

Title of Book

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PREFACE

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V. MULTIPLICATION

- 59.** 1. **Multiplication** is taking one number as many times as there are units in another; or,
 2. Multiplication is a short method of adding numbers that are equal.
- 60.** The number to be taken, is called the *Multiplicand*; the other number, the *Multiplier*; and the result obtained, the *Product*. The Multiplicand and Multiplier are together called *Factors* (makers), because they *make* the Product.

PROBLEM.—How many trees in 3 rows, each containing 42 trees?

SOLUTION.—Since 3 rows contain 3 *times* as many trees as *one* row, take 42 *three* times. This may be done by writing 42 three times, and then *adding*. This gives 126 trees for the whole number of trees.

Instead, however, of writing 42 *three* times, write it *once*; then placing under it the figure 3, the *number of times* it is to be taken, say, 3 times 2 are 6, and 3 times 1 are 12. This process is *Multiplication*.

OPERATION

First row,	42	trees
Second row,	42	trees
Third row,	42	trees
	126	trees
	42	trees
	3	
	126	trees

PRINCIPLES.—1. *The multiplicand may be either concrete or abstract.*

- 2.** *The multiplier must always be an abstract number.*
- 3.** *The product is the same in kind as the multiplicand.*
- 4.** *The product is the same, whichever factor is taken as the multiplier.*
- 5.** *The partial products are the same in kind as the multiplicand.*
- 6.** *The sum of the partial products is equal to the total product.*

MULTIPLICATION TABLE.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

- 61.** Multiplication is divided into two cases:

1. When the multiplier does not exceed 12.
2. When the multiplier exceeds 12.

CASE I

62. When the multiplier does not exceed 12.

PROBLEM.—At the rate of 53 miles an hour, how far will a railroad car run in four hours?

SOLUTION.—Here say, 4 times 3 (units) are 12 (units); write the 2 in units' place, and carry the 1 (ten); then, 4 times 5 are 20, and 1 carried makes 21 (tens), and the work is complete.

OPERATION	
53 miles.	
4	
<hr/>	
212 miles	

DEMONSTRATION.— The multiplier being written under the multiplicand for convenience, begin with units, so that if the product should contain tens, they may be carried to the tens; and so on for each successive order.

Since every figure of the multiplicand is multiplied, therefore, the *whole* multiplicand is multiplied.

Rule.—1. *Write the multiplicand, and place the multiplier under it, so that units of the same order shall stand in the same column, and draw a line beneath.*

2. *Begin with units; multiply each figure of the multiplicand by the multiplier, carrying as in Addition.*

PROOF.—Separate the multiplier into any two parts; multiply by these separately. The sum of the products must be equal to the first product.

EXAMPLES FOR PRACTICE

1. $195 \times 3.$ *Ans.*
2. $3823 \times 4.$ *Ans.*
3. $8765 \times 5.$ *Ans.*
4. $98374 \times 6.$ *Ans.*
5. $64382 \times 7.$ *Ans.*
6. $58765 \times 8.$ *Ans.*
7. $837941 \times 9.$ *Ans.*
8. $645703 \times 10.$ *Ans..*
9. $407649 \times 11.$ *Ans.*
10. If 4 men can perform a certain piece of work in 15 days, how long will it require 1 man?

SOLUTION.—One man must work four times as long as four men.

$$4 \times 15 \text{ days} = 60 \text{ days.}$$

11. How many pages in a half-dozen books, each containing 336 pages?
12. How far can an ocean steamer travel in a week, at the rate of 245 miles a day?

13. What is the yearly expense of a cotton-mill, if \$32053 are paid out every month?
14. A receives from his business an average of \$45 a day. He pays three clerks \$3; three, \$9; and three, \$12 a week; other expenses amount to \$4 a day; what are his profits for one week?

CASE II

63. When the multiplier exceeds 12.

PROBLEM.—Multiply 246 by 235.

SOLUTION.—first Multiply by 5 (units), and place the first figure of the product, 1230, under the 5 (units). Then multiply by 3 (tens), and place the product, 738, under the 3 (tens). Lastly, multiply by 2 (hundreds), and place the first figure of the product, 492, under the 2 (hundreds). Then add these several products for the entire product.

		OPERATION
	246	
	235	
1230	product by	5
738	product by	30
492	product by	200
57810	product by	235

DEMONSTRATION.—The 0 of the first product, 1230, is *units* (Art. 62). The 8 of the second product, 738, is *tens*, because 3 (tens) times 6 = 6 times 3 (tens) = 18 (tens); giving 8 (tens) to be written in the tens' column. The 2 of the third product, 492, is hundreds, because 2 (hundreds) times 6 = 6 times 2 (hundreds) = 12 (hundreds), giving 2 (hundreds) to be written in the hundreds' column. The right-hand figure of each product being in its proper column, the other figures will fall in their proper columns; and each line being the product of the multiplicand by a *part* of the multiplier, their sum will be the product by all the parts or the *whole* of the multiplier.

Rule.—1. Write the multiplier under the multiplicand, placing figures of the same order in the same column, and draw a line beneath.

2. Multiply each figure of the multiplicand by each figure of the multiplier successively; first by the units' figure, then by the tens' figure, etc.; placing the right-hand figure of each product under that figure of the multiplier which produces it, then draw a line beneath.

3. Add the several partial products together; their sum will be the required product.

METHODS OF PROOF.—1. Multiply the multiplier by the multiplicand; this product must be the same as the first product.

2. The same as when the multiplier does not exceed 12.

NOTE.—For proof by casting out the 9's, see Art. 105.

REMARK.—Although it is customary to use the figures of the multiplier in regular order beginning with units, it will give the same product to use them in any order, observing that the right-hand figure of each partial product must be placed under the figure of the multiplier which produces it.

		OPERATION
	246	
	235	
738	product by	30
492	product by	200
1230	product by	5
57810	product by	235

EXAMPLES FOR PRACTICE

1. $7198 \times 216.$ *Ans.*
2. $8862 \times 189.$ *Ans.*
3. $7575 \times 7575.$ *Ans.*
4. $15607 \times 3094.$ *Ans.*
5. $93186 \times 4455.$ *Ans.*
6. $135790 \times 24680.$ *Ans.*
7. $3523725 \times 2583.$ *Ans.*
8. $4687319 \times 1987.$ *Ans.*
9. $9264397 \times 9584.$ *Ans.*
10. $9507340 \times 7071.$ *Ans.*
11. $1644405 \times 7749.$ *Ans.*
12. $1389294 \times 8900.$ *Ans.*
13. $2778588 \times 9867.$ *Ans.*
14. $204265 \times 562402.$ *Ans.*

PRACTICAL PROBLEMS

1. In a mile are 63360 inches: how many inches are there in the circumference of the earth at the equator if the distance be 25000 miles?
2. The flow of the Mississippi at Memphis is about 434000 cubic feet a second: required the weight of water passing that point in one day of 86400 seconds, if a cubic foot of water weigh 62 pounds?
3. John Sexton sold 25625 bushels of wheat, at \$1.20 a bushel, and received in payment 320 acres of land, valued at \$50 an acre; 60 head of horses, valued at \$65 a head; 10 town lots, worth \$150 each; and the remainder in money: how much money did he receive?
4. If light comes from the sun to the earth in 495 seconds, what is the distance from the earth to the sun, light moving 192500 miles a second?
5. If 3702754100 cubic feet of solid matter is deposited in the Gulf of Mexico by the Mississippi every year, what is the deposit for 6000 years?
6. The area of Missouri is 65350 square miles: how many acres are there in the State, allowing 640 acres to each square mile?
7. In the United States, at the close of 1878, there were 81841 miles of railroad: if the average cost of building be \$50000 a mile, what has been the total cost of building the railroads in this country?

8 RAY'S NEW INTELLECTUAL ARITHMETIC

8. The number of pounds of tobacco produced in this country in 1870 was 260000000. If this were manufactured into plugs one inch wide and six inches long, and four plugs weigh a pound, what would be the length in inches of the entire crop?

BUSINESS TERMS AND EXPLANATIONS

64. A **Bill** is an account of goods sold or delivered, services rendered, or work done. Usually the price or value is annexed to each article, and the date of purchase given.

It is customary to write the total amount off to the right, and not directly under the column of amounts added.

65. A **Receipt** is a written acknowledgment of payment. The common form consists in signing the name after the words "Received Payment" written at the foot of the bill.

1. Joseph Allen bought of Seth Ward, at Springfield, Ill., Jan. 2, 1879, 30 barrels of flour, at \$3.60 a barrel; 48 barrels of mess pork, at \$16.25 a barrel; 16 boxes of candles, at \$3.50 a box; 23 barrels of molasses, at \$28.75 a barrel; and 64 sacks of coffee, at \$47.50 a sack. Place the purchases in bill form.

SOLUTION.

SPRINGFIELD, ILI., Jan. 2, 1879.

JOSEPH ALLEN,

1879

Bought of SETH WARD.

Jan	2	To 30 bl. flour,	@ \$3.60 a bl.	108	00		
"	2	" 48 " mess pork,	" 16.25 "	780	00		
"	2	" 16 Boxes candles,	" 3.50 " box	780	00		
"	2	" 23 bl. molasses,	" 28.75 " bl.	661	25		
"	2	" 64 sacks coffee,	" 47.50 " sack	3040	00		
						\$4645	25

2.

1. At St. Louis, March 1, 1879, Chester Snyder bought of Thomas Glenn, 4 lb. of tea, at 40 ct.; 21 lb. of butter, at 21 ct.; 58 lb. of bacon, at 13 ct.; 16 lb. of lard, at 9 ct.; 30 lb. of cheese, at 12 ct.; 4 lb. of raisins, at 20 ct.; and 9 doz. of eggs, at 15 ct. Place these purchases in the form of a receipted bill.

66. A **Statement of Account** is a written form rendered to a customer, showing his debits and credits as they appear on the books. The following is an example:

CINCINNATI, Feb. 2, 1880.

JOHN SNITH,

1880

In Account *with* VAN ANTWERP, BRAGG & CO.

Jan	2	To 525 McGuffey's Revised First Readers, @ 16c.	84				
"	10	" 50 Ray's New Higher Arithmetics, " 75c.	37	50			
		Cr.			121	50	
"	20	By Cash	20				
"	31	" Merchandise	12	75	32	75	
					\$88	75	

3. James Wilson & Co. bought of the Alleghany Coal Co., March 2, 1880, five hundred tons of coal, at \$2.75 a ton, and sold the same Company during the month, as follows: March 3d, 14 barrels of flour, at \$6.55 a barrel; March 10th, 6123 pounds of sugar, at 5 ct. a pound; they also paid them on account, on March 15th, cash, \$687.50. Make out a statement of account in behalf of the Alleghany Coal Co. under date of April 1, 1880.

CONTRACTIONS IN MULTIPLICATION

CASE I

67. When the multiplier is a composite number.

A **Composite Number** is the product of two or more whole numbers, each greater than 1, called its *factors*. Thus, 10 is a composite number, whose factors are 2 and 5; and 30 is one whose factors are 2, 3, and 5.

PROBLEM.—At 7 cents a piece, what will 6 melons cost?

ANALYSIS.—Three times 2 times are 6 times. Hence, it is the same to take 2 times 7, and then take this product 3 times, as to take 6 times 7. The same may be shown of any other composite number.

OPERATION

7	cents, cost of 1 melon.
2	
<hr/>	14 cents, cost of 2 melons.
3	
<hr/>	42 cents, cost of 6 melons.

Rule.—Separate the multiplier into two or more factors. Multiply first by one of the factors, then this product by another factor, and so on till each factor has been used as a multiplier. The last product will be the result required.

EXAMPLES FOR PRACTICE

1. At the rate of 37 miles a day, how far will a man walk in 28 days?
2. Sound moves about 1130 feet per second: how far will it move in 54 seconds?
3. If an engine travel at an average speed of 25 miles an hour, how far can it travel in a week, or 168 hours?

CASE II

68. When the multiplier is 1 with ciphers annexed, as 10, 100, 1000, etc.

DEMONSTRATION.—By the principles of Notation (Art. 43), placing *one* cipher on the right of a number, changes the units into tens, the tens into hundreds, and so on, and, therefore, *multiplies the number by 10*.

Annexing *two* ciphers to a number changes the units into hundreds, the tens into thousands, and so on, and, therefore, *multiplies the number by 100*. Annexing *three* ciphers multiplies the number by 1000, etc.

Rule.—*Annex to the multiplicand as many ciphers as there are in the multiplier; the result will be the required product.*

EXAMPLES FOR PRACTICE

- | | |
|---------------------------|-------------|
| (a) Multiply 743 by 10. | <i>Ans.</i> |
| (b) Multiply 375 by 100. | <i>Ans.</i> |
| (c) Multiply 207 by 1000. | <i>Ans.</i> |

CASE III

69. When ciphers are on the right in one or both factors.

PROBLEM.—Find the product of 5400 by 130.

SOLUTION.—Find the product of 54 by 13, and then annex three ciphers; that is, as many as there are on the right in both the factors.

OPERATION
5400
130
<hr/>
162
54
<hr/>
702000

ANALYSIS.—Since 13 times 54 = 702, it follows that 13 times 54 hundreds (5400) = 702 hundreds (70200); and 130 times 5400 = 10 times 13 times 5400 = 10 times 70200 = 702000.

Rule.—*Multiply as if there were no ciphers on the right in the numbers; then annex to the product as many ciphers as there are on the right in both the factors.*

EXAMPLES FOR PRACTICE

- | | |
|----------------------------|-------------|
| 1. $15460 \times 3200.$ | <i>Ans.</i> |
| 2. $30700 \times 5904000.$ | <i>Ans.</i> |

CASE IV

70. When the multiplier is a little less or a little greater than 10, 100, 1000, etc.

PROBLEM.—Multiply 3046 by 997.

ANALYSIS.—Since 997 is equal to 1000 diminished by 3, to multiply by it is the same as to multiply by 1000 (that is, to annex 3 ciphers) and by 3, and take the difference of the products; and the same can be shown in any similar case.

OPERATION
30046
997
<hr/>
3046000
9138
<hr/>
3036862

NOTE.—Where the number is a little *greater* than 10, 100, 1000, etc., the two products must be *added*.

Rule.—Annex to the multiplicand as many ciphers as there are figures in the multiplier; multiply the multiplicand by the difference between the multiplier and 100, 1000, etc., and add or subtract the smaller result as the multiplier is greater or less than 100, 1000, etc.

EXAMPLES FOR PRACTICE

1. 7023×99 . *Ans.*
2. 16642×996 . *Ans.*
3. 372051×1002 . *Ans.*

CASE V

71. When one part taken as units, in the multiplier, is a factor of another part so taken.

PROBLEM.—Multiply 387295 by 216324.

SOLUTION.—Commence with the 3 of the multiplier, and obtain the first partial product, 1161885; then multiply this product by 8, which gives the product of the multiplicand by 24 at once (since 8 times 3 times any number make 24 times it). Set the right-hand figure under the right-hand figure 4 of the multiplier in use. Multiply the second partial product by 9, which gives the product of the multiplicand by 216 (since 9 times 24 times a number make 216 times that number). Set the right-hand figure of this partial product under the 6 of the multiplicand; and, finally, add to obtain the total product.

OPERATION
387295
216324
<hr/>
1161885
9295080
83655720
<hr/>
8378123580

Rule.—1. *Multiply the multiplicand by some figure or figures of the multiplier, which are a factor of one or more parts of the multiplier.*

2. *Multiply this partial product by a factor of some other figure or figures of the multiplier, and write the right-hand figure thus obtained under the right-hand figure of the multiplier thus used.*

3. *Continue thus until the entire multiplier is used, and then add the partial products.*

EXAMPLES FOR PRACTICE

- (a) $38057 \times 48618.$ *Ans.*
 - (b) $267388 \times 14982.$ *Ans.*
 - (c) $481063 \times 63721.$ *Ans.*
 - (d) $66917 \times 849612.$ *Ans.*
 - (e) $102735 \times 273162.$ *Ans.*
 - (f) $536712 \times 729981.$ *Ans.*
-

Topical Outline.

MULTIPLICATION.

1. Definitions.

2. Terms. $\left\{ \begin{array}{l} 1. \text{ Multiplicand.} \\ 2. \text{ Multiplier.} \\ 3. \text{ Partial Product.} \\ 4. \text{ Product.} \end{array} \right.$

3. Sing.

4. Principles.

5. Operation. $\left\{ \begin{array}{l} 1. \text{ Writing Numbers.} \\ 2. \text{ Drawing Line Beneath.} \\ 3. \text{ Finding Partial Products.} \\ 4. \text{ Drawing a Line Beneath Partial Products.} \\ 5. \text{ Adding the Partial Products.} \end{array} \right.$

6. Rule.

7. Proof.

8. Applications.

9. Contractions.

MULTIPLICATION.—BILLS AND ACCOUNTS

Art. 65.

(2.)

ST. LOUIS, March 1, 1879.

CHESTER SNYDER,

1879.

Bought of Thomas Glenn.

			\$			
March	1	4 lb. tea @ 40 ct. a lb.,		1	60	
"	1	21 " butter, @ 21 ct. a lb.		4	41	
"	1	58 " bacon, " 13 ct. "		7	54	
"	1	16 " lard, " 9 ct. "		1	44	
"	1	30 " cheese, " 12 ct. "		3	60	
"	1	4 " raisins, " 20 ct. "			80	
"	1	9 doz. eggs, " 15 ct. a doz.,		1	35	
						\$20 74

Received payment,

THOMAS GLENN.

Art. 66.

(3.)

Allegheny, April 1, 1880.

JAMES WILSON & CO.,

1880.

In Acc't with ALLEGHENY COAL CO.

		DR.		\$	
March	2	To 500 tons coal, @ \$2.75 a ton,		1375	00
		CR.	\$		
"	3	By 14 bbl. flour, @ \$6.55 a bbl.,		91	70
"	10	" 6123 lb. sugar, @ 8 ct. a lb.,		489	84
"	15	" cash on acc't,		687	50
		Balance due Allegheny Coal Co.,		1269	04
				\$105	96

CONTRACTIONS IN MULTIPLICATION

Case IV.

Art. 70.

(1.)	(2.)	(3.)
$\begin{array}{r} 7023 \\ \times 99 \\ \hline 702300 \\ 7023 \\ \hline 695277 \end{array}$	$\begin{array}{r} 16642 \\ \times 996 \\ \hline 16642000 \\ 66568 \\ \hline 16575432 \end{array}$	$\begin{array}{r} 372051000 \\ \times 744102 \\ \hline 372795102 \end{array}$

Case V.

Art. 71.

(1.)	(2.)	(3.)
$\begin{array}{r} 38057 \\ \times 48618 \\ \hline 228342 \\ 685026 \\ \hline 1826736 \\ 1850255226 \end{array}$	$\begin{array}{r} 267388 \\ \times 14982 \\ \hline 5347776 \\ 3743432 \\ \hline 26204024 \\ 4006007016 \end{array}$	$\begin{array}{r} 481063 \\ \times 63721 \\ \hline 3367441 \\ 10102323 \\ \hline 30306969 \\ 30653815423 \end{array}$
(4.)	(5.)	(6.)
$\begin{array}{r} 66917 \\ \times 849612 \\ \hline 803004 \\ 6424032 \\ \hline 5621028 \\ 56853486204 \end{array}$	$\begin{array}{r} 102735 \\ \times 273162 \\ \hline 308205 \\ 2773845 \\ \hline 16643070 \\ 28063298070 \end{array}$	$\begin{array}{r} 536712 \\ \times 729981 \\ \hline 4830408 \\ 43473672 \\ \hline 391263048 \\ 391789562472 \end{array}$

ARITHMETICAL SIGNS

Art. 86.

$$(3.) \quad \left. \begin{array}{lcl} 21 \div 3 \times 7 & = +49 \\ -1 \times 1 \div 1 \times 4 \div 2 & = -2 \\ 18 \div 3 \times 6 \div 4 & = +9 \\ 1 \times 4 \times 6 \div 8 & = +3 \end{array} \right\} = 59, \text{ Ans.}$$

$$(4.) \quad \left. \begin{array}{lcl} 16 \times 4 \div 8 & = +8 \\ -7 + 48 \div 16 & = -4 \\ -3 - 28 \times 0 & = -3 \\ 24 \times 6 \div 48 & = +3 \\ -4 \times 9 \div 12 & = -3 \end{array} \right\} = 1, \text{ Ans.}$$

$$(5.) \quad \begin{array}{rcl} 16 \div 16 \times 96 \div 8 & = +12 \\ -7 - 5 + 3 & = -9 \end{array} \left. \begin{array}{l} \\ \end{array} \right\} = +3.$$

$$(27 \div 9) \div 3 - 1 = 0.$$

$$91 \div 13 \times 7 - 45 - 3 = 1.$$

Then, $3 \times 0 + 1 \times 9 = 9$, Ans.

CONTRACTIONS IN MULTIPLICATION AND DIVISION

Case I.

Art. 88.

$$\begin{array}{r} (1.) \\ 3)42200 \\ \hline 14066\frac{2}{3}, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (2.) \\ 656400 \\ \hline 8)3282000 \\ \hline 410250, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (3.) \\ 6)1072400 \\ \hline 178733\frac{2}{6}, \text{ Ans.} \end{array}$$

Case II.

Art. 89.

$$\begin{array}{r} (1.) \\ 4514020000 \\ \hline 451402 \\ \hline 3)4513568598 \\ \hline 1504522866, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (2.) \\ 281257000000 \\ \hline 281257 \\ \hline 281256718743 \\ \hline 9)1406283593715 \\ \hline 156253732635, \text{ Ans.} \end{array}$$

$$\begin{array}{r} (3.) \\ 63022400000000 \\ \hline 630224000 \\ \hline 9)6301609776000 \\ \hline 700178864000 \\ \hline 4 \\ \hline 2800715456000, \text{ Ans.} \end{array}$$

Case III.

Art. 90.

$$\begin{array}{r} (1.) \quad 225) \quad 300521761 \\ \hline 4 \\ \hline 9|00) \quad 12020870|44 \\ \text{Quot.} \quad 1335652, \text{ Ans.} \\ 244 \div 4 = 61, \text{ Rem.} \end{array}$$

$$\begin{array}{r} (2.) \quad 43750) \quad 1510337264 \\ \hline 4 \\ \hline 175000) \quad 6041349056 \\ \hline 4 \\ \hline 7|00000) \quad 241653|96224 \\ \text{Quot.} = 34521, \text{ Ans.} \\ 696224 \div 4 \div 4 = 43514, \text{ Rem.} \end{array}$$

$$\begin{array}{r}
 (3.) \quad 1406250) \quad 22500712361 \\
 \qquad \qquad \qquad 8 \qquad \qquad \qquad 8 \\
 \hline
 11250000) \quad 180005698888 \\
 \qquad \qquad \qquad 8 \qquad \qquad \qquad 8 \\
 \hline
 9|0000000) \underline{144004} |5591104 \\
 \text{Quot.} = 16000, \text{Ans.} \\
 45591104 \div 8 \div 8 = \\
 712361 = \text{Rem.}
 \end{array}$$

$$\begin{array}{r}
 (4.) \quad 20833\frac{1}{8}) \quad 620712480 \\
 \qquad \qquad \qquad 3 \qquad \qquad \qquad 3 \\
 \hline
 62500) \quad 1862137440 \\
 \qquad \qquad \qquad 4 \qquad \qquad \qquad 4 \\
 \hline
 250000) \quad 7448549760 \\
 \qquad \qquad \qquad 4 \qquad \qquad \qquad 4 \\
 \hline
 1|000000) \underline{29794} |199040 \\
 \text{Quot.} = 29794, \text{Ans.} \\
 199040 \div 4 \div 4 \div 3 = 4146\frac{2}{3}, \text{Rem.}
 \end{array}$$

$$\begin{array}{r}
 (5.) \quad 2916\frac{2}{3}) \quad 742851692 \\
 \qquad \qquad \qquad 3 \qquad \qquad \qquad 3 \\
 \hline
 8750) \quad 2228555076 \\
 \qquad \qquad \qquad 8 \qquad \qquad \qquad 8 \\
 \hline
 7|0000) \underline{1782844} |0608 \\
 \text{Quot.} = 254692, \text{Ans.} \\
 608 \div 8 \div 3 = 25\frac{1}{3}, \text{Rem.}
 \end{array}$$

Miscellaneous Exercises.

- (1.) $\$6 \times 153 = \918 ; $\$918 \div 54 = \17 , Ans.
- (2.) $217 \times 35 + 25 = 7620$, Ans.
- (3.) $48794 \div 41 = 119$, Ans.
- (4.) $103 \times 103 = 10609$, Ans.
- (5.) $53815 \div 375 = 143$ and 190 rem.; 144 times 375 = 54000, this is 185 more than 53815, and since the next lower product is 190 less than 53815, 54000 is the nearest. Ans. 54000.
- (6.) $\$2675 \div 25 = \107 ; $\$107 \times 19 = \2033 , Ans.
- (7.) $\$210 \div 15 = \14 , gain on each; $\$75 + \$14 = \$89$, Ans.
- (8.) $391 \text{ mi.} - 139 \text{ mi.} = 252 \text{ mi.}$; $11 \text{ hr.} - 4 \text{ hr.} = 7 \text{ hr.}$; $252 \text{ mi.} \div 7 = 36 \text{ miles}$, Ans.
- (9.) $235 \text{ yd.} - 12 \text{ yd.} = 223 \text{ yd.}$; $\$5 \times 235 = \1175 ; $\$7 \times 223 = \1561 ; $\$1561 - \$1175 = \$386$, Ans.
- (10.) $135 \text{ bl.} - 83 \text{ bl.} = 52 \text{ bl.}$; $\$2 \times 52 = \104 , Ans.
- (11.) $\$75 \times 5 = \375 ; $\$68 \times 12 = \816 ; $\$73 \times 17 = \1241 ; $\$375 + \$816 = \$1191$; $\$1241 - \$1191 = \$50$ gain, Ans.
Also $\$1191 + \$118 = \$1309$; $\$1309 \div 17 = \77 , Ans.
- (12.) $\$240 - \$24 = \$216$, whole cost; $\$216 \div 3 = \72 , the cost of 1 piece; $\$72 \div \$4 = 18$. Ans. 18 yards.

- (13.) 13 men $- 8$ men = 5 men; 1 man can do it in 13 times 15 days, which is 195 days; and 5 men, in $\frac{1}{5}$ of 195 days, or 39 days, *Ans.*
- (14.) $14 \text{ men} \div 7 \text{ men} = 21 \text{ men}$; 1 man can do it in 14 times 24 days, that is, 336 days; and 21 men, in $\frac{1}{21}$ of 336 days, or 16 days, *Ans.*
- (15.) For 1 day, the provisions will support 30 times 45 men, or 1350 men; for 50 days, one fiftieth of 1350 men = 27 men; 45 men $- 27$ men = 18 men, *Ans.*
- (16.) $\$18 \times 3 = \54 ; $\$85 + \$54 = \$139$, total value; $\$139 - \$41 = \$98$; 1 sheep cost one fourteenth of \$98, or \$7, *Ans.*
- (17.) A sheep and a hog cost \$13; therefore, he will buy as many of each as \$13 is contained times in \$1482; $1482 \div 13 = 114$, *Ans.*
- (18.) 1 horse and 2 oxen cost \$84; therefore, he bought as many horses as \$84 is contained times in \$1260; $1260 \div 84 = 15$; twice 15 equals 30. *Ans.* 15 horses and 30 oxen.
- (19.) One seventh of 1050 ct. = 150 ct.; $150 \div 25 = 6$, of the 25-cent pieces. 1050 ct. $- 150$ ct. = 900 ct.; 1 of each of the others would make 10 ct. + 5 ct. + 3 ct. = 18 ct.; $900 \div 18 = 50$, the number of each of the others. *Ans.* Of 25-cent pieces, 6; of the others, 50 each.
- (20.) $\$6300 \div 140 = \45 , gain per acre: $\$210 - \$45 = \$165$, the cost; $\$5600 \div 140 = \40 , loss per acre; $\$165 - \$40 = \$125$, sold for per acre. *Ans.* \$165, cost; \$125, sold for.

Art. 134.**Case IV.**

- (1.) $\$ \frac{37}{8} = \$37 \div 8 = \$4\frac{5}{8}$, *Ans.*
- (2.) $\frac{137}{4} \text{ bu.} = 137 \text{ bu.} \div 4 = 34\frac{1}{4} \text{ bu.}$, *Ans.*
- (3.) $\frac{785}{60} \text{ hr.} = 785 \text{ hr.} \div 60 = 13\frac{1}{12} \text{ hr.}$, *Ans.*
- (4.) $\frac{1295}{37} = 1295 \div 37 = 35$, *Ans.*
- (5.) $\frac{800}{9} = 800 \div 9 = 88\frac{8}{9}$, *Ans.*
- (6.) $\frac{1162}{11} = 1162 \div 11 = 105\frac{7}{11}$, *Ans.*
- (7.) $\frac{4260}{13} = 4260 \div 13 = 327\frac{9}{13}$, *Ans.*
- (8.) $\frac{15780}{31} = 15780 \div 31 = 509\frac{1}{31}$, *Ans.*

Case V.**Art. 135.**

3

$$(1.) \frac{1}{3} \times \frac{3}{4} \times \frac{4}{7} = \frac{1}{7}, \text{ } \textit{Ans.} \qquad (2.) \frac{\frac{2}{5} \times \frac{4}{7} \times \frac{21}{8}}{\frac{4}{5}} = \frac{8}{5}, \text{ } \textit{Ans.}$$

$$(3.) \frac{\cancel{A} \times \cancel{1}\cancel{5} \times \cancel{8}}{\cancel{3} \times \cancel{1}\cancel{6} \times \cancel{3}} = \frac{2}{1} = 2, \text{ Ans.}$$

\cancel{A}

$$(4.) \frac{1 \times \cancel{A} \times \cancel{1}\cancel{5}}{2 \times \cancel{3} \times \cancel{4}} = \frac{3}{2} = 1\frac{1}{2}, \text{ Ans.}$$

$$(5.) \frac{\cancel{3} \times \cancel{8} \times \cancel{4} \times \cancel{3}\cancel{5}}{\cancel{4} \times \cancel{9} \times \cancel{7} \times \cancel{4}} = \frac{10}{3} = 3\frac{1}{3}, \text{ Ans.}$$

3

$$(6.) \frac{1 \times \cancel{3} \times \cancel{6} \times 3 \times \cancel{1}\cancel{4}}{\cancel{3} \times \cancel{5} \times \cancel{7} \times \cancel{4} \times \cancel{3}} = \frac{3}{5}, \text{ Ans.}$$

2

$$(7.) \frac{\cancel{8} \times \cancel{3} \times \cancel{4} \times \cancel{7}\cancel{7} \times \cancel{5}\cancel{7}}{\cancel{1}\cancel{1} \times \cancel{7} \times \cancel{1}\cancel{9} \times \cancel{2}\cancel{4} \times \cancel{8}} = \frac{3}{2} = 1\frac{1}{2}, \text{ Ans.}$$

8 2

$$(8.) \frac{\cancel{1}\cancel{2} \times \cancel{9} \times \cancel{7} \times \cancel{1}\cancel{0} \times \cancel{3}\cancel{9}}{\cancel{1}\cancel{3} \times \cancel{1}\cancel{6} \times \cancel{1}\cancel{8} \times \cancel{2}\cancel{1} \times \cancel{3}\cancel{5}} = \frac{3}{28}, \text{ Ans.}$$

4 2 3 7

Case VI.

Art. 138.

$$\begin{aligned} (1.) \\ 1 \times 3 \times 5 &= 15 \\ 2 \times 2 \times 5 &= 20 \\ 3 \times 2 \times 3 &= 18 \\ 2 \times 3 \times 5 &= 30 \quad \text{Den.} \\ \text{Ans. } \frac{15}{30}, \frac{20}{30}, \frac{18}{30}. \end{aligned}$$

$$\begin{aligned} (3.) \\ 2 \times 7 \times 8 &= 112 \\ 3 \times 3 \times 8 &= 72 \\ 5 \times 3 \times 7 &= 105 \\ 3 \times 7 \times 8 &= 168 \quad \text{Den.} \\ \text{Ans. } \frac{112}{168}, \frac{72}{168}, \frac{105}{168}. \end{aligned}$$

$$\begin{aligned} (2.) \\ 1 \times 5 \times 6 &= 30 \\ 1 \times 4 \times 6 &= 24 \\ 1 \times 4 \times 5 &= 20 \\ 4 \times 5 \times 6 &= 120 \quad \text{Den.} \\ \text{Ans. } \frac{30}{120}, \frac{24}{120}, \frac{20}{120}. \end{aligned}$$

$$\begin{aligned} (4.) \\ 1 \times 5 \times 6 \times 8 &= 240 \\ 3 \times 2 \times 6 \times 8 &= 288 \\ 5 \times 2 \times 5 \times 8 &= 400 \\ 7 \times 2 \times 5 \times 6 &= 420 \\ 2 \times 5 \times 6 \times 8 &= 480 \quad \text{Den.} \\ \text{Ans. } \frac{240}{480}, \frac{288}{480}, \frac{400}{480}, \frac{420}{480}. \end{aligned}$$

$$(5.) \begin{aligned} \frac{1}{2} \times \frac{7}{2} &= \frac{7}{4}; & 2 \times 4 \times 5 &= 40 \\ \frac{2}{3} \times \frac{3}{5} &= \frac{2}{5}; & 7 \times 3 \times 5 &= 105 \\ && 2 \times 3 \times 4 &= 24 \\ && 3 \times 5 \times 5 &= 60 \quad \text{Den.} \\ && \text{Ans. } \frac{40}{60}, \frac{105}{60}, \frac{24}{60}. \end{aligned}$$

$$(6.) \begin{aligned} &\begin{array}{cc} 2 & 2 \\ \frac{2}{3} \times \frac{6}{7} & = \frac{4}{7}; \frac{3}{4} \times \frac{8}{9} = \frac{2}{3}; \\ & 3 \\ & 3 \end{array} & 4 \times 3 \times 20 &= 240 \\ && 2 \times 7 \times 20 &= 280 \\ && 9 \times 3 \times 7 &= 189 \\ && 7 \times 3 \times 20 &= 420 \quad \text{Den.} \\ && \text{Ans. } \frac{240}{420}, \frac{280}{420}, \frac{189}{420}. \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \times \frac{4}{5} \times \frac{3}{7} \times \frac{2}{8} &= \frac{9}{20}. \\ & 2 \end{aligned}$$

$$(1.) \begin{aligned} \frac{1}{2} \times \frac{4}{4} &= \frac{4}{8}. \\ \frac{3}{4} \times \frac{2}{2} &= \frac{6}{8}. \\ \text{Ans. } \frac{4}{8}, \frac{6}{8}, \frac{5}{8}. \end{aligned}$$

$$(2.) \begin{aligned} \frac{2}{3} \times \frac{4}{4} &= \frac{8}{12}. \\ \frac{5}{6} \times \frac{2}{2} &= \frac{10}{12}. \\ \text{Ans. } \frac{8}{12}, \frac{10}{12}, \frac{7}{12}. \end{aligned}$$

$$(3.) \begin{aligned} \frac{3}{4} \times \frac{5}{5} &= \frac{15}{20}. \\ \frac{4}{5} \times \frac{4}{4} &= \frac{16}{20}. \\ \frac{9}{10} \times \frac{2}{2} &= \frac{18}{20}. \\ \text{Ans. } \frac{15}{20}, \frac{16}{20}, \frac{18}{20}, \frac{11}{20}. \end{aligned}$$

CASE VII.

Art. 139.

(1.) The L. C. M. of 3, 4, and 6 = 12 :

$$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}; \quad \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}; \quad \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}. \quad \text{Ans. } \frac{4}{12}, \frac{9}{12}, \frac{10}{12}.$$

(2.) The L. C. M. of 2, 5, 10, and 4 is 20 :

$$\frac{1}{2} \times \frac{10}{10} = \frac{10}{20}; \quad \frac{3}{5} \times \frac{4}{4} = \frac{12}{20}; \quad \frac{9}{10} \times \frac{2}{2} = \frac{18}{20}; \quad \frac{3}{4} \times \frac{5}{5} = \frac{15}{20}. \quad \text{Ans. } \frac{10}{20}, \frac{12}{20}, \frac{18}{20}, \frac{15}{20}.$$

(3.) The L. C. M. of 7, 8, and 14 is 56 :

$$\frac{3}{7} \times \frac{8}{8} = \frac{24}{56}; \quad \frac{5}{8} \times \frac{7}{7} = \frac{35}{56}; \quad \frac{11}{14} \times \frac{4}{4} = \frac{44}{56}. \quad \text{Ans. } \frac{24}{56}, \frac{35}{56}, \frac{44}{56}.$$

$$(4.) 2) \frac{6}{8} = \frac{3}{4}; \quad 3) \frac{9}{12} = \frac{3}{4}; \quad 5) \frac{15}{20} = \frac{3}{4}. \quad \text{Ans. } \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}.$$

$$(5.) 3) \frac{6}{9} = \frac{2}{3}; \quad 3) \frac{9}{12} = \frac{3}{4}; \quad 4) \frac{12}{20} = \frac{3}{5};$$

$$\text{L. C. M. of 3, 4, 5, and 10, is } 60: \frac{2}{3} \times \frac{20}{20} = \frac{40}{60};$$

$$\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}; \quad \frac{3}{5} \times \frac{12}{12} = \frac{36}{60}; \quad \frac{7}{10} \times \frac{6}{6} = \frac{42}{60}. \quad \text{Ans. } \frac{40}{60}, \frac{45}{60}, \frac{36}{60}, \frac{42}{60}.$$

$$(6.) \quad 1\frac{3}{4} = \frac{7}{4}; \quad 3\frac{2}{3} = \frac{11}{3}; \quad \frac{3}{10} \times \frac{25}{7} = \frac{15}{14};$$

2

L. C. M. of 4, 3, and 14, is 84; $\frac{7 \times 21}{4 \times 21} = \frac{147}{84}$;

$$\frac{11 \times 28}{3 \times 28} = \frac{308}{84}; \quad \frac{15 \times 6}{14 \times 6} = \frac{90}{84}. \quad Ans. \frac{147}{84}, \frac{308}{84}, \frac{90}{84}.$$

ALLIGATION ALTERNATE

CASE I.

Art. 363.

(1.)

		Balance.	lb.		Balance	lb.		Balance	lb.
	25	3	17	2	19		6	17	17
	27	1	4		4		17	4	6
28	30	2	3	3 or,	3	3 or,	3	1	1
	32	4	1		1		3		1
	45	17	3	3			1	3	3
					<i>Ans.</i>		<i>Ans.</i>		<i>Ans.</i>

(2.)

		Balance.	lb.		Balance	lb.		
	5	$\frac{7}{4}$	1		1	5		
	$5\frac{1}{2}$	$\frac{5}{4}$	5		5	1		
	6	$\frac{3}{4}$	5	5 or,	5	1		
	7	$\frac{1}{4}$	7		5	3		
	8	$\frac{5}{4}$	5	3	7	7		
					<i>Ans.</i>		<i>Ans.</i>	

(3.)

		Balance.	Gal.		Balance	Gal.		
	84	3	9	1	10	7		
	86	1	7		7	9		
87	88	1	3	3 or,	3	1		
	94	7	1		1	1		
	96	9	3		3	1		
					<i>Ans.</i>		<i>Ans.</i>	

OPERATION

Bulks per lb.	Diff	Bal.
$\frac{2}{21}$	$\frac{3487}{97251}$	723
$\frac{4}{77}$	$\frac{723}{97251}$	3487

Explanation.—Since silver has sp. gr. $\frac{21}{2}$, one lb. silver has $\frac{2}{21}$ the bulk of one lb. water; since gold has sp. gr. $\frac{77}{4}$, one lb. gold has $\frac{4}{77}$ the bulk of one lb. water; the combination having sp. gr. $\frac{421}{25}$, one lb. of the combination must have $\frac{25}{421}$ the bulk of one lb. water. If we take the whole in silver, each pound will have $\frac{1}{\frac{3487}{97251}}$ bulk *too great*;

if the whole be taken gold, each pound will have $\frac{723}{97251}$ bulk *too small*; hence, balance in ratio of 723 lb. silver to 3487 lb. gold, *Ans.*

Remark.—If the sp. gr. of a substance be 4, one pound of it will have a bulk equal to $\frac{1}{4}$ the bulk of a pound of water; if sp. gr. be $\frac{21}{2}$, a pound of the substance will have the bulk of $\frac{2}{21}$ of a pound of water. By thus *inverting* the numbers expressing the sp. gravities of different things, their bulks may be directly compared, just as we compare the *prices per pound* in other examples. When we compare *prices*, in a common example, we balance the losses against the gains, *calling* the units in the balancing, "*pounds*," though they may be transferred from a column of prices, named in "*cents*."

Illustration.—Suppose, where the average price is $\frac{2}{9}$ ct. we find a loss of $\frac{714}{189}$ cents on one kind, and gain on another kind of a lb. $\frac{24}{189}$ cents, we take 714 of the latter kind of pounds, and 24 pounds of the former. The specific gravity came would read: We lose on a pound $\frac{714}{189}$ bulks, and gain on another pound $\frac{24}{189}$ bulks; hence balance by 714 of latter to 24 of former in pounds.

$$\begin{array}{c|c|c|c}
 & \frac{2}{21} & \frac{24}{189} & 714 \\
 \frac{2}{9} & & \hline
 & \frac{4}{1} & \frac{714}{189} & 24
 \end{array}$$

$$(5.) \quad \begin{array}{c|c||c|c} & & \text{Bal.} & \text{lb.} \\ \frac{3}{4} & \frac{1}{8} & \frac{1}{40} & 1 \\ \hline \frac{9}{10} & \frac{1}{40} & \frac{1}{8} & \text{or, } 5 \end{array}$$

		(6.)			Bal.	Parts.
22		24	2	2	1	3
		20	2		1	1
		18	4	1		1

Ans.

CASE II.

Art. 364.

$$\begin{array}{c}
 (1.) \\
 \text{Bal.} \\
 \hline
 \begin{array}{r|rr|r}
 & 50 & 10 & 12 & \times 8 = 96, \\
 60 & & & & \textit{Ans.} \\
 \hline
 & 72 & 12 & 10 & \times 8 = 80.
 \end{array}
 \end{array}$$

			(2.)	
			Bal.	
65	40	25	2	
	50	15		2 × 50 =
	60	5	2	
	75	10	5	<i>Ans.</i>
			3	4 + (5 ×
			1	= 254,

				(S.)
Bal.				
56	0	56	5	$\times \frac{29}{8} = \frac{145}{8}$ pt. = 2 gal. 1 qt. $\frac{1}{8}$ pr., Ans.
	91	35	8	$\times \frac{29}{8} = 29$ pt.

$$(4.) \quad \begin{array}{c} \text{gr.} \\ 18 \left| \begin{array}{c|c|c|c} 16 & 2 & 1.8 & \times \frac{81}{1.8} = 3 \\ \hline 21.6 & 3.6 & 1 & \text{pwt. } 9 \text{ gr.} \\ \hline 1 & \end{array} \right. \\ 1 \text{ pwt. } 21 \text{ gr., Ans.} \end{array} \quad (5.) \quad \begin{array}{c} \text{pt.} \\ 60 \left| \begin{array}{c|c|c|c} 0 & 60 & 3 & 3 \\ \hline 78 & 18 & 10 & \left| \begin{array}{c|c} \frac{9}{5} & \frac{12}{5} \\ \hline 8 & \end{array} \right. \\ \hline 96 & 36 & 3 & \end{array} \right. \\ \text{pt.} = 4\frac{1}{5} \text{ pt.} \\ \text{Ans.} \end{array}$$

$$(6.) \quad \begin{array}{rcl} \text{Part.} & \% & \text{acidity} \\ 12\frac{1}{2} \text{ pt.} & \times & 0 = 0 \\ 7\frac{1}{2} " & \times & 100 = 750 \\ \hline 20 &)750(37\frac{1}{2} \% & \text{obtained.} \\ \text{desired,} & \overline{22\frac{1}{2}} & \begin{array}{c} 15 & \text{pt.} \\ 0 \left| \begin{array}{c|c|c|c} & 22.5 & 31 & \times \frac{7.5}{9} = 25\frac{5}{6} \text{ pt.} \\ \hline & \text{---} & & = 12\frac{1}{2} \\ 100 & 77.5 & 9 & \frac{1}{3} \text{ pt.} \\ \hline & & & 1 \text{ gal. } 2 \text{ qt. } 1\frac{1}{3} \text{ pt., Ans.} \end{array} \right. \end{array} \end{array}$$

Explanation.—The combination is to *displace* water equal to *once* its weight, and, hence, 1 is the average. The lead, while in the water, displaces $\frac{1}{11}$ of its own weight; the copper displaces $\frac{1}{9}$ of its own weight; the cork, when wholly in water, displaces 4 times its own weight.

Hence, the piece, say 1 oz., of lead *lacks* displacing $\frac{10}{11}$ of an oz.; 1 oz. of copper lacks displacing $\frac{8}{9}$; a cork oz. displaces *too much*, by 3 times its weight; hence, balancing, we take 3 oz. of lead for each $\frac{10}{11}$ oz. of cork, and 3 oz. of copper for each $\frac{8}{9}$ oz. of cork. But the conditions require only $\frac{1}{6}$ of 3 oz. of copper; hence, to balance that requires $\frac{1}{5}$ of $\frac{8}{9}$, or $\frac{8}{54}$ oz. cork. The conditions also require 12 oz. cork *in all*; therefore, $12 - \frac{8}{54}$, or $\frac{320}{27}$ oz. cork are yet required, and as this contains $\frac{10}{11}$, $\frac{352}{27}$ times, there must be, by first balancing, $\frac{352}{27} \times 3$, or $\frac{352}{9}$ oz. lead, which is 2 lb. $7\frac{1}{9}$ oz., *Ans.*

Explanation.—First, balancing in proportion to bulk, 40 of cork to 3 of lead, 32 of cork to 3 of copper. This makes their actual weights, as $40 \times \frac{1}{4}$ to 11×3 , and $32 \times \frac{1}{4}$ to 9×3 . Then the $\frac{1}{2}$ oz. copper requiring $\frac{8}{54}$ oz. cork, the remaining $\frac{640}{54}$ oz. cork require 2 lb. $7\frac{1}{9}$ oz. lead, *Ans.*

$$(7.) \quad \begin{array}{c} \text{OPERATION.} \\ 1 \left| \begin{array}{c|c|c|c} 1 & \frac{10}{11} & 3 & \left| \begin{array}{c} \frac{1}{2} \\ \hline 3 \end{array} \right. \\ \hline \frac{1}{9} & \frac{8}{9} & & \end{array} \right. \\ \times \frac{352}{27} = \frac{352}{9} \text{ oz.,} \\ \text{Ans.} \\ 12 - \frac{8}{54} = \frac{640}{54}; \frac{640}{54} \div \frac{10}{11} = \frac{352}{27}. \end{array} \quad (1\text{st.})$$

$$\begin{array}{c} \text{OPERATION.} \\ 1 \left| \begin{array}{c|c|c|c} \frac{1}{4} & \frac{3}{4} & \left| \begin{array}{c|c} 40 & 32 \\ \hline 3 & \end{array} \right. \\ \hline 9 & 8 & & \end{array} \right. \\ = \left| \begin{array}{c|c} 10 & 8 \\ \hline 27 & \end{array} \right. \\ \frac{8}{54} + \frac{640}{54} \\ \frac{1}{2} \\ \frac{352}{9} \text{ oz.,} \end{array} \quad (2\text{d.})$$

(8.)

120 × 74 = 8880		Since 60 is 120% of that desired, 60 ÷ 120, or 50% is the required average
150 × 68 = 10200		
130 × 54 = 7020		
<hr style="border-top: 1px solid black; border-bottom: none; border-left: none; border-right: none; margin: 0; padding: 0; width: 100%; height: 1px; display: inline-block; vertical-align: middle;"/> 400)	<hr style="border-top: 1px solid black; border-bottom: none; border-left: none; border-right: none; margin: 0; padding: 0; width: 100%; height: 1px; display: inline-block; vertical-align: middle;"/> 26100 (65 $\frac{1}{4}$ %)	50 65 $\frac{1}{4}$ 15 $\frac{1}{4}$ 10 × 40 = 400
		40 10 15 $\frac{1}{4}$ × 40 = 610
		610 shares, <i>Ans.</i>

(9.)

	\$	bbl.
400 × 7.50 = 3000		7.06 .56 100 × 20 = 2000
640 × 7.25 = 4640	req. av. 6.50	5.50 1 56 × 20 = 1120
960 × 6.75 = 6480		
<hr style="border-top: 1px solid black; border-bottom: none; border-left: none; border-right: none; margin: 0; padding: 0; width: 100%; height: 1px; display: inline-block; vertical-align: middle;"/> 2000)14120 (7.06	bbl., <i>Ans.</i>
		average now.

Case III.

Art. 365.

(1.)

The given lbs. and prices make an average of $5\frac{1}{7}$ ct. Then,

6 3 3 1 $\frac{3}{4}$ lb., <i>Ans.</i>	The balancing requires 1 of the first for 3 of the third; and 7 of the second for 6 of the third. This gives the required 7; but as there are 16 – 13, or 3 yet required, and as these must be taken
5 $\frac{1}{7}$ $\frac{6}{7}$ 7	
<hr style="border-top: 1px solid black; border-bottom: none; border-left: none; border-right: none; margin: 0; padding: 0; width: 100%; height: 1px; display: inline-block; vertical-align: middle;"/> 7 1 3 6 $6 + 2\frac{1}{4} = 8\frac{1}{4}$ lb., <i>Ans.</i>	

in proportion, *as one* of the first to *three* of the third, we take $\frac{3}{4}$ of the first balance column and add it to the second; having $\frac{3}{4} + 7 + 8\frac{1}{4} = 16$.

(2.)

bbl.	pr.	\$
300 × 7.50 = 2250		
800 × 7.80 = 6240		
<hr style="border-top: 1px solid black; border-bottom: none; border-left: none; border-right: none; margin: 0; padding: 0; width: 100%; height: 1px; display: inline-block; vertical-align: middle;"/> 400 × 7.65 = 3060		
<hr style="border-top: 1px solid black; border-bottom: none; border-left: none; border-right: none; margin: 0; padding: 0; width: 100%; height: 1px; display: inline-block; vertical-align: middle;"/> 1500)11550 (\$7.70, av. for 1500 bbl.	

As there are to be 2000 bbl., there are yet 500 bbl. required, and their price must be \$(2000 × 7.85 – 11550) ÷ 500 = \$8.30, the av. Hence,

$$8.30 \left| \begin{array}{c} 8 \\ 8.50 \end{array} \right| \begin{array}{c}.30 \\ .20 \end{array} \left| \begin{array}{c} 2 \\ 3 \end{array} \right| \times 100 = \left\{ \begin{array}{l} 200 \text{ bbl.} \\ 300 \text{ bbl.} \end{array} \right\} \text{Ans.}$$

(3.)

$$\begin{array}{rcl}
 \text{lb.} & \$ & 56 \text{ lb.} - 40 \text{ lb.} = 26 \text{ lb. desired.} \\
 14 \times .30 = & 4.20 & (56 \times .40 - 17.80) \div 16 = 28\frac{3}{4} \text{ ct. av. for 16} \\
 6 \times .60 = & 3.60 & \text{lb.} \\
 \hline
 40) & 17.80(44 \frac{1}{2} \text{ ct., av.} & \\
 28\frac{3}{4} \left| \begin{array}{c|c|c|c} 25 & 3\frac{3}{4} & 5 & \\ \hline 35 & 6\frac{1}{4} & 3 & \end{array} \right. \times 2 = \left\{ \begin{array}{l} 10 \text{ lb.} \\ 6 \text{ lb.} \end{array} \right\} \text{Ans.} & \\
 & & (4.)
 \end{array}$$

If the specific gravity of a body be $\frac{7}{1}$, it loses $\frac{1}{7}$ in water; so, copper loses $\frac{4}{31}$, and silver $\frac{2}{21}$, while the required loss of combination weight is $\frac{5}{43}$; hence,

$$\frac{5}{43} \left| \begin{array}{c|c|c|c} \frac{4}{31} & \frac{357}{27993} & 589 & \\ \hline \frac{2}{21} & \frac{589}{27993} & \frac{357}{27993} & \\ & & 946 & \end{array} \right. \times \frac{12}{946} = \left\{ \begin{array}{l} 7\frac{223}{473} \text{ oz.} \\ 4\frac{250}{473} \text{ oz.} \end{array} \right\} \text{Ans.}$$

(5.)

$$\begin{array}{rcl}
 1 \text{ st.} & \text{gr.} & \\
 18 \left| \begin{array}{c|c|c|c|c|c} 15 & 3 & 6 & 2 & 64 & 2 \text{ pwt. } 16 \text{ gr., Ans.} \\ \hline 20 & 2 & & 3 & 24 & 1 \text{ pwt., Ans.} \\ \hline 24 & 6 & & 3 & 24 & 1 " Ans. \\ & & & 9 & & 112 \div 14 = 8. \end{array} \right. & &
 \end{array}$$

Taking 3's and 5's to make 112 gr., we proceed thus to find other answers.

$$2d. \left| \begin{array}{c|c||c|c|c|c|c|c|c|c} 2 & 2 & 72 & 68 & 64 & 60 & 56 & 52 & 48 & \\ \hline 3 & 6 & 15 & 24 & 33 & 42 & 51 & 60 & & \\ \hline 1 & 3 & 34 & 29 & 24 & 19 & 14 & 9 & 4 & \\ \hline 3 & 5 & & & & & & & & \end{array} \right|$$

$$4 \left| \begin{array}{c|c||c|c|c|c|c|c|c|c} 9 & 5 & 3 & 2 & 29 & 30 & 31 & 32 & 33 & 34 \\ \hline 2 & 2 & 5 & 5 & 68 & 60 & 52 & 44 & 36 & 28 \\ \hline 1 & 3 & 5 & 7 & 3 & 10 & 17 & 24 & 31 & 38 \\ \hline 8 & 7 & & & & & & & & \end{array} \right. \text{(6.)} \quad \begin{array}{l} \text{calves,} \\ \text{hogs,} \\ \text{lams.} \end{array}$$

Take 5ths of 8 and 5ths of 7 to make 100; or whole 8's and whole 7's to make 500.

(7.)

If a body have a specific gravity of 2, in water it displaces $\frac{1}{2}$ its own weight; if its sp. gr. be $\frac{4}{3}$, it displaces in like manner $\frac{3}{4}$; so the

crown, sp. gr. $\frac{117}{8}$, displaced $\frac{8}{117}$, and thus with the two metals. Hence, the question is,—*If gold displace, in water*

$\frac{4}{77}$ of its own weight, and silver $\frac{2}{21}$ of its own weight, how should these be combined so as to displace $\frac{8}{117}$ of their weight? The above balancing shows their actual weights should combine as 74 to 121; i.e., the gold should weigh $\frac{121}{195}$ of the combined weights. The whole weight being $17\frac{1}{2}$ lb., the weight of the gold must be $17\frac{1}{2}$ lb. $\times \frac{121}{195} = 10\frac{67}{78}$ lb., Ans.

$$\left| \begin{array}{c|c|c|c} \frac{2}{21} & \frac{726}{27027} & 74 & \\ \hline \frac{4}{77} & \frac{444}{27027} & 121 & \\ \hline & & 195 & \end{array} \right. \times \frac{17.5}{195} = \left\{ \begin{array}{l} 6\frac{25}{39} \text{ silver.} \\ 10\frac{67}{78} \text{ gold.} \end{array} \right\} \text{Ans.}$$

INVOLUTION

Art. 370.

- (1.) $(5)^2 = 5 \times 5 = 25$, *Ans.*
- (2.) $(14)^3 = 14 \times 14 \times 14 = 2744$, *Ans.*
- (3.) $(6)^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776$, *Ans.*
- (4.) $(192)^2 = 192 \times 192 = 36864$, *Ans.*
- (5.) $1 \times 1 = 1$, *Ans.*
- (6.) $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{81}{625}$, *Ans.*
- (7.) $\left(2\frac{1}{4}\right)^3 = \frac{9}{4} \times \frac{9}{4} \times \frac{9}{4} = \frac{729}{64} = 11\frac{25}{64}$, *Ans.*
- (8.) $\left(\frac{7}{8}\right)^5 = \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = \frac{16807}{32768}$, *Ans.*
- (9.) $(.02)^3 = .02 \times .02 \times .02 = .000008$, *Ans.*
- (10.) $(5)^4 = 5 \times 5 \times 5 \times 5 = 625$; $\therefore (5^4)^2$, or $5^8 = 625 \times 625 = 390625$, *Ans.*
- (11.) $(.046)^3 = .046 \times .046 \times .046 = .000097336$, *Ans.*
- (12.) $\left(\frac{1}{9}\right)^7 = \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} = \frac{1}{4782969}$, *Ans.*
- (13.) $(2056)^2 = 2056 \times 2056 = 4227136$, *Ans.*
- (14.) $(7.62\frac{1}{2})^2 = 7.62\frac{1}{2} \times 7.62\frac{1}{2} = 58.1406\frac{1}{4}$, *Ans.*

Art. 371.

- (1.) $19^2 = (10 + 9)^2 = 100 + 2(10 \times 9) + 81 = 361$, *Ans.*
- (2.) $29^2 = (20 + 9)^2 = 400 + 2(20 \times 9) + 81 = 841$, *Ans.*
- (3.) $4^2 = (1 + 3)^2 = 1 + 2(1 \times 3) + 9 = 16$, *Ans.*
- (4.) $40^2 = (30 + 10)^2 = 900 + 2(30 \times 10) + 100 = 1600$, *Ans.*
- (5.) $125^2 = (100 + 25)^2 = 10000 + 2(100 \times 25) + 625 = 15625$, *Ans.*
- (6.) $59^2 = (50 + 9)^2 = 2500 + 2(50 \times 9) + 81 = 3481$, *Ans.*

Art. 372.

- (1.) $19^3 = (10 + 9)^3 = 1000 + 3(100 \times 9) + 3(10 \times 81) + 729 = 6859$, *Ans.*
- (2.) $29^3 = (20 + 9)^3 = 8000 + 3(400 \times 9) + 3(20 \times 81) + 729 = 24389$, *Ans.*
- (3.) $4^3 = (1 + 3)^3 = 1 + 3(1 \times 3) + 3(1 \times 9) + 27 = 64$, *Ans.*
- (4.) $40^3 = (20 + 20)^3 = 8000 + 3(400 \times 20) + 3(20 \times 400) + 8000 = 64000$, *Ans.*
- (5.) $125^3 = (120 + 5)^3 = 1728000 + 3(14400 \times 5) + 3(120 \times 25) + 125 = 1953125$, *Ans.*
- (6.) $216^3 = (200 + 16)^3 = 8000000 + 3(40000 \times 16) + 3(200 \times 256) + 4096 = 10077696$, *Ans.*

EVOLUTION

EXTRACTION OF THE SQUARE ROOT

Art. 375.

$(1.)$ $\begin{array}{r} 2809(53, \text{Ans.}) \\ 25 \\ \hline 103 \end{array}$	$(2.)$ $\begin{array}{r} 1444(38, \text{Ans.}) \\ 9 \\ \hline 68 \end{array}$	$(3.)$ $\begin{array}{r} 11881(109, \text{Ans.}) \\ 1 \\ \hline 209 \end{array}$
$\boxed{309}$	$\boxed{544}$	$\boxed{1881}$
$\boxed{309}$	$\boxed{544}$	$\boxed{1881}$

$(4.)$ $\begin{array}{r} 185640625(13625, \text{Ans.}) \\ 1 \\ \hline 23)85 \\ 69 \\ \hline 266)1664 \\ 1596 \\ \hline 2722) 6806 \\ 5444 \\ \hline 27245) 136225 \\ 136225 \\ \hline \end{array}$	$(5.)$ $\begin{array}{r} 80012304(8944.9-, \text{Ans.}) \\ 64 \\ \hline 169 \end{array}$
$\boxed{1601}$	$\boxed{1521}$
$\boxed{8023}$	$\boxed{7136}$
$\boxed{88704}$	$\boxed{71536}$
$\boxed{1716800}$	$\boxed{1610001}$
	$\boxed{6799}$

$(6.)$ $\begin{array}{r} 6203794(2490.74, \text{Ans.}) \\ 4 \\ \hline 44 \end{array}$	$(7.)$ $\begin{array}{r} 3444736(1856, \text{Ans.}) \\ 1 \\ \hline 28 \end{array}$
$\boxed{220}$	$\boxed{244}$
$\boxed{176}$	$\boxed{224}$
$\boxed{4437}$	$\boxed{2047}$
$\boxed{4401}$	$\boxed{1825}$
$\boxed{369400}$	$\boxed{22236}$
$\boxed{348649}$	
$\boxed{20751}$	
$\boxed{19926-}$	

(8.)
 $\dot{5}7\dot{6}0\dot{0}(240, \text{Ans.})$

$$\begin{array}{r} 4 \\ 44 \overline{)176} \\ 176 \\ \hline 00 \end{array}$$

(9.)
 $\dot{1}\dot{6}4\dot{9}9\dot{8}4\dot{4}(4062, \text{Ans.})$

$$\begin{array}{r} 1600 \\ 806 \overline{)4998} \\ 4836 \\ \hline 16244 \\ 16244 \end{array}$$

(10.)
 $\dot{4}9\dot{0}9\dot{8}0\dot{4}9(7007, \text{Ans.})$

$$\begin{array}{r} 49 \\ 14007 \overline{)98049} \\ 98049 \end{array}$$

(13.)
 $\dot{3}0\dot{0}0\dot{0}0\dot{0}(1.7320508, \text{Ans.})$

$$\begin{array}{r} 1 \\ 27 \overline{)200} \\ 189 \\ \hline 343 \overline{)1100} \\ 1029 \\ \hline 3462 \overline{)7100} \\ 6924 \\ \hline 3464 + \overline{)17600} \\ 17320 + \\ \hline 279 \\ 277 \end{array}$$

(11.)
 $\dot{7}3\dot{0}0\dot{5}(270.194, \text{Ans.})$

$$\begin{array}{r} 4 \\ 47 \overline{)330} \\ 329 \\ \hline 5401 \overline{)10500} \\ 5401 \\ \hline 54029 \overline{)509900} \\ 486261 \\ \hline 540384 \overline{)2363900} \\ 2161536 \end{array}$$

(14.)
 $\dot{9}.8\dot{6}9\dot{6}0\dot{4}4\dot{0}1\dot{0}(3.1415926, \text{Ans.})$

$$\begin{array}{r} 9 \\ 61 \overline{)86} \\ 61 \\ \hline 624 \overline{)2596} \\ 2496 \\ \hline 6281 \overline{)10004} \\ 6281 \\ \hline 62825 \overline{)372340} \\ 314125 \\ \hline 58215 \\ 56543 \\ \hline 1672 \\ 1257 \\ \hline 415 \\ 377 \end{array}$$

(12.)
 $386^3 = 575\dot{1}245\dot{6}(7583.69, \text{Ans.})$

$$\begin{array}{r} 49 \\ 45 \overline{)851} \\ 725 \\ \hline 1508 \overline{)12624} \\ 12064 \\ \hline 15163 \overline{)56056} \\ 45489 \\ \hline 10567 \\ 9098 \\ \hline 1469 \\ 1365 \end{array}$$

(15.) $\sqrt{.030625} = .175$; $\sqrt{40.96} = 6.4$; $\sqrt{.00000625} = .0025$; $.175 \times .0025 \times 6.4 = .0028$, Ans.

(16.) $126 \times 58 \times 604 = 4414032$, Ans.

$$(17.) \sqrt{12.96} \times \sqrt{\frac{5}{6}} = \sqrt{10.8} = 3.2863, \text{ Ans.}$$

Art. 377.

$$(1.) \sqrt{\frac{6}{7}} = \sqrt{\frac{42}{49}} = \frac{1}{7}\sqrt{42} = .92682+, \text{ Ans.}$$

$$(2.) \sqrt{34\frac{5}{8}} = \sqrt{34.625} = 5.8843+, \text{ Ans.}$$

$$(3.) \sqrt{\frac{4}{7}} = \sqrt{\frac{28}{49}} = \frac{1}{7}\sqrt{28} \text{ and not } \frac{1}{7}\sqrt{36}; \text{ hence, } \frac{5}{7} \text{ more nearly than } \frac{6}{7}, \text{ Ans.}$$

$$(4.) \sqrt{272.25} = 16.5, \text{ Ans.}$$

$$(5.) \sqrt{6.40} = 2.5298+, \text{ Ans.}$$

$$(6.) \frac{28}{57} \times \frac{392}{2527} \times \frac{35}{38} \times \frac{3}{1} = \frac{784}{361 \times 361} \times 35; \text{ hence, sq. rt.} = \frac{28}{361}\sqrt{35} = 5.9160798 \times \frac{28}{361} = .45886+, \text{ Ans.}$$

$$(7.) \sqrt{123.454321} \times .81 = 11.111 \times .9 = 9.9999, \text{ Ans.}$$

$$(8.) \sqrt{1.728 \times 4.8 \times \frac{3}{7}} = \sqrt{1.44 \times 1.44 \times 4 \times \frac{3}{7}} = 1.2 \times 1.2 \times 2 \times \frac{1}{7}\sqrt{21} = \frac{2.88}{7}\sqrt{21}, \text{ Ans.}$$

EXTRACTION OF THE CUBE ROOT

Art. 380.

(1.)	(2.)	19683(27, Ans. 8 11683 11683 11683)
512(8, Ans. 512 —)	4 × 300 = 1200 2 × 7 × 30 = 420 7 × 7 = 49 1669	

(3.)	(4.)
7301384(194, Ans. 1 6301 300 270 81 651 5859 108300 442384 2280 16 110596 442384)	94818816(456, Ans. 64 30818 4800 600 25 5425 27125 607500 3693816 8100 36 615636 3693816)

(5.)

 $\dot{1}06\dot{7}46\dot{2}64\dot{8}$ (1022, Ans.

1

$$\begin{array}{r}
 1 \times 300 = 300) \quad 67 \\
 \hline
 100 \times 300 = 30000 \quad | \quad 67462 \\
 10 \times 2 \times 30 = \quad 600 \quad | \\
 2 \times 2 = \quad \quad \quad 4 \quad | \\
 \hline
 30604 \quad | \quad 61208 \\
 \hline
 10404 \times 300 = 3121200 \quad | \quad 6254648 \\
 102 \times 2 \times 30 = \quad 6120 \quad | \\
 2 \times 2 = \quad \quad \quad 4 \quad | \quad 6254648 \\
 \hline
 3127324 \quad | \\
 \end{array}$$

(6.)

 $\dot{5}.08844\dot{8}$ (1.72, Ans.

$$\begin{array}{r}
 1 \\
 300 \overline{)4088} \\
 210 \\
 49 \overline{)3913} \\
 559 \quad | \quad 175448 \\
 \hline
 86700 \\
 1020 \\
 4 \\
 \hline
 87724 \quad | \quad 175448
 \end{array}$$

(7.)

 $2\dot{2}188.\dot{0}4\dot{1}$ (28.1, Ans.

$$\begin{array}{r}
 8 \\
 1200 \overline{)14188} \\
 480 \\
 64 \quad | \quad 13952 \\
 1744 \quad | \quad 236041 \\
 \hline
 235200 \\
 840 \\
 1 \\
 \hline
 2360411 \quad | \quad 236041
 \end{array}$$

(8.)

 $3\dot{2}.650$ (3.196154+, Ans.

$$\begin{array}{r}
 27 \\
 2700 \overline{)5650} \\
 90 \\
 1 \quad | \quad 2791 \\
 \hline
 2791 \quad | \quad 2859000 \\
 288300 \\
 8370 \\
 81 \quad | \quad 2670759 \\
 296751 \quad | \quad 188241000 \\
 30528300 \\
 57420 \\
 36 \quad | \quad 183514536 \\
 30585756 \quad | \quad 4726464 \\
 305858 \\
 \hline
 166788 \\
 15292 \\
 1386
 \end{array}$$

(9.)

 $.007900$ (.1991632+, Ans.

$$\begin{array}{r}
 1 \\
 300 \overline{)6900} \\
 270 \\
 81 \quad | \quad 5859 \\
 651 \quad | \quad 1041000 \\
 108300 \\
 5130 \\
 81 \quad | \quad 1021599 \\
 113511 \quad | \quad 19401000 \\
 11880300 \\
 5970 \\
 1 \\
 11886271 \quad | \quad 11886271 \\
 7514729 \\
 713176 \\
 \hline
 38296 \\
 3565 \\
 264
 \end{array}$$

(10.)

$$\begin{array}{r} \dot{3}.009200(1.443724, Ans. \\ \begin{array}{r} 1 \\ 300 \overline{)2009} \\ 120 \\ 16 \overline{)1744} \\ 436 \overline{)265200} \\ \hline 58800 \\ 1680 \\ 16 \overline{)241984} \\ 60496 \overline{)23216000} \\ \hline 6220800 \\ 12960 \\ 9 \overline{)18701307} \\ 6233769 \overline{)4514693} \\ \hline 436364, \text{ etc., as above.} \end{array} \end{array}$$

(11.)

$$\begin{array}{r} \frac{23}{729} = .03\dot{1}55006858\dot{7}(.315985, Ans. \\ \begin{array}{r} 27 \\ 2700 \overline{)4550} \\ 90 \\ 1 \overline{)2791} \\ 2791 \overline{)1759068} \\ \hline 288300 \\ 4650 \\ 25 \overline{)1464875} \\ 292975 \overline{)294193587} \\ \hline 29767500 \\ 85050 \\ 81 \overline{)268673679} \\ 29852631 \overline{)25519908} \\ \hline 2388210, \text{ etc., etc.} \end{array} \end{array}$$

(12.)

$$\begin{array}{r} \dot{2}5(2.924018, Ans. \\ \begin{array}{r} 8 \\ 1200 \overline{)1700} \\ 540 \\ 81 \overline{)16389} \\ 1821 \overline{)611000} \\ \hline 252300 \\ 1740 \\ 4 \overline{)508088} \\ 254044 \overline{)102912000} \\ \hline 25579200 \\ 35040 \\ 16 \overline{)102457024} \\ 25614256 \quad | \quad 454976 \\ \hline 2561 \quad | \quad 1988 \end{array} \end{array}$$

(13.)

$$\begin{array}{r} \dot{1}1(2.22398, Ans. \\ \begin{array}{r} 8 \\ 1200 \overline{)3000} \\ 120 \\ 4 \overline{)2648} \\ 1324 \overline{)352000} \\ \hline 145200 \\ 1320 \\ 4 \overline{)293048} \\ 146524 \overline{)58952000} \\ \hline 14785200 \\ 19980 \\ 9 \overline{)44415567} \\ 14805189 \overline{)14536433} \\ \hline 1332467 \\ 121176 \\ 11844 \end{array} \end{array}$$

(14.)

$$\frac{2}{3} = .66\dot{6} (.87358, \text{Ans.})$$

$$\begin{array}{r} 512 \\ 19200 \quad \boxed{154666} \\ 1680 \\ 49 \quad \boxed{146503} \\ \hline 20929 \quad 8163666 \\ \hline 2270700 \\ 7830 \\ 9 \quad \boxed{6835817} \\ 2278\cancel{5}39 \quad \boxed{1328049} \\ 113927 \\ \hline 18877 \\ 1823 \\ \hline \end{array}$$

(15.)

$$\frac{4}{15} = .26\dot{6} (.64366, \text{Ans.})$$

$$\begin{array}{r} 216 \\ 10800 \quad \boxed{50666} \\ 720 \\ 16 \quad \boxed{46144} \\ \hline 11536 \quad 4522667 \\ \hline 1228800 \\ 5760 \\ 9 \quad \boxed{3703707} \\ 1234\cancel{5}69 \quad \boxed{818960} \\ 74074 \\ \hline 7822 \\ \hline \end{array}$$

(16.)

 $171.41\dot{6}32\dot{8}87\dot{5} (5.555, \text{Ans.})$

$$\begin{array}{r} 125 \\ 25 \times 300 = 7500 \quad \boxed{4616} \\ 5^2 \times 30 = 750 \\ 5^2 = \underline{25} \\ \hline 8275 \quad \boxed{41375} \\ \hline 5041328 \\ 55^2 \times 300 = 907500 \\ 55 \times 5 \times 30 = 8250 \\ 5 \times 5 = \underline{25} \\ \hline 915775 \quad \boxed{4578875} \\ \hline 462453875 \\ 555^2 \times 300 = 92407500 \\ 555 \times 5 \times 30 = 83250 \\ 5 \times 5 \times \underline{25} \\ \hline 92490775 \quad \boxed{462453875} \\ \hline \end{array}$$

(17.) $\dot{7}01\dot{1}$ (19.1393267, *Ans.*)

300	1
270	6011
81	5859
651	152000
$19^2 \times 300 = 108300$	
$19 \times 30 = 570$	
	1
	108871
	108871
	43129000
$191^2 \times 300 = 10944300$	
$191 \times 3 \times 30 = 17190$	
	9
	10961499
	32884497
	10244503000
$1913^2 \times 300 = 1097870700$	
$1913 \times 30 \times 9 = 516510$	
$9 \times 9 = 81$	
	1098387291
	9885485619, etc., et.

(18.) $\sqrt[3]{\frac{48}{4394}} = \sqrt[3]{\frac{24}{2197}} = \frac{1}{13} \sqrt[3]{24} = \frac{1}{13}$ of $2.8844991 = .2218845$, *Ans.*

(19.) $\sqrt[3]{\frac{2}{3}}$ of $\frac{4}{11} = \sqrt[3]{.24242424+} = .6235319$, *Ans.*

EXTRACTION OF ANY ROOT

Art. 384.

(1.) $15625(125, \text{Ans.})$

0	1
1	*5625
1	44
$\frac{1}{*2}$	*1225
2	1225
22	
$\frac{2}{*24}$	
5	
245	

(2.)

$$\begin{array}{r}
 0 & 0 & 68719476736(4096, Ans. \\
 4 & 16 & 64 \\
 \hline
 4 & 16 & * 4719476 \\
 4 & 32 & 4417929 \\
 \hline
 8 & *48 & * 302547736 \\
 4 & 10881 & 301547736 \\
 \hline
 *12 & 490881 & \\
 09 & 10962 & \\
 \hline
 1209 & *501843 & \\
 9 & 73656 & \\
 \hline
 1218 & *50257956 & \\
 9 & & \\
 \hline
 *1227 & & \\
 6 & & \\
 \hline
 12276 & &
 \end{array}$$

(3.)

$$\begin{array}{rrrr}
 0 & 0 & 0 & 0 & 14348907(27, Ans. \\
 2 & 4 & 8 & 16 & *11148907 \\
 4 & 12 & 32 & *80 & 11148907 \\
 6 & 24 & *80 & 1592701 & \\
 8 & *40 & 113243 & & \\
 \hline
 *10 & 4749 & & & \\
 107 & & & &
 \end{array}$$

(4.)

0	0	151(5.325074, <i>Ans.</i>
5	25	<u>125</u>
5	50	* 26.00
10	*75	<u>23.877</u>
5	4.59	* 2.123000
*15	79.59	<u>1.691768</u>
.3	4.68	* .431232000
15.3	*84.27	<u>424935125</u>
.3	.3184	* .006296875
15.6	84.5884	<u>595467</u>
.3	.3188	34220
*15.9	*84.9072	
.02	.079825	
15.92	84.987025	
.02	.079850	
15.94	*85.066875	
.02		
*15.96		
.005		
15.965		
15.970		
*15.975		

(5.) Proceeding by Art. 382, we have $\sqrt[4]{97.41} = \text{sq. rt. of } \sqrt{97.41}$, which $= \sqrt{9.86965045} = 3.1415\varnothing$, or, 3.1416, *Ans.*

(6.) $\sqrt[4]{1.08} = \sqrt{\text{sq. rt. of } 1.08} = \sqrt{1.03923048} = 1.01943$, *Ans.*

(7.)

		$\frac{5}{12} = .41\bar{6}$:		
0	0	0	0	.4166(.83938, <i>Ans.</i>
8	64	512	4096	889866667
16	192	2048	204800000	227626024
24	384	5120000	220746881	9381865
32	64000	5315627	237291605	1944016
400	65209	5514908	24249351	
403	66427	5717870	24775122	
406	67654	5799	2479283	
409	68890	58419	2481054	
412		59039		

(8.)

$$35.2 = 32 \times 1.10. \quad \text{Hence root} = 2\sqrt[5]{1.10}$$

0	0	0	0	1.10	(1.01924)
1	1	1	1	*.10	
2	3	4	*5	*.0489899499	
3	6	*10	5.10100501	*.0013207560	
4	*10.	10.100501	*5.20302005	<u>109721</u>	
*5.01	10.0501	10.201504	5.2965771	22354	
5.02	10.1003	*10.303010	5.390967+	<u>21944</u>	
5.03	*10.1506	10.395228	<u>951</u>	410	
5.04	10.2010	10.487854	5.4860		
*5.05	10.2464	<u>92</u>	&c.,		
&c.,	10.2918	10.579			

$$1.01924 \times 2 = 2.03848, \text{ Ans.}$$

$$(8.) \sqrt[4]{782757789696} = \sqrt[4]{\text{cube rt.}} = \sqrt[4]{9216} = \sqrt{96} = 9.79795897, \text{ Ans.}$$

$$(9.) \sqrt{1367631} = \sqrt[3]{\text{cube rt.}} = \sqrt[3]{111} = 4.8058955, \text{ Ans.}$$

APPLICATIONS OF SQUARE AND CUBE ROOT

Art. 388.

$$(1.) \sqrt{30^2 + 12^2} = \sqrt{1044} = 32.31+ \text{ ft., Ans.}$$

$$(2.) \sqrt{10^2 \times 2} = 10\sqrt{2} = 14.142+ \text{ ft., Ans.}$$

$$(3.) \sqrt{69^2 + 92^2} = \sqrt{13225} = 115; \text{ and } (69 + 92) - 115 = 46 \text{ rd., Ans.}$$

$$(4.) \sqrt{500^2 + 360^2} = \sqrt{379600} = 616+ \text{ yd., Ans.}$$