Homework 1: Direction of Arrival estimation

Author: **Iman Javaheri Neyestanak**

Student ID: 10976128
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1.Introduction

This report focuses on estimating the Direction of Arrival (DoA) and range of targets using radar systems. Various scenarios, including a single-target, multi-target, and MIMO configurations, are considered. The MATLAB implementation simulates the scenarios with theoretical justification.

In this simulation, we must design the ULA, comprising 58 antennas with appropriate spacing between the elements, positioned around a single target situated at an angular position θ.

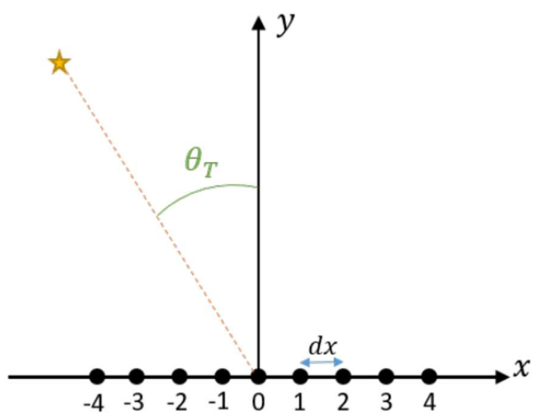

Figure 1: System geometry

2.Direction of Arrival estimation

In this section of the homework, we will design and simulate a ULA able to detect the angular position of the target in Figure 1 with a resolution at the boresight (θ = 0) of 2 degrees.

2.1. System model

Each antenna in the ULA transmits a continuous wave signal given by:

$$g(t) = e^{j2\pi f_0 t}$$

In a monostatic configuration, each antenna also receives the echo of its transmitted signal reflected from the target.

Consider a target located at an angle θ_T relative to the array's normal (broadside direction). The distance difference between the paths from the target to each antenna causes a phase difference in the received signals.

The path difference Δd between adjacent antennas is:

$$\Delta d = d_x \sin\theta_T$$

The corresponding phase difference $\Delta\phi$ is:

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta d = \frac{2\pi}{\lambda} d_x \sin\theta_T$$

The received signal at the n-th antenna can be modelled as:

$$r_n(t) = Ae^{j(2\pi f_0 t + \phi_n)}$$

2.2. Array design and DoA estimation

The angular resolution $\Delta\theta$ for a Uniform Linear Array (ULA) is given by:

$$\Delta\theta = \frac{\lambda}{L}$$

Rearrange to find the required array length:

$$L = \frac{\lambda}{\Delta\theta}$$

For a frequency $f_0 = 77GHz$:

$$\lambda = \frac{c}{f_0} = \frac{3*10^8 m/s}{77*10^9 Hz} \approx 3.9mm$$

Convert $\Delta\theta = 2^o$ to radians:

$$\Delta\theta = 2*\frac{\pi}{180} \approx 0.035 radians$$

Substituting the values:

$$L = \frac{\lambda}{\Delta\theta} = \frac{3.9*10^{-3}}{0.035} \approx 0.111m$$

To avoid spatial aliasing, the spacing must satisfy:

$$d_x \le \frac{\lambda}{2} = \frac{3.9*10^{-3}}{2} \approx 1.95mm$$

The number of antennas N is related to the total array length L and the spacing d_x:

$$N = \frac{L}{d_x} + 1$$

Substituting L = 0.111 m and d_x = 1.95 mm:

$$N = \frac{0.111}{0.00195} + 1 \approx 58$$

1. What should be the spacing between antennas?

$$d_x \le \frac{\lambda}{2} = \frac{3.9*10^{-3}}{2} \approx 1.95mm$$

2. Can you propose a way to detect the direction of arrival?

To estimate θ_T, we can measure the phase difference between the received signals at different antennas by computing the phase difference between adjacent antennas:

$$\Delta\phi = angle(r_{n+1}(t) r_n^*(t))$$

Solve for θ_T using the phase difference formula:

$$\theta_T = \arcsin(\frac{\Delta\phi\lambda}{2\pi d_x})$$

By measuring the phase differences between the received signals at different antennas in the ULA, we can estimate the angle of arrival of the target's echo.

3. Can you design the total array length to obtain the desired finest resolution of 2 degrees?

$$L = \frac{\lambda}{\Delta\theta} = \frac{3.9*10^{-3}}{0.035} \approx 0.111m$$

3.MATLAB implementation

With the design now complete, the next step is to implement it in MATLAB. First, we define antenna positions by creating a vector representing the spatial positions of the antennas along the x-axis. After that, we define the target's location by setting the angle(s) at which the target(s) is located. Finally, we generate the received signal vector. In this step, we calculate the phase shift for each antenna due to the target's angle and generate the received signal at each antenna, assuming a plane wave.

For estimating the DoA, we calculate the phase differences between adjacent antennas. To reduce the noise, we can compute the average phase difference. And, finally, we use this phase difference to estimate the angle.

1. Compare the estimated DoA with the true angular position of the target. Is it the same?

```
True Angles: 35.00 degrees
Estimated Angles: 35.10 degrees
```

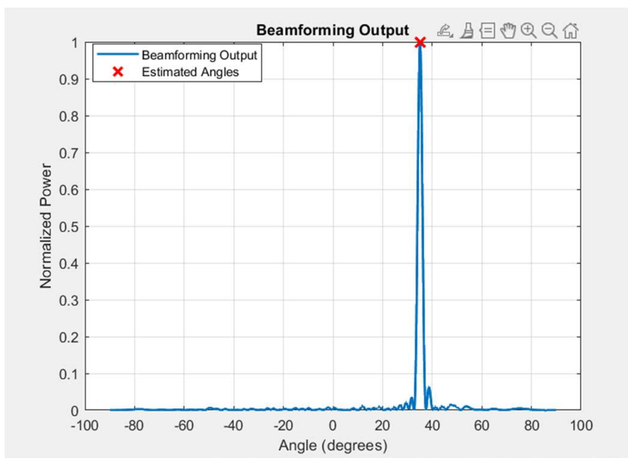

Figure 2: Single target estimation

It can be seen that the estimated DoA is very close to the true angle of 35 degrees. Due to factors such as numerical precision and approximations, there may be small differences in the results.

2. Is the resolution respected?

If we choose the targets very close to each other (less than 2 degrees), it may arise that the system is not capable of distinguishing both. Instead, one single target is detected at a angle of between them which is middle in fact.

```
True Angles: 15.00 degrees
True Angles: 16.00 degrees
Estimated Angles: 15.50 degrees
```

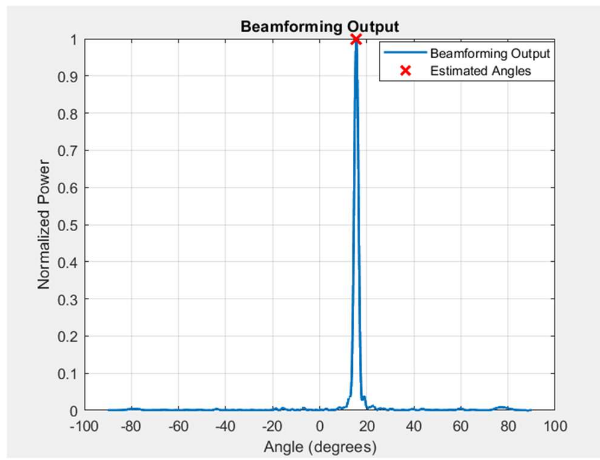

Figure 3: Resolution test

In case of angle of 2 degrees, however, the system is able to detect both the targets.

```
True Angles: 15.00 degrees
True Angles: 17.00 degrees
Estimated Angles: 14.60 degrees
Estimated Angles: 17.40 degrees
```

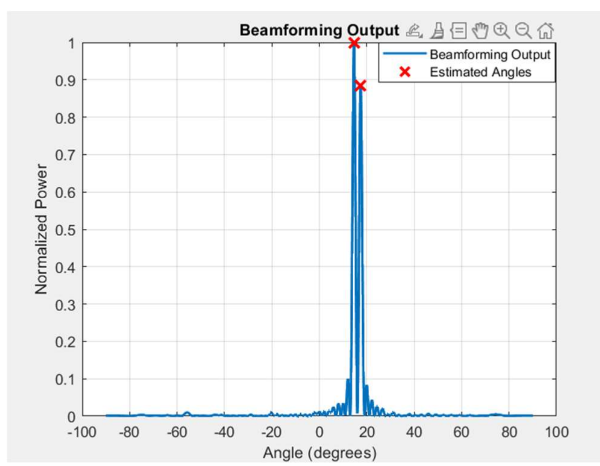

Figure 4: Resolution test

So, we can conclude that the resolution is respected.

3. Repeat the simulation with more than one target in the scene. What happens?

```
True Angles: -10.00 degrees
True Angles: 15.00 degrees
Estimated Angles: -9.90 degrees
Estimated Angles: 15.00 degrees
```

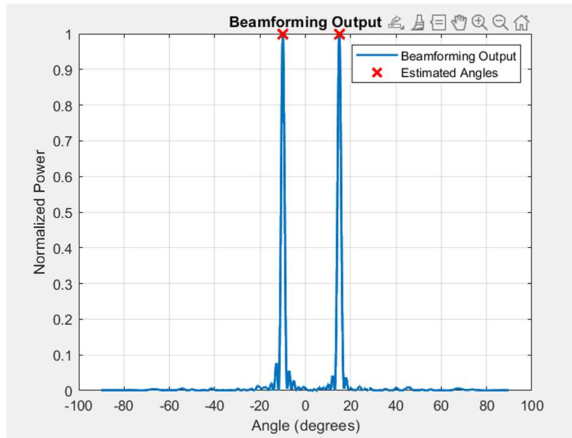

Figure 5: Multiple target

The system can detect the targets successfully, as expected.

4. What happens if you increase the spacing between antennas (dx)?

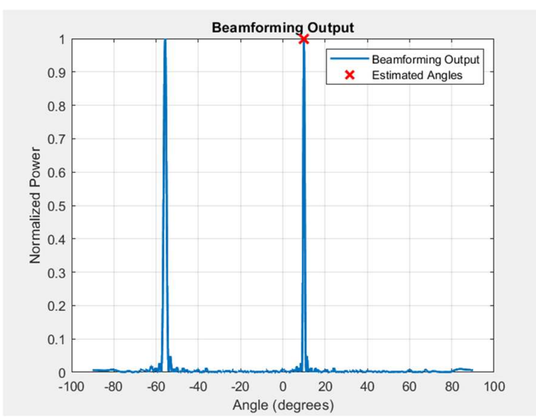

Figure 6: Effect of increasing antenna spacing ($dx = \lambda$)

The antenna spacing was increased to $dx = \lambda$ with a single target located at 10 degrees, which led to the emergence of grating lobes. These lobes appear as extra false peaks in the response, negatively impacting performance and reducing the accuracy of target localization. In conclusion, a half-wavelength spacing is ideal to prevent the formation of grating lobes and ensure precise Direction of Arrival (DoA) estimation.

4. 2D position estimation

In the previous section, the position estimation is just angular, meaning that the system is not able to estimate the distance between the array and the target (in radar jargon, this distance is called range). By changing the transmitted signal, it is possible to estimate the 2D position of the target (i.e., both in range and angle).

4.1. System model

In the previous section, we estimated only the angular position (DoA) of the target. To estimate both the angle and range of the target we should modify the transmitted signal as below:

$$g(t) = \sin c(Bt)e^{j2\pi f_0 t}$$

This signal has a wide bandwidth, which allows us to achieve high resolution in range.

1. What is the time and space resolution of such a signal?

The relation between time resolution of a signal and its bandwidth is as follow:

$$\Delta t = \frac{1}{B} = \frac{1}{1*10^9\,Hz} = 1*10^{-9}\,s$$

So, the time resolution is 1 ns.

To calculate space resolution, we have:

$$\Delta r = \frac{c\Delta t}{2} = \frac{(3*10\,^\wedge 8m/s)*(1*10^{-9}\,s)}{2} = 0.15m$$

So, the range resolution is 15 cm.

2. What is the expression of the received and demodulated signal at each antenna given a generic target in the scene? The signal is no longer monodimensional (where the only independent variable was the antenna position), but it is bi-dimensional. The two variables are the time t (or equivalently, the range r, related to the time by t = 2r/c) and the antenna position.

When the transmitted signal g(t) is reflected by a target at range r and angle θ, the received signal at the n-th antenna is:

$$r_n(t) = \alpha \sin c[B(t-\tau)]e^{j2\pi f_0(t-\tau)}e^{-jkx_n \sin\theta}$$

Where:

α is the round-trip time delay

$$\tau = \frac{2r}{c}$$

k is wavenumber:

$$k = \frac{2\pi f_0}{c}$$

x_n is the position of the n-th antenna.

$e^{-jkx_n \sin\theta}$ is considered for the phase shift due to the angle θ.

To demodulate the signal and bring it to baseband, first, we multiply by the complex conjugate of the carrier:

$$\tilde{r}_n(t) = r_n(t) * e^{-j2\pi f_0 t}$$

By simplifying we reach to the expression

$$\tilde{r}_n(t) = \alpha \sin c[B(t-\tau)]e^{-j2\pi f_0 \tau}e^{-jkx_n \sin\theta}$$

By using a low pass filter, the high frequency components are filtered out, leaving the baseband signal.

4.2. MATLAB Implementation

With the expression derived, we can implement the simulation in MATLAB by generating the transmitted and received signals and simulate the time delays and phase shifts due to the target’s range and angle.

To estimate the range, we use matched filtering or cross correlation to estimate τ from the sinc function. To estimate the angle, we use the spatial phase shifts across the antennas to estimate θ. After all, we combine the range and angle estimates to determine the target's position.

The results are as follows:

```
True Range: 83.00 meters
Estimated Range: 83.01 meters
True Angle: 26.00 degrees
Estimated Angle: 26.00 degrees
```

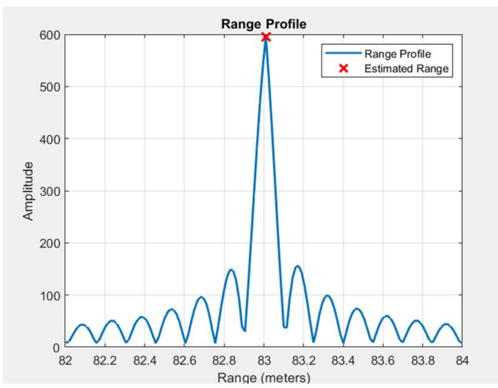

Figure 7: Range profile

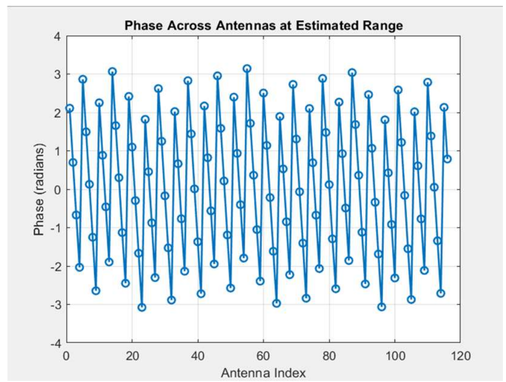

Figure 8: Phase profile

5.MIMO array

In practical applications, arrays are often realized using Multiple-Input Multiple-Output (MIMO) radar technology. In the setup described earlier, the N antennas are used for both transmitting and receiving signals, a configuration referred to as monostatic. On the other hand, a MIMO system involves N_{Tx} transmitting antennas and N_{Rx} receiving antennas, which are separate.

In particular, we will design a MIMO array comprising 2 transmitting antennas ($N_{Tx} = 2$) and 29 receiving antennas ($N_{Rx} = 29$). This configuration creates a total of 58 virtual elements ($N_{Tx} * N_{Rx} = 58$) using only 31 physical antennas ($N_{Tx} + N_{Rx} = 31$). This design replicates the virtual array presented in Section 2, maintaining uniform spacing to ensure precise estimation of both the direction of arrival (DoA) and range.

The transmitted signal is equal to:

$$g_{tx}(t) = \sin c(Bt).e^{j2\pi f_0 t}$$

And the received signal is

$$receive\_signal(j,:) = \sum_{i=1}^{NTx} g_{tx}(t-\tau).e^{j\phi_{ij}}$$

Where ϕ_{ij} is phase shifts calculated for each transmitter-receiver pair, based on their relative positions and the target's angle as follow:

$$\phi_{ij} = -k(x_{Rx_j} + x_{Tx_i})\sin(\theta)$$

The received signals are multiplied by the complex conjugate of the carrier signal to shift them back to the baseband frequency.

Range estimation using matched filtering involves cross-correlating the demodulated received signal with a reference sinc pulse to calculate the time delay (τ), which is used to determine the range. This process is efficiently implemented using FFT-based convolution, expressed as

$$R = ifft(fft(received\_signal\_de\mathrm{mod}(j,:)).conj(fft(ref\_signal))) \; .$$

The range profiles from all receive antennas are then summed to improve the Signal-to-Noise Ratio (SNR). Finally, the target's range is determined by identifying the peak in the summed range profile.

Direction of Arrival (DoA) estimation at the estimated range begins by extracting the demodulated received signal at the estimated time delay across all receive antennas. Next, the phase differences between adjacent receive antennas are calculated to determine the angle of arrival, using the formula

$$\Delta\phi = \angle(signal\_at\_tau(j+1).conj(signal\_at\_tau(j)))$$

To reduce noise effects, these phase differences are averaged.

Finally, the angle is estimated using the formula

$$\theta_{est} = \arcsin\left(\frac{-\Delta\phi_{avg}.\lambda}{2\pi d_x}\right),$$

where the negative sign corrects the angle estimation based on phase shift sign conventions.

By running the simulation, we reach to following results

```
True Range: 78.00 meters
Estimated Range: 78.00 meters
True Angle: 28.00 degrees
Estimated Angle: 28.00 degrees
```

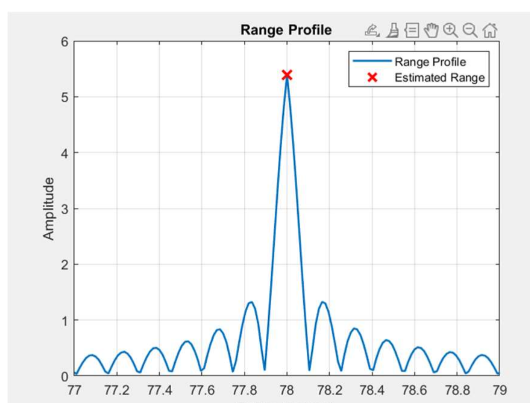

Figure 9: Range profile

6.Orthogonal waveform

In this section, we will design a MIMO radar system where all antennas transmit orthogonal waveforms simultaneously. This method prevents mutual interference and enables the receiver to distinguish signals from each transmitter using matched filtering for range compression.

When multiple antennas transmit at the same time, their signals can interfere with one another at the receiver, which complicates accurate range and angle estimation. To address this, we must design waveforms that are orthogonal or nearly orthogonal, ensuring their cross-correlation is either zero or very small.

$$r(t) = s_1(t) * s_2^*(-t) = \int s_1(\tau) s_2^*(\tau + t) d\tau \approx 0$$

By maintaining orthogonality, we can apply matched filtering at the receiver to separate the contribution of each transmitter.

For this implementation, we will use Orthogonal Frequency Division Multiplexing (OFDM) waveforms, where each transmitter is assigned a distinct subcarrier frequency. This approach enables simultaneous transmission while preserving orthogonality between the signals.

The system implementation begins by defining the system parameters, including the number of transmit and receive antennas, carrier frequency, bandwidth, and OFDM subcarrier frequencies. Next, orthogonal waveforms are generated by assigning a unique subcarrier frequency to each transmitter and modulating signals for each transmitter. The signal propagation is then simulated by modeling the transmitted signals as they travel to the target and return to the receivers, accounting for phase shifts caused by the target's angle and range. Additive white Gaussian noise (AWGN) is introduced to simulate realistic conditions. At the receiver, matched filtering is performed using the known transmitted waveforms to separate each transmitter's contribution. Range and angle estimation follows, where the time delay (range) for each transmitter's signal is estimated, and Direction of Arrival (DoA) is estimated using the virtual array formed by the transmit-receive pairs. Finally, orthogonality is validated by ensuring that the cross-correlation between the signals from different transmitters is minimal.

By using OFDM to implement orthogonal waveforms, we successfully simulated a MIMO radar system where multiple antennas transmit simultaneously without causing mutual interference. The matched filtering process effectively isolates each transmitter's contribution at the receiver, enabling precise range and angle estimation.

```
True Range: 100.00 meters
Estimated Range: 124.12 meters
True Angle: 25.00 degrees
Estimated Angle: 24.77 degrees
```

It is worth noting that, the angle estimation is almost the same, while the range estimation not and it has a large error.