Makassar, 7 Maret 2021

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Masalah Syarat Batas / Pertemuan ke-4 / Tugas

Hal. 50

D. Latihan 2

A. Selesaikanlah soal-soal berikut!

1.) Fungsi f yang periodik dengan periode 2π ditentukan oleh $f(x) = x(\pi - x)$ dalam selang $0 < x < \pi$.

Tentukan :

a.) Deret Kosinus Fourier $f(x)$

b.) Grafiknya $f(x)$.

Penyelesaian :

a.) Karena $F(x)$ fungsi genap, maka $b_n = 0$, sehingga kita cukup menentukan a_0 dan a_n saja. Kita peroleh:

$$a_0 = \frac{2}{\pi} \cdot \int_0^{\pi} (x(\pi - x))\,dx$$

$$= \frac{2}{\pi} \cdot \int_0^{\pi} (x\pi - x^2)\,dx$$

$$= \frac{2}{\pi} \cdot \left[\frac{x^2\pi}{2} - \frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{\pi} \cdot \left[\frac{\pi^3}{2} - \frac{\pi^3}{3}\right]$$

$$= \frac{2}{\pi} \cdot \left[\frac{3\pi^3 - 2\pi^3}{6}\right]$$

$$= \frac{2}{\pi} \cdot \left[\frac{\pi^3}{6}\right]$$

$$= \frac{2\pi^2}{6}$$

$$= \frac{\pi^2}{3}$$

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$$a_n = \frac{2}{\pi} \cdot \int_0^{\pi} (x(\pi - x)) \cdot \cos nx \cdot dx$$

$$= \frac{2}{\pi} \cdot \underbrace{\int_0^{\pi} (x\pi - x^2) \cdot \cos nx \cdot dx}_{*}$$

$$= \ldots$$

$(*)$ $\int_0^{\pi} (x\pi - x^2) . \cos nx \cdot dx =$ misal $u = x\pi - x^2$, $du = (\pi - 2x)dx$; misal $dv = \cos nx \, dx$, $v = \int \cos nx \, dx$, $v = \frac{\sin nx}{n}$

Integral parsial

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= \left[\left((x\pi - x^2) . \frac{\sin nx}{n} \right) - \left(\int \frac{\sin nx}{n} \cdot (\pi - 2x) \cdot dx \right) \right]_0^{\pi}$$

$$= \left[\left((x\pi - x^2) . \frac{\sin nx}{n} \right) - \left(\frac{1}{n} \cdot \int (\pi \cdot \sin nx - 2x \cdot \sin nx) dx \right) \right]_0^{\pi}$$

$$= \left[\left((x\pi - x^2) . \frac{\sin nx}{n} \right) - \left(\frac{1}{n} \cdot \left(\int \pi \cdot \sin nx \, dx - \int 2x \cdot \sin nx \, dx \right) \right) \right]_0^{\pi}$$

$$= \left[\left((x\pi - x^2) . \frac{\sin nx}{n} \right) - \left(\frac{1}{n} \left(\frac{-\pi \cdot \cos nx}{n} - \underbrace{\int 2x \cdot \sin nx \, dx}_{**} \right) \right) \right]_0^{\pi}$$

$(**)$ $\int 2x \cdot \sin nx \cdot dx =$ misal $u = 2x$, $du = 2 \, dx$; misal $dv = \sin nx \, dx$, $v = \int \sin nx \, dx$, $v = -\frac{\cos nx}{n}$

Integral Parsial

$$\int u \cdot dv = uv - \int v du$$

$$= \left(2x \cdot - \frac{\cos nx}{n} \right) - \left(\int - \frac{\cos nx}{n} \cdot 2 \, dx \right)$$

$$= \left(-2x \cdot \frac{\cos nx}{n} \right) - \left(-\frac{2}{n} \int \cos nx \cdot dx \right)$$

$$= \left(-2x \cdot \frac{\cos nx}{n} \right) - \left(-\frac{2}{n} \cdot \frac{\sin nx}{n} \right)$$

$$= \left(-2x \cdot \frac{\cos nx}{n} \right) + \left(\frac{2 \sin nx}{n^2} \right)$$

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Maka, (*) menjadi

$$\int_0^{\pi} (x\pi - x^2)\cdot \cos nx \, dx = \left[\left((x\pi - x^2)\cdot\frac{\sin nx}{n}\right) - \left(\frac{1}{n}\left(-\frac{\pi\cdot\cos nx}{n} - \int 2x\cdot \sin nx \, dx\right)\right)\right]_0^{\pi}$$

$$= \left[\left((x\pi - x^2)\cdot\frac{\sin nx}{n}\right) - \left(\frac{1}{n}\cdot\left(\left(-\frac{\pi\cdot\cos nx}{n}\right) - \left(-2x\cdot\frac{\cos nx}{n} + \frac{2\sin nx}{n^2}\right)\right)\right)\right]_0^{\pi}$$

$$= \left[\left((x\pi - x^2)\cdot\frac{\sin nx}{n}\right) - \left(\frac{1}{n}\left(-\frac{\pi\cdot\cos nx}{n} + \frac{2x\cdot\cos nx}{n} - \frac{2\sin nx}{n^2}\right)\right)\right]_0^{\pi}$$

$$= \left[\left((\pi\pi - \pi^2)\cdot\frac{\sin n\pi}{n}\right) - \left(\frac{1}{n}\left(-\frac{\pi\cdot\cos n\pi}{n} + \frac{2\pi\cdot\cos n\pi}{n} - \frac{2\sin n\pi}{n^2}\right)\right)\right]$$

$$- \left[\, 0 \,\right]$$

$$= \left[\left((0)\cdot(0)\right) - \left(\frac{1}{n}\left(\frac{2\pi\cdot\cos n\pi - \pi\cdot\cos n\pi}{n} - \frac{2\sin n\pi}{n^2}\right)\right)\right]$$

$$= \left[-\frac{1}{n}\left(\frac{(2\pi - \pi)(\cos n\pi)}{n} - \frac{2\sin n\pi}{n^2}\right)\right]$$

$$= \left[-\frac{1}{n}\cdot\left(\frac{\pi\cdot\cos n\pi}{n} - \frac{2\sin n\pi}{n^2}\right)\right]$$

$$= \left[-\frac{1}{n}\left(\frac{n\cdot\pi\cdot\cos n\pi - 2\sin n\pi}{n^2}\right)\right]$$

$$= \left(\frac{2\sin n\pi - n\pi\cdot\cos n\pi}{n^3}\right)$$

$$\Rightarrow a_n = \frac{2}{\pi}\cdot\left(\frac{2\sin n\pi - n\pi\cos n\pi}{n^3}\right)$$

$\therefore$ Deret Kosinus Fourier $f(x)$ adalah

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos nx$$

$$= \frac{1}{2}\cdot\left(\frac{\pi^2}{3}\right) + \sum_{n=1}^{\infty}\frac{2}{\pi}\cdot\left(\frac{2\sin n\pi - n\pi\cos n\pi}{n^3}\right)\cdot\cos nx \quad //$$

$$= \frac{1}{2}\cdot\left(\frac{\pi^2}{3}\right) + \sum_{n=1}^{\infty}\frac{2}{\pi}\left(\frac{2\cdot(0) - n\pi\cdot\cos n\pi}{n^3}\right)\cdot\cos nx$$

$$= \frac{1}{2}\cdot\left(\frac{\pi^2}{3}\right) + \sum_{n=1}^{\infty}\frac{2}{\pi}\left(\frac{-n\pi\cdot\cos n\pi}{n^3}\right)\cdot\cos nx.$$

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b.) Karena ditanyakan deret Kosinus Fourier $f(x)$ maka dapat dibentuk fungsi $F(x)$ yang memenuhi :

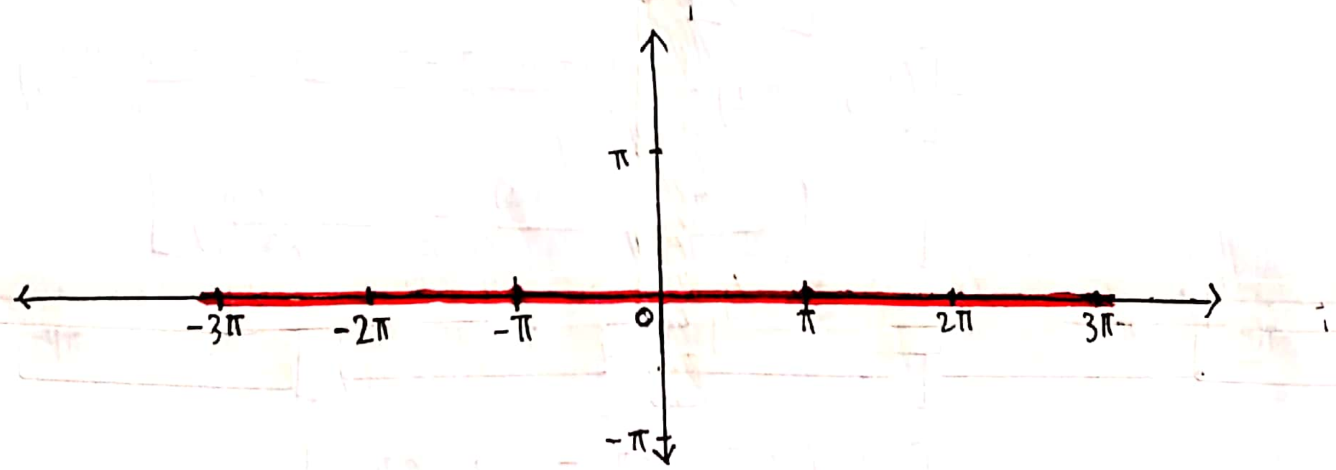
$$F(x) \equiv x(\pi - x) \text{ dalam selang } 0 < x < \pi ,$$

$$F(x) \equiv -x(\pi - (-x)) \text{ dalam selang } -\pi < x < 0$$

$$\equiv -x(\pi + x) \text{ dalam selang } -\pi < x < 0$$

maka, $F(x)$ fungsi genap dalam selang $-\pi < x < \pi$, yang mempunyai deret Kosinus Fourier.

Grafiknya dapat dilihat seperti berikut ini,



$u/x = \pi \Rightarrow F(x) \equiv \pi(\pi - \pi) \equiv \pi(0) \equiv 0$

$u/x = 2\pi \Rightarrow F(x) \equiv 2\pi(\pi - 2\pi) \equiv 2\pi(-\pi) \equiv -2\pi^2$

$u/x = 3\pi \Rightarrow F(x) \equiv 3\pi(\pi - 3\pi) \equiv 3\pi(-2\pi) \equiv -6\pi^2$

$u/x = 4\pi \Rightarrow F(x) \equiv 4\pi(\pi - 4\pi) \equiv 4\pi(-3\pi) \equiv -12\pi^2$

$u/x = -\pi \Rightarrow F(x) \equiv -\pi(\pi + (-\pi)) \equiv -\pi(0) \equiv 0$

$u/x = -2\pi \Rightarrow F(x) \equiv -(-2\pi)(\pi + (-2\pi)) \equiv 2\pi(-\pi) \equiv -2\pi^2$

$u/x = -3\pi \Rightarrow F(x) \equiv -(-3\pi)(\pi + (-3\pi)) \equiv 3\pi(-2\pi) \equiv -6\pi^2$

$u/x = -4\pi \Rightarrow F(x) \equiv -(-4\pi)(\pi + (-4\pi)) \equiv 4\pi(-3\pi) \equiv -12\pi^2$