Analisis Real Lanjut Presentasi Pertemuan IX

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Lebesgue Space

Misalkan

$$f(x) = e^{-|x|}, \quad x \in \mathbb{R}$$

Tentukan apakah $f(x) \in L^1(\mathbb{R})$

Penyelesaian :

$$\begin{aligned}
\int_{-\infty}^{\infty} |f(x)|\,dx &= \int_{-\infty}^{0} |f(x)|\,dx + \int_{0}^{\infty} |f(x)|\,dx \\
&= \int_{-\infty}^{0} |e^{-(-x)}|\,dx + \int_{0}^{\infty} |e^{-x}|\,dx \\
&= \int_{-\infty}^{0} |e^{x}|\,dx + \int_{0}^{\infty} |e^{-x}|\,dx \\
&= \lim_{t\to-\infty} \int_{t}^{0} |e^{x}|\,dx + \lim_{t\to\infty} \int_{0}^{t} |e^{-x}|\,dx \\
&= \lim_{t\to-\infty} (e^{x}|_{t}^{0}) + \lim_{t\to\infty} (-e^{-x}|_{0}^{t}) \\
&= \lim_{t\to-\infty} (e^{x}|_{t}^{0}) - \lim_{t\to\infty} (e^{-x}|_{0}^{t}) \\
&= \lim_{t\to-\infty} (e^{0} - e^{t}) - \lim_{t\to\infty} (e^{-t} - e^{0}) \\
&= \lim_{t\to-\infty} e^{0} - \lim_{t\to-\infty} e^{t} - (\lim_{t\to\infty} (e^{-t}) - \lim_{t\to\infty} e^{0}) \\
&= (1-0) - (0-1) \\
&= 1 + 1 \\
&= 2
\end{aligned}$$

Jadi, karena $\int |f(x)|\dot{d}x = 2 < \infty$ maka $f(x) \in L^1(\mathbb{R})$.