۹۹۱۰۲۲۰۷ ایمان محمدی کوییز ۴ و آمار

$X \sim MirGoli(\lambda) \qquad \lambda > 2$

$$f_x(x) = \frac{1}{\Gamma(0, \ln(2)) - \Gamma(0, \ln(\lambda))} \times \frac{1}{x^2 \ln(x)} \qquad 2 < x < \lambda$$

$$f_{(x)}(x) = g(\lambda) \times \frac{1}{x^2 \ln(x)}$$

$$\lim_{n\to\infty} (X_1 X_2 \dots X_n)^{\frac{1}{n}} = ?$$

$$(X_1 X_2 \dots X_n)^{\frac{1}{n}} = e^{\frac{Y_1 + \dots + Y_n}{n}} \Rightarrow \begin{cases} \bar{Y} = \frac{Y_1 + \dots + Y_n}{n} \\ \text{Law of Large Numbers} \Rightarrow \lim_{n\to\infty} \bar{Y} = \mu \end{cases}$$

$$\Rightarrow e^{\bar{Y}}$$

حالا μ را محاسبه می کنیم $\Leftarrow$

$$\mu = E(y_i) = E(\ln(X_i)) = \int_2^{\lambda} \ln x \times g(\lambda) \times \frac{1}{x^2 \ln x} dx$$

$$\Rightarrow \mu = \int_2^{\lambda} g(\lambda) \frac{dx}{x^2} \Rightarrow \mu = g(\lambda) \int_2^{\lambda} \frac{dx}{x^2}$$

$$\Rightarrow \mu = \frac{-g(\lambda)}{x} \Big|_2^{\lambda} \Rightarrow \mu = -g(\lambda) \left(\frac{1}{\lambda} - \frac{1}{2} \right)$$

$$\xrightarrow[\lambda = 8]{\lambda = 8 \Rightarrow g(\lambda) \simeq 3} \mu = -3 \left(\frac{1}{8} - \frac{1}{2} \right) = \frac{9}{8}$$

$$\Rightarrow \lim_{n\to\infty} (X_1 X_2 \dots X_n)^{\frac{1}{n}} = e^{\frac{9}{8}}$$