}$$

$$\begin{array}{r}
 (6.) \\
 536712 \\
 \underline{729981} \\
 4830408 \\
 43473672 \\
 391263048 \\
 \hline
 391789562472
 \end{array}$$

## ARITHMETICAL SIGNS

## Art. 86.

$$(3.) \quad \left. \begin{array}{rcl}
 21 \div 3 \times 7 & = & +49 \\
 -1 \times 1 \div 1 \times 4 \div 2 & = & -2 \\
 18 \div 3 \times 6 \div 4 & = & +9 \\
 1 \times 4 \times 6 \div 8 & = & +3
 \end{array} \right\} = 59, \text{ Ans.}$$

$$(4.) \quad \left. \begin{array}{rcl}
 16 \times 4 \div 8 & = & +8 \\
 -7 + 48 \div 16 & = & -4 \\
 -3 - 28 \times 0 & = & -3 \\
 24 \times 6 \div 48 & = & +3 \\
 -4 \times 9 \div 12 & = & -3
 \end{array} \right\} = 1, \text{ Ans.}$$

$$(5.) \quad \left. \begin{array}{rcl} 16 \div 16 \times 96 \div 8 & = & +12 \\ -7 - 5 + 3 & = & -9 \end{array} \right\} = +3.$$

$$(27 \div 9) \div 3 - 1 = 0.$$

$$91 \div 13 \times 7 - 45 - 3 = 1.$$

Then,  $3 \times 0 + 1 \times 9 = 9$ , Ans.

## CONTRACTIONS IN MULTIPLICATION AND DIVISION

### Case I.

Art. 88.

$$\begin{array}{r} (1.) \\ 3 \overline{)42200} \\ 14066\frac{2}{3}, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (2.) \\ 656400 \\ \underline{\phantom{00}5} \\ 8 \overline{)3282000} \\ 410250, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (3.) \\ 6 \overline{)1072400} \\ 178733\frac{2}{6}, \text{ Ans.} \end{array}$$

### Case II.

Art. 89.

$$\begin{array}{r} (1.) \\ 4514020000 \\ \underline{451402} \\ 3 \overline{)4513568598} \\ 1504522866, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (2.) \\ 281257000000 \\ \underline{281257} \\ 281256718743 \\ \underline{\phantom{00}5} \\ 9 \overline{)1406283593715} \\ 156253732635, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (3.) \\ 6302240000000 \\ \underline{630224000} \\ 9 \overline{)6301609776000} \\ 700178864000 \\ \underline{\phantom{00}4} \\ 2800715456000, \text{ Ans.} \end{array}$$

### Case III.

Art. 90.

$$\begin{array}{r} (1.) \quad 225) \quad 300521761 \\ \quad \quad 4 \qquad \qquad \quad 4 \\ \hline \quad 9|00) \quad 12020870|44 \\ \text{Quot.} \quad 1335652, \text{ Ans.} \\ 244 \div 4 = 61, \text{ Rem.} \end{array}$$

$$\begin{array}{r} (2.) \quad 43750) \quad 1510337264 \\ \quad \quad 4 \qquad \qquad \quad 4 \\ \hline \quad 175000) \quad 6041349056 \\ \quad \quad 4 \qquad \qquad \quad 4 \\ \hline \quad 7|00000) \quad 241653|96224 \\ \text{Quot.} = 34521, \text{ Ans.} \\ 696224 \div 4 \div 4 = 43514, \text{ Rem.} \end{array}$$



- (13.) 13 men  $-$  8 men = 5 men; 1 man can do it in 13 times 15 days, which is 195 days; and 5 men, in  $\frac{1}{5}$  of 195 days, or 39 days, *Ans.*
- (14.) 14 men  $\div$  7 men = 2 men; 1 man can do it in 14 times 24 days, that is, 336 days; and 2 men, in  $\frac{1}{2}$  of 336 days, or 168 days, *Ans.*
- (15.) For 1 day, the provisions will support 30 times 45 men, or 1350 men; for 50 days, one fiftieth of 1350 men = 27 men; 45 men  $-$  27 men = 18 men, *Ans.*
- (16.)  $\$18 \times 3 = \$54$ ;  $\$85 + \$54 = \$139$ , total value;  $\$139 - \$41 = \$98$ ; 1 sheep cost one fourteenth of  $\$98$ , or  $\$7$ , *Ans.*
- (17.) A sheep and a hog cost  $\$13$ ; therefore, he will buy as many of each as  $\$13$  is contained times in  $\$1482$ ;  $1482 \div 13 = 114$ , *Ans.*
- (18.) 1 horse and 2 oxen cost  $\$84$ ; therefore, he bought as many horses as  $\$84$  is contained times in  $\$1260$ ;  $1260 \div 84 = 15$ ; twice 15 equals 30. *Ans.* 15 horses and 30 oxen.
- (19.) One seventh of 1050 ct. = 150 ct.;  $150 \div 25 = 6$ , of the 25-cent pieces. 1050 ct.  $-$  150 ct. = 900 ct.; 1 of each of the others would make 10 ct. + 5 ct. + 3 ct. = 18 ct.;  $900 \div 18 = 50$ , the number of each of the others. *Ans.* Of 25-cent pieces, 6; of the others, 50 each.
- (20.)  $\$6300 \div 140 = \$45$ , gain per acre:  $\$210 - \$45 = \$165$ , the cost;  $\$5600 \div 140 = \$40$ , loss per acre;  $\$165 - \$40 = \$125$ , sold for per acre. *Ans.*  $\$165$ , cost;  $\$125$ , sold for.

**Art. 134.**

**Case IV.**

- (1.)  $\$ \frac{37}{8} = \$37 \div 8 = \$4 \frac{5}{8}$ , *Ans.*
- (2.)  $\frac{137}{4}$  bu. = 137 bu.  $\div$  4 =  $34 \frac{1}{4}$  bu., *Ans.*
- (3.)  $\frac{785}{60}$  hr. = 785 hr.  $\div$  60 =  $13 \frac{1}{12}$  hr., *Ans.*
- (4.)  $\frac{1295}{37} = 1295 \div 37 = 35$ , *Ans.*
- (5.)  $\frac{800}{9} = 800 \div 9 = 88 \frac{8}{9}$ , *Ans.*
- (6.)  $\frac{1162}{11} = 1162 \div 11 = 105 \frac{7}{11}$ , *Ans.*
- (7.)  $\frac{4260}{13} = 4260 \div 13 = 327 \frac{9}{13}$ , *Ans.*
- (8.)  $\frac{15780}{31} = 15780 \div 31 = 509 \frac{1}{31}$ , *Ans.*

**Case V.**

**Art. 135.**

- (1.)  $\frac{1}{3} \times \frac{3}{4} \times \frac{4}{7} = \frac{1}{7}$ , *Ans.*
- (2.)  $\frac{2}{5} \times \frac{4}{7} \times \frac{21}{8} = \frac{8}{5}$ , *Ans.*

$$(3.) \frac{\overset{3}{4} \times \overset{2}{15} \times \overset{2}{8}}{\underset{4}{5} \times \underset{4}{16} \times \underset{4}{3}} = \frac{2}{1} = 2, \text{ Ans.}$$

$$(4.) \frac{1 \times \overset{3}{4} \times \overset{3}{15}}{\underset{2}{2} \times \underset{5}{5} \times \underset{4}{4}} = \frac{3}{2} = 1\frac{1}{2}, \text{ Ans.}$$

$$(5.) \frac{\overset{2}{3} \times \overset{5}{8} \times \overset{2}{4} \times \overset{5}{35}}{\underset{3}{4} \times \underset{3}{9} \times \underset{7}{7} \times \underset{4}{4}} = \frac{10}{3} = 3\frac{1}{3}, \text{ Ans.}$$

$$(6.) \frac{\overset{2}{1} \times \overset{2}{3} \times \overset{2}{6} \times \overset{2}{3} \times \overset{2}{14}}{\underset{2}{3} \times \underset{5}{5} \times \underset{7}{7} \times \underset{4}{4} \times \underset{3}{3}} = \frac{3}{5}, \text{ Ans.}$$

$$(7.) \frac{\overset{7}{8} \times \overset{3}{3} \times \overset{7}{4} \times \overset{3}{77} \times \overset{3}{57}}{\underset{8}{11} \times \underset{7}{7} \times \underset{19}{19} \times \underset{24}{24} \times \underset{8}{8}} = \frac{3}{2} = 1\frac{1}{2}, \text{ Ans.}$$

$$(8.) \frac{\overset{4}{12} \times \overset{5}{9} \times \overset{3}{7} \times \overset{3}{10} \times \overset{3}{39}}{\underset{4}{13} \times \underset{2}{16} \times \underset{2}{18} \times \underset{3}{21} \times \underset{7}{35}} = \frac{3}{28}, \text{ Ans.}$$

## Case VI.

## Art. 138.

$$(1.)$$

$$1 \times 3 \times 5 = 15$$

$$2 \times 2 \times 5 = 20$$

$$3 \times 2 \times 3 = 18$$

$$2 \times 3 \times 5 = 30 \quad \text{Den.}$$

$$\text{Ans. } \frac{15}{30}, \frac{20}{30}, \frac{18}{30}.$$

$$(3.)$$

$$2 \times 7 \times 8 = 112$$

$$3 \times 3 \times 8 = 72$$

$$5 \times 3 \times 7 = 105$$

$$3 \times 7 \times 8 = 168 \quad \text{Den.}$$

$$\text{Ans. } \frac{112}{168}, \frac{72}{168}, \frac{105}{168}.$$

$$(2.)$$

$$1 \times 5 \times 6 = 30$$

$$1 \times 4 \times 6 = 24$$

$$1 \times 4 \times 5 = 20$$

$$4 \times 5 \times 6 = 120 \quad \text{Den.}$$

$$\text{Ans. } \frac{30}{120}, \frac{24}{120}, \frac{20}{120}.$$

$$(4.)$$

$$1 \times 5 \times 6 \times 8 = 240$$

$$3 \times 2 \times 6 \times 8 = 288$$

$$5 \times 2 \times 5 \times 8 = 400$$

$$7 \times 2 \times 5 \times 6 = 420$$

$$2 \times 5 \times 6 \times 8 = 480 \quad \text{Den.}$$

$$\text{Ans. } \frac{240}{480}, \frac{288}{480}, \frac{400}{480}, \frac{420}{480}.$$

$$(5.) \quad \frac{1 \times 7}{2 \times 2} = \frac{7}{4};$$

$$\frac{2 \times 3}{3 \times 5} = \frac{2}{5};$$

$$2 \times 4 \times 5 = 40$$

$$7 \times 3 \times 5 = 105$$

$$2 \times 3 \times 4 = 24$$

$$3 \times 5 \times 5 = 60 \quad \text{Den.}$$

$$\text{Ans. } \frac{40}{60}, \frac{105}{60}, \frac{24}{60}.$$

$$(6.) \quad \frac{2 \times 6}{3 \times 7} = \frac{4}{7}; \quad \frac{3 \times 8}{4 \times 9} = \frac{2}{3};$$

$$4 \times 3 \times 20 = 240$$

$$2 \times 7 \times 20 = 280$$

$$9 \times 3 \times 7 = 189$$

$$7 \times 3 \times 20 = 420 \quad \text{Den.}$$

$$\text{Ans. } \frac{240}{420}, \frac{280}{420}, \frac{189}{420}.$$

$$\frac{1 \times 4 \times 3 \times 21}{2 \times 5 \times 7 \times 8} = \frac{9}{20}.$$

$$(1.) \quad \frac{1 \times 4}{2 \times 4} = \frac{4}{8}.$$

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

$$\text{Ans. } \frac{4}{8}, \frac{6}{8}, \frac{5}{8}.$$

$$(2.) \quad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}.$$

$$\frac{5 \times 2}{6 \times 2} = \frac{10}{12}.$$

$$\text{Ans. } \frac{8}{12}, \frac{10}{12}, \frac{7}{12}.$$

$$(3.) \quad \frac{3 \times 5}{4 \times 5} = \frac{15}{20}.$$

$$\frac{4 \times 4}{5 \times 4} = \frac{16}{20}.$$

$$\frac{9 \times 2}{10 \times 2} = \frac{18}{20}.$$

$$\text{Ans. } \frac{15}{20}, \frac{16}{20}, \frac{18}{20}, \frac{11}{20}.$$

## CASE VII.

### Art. 139.

(1.) The L. C. M. of 3, 4, and 6 = 12 :

$$\frac{1 \times 4}{3 \times 4} = \frac{4}{12}; \quad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}; \quad \frac{5 \times 2}{6 \times 2} = \frac{10}{12}. \quad \text{Ans. } \frac{4}{12}, \frac{9}{12}, \frac{10}{12}.$$

(2.) The L. C. M. of 2, 5, 10, and 4 is 20 :

$$\frac{1 \times 10}{2 \times 10} = \frac{10}{20}; \quad \frac{3 \times 4}{5 \times 4} = \frac{12}{20}; \quad \frac{9 \times 2}{10 \times 2} = \frac{18}{20}; \quad \frac{3 \times 5}{4 \times 5} = \frac{15}{20}. \quad \text{Ans. } \frac{10}{20}, \frac{12}{20}, \frac{18}{20}, \frac{15}{20}.$$

(3.) The L. C. M. of 7, 8, and 14 is 56 :

$$\frac{3 \times 8}{7 \times 8} = \frac{24}{56}; \quad \frac{5 \times 7}{8 \times 7} = \frac{35}{56}; \quad \frac{11 \times 4}{14 \times 4} = \frac{44}{56}. \quad \text{Ans. } \frac{24}{56}, \frac{35}{56}, \frac{44}{56}.$$

$$(4.) \quad 2)\frac{6}{8} = \frac{3}{4}; \quad 3)\frac{9}{12} = \frac{3}{4}; \quad 5)\frac{15}{20} = \frac{3}{4}. \quad \text{Ans. } \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}.$$

$$(5.) \quad 3)\frac{6}{9} = \frac{2}{3}; \quad 3)\frac{9}{12} = \frac{3}{4}; \quad 4)\frac{12}{20} = \frac{3}{5};$$

$$\text{L. C. M. of 3, 4, 5, and 10, is 60: } \frac{2 \times 20}{3 \times 20} = \frac{40}{60};$$

$$\frac{3 \times 15}{4 \times 15} = \frac{45}{60}; \quad \frac{3 \times 12}{5 \times 12} = \frac{36}{60}; \quad \frac{7 \times 6}{10 \times 6} = \frac{42}{60}. \quad \text{Ans. } \frac{40}{60}, \frac{45}{60}, \frac{36}{60}, \frac{42}{60}.$$

$$(6.) \quad 1\frac{3}{4} = \frac{7}{4}; \quad 3\frac{2}{3} = \frac{11}{3}; \quad \frac{3 \times 25}{10 \times 7} = \frac{15}{14};$$

$$\text{L. C. M. of 4, 3, and 14, is 84; } \frac{7 \times 21}{4 \times 21} = \frac{147}{84};$$

$$\frac{11 \times 28}{3 \times 28} = \frac{308}{84}, \quad \frac{15 \times 6}{14 \times 6} = \frac{90}{84}. \quad \text{Ans. } \frac{147}{84}, \frac{308}{84}, \frac{90}{84}.$$

## ALLIGATION ALTERNATE

### CASE I.

Art. 363.

(1.)

			Balance.	lb.		Balance	lb.		Balance	lb.
28	25	3	17	2	19	2	4	6	17	17
	27	<u>1</u>	4	4	4	17		17	4	2
	30	2		3	3 or,	3		3 or,	1	1
	32	4	1	1	1	3	3	3	1	1
	45	17	3	3	3		1	1	3	3
					Ans.			Ans.		Ans.

(2.)

			Balance.	lb.		Balance	lb.
$6\frac{3}{4}$	5	$\frac{7}{4}$		1	1	5	5
	$5\frac{1}{2}$	$\frac{5}{4}$	5		5	1	1
	6	$\frac{3}{4}$		5	5 or,	1	1
	7	$\frac{1}{4}$		7	7	5	3
	8	$\frac{5}{4}$	5	3	7	7	7
					Ans.		Ans.

(3.)

			Balance.	Gal.		Balance	Gal.
87	84	3	9	1	10	7	7
	86	<u>1</u>		7	7	9	1
	88	1		3	3 or,	1	1
	94	7	1		1	3	3
	96	9	3		3	1	1
					Ans.		Ans.

OPERATION			
	Bulks per lb.	Diff	Bal.
$\frac{25}{421}$	$\frac{2}{21}$	$\frac{3487}{97251}$	723
	$\frac{4}{77}$	$\frac{723}{97251}$	3487

**Explanation.**—Since silver has sp. gr.  $\frac{21}{2}$ , *one lb.* silver has  $\frac{2}{21}$  the bulk of one lb. water; since gold has sp. gr.  $\frac{77}{4}$ , *one lb.* gold has  $\frac{4}{77}$  the bulk of one lb. water; the combination having sp. gr.  $\frac{421}{25}$ , *one lb.* of the combination must have  $\frac{25}{421}$  the bulk of one lb. water. If we take the *whole* in silver, each pound will have  $\frac{1}{\frac{3487}{97251}}$  bulk *too great*;

if the whole be taken gold, each pound will have  $\frac{723}{97251}$  bulk *too small*; hence, balance in ratio of 723 lb. silver to 3487 lb. gold, *Ans.*

**Remark.**—If the sp. gr. of a substance be 4, one pound of it will have a bulk equal to  $\frac{1}{4}$  the bulk of a pound of water; if sp. gr. be  $\frac{21}{2}$ , a pound of the substance will have the bulk of  $\frac{2}{21}$  of a pound of water. By thus *inverting* the numbers expressing the sp. gravities of different things, their bulks may be directly compared, just as we compare the *prices per pound* in other examples. When we compare *prices*, in a common example, we balance the losses against the gains, *calling* the units in the balancing, "*pounds*," though they may be transferred from a column of prices, named in "*cents*."

**Illustration.**—Suppose, where the average price is  $\frac{2}{9}$  ct. we find a *loss* of  $\frac{714}{189}$  *cents on one kind*, and gain on *another kind* of a lb.  $\frac{24}{189}$  *cents*, we take 714 of the latter kind of *pounds*, and 24 *pounds* of the former. The specific gravity came would read: We lose on a pound  $\frac{714}{189}$  bulks, and gain on another pound  $\frac{24}{189}$  *bulks*; hence balance by 714 of latter to 24 of former in *pounds*.

$\frac{2}{9}$	$\frac{2}{21}$	$\frac{24}{189}$	714
	$\frac{4}{1}$	$\frac{714}{189}$	24

(5.)			
	Bal.	lb.	
$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{8}$	or,
	$\frac{9}{10}$	$\frac{1}{40}$	
	$\frac{1}{40}$	$\frac{1}{8}$	<i>Ans.</i>

(6.)			
	Bal.	Parts.	
22	24	$\frac{2}{2}$	3
	20	$\frac{2}{2}$	1
	18	$\frac{2}{4}$	1
			<i>Ans.</i>

## CASE II.

### Art. 364.

(1.)			
	Bal.		
60	50	10	$\times 8 = 96,$
	72	12	$\times 8 = 80.$
			<i>Ans.</i>

(2.)			
	Bal.		
65	40	25	$2 \times 50 =$
	50	15	2
	60	5	2
	75	10	5
			$4 + (5 \times 2) = 254,$
			<i>Ans.</i>

(3.)			
	Bal.		
56	0	56	$\times \frac{29}{8} = \frac{145}{8}$ pt. = 2 gal. 1 qt. $\frac{1}{8}$ pr., <i>Ans.</i>
	91	35	$\times \frac{29}{8} = 29$ pt.

$$(4.)$$

			gr.			
18	16	2	1.8	× $\frac{81}{1.8}$ = 3		
				pwt. 9 gr.		
	21.6	3.6	1	× $\frac{81}{1.8}$ =		
	1 pwt. 21 gr.,			Ans.		

$$(5.)$$

						pt.	
60	0	$\frac{60}{18}$	3	3	$\frac{9}{5}$	$\frac{12}{5}$	= $\frac{21}{5}$
	78	$\frac{18}{36}$	5	10	3	8	pt. =
	96	$\frac{36}{5}$					$4\frac{1}{5}$ pt.
							Ans.

$$(6.)$$

Part.	%	acidity	
$12\frac{1}{2}$ pt. ×	0	=	0
$7\frac{1}{2}$ “ ×	100	=	<u>750</u>
20			)750(37 $\frac{1}{2}$ %
			obtained.

  

	15		pt.	
	22 $\frac{1}{2}$	0	22.5	31
desired,				× $\frac{7.5}{9}$ = 25 $\frac{5}{6}$ pt.
				= 12 $\frac{1}{2}$
		100	77.5	9
				13 $\frac{1}{3}$ pt.
				1 gal. 2 qt. 1 $\frac{1}{3}$ pt., Ans.

**Explanation.**—The combination is to *displace* water equal to *once* its weight, and, hence, 1 is the average. The lead, while in the water, displaces  $\frac{1}{11}$  of its own weight; the copper displaces  $\frac{1}{9}$  of its own weight; the cork, when wholly in water, displaces 4 times its own weight.

Hence, the piece, say 1 oz., of lead *lacks* displacing  $\frac{10}{11}$  of an oz.; 1 oz. of copper lacks displacing  $\frac{8}{9}$ ; a cork oz. displaces *too much*, by 3 times its weight; hence, balancing, we take 3 oz. of lead for each  $\frac{10}{11}$  oz. of cork, and 3 oz. of copper for each  $\frac{8}{9}$  oz. of cork. But the conditions require only  $\frac{1}{6}$  of 3 oz. of copper; hence, to balance that requires  $\frac{1}{5}$  of  $\frac{8}{9}$ , or  $\frac{8}{54}$  oz. cork. The conditions also require 12 oz. cork *in all*; therefore,  $12 - \frac{8}{54}$ , or  $\frac{320}{27}$  oz. cork are yet required, and as this contains  $\frac{10}{11}$ ,  $\frac{352}{27}$  times, there must be, by first balancing,  $\frac{352}{27} \times 3$ , or  $\frac{352}{9}$  oz. lead, which is 2 lb. 7 $\frac{1}{9}$  oz., *Ans.*

**Explanation.**—First, balancing in proportion to bulks, 40 of cork to 3 of lead, 32 of cork to 3 of copper. This makes their actual weights, as  $40 \times \frac{1}{4}$  to  $11 \times 3$ , and  $32 \times \frac{1}{4}$  to  $9 \times 3$ . Then the  $\frac{1}{2}$  oz. copper requiring  $\frac{8}{54}$  oz. cork, the remaining  $\frac{640}{54}$  oz. cork require 2 lb. 7 $\frac{1}{9}$  oz. lead, *Ans.*

$$(7.)$$

OPERATION.

1	1	$\frac{10}{11}$	3	3	$\frac{1}{2}$	× $\frac{352}{27}$ = $\frac{352}{9}$	(1st.)
	$\frac{1}{9}$	$\frac{8}{9}$		3	$\frac{8}{54}$	= $\frac{352}{9}$ oz.,	Ans.
	4	3	$\frac{10}{11}$	$\frac{8}{9}$	$\frac{8}{54}$	$\frac{640}{54}$	
	$12 - \frac{8}{54} = \frac{640}{54}$ , $\frac{640}{54} \div \frac{10}{11} = \frac{352}{27}$ .						

OPERATION. (2d.)

		proper. bulks.		proper. wrights.	
1	$\frac{1}{4}$	$\frac{3}{4}$	40	32	10
	9	8		3	8
	11	10	3	3	27
	=				$\frac{8}{54} + \frac{640}{54}$
					$\frac{1}{2}$
					$\frac{352}{9}$ oz.,

(8.)

$\begin{array}{rcl} 120 \times 74 & = & 8880 \\ 150 \times 68 & = & 10200 \\ 130 \times 54 & = & 7020 \\ \hline 400 & & 26100 \end{array}$	<p>Since 60 is 120% of that desired,  <math>60 \div 120</math>, or 50% is the required          average</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;">50</div> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"><math>65\frac{1}{4}</math></td> <td style="border-right: 1px solid black; padding: 0 5px;"><math>15\frac{1}{4}</math></td> <td style="padding: 0 5px;">10</td> <td><math>\times 40 = 400</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">40</td> <td style="border-right: 1px solid black; padding: 0 5px;">10</td> <td style="padding: 0 5px;"></td> <td><math>15\frac{1}{4} \times 40 = 610</math></td> </tr> </table> </div> <p style="text-align: right;">610 shares, <i>Ans.</i></p>	$65\frac{1}{4}$	$15\frac{1}{4}$	10	$\times 40 = 400$	40	10		$15\frac{1}{4} \times 40 = 610$
$65\frac{1}{4}$	$15\frac{1}{4}$	10	$\times 40 = 400$						
40	10		$15\frac{1}{4} \times 40 = 610$						

(9.)

	\$				bbl.										
$\begin{array}{rcl} 400 \times 7.50 & = & 3000 \\ 640 \times 7.25 & = & 4640 \\ 960 \times 6.75 & = & 6480 \\ \hline 2000 & & 14120 \end{array}$			req. av. 6.50												
					<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">7.06</td> <td style="border-right: 1px solid black; padding: 0 5px;">.56</td> <td style="padding: 0 5px;">100</td> <td><math>\times 20 = 2000</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">5.50</td> <td style="border-right: 1px solid black; padding: 0 5px;">1</td> <td style="padding: 0 5px;">56</td> <td><math>\times 20 = 1120</math></td> </tr> </table>	7.06	.56	100	$\times 20 = 2000$	5.50	1	56	$\times 20 = 1120$		
7.06	.56	100	$\times 20 = 2000$												
5.50	1	56	$\times 20 = 1120$												
						bbl., <i>Ans.</i>									
						average now.									

### Case III.

#### Art. 365.

(1.)

The given lbs. and prices make an average of  $5\frac{1}{7}$  ct. Then,

$6 \left  \begin{array}{c} 3 \\ 5\frac{1}{7} \\ 7 \end{array} \right  \left  \begin{array}{c} 3 \\ \frac{6}{7} \\ 1 \end{array} \right  \left  \begin{array}{c} 1 \\ 7 \\ 3 \end{array} \right  \left  \begin{array}{c} 7 \\ 6 \end{array} \right $	<p><math>\frac{3}{4}</math> lb., <i>Ans.</i>      The balancing requires 1 of the first for 3 of the third; and 7 of the second for 6 of the third.</p> <p><math>6 + 2\frac{1}{4} = 8\frac{1}{4}</math> lb., <i>Ans.</i>      This gives the required 7; but as there are 16 – 13, or 3 yet required, and as these must be taken</p>
---	--

in proportion, *as one* of the first to *three* of the third, we take  $\frac{3}{4}$  of the first balance column and add it to the second; having  $\frac{3}{4} + 7 + 8\frac{1}{4} = 16$ .

(2.)

	bbl.	pr.	\$		
	300	$\times 7.50$	=	2250	
	800	$\times 7.80$	=	6240	
	400	$\times 7.65$	=	3060	
	<u>1500</u>			<u>11550</u>	(\$7.70, av. for 1500 bbl.)

As there are to be 2000 bbl., there are yet 500 bbl. required, and their price must be  $\$(2000 \times 7.85 - 11550) \div 500 = \$8.30$ , the av. Hence,

(3.)

$$8.30 \left| \begin{array}{c} 8 \\ 8.50 \end{array} \right| \left| \begin{array}{c} .30 \\ .20 \end{array} \right| \left| \begin{array}{c} 2 \\ 3 \end{array} \right| \times 100 = \left\{ \begin{array}{c} 200 \text{ bbl.} \\ 300 \text{ bbl.} \end{array} \right\} \text{Ans.}$$

lb.	\$	
$14 \times .30 =$	4.20	56 lb. - 40 lb. = 26 lb. desired.
$6 \times .60 =$	3.60	$(56 \times .40 - 17.80) \div 16 = 28\frac{3}{4}$ ct. av. for 16 lb.
40)	17.80	
	$(44\frac{1}{2}$ ct., av.	

$$28\frac{3}{4} \left| \begin{array}{c|c|c|c} 25 & 3\frac{3}{4} & 5 & \\ \hline 35 & 6\frac{1}{4} & 3 & \end{array} \right| \times 2 = \left\{ \begin{array}{c} 10 \text{ lb.} \\ 6 \text{ lb.} \end{array} \right\} \text{Ans.}$$

(4.)

If the specific gravity of a body be  $\frac{7}{1}$ , it loses  $\frac{1}{7}$  in water; so, copper loses  $\frac{4}{31}$ , and silver  $\frac{2}{21}$ , while the required loss of combination weight is  $\frac{5}{43}$ ; hence,

$$\frac{5}{43} \left| \begin{array}{c|c|c|c} \frac{4}{31} & \frac{357}{27993} & 589 & \\ \hline \frac{2}{21} & \frac{589}{27993} & 357 & \\ \hline & & 946 & \end{array} \right| \times \frac{12}{946} = \left\{ \begin{array}{c} 7\frac{223}{473} \text{ oz.} \\ 4\frac{250}{473} \text{ oz.} \end{array} \right\} \text{Ans.}$$

(5.)

1 st.	gr.	
18	$\left  \begin{array}{c c c c c} 15 & 3 & 6 & 2 & 64 \\ \hline 20 & 2 & & 3 & 24 \\ \hline 24 & 6 & & 3 & 24 \end{array} \right $	2 pwt. 16 gr., <i>Ans.</i> 1 pwt., <i>Ans.</i> 1 “ <i>Ans.</i>
	$\begin{array}{cc} 9 & 5 \end{array}$	$112 \div 14 = 8.$

Taking 3's and 5's to make 112 gr., we proceed thus to find other answers.

2d.

$\left  \begin{array}{c c} 2 & 2 \\ \hline 3 & 3 \\ \hline 1 & 3 \end{array} \right $	$\left  \begin{array}{c c} 72 & 68 \\ \hline 6 & 15 \\ \hline 34 & 29 \end{array} \right $	$\left  \begin{array}{c c} 64 & 60 \\ \hline 24 & 33 \\ \hline 24 & 19 \end{array} \right $	$\left  \begin{array}{c c} 56 & 52 \\ \hline 42 & 51 \\ \hline 14 & 9 \end{array} \right $	$\left  \begin{array}{c c} 48 & 40 \\ \hline 60 & 30 \\ \hline 4 & 1 \end{array} \right $
$\begin{array}{cc} 3 & 5 \end{array}$				

(6.)

4	$\left  \begin{array}{c c} 9 & 5 \\ \hline 2 & 2 \\ \hline 1 & 3 \end{array} \right $	$\left  \begin{array}{c c} 3 & 2 \\ \hline & 5 \\ \hline 5 & \end{array} \right $	$\left  \begin{array}{c c} 29 & 30 \\ \hline 68 & 60 \\ \hline 3 & 10 \end{array} \right $	$\left  \begin{array}{c c} 31 & 32 \\ \hline 52 & 44 \\ \hline 17 & 24 \end{array} \right $	$\left  \begin{array}{c c} 33 & 34 \\ \hline 36 & 28 \\ \hline 31 & 38 \end{array} \right $	$\left  \begin{array}{c c} 35 & 36 \\ \hline 20 & 12 \\ \hline 45 & 52 \end{array} \right $	$\left  \begin{array}{c c} 37 & 4 \\ \hline & \end{array} \right $	calves,
								hogs,
								lambs.
		$\begin{array}{cc} 8 & 7 \end{array}$						

Take 5ths of 8 and 5ths of 7 to make 100; *or whole 8's and whole 7's to make 500.*

(7.)

If a body have a specific gravity of 2, in water it displaces  $\frac{1}{2}$  its own weight; if its sp. gr. be

$\frac{4}{3}$ , it displaces in like manner  $\frac{3}{4}$ ; so the crown, sp. gr.  $\frac{117}{8}$ , displaced  $\frac{8}{117}$ , and thus with the two metals. Hence, the question is,—*If gold displace, in water*

$$\frac{4}{77} \left| \begin{array}{c|c|c|c} \frac{2}{21} & \frac{726}{27027} & 74 & \\ \hline \frac{4}{77} & \frac{444}{27027} & 121 & \\ \hline & & 195 & \end{array} \right| \times \frac{17.5}{195} = \left\{ \begin{array}{c} 6\frac{25}{39} \text{ silver.} \\ 10\frac{67}{78} \text{ gold.} \end{array} \right\} \text{Ans.}$$

$\frac{4}{77}$  of its own weight, and silver  $\frac{2}{21}$  of its own weight, how should these be combined so as to displace  $\frac{8}{117}$  of their weight? The above balancing shows their actual weights should combine as 74 to 121; i.e., the gold should weigh  $\frac{121}{195}$  of the *combined* weights. The whole weight being  $17\frac{1}{2}$  lb., the weight of the gold must be  $17\frac{1}{2}$  lb.  $\times \frac{121}{195} = 10\frac{67}{78}$  lb., *Ans.*

## INVOLUTION

**Art. 370.**

- (1.)  $(5)^2 = 5 \times 5 = 25$ , *Ans.*  
 (2.)  $(14)^3 = 14 \times 14 \times 14 = 2744$ , *Ans.*  
 (3.)  $(6)^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776$ , *Ans.*  
 (4.)  $(192)^2 = 192 \times 192 = 36864$ , *Ans.*  
 (5.)  $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$ , *Ans.*  
 (6.)  $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{81}{625}$ , *Ans.*  
 (7.)  $\left(2\frac{1}{4}\right)^3 = \frac{9}{4} \times \frac{9}{4} \times \frac{9}{4} = \frac{729}{64} = 11\frac{25}{64}$ , *Ans.*  
 (8.)  $\left(\frac{7}{8}\right)^5 = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = \frac{16807}{32768}$ , *Ans.*  
 (9.)  $(.02)^3 = .02 \times .02 \times .02 = .000008$ , *Ans.*  
 (10.)  $(5)^4 = 5 \times 5 \times 5 \times 5 = 625$ ;  $\therefore (5^4)^2$ , or  $5^8 = 625 \times 625 = 390625$ , *Ans.*  
 (11.)  $(.046)^3 = .046 \times .046 \times .046 = .000097336$ , *Ans.*  
 (12.)  $\left(\frac{1}{9}\right)^7 = \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} = \frac{1}{4782969}$ , *Ans.*  
 (13.)  $(2056)^2 = 2056 \times 2056 = 4227136$ , *Ans.*  
 (14.)  $(7.62\frac{1}{2})^2 = 7.62\frac{1}{2} \times 7.62\frac{1}{2} = 58.1406\frac{1}{4}$ , *Ans.*

**Art. 371.**

- (1.)  $19^2 = (10 + 9)^2 = 100 + 2(10 \times 9) + 81 = 361$ , *Ans.*  
 (2.)  $29^2 = (20 + 9)^2 = 400 + 2(20 \times 9) + 81 = 841$ , *Ans.*  
 (3.)  $4^2 = (1 + 3)^2 = 1 + 2(1 \times 3) + 9 = 16$ , *Ans.*  
 (4.)  $40^2 = (30 + 10)^2 = 900 + 2(30 \times 10) + 100 = 1600$ , *Ans.*  
 (5.)  $125^2 = (100 + 25)^2 = 10000 + 2(100 \times 25) + 625 = 15625$ , *Ans.*  
 (6.)  $59^2 = (50 + 9)^2 = 2500 + 2(50 \times 9) + 81 = 3481$ , *Ans.*

**Art. 372.**

- (1.)  $19^3 = (10 + 9)^3 = 1000 + 3(100 \times 9) + 3(10 \times 81) + 729 = 6859$ , *Ans.*  
 (2.)  $29^3 = (20 + 9)^3 = 8000 + 3(400 \times 9) + 3(20 \times 81) + 729 = 24389$ , *Ans.*  
 (3.)  $4^3 = (1 + 3)^3 = 1 + 3(1 \times 3) + 3(1 \times 9) + 27 = 64$ , *Ans.*  
 (4.)  $40^3 = (20 + 20)^3 = 8000 + 3(400 \times 20) + 3(20 \times 400) + 8000 = 64000$ , *Ans.*  
 (5.)  $125^3 = (120 + 5)^3 = 1728000 + 3(14400 \times 5) + 3(120 \times 25) + 125 = 1953125$ , *Ans.*  
 (6.)  $216^3 = (200 + 16)^3 = 8000000 + 3(40000 \times 16) + 3(200 \times 256) + 4096 = 10077696$ , *Ans.*

## EVOLUTION

## EXTRACTION OF THE SQUARE ROOT

Art. 375.

$$\begin{array}{r}
 \text{(1.)} \\
 2\dot{8}0\dot{9}(53, \text{ Ans.} \\
 25 \\
 103 \overline{) \begin{array}{l} 309 \\ 309 \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(2.)} \\
 1\dot{4}4\dot{4}(38, \text{ Ans.} \\
 9 \\
 68 \overline{) \begin{array}{l} 544 \\ 544 \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(3.)} \\
 \dot{1}1\dot{8}\dot{8}\dot{1}(109, \text{ Ans.} \\
 1 \\
 209 \overline{) \begin{array}{l} 1881 \\ 1881 \end{array}}
 \end{array}$$

$$\begin{array}{r}
 \text{(4.)} \\
 1\dot{8}\dot{5}\dot{6}\dot{4}0\dot{6}\dot{2}\dot{5}(13625, \text{ Ans.} \\
 1 \\
 23 \overline{) 85} \\
 69 \\
 266 \overline{) 1664} \\
 1596 \\
 2722 \overline{) \quad 6806} \\
 5444 \\
 27245 \overline{) 136225} \\
 136225
 \end{array}$$

$$\begin{array}{r}
 \text{(5.)} \\
 8\dot{0}0\dot{1}2\dot{3}0\dot{4}(8944.9-, \text{ Ans.} \\
 64 \\
 169 \overline{) \begin{array}{l} 1601 \\ 1521 \end{array}} \\
 1784 \overline{) \quad 8023} \\
 7136 \\
 17884 \overline{) \quad 88704} \\
 71536 \\
 178889 \overline{) 1716800} \\
 1610001 \\
 6799
 \end{array}$$

$$\begin{array}{r}
 \text{(6.)} \\
 \dot{6}2\dot{0}3\dot{7}9\dot{4}(2490.74, \text{ Ans.} \\
 4 \\
 44 \overline{) \begin{array}{l} 220 \\ 176 \end{array}} \\
 489 \overline{) \begin{array}{l} 4437 \\ 4401 \end{array}} \\
 49807 \overline{) \quad 369400} \\
 348649 \\
 498144 \overline{) \quad 20751} \\
 19926-
 \end{array}$$

$$\begin{array}{r}
 \text{(7.)} \\
 \dot{3}4\dot{4}\dot{4}\dot{7}3\dot{6}(1856, \text{ Ans.} \\
 1 \\
 28 \overline{) \begin{array}{l} 244 \\ 224 \end{array}} \\
 365 \overline{) \quad 2047} \\
 1825 \\
 3706 \overline{) \quad 22236} \\
 22236
 \end{array}$$

$$\begin{array}{r}
 \text{(8.)} \\
 57\dot{6}0\dot{0}(240, \text{ Ans.}) \\
 4 \\
 44 \overline{) 176} \\
 \underline{176} \\
 00
 \end{array}$$

$$\begin{array}{r}
 \text{(9.)} \\
 1649984\dot{4}(4062, \text{ Ans.}) \\
 1600 \\
 806 \overline{) 4998} \\
 \underline{4836} \\
 8122 \overline{) 16244} \\
 \underline{16244}
 \end{array}$$

$$\begin{array}{r}
 \text{(10.)} \\
 4909804\dot{9}(7007, \text{ Ans.}) \\
 49 \\
 14007 \overline{) 98049} \\
 \underline{98049}
 \end{array}$$

$$\begin{array}{r}
 \text{(13.)} \\
 3.00000\dot{0}(1.7320508, \text{ Ans.}) \\
 1 \\
 27 \overline{) 200} \\
 \underline{189} \\
 343 \overline{) 1100} \\
 \underline{1029} \\
 3462 \overline{) 7100} \\
 \underline{6924} \\
 3464+ \overline{) 17600} \\
 \underline{17320+} \\
 279 \\
 \underline{277}
 \end{array}$$

$$\begin{array}{r}
 \text{(11.)} \\
 73005\dot{5}(270.194, \text{ Ans.}) \\
 4 \\
 47 \overline{) 330} \\
 \underline{329} \\
 5401 \overline{) 10500} \\
 \underline{5401} \\
 54029 \overline{) 509900} \\
 \underline{486261} \\
 540384 \overline{) 2363900} \\
 \underline{2161536}
 \end{array}$$

$$\begin{array}{r}
 \text{(14.)} \\
 9.869604401\dot{0}(3.1415926, \text{ Ans.}) \\
 9 \\
 61 \overline{) 86} \\
 \underline{61} \\
 624 \overline{) 2596} \\
 \underline{2496} \\
 6281 \overline{) 10004} \\
 \underline{6281} \\
 62825 \overline{) 372340} \\
 \underline{314125} \\
 58215 \\
 \underline{56543} \\
 1672 \\
 \underline{1257} \\
 415 \\
 \underline{377}
 \end{array}$$

$$\begin{array}{r}
 \text{(12.)} \\
 386^3 = 5751245\dot{6}(7583.69, \text{ Ans.}) \\
 49 \\
 45 \overline{) 851} \\
 \underline{725} \\
 1508 \overline{) 12624} \\
 \underline{12064} \\
 15163 \overline{) 56056} \\
 \underline{45489} \\
 10567 \\
 \underline{9098} \\
 1469 \\
 \underline{1365}
 \end{array}$$

(15.)  $\sqrt{.030625} = .175$ ;  $\sqrt{40.96} = 6.4$ ;  $\sqrt{.00000625} = .0025$ ;  $.175 \times .0025 \times 6.4 = .0028, \text{ Ans.}$

(16.)  $126 \times 58 \times 604 = 4414032, \text{ Ans.}$

$$(17.) \sqrt{12.96} \times \sqrt{\frac{5}{6}} = \sqrt{10.8} = 3.2863, \text{ Ans.}$$

**Art. 377.**

$$(1.) \sqrt{\frac{6}{7}} = \sqrt{\frac{42}{49}} = \frac{1}{7}\sqrt{42} = .92682+, \text{ Ans.}$$

$$(2.) \sqrt{34\frac{5}{8}} = \sqrt{34.625} = 5.8843+, \text{ Ans.}$$

$$(3.) \sqrt{\frac{4}{7}} = \sqrt{\frac{28}{49}} = \frac{1}{7}\sqrt{28} \text{ and not } \frac{1}{7}\sqrt{36}; \text{ hence, } \frac{5}{7} \text{ more nearly than } \frac{6}{7}, \text{ Ans.}$$

$$(4.) \sqrt{272.25} = 16.5, \text{ Ans.}$$

$$(5.) \sqrt{6.40} = 2.5298+, \text{ Ans.}$$

$$(6.) \frac{28}{57} \times \frac{392}{2527} \times \frac{35}{38} \times \frac{3}{1} = \frac{784}{361 \times 361} \times 35; \text{ hence, sq. rt.} = \frac{28}{361}\sqrt{35} = 5.9160798 \times \frac{28}{361} = .45886+, \text{ Ans.}$$

$$(7.) \sqrt{123.454321} \times .81 = 11.111 \times .9 = 9.9999, \text{ Ans.}$$

$$(8.) \sqrt{1.728 \times 4.8 \times \frac{3}{7}} = \sqrt{1.44 \times 1.44 \times 4 \times \frac{3}{7}} = 1.2 \times 1.2 \times 2 \times \frac{1}{7}\sqrt{21} = \frac{2.88}{7}\sqrt{21}, \text{ Ans.}$$

**EXTRACTION OF THE CUBE ROOT****Art. 380.**

(1.)

$$\begin{array}{r} 51\dot{2}(8, \text{ Ans.} \\ \underline{512} \end{array}$$

(2.)

$$19\dot{6}8\dot{3}(27, \text{ Ans.}$$

8

$$\begin{array}{r} 4 \times 300 = 1200 \\ 2 \times 7 \times 30 = 420 \\ 7 \times 7 = \underline{49} \\ 1669 \\ \hline 11683 \end{array}$$

(3.)

$$\dot{7}30\dot{1}38\dot{4}(194, \text{ Ans.}$$

1

$$\begin{array}{r} 1 \times 300 = 300 \\ 1 \times 9 \times 30 = 270 \\ 9 \times 9 = 81 \\ \hline 651 \\ 361 \times 300 = 108300 \\ 19 \times 4 \times 30 = 2280 \\ 4 \times 4 = 16 \\ \hline 110596 \end{array} \begin{array}{r} \hline 6301 \\ \\ \\ \hline 5859 \\ 442384 \\ \\ \hline 442384 \end{array}$$

(4.)

$$9\dot{4}81\dot{8}81\dot{6}(456, \text{ Ans.}$$

64

$$\begin{array}{r} 16 \times 300 = 4800 \\ 4 \times 5 \times 30 = 600 \\ 5 \times 5 = 25 \\ \hline 5425 \\ 2025 \times 300 = 607500 \\ 45 \times 6 \times 30 = 8100 \\ 6 \times 6 = 36 \\ \hline 615636 \end{array} \begin{array}{r} \hline 30818 \\ \\ \\ \hline 27125 \\ 3693816 \\ \\ \hline 3693816 \end{array}$$

(5.)

$\dot{1}0\dot{6}\dot{7}4\dot{6}\dot{2}64\dot{8}(1022, Ans.$

$$\begin{array}{r|l}
 1 & \\
 \hline
 1 \times 300 = 300 & 67 \\
 \hline
 100 \times 300 = 30000 & 67462 \\
 10 \times 2 \times 30 = & 600 \\
 2 \times 2 = & 4 \\
 \hline
 30604 & 61208 \\
 \hline
 10404 \times 300 = 3121200 & 6254648 \\
 102 \times 2 \times 30 = & 6120 \\
 2 \times 2 = & 4 \\
 \hline
 3127324 & 6254648
 \end{array}$$

(6.)

$\dot{5}.08\dot{8}44\dot{8}(1.72, Ans.$

$$\begin{array}{r|l}
 1 & \\
 \hline
 300 & 4088 \\
 210 & \\
 49 & 3913 \\
 \hline
 559 & 175448 \\
 \hline
 86700 & \\
 1020 & \\
 4 & \\
 \hline
 87724 & 175448
 \end{array}$$

(7.)

$2\dot{2}18\dot{8}.04\dot{1}(28.1, Ans.$

$$\begin{array}{r|l}
 8 & \\
 \hline
 1200 & 14188 \\
 480 & \\
 64 & 13952 \\
 \hline
 1744 & 236041 \\
 \hline
 235200 & \\
 840 & \\
 1 & \\
 \hline
 2360411 & 236041
 \end{array}$$

(8.)

$3\dot{2}.6\dot{5}\dot{0}(3.196154+, Ans.$

$$\begin{array}{r|l}
 27 & \\
 \hline
 2700 & 5650 \\
 90 & \\
 1 & 2791 \\
 \hline
 2791 & 2859000 \\
 \hline
 288300 & \\
 8370 & \\
 81 & 2670759 \\
 \hline
 296751 & 188241000 \\
 \hline
 30528300 & \\
 57420 & \\
 36 & 183514536 \\
 \hline
 30585756 & 4726464 \\
 & 305858 \\
 \hline
 & 166788 \\
 & 15292 \\
 \hline
 & 1386
 \end{array}$$

(9.)

$.00\dot{7}90\dot{0}(.1991632+, Ans.$

$$\begin{array}{r|l}
 1 & \\
 \hline
 300 & 6900 \\
 270 & \\
 81 & 5859 \\
 \hline
 651 & 1041000 \\
 \hline
 108300 & \\
 5130 & \\
 81 & 1021599 \\
 \hline
 113511 & 19401000 \\
 \hline
 11880300 & \\
 5970 & \\
 1 & \\
 \hline
 11886271 & 11886271 \\
 & 7514729 \\
 & 713176 \\
 \hline
 & 38296 \\
 & 3565 \\
 \hline
 & 264
 \end{array}$$

(10.)

$$\begin{array}{r} 3.009200(1.443724, \text{ Ans.} \\ 1 \\ 300 \overline{)2009} \\ 120 \\ 16 \overline{)1744} \\ 436 \overline{)265200} \\ 58800 \\ 1680 \\ 16 \overline{)241984} \\ 60496 \overline{)23216000} \\ 6220800 \\ 12960 \\ 9 \overline{)18701307} \\ 6233769 \overline{)4514693} \\ \underline{436364}, \text{ etc., as above.} \end{array}$$

(11.)

$$\begin{array}{r} \frac{23}{729} = .031550068587(.315985, \text{ Ans.} \\ 27 \\ 2700 \overline{)4550} \\ 90 \\ 1 \overline{)2791} \\ 2791 \overline{)1759068} \\ 288300 \\ 4650 \\ 25 \overline{)1464875} \\ 292975 \overline{)294193587} \\ 29767500 \\ 85050 \\ 81 \overline{)268673679} \\ 29852631 \overline{)25519908} \\ \underline{2388210}, \text{ etc., etc.} \end{array}$$

(12.)

$$\begin{array}{r} 25(2.924018, \text{ Ans.} \\ 8 \\ 1200 \overline{)1700} \\ 540 \\ 81 \overline{)16389} \\ 1821 \overline{)611000} \\ 252300 \\ 1740 \\ 4 \overline{)508088} \\ 254044 \overline{)102912000} \\ 25579200 \\ 35040 \\ 16 \overline{)102457024} \\ 25614256 \overline{)454976} \\ \underline{2561} \\ 1988 \end{array}$$

(13.)

$$\begin{array}{r} 11(2.22398, \text{ Ans.} \\ 8 \\ 1200 \overline{)3000} \\ 120 \\ 4 \overline{)2648} \\ 1324 \overline{)352000} \\ 145200 \\ 1320 \\ 4 \overline{)293048} \\ 146524 \overline{)58952000} \\ 14785200 \\ 19980 \\ 9 \overline{)44415567} \\ 14805189 \overline{)14536433} \\ \underline{1332467} \\ 121176 \\ \underline{11844} \end{array}$$

(14.)

$$\frac{2}{3} = .66\dot{6}(.87358, \text{ Ans.})$$

19200	154666
1680	
49	146503
20929	8163666
2270700	
7830	
9	6835817
2278539	1328049
	113927
	18877
	1823

(15.)

$$\frac{4}{15} = .26\dot{6}(.64366, \text{ Ans.})$$

10800	50666
720	
16	46144
11536	4522667
1228800	
5760	
9	3703707
1234569	818960
	74074
	7822

(16.)

$$171.41\dot{6}32887\dot{5}(5.555, \text{ Ans.})$$

25 × 300 = 7500	4616
5 <sup>2</sup> × 30 = 750	
5 <sup>2</sup> = 25	
8275	41375
55 <sup>2</sup> × 300 = 907500	5041328
55 × 5 × 30 = 8250	
5 × 5 = 25	
915775	4578875
555 <sup>2</sup> × 300 = 92407500	462453875
555 × 5 × 30 = 83250	
5 × 5 × 25	
92490775	462453875

(17.)

300	7011̄ (19.1393267, <i>Ans.</i> )
270	1
81	6011
651	5859
$19^2 \times 300 = 108300$	152000
$19 \times 30 = 570$	
<u>1</u>	
108871	108871
	43129000
$191^2 \times 300 = 10944300$	
$191 \times 3 \times 30 = 17190$	
<u>9</u>	
10961499	32884497
	10244503000
$1913^2 \times 300 = 1097870700$	
$1913 \times 30 \times 9 = 516510$	
$9 \times 9 = 81$	
1098387291	9885485619, etc., et.

(18.)  $\sqrt[3]{\frac{48}{4394}} = \sqrt[3]{\frac{24}{2197}} = \frac{1}{13} \sqrt[3]{24} = \frac{1}{13}$  of 2.8844991 = .2218845, *Ans.*

(19.)  $\sqrt[3]{\frac{2}{3}} \text{ of } \frac{4}{11} = \sqrt[3]{.24242424+} = .6235319, \text{ } Ans.$

## EXTRACTION OF ANY ROOT

**Art. 384.**

(1.)

0	15625(125, <i>Ans.</i> )
$\frac{1}{1}$	$\frac{1}{1}$
1	*5625
<u>1</u>	<u>44</u>
*2	*1225
<u>2</u>	<u>1225</u>
22	
<u>2</u>	
*24	
<u>5</u>	
245	

(2.)

0	0	68719476736(4096, <i>Ans.</i>
4	16	64
<hr/> 4	16	* 4719476
4	32	4417929
<hr/> 8	*48	* 302547736
4	10881	301547736
<hr/> *12	490881	
09	10962	
<hr/> 1209	*501843	
9	73656	
<hr/> 1218	*50257956	
9		
<hr/> *1227		
6		
<hr/> 12276		

(3.)

0	0	0	0	14348907(27, <i>Ans.</i>
2	4	8	16	*11148907
4	12	32	*80	11148907
6	24	*80	1592701	
8	*40	113243		
*10	4749			
107				

(4.)

0	0	151(5.325074, <i>Ans.</i>
5	25	125
5	50	* 26.00
10	*75	23.877
5	4.59	* 2.123000
*15	79.59	1.691768
.3	4.68	* .431232000
15.3	*84.27	424935125
.3	.3184	* .006296875
15.6	84.5884	595467
.3	.3188	34220
*15.9	*84.9072	
.02	.079825	
15.92	84.987025	
.02	.079850	
15.94	*85.066875	
.02		
*15.96		
.005		
15.965		
15.970		
*15.975		

(5.) Proceeding by Art. 382, we have  $\sqrt[4]{97.41} = \text{sq. rt. of } \sqrt{97.41}$ , which =  $\sqrt{9.86965045} = 3.14159$ , or, 3.1416, *Ans.*

(6.)  $\sqrt[4]{1.08} = \sqrt{\text{sq. rt. of } 1.08} = \sqrt{1.03923048} = 1.01943$ , *Ans.*

(7.)

		$\frac{5}{12} = .41\dot{6} :$			
0	0	0	0	.4166(.83938,	<i>Ans.</i>
8	64	512	4096	889866667	
16	192	2048	204800000	227626024	
24	384	5120000	220746881	9381865	
32	64000	5315627	237291605	1944016	
400	65209	5514908	24249351		
403	66427	5717870	24775122		
406	67654	5799	2479283		
409	<del>68890</del>	58419	2481054		
412		59039			

(8.)

$$35.2 = 32 \times 1.10. \quad \text{Hence root} = 2\sqrt[5]{1.10}$$

0	0	0	0	1.10	(1.01924
1	1	1	1	*.10	
2	3	4	*5	*.0489899499	
3	6	*10	5.10100501	*.0013207560	
4	*10.	10.100501	*5.20302005	109721	
*5.01	10.0501	10.201504	5.2965771	22354	
5.02	10.1003	*10.303010	5.390967+	21944	
5.03	*10.1506	10.395228	951	410	
5.04	10.2010	10.487854	5.4860		
*5.05	10.2464	92	&c.,		
&c.,	10.2918	10.579			

$$1.01924 \times 2 = 2.03848, \text{ Ans.}$$

$$(8.) \sqrt[12]{782757789696} = \sqrt[4]{\text{cube rt.}} = \sqrt[4]{9216} = \sqrt{96} = 9.79795897, \text{ Ans.}$$

$$(9.) \sqrt{1367631} = \sqrt[3]{\text{cube rt.}} = \sqrt[3]{111} = 4.8058955, \text{ Ans.}$$

## APPLICATIONS OF SQUARE AND CUBE ROOT

**Art. 388.**

$$(1.) \sqrt{30^2 + 12^2} = \sqrt{1044} = 32.31+ \text{ ft.}, \text{ Ans.}$$

$$(2.) \sqrt{10^2 \times 2} = 10\sqrt{2} = 14.142+ \text{ ft.}, \text{ Ans.}$$

$$(3.) \sqrt{69^2 + 92^2} = \sqrt{13225} = 115; \text{ and } (69 + 92) - 115 = 46 \text{ rd.}, \text{ Ans.}$$

$$(4.) \sqrt{500^2 + 360^2} = \sqrt{379600} = 616+ \text{ yd.}, \text{ Ans.}$$