Sémantique des Langages de Programmation (SemLP)
TD n° 3 : Unification

Exercice 1 : (Ex. 52 in the course notes)

Apply the unification algorithm viewed in the course to the following systems of equations :

1. $\{\ f(x, f(x, y)) = f(g(y), f(g(a), z))\ \}$

2. $\{\ f(x, g(y)) = f(y, g(g(x)))\ \}$

Exercice 2 : * (Ex. 55 in the course notes)**

1. Propose a method to transform a unification problem of the shape :

$$E = \{\ t_1 = s_1,\ \ldots,\ t_n = s_n\ \}$$

over the signature $\Sigma = \{g_1, g_n\}$ with $n, m \geqslant 1$ into a unification problem E' with the following properties :

1. E' contains exactly one equation,
2. the terms in E' range over the signature $\Sigma' = \{f\}$, where f is binary,
3. E has a solution if and only if E' has a solution, and

2. Apply the method to the system below, where x, y and z are variables.

$$E = \{\ x = h(y),\ g(c, x, y) = g(y, z, z)\ \}$$

Exercice 3 : (Ex. 56 in the course notes)

Let $t, s, \ldots$ be terms over the signature Σ. We say that t is a *filter* for s if there exists a substitution S with $S\,t = s$. We denote this fact as $t \leqslant s$[1]. Prove or disprove the following assertions :

1. If $t \leqslant s$, then t and s are unifiable.

2. If t and s are unifiable, then $t \leqslant s$ and $s \leqslant t$.

3. it $t \leqslant s$ and $s \leqslant t$, then s and t are unifiable.

4. For all t, s there exists an r with $r \leqslant t$, and $r \leqslant s$.

5. For all t, s, there exists an r with $t \leqslant r$ and $s \leqslant r$.

1. Not to be confused with the notation $S \leqslant S'$ over substitutions.

Termination

Exercice 4 : (Ex. 63 in the course notes)

Find a polynomial interpretation to show termination of the following TRS :

$$f(f(x,y),z) \to f(x,f(y,z)), \qquad f(x,f(y,z)) \to f(y,y)$$

Exercice 5 : (Ex. 64 in the course notes)

1. Find a polynomial interpretation for the following TRS :

$$\begin{aligned} x+\mathsf{0} &\to x, & x+\mathsf{s}(y) &\to \mathsf{s}(x+y), && \text{(addition)}\\ d(\mathsf{0}) &\to \mathsf{0}, & d(\mathsf{s}(x)) &\to \mathsf{s}(\mathsf{s}(d(x))), && \text{(double)}\\ q(\mathsf{0}) &\to \mathsf{0}, & q(\mathsf{s}(x)) &\to q(x)+\mathsf{s}(d(x)), && \text{(square)} \end{aligned}$$

2. Consider the term $t = q^{n+1}(s^2\mathsf{0})$ whose size is linear in n. Show that there is a reduction as shown below with length doubly exponential in n.

$$t \to^* q^{n+1}(s^{2^{2^n}})$$