Main Consepts about Game Theory

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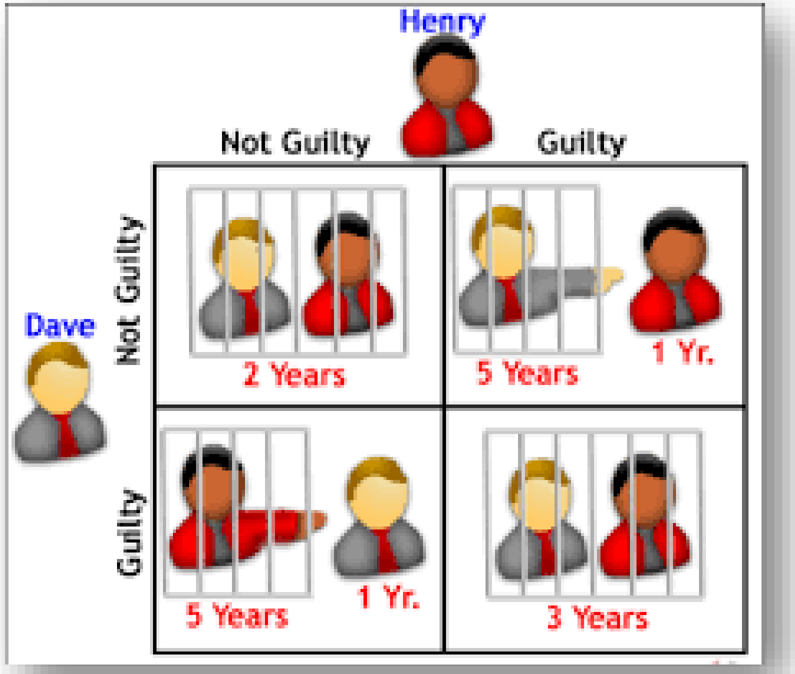
Spring 2023

Prisoner’s Dilemma (PD)

- If both silent: 2 year
- If both confess: 3 year
- If one silent & the other confess: 5, 1 year!



Prisoner’s Dilemma (PD)

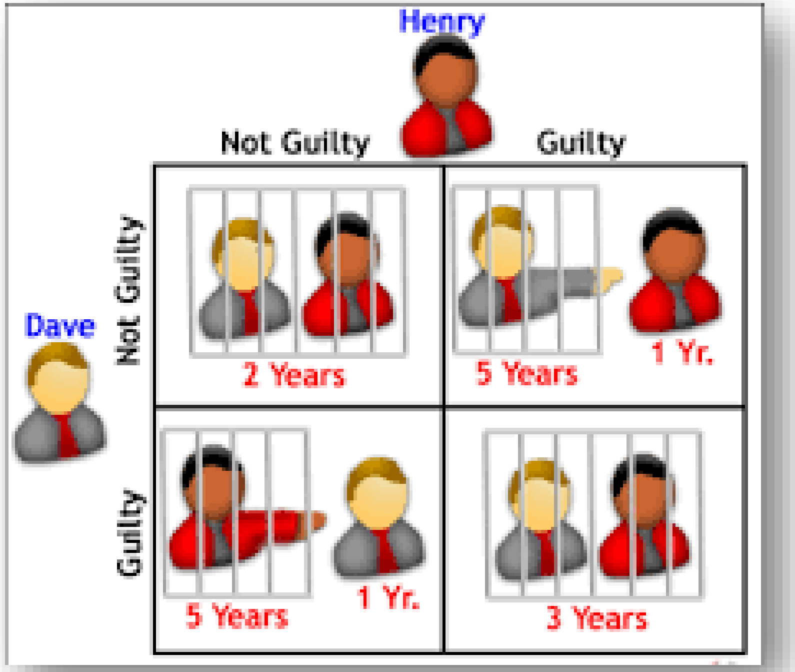
- Mathematical Form: in distinct tables

-2	-5
-1	-3

Happiness for Dave

-2	-1
-5	-3

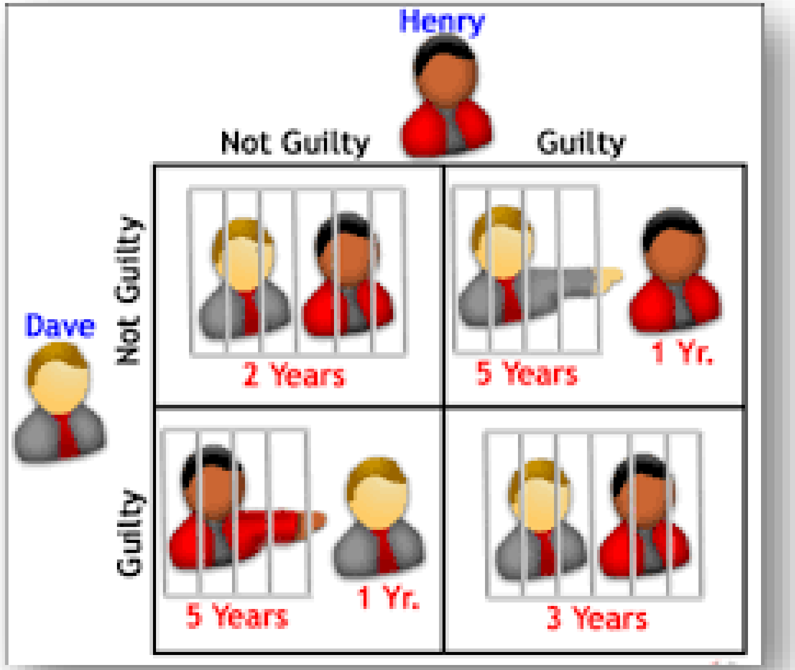
Happiness for Henry



Prisoner's Dilemma (PD)

- Normal Form Games: in common table

		Henry	
		Silent	Confess
Dave	Silent	-2,-2	-5,-1
	Confess	-1,-5	-3,-3



An important assumption: Rationality

- You Only care about your own decisions (not others).

All Agents are rational!
(Maximize your utility)

First Lesson

Put Yourself in Others' Shoes and Try to Figure Out What They Will Do!

"Think Strategically"

TCP Packet Game

- TCP or UDP?

TCP Packet Game

- TCP or UDP?

- Game rules
  - both TCP: both get 1 ms delay,
  - both UDP: both get 3 ms delay,
  - one TCP, one UDP: 4 ms , 0 ms delay!

- Please model this situations in normal form.

TCP Packet Game

Alice

		TCP	UDP
Bob	TCP	-1,-1	-4,0
	UDP	0,-4	-3,-3

A Question?

- Prisoner's dilemma vs TCP packet game?

Henry

		Silent	Confess
Dave	Silent	-2,-2	-5,-1
	Confess	-1,-5	-3,-3

Alice

		TCP	UDP
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- Is the same?

A Question?

- Prisoner’s dilemma vs TCP packet game?

Henry

Dave		Silent	Confess
	Silent	**-2,-2**	**-5,-1**
	Confess	**-1,-5**	**-3,-3**

Alice

Bob		TCP	UDP
	TCP	**-1,-1**	**-4,0**
	UDP	**0,-4**	**-3,-3**

- Is the same? Yes!

Pay attention to preferences!

- Any game with this form is PD.

		Player 2	
		Cooperation	Defection
Player1	Cooperation	a,a	b,c
	Defection	c,b	d,d

with c>a>d>b.

Second Lesson

Pay attention to preferences between payoffs!

Game Definition

- Finite, n-person normal form game: $\langle N, A, u \rangle$:
  - Players: $N = \{1, \ldots, n\}$ is a finite set of n , indexed by i
  - Action set for player i A_i
    - $a = (a_1, \ldots, a_n) \in A = A_1 \times \ldots \times A_n$ is an action profile
  - Utility function or Payoff function for player i: $u_i : A \to \mathbb{R}$
    - $u = (u_1, \ldots, u_n)$, is a profile of utility functions

Game Representation

- Writing a 2-player game as a matrix:
  - "row" player is player 1, "column" player is player 2
  - rows correspond to actions $a_1 \in A_1$, columns correspond to actions $a_2 \in A_2$
  - cells listing utility or payoff values for each player: the row player first, then the column

Dominated Strategies

Definition:

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- Please pay attention to "dominant" and "dominated"!

A Question?

- Is there any dominated strategy in these games? What?

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A Question?

- Is there any dominated strategy in PD game? What?

		Henry	
		Silent	Confess
Dave	Silent	-2,-2	-5,-1
	Confess	-1,-5	-3,-3

- Yes! Silent

Third Lesson

Do Not Play
Strictly Dominat<u>ed</u> Strategies!

Forth Lesson

Rational Choice
(i.e., Not Choosing a Dominat<u>ed</u> Strategy)
Can Lead to Outcomes that Suck!

Pick a number game!

- Without showing your neighbor what you’re doing, write down an integer number between 1 and 100.
- The winner is the person whose number is closest to 2/3 of the average in the class.
- The winner will win 10 $ minus the difference in cents between her choice and that 2/3 of the average.

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- Example: 3 students.
- Numbers: 25,5,60.
- Who win?
- How much?

Some Notations

	Notation	Pick a Number Game
Players	i, j, …	You all
Strategy	s_i: a particular strategy of player i s_{-i}: the strategy of everybody else except player i	S_4=12, s_8=22
Strategy Set	S_i: the set of possible strategies of player i	{1, 2, …, 100}
Strategy Profile	s: a particular play of the game "strategy profile" (vector, or list)	The collection of your pieces of paper
Payoffs	$u_i(s_1,\ldots, s_i,\ldots, s_N) = u_i(s)$	$u_i(s) = \begin{cases} \$10 - .01^*\Delta & \textit{if you win} \\ 0 & \textit{otherwise} \end{cases}$

Pay attention to Information!

- We assume all the agents of the game to be known
  - Everybody knows the possible strategies everyone else could choose
  - Everybody knows everyone else's payoffs

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Complete Information Game!

- This is not very realistic, but we start from this class of games.

Example

A not symmetric game.

		Player 2 L	C	R
Player 1	T	5, -1	11, 3	0,0
	B	6, 4	0, 2	2,0

Players		
Strategy sets		
Payoffs		

Example

A not symmetric game.

		L	C	R
			Player 2	
Player 1	**T**	5, -1	11, 3	0,0
	B	6, 4	0, 2	2,0

Players	**1, 2**	
Strategy sets	S_1={T,B}	S_2={L,C,R}
Payoffs	U_1(T,C) = 11	U_2(T,C) = 3

Game Analysis

- Thinking Strategically
- How is the game going to be played?
- You are the player 1 : what would you do?
  - Does player 1 have a dominated strategy?
- You are the player 2 : what would you do?
  - Does player 2 have a dominated strategy?

Game Analysis

- Thinking Strategically
- How is the game going to be played?
- You are the player 1 : what would you do?
  - Does player 1 have a dominated strategy? No
- You are the player 2 : what would you do?
  - Does player 2 have a dominated strategy? Yes, R is Strictly dominated by C.

Some Formal Definitions

Definition: Strict dominance

We say player i's strategy s_i' is strictly dominated by player i's strategy s_i if:

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}) \text{ for all } s_{-i}$$

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- No matter what other people do, by choosing s_i instead of s'_i, player i will always obtain a higher payoff.

Another example

- "Hannibal" game!

Attacker

		e	h
Defender	**E**	1, 1	1, 1
	H	0, 2	2, 0

Strategies

1. e, E = Easy Path ;
2. h,H = Hard Path

Payoffs:

1. **Attacker:** Number of battalions in your country
2. **Defender:** Number of attacker's lost battalions

Game Analysis

- Thinking Strategically
- You are the defender: what would you do?
  - Any dominated strategy?
- You are the attacker: what would you do?
  - Any dominated strategy?

Game Analysis

- Thinking Strategically
- You are the defender: what would you do?
  - Any dominated strategy? No
- You are the attacker: what would you do?
  - Any dominated strategy? Yes e, but not strictly dominated!

Another Formal Definitions

Definition: Weak dominance

We say player i's strategy s_i' is weakly dominated by player i's strategy s_i if:

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \text{ for all } s_{-i}$$
$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}) \text{ for some } s_{-i}$$

- No matter what other people do, by choosing s_i instead of s'_i, player i will always do at least well, and in some cases she does strictly better.

Pick a number game again!

- Without showing your neighbor what you're doing, write down an integer number between 1 and 100.
- The winner is the person whose number is closest to 2/3 of the average in the class.
- The winner will win 10 $ minus the difference in cents between her choice and that 2/3 of the average.

Game analysis

- What we know?
  1. Do not choose a strictly dominated strategy
  2. Also, do not choose a weakly dominated strategy
  3. You should put yourself in others' shoes, try to figure out what they are going to play, and respond appropriately

Dominated strategy?

- If everyone would chose 100, then the winning number would be 67

Dominated strategy?

- If everyone would chose 100, then the winning number would be 67
- Numbers bigger than 67 are weakly dominated by 67
- Rationality tells not to choose numbers bigger than 67

New game

- Now we’ve eliminated dominated strategies, it’s like a new game played over the set [1, …, 67]

- Once you figured out that nobody is going to choose a number above 67, the conclusion is Also strategies above 45 are ruled out.

- And so on!

Iterated elimination

- Eventually, we can show that also strategies above 30 are weakly dominated, once we delete previously dominated strategies.
- We can go on with this line of reasoning and end up with the conclusion that:

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- Eventually, we can show that also strategies above 30 are weakly dominated, once we delete previously dominated strategies.
- We can go on with this line of reasoning and end up with the conclusion that:

1 is the winning strategy!

- Suppose a player believes the average play will be X (including his or her own integer)
- That player's optimal strategy is to say the closest integer to $\frac{2}{3}X$.
- X has to be less than 100, so the optimal strategy of any player has to be no more than **67**.
- If X is no more than **67**, then the optimal strategy of any player has to be no more than $\frac{2}{3}67$.
- If X is no more than $\frac{2}{3}67$, then the optimal strategy of any player has to be no more than $(\frac{2}{3})^2 67$.
- Iterating, the unique Nash equilibrium of this game is for every player to announce 1!

Summary

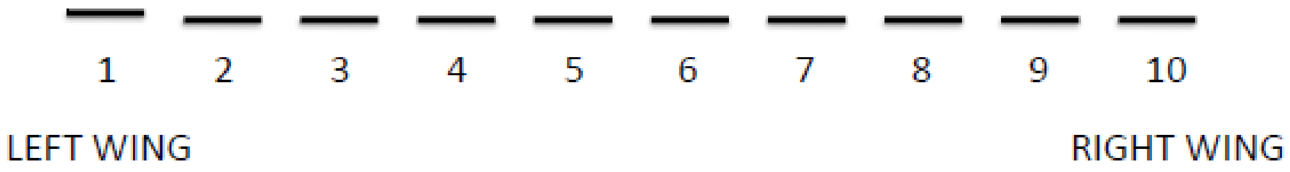
- Look at a game
- Figure out which strategies are dominated and delete them
- Look at the game again
- Look at which strategies are dominated now
- … and so on …

Summary

- Iterative deletion of dominated strategies seems a powerful idea
- but it’s also dangerous if you take it literally
- In some games, iterative deletion converges to a single choice, in others it may not

Election game

- 2 candidates as players
  - Choosing their political positions on a spectrum
- Assume the spectrum has 10 positions



Election game

- Voters are uniformly distributed and they will eventually vote for the closest candidate
- We assume that the candidates aim to **maximize** their share of vote (Win the Election)

Votes:	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
	1	2	3	4	5	6	7	8	9	10

LEFT WING RIGHT WING

Election game

- Are there any dominated strategies here?
- What's the prediction that game theory suggests here?
- Solve at home!