# Midterm Answers

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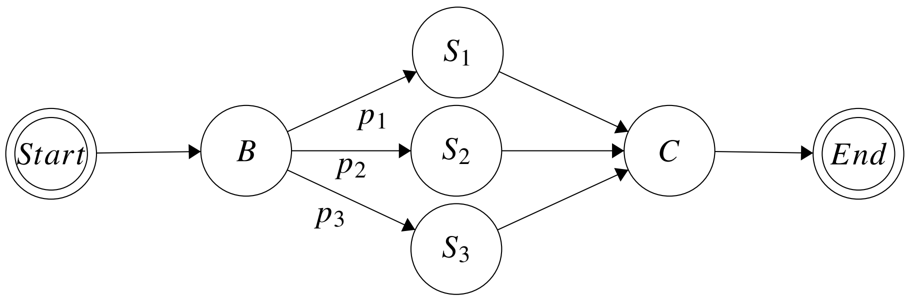
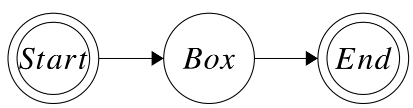
1. One of the main challenges in wireless sensor networks is load imbalance in the network, which means that some nodes need to send more packets than others. This issue has various reasons and can increase the energy drain of the nodes, turn them off, and bring irreparable consequences in some applications. Provide an arbitrary proposed solution and compare it with the network without load balancing.




To solve the problem, we place a load balancer node along the service route. This node calculates the distribution according to the utility of other nodes and sends each packet with a certain probability obtained from the said distribution to one of the other nodes. Then each node processes its packet and sends the result to the collecting node, and the result is then sent out of the service network by this node.




As can be seen from the figures above, people outside the service network will not notice its internal structure, but this change in its structure will increase its efficiency by using a larger number of service nodes.

2. A CAT5 cable manufacturer has stated on the packaging of their products that the resistance of these cables is a random variable with a normal distribution, the mean of which is $0.01\Omega$ and the standard deviation is $0.001\Omega$. If the computer engineering faculty wants to buy 1000 cables from this company, how many of these cables have a resistance between $0.009\Omega$ and $0.011\Omega$?

$$\begin{aligned} P(0.009 < x < 0.011) &= F_x(0.011) - F_x(0.009) \\ &= \Phi(\frac{0.011 - 0.01}{0.001}) - \Phi(\frac{0.009 - 0.01}{0.001}) \\ &= \Phi(1) - \Phi(-1) \\ &= 2\Phi(1) - 1 = 0.68 \end{aligned} \tag{1}$$

3. Obtain the response of the transient and permanent state State(2) in the following Markov chain. Suppose the vector Initial is [1, 0, 0].

   We know that the derivative of the vector of states of the given system is equal to $\pi(t)Q$, where $\Pi$ is the probability of each state at time $t$ and $Q$ is the transition matrix between the states.

$$Q = \begin{bmatrix} -0.3 & 0.3 & 0 \\ 0 & -0.3 & 0.3 \\ 0.5 & 0.5 & -1 \end{bmatrix} \tag{2}$$

$$\pi'(t) = \pi(t)Q \tag{3}$$

   To get the answer to the transient state, we first form the following system of equations from the above expression.

$$\begin{cases} \pi_1'(t) = -0.3\pi_1(t) + 0.5\pi_3(t) \\ \pi_2'(t) = +0.3\pi_1(t) - 0.3\pi_2(t) + 0.5\pi_3(t) \\ \pi_3'(t) = +0.3\pi_2(t) - 1.0\pi_3(t) \end{cases} \tag{4}$$

   Then we take this system of equations to the Laplace space. In this transformation, $\pi(t)$ and $\pi'(t)$ become $\pi(s)$ and $s\pi'(s)$.

$$\begin{cases} s\pi_1(s) - 1 = -0.3\pi_1(s) + 0.5\pi_3(s) \\ s\pi_2(s) - 0 = +0.3\pi_1(s) - 0.3\pi_2(s) + 0.5\pi_3(s) \\ s\pi_3(s) - 0 = +0.3\pi_2(s) - 1.0\pi_3(s) \end{cases} \tag{5}$$

   Now, the answer to the transient state is reached by solving this device and returning it from the Laplace space.

$$\pi_2(s) = \frac{15(s+1)}{s(50s^2 + 80s + 27)} \tag{6}$$

$$\begin{aligned} \pi_2(s) = \frac{5}{9} &+ (\frac{13}{18\sqrt{10}} - \frac{5}{18}) \exp((\frac{-4}{5} - \frac{1}{\sqrt{10}})t) \\ &+ (\frac{-13}{18\sqrt{10}} - \frac{5}{18}) \exp((\frac{1}{\sqrt{10}} - \frac{4}{5})t) \end{aligned} \tag{7}$$

   Finally, it is enough to take the limit from the transient state equation to calculate the permanent state. In this case, the final answer will be as follows

$$\lim_{x \to \infty} \pi_2(s) = \frac{5}{9} \tag{8}$$

4. In queue-based systems, the time interval between a customer arriving at a server Idle and the first moment that the server becomes idle again is Period Busy or called the busy period. If we assume that

this system uses a queue M/G/1 is modeled and the average service time in it is equal to $\bar{\mu}$, Show that the average busy period in this system is obtained from the following equation.

$$\frac{\bar{\mu}}{1-\rho}$$

If $\lambda$ is the input rate and $\bar{\mu}$ is the average service time for each customer, we know

$$\rho = \lambda E(s) = \lambda \bar{\mu} \tag{9}$$

We also know that $\rho$, which shows the system's utility, is equal to the ratio of the system's busy time to the total time. And since the input rate is equal to $\lambda$, the average idle time of the server (the average time required for the next customer to arrive) is equal to $\frac{1}{\lambda}$.

$$\rho = \frac{T_{busy}}{T_{busy} + T_{idle}} = \frac{T_{busy}}{T_{busy} + \frac{1}{\lambda}} \tag{10}$$

$$\rho = \lambda \bar{\mu} = \frac{T_{busy}}{T_{busy} + \frac{1}{\lambda}} \tag{11}$$

As a result of the above equation, the requested equation in the case of the question is proved as follows.

$$T_{busy} = \frac{\bar{\mu}}{1-\rho} \tag{12}$$

5. Consider an airport with one runway. Each of the planes to sit on this band must be in a queue. Suppose the service distribution is exponential at the rate of 22 planes per hour ($\mu$) and the arrival of planes at the airport is Poisson with a rate of 21 planes Consider time ($\lambda$). Calculate parameters $L, Q, L_w$ and $Q_w$ for this airport.

$$\rho = \frac{\lambda}{\mu} = \frac{20}{27} \approx 0.74 \tag{13}$$

$$L = \frac{\rho}{1-\rho} = \frac{20}{7} \approx 2.86, W = \frac{\frac{1}{\mu}}{1-\rho} = \frac{1}{7} \approx 0.14 \tag{14}$$

$$L_Q = \frac{\rho^2}{1-\rho} = \rho L = \frac{400}{189} \approx 2.12, W_Q = \frac{\frac{\rho}{\mu}}{1-\rho} = \frac{1}{7} \approx 0.14 \tag{15}$$