Teori Fuzzy / Pertemuan ke-6 / Tugas

Imanuel AS
1811141008

Imanuel

Makassar, 19 Maret 2021

① Buktikan bahwa operator-operator t-norm pada halaman 43 memenuhi keempat aksioma pada Definisi 3.6 hal 42.

Penyelesaian:

Diketahui : Operator-operator t-norm, such as : minimum, hasil kali aljabar, hasil kali terbatas, hasil kali Einstein, hasil kali Hamacher, hasil kali drastis (Follow page 43)

Akan ditunjukkan : operator-operator t-norm diatas memenuhi Definisi 3.6 berikut

Suatu operator t-norm adalah fungsi dua variabel $t(\ldots,\ldots)$ yang memenuhi :

(i) $t(0,0) = 0;\ t(a,1) = t(1,a) = a$ [Batas]

(ii) $t(a,b) \leq t(c,d)$ jika $a \leq c$ dan $b \leq d$ [Monotonisitas]

(iii) $t(a,b) = t(b,a)$ [Komutatifitas]

(iv) $t(a, t(b,c)) = t(t(a,b), c)$ [Assosiatifitas]

Ambil sebarang a, b, c, dimana a, b, c masing-masing adalah fungsi keanggotaan pada masing-masing himpunan kabur yang diberikan.

→ next

Imanuel AS / 1811141008 Imanuel Makassar, 19 Maret 2024

a.) Adb. Minimum : $t_{min}(a,b) = min(a,b) = a \wedge b$ memenuhi keempat aksioma diatas.

(i) Adb. $t_{min}(0,0) = 0$; $t_{min}(a,1) = t_{min}(1,a) = a$

⇒ Untuk $t_{min}(0,0) = 0$

Jelas bahwa, $t_{min}(0,0) = min(0,0) = 0$ [obvious]

⇒ Untuk $t_{min}(a,1) = t_{min}(1,a) = a$

Ambil sebarang $\mu(x)$, tulis $a = \mu(x)$, dalam kasus ini $b = 1$

Perhatikan bahwa,

Kasus I : $\forall a = 1$

Jelas bahwa, $t_{min}(1,1) = 1$

Kasus II : $\forall a < 1$

Jelas bahwa, $t_{min}(1,a) = t_{min}(a,1) = a$ [obvious]

$\therefore t_{min}(0,0) = 0$; $t_{min}(a,1) = t_{min}(1,a) = a$. ☒

Imanuel A S / 18111410008 Imanuel Makassar, 19 Maret 2024

(ii) Adb. $t_{min}(a,b) \leq t_{min}(c,d)$ jika $a \leq c$ dan $b \leq d$.
Menggunakan bukti kontradiksi:

Jika $a \leq c$ dan $b \leq d$ maka $t_{min}(a,b) > t_{min}(c,d)$.

Kasus I: Misalkan, $t_{min}(a,b) = a$

Maka, $a > t_{min}(c,d)$

Hal ini tidak mungkin terjadi, sebab:

$a > c$ kontradiksi, karena definisi awal $a \leq c$.

$a > d$ kontradiksi, karena berdasarkan $t_{min}(a,b) = a$ dan $b \leq d$ diperoleh $a < b \leq d$, jadi $a \leq d$.

Kasus II: Misalkan, $t_{min}(a,b) = b$

Maka, $b > t_{min}(c,d)$

Hal ini tidak mungkin terjadi, sebab:

$b > c$ kontradiksi, karena berdasarkan $t_{min}(a,b) = b$ dan $a \leq c$ diperoleh $b < a \leq c$, jadi $b \leq c$.

$b > d$ kontradiksi, karena definisi awal $b \leq d$.

∴ Berdasarkan kasus I dan kasus II, diperoleh

Jika $a \leq c$ dan $b \leq d$ maka $t_{min}(a,b) \leq t_{min}(c,d)$.

▨

Imanuel AS / 18111410008 Imanuel Makassar, 22 Maret 2024

(iii) Adb. $t_{min}(a,b) = t_{min}(b,a)$

Perhatikan bahwa,

- $t_{min}(a,b) = min(a,b)$ [Definisi]
  $= a \wedge b$ ....(*) [Definisi]
- $t_{min}(b,a) = min(b,a)$ [Definisi]
  $= a \wedge b$ ....(**) [Definisi]

∴ Karena (*) = (**) maka diperoleh,

$t_{min}(a,b) = t_{min}(b,a)$. ∎

(iv) Adb. $t_{min}(a, t_{min}(b,c)) = t_{min}(t_{min}(a,b), c)$.

Perhatikan bahwa,

$t_{min}(a, t_{min}(b,c)) = t_{min}(a, (b \wedge c))$ $[t_{min}(b,c) = b \wedge c]$

$= min(a,b,c)$

$= a \wedge b \wedge c$ ....(*)

$t_{min}(t_{min}(a,b), c) = t_{min}((a \wedge b), c)$ $[t_{min}(a,b) = a \wedge b]$

$= min(a,b,c)$

$= a \wedge b \wedge c$ ....(**)

∴ Karena (*) = (**) maka diperoleh,

$t_{min}(a, t_{min}(b,c)) = t_{min}(t_{min}(a,b), c)$ ∎

∴ $t_{min}(a,b) = min(a,b) = a \wedge b$

Imanuel AS / 1811141008 Malaysia, 26 Maret 2021

(2) Buktikan hukum de Morgan tergeneralisasi (dengan tetap menggunakan rumus komplemen dasar/klasik yakni, $N(a) = 1-a$) dengan menggunakan beberapa pasangan-pasangan dari beberapa operator yang ada. (Hal 47-48)

Penyelesaian :

1.) Misalnya, kita mengambil pasangan dual operator, hasil kali terbatas dan jumlah terbatas.

Perhatikan bahwa,

$\Rightarrow t_{bp}(a,b) = N\left(S_{bs}(N(a), N(b))\right)$

$a * b = N\left(N(a) \pm N(b)\right)$,

$\max[0, (a+b-1)] = N\left((1-a) \pm (1-b)\right)$

$0 \vee (a+b-1) = N\left(\min(1, (1-a)+(1-b))\right) - 1$

$= N\left(\min(1, 2-a-b)\right)$

$= 1 - \left(\min(1, 2-a-b)\right)$

Kasus I : $\because \min(1, 2-a-b) = 1$

$\Rightarrow 0 \vee (a+b-1) = 1 - 1$

$= 0$

Kasus II : $\because \min(1, 2-a-b) = 2-a-b$

$\Rightarrow 0 \vee (a+b-1) = 1 - (2-a-b)$

$= -1+a+b$

$= a+b-1$

$\therefore t_{bp}(a,b) = 0 \vee (a+b-1)$ (Terbukti) ▨

$\Rightarrow S_{bs}(a,b) = N\left(t_{bp}(N(a), N(b))\right)$

$a \pm b = N\left(N(a) * N(b)\right)$

$\min(1, a+b) = N\left((1-a) * (1-b)\right)$

$1 \wedge (a+b) = N\left(\max[0, (1-a)+(1-b)-1]\right)$

$= N\left(\max[0, 1-a-b]\right)$

$= 1 - \left(\max[0, 1-a-b]\right)$

Kasus I : $\because \max[0, 1-a-b] = 0$

$\Rightarrow 1 \wedge (a+b) = 1 - 0$

$= 1$

Kasus II : $\because \max[0, 1-a-b] = 1-a-b$

$\Rightarrow 1 \wedge (a+b) = 1 - (1-a-b)$

$= a+b$

$\therefore S_{bs}(a,b) = 1 \wedge (a+b)$ (Terbukti) ▨

Imanuel AS / 18111410008 Magister Makassar, 26 Maret 2024

2) Misalnya kita mengambil pasangan dual operator, Hasil kali Einstein dan Jumlah Einstein.

Perhatikan bahwa,

• $t_{ep}(a,b) = N(S_{es}(N(a), N(b)))$

$a * b = N(N(a) \pm N(b))$

$\frac{ab}{2-[a+b-ab]} = N((1-a) \pm (1-b))$

$\frac{ab}{2-[a+b-ab]} = N\left(\frac{(1-a)+(1-b)}{1+[(1-a)(1-b)]}\right)$

$= N\left(\frac{2-a-b}{1+(1-b-a+ab)}\right)$

$= N\left(\frac{2-a-b}{2-b-a+ab}\right)$

$= 1 - \left(\frac{2-a-b}{2-b-a+ab}\right)$

$= \frac{2-b-a+ab}{2-b-a+ab} - \left(\frac{2-a-b}{2-b-a+ab}\right)$

$= \frac{ab}{2-b-a+ab}$

$\frac{ab}{2-[a+b-ab]} = \frac{ab}{2-[a+b-ab]}$ (Terbukti) ▨

• $S_{es}(a,b) = N(t_{ep}(N(a), N(b)))$

$a \pm b = N(N(a) * N(b))$

$\frac{a+b}{1+ab} = N((1-a) * (1-b))$

$\frac{a+b}{1+ab} = N\left(\frac{(1-a)(1-b)}{2-[(1-a)+(1-b)-(1-a)(1-b)}\right)$

$= N\left(\frac{1-b-a+ab}{2-[(1-a)+(1-b)-(1-b-a+ab)]}\right)$

$= N\left(\frac{1-b-a+ab}{2-[2-a-b-1+b+a-ab]}\right)$

$= N\left(\frac{1-b-a+ab}{2-[2-1-ab]}\right)$

$= N\left(\frac{1-b-a+ab}{1+ab}\right)$

$= 1 - \left(\frac{1-b-a+ab}{1+ab}\right)$

$= \frac{1+ab}{1+ab} - \left(\frac{1-b-a+ab}{1+ab}\right)$

$\frac{a+b}{1+ab} = \frac{a+b}{1+ab}$ (Terbukti) ▨

3.) Misalnya kita mengambil pasangan dual operator, Hasil kali Hamacher dan jumlah Hamacher.

Perhatikan bahwa,

• $t_{hp}(a,b) = N\left(S_{hs}(N(a), N(b))\right)$

$a * b = N\left(N(a) \perp N(b)\right)$

$\frac{ab}{a+b-ab} = N\left((1-a) \perp (1-b)\right)$

$\frac{ab}{a+b-ab} = N\left(\frac{(1-a)+(1-b)-2(1-a)(1-b)}{1-(1-a)(1-b)}\right)$

$= N\left(\frac{(1-a)+(1-b)-2(1-b-a+ab)}{1-(1-b-a+ab)}\right)$

$= N\left(\frac{2-a-b-2+2b+2a-2ab}{1-1+b+a-ab}\right)$

$= N\left(\frac{a+b-2ab}{a+b-ab}\right)$

$= 1-\left(\frac{a+b-2ab}{a+b-ab}\right)$

$= \frac{a+b-ab}{a+b-ab} - \left(\frac{a+b-2ab}{a+b-ab}\right)$

$\frac{ab}{a+b-ab} = \frac{ab}{a+b-ab}$ (Terbukti) ▨

• $S_{hs}(a,b) = N\left(t_{hp}(N(a), N(b))\right)$

$a \perp b = N\left(N(a) * N(b)\right)$

$\frac{a+b-2ab}{1-ab} = N\left((1-a) * (1-b)\right)$

$\frac{a+b-2ab}{1-ab} = N\left(\frac{(1-a)(1-b)}{(1-a)+(1-b)-(1-a)(1-b)}\right)$

$= N\left(\frac{1-b-a+ab}{(1-a)+(1-b)-(1-b-a+ab)}\right)$

$= N\left(\frac{1-b-a+ab}{2-a-b-1+b+a-ab}\right)$

$= N\left(\frac{1-b-a+ab}{1-ab}\right)$

$= 1-\left(\frac{1-b-a+ab}{1-ab}\right)$

$= \frac{1-ab}{1-ab} - \left(\frac{1-b-a+ab}{1-ab}\right)$

$\frac{a+b-2ab}{1-ab} = \frac{a+b-2ab}{1-ab}$ (Terbukti) ▨

Imanuel AS / 18111411008 Matematika, 26 Maret 2021

4.) Misalnya kita mengambil pasangan dual operator, Hasil kali drastis dan Jumlah drastis.

Perhatikan bahwa,

• $t_{dp}(a,b) = N(S_{dp}(N(a), N(b)))$

$a * b = N(N(a) \pm N(b))$

$\left.\begin{array}{l} a\,;b=1 \\ b\,;a=1 \\ 0\,;a\wedge b<1 \end{array}\right\} = N((1-a) \pm (1-b))$

$= N\begin{pmatrix} (1-a)\,; & (1-b)=0 \\ (1-b)\,; & (1-a)=0 \\ 1\,; & (1-a),(1-b)>0 \end{pmatrix}$

Kasus I : Misalkan $(1-a)=0$ dan $(1-b)=0$ $\Rightarrow 1=a$ dan $1=b$

$\Rightarrow 1 = N((1-a) \pm (1-b))$

$1 = N(0)$

$1 = 1-0$

$1 = 1$ (Terbukti) ▨

Kasus II : Misalkan $(1-a)=0$ dan $(1-b)>0$ $\Rightarrow 1=a$ dan $1>b$

$\Rightarrow b = N((1-a) \pm (1-b))$

$= N(1-b)$

$= 1-(1-b)$

$b = b$ (Terbukti) ▨

Kasus III : Misalkan $(1-a)>0$ dan $(1-b)=0$ $\Rightarrow 1>a$ dan $b=1$

$\Rightarrow a = N((1-a) \pm (1-b))$

$= N(1-a)$

$= 1-(1-a)$

$a = a$ (Terbukti) ▨

Kasus IV : Misalkan $(1-a)>0$ dan $(1-b)>0$ $\Rightarrow 1>a$ dan $1>b$

$\Rightarrow 0 = N((1-a) \pm (1-b))$

$= N(1)$

$= 1-1$

$0 = 0$ (Terbukti) ▨

selanjutnya, diperhatikan juga bahwa,

$\Rightarrow S_{dp}(a,b) = N(t_{dp}(N(a), N(b))$

$a \pm b = N(N(a) * N(b))$

$\left.\begin{matrix} a\,;\ b=0 \\ b\,;\ a=0 \\ 1\,;\ a,b>0 \end{matrix}\right\} = N((1-a) * (1-b))$

$$= N\begin{pmatrix} 1-a & ;\ (1-b)=1 \\ 1-b & ;\ (1-a)=1 \\ 0 & ;\ (1-a),(1-b)<1 \end{pmatrix}$$

Kasus I: Misalkan $(1-a)=1$ dan $(1-b)=1$ $\Rightarrow a=0$ dan $b=0$

$\Rightarrow 0 = N((1-a) * (1-b))$

$0 = N(1)$

$0 = 1-1$

$0 = 0$ (Terbukti) ▨

Kasus II: Misalkan $(1-a)=1$ dan $(1-b)<1$ $\Rightarrow a=0$ dan $b>0$

$\Rightarrow b = N((1-a) * (1-b))$

$b = N(1-b)$

$b = 1-(1-b)$

$b = b$ (Terbukti) ▨

Kasus III: Misalkan $(1-a)<1$ dan $(1-b)=1$ $\Rightarrow a>0$ dan $b=0$

$\Rightarrow a = N((1-a) * (1-b))$

$a = N(1-a)$

$a = 1-(1-a)$

$a = a$ (Terbukti) ▨

Kasus IV: Misalkan $(1-a)<1$ dan $(1-b)<1$ $\Rightarrow a>0$ dan $b>0$

$\Rightarrow 1 = N((1-a) * (1-b))$

$1 = N(0)$

$1 = 1-0$

$1 = 1$ (Terbukti) ▨

5.) Misalkan kita mengambil pasangan dual operator, Minimum dan Maximum. Perhatikan bahwa,

• $t_{min}(a,b) = N(S_{max}(N(a), N(b)))$

$a * b = N(N(a) \pm N(b))$

$min(a,b) = N((1-a) \pm (1-b))$

$a \wedge b = N(max((1-a),(1-b)))$

$= N((1-a) \vee (1-b))$

Kasus I : Misalkan $max((1-a),(1-b)) = (1-a) \Rightarrow (1-a) > (1-b)$
$a < b$

$\Rightarrow a = N(max((1-a),(1-b)))$

$= N(1-a)$

$= 1-(1-a)$

$a = a$ (Terbukti) ▨

Kasus II : Misalkan $max((1-a),(1-b)) = (1-b) \Rightarrow (1-a) < (1-b)$
$a > b$

$\Rightarrow b = N(max((1-a),(1-b)))$

$= N(1-b)$

$= 1-(1-b)$

$b = b$ (Terbukti) ▨

• $S_{max}(a,b) = N(t_{min}(N(a), N(b)))$

$a \pm b = N(N(a) * N(b))$

$max(a,b) = N((1-a) * (1-b))$

$a \vee b = N(min((1-a),(1-b)))$

$= N((1-a) \wedge (1-b))$

Kasus I : Misalkan $min((1-a),(1-b)) = (1-a) \Rightarrow (1-a) < (1-b)$
$a > b$

$\Rightarrow a = N(min((1-a),(1-b)))$

$= N(1-a)$

$= 1-(1-a)$

$a = a$ (Terbukti) ▨

Kasus II : Misalkan $min((1-a),(1-b)) = (1-b) \Rightarrow (1-a) > (1-b)$
$a < b$

$\Rightarrow b = N(min((1-a),(1-b)))$

$= N(1-b)$

$= 1-(1-b)$

$b = b$ (Terbukti) ▨

Imanuel AS / 1811141008 Imanuel Makassar, 27 Maret 2021

(3.) Kerjakan Latihan di Bab III hal 51

Penyelesaian: Latihan 3.4 hal 51 (Langsung Jawab)

1.) a.) $\tilde{A}^c = \{(5,0), (10,0), (15,0.1), (20,0.3), (25,0.5), (30,0.7), (35,0.9), (40,1), (45,1), (50,1)\}$

$= \{(15,0.1), (20,0.3), (25,0.5), (30,0.7), (35,0.9), (40,1), (45,1), (50,1)\}$

b.) $\tilde{A} \cap \tilde{B} = \{(5,0.1), (10,0.2), (15,0.8), (20,0.7), (25,0.5), (30,0.3), (35,0.1)\}$

c.) $\tilde{A} \cup \tilde{B} = \{(5,1), (10,1), (15,0.9), (20,1), (25,1), (30,0.7), (35,0.4)\}$

d.) $\tilde{C} \cup \tilde{D} = \{(5,0.1), (10,0.4), (15,0.8), (20,1), (25,1), (30,0.8), (35,0.8), (40,1), (45,1), (50,1)\}$

$\tilde{B} \cap (\tilde{C} \cup \tilde{D}) = \{(5,0.1), (10,0.2), (15,0.8), (20,1), (25,1), (30,0.7), (35,0.4)\}$

e.) $\tilde{D}^c = \{(5,0.9), (10,0.6), (15,0.2), (20,0), (25,0), (30,0.2), (35,0.6), (40,1), (45,1), (50,1)\}$

$\tilde{B} \cap \tilde{D}^c = \{(5,0.1), (10,0.2), (15,0.2), (20,0), (25,0), (30,0.2), (35,0.4), (40,0), (45,0), (50,0)\}$

$\tilde{C} \cup (\tilde{B} \cap \tilde{D}^c) = \{(5,0.1), (10,0), (15,0.2), (20,0.2), (25,0.4), (30,0.7), (35,0.8), (40,1), (45,1), (46,1)\}$

f.) $\tilde{B} \cup \tilde{C} = \{(5,0.1), (10,0.2), (15,0.8), (20,1), (25,1), (30,0.7), (35,0.8), (40,1), (45,1), (50,1)\}$

$\tilde{D}^c \setminus (\tilde{B} \cup \tilde{C}) = \{??\}$

Imanuel AS / 1811141008 Imanuel Makassar, 27 Maret 202

2) $\tilde{B} \cap \tilde{C} = \{(1,0), (2,0.4), (3,0), (4,0.8), (5,1), (6,0), (7,0.6), (8,0), (9,0), (10,0)\}$.

$\tilde{B} \cup \tilde{C} = \{(1,0), (2,0.4), (3,0.6), (4,0.8), (5,1), (6,0.8), (7,0.6), (8,0.4), (9,0), (10,0)\}$.

3.) a.) $\tilde{A} \cap \tilde{B} = \{(1,0), (2,0), (3,0.2), (4,0.4), (5,0.6), (6,0.3), (7,0), (8,0), (9,0), (10,0)\}$

b.) $\tilde{A} \cup \tilde{B} = \{(1,0.2), (2,0.5), (3,0.8), (4,1), (5,0.7), (6,0.8), (7,1), (8,1), (9,0), (10,0)\}$

c.) $\tilde{A}^c = \{(1,0.8), (2,0.5), (3,0.2), (4,0), (5,0.3), (6,0.7), (7,1), (8,1), (9,1), (10,1)\}$

$\tilde{B}^c = \{(1,1), (2,1), (3,0.8), (4,0.6), (5,0.4), (6,0.2), (7,0), (8,0), (9,1), (10,1)\}$

$\tilde{A}^c \cap \tilde{B}^c = \{(1,0.8), (2,0.5), (3,0.2), (4,0), (5,0.3), (6,0.2), (7,0), (8,0), (9,1), (10,1)\}$

d.) $\tilde{A}^c \cup \tilde{B}^c = \{(1,1), (2,1), (3,0.8), (4,0.6), (5,0.4), (6,0.7), (7,1), (8,1), (9,1), (10,1)\}$.

Imanuel AS / 1811141008 Makassar, 27 Maret 202

4.) a.) $\tilde{B} \cup \tilde{C} = \{(1,0), (2, 0.64), (3, 0.6), (4, 0.96), (5, 1), (6, 0.8), (7, 0.84), (8, 0.4), (9, 0), (10, 0)\}$.

b.) $\tilde{B} \cup \tilde{C} = \{(1,0), (2, 0.8), (3, 0.6), (4, 1), (5, 1), (6, 0.8), (7, 1), (8, 0.4), (9, 0), (10, 0)\}$

c.) $\tilde{B} \cup \tilde{C} = \{(1,0), (2, 1), (3, 0.6), (4, 1), (5, 1), (6, 0.8), (7, 1), (8, 0.4), (9, 0), (10, 0)\}$

d.) $\tilde{B} \cup \tilde{C} = \{(1,0), (2, 0.68), (3, 0.6), (4, 0.97), (5, 1), (6, 0.8), (7, 0.88), (8, 0.4), (9, 0), (10, 0)\}$

e) $\tilde{B} \cup \tilde{C} = \{(1,0), (2, 0.57), (3, 0.6), (4, 0.88), (5, 0), (6, 0.8), (7, 0.75), (8, 0.4), (9, 0), (10, 0)\}$

5.) a.) $\tilde{A} \cap \tilde{B} = \{(1,0), (2, 0), (3, 0.16), (4, 0.4), (5, 0.42), (6, 0.24), (7, 0), (8, 0), (9, 0), (10, 0)\}$

b.) $\tilde{A} \cap \tilde{B} = \{(1,0), (2, 0.5), (3, 0), (4, 0.4), (5, 0.3), (6, 0.1), (7, 0), (8, 0), (9, 0), (10, 0)\}$

c) $\tilde{A} \cap \tilde{B} = \{(1,0), (2, 0), (3, 0), (4, 0.4), (5, 0), (6, 0), (7, 1), (8, 1), (9, 0), (10, 0)\}$

d.) $\tilde{A} \cap \tilde{B} = \{(1,0), (2, 0), (3, 0.09), (4, 0.2), (5, 0.23), (6, 0.13), (7, 0), (8, 0), (9, 0), (10, 0)\}$

e) $\tilde{A} \cap \tilde{B} = \{(1,0), (2, 0), (3, 0.19), (4, 0.4), (5, 0.47), (6, 0.33), (7, 0), (8, 0), (9, 0), (10, 0)\}$

Imanuel AS / 1811141008 Manuel Makassar, 27 Maret 20

6.) a.) $\tilde{A}^c = \{(1, 0.57), (2, 0.25), (3, 0.07), (4, 0.33), (5, 0.13), (6, 0.43), (7, 1), (8, 1), (9, 1), (10, 1)\}$

$\tilde{B}^c = \{(1, 1), (2, 1), (3, 0.57), (4, 0.33), (5, 0.10), (6, 0.07), (7, 0), (8, 0), (9, 1), (10, 1)\}$

b.) $\tilde{A}^c = \{(1, 0.3), (2, 0.08), (3, 0.01), (4, 0), (5, 0.02), (6, 0.2), (7, 1), (8, 1), (9, 1), (10, 1)\}$

$\tilde{B}^c = \{(1, 1), (2, 1), (3, 0.3), (4, 0.13), (5, 0.05), (6, 0.01), (7, 0), (8, 0), (9, 1), (10, 1)\}$

7.) a.) hal 5 (solved)
b.) hal 8-9 (solved)
c.) hal 6 (solved)
d.) hal 7 (solved)