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TUGAS V

Teori Modul

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LATIHAN SOAL

(1.) Dik. $\mathbb{Z}_{12}$, $\mathbb{Z}$- modul

$N_1 = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}$

$N_2 = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}\}$

$N_3 = \{\bar{0}, \bar{4}, \bar{8}\}$

Periksa apakah $\mathbb{Z}_{12} = N_1 \oplus N_2 \oplus N_3$??

Penyelesaian:

(i) $N_1 + N_2 + N_3 = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} + \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}\} + \{\bar{0}, \bar{4}, \bar{8}\}$

$= \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \bar{3}, \bar{5}, \bar{7}, \bar{9}, \overline{11}, \bar{1}\}$

$= \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \overline{10}, \overline{11}\}$

$= \mathbb{Z}_{12}$

(ii) $N_1 \cap (N_2 + N_3) = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} \cap [\{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}\} + \{\bar{0}, \bar{4}, \bar{8}\}]$

$= \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} \cap \{\bar{0}, \bar{4}, \bar{8}, \bar{2}, \bar{6}, \overline{10}\}$

$= \{\bar{0}, \bar{6}\}$

$\neq \{\bar{0}\}$

Jadi $N_1 \cap (N_2 + N_3) \neq \{\bar{0}\}$

$\therefore \mathbb{Z}_{12} \neq N_1 \oplus N_2 \oplus N_3$

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② Dik $\mathbb{Z}_{12}$ $\mathbb{Z}$- modul.

$N_1 = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}$; $N_2 = \{\bar{0}, \bar{4}, \bar{8}\}$ adalah submodul-submodul di $\mathbb{Z}_{12}$.

Perlihatkan $\mathbb{Z}_{12} = N_1 \oplus N_2$

Penyelesaian :

(i) $N_1 + N_2 = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} + \{\bar{0}, \bar{4}, \bar{8}\}$

$= \{\bar{0}, \bar{4}, \bar{8}, \bar{3}, \bar{7}, \bar{11}, \bar{6}, \bar{10}, \bar{2}, \bar{9}, \bar{1}, \bar{5}\}$

$= \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \bar{10}, \bar{11}\}$

$= \mathbb{Z}_{12}$

(ii) $N_1 \cap N_2 = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} \cap \{\bar{0}, \bar{4}, \bar{8}\}$

$= \{\bar{0}\}$

$\therefore \mathbb{Z}_{12} = N_1 \oplus N_2$

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(3) Dik $\mathbb{Z}_{16}$ $\mathbb{Z}$-modul

$N_1 = \{\bar{0}, \bar{8}\}$

$N_2 = \{\bar{0}, \bar{4}, \bar{8}, \overline{12}\}$

$N_3 = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}\}$

adalah submodul-submodul dari $\mathbb{Z}_6$.

Perlihatkan $\mathbb{Z}_{16} = N_1 \oplus N_2 \oplus N_3$

Penyelesaian:

(i) $N_1 + N_2 + N_3 = \{\bar{0}, \bar{8}\} + \{\bar{0}, \bar{4}, \bar{8}, \overline{12}\} + \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}\}$

$= \{\bar{0}, \bar{4}, \bar{8}, \overline{12}\} + \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}\}$

$= \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}\}$

$\neq \mathbb{Z}_{16}$

$\therefore \mathbb{Z}_{16} \neq N_1 \oplus N_2 \oplus N_3$

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(4.) $\mathbb{Z}_{20}$ $\mathbb{Z}$ - modul

$N_1 = \{ \bar{0}, \bar{5}, \overline{10}, \overline{15} \}$

$N_2 = \{ \bar{0}, \bar{4}, \bar{8}, \overline{12}, \overline{16} \}$

adalah submodul - submodul dari $\mathbb{Z}_{20}$

Tunjukkan $\mathbb{Z}_{20} = N_1 \oplus N_2$

<u>Penyelesaian</u>:

(i) $N_1 + N_2 = \{ \bar{0}, \bar{5}, \overline{10}, 15 \} + \{ \bar{0}, \bar{4}, \bar{8}, \overline{12}, \overline{16} \}$

$= \{ \bar{0}, \bar{4}, \bar{8}, \overline{12}, \overline{16}, \bar{5}, \bar{9}, \overline{13}, \overline{17}, \bar{1}, \overline{10}, \overline{14}, \overline{18}, \bar{2}, \bar{6}, \overline{15}, \overline{19}, \bar{3}, \bar{7}, \overline{11} \}$

$= \{ \bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}, \overline{16}, \overline{17}, \overline{18}, \overline{19} \}$

$= \mathbb{Z}_{20}$

(ii) $N_1 \cap N_2 = \{ \bar{0}, \bar{5}, \overline{10}, \overline{15} \} \cap \{ \bar{0}, \bar{4}, \bar{8}, \overline{12}, \overline{16} \}$

$= \{ \bar{0} \}$

$\therefore \mathbb{Z}_{20} = N_1 \oplus N_2$