Makassar, 25 September 2020

Teori Modul / Pertemuan ke-3 / Catatan

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Teori Modul : Catatan Pertemuan ke-3

Ruang Vektor Atas Lapangan

$\boxed{D}$

Misalkan V suatu himpunan, $V \neq \phi$ dengan F suatu lapangan, V dilengkapi dengan operasi penjumlahan

$$+ : V \times V \longrightarrow V$$
$$(\vec{a}, \vec{b}) \longmapsto \vec{a} + \vec{b}$$

dan operasi perkalian skalar

$$\bullet : F \times V \longrightarrow V$$
$$(\alpha, \vec{a}) \longmapsto \alpha\vec{a}$$

disebut Ruang Vektor atas Lapangan F jika memenuhi :

(1) $\forall \vec{a}, \vec{b} \in V \Rightarrow \vec{a} + \vec{b} \in V$

(2) $\forall \vec{a}, \vec{b}, \vec{c} \in V \Rightarrow \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

(3) Terdapat $\vec{0} \in V$ sehingga untuk setiap $\vec{a} \in V$ memenuhi :

$$\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$$

(4) $\forall \vec{a} \in V$, $\exists -\vec{a} \in V$ sehingga $\vec{a} + (-\vec{a}) = -\vec{a} + \vec{a} = \vec{0}$

(5) $\forall \vec{a}, \vec{b} \in V \Rightarrow \vec{a} + \vec{b} = \vec{b} + \vec{a}$

(6) $\forall \vec{a} \in V$, $\alpha \in F \Rightarrow \alpha\vec{a} \in V$

(7) $\forall \vec{a}, \vec{b} \in V$, $\alpha \in F \Rightarrow \alpha(\vec{a} + \vec{b}) = \alpha\vec{a} + \alpha\vec{b}$

(8) $\forall \vec{a} \in V$, $\alpha, \beta \in F \Rightarrow (\alpha + \beta)\vec{a} = \alpha\vec{a} + \beta\vec{a}$

(9) $\forall \vec{a} \in V$, $\alpha, \beta \in F \Rightarrow (\alpha.\beta).\vec{a} = \alpha.(\beta.\vec{a})$

(10) $\exists\, 1 \in F$. $\forall \vec{a} \in V \Rightarrow 1.\vec{a} = \vec{a}$

Note:
Aksioma (1) - (5) merupakan penjumlahan-nya dan
Aksioma (6) - (10) merupakan aksi skalarnya.

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$\boxed{E}$

① Himpunan $\mathbb{R}^2 = \{(a,b) \mid a,b \in \mathbb{R}\}$ dengan operasi penjumlahan dan perkalian skalar

$$\vec{x} + \vec{y} = (a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$$
$$\alpha \vec{x} = \alpha (a_1, b_1) = (\alpha a_1, \alpha b_1)$$

Buktikan $\mathbb{R}^2$ ruang vektor atas $\mathbb{R}$

Bukti :

Ambil sebarang $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^2$, $\alpha, \beta \in \mathbb{R}$ sebarang

Tulis
$\vec{x} = (a_1, b_1)$ untuk suatu $a_1, b_1 \in \mathbb{R}$
$\vec{y} = (a_2, b_2)$ untuk suatu $a_2, b_2 \in \mathbb{R}$
$\vec{z} = (a_3, b_3)$ untuk suatu $a_3, b_3 \in \mathbb{R}$

(1) Adb. $\mathbb{R}^2 \neq \phi$

Misal $(1,2) \in \mathbb{R}^2$. Jadi $\mathbb{R}^2 \neq \phi$

(2) Adb. $\vec{x} + \vec{y} \in \mathbb{R}^2$

$\vec{x} + \vec{y} = (a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2) \in \mathbb{R}_2$

(3) Adb. $(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z})$

Note that,

$$\begin{aligned}
(\vec{x} + \vec{y}) + \vec{z} &= [(a_1, b_1) + (a_2, b_2)] + (a_3, b_3) \\
&= (a_1 + a_2, b_1 + b_2) + (a_3, b_3) \\
&= (a_1 + a_2 + a_3, b_1 + b_2 + b_3) \\
&= (a_1 + (a_2 + a_3), b_1 + (b_2 + b_3)) \\
&= (a_1, b_1) + (a_2 + a_3, b_2 + b_3) \\
&= (a_1, b_1) + [(a_2, b_2) + (a_3, b_3)] \\
&= \vec{x} + (\vec{y} + \vec{z})
\end{aligned}$$

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(4) Pilih $\vec{0} = (0,0) \in \mathbb{R}^2$ sehingga untuk setiap $\vec{x} = (a_1, b_1) \in \mathbb{R}^2$ + $a_1, b_1 \in \mathbb{R}$ berlaku

$$\vec{x} + \vec{0} = (a_1, b_1) + (0,0) = (a_1 + 0, b_1 + 0) = (a_1, b_1) = \vec{x}$$

dilain pihak

$$\vec{0} + \vec{x} = (0,0) + (a_1, b_1) = (0 + a_1, 0 + b_1) = (a_1, b_1) = \vec{x}$$

sehingga

$$\vec{x} + \vec{0} = \vec{0} + \vec{x} = \vec{x}$$

(5) Ambil $\vec{x} = (a_1, b_1) \in \mathbb{R}^2$, $a_1, b_1 \in \mathbb{R}$
Pilih $-\vec{x} = (-a_1, -b_1) \in \mathbb{R}^2$ sehingga

$$\begin{aligned} \vec{x} + (-\vec{x}) &= (a_1, b_1) + (-a_1, -b_1) \\ &= (a_1 - a_1, b_1 - b_1) \\ &= (0,0) \\ &= \vec{0} \end{aligned}$$

dilain pihak

$$\begin{aligned} (-\vec{x}) + \vec{x} &= (-a_1, -b_1) + (a_1, b_1) \\ &= (-a_1 + a_1, -b_1 + b_1) \\ &= (0,0) \\ &= \vec{0} \end{aligned}$$

Jadi, $\forall\ x \in \mathbb{R}^2, \exists\ \vec{0} \in \mathbb{R}^2$ sehingga $\vec{x} + \vec{0} = \vec{0} + \vec{x} = \vec{x}$

(6) Adb. $\vec{x} + \vec{y} = \vec{y} + \vec{x}$

$$\begin{aligned} \vec{x} + \vec{y} = (a_1, b_1) + (a_2, b_2) &= (a_1 + a_2, b_1 + b_2) \\ &= (a_2 + a_1, b_2 + b_1) \\ &= (a_2, b_2) + (a_1, b_1) \\ &= \vec{y} + \vec{x} \end{aligned}$$

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(7) Adb. $\alpha\vec{x} \in \mathbb{R}^2$

$\alpha \cdot \vec{x} = \alpha \cdot (a_1, b_1) = (\alpha a_1, \alpha b_1) \in \mathbb{R}^2$

(8) Adb. $\alpha(\vec{x}+\vec{y}) = \alpha\vec{x} + \alpha\vec{y}$

$$
\begin{aligned}
\alpha(\vec{x}+\vec{y}) &= \alpha((a_1,b_1)+(a_2,b_2)) \\
&= \alpha(a_1+a_2, b_1+b_2) \\
&= (\alpha(a_1+a_2), \alpha(b_1+b_2)) \\
&= (\alpha a_1+\alpha a_2, \alpha b_1+\alpha b_2) \\
&= ((\alpha a_1, \alpha b_1)+(\alpha a_2, \alpha b_2)) \\
&= \alpha(a_1,b_1) + \alpha(a_2,b_2) \\
&= \alpha\vec{x} + \alpha\vec{y}
\end{aligned}
$$

(9) Adb. $(\alpha+\beta)\vec{x} = \alpha\vec{x} + \beta\vec{x}$

$$
\begin{aligned}
(\alpha+\beta)\vec{x} &= (\alpha+\beta) \cdot (a_1, b_1) \\
&= ((\alpha+\beta)a_1, (\alpha+\beta)b_1) \\
&= (\alpha a_1+\beta a_1, \alpha b_1+\beta b_1) \\
&= \alpha(a_1,b_1) + \beta(a_1,b_2) \\
&= \alpha\vec{x} + \beta\vec{y}
\end{aligned}
$$

(10) Adb $(\alpha\beta)\vec{x} = \alpha(\beta\vec{x})$

$$
\begin{aligned}
(\alpha\beta)\vec{x} &= (\alpha\beta) \cdot (a_1, b_1) \\
&= ((\alpha\beta)a_1, (\alpha\beta)b_1) \\
&= (\alpha \cdot (\beta a_1), \alpha \cdot (\beta b_1)) \\
&= \alpha(\beta a_1, \beta b_1) \\
&= \alpha \cdot [\beta(a_1,b_1)] \\
&= \alpha \cdot [\beta\vec{x}]
\end{aligned}
$$

(11) Terdapat $1 \in \mathbb{R}$ sehingga untuk setiap $\vec{x} \in \mathbb{R}^2$ berlaku $1\vec{x} = 1(a_1,b_1) = (a_1,b_1) = \vec{x}$

$\therefore$ $\mathbb{R}^2$ ruang vektor atas $\mathbb{R}$

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(2) Periksa yang manakah diantara himpunan berikut yang merupakan ruang vektor.

(a) $\mathbb{Z}$ ruang vektor atas $\mathbb{R}$?

Jawab :

Bukan, karena $\alpha = \sqrt{2}$, $a = 2 \in \mathbb{Z}$.

tetapi $\alpha a = 2\sqrt{2} \notin \mathbb{Z}$

$\therefore$ $\mathbb{Z}$ bukan ruang vektor atas $\mathbb{R}$

(b) $\mathbb{R}$ ruang vektor atas $\mathbb{Z}$?

Jawab :

Bukan, karena $\mathbb{Z}$ bukan lapangan

(c) $M_2(\mathbb{R})$ ruang vektor atas $\mathbb{R}$?

Jawab :

Ya, bukti detail ada di halaman selanjutnya

(Tugas pertemuan 3)

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Tugas :

Buktikan bahwa $\mathbb{M}_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{R} \right\}$

merupakan ruang vektor atas lapangan $\mathbb{R}$

Penyelesaian :

Akan dibuktikan : $M_2(\mathbb{R})$ adalah ruang vektor atas lapangan $\mathbb{R}$

(1) Adb. $\mathbb{M}_2(\mathbb{R}) \neq \phi$

Misal $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in \mathbb{M}_2(\mathbb{R})$. Jadi, $\mathbb{M}_2(\mathbb{R}) \neq \phi$ .... $\blacksquare$

(2). Adb. $\forall A, B \in \mathbb{M}_2(\mathbb{R}) \Rightarrow A.B \in M_2(\mathbb{R})$

Ambil sebarang $A, B \in M_2(\mathbb{R})$

Tulis $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$B = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ untuk suatu $a_2, b_2, c_2, d_2 \in \mathbb{R}$

Note that,

$$A + B = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix} \in \mathbb{M}_2(\mathbb{R}) \text{ .... } \blacksquare$$

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(3) Adb. $\forall A, B, C \in M_2(\mathbb{R}) \Rightarrow A+(B+C) = (A+B)+C$

Ambil sebarang $A, B, C \in M_2(\mathbb{R})$

Tulis, $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$B = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ untuk suatu $a_2, b_2, c_2, d_2 \in \mathbb{R}$

$C = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$ untuk suatu $a_3, b_3, c_3, d_3 \in \mathbb{R}$

Note that

$$A+(B+C) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1+(a_2+a_3) & b_1+(b_2+b_3) \\ c_1+(c_2+c_3) & d_1+(d_2+d_3) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1+a_2)+a_3 & (b_1+b_2)+b_3 \\ (c_1+c_2)+c_3 & (d_1+d_2)+d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right] + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= (A+B)+C \quad \ldots\ldots\ldots\ldots \blacksquare$$

(4) Adb. $\exists\, O \in M_2(\mathbb{R}) \ni \forall A \in M_2(\mathbb{R}) \Rightarrow A + O = O + A = A$

Pilih $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in M_2(\mathbb{R})$

Ambil sebarang $A \in M_2(\mathbb{R})$

Tulis $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Note that,

(*) $A + O = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a_1 + 0 & b_1 + 0 \\ c_1 + 0 & d_1 + 0 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = A$

(**) $A + O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} 0 + a_1 & 0 + b_1 \\ 0 + c_1 & 0 + d_1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = A$

Karena dari (*) dan (**) diperoleh $A + O = A = O + A$ maka $\exists\, O \in M_2(\mathbb{R}) \ni \forall A \in M_2(\mathbb{R}) \Rightarrow A + \vec{O} = \vec{O} + A = A$ terbukti. .... ▨

(5) Adb. $\forall A \in M_2(\mathbb{R})$, $\exists -A \in M_2(\mathbb{R}) \ni +(-A) = -A + A = 0$

Ambil sebarang $A, \dot{B} \in M_2(\mathbb{R})$

Tulis, $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Pilih $-A = \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} \in M_2(\mathbb{R})$

Note that,

(*) $A + (-A) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} = \begin{pmatrix} a_1 + (-a_1) & b_1 + (-b_1) \\ c_1 + (-c_1) & d_1 + (-d_1) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$

(**) $-A + A = \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} -a_1 + a_1 & -b_1 + b_1 \\ -c_1 + c_1 & -d_1 + d_1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$

Karena dari (*) dan (**) diperoleh $A + (-A) = O = -A + A$ maka $\forall A \in M_2(\mathbb{R})$, $\exists -A \in M_2(\mathbb{R}) \ni A + (-A) = -A + A = O$ terbukti. .... ▨

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(6) Adb. $\forall A, B \in \mathbb{M}_2(\mathbb{R}) \Rightarrow A + B = B + A$

Ambil sebarang $A, B \in \mathbb{M}_2(\mathbb{R})$

Tulis $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$B = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ untuk suatu $a_2, b_2, c_2, d_2 \in \mathbb{R}$

Note that,

$$A + B = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 + a_1 & b_2 + b_1 \\ c_2 + c_1 & d_2 + d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= B + A \quad \ldots\ldots\ldots\ldots\ldots\ldots \boxtimes$$

(7) Adb. $\forall A \in \mathbb{M}_2(\mathbb{R}), \alpha \in \mathbb{R} \Rightarrow \alpha . A \in \mathbb{M}_2(\mathbb{R})$

Ambil sebarang $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \in \mathbb{M}_2(\mathbb{R})$, untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Ambil sebarang $\alpha \in \mathbb{R}$

Note that,

$$\alpha . A = \alpha . \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha . a_1 & \alpha . b_1 \\ \alpha c_1 & \alpha . d_1 \end{pmatrix} \in \mathbb{M}_2(\mathbb{R}) \quad \ldots\ldots\ldots\ldots \boxtimes$$

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(8) Adb. $\forall A, B \in M_2(\mathbb{R})$, $\alpha \in \mathbb{R} \Rightarrow \alpha(A+B) = \alpha A + \alpha B$

Ambil sebarang $A, B \in M_2(\mathbb{R})$

Tulis $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$B = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ untuk suatu $a_2, b_2, c_2, d_2 \in \mathbb{R}$

Ambil sebarang $\alpha \in \mathbb{R}$

Note that,

$$\alpha(A+B) = \alpha\left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right]$$

$$= \alpha \cdot \left[\begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix}\right]$$

$$= \begin{pmatrix} \alpha \cdot (a_1+a_2) & \alpha(b_1+b_2) \\ \alpha(c_1+c_2) & \alpha(d_1+d_2) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha . a_1 + \alpha . a_2 & \alpha b_1 + \alpha b_2 \\ \alpha . c_1 + \alpha . c_2 & \alpha . d_1 + \alpha . d_2 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha . a_1 & \alpha . b_1 \\ \alpha . c_1 & \alpha . d_1 \end{pmatrix} + \begin{pmatrix} \alpha . a_2 & \alpha . b_2 \\ \alpha . c_2 & \alpha d_2 \end{pmatrix}$$

$$= \alpha\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \alpha\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$= \alpha . A + \alpha . B$$ .... ∴ ▨

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(g) Adb. $\forall A \in M_2(\mathbb{R})$, $\alpha, \beta \in \mathbb{R} \Rightarrow (\alpha+\beta)A = \alpha A + \beta A$

Ambil sebarang $A \in M_2(\mathbb{R})$

Tdb $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Ambil sebarang $\alpha, \beta \in \mathbb{R}$

Note that,

$$(\alpha+\beta) A = (\alpha+\beta) \cdot \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} (\alpha+\beta) a_1 & (\alpha+\beta) b_1 \\ (\alpha+\beta) c_1 & (\alpha+\beta) d_1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha . a_1 + \beta . a_1 & \alpha . b_1 + \beta . b_1 \\ \alpha . c_1 + \beta . c_1 & \alpha . d_1 + \beta . d_1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha . a_1 & \alpha . b_1 \\ \alpha . c_1 & \alpha . d_1 \end{pmatrix} + \begin{pmatrix} \beta . a_1 & \beta . b_1 \\ \beta . c_1 & \beta . d_1 \end{pmatrix}$$

$$= \alpha . \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \beta . \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \alpha . A + \beta . A \quad \ldots\ldots\ldots\ldots\ldots\ldots \boxed{/\!/}$$

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(10) Adb. $\forall A \in M_2(\mathbb{R})$, $\alpha, \beta \in \mathbb{R} \Rightarrow (\alpha.\beta)A = \alpha(\beta.A)$

Ambil sebarang $A \in M_2(\mathbb{R})$

Tulis $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Ambil sebarang $\alpha, \beta \in \mathbb{R}$

Note that,

$$(\alpha.\beta).A = (\alpha.\beta).\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} (\alpha.\beta).a_1 & (\alpha.\beta).b_1 \\ (\alpha.\beta).c_1 & (\alpha.\beta)\,d_1 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha.(\beta.a_1) & \alpha.(\beta.b_1) \\ \alpha.(\beta.c_1) & \alpha.(\beta.d_1) \end{pmatrix}$$

$$= \alpha.\begin{pmatrix} \beta.a_1 & \beta.b_1 \\ \beta.c_1 & \beta.d_1 \end{pmatrix}$$

$$= \alpha.\left[\beta.\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\right]$$

$$= \alpha.(\beta.A) \quad \ldots\ldots\ldots\ldots\ldots\ldots \blacksquare$$

(11) Adb. $\exists I \in \mathbb{R}$, $\forall A \in M_2(\mathbb{R}) \Rightarrow I.A = A$.

Pilih $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Ambil sebarang $A = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \in M_2(\mathbb{R})$;untuk suatu $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Note that, $I.A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$

$$= \begin{pmatrix} a_1 + 0 & b_1 + 0 \\ 0 + c_1 & 0 + d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= A \quad \ldots\ldots\ldots\ldots\ldots\ldots \blacksquare$$

$\therefore$ Karena $M_2(\mathbb{R})$ suatu himpunan, $M_2(\mathbb{R}) \neq \emptyset$ dengan $\mathbb{R}$ suatu lapangan, $M_2(\mathbb{R})$ dilengkapi dengan dengan operasi penjumlahan ~~matriks~~ standar dan operasi perkalian matriks standar, dan memenuhi ke-10 aksioma Ruang Vektor atas Lapangan maka $M_2(\mathbb{R})$ adalah Ruang Vektor diatas Lapangan $\mathbb{R}$. (Terbukti) $\blacksquare$