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TUGAS Masalah Syarat Batas

Makassar, 26 Februari 2021

(1.) $$\boxed{f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cdot \cos nx + b_n \cdot \sin nx)} \dots (2) \rightarrow \text{hal. } 7$$

Jika kedua ruas persamaan (2) dikalikan dengan $\sin mx$ dimana m adalah bilangan bulat positif, kemudian diintegralkan dari $-\pi$ sampai $+\pi$ maka dengan cara yang sama seperti diatas kita peroleh :

$$\boxed{b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \; dx} \dots (4) \quad ; n \in \text{bilangan asli}$$

Tunjukkan!

Penyelesaian:

$$\int_{-\pi}^{\pi} f(x) \cdot \sin mx \; dx = \left[\frac{1}{2} a_0 \cdot \int_{-\pi}^{\pi} \sin mx \; dx\right] + \left[\sum_{n=1}^{\infty} \left(a_n \cdot \int_{-\infty}^{\infty} \cos nx \cdot \sin mx \; dx + b_n \cdot \int_{-\infty}^{\infty} \sin nx \cdot \sin mx \; dx\right)\right]$$

Agar lebih mudah dan jelas, maka akan dihitung terlebih dahulu tiga buah integral pada ruas kanan, satu per satu.

$$\Rightarrow \int_{-\pi}^{\pi} \sin mx \; dx = \frac{1}{m} \left[\cos mx\right]_{-\pi}^{\pi}$$

$$= \frac{1}{m} \left[\cos m\pi - \cos(-m\pi)\right]$$

$$= \frac{1}{m} \left[\cos m\pi - \cos(m\pi)\right]$$

$$= \frac{1}{m} [0] \quad ; m \in \mathbb{N} \Rightarrow m \neq 0$$

$$= 0$$

Rumus Sudut Negatif 2 :
$[\cos(-a^\circ) = \cos a^\circ]$

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Makassar, 26 Febru 2024

→ untuk menghitung nilai $\int_{-\pi}^{\pi} \cos nx \cdot \sin mx \, dx$

kita akan bedakan dua kemungkinan, yaitu bila $n \neq m$, kita peroleh

$$\int_{-\pi}^{\pi} \cos nx \cdot \sin mx \, dx = \frac{1}{2} \cdot \int_{-\pi}^{\pi} \left[\sin(n+m)x - \sin(n-m)x \right] dx$$

[Rumus Jumlah dan Selisih Fungsi Trigonometri]

$$= \frac{1}{2} \cdot \left(\left[\int_{-\pi}^{\pi} \sin(n+m)x \, dx \right] - \left[\int_{-\pi}^{\pi} \sin(n-m)x \, dx \right] \right)$$

$$= \frac{1}{2} \left(\left[\frac{-1}{n+m} \cdot \cos(n+m)x \right]_{-\pi}^{\pi} - \left[\frac{-1}{n-m} \cdot \cos(n-m)x \right]_{-\pi}^{\pi} \right)$$

$$= \frac{1}{2} \cdot \left(\left[\frac{-1}{n+m} \cdot \left(\cos(n+m)x \right)_{-\pi}^{\pi} \right] - \left[\frac{-1}{n-m} \cdot \left(\cos(n-m)x \right)_{-\pi}^{\pi} \right] \right)$$

$$= \frac{1}{2} \cdot \left(\left[-\frac{1}{n+m} \left(\left(\cos(n+m)\pi \right) - \left(\cos(n+m) \cdot -\pi \right) \right) \right] - \left[-\frac{1}{n-m} \left(\left(\cos(n-m)\pi \right) - \left(\cos(n-m) \cdot -\pi \right) \right) \right] \right)$$

$$= \frac{1}{2} \cdot \left(\left[-\frac{1}{n+m} \left(\left(\cos(n+m)\pi \right) - \left(\cos(n+m)\pi \right) \right) \right] - \left[-\frac{1}{n-m} \left(\left(\cos(n-m) \cdot \pi \right) - \left(\cos(n-m)\pi \right) \right) \right] \right)$$

$$= \frac{1}{2} \cdot \left(\left[-\frac{1}{n+m} (0) \right] - \left[-\frac{1}{n-m} (0) \right] \right)$$

$$= \frac{1}{2} \cdot \left([0] - [0] \right)$$

$$= \frac{1}{2} \cdot (0)$$

$$= 0$$

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Dipindai dengan CamScanner

Makassar, 27 Febr 2021

Disisi lain, bila $n = m$, maka diperoleh sebagai berikut

$$\int_{-\pi}^{\pi} \cos nx \cdot \sin mx \, dx = \int_{-\pi}^{\pi} \cos nx \cdot \sin nx \cdot dx$$

Miss $u = \sin nx$

$du = \cos (nx) \cdot (n) \, dx$

$\frac{du}{n} = \cos (nx) \, dx$

$$= \int_{-\pi}^{\pi} \sin nx \cdot \cos nx \cdot dx$$

$$= \int_{-\pi}^{\pi} u \cdot \frac{du}{n}$$

$$= \int_{-\pi}^{\pi} \frac{u}{n} \, du$$

$$= \frac{1}{n} \cdot \int_{-\pi}^{\pi} u \, du$$

$$= \frac{1}{n} \cdot \left[\frac{1}{2} u^2\right]_{-\pi}^{\pi}$$

$$= \frac{1}{n} \cdot \left[\frac{1}{2} \cdot (\sin nx)^2\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2n} \cdot \left[(\sin nx)^2\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2n} \left[(\sin (n \pi))^2 - (\sin (n \cdot -\pi))^2\right]$$

$$= \frac{1}{2n} \cdot \left[(\sin (n\pi))^2 - (-\sin (n\pi))^2\right]$$

$$= \frac{1}{2n} \cdot \left[(\sin (n\pi))^2 - (\sin (n\pi))^2\right]$$

$$= \frac{1}{2n} \cdot [0]$$

$$= 0$$

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→ Untuk menghitung nilai $\int_{-\pi}^{\pi} \sin nx \cdot \sin mx \cdot dx$

kita akan bedakan dua kemungkinan, yaitu bila $n \neq m$ kita peroleh

$$\int_{-\pi}^{\pi} \sin nx \cdot \sin mx \cdot dx = -\frac{1}{2} \cdot \int_{-\pi}^{\pi} \left[\cos(n+m)x - \cos(n-m)x\right] dx$$

[Rumus Jumlah dan Selisih Fungsi Trigon]

$$= \frac{1}{2} \cdot \int_{-\pi}^{\pi} \left[\cos(n-m)x - \cos(n+m)x\right] dx$$

$$= \frac{1}{2} \cdot \left[\int_{-\pi}^{\pi} \cos(n-m)x \, dx - \int_{-\pi}^{\pi} \cos(n+m)x \, dx\right]$$

$$= \frac{1}{2}\left(\left[\frac{1}{(n-m)} \sin(n-m)x\right]_{-\pi}^{\pi} - \left[\frac{1}{(n+m)} \cdot \sin(n+m)x\right]_{-\pi}^{\pi}\right)$$

$$= \left[\frac{\sin(n-m)x}{2n-2m}\right]_{-\pi}^{\pi} - \left[\frac{\sin(n+m)x}{2n+2m}\right]_{-\pi}^{\pi}$$

$$= [0] - [0]$$

[Krn kelipatan $\sin \pi$ selalu $= 0$]

$$= 0$$

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Manu

Makassar, 27 Februari 2021

Disisi lain, bila $n=m$, maka diperoleh sebagai berikut

$$\int_{-\pi}^{\pi} \sin nx \cdot \sin mx \cdot dx = \int_{-\pi}^{\pi} \sin nx \cdot \sin nx \cdot dx$$ [Asumsi $n=m$]

$$= \int_{-\pi}^{\pi} (\sin nx)^2 \, dx$$

$$= \int_{-\pi}^{\pi} \sin^2 nx \, dx$$

$$= \int_{-\pi}^{\pi} \frac{1}{2}(1-\cos 2nx)\, dx$$ [$1-\cos 2x = 2\sin^2 x$]

$$= \frac{1}{2} \cdot \int_{-\pi}^{\pi} (1-\cos 2nx)\, dx$$

$$= \frac{1}{2} \cdot \left[\int_{-\pi}^{\pi} 1\, dx - \int_{-\pi}^{\pi} \cos 2nx \, dx\right]$$

$$= \frac{1}{2} \cdot \left[[x]_{-\pi}^{\pi} - \left[\frac{1}{2n} \cdot \sin(2nx)\right]_{-\pi}^{\pi} \right]$$

$$= \frac{1}{2} \cdot \left[(\pi - (-\pi)) - (0)\right]$$ [kelipatan sin π selalu $= 0$]

$$= \frac{1}{2} \cdot [2\pi]$$

$$= \pi$$

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∴ Untuk setiap bilangan m, persamaan (2) menjadi

$$\int_{-\pi}^{\pi} f(x) \cdot \sin mx \, dx = \left[\frac{1}{2} \cdot a_0 \cdot \int_{-\pi}^{\pi} \sin mx \, dx\right] + \left[\sum_{n=1}^{\infty} \left(a_n \cdot \int_{-\pi}^{\pi} \cos nx \cdot \sin mx \cdot dx + b_n \cdot \int_{-\pi}^{\pi} \sin nx \cdot \sin mx \cdot dx\right)\right]$$

$$= \left[\frac{1}{2} \cdot a_0 \cdot (0)\right] + \left[\sum_{n=1}^{\infty} \left(a_n \cdot (0) + b_n \cdot (\pi)\right)\right]$$

$$= \sum_{n=1}^{\infty} b_n \cdot \pi$$

$$\frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \sin mx \, dx = \sum_{n=1} b_n$$

$$\frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \sin nx \, dx = \sum_{n=1}^{\infty} b_n \qquad [n = m]$$

$$\boxed{\frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \sin nx \cdot dx = b_n} \quad \ldots (4) \quad ; n \in \mathbb{N}$$

∎

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(2.) Hal. 16 Contoh 2

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Fungsi f yang periodik dengan periode 2π ditentukan sebagai berikut:

$f(x) = x$ dalam selang $-\pi < x < 0$

$f(x) = \pi$ dalam selag $0 < x < \pi$

Ditanyakan :

b.) Deret Fourier $f(x)$

Penyelesaian :

b.) Koefisien a_0, a_n, dan b_n kita peroleh sebagai berikut :

$$a_0 = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right)$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} x\, dx + \int_{0}^{\pi} \pi\, dx \right)$$

$$= \frac{1}{\pi} \left(\frac{1}{2} x^2 \Big|_{-\pi}^{0} + \pi x \Big|_{0}^{\pi} \right)$$

$$= \frac{1}{\pi} \left(\frac{1}{2} \cdot -(\pi)^2 + (\pi)^2 \right)$$

$$= \frac{1}{\pi} \left(\frac{-(\pi)^2 + 2\pi^2}{2} \right)$$

$$= \frac{-\pi + 2\pi}{2}$$

$$= \frac{\pi}{2}$$

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$$a_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \cos nx \cdot dx$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} f(x) \cdot \cos nx \cdot dx + \int_{0}^{\pi} f(x) \cdot \cos nx \cdot dx \right)$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} x \cdot \cos nx \cdot dx + \int_{0}^{\pi} \pi \cdot \cos nx \cdot dx \right)$$

$\circ$) $\int_{-\pi}^{0} x \cdot \cos nx \cdot dx =$

misal $u = x$, $du = dx$

misal $dv = \cos nx \, dx$, $v = \int \cos nx \, dx$, $v = \frac{\sin nx}{n}$

$$\boxed{\text{Integral Parsial} \quad \int u \, dv = uv - \int v \, du}$$

$$= \left[\left(x \cdot \frac{\sin nx}{n} \right) - \left(\int \frac{\sin nx}{n} \cdot dx \right) \right]_{-\pi}^{0}$$

$$= \left[\left(x \cdot \frac{\sin nx}{n} \right) - \left(\frac{1}{n} \cdot \int \sin nx \, dx \right) \right]_{-\pi}^{0}$$

$$= \left[\left(x \cdot \frac{\sin nx}{n} \right) - \left(\frac{1}{n} \cdot \left(- \frac{\cos (nx)}{n} \right) \right) \right]_{-\pi}^{0}$$

$$= \left[\left(x \cdot \frac{\sin nx}{n} \right) + \left(\frac{1}{n^2} \cdot \cos (nx) \right) \right]_{-\pi}^{0}$$

$$= \left[(0) + \left(\frac{1}{n^2} \right) \right] - \left[(0) + \frac{\cos (n\pi)}{n^2} \right]$$

$$= \frac{1 - \cos (n\pi)}{n^2}$$

$\circ$) $\int_{0}^{\pi} \pi \cdot \cos nx \cdot dx = \pi \cdot \int_{0}^{\pi} \cos nx \, dx$

$$= \pi \cdot \left[\frac{\sin nx}{n} \right]_{0}^{\pi}$$

$$= \pi \left[\frac{\sin n(\pi)}{n} - \frac{\sin n(0)}{n} \right]$$

$$= \pi \left[0 - 0 \right]$$

$$= 0$$

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Maka,

$$a_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} x \cdot \cos nx \cdot dx + \int_{0}^{\pi} \pi \cdot \cos nx \cdot dx\right)$$

$$= \frac{1}{\pi}\left(\frac{1-\cos(n\pi)}{n^2} + 0\right)$$

$$= \frac{1}{\pi}\left(\frac{1-\cos(n\pi)}{n^2}\right)$$

$$= \frac{1-\cos(n\pi)}{\pi n^2}$$

Jika n genap, maka $\cos n\pi = 1$

sedangkan jika n ganjil, maka $\cos n\pi = -1$

yang dapat ditulis $(-1)^n$.

Jadi, $a_n = 0$ bila n genap

dan $a_n = \frac{2}{\pi n^2}$ bila n ganjil.

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$$b_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \sin nx \cdot dx$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} f(x) \cdot \sin nx \cdot dx + \int_{0}^{\pi} f(x) \cdot \sin nx \cdot dx \right)$$

$$= \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} x \cdot \sin nx \cdot dx + \int_{0}^{\pi} \pi \cdot \sin nx \cdot dx \right)$$

$\Rightarrow \int_{-\pi}^{0} x \cdot \sin nx \cdot dx =$ | miss $u = x$ $du = dx$ miss $dV = \sin nx \, dx$

$V = \int \sin nx \, dx$

$V = -\frac{\cos nx}{n}$

$\boxed{\text{Integral Parsial} \quad \int u \, dv = uv - \int v \, du}$

$$= \left[\left(x \cdot -\frac{\cos nx}{n} \right) - \left(\int -\frac{\cos nx}{n} \cdot dx \right) \right]_{-\pi}^{0}$$

$$= \left[\left(-x \cdot \frac{\cos nx}{n} \right) + \left(\frac{1}{n} \int \cos nx \cdot dx \right) \right]_{-\pi}^{0}$$

$$= \left[\left(-x \cdot \frac{\cos nx}{n} \right) + \left(\frac{1}{n} \cdot \frac{\sin nx}{n} \right) \right]_{-\pi}^{0}$$

$$= \left[\left(-x \cdot \frac{\cos nx}{n} \right) + \left(\frac{\sin nx}{n^2} \right) \right]_{-\pi}^{0}$$

$$= [(0) + (0)] - \left[\left(-(-\pi) \cdot \frac{\cos n(-\pi)}{n} \right) + \left(\frac{\sin n(-\pi)}{n^2} \right) \right]$$

$$= [0] - \left[\pi \left(\frac{\cos (n\pi)}{n} \right) + \left(-\frac{\sin (n\pi)}{n^2} \right) \right]$$

$$= -\pi \cdot \left(\frac{\cos n\pi}{n} \right) + (0)$$

$$= -\pi \frac{\cos (n\pi)}{n}$$

$\Rightarrow \int_{0}^{\pi} \pi \cdot \sin nx \, dx = \pi \int_{0}^{\pi} \sin nx \cdot dx$

$$= \pi \left[-\frac{\cos nx}{n} \right]_{0}^{\pi}$$

$$= \pi \left[\left(-\frac{\cos n\pi}{n} \right) - \left(-\frac{\cos n(0)}{n} \right) \right]$$

$$= \pi \left[-\frac{\cos n\pi}{n} + \frac{1}{n} \right]$$

$$= -\frac{\pi \cdot \cos n\pi}{n} + \frac{\pi}{n}$$

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Maka,

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} x \cdot \sin nx \cdot dx + \int_{0}^{\pi} \pi \cdot \sin nx \cdot dx\right)$$

$$= \frac{1}{\pi}\left(-\pi \cdot \frac{\cos(n\pi)}{n} + \left(-\frac{\pi \cdot \cos(n\pi)}{n} + \frac{\pi}{n}\right)\right)$$

$$= \frac{1}{\pi}\left(-\pi\frac{\cos(n\pi)}{n} - \frac{\pi \cdot \cos(n\pi)}{n} + \frac{1}{n}\right)$$

$$= -\frac{\cos(n\pi)}{n} - \frac{(\cos(n\pi))}{n} + \frac{1}{n}$$

$$= -2\left(\frac{\cos(n\pi)}{n}\right) + \frac{1}{n}$$

$$= -\frac{2}{n} \cdot \cos(n\pi) + \frac{1}{n}$$

//

∴ Deret Fourier untuk $f(x)$ adalah

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cdot \cos nx + b_n \cdot \sin nx)$$

$$= \frac{1}{2}\cdot\left(\frac{1}{2}\pi\right) + \sum_{n=1}^{\infty}\left(\left(\frac{1-\cos(n\pi)}{\pi n^2} \cdot \cos nx\right) + \left(\left(-\frac{2}{n}\cdot\cos(n\pi) + \frac{1}{n}\right)\cdot \sin nx\right)\right)$$

$$= \frac{1}{4}\pi + \frac{1}{\pi}\cdot\sum_{n=1}^{\infty}\left(\left(\frac{1-\cos(n\pi)}{n^2} \cdot \cos nx\right) + \left(-2\cos(n\pi) + 1\right)\cdot\frac{\sin nx}{n}\right)$$

$$= \frac{1}{4}\pi + \frac{1}{\pi}\left(\left(\sum_{n=1}^{\infty}\left(\frac{1-\cos(n\pi)}{n^2}\cdot \cos nx\right)\right) - \left(\sum_{n=1}^{\infty}\left(\left(1 - 2\cos(n\pi)\right)\cdot\frac{\sin nx}{n}\right)\right)\right)$$

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