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Tentukan deret Fourier berikut

$$f(x) = \begin{cases} 0 & ; \quad -\pi < x < 0 \\ x & ; \quad 0 < x < \pi \end{cases}$$

Penyelesaian :

Periode = |batas atas| + |batas bawah|

$= \quad |\pi| \quad + \quad |-\pi|$

$= \quad \pi \quad + \quad \pi$

$= \quad 2\pi$

- Tes Fungsi genap $f(x) = f(-x)$

Misal, $x = \frac{1}{2}\pi$

$\Rightarrow \left.\begin{array}{c} f\left(\frac{1}{2}\pi\right) \neq f\left(-\frac{1}{2}\pi\right) \\ x \neq 0 \end{array}\right\} \therefore$ Bukan fungsi genap ... (*)

- Tes Fungsi ganjil $f(-x) = -f(x)$

Misal, $x = \frac{1}{2}\pi$

$\Rightarrow \left.\begin{array}{c} f\left(-\frac{1}{2}\pi\right) \neq -f\left(\frac{1}{2}\pi\right) \\ 0 \neq -(x) \end{array}\right\} \therefore$ Bukan fungsi ganjil ... (**)

$\therefore$ Dari (*) dan (**) fungsi $f(x)$ bukan fungsi genap maupun fungsi ganjil.

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Selanjutnya, akan dicari nilai a_n, a_0 dan b_n.

$$a_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \cos nx \, dx$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(x) \cdot \cos nx \cdot dx + \int_{0}^{\pi} f(x) \cdot \cos nx \, dx \right)$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} 0 \cdot \cos nx \, dx + \int_{0}^{\pi} x \cdot \cos nx \, dx \right)$$

$$= \frac{1}{\pi} \cdot \left(0 + \int_{0}^{\pi} x \cdot \cos nx \, dx \right)$$

= M.P.S $u = x$ $\quad dv = \cos nx \, dx$

$du = dx$ $\quad v = \int \cos nx \, dx$

$v = \frac{1}{n} \cdot \sin nx$

$$= \frac{1}{\pi} \left(\left(x \cdot \frac{1}{n} \cdot \sin nx \right) - \left(\int \frac{1}{n} \cdot \sin nx \cdot dx \right) \right) \Big|_0^{\pi}$$

$$= \frac{1}{\pi} \left(\left(x \cdot \frac{1}{n} \cdot \sin nx \right) - \left(\frac{1}{n} \cdot \int \sin nx \, dx \right) \right) \Big|_0^{\pi}$$

$$= \frac{1}{\pi} \left(\left(\frac{x}{n} \cdot \sin nx \right) - \left(\frac{1}{n} \cdot \frac{1}{n} \cdot (-\cos nx) \right) \right) \Big|_0^{\pi}$$

$$= \frac{1}{\pi} \left(\frac{x}{n} \cdot \sin nx + \frac{1}{n^2} \cdot \cos nx \right) \Big|_0^{\pi}$$

$$= \frac{1}{\pi} \left[\left(\left(\frac{\pi}{n} \cdot \sin n\pi \right) + \left(\frac{1}{n^2} \cdot \cos n\pi \right) \right) - \left(\left(\frac{0}{n} \cdot \sin n.0 \right) + \left(\frac{1}{n^2} \cdot \cos n.0 \right) \right) \right]$$

$$= \frac{1}{\pi} \left[\left((0) + \left(\frac{1}{n^2} \cdot \cos n\pi \right) \right) - \left((0) + \left(\frac{1}{n^2} \cdot (1) \right) \right) \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n^2} \cdot \cos n\pi - \frac{1}{n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n^2} (\cos n\pi - 1) \right]$$

$$\boxed{a_n = \frac{1}{\pi n^2} \cdot (\cos n\pi - 1)}$$

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$$a_0 = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right)$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} 0\, dx + \int_{0}^{\pi} x\, dx \right)$$

$$= \frac{1}{\pi} \left(0 + \left[\frac{1}{2} x^2 \right]_0^{\pi} \right)$$

$$= \frac{1}{\pi} \left(\left(\frac{1}{2} \pi^2 \right) - \left(\frac{1}{2} 0^2 \right) \right)$$

$$= \frac{1}{\pi} \left(\frac{1}{2} \pi^2 \right)$$

$$\boxed{a_0 = \frac{\pi}{2}}$$

Dipindai dengan CamScanner

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$$b_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \sin nx \, dx$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(x) \cdot \sin nx \, dx + \int_{0}^{\pi} f(x) \cdot \sin nx \, dx \right)$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} 0 \cdot \sin nx \, dx + \int_{0}^{\pi} x \cdot \sin nx \, dx \right)$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} 0 \, dx + \int_{0}^{\pi} x \cdot \sin nx \, dx \right)$$

$$= \frac{1}{\pi} \left(0 + \int_{0}^{\pi} x \cdot \sin nx \, dx \right)$$

$$= \frac{1}{\pi} \left(\int_{0}^{\pi} x \cdot \sin nx \, dx \right)$$

= Misal $u = x$ $dV = \sin nx \, dx$

$du = dx$ $V = \int \sin nx \, dx$

$V = \frac{1}{n} \cdot (-\cos nx)$

$$= \frac{1}{\pi} \left(\left(x \cdot \frac{1}{n} \cdot (-\cos nx) \right) - \left(\int \frac{1}{n} \cdot (-\cos nx) \, dx \right) \right)_0^{\pi}$$

$$= \frac{1}{\pi} \left(\left(\frac{x}{n} (-\cos nx) \right) + \left(\frac{1}{n} \int \cos nx \, dx \right) \right)_0^{\pi}$$

$$= \frac{1}{\pi} \left(-\frac{x}{n} \cdot \cos nx + \frac{1}{n} \cdot \frac{1}{n} \cdot \sin nx \right)_0^{\pi}$$

$$= \frac{1}{\pi} \left[\left(-\frac{\pi}{n} \cdot \cos n\pi + \frac{1}{n^2} \cdot \sin n\pi \right) - \left(-\frac{0}{n} \cdot \cos n \cdot 0 + \frac{1}{n^2} \cdot \sin n0 \right) \right]$$

$$= \frac{1}{\pi} \left[\left(-\frac{\pi}{n} \cdot \cos n\pi + \frac{1}{n^2} \cdot (0) \right) - \left((0) + \frac{1}{n^2} (0) \right) \right]$$

$$= \frac{1}{\pi} \left[\left(-\frac{\pi}{n} \cdot \cos n\pi \right) \right]$$

$$\boxed{b_n = -\frac{1}{n} \cdot \cos n\pi}$$

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∴ Deret Fourier $f(x)$ adalah :

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cdot \cos nx + b_n \cdot \sin nx \right) =$$

$$\frac{1}{2} \cdot \left(\frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \left(\left(\frac{1}{\pi n^2} (\cos n\pi - 1) \cdot \cos nx \right) + \left(-\frac{1}{n} \cdot \cos n\pi \cdot \sin nx \right) \right) =$$

$$\frac{\pi}{4} + \sum_{n=1}^{\infty} \left(\left(\frac{1}{\pi n^2} \cdot (\cos n\pi - 1) \cdot (\cos nx) \right) - \left(\frac{1}{n} \cdot \cos n\pi \cdot \sin nx \right) \right) //$$