Masalah Syarat Batas / Pertemuan ke - 10 / latihan soal   Makassar, 21 April 2021

Imanuel AS / 1811141008

Ttd:

Imanuel

Contoh Soal :

1. $\int_0^1 x^2 (\ln x)^4 \, dx$

Penyelesaian :

Dengan menggunakan Formula I

$$\int_0^1 x^m (\ln x)^n \, dx = \frac{(-1)^n \cdot \Gamma(n+1)}{(m+1)^{n+1}} \quad ; n \in \mathbb{N}, \ m > -1$$

dimana $m = 2$ dan $n = 4$, diperoleh

$$\int_0^1 x^2 (\ln x)^4 \, dx = \frac{(-1)^4 \cdot \Gamma(4+1)}{(2+1)^{4+1}}$$

$$= \frac{(1) \cdot \Gamma(5)}{(3)^5}$$

$$= \frac{\Gamma(5)}{243} \longrightarrow \Gamma(n) = \int_0^\infty x^{n-1} \cdot e^{-x} \, dx$$

$$= \frac{\int_0^\infty x^{5-1} \cdot e^{-x} \, dx}{243}$$

$$= \frac{\int_0^\infty x^4 \cdot e^{-x} \, dx}{243} \longrightarrow \int_0^\infty x^n \cdot e^{-x} \, dx = n!$$

$$= \frac{4!}{243}$$

$$= \frac{24}{243}$$

$$= \frac{8}{81} \quad //$$

Imanuel AS / 1811141008 Makassar, 21 April 2020.

② $\int_0^1 (\ln x)^3 \, dx$?

Penyelesaian

$$\int_0^1 x^m \cdot (\ln x)^n \, dx = \frac{(-1)^n \cdot \Gamma(n+1)}{(m+1)^{n+1}} \quad ; n \in \mathbb{N}, m > -1$$

$$\int_0^1 x^0 \cdot (\ln x)^3 \, dx = \frac{(-1)^3 \cdot \Gamma(3+1)}{(0+1)^{3+1}}$$

$$\int_0^1 (\ln x)^3 \, dx = \frac{(-1) \cdot \Gamma(4)}{(1)^4}$$

$$\Gamma(n) = \int_0^\infty x^{n-1} \cdot e^{-x} \, dx$$

$$= (-1) \cdot \left[\int_0^\infty x^{4-1} \cdot e^{-x} \, dx \right]$$

$$= -\left[\int_0^\infty x^3 \cdot e^{-x} \, dx \right] \qquad \int_0^\infty x^n \cdot e^{-x} dx = n!$$

$$= -[3!]$$

$$= -6$$

Imanuel AS / 1811141008 Imanuel   Makassar, 21 April 2021

(3.) $\int_0^{\frac{\pi}{2}} \cos^3\theta \cdot \sin^5\theta \cdot d\theta = \ldots$

Penyelesaian

$$\int_0^{\frac{\pi}{2}} \cos^{2n-1}\theta \cdot \sin^{2m-1}\theta \cdot d\theta = \frac{\Gamma(n).\Gamma(m)}{2.\Gamma(m+n)} \quad ; n,m > 0$$

$$\int_0^{\frac{\pi}{2}} \cos^{2(2)-1}\theta \cdot \sin^{2(3)-1}\theta \cdot d\theta = \frac{\Gamma(2).\Gamma(3)}{2.\Gamma(2+3)}$$

$$\int_0^{\frac{\pi}{2}} \cos^3\theta \cdot \sin^5\theta \, d\theta = \frac{(1).(2)}{2.(24)}$$

$$= \frac{1}{24}$$

$$\Gamma(2) = \int_0^\infty x^{2-1} \cdot e^{-x} dx$$

$$= \int_0^\infty x^1 \cdot e^{-x} dx$$

$$= 1!$$

$$= 1$$

$$\Gamma(3) = \int_0^\infty x^{3-1} \cdot e^{-x} dx$$

$$= \int_0^\infty x^2 \cdot e^{-x} dx$$

$$= 2!$$

$$= 2$$

$$\Gamma(2+3) = \Gamma(5)$$

$$= \int_0^\infty x^{5-1} \cdot e^{-x} dx$$

$$= \int_0^\infty x^4 \cdot e^{-x} dx$$

$$= 4!$$

$$= 4 \times 3 \times 2 \times 1$$

$$= 24$$

Imanuel AS / 181114 1008

Makassar, 21 April 2021

(4.) $\int_0^{\frac{\pi}{2}} \cos^7 \theta \, d\theta = \ldots$

Penyelesaian

$$\int_0^{\frac{\pi}{2}} \cos^{2n-1} \theta \cdot \sin^{2m-1} \theta \, d\theta = \frac{\Gamma(n) \cdot \Gamma(m)}{2 \cdot \Gamma(m+n)} \quad ; n, m > 0$$

$$\int_0^{\frac{\pi}{2}} \cos^{2(4)-1} \theta \cdot \sin^{2(\frac{1}{2})-1} \theta \, d\theta = \frac{\Gamma(4) \cdot \Gamma(\frac{1}{2})}{2 \cdot \Gamma(\frac{1}{2}+4)}$$

$$\int_0^{\frac{\pi}{2}} \cos^7 \theta \, d\theta \longrightarrow = \frac{6 \cdot \sqrt{\pi}}{2 \cdot \frac{105}{16} \cdot \sqrt{\pi}}$$

$$= \frac{3\sqrt{\pi}}{\sqrt{\pi}} \cdot \frac{16}{105}$$

$$= \frac{16}{35} \,//$$

$$\Gamma(4) = \int_0^\infty x^{4-1} \cdot e^{-x} \, dx$$

$$= \int_0^\infty x^3 \cdot e^{-x} \, dx$$

$$= 3!$$

$$= 6$$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$ (Diketahui)

$$\Gamma(\tfrac{1}{2}+4) = \Gamma(\tfrac{9}{2})$$

$$= \Gamma(\tfrac{7}{2}+1)$$

$$= \tfrac{7}{2} \cdot \Gamma(\tfrac{7}{2})$$

$$= \tfrac{7}{2} \cdot \left[\Gamma(\tfrac{5}{2}+1)\right]$$

$$= \tfrac{7}{2} \left[\tfrac{5}{2} \cdot \Gamma(\tfrac{5}{2})\right]$$

$$= \tfrac{7}{2} \left[\tfrac{5}{2} \cdot \left(\Gamma(\tfrac{3}{2}+1)\right)\right]$$

$$= \tfrac{7}{2} \left[\tfrac{5}{2} \cdot \left(\tfrac{3}{2} \cdot \Gamma(\tfrac{3}{2})\right)\right]$$

$$= \frac{105}{8} \cdot \left(\Gamma(\tfrac{1}{2}+1)\right)$$

$$= \frac{105}{8} \cdot \left(\tfrac{1}{2} \cdot \Gamma(\tfrac{1}{2})\right)$$

$$= \frac{105}{8} \cdot \left(\tfrac{1}{2} \cdot \sqrt{\pi}\right)$$

$$= \frac{105}{16} \sqrt{\pi}$$

Imanuel AS / 1811141008 Makassar, 21 April 2021

⑤ $$\int_0^1 \frac{dx}{\sqrt{1-x^3}} = \ldots$$

Penyelesaian :

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n}+\frac{1}{2})} \quad ; n>0$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^3}} = \frac{\sqrt{\pi}}{3} \cdot \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{1}{3}+\frac{1}{2})}$$

$$= \frac{\sqrt{\pi}}{3} \cdot \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{5}{6})} \quad //$$

$\Gamma(\frac{1}{3}) \approx 2,678$ (Wikipedia)

$\Gamma(\frac{5}{6}) = \Gamma(\quad +1$

Imanuel AS / 1811141008 ~~IFF~~ Imanuel Makassar, 21 April 2021

(6.) $\int_0^1 3(1-x^4)^{-\frac{1}{2}}\, dx = \ldots$

Penyelesaian :

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)} \quad ; n > 0$$

$$3\int_0^1 \frac{dx}{\sqrt{1-x^4}} = 3 \cdot \frac{\sqrt{\pi}}{4} \cdot \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}$$

$$\int_0^1 3 \cdot (1-x^4)^{-\frac{1}{2}}\, dx = \frac{3\sqrt{\pi}}{4} \cdot \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} \quad //$$

MSB / Pertemuan Ke-10 / latihan Soal   Imanuel AS / 1811141008   Makassar, 21 April 2021

Contoh Soal

1. $\int_0^1 x^2 \left(\ln \frac{1}{x}\right)^3 dx = \ldots$

Penyelesaian

$$\int_0^1 x^m \cdot (\ln x)^n \, dx = \frac{(-1)^n \cdot \Gamma(n+1)}{(m+1)^{n+1}} \quad ; n \in \mathbb{N} \;, m > -1$$

$$\begin{aligned}
\int_0^1 x^2 \left(\ln \frac{1}{x}\right)^3 dx &= \int_0^1 x^2 (\ln x^{-1})^3 \, dx \\
&= \int_0^1 x^2 (-\ln x)^3 \, dx \\
&= -\int_0^1 x^2 (\ln x)^3 \, dx \\
&= \frac{(-1)^3 \cdot \Gamma(3+1)}{(2+1)^{3+1}} \\
&= \frac{(-1) \cdot \Gamma(4)}{(3)^4} \\
&= -\frac{3!}{81} \\
&= -\frac{6}{81} \\
&= -\frac{2}{27} \;//
\end{aligned}$$

Imanuel AS / 1811141008 Makassar, 21 April 2021

(2) $\int_0^1 \frac{dx}{\sqrt{-\ln x}} = \dots$

Penyelesaian :

$\int_0^1 \frac{dx}{\sqrt{1-x^n}} \neq \frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n}+\frac{1}{2})}$

$\int_0^1 \frac{dx}{\sqrt{-\ln x}} =$

Misal $u = -\ln x \Rightarrow -\ln x = u$

$\ln x = -u$

${}^{e}\log x = -u \iff e^{-u} = x$

$-e^{-u}\,du = dx$

Jika $x = 1 \Rightarrow u = 0$

Jika $x = 0 \Rightarrow u = \infty$

Maka,

$= \int_\infty^0 \frac{-e^{-u}}{\sqrt{u}}\,du$

$= -\int_\infty^0 \frac{e^{-u}}{\sqrt{u}}\,du$

$= \int_0^\infty \frac{e^{-u}}{\sqrt{u}}\,du$

$= \int_0^\infty u^{-\frac{1}{2}} \cdot e^{-u}\,du \rightsquigarrow \int_0^\infty x^{n-1} \cdot e^{-x}\,dx = \Gamma(n)$

$= \int_0^\infty u^{\frac{1}{2}-1} \cdot e^{-u}\,du$

$= \Gamma\left(\frac{1}{2}\right)$

$= \sqrt{\pi}$ //

Dipindai dengan CamScanner

Imanuel AS / 1811141008 Imanuel   Makassar, 21 April 2021

③ $\int_0^{\infty} 3^{-4x^2}\, dx = \dots$

Penyelesaian:

$$\int_0^{\infty} 3^{-4x^2}\, dx = \int_0^{\infty} \left(e^{{}^{e}\log 3}\right)^{-4x^2} dx$$

$$= \int_0^{\infty} \left(e^{\ln 3}\right)^{-4x^2} dx$$

$$= \int_0^{\infty} e^{-4(\ln 3).x^2}\, dx \rightsquigarrow \int_0^{\infty} x^{n-1} . e^{-x}\, dx = \Gamma(n)$$

Misal $4(\ln 3)x^2 = u \longrightarrow x^2 = \frac{u}{4(\ln 3)}$, $x = \frac{\sqrt{u}}{\sqrt{4 \ln 3}}$

$$(4.\ln 3)2x + x^2.\left(4.\left(\tfrac{1}{3}.0\right) + (\ln 3).0\right) dx = du$$

$$(4.\ln 3)2x + x^2.(0+0)\, dx = du$$

$$(4.\ln 3)2x\, dx = du$$

$$dx = \frac{du}{4(\ln 3)2x}$$

$$dx = \frac{du}{8x(\ln 3)}$$

Maka

$$= \int_0^{\infty} e^{-u} . \frac{1}{8x(\ln 3)} . du$$

$$= \int_0^{\infty} e^{-u} \frac{1}{8.\frac{\sqrt{u}}{\sqrt{4\ln 3}}.\ln 3} . du$$

$$= \int_0^{\infty} e^{-u} \frac{1}{8\ln 3} . \frac{\sqrt{4 \ln 3}}{\sqrt{u}}\, du$$

$$= \frac{1}{8.\ln 3} . \sqrt{4\ln 3} . \int_0^{\infty} e^{-u} . u^{-\frac{1}{2}}\, du \rightsquigarrow \int_0^{\infty} x^{n-1} . e^{-x} dx = \Gamma(n)$$

$$= \frac{2\sqrt{\ln 3}}{8.\ln 3} . \int_0^{\infty} u^{\frac{1}{2}-1} . e^{-u}\, du$$

$$= \frac{1}{4} . (\ln 3)^{\frac{1}{2}-1} . \Gamma\left(\tfrac{1}{2}\right)$$

$$= \frac{1}{4} . (\ln 3)^{-\frac{1}{2}} . \sqrt{\pi}$$

$$= \frac{1}{4} . \frac{1}{\sqrt{\ln 3}} . \sqrt{\pi}$$

//