Binomial: $p(x)=\binom{n}{x}p^x q^{n-x}$; Geo: $p(x)=q^{x-1}p$; Gamma: $\Gamma(\beta)=(\beta-1)! = \int_0^\infty x^{\beta-1}e^{-x}dx$; $f(x)=\frac{\beta\theta}{\Gamma(\beta)}(\beta\theta x)^{\beta-1}e^{-\beta\theta x}$;

Weibull: $f(x)=\frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], x\ge\nu$;

$N(\mu,\sigma^2)$; Tri: $x\in[a,b]\cup[b,c]\to h_b=2/(c-a)$;

$B(\beta_1,\beta_2)=\Gamma(\beta_1)\Gamma(\beta_2)/\Gamma(\beta_1+\beta_2)$; NB: $\binom{y-1}{k-1}q^{y-k}p^k, y\ge k$

Erlang: $F(x)=1-\sum_{i=0}^{k-1} e^{-k\theta x}(k\theta x)^i/i!, x>0$; Poisson: $\frac{e^{-\alpha}\alpha^x}{x!}$

$\boxed{P_n(t)}$ = prob ∃n customer at time t, $\boxed{\lambda}$ = arrival rate, $\boxed{\mu}$ = service rate 1 server,
$\boxed{\rho}$ = server utilization, $\boxed{A}_n$ = interarrival time between customer $n, n-1$,
$\boxed{S_n}$ = service time for n_{th} customer, $\boxed{W_n}$ = time spent in system by n_{th} customer
$\boxed{W_n^Q}$ = time spent in Queue, $\boxed{L(t)}$ = #cusomers at t, $\boxed{L_Q(t)}$ = #customers in Q at t,
$\boxed{w}$ = avrg time in system per cusomer, $\boxed{w_Q}$ = avrg … in Queue …, $\hat L = \hat\lambda\,\hat w$: little law

$\hat L=\frac{1}{T}\int_0^T L(t)dt, \rho=\frac{\lambda}{c\mu}, L=\sum_{n=0}^{\infty} nP(L(t\to\infty)=n), w_Q = w-\frac{1}{\mu}$

M/G/1 : $L=\rho+\rho^2(1+\sigma^2\mu^2)/2(1-\rho), P_0=1-\rho$

M/M/1: $P_n=(1-\rho)\rho^n, L=\lambda/\mu-\lambda, \sigma\mu=1$

LCM: Gen $(X_1,\ldots)\in[0,m-1]$ Using $X_{i+1}=(aX_i+c) \bmod m$ for $i\in\mathbb{W}, X_0=$ seed

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$\boxed{c\ne0, c=2^b, (c,m)=1, a=4k+1\to P=m}$ $\boxed{c=0, X_0 \bmod 2=1, a=8k+3\to P=2^{b-2}}$

$\boxed{c=0, m \text{ prime}, a^{m-1}\overset{m}{\equiv}1\to P=m-1}$ → multiplicative

$X_i=\left(\sum_{j=1}^{k}(-1)^{j-1}X_{i,j}\right) \bmod (m_1-1),\quad P=\frac{\prod_{j=1}^{k}(m_j-1)}{2^{k-1}},\quad R_i=\begin{cases}\frac{X_i}{m_1}, & X_i>0\\ 1-\frac{1}{m_1}, & X_i=0\end{cases}$

RNS → $S_i=X_{b(i-1)}, i\in\left\{1,\ldots,\left\lfloor\frac{P}{b}\right\rfloor\right\}$

Level of significance: $\alpha=P(\text{reject } H_0|H_0 \text{ is true})$

KS Test: $S_N(x)=\frac{count(R_i)\text{ where } R_i\le x}{N}, D=\max|F(x)-S_N(x)|, D<D_\alpha\to H_0$

$\{R_i\}:=Sort\{R_i\}\to D^+=\max\left\{\frac{i}{N}-R_i\right\}, D^-=\max\left\{R_i-\frac{(i-1)}{N}\right\}\to D=\max(D^+,D^-)$

Chi − Square: $N\ge50,\ X_0^2=\sum_{i=1}^{n}\frac{(O_i-E_i)^2}{E_i}, E_i=\frac{N}{n}, \nu=n-1, X_0^2<X_\alpha^2$

Autocorrelation: $M=\left\lfloor\frac{N-i}{m}-1\right\rfloor, Z_0=\frac{\hat\rho_{im}}{\sigma_{\hat\rho_{im}}}\sim N(0,1),\quad \sigma_{\hat\rho_{im}}=\frac{\sqrt{13M+7}}{12(M+1)},\quad \hat\rho_{im}=\frac{1}{M+1}\left[\sum_{k=0}^{M}R_{i+km}R_{i+(k+1)m}\right]-0.25,\ H_0\leftrightarrow|Z_0|<z_{\frac{\alpha}{2}},\ \phi\left(z_{\frac{\alpha}{2}}\right)=1-\frac{\alpha}{2}\to M\gg, R_i\gg0$

$x=F^{-1}(r), \forall x_0\ P(r\le F(x_0))=F(x_0), r\in U(0,1)\to$ exponential: $x=-\frac{\ln r}{\lambda}, \boxed{r\equiv1-r}$

Weibull: $X=\alpha(-\ln 1-R)^{1/\beta}$, Tri: $X=\begin{cases}\sqrt{2r}, r\in\left[0,\frac{1}{2}\right]\\ 2-\sqrt{2-2r}, r\in\left(\frac{1}{2},1\right]\end{cases}$

$n\ll: X=\hat F^{-1}(R)=x_{i-1}+a_i\left(R-\frac{i-1}{n}\right), a_i=n(x_i-x_{i-1}), P(x_{i-1}\le x\le x_i)=\frac{1}{n}\to i=\lceil nR\rceil$

$n\gg: X=x_{i-1}+a_i(R-c_{i-1}), a_i=\frac{x_i-x_{i-1}}{c_i-c_{i-1}}, c_{i-1}<R<c_i,\ c_i=\sum_{j=1}^{i}\frac{freq_j}{n}$

Poisson: $P(N=n)=\frac{e^{-\alpha}\alpha^n}{n!}, n\in\mathbb{W}, E(N)=\frac{1}{\alpha}\to A_i=-\frac{1}{\alpha}\ln R_i\to\sum_{i=1}^{n}A_i\le1<\sum_{i=1}^{n+1}A_i\to \prod_{i=1}^{n}R_i\ge\boxed{e^{-\alpha}>\prod_{i=1}^{n+1}R_i}\to R_0=1\to n:=n+1; \prod_{i=0}^{n+1}R_i<e^{-\alpha}\equiv N=n$

$Z=\frac{N-\alpha}{\sqrt{\alpha}}\to Z_1=(-2\ln R_1)^{0.5}\cos 2\pi R_2, Z_2(-2\ln R_1)^{0.5}\sin 2\pi R_2\to N=\left\lceil\alpha+\sqrt{\alpha}Z-\frac{1}{2}\right\rceil$

NSSP: $\lambda^*=\max\lambda(t)\to E\sim Exp(\lambda^*); t:=t+E, i:=i+1; R\le\lambda(t)/\lambda^*\equiv\tau_i=t$

$X\sim$ Erlang $\to X=\sum_{i=1}^{k}-\frac{1}{k\theta}\ln R_i$

Q − Q: $F(\gamma)=P(X\le\gamma)=q\to X_i\Rightarrow sort\{Y_j\}\equiv y_j\cong F^{-1}\left(\frac{j-0.5}{n}\right)$

$\bar X=\frac{\sum_{j=1}^{n}f_jX_j}{n}, S^2=\frac{\sum_{j=1}^{n}f_jX_j^2-n\bar X^2}{n-1}, f_j=\quad X_i; \bar X=\frac{\sum_{j=1}^{c}f_jm_j}{n}, S^2=\frac{\sum_{j=1}^{c}f_jm_j^2-n\bar X^2}{n-1}$

Poisson: $\alpha\cong\bar X$; Exp: $\lambda\cong\frac{1}{\bar X}$; Gamma: $\beta,\theta\to\beta\leftarrow M=\ln\bar X-\frac{1}{n}\sum_{i=1}^{n}\ln X_i, \theta=\frac{1}{\bar X}$;

$\chi_0^2=\sum_{i=1}^{k}\frac{(O_i-E_i)^2}{E_i}, 5\le E_i=n\times p_i, p_i=p(interval_i), \nu=(k-s)-1$

$\hat\lambda(t)=\frac{1}{n\Delta t}\sum_{j=1}^{n}C_{ij}, (i-1)\Delta t<t\le i\Delta t, \Delta t=\frac{t}{k}$

C_{ij} = # of arrivals during the i^{th} time interval on the j^{th} observation period

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$(X_1-\mu_1)=\beta(X_2-\mu_2)+\varepsilon, E(\varepsilon)=0, sgn(cov(X_1,X_2))=sgn(\beta)$

$cov(X_1,X_2)=E[(X_1-\mu_1)(X_2-\mu_2)]=E(X_1X_2)-\mu_1\mu_2$

$\rho=corr(X_1,X_2)=\frac{cov(X_1,X_2)}{\sigma_1\sigma_2}\in[-1,1]$

$t_0=\left|\frac{\bar Y-\mu_0}{S/\sqrt{n}}\right|,\quad$ 2sided: $t_{\frac{\alpha}{2},n-1}$, 1sided: $t_{\alpha,n-1}$

$\delta=\frac{|\bar Y-\mu|}{\sigma}$, C.I. $=\bar Y\pm t_{\frac{\alpha}{2},n-1}S/\sqrt{n}$

Weibull with $\nu=0$ $\quad\alpha,\beta\quad$ $\widehat{\beta}_0=\frac{\bar X}{S}$

$\widehat{\beta}_j=\widehat{\beta}_{j-1}-\frac{f(\widehat{\beta}_{j-1})}{f'(\widehat{\beta}_{j-1})}$

See Equations (11) and (14) for $f(\widehat{\beta})$ and $f'(\widehat{\beta})$

Iterate until convergence

$\widehat{\alpha}=\left(\frac{1}{n}\sum_{i=1}^{n}X_i^{\widehat{\beta}}\right)^{1/\widehat{\beta}}$

$$f(\beta)=\frac{n}{\beta}+\sum_{i=1}^{n}\ln X_i-\frac{n\sum_{i=1}^{n}X_i^{\beta}\ln X_i}{\sum_{i=1}^{n}X_i^{\beta}}$$

$$f'(\beta)=-\frac{n}{\beta^2}-\frac{n\sum_{i=1}^{n}X_i^{\beta}(\ln X_i)^2}{\sum_{i=1}^{n}X_i^{\beta}}+\frac{n\left(\sum_{i=1}^{n}X_i^{\beta}\ln X_i\right)^2}{\left(\sum_{i=1}^{n}X_i^{\beta}\right)^2}$$