$$X_{i+1} = (aX_i + c) \bmod m,$$

$c \neq 0$, (m,c) = 1, m = 2^b, a = 1+4k → p = m

$c = 0$, x_0 is odd, a=3+8k | | a=5+8k → p = 2^{b-2} = m/4

m is prime, c = 0, $(a^k - 1) \% m = 0$ → p = min(k) = m-1

m prime, p = m-1: $X_i = \left(\sum_{j=1}^{k} (-1)^{j-1} X_{i,j}\right) \bmod m_1 - 1$

$$R_i = \begin{cases} \dfrac{X_i}{m_1}, & X_i \succ 0 \\ \dfrac{m_1 - 1}{m_1}, & X_i = 0 \end{cases}$$

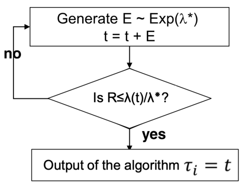
$$P = \frac{(m_1 - 1)(m_2 - 1)\ldots(m_k - 1)}{2^{k-1}}$$

Poisson Distribution $p(n) = P(N = n) = \dfrac{e^{-\alpha}\alpha^n}{n!}, \quad n = 0, 1, 2, \ldots$

$$\sum_{i=1}^{n} -\frac{1}{\alpha}\ln R_i \le 1 < \sum_{i=1}^{n+1} -\frac{1}{\alpha}\ln R_i \qquad \prod_{i=1}^{n} R_i \ge e^{-\alpha} > \prod_{i=1}^{n+1} R_i$$

$$Z_1 = (-2\ln R_1)^{1/2}\cos(2\pi R_2)$$
$$Z_2 = (-2\ln R_1)^{1/2}\sin(2\pi R_2)$$
$$N = \lceil \alpha + \sqrt{\alpha} Z - 0.5 \rceil$$

Non-stationary Poisson Process



Kolmogorov-Smirnov Test $S_N(x) = \dfrac{\text{number of } R_1, R_2, \ldots, R_n \text{ which are} \le x}{N}$

D = max| F(x) - $S_N(x)$|

Chi-Square Test

$$X_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

n is the # of classes

E_i is the expected # in the i^{th} class

O_i is the observed # in the i^{th} class

erlang $X = \sum_{i=1}^{k} X_i \longrightarrow X = \sum_{i=1}^{k} -\frac{1}{k\theta}\ln R_i = -\frac{1}{k\theta}\ln\left(\prod_{i=1}^{k} R_i\right)$

Tests for Autocorrelation

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} R_{i+(k+1)m}\right] - 0.25$$

$$\hat{\sigma}_{\rho_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$

Q-Q plot $F(\gamma) = P(X \le \gamma) = q$, for $0 < q < 1$ $\quad \gamma = F^{-1}(q)$

y_j is approximately $F^{-1}\left(\dfrac{j - 0.5}{n}\right)$ **and y_j is an estimate of the *(j - 1/2)/n* quantile of *X*.**

Plotting y_j versus $F^{-1}((j - 1/2)/n)$ as our Q-Q plot

$$S^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1} \qquad \bar{X} = \frac{\sum_{j=1}^{c} f_j m_j^2}{n} \qquad S^2 = \frac{\sum_{j=1}^{c} f_j m_j^2 - n\bar{X}^2}{n-1}$$

Exponential Distribution $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$

$$F(x) = \int_{-\infty}^{+\infty} f(t)\,dt = \begin{cases} 1 - e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases} \qquad X_i = -\frac{1}{\lambda}\ln R_i$$

Distribution	*Parameter(s)*	*Suggested Estimator(s)*
Poisson	α	$\hat{\alpha} = \bar{X}$
Exponential	λ	$\hat{\lambda} = \frac{1}{\bar{X}}$
Gamma	β, θ	$\hat{\beta}$ (see Table A.9) $\hat{\theta} = \frac{1}{\bar{X}}$
Normal	μ, σ^2	$\hat{\mu} = \bar{X}$ $\hat{\sigma}^2 = S^2$ (unbiased)
Lognormal	μ, σ^2	$\hat{\mu} = \bar{X}$ (after taking ln of the data) $\hat{\sigma}^2 = S^2$ (after taking ln of the data)
Weibull with $\nu = 0$	α, β	$\hat{\beta}_0 = \frac{\bar{X}}{S}$ $\hat{\beta}_j = \hat{\beta}_{j-1} - \frac{f(\hat{\beta}_{j-1})}{f'(\hat{\beta}_{j-1})}$ See Equations (11) and (14) for $f(\hat{\beta})$ and $f'(\hat{\beta})$ Iterate until convergence $\hat{\alpha} = \left(\frac{1}{n}\sum_{i=1}^{n} X_i^{\hat{\beta}}\right)^{1/\hat{\beta}}$
Beta	β_1, β_2	$\Psi(\hat{\beta}_1) + \Psi(\hat{\beta}_1 - \hat{\beta}_2) = \ln(G_1)$ $\Psi(\hat{\beta}_2) + \Psi(\hat{\beta}_1 - \hat{\beta}_2) = \ln(G_2)$ where Ψ is the digamma function, $G_1 = \left(\prod_{i=1}^{n} X_i\right)^{1/n}$ and $G_2 = \left(\prod_{i=1}^{n}(1 - X_i)\right)^{1/n}$

Weibull distribution (v = 0) $f(x) = \begin{cases} \dfrac{\beta}{\alpha^\beta} x^{\beta-1} e^{-(x/\alpha)^\beta}, & x \ge 0 \\ 0, & otherwise \end{cases}$

$$F(X) = 1 - e^{-(X/\alpha)^\beta}, x \ge 0$$

$$\Rightarrow \quad X = \alpha[-\ln(1-R)]^{1/\beta}$$

Triangular distribution

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{x^2}{2}, & 0 < x \le 1 \\ 1 - \dfrac{(2-x)^2}{2}, & 1 < x \le 2 \\ 1, & x > 2 \end{cases}$$

$$\Rightarrow \quad X = \begin{cases} \sqrt{2R}, & 0 \le R \le \frac{1}{2} \\ 2 - \sqrt{2(1-R)}, & \frac{1}{2} \prec R \le 1 \end{cases}$$

$$\boxed{X = \hat{F}^{-1}(R) = x_{(i-1)} + a_i\left(R - \frac{(i-1)}{n}\right)}$$

where $a_i = \dfrac{x_{(i)} - x_{(i-1)}}{i/n - (i-1)/n} = \dfrac{x_{(i)} - x_{(i-1)}}{1/n}$

if $c_{i-1} < R \le c_i$ (ci : cumulative frequency)

$$X = \hat{F}^{-1}(R) = x_{(i-1)} + a_i(R - c_{i-1})$$

Where

$$a_i = \frac{x_{(i)} - x_{(i-1)}}{c_i - c_{i-1}}$$

$$\operatorname{cov}(X_1, X_2) = E[(X_1 - \mu_1)(X_2 - \mu_2)] = E(X_1 X_2) - \mu_1\mu_2 \qquad \rho = \operatorname{corr}(X_1, X_2) = \frac{\operatorname{cov}(X_1, X_2)}{\sigma_1\sigma_2}$$

Hypothesis Testing $\bar{Y}_2 = \dfrac{1}{n}\sum_{i=1}^{n} Y_{2i} \qquad S = \left(\dfrac{\sum_{i=1}^{n}(Y_{2i} - \bar{Y}_2)^2}{n-1}\right)^{1/2}$

Conduct the *t* test:

$$|t_0| = \left|\frac{\bar{Y}_2 - \mu_0}{S/\sqrt{n}}\right| > t_{critical}$$

Confidence Interval Testing $\bar{Y} \pm t_{\alpha/2, n-1} S/\sqrt{n}$