

سؤال (1) X و Y مستقل هستند
(الف)

$$P(X \cap Y) = f_{xy}(x, y), P(x) = f_x(x), P(y) = f_y(y)$$

$$f_x(x) = \int_0^{\infty} f_{xy}(x, y) dy = \int_0^{\infty} 4e^{-2x} e^{-2y} dy = 2e^{-2x}$$

$$f_y(y) = \int_0^{\infty} f_{xy}(x, y) dx = \int_0^{\infty} 4e^{-2x} e^{-2y} dx = 2e^{-2y}$$

$$\Rightarrow f_x(x) \cdot f_y(y) = 4e^{-2x-2y} = f_{xy}(x, y)$$

$$\Rightarrow P(X \cap Y) = P(X) P(Y) \Rightarrow \text{مستقل هستند}$$

$$E(Y|X > r) = E(Y) \quad (\Leftrightarrow \text{Independence } Y \text{ and } X) \quad (1)$$

$$Y \sim \text{exp}(\lambda) \Rightarrow E(Y) = \frac{1}{\lambda} = \frac{1}{\mu}$$

$$f_Y(y) = \mu e^{-\mu y}$$

$$P(X > Y) = \int_{y=0}^{\infty} \int_{n=y}^{\infty} f_{X,Y}(n, y) dn dy \quad (2)$$

$$= \int_0^{\infty} \int_y^{\infty} \mu e^{-\mu n} e^{-\mu y} dn dy = \int_0^{\infty} \mu e^{-\mu y} e^{-\mu y} dy$$

$$= \frac{\mu}{\mu^2}$$

$$f_{xy}(x,y) = \begin{cases} 1 & 0 \leq y \leq 1 \text{ and } |x| \leq y \\ 0 & \text{otherwise} \end{cases}$$

سوال (۲)
الف)

$$\text{COV}(X,Y) = E(XY) - E(X)E(Y)$$

$$f_x(x) = \int_{-\infty}^{+\infty} f_{xy}(x,y) dy = \int_{|x|}^1 dy = 1 - |x|$$

$$0 \leq y \leq 1$$

$$f_y(y) = \int_{-\infty}^{+\infty} f_{xy}(x,y) dx = 2y \quad -1 \leq x \leq 1$$

$$E(X) = \int_{-1}^1 f_x(x) x dx = \int_{-1}^1 x(1-|x|) dx = 0$$

$$E(XY) = \int_0^1 \int_{-y}^y xy dx dy = \int_0^1 y \int_{-y}^y x dy dx = 0 \quad \text{تابع فرد} \rightarrow 1-|x| \text{ و } x$$

$$\Rightarrow \text{COV}(X,Y) = 0 - 0 \cdot (E(Y)) = \underline{\underline{0}}$$

مسئله ~~فرض~~ فرض می کنیم $x=y=1$ و $f_{xy}(x,y)=1$ پس

$$\text{but } f_y(x) f_x(x) = 2y - 2y|x| = 0 \neq f_{xy}(x,y)$$

پس X و Y مستقل نیستند \Leftarrow