Extensive Form Games II

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Introduction

- The extensive form is an alternative representation that makes the **Sequential** structure explicit.
- Two variants:
  - perfect information extensive-form games
  - imperfect-information extensive-form games **(Hidden Information)**

Example: Pocker

- Sequential play
- See some cards but not all
- See bets and react to them

Example: Pocker

- Many possible hands!
- Many betting strategies!
- Impossible to draw the tree... However, there is much we can learn about such games.

Introduction

- So far, we've allowed players to choose an action at every choice node.
  - This implies that players know the node they are in and all the prior choices, including those of other agents.
  - We may want to model agents needing to act with partial or no knowledge of the actions taken by others, or even themselves.

Introduction

- So far, we've allowed players to choose an action at every choice node.
  - This implies that players know the node they are in and all the prior choices, including those of other agents.
  - We may want to model agents needing to act with partial or no knowledge of the actions taken by others, or even themselves.
- Imperfect information extensive-form games:
  - each player's choice nodes partitioned into information sets
  - agents cannot distinguish between choice nodes in the same information set.

Formal Definition

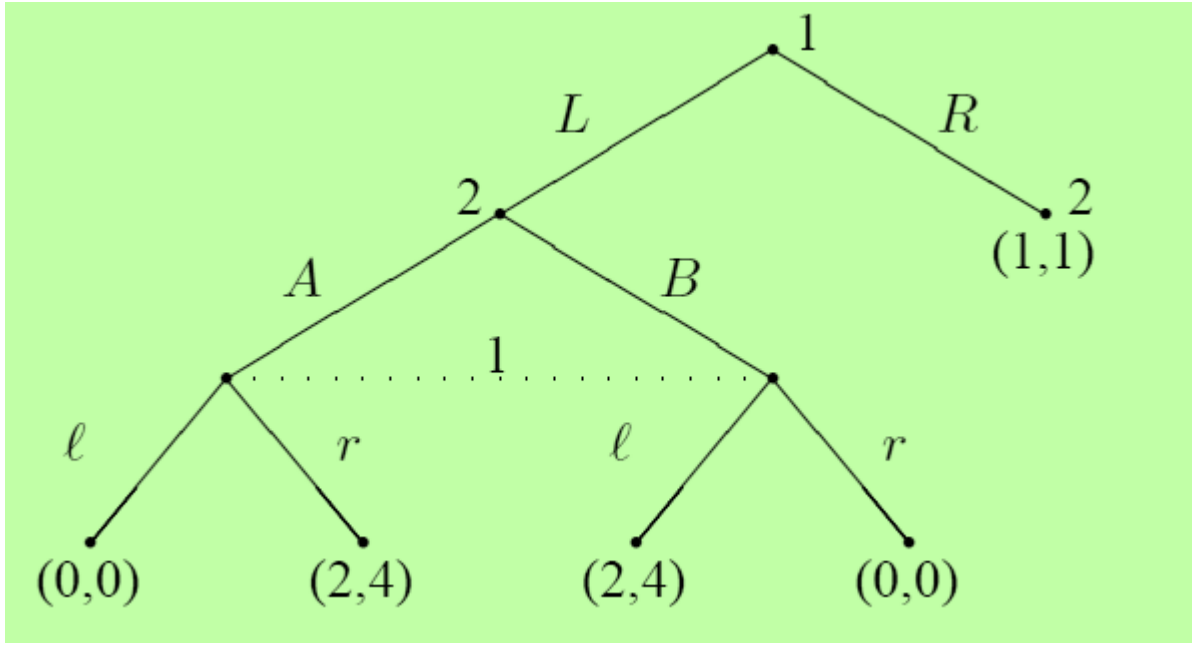
Definition

An imperfect-information game (in extensive form) is a tuple $(N, A, H, Z, \chi, \rho, \sigma, u, I)$, where

- $(N, A, H, Z, \chi, \rho, \sigma, u)$ is a perfect-information extensive-form game, and
- $I = (I_1, \ldots, I_n)$, where $I_i = (I_{i,1}, \ldots, I_{i,k_i})$ is an information set (that is, a partition of) $\{h \in H : \rho(h) = i\}$ with the property that $\chi(h) = \chi(h')$ and $\rho(h) = \rho(h')$ whenever there exists a j for which $h \in I_{i,j}$ and $h' \in I_{i,j}$.

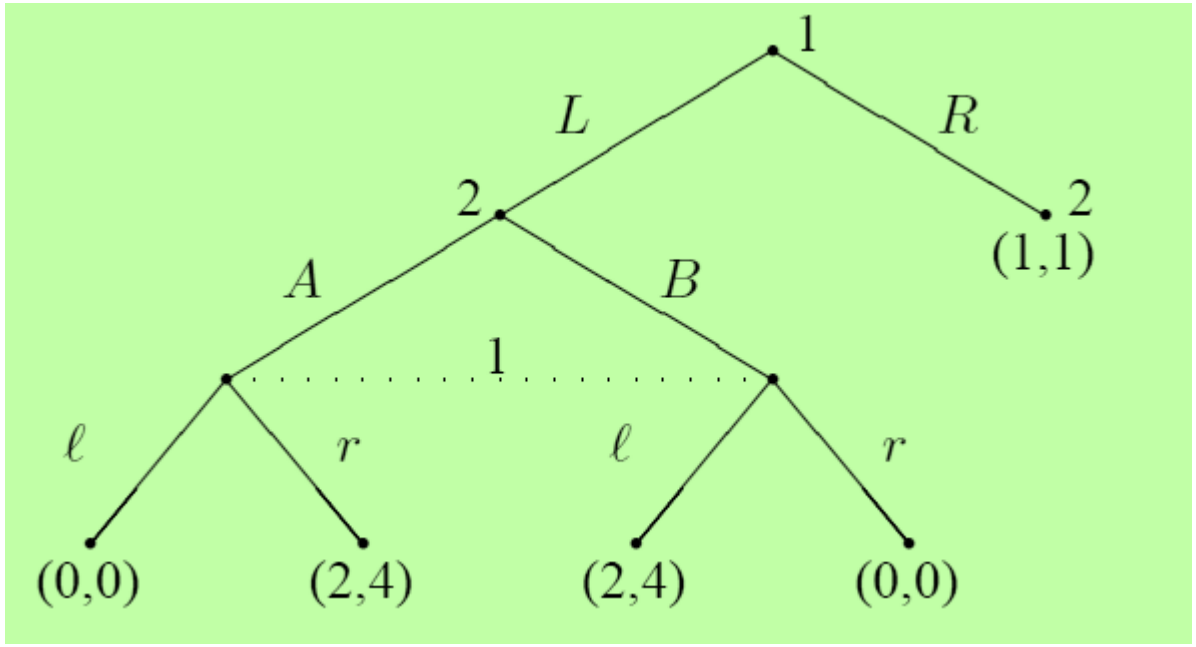
Example

- Pure strategies?
- Information sets?



Example

- Pure strategies?
- S_2 = {A,B}
- S_1 = {(L,l), (L,r), (R,l), (R,r)}



- Information sets?
- Player2 has 1 information set.
- Player1 has 2 information set.

Formal Definition

- In Perfect Info Game:

Definition (pure strategies)

Let $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ be a perfect-information extensive-form game. Then the pure strategies of player i consist of the cross product

$$\prod_{h \in H, \rho(h)=i} \chi(h)$$

- In Imperfect Info Game:

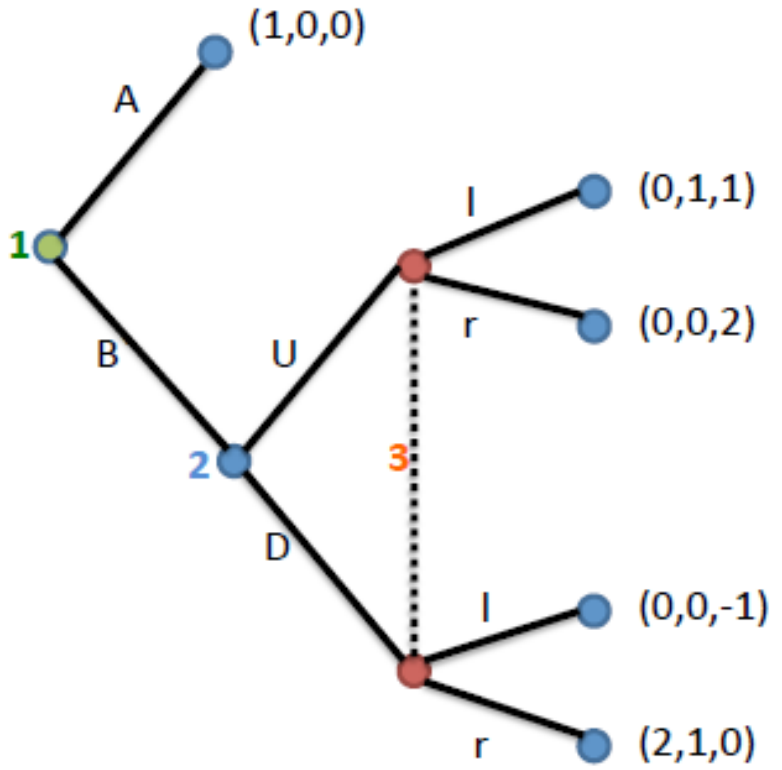
Formally, the pure strategies of player i consist of the cross product $\prod_{I_{i,j} \in I_i} \chi(I_{i,j})$.

Edited Definition of Subgame

- In Imperfect Info Game:

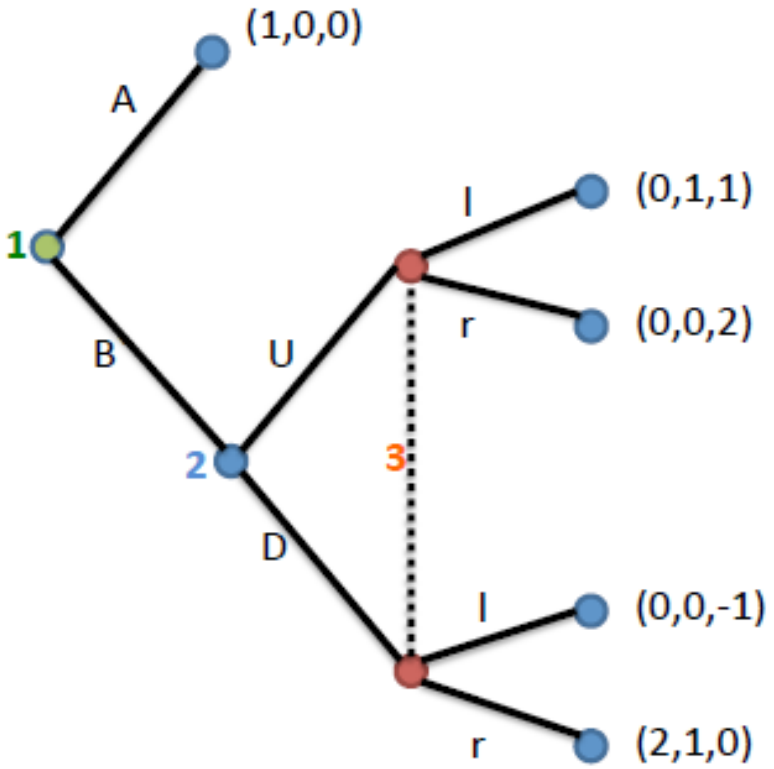
> ➢ A **sub-game** is a part of the game that looks like a game within the tree. It satisfies the three following properties:
>
> 1. It starts from a **single node**
> 2. It **comprises all successors** to that node
> 3. It **does not** break up any information set

Example: 3 player game



Example: 3 player game

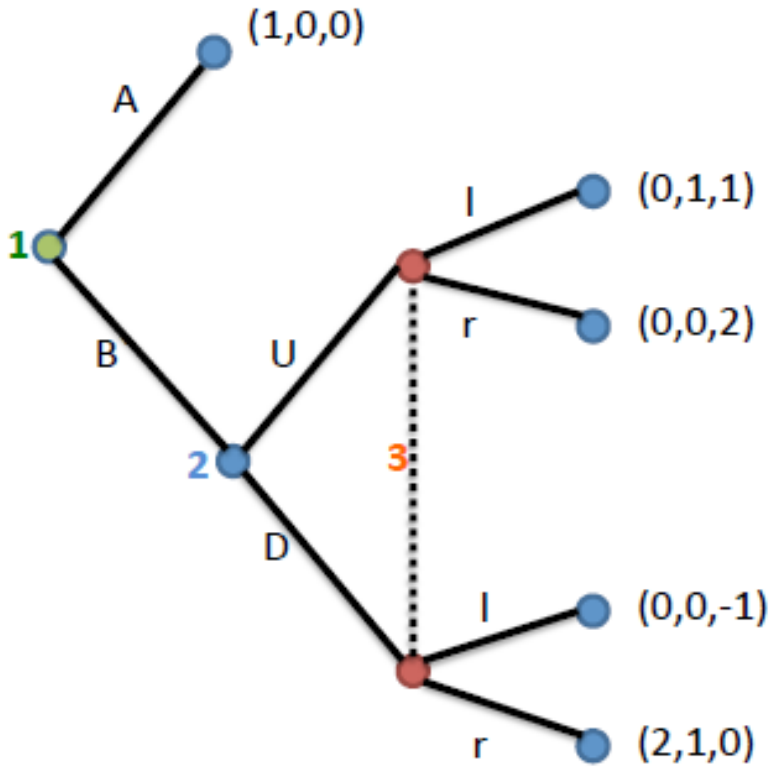
- What is SPNE?



Example: 3 player game

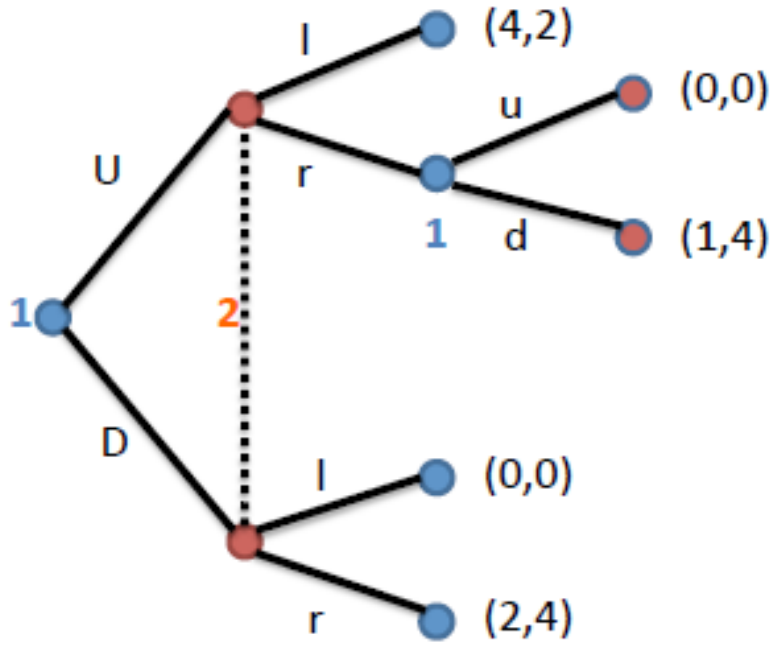
- What is SPNE?

SPNE is (B,D,r),



Another Example

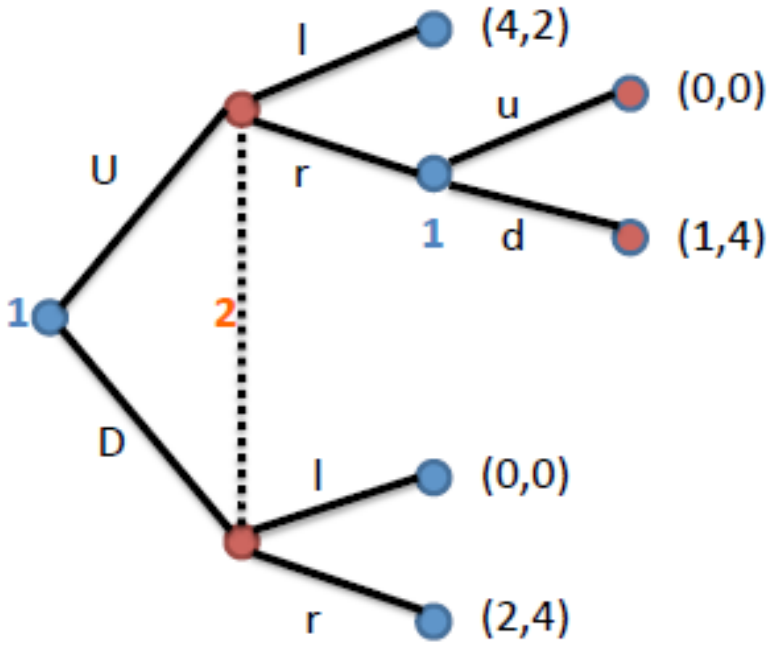
- What is SPNE?



Another Example

- What is SPNE?

SPNE is (Dd,r)



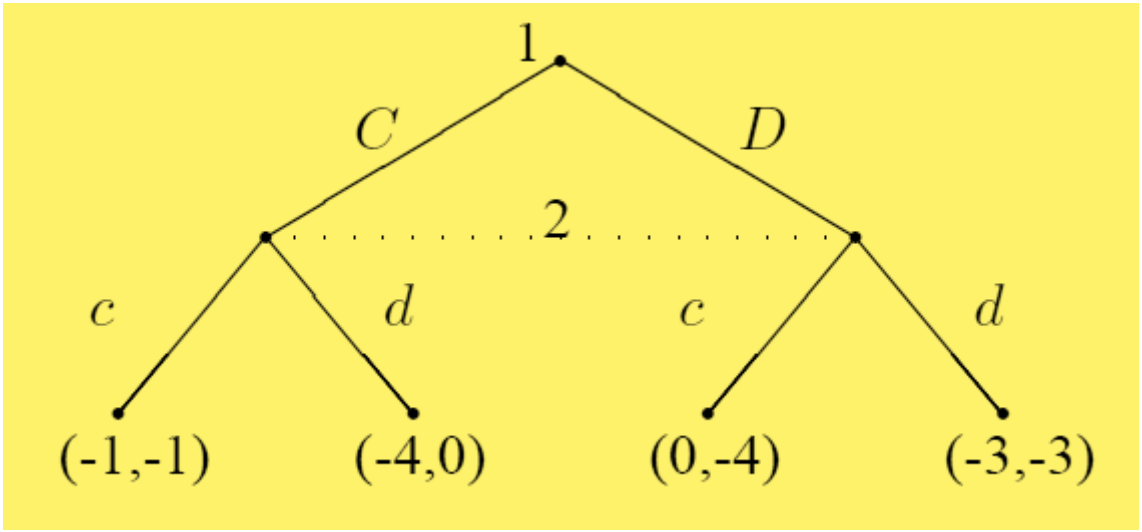
Converting Normal Form Games into IIG

- Prisoner's Dilemma Game:

	C	D
C	-1,-1	-4,0
D	0,-4	-3,-3

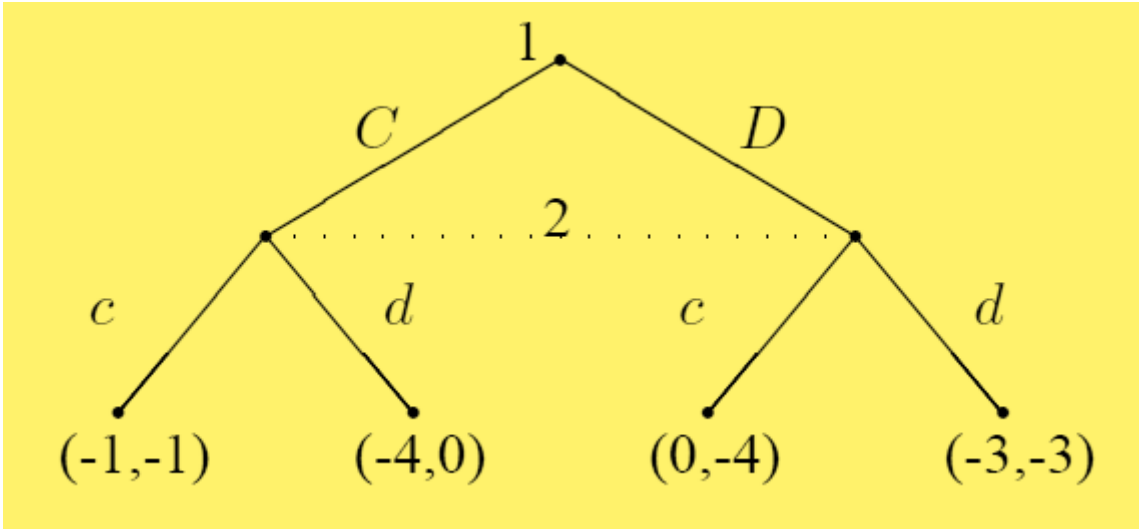
Converting Normal Form Games into IIG

- Conversion of PD game into IIG.



Converting Normal Form Games into IIG

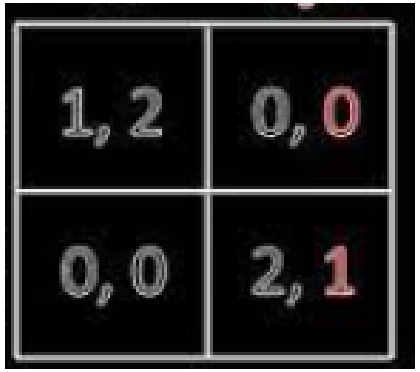
- Conversion of PD game into IIG.



- It would be the same if we put player 2 at the root node.

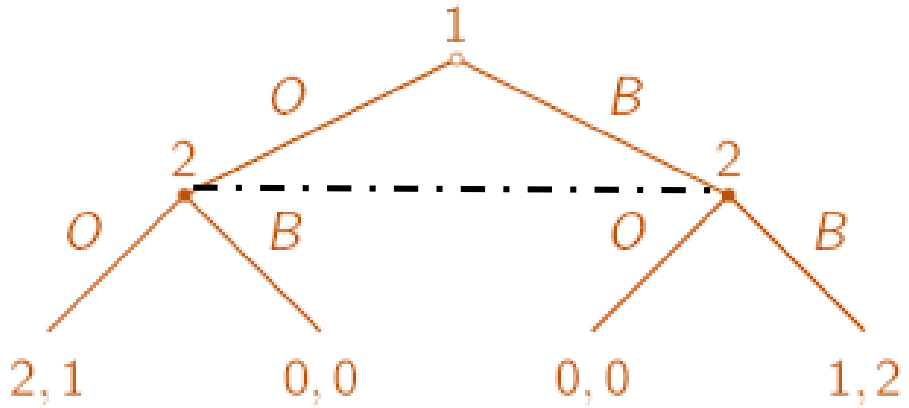
Converting Normal Form Games into IIG

- Battle of sexes game in extensive form?



Converting Normal Form Games into IIG

- Battle of sexes game



1, 2	0, 0
0, 0	2, 1

Some Points

- We can convert any normal form game into Imperfect Info Game (IIG).

- We've now defined two conversion NF $\rightarrow$ IIEF and IIEF $\rightarrow$ NF.

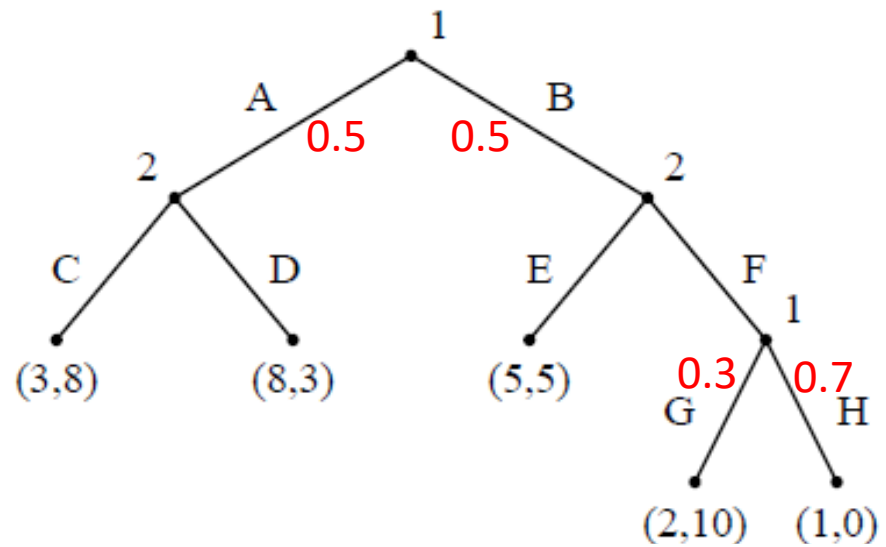
Randomized Strategies

- There are two meaningfully different kinds of randomized strategies in imperfect information extensive form games

- Mixed strategy: randomize over pure strategies
- Behavioral strategy: independent coin toss when an information set is encountered

Example

- Example of a behavioral strategy:
  - A with probability .5 and G with probability .3

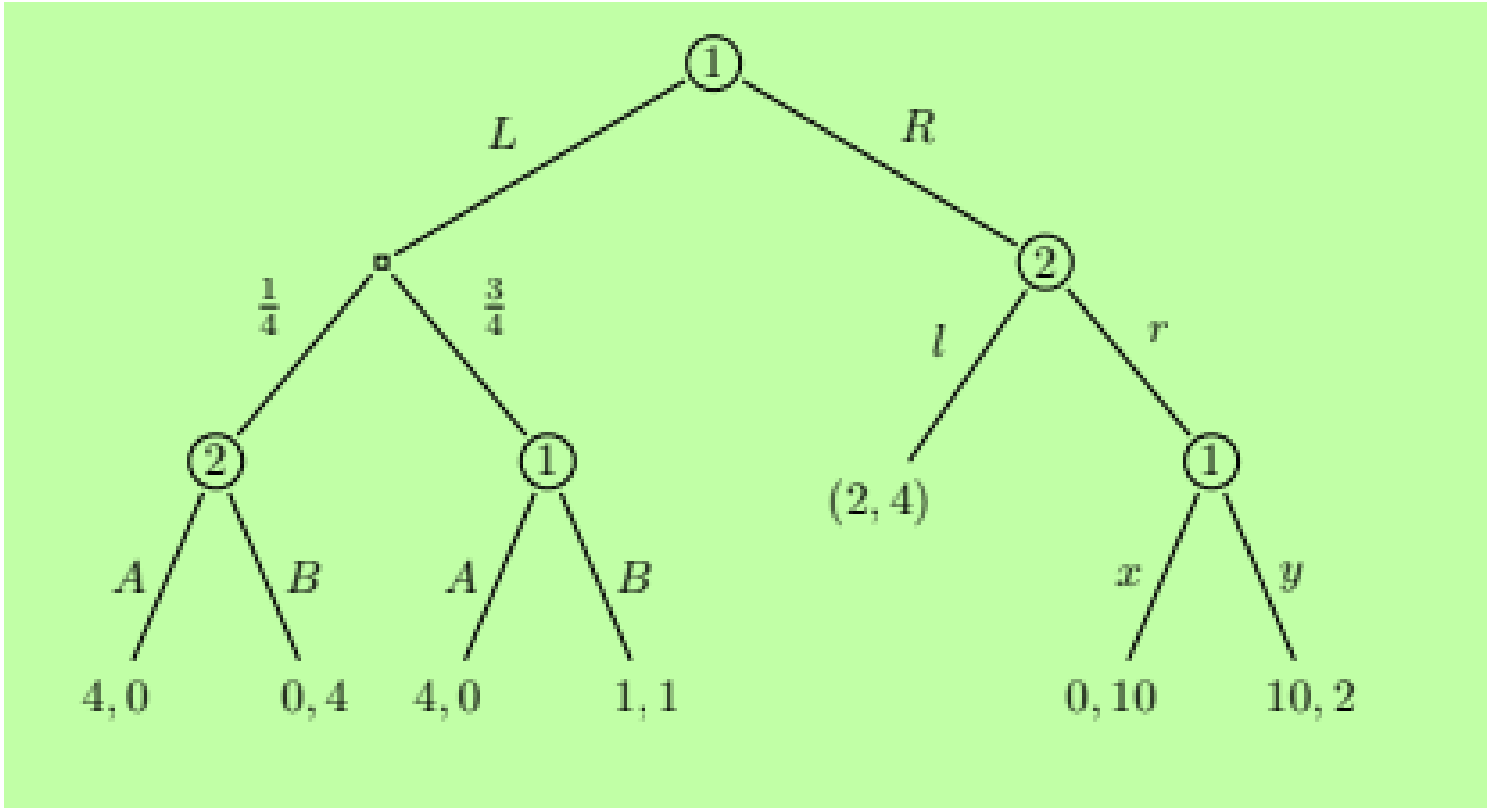


- Example of a mixed strategy
  - $(.6(A,G),.4(B,H))$

		CE	CF	DE	DF
0.6	AG	$3,8$	$3,8$	$8,3$	$8,3$
	AH	$3,8$	$3,8$	$8,3$	$8,3$
	BG	$5,5$	$2,10$	$5,5$	$2,10$
0.4	BH	$5,5$	$1,0$	$5,5$	$1,0$

Another Example

- What is the Nash Equilibria?



Another Example

- What is the Nash Equilibria?
- SPNE = (LAy , Bl)