

Sémantique des Langages de Programmation (SemLP)

TD n° 3 : Unification

Exercice 1 : (Ex. 52 in the course notes)

Apply the unification algorithm viewed in the course to the following systems of equations :

1. $\{ f(x, f(x, y)) = f(g(y), f(g(a), z)) \}$
2. $\{ f(x, g(y)) = f(y, g(g(x))) \}$

Exercice 2 : *** (Ex. 55 in the course notes)

1. Propose a method to transform a unification problem of the shape :

$$E = \{ t_1 = s_1, \dots, t_n = s_n \}$$

over the signature $\Sigma = \{g_1, g_n\}$ with $n, m \geq 1$ into a unification problem E' with the following properties :

1. E' contains exactly one equation,
 2. the terms in E' range over the signature $\Sigma' = \{f\}$, where f is binary,
 3. E has a solution if and only if E' has a solution, and
2. Apply the method to the system below, where x, y and z are variables.

$$E = \{ x = h(y), g(c, x, y) = g(y, z, z) \}$$

Exercice 3 : (Ex. 56 in the course notes)

Let t, s, \dots be terms over the signature Σ . We say that t is a *filter* for s if there exists a substitution S with $S t = s$. We denote this fact as $t \leq s$ ¹. Prove or disprove the following assertions :

1. If $t \leq s$, then t and s are unifiable.
2. If t and s are unifiable, then $t \leq s$ and $s \leq t$.
3. if $t \leq s$ and $s \leq t$, then s and t are unifiable.
4. For all t, s there exists an r with $r \leq t$, and $r \leq s$.
5. For all t, s , there exists an r with $t \leq r$ and $s \leq r$.

1. Not to be confused with the notation $S \leq S'$ over substitutions.

Termination

Exercice 4 : (Ex. 63 in the course notes)

Find a polynomial interpretation to show termination of the following TRS :

$$f(f(x, y), z) \rightarrow f(x, f(y, z)), \quad f(x, f(y, z)) \rightarrow f(y, y)$$

Exercice 5 : (Ex. 64 in the course notes)

1. Find a polynomial interpretation for the following TRS :

$$\begin{array}{lll} x + 0 \rightarrow x, & x + s(y) \rightarrow s(x + y), & \text{(addition)} \\ d(0) \rightarrow 0, & d(s(x)) \rightarrow s(s(d(x))), & \text{(double)} \\ q(0) \rightarrow 0, & q(s(x)) \rightarrow q(s(x)) + s(d(x)), & \text{(square)} \end{array}$$

2. Consider the term $t = q^{n+1}(s^2 0)$ whose size is linear in n . Show that there is a reduction as shown below with length doubly exponential in n .

$$t \rightarrow^* q^{n+1}(s^{2^{2^n}})$$