

Sémantique des Langages de Programmation (SemLP)

TD n° 2 : WP + Rewriting

Exercise 1 : Weakest Precondition for Imp (Ex. 12 in the course notes)

Let S be a statement and B an assertion. The weakest precondition of S with respect to B is a predicate that we denote with $wp(S, B)$ such that :

- (i) $\{wp(S, B)\} S \{B\}$ is valid, and
- (ii) if $\{A\} S \{B\}$ is valid then $A \supset wp(S, B)$ is valid.

Assuming that the statement S does not contain while loops. Propose a strategy to compute $wp(S, B)$ and derive a method to reduce the validity of the **pca** triple $\{A\} S \{B\}$ to the validity of a logical assertion.

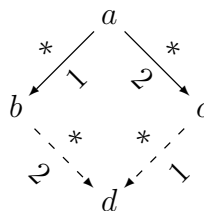
Exercise 2 :

Provide *simple* examples of rewriting systems satisfying, if possible, the following properties. Justify your answer in the case that no such system exists.

1. Terminating and not normalizing,
2. Normalizing and non-terminating,
3. Terminating and normalizing,
4. Confluent and non-terminating,
5. Church-Rosser and terminating.

Exercise 3 : (Ex. 27 in the course notes)

Let (A, \rightarrow_1) and (A, \rightarrow_2) be two rewriting systems. We say that they *commute* if $a \xrightarrow{*}_1 b$ and $a \xrightarrow{*}_2 c$ implies $\exists d, (b \xrightarrow{*}_2 d \text{ and } c \xrightarrow{*}_1 d)$ as shown below.



Show that if \rightarrow_1 and \rightarrow_2 are *confluent* and *commute* then $\rightarrow_1 \cup \rightarrow_2$ is confluent too.

Exercise 4 :

Consider the following **Imp** command (extended with integer addition and division) where b is an arbitrary boolean condition :

while $(u > l + 1)$ do $(r := (u + l)/2$; if b then $u := r$ else $l := r)$

Show that the evaluation of the command starting from any state where $u, l \in \mathbf{N}$ terminates.

Exercice 5 :

Does the evaluation of the following Imp commands terminate assuming that initially $m, n \in \mathbf{N}$?

1. while $(m \neq n)$ do (if $(m > n)$ then $m := m - n$ else $n := n - m$)
2. while $(m \neq n)$ do (if $(m > n)$ then $m := m - n$ else $(h := m; m := n; n := h)$)

Exercice 6 :

Let Σ^* denote the set of finite words over the alphabet $\Sigma = \{f, g_1, g_2\}$ with generic elements w, w', \dots . As usual, ϵ denotes the empty word. Let \rightarrow denote the smallest binary relation on Σ^* such that for all $w \in \Sigma^*$:

$$(1) fg_1w \rightarrow g_1g_1ffw, \quad (2) fg_2w \rightarrow g_2fw, \quad (3) f \rightarrow \epsilon,$$

and such that if $w \rightarrow w'$ and $a \in \Sigma$ then $aw \rightarrow aw'$. Prove or give a counter-example to the following assertions :

1. If $w \xrightarrow{*} w_1$ and $w \xrightarrow{*} w_2$ then there exists w' such that $w_1 \xrightarrow{*} w'$ and $w_2 \xrightarrow{*} w'$.
2. The rewriting system (Σ^*, \rightarrow) is terminating.
3. Replacing rule (1) with the rule $fg_1w \rightarrow g_1g_1fw$, the answers to the previous questions are unchanged.