Analisis Real II / Pertemuan ke-4 / Tugas - diskusi

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Analisis Real II

$$\left.\begin{array}{l}\lim\limits_{x \to c} f(x) = L \\ \\ \lim\limits_{x \to c} g(x) = M\end{array}\right\} \Rightarrow \lim_{x \to c} f(x).g(x) = L.M$$

$\Downarrow$

Buktikan $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$, $M \neq 0$

Penyelesaian:

(1) Analisis Pendahuluan: Adb. $\forall \varepsilon > 0, \exists \delta > 0 \ni 0 |x - c| < \delta \Rightarrow |(f(x)g(x)) - LM| < \varepsilon$

Note that,

$$\begin{aligned}|(f(x).g(x)) - L.M| &= |f(x).g(x) - M.f(x) + M.f(x) - L.M| \\ &\leq |f(x).g(x) - M.f(x)| + |M.f(x) - L.M| \\ &\leq |f(x)|.|g(x) - M| + |M|.|f(x) - L|\end{aligned}$$

Karena diketahui $\lim\limits_{x \to c} f(x) = L$ ada, maka jika diberikan $\delta_1 > 0$ maka sekarang kita mempunyai $|f(x) - L| < \dfrac{\varepsilon}{2|M|}$

Adapun $\lim\limits_{x \to c} g(x) = M$ ada, maka jika diberikan $\delta_2 > 0$ maka sekarang kita mempunyai $|g(x) - M| < \dfrac{\varepsilon}{2|M|}$

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Bukti Formal :

Ambil sebarang $\varepsilon > 0$

Pilih $\delta = \min\{\delta_1, \delta_2\}$

Maka untuk $0 < |x-c| < \delta$ diperoleh

$$|(f(x) \cdot g(x)) - L.M| = |f(x) \cdot g(x) - M.f(x) + M.f(x) - LM|$$

$$\leq |f(x) \cdot g(x) - M.f(x)| + |M.f(x) - LM|$$

$$\leq |f(x)| \cdot |g(x) - M| + |M| \cdot |f(x) - L|$$

$$\leq |L| \cdot \frac{\varepsilon}{2|L|+1} + |M| \cdot \frac{\varepsilon}{2|M|+1}$$

$$\leq |L| \cdot \frac{\varepsilon}{2|L|} + |M| \cdot \frac{\varepsilon}{2|M|}$$

$$\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

(Terbukti) $\blacksquare$

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(2) Analysis Pendahuluan: Adb. $\forall \varepsilon > 0, \exists \delta > 0 \ni 0 < |x - c| < \delta \Rightarrow \left| \left(\frac{f(x)}{g(x)} \right) - \frac{L}{M} \right| < \varepsilon$

Adb. $\lim_{x \to c} g(x) = M$ dan $M \neq 0 \Rightarrow \lim_{x \to c} \frac{1}{g(x)} = \frac{1}{M}$

Karena $\lim_{x \to c} g(x) = M$ ada, maka terdapat $\delta_2 > 0$

sehingga $|g(x) - M| < \varepsilon$

Note that,

$$\left| \frac{1}{g(x)} - \frac{1}{M} \right| = \left| \frac{M \cdot g(x)}{g(x) \cdot M} \right|$$

$$= \frac{|M \cdot g(x)|}{|g(x) \cdot M|}$$

$$< \varepsilon \cdot \frac{|M \cdot g(x)|}{|g(x) \cdot M|} = \varepsilon$$

sehingga $\lim_{x \to c} \frac{1}{g(x)} = \frac{1}{M}$ ada

Diketahui pula $\lim_{x \to c} f(x) = L$ ada, maka terdapat $\delta_1 > 0$ sehingga

$|g(x) - L| < \varepsilon \cdot \frac{|M|}{2}$

Diketahui pula $\lim_{x \to c} \frac{1}{g(x)} = \frac{1}{M}$ ada, maka $\exists \delta_2 > 0 \ni \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \frac{\varepsilon}{2 \cdot |L|}$

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Bukti Formal:

Ambil $\varepsilon > 0$

Pilih $\delta = \min\{\delta_1, \delta_2\}$

Maka, untuk $0 < |x-c| < \delta$ diperoleh

$$\left|\frac{f(x)}{g(x)} - \frac{L}{M}\right| = \left|f(x) \cdot \frac{1}{g(x)} - L \cdot \frac{1}{g(x)} + L \cdot \frac{1}{g(x)} - \frac{L}{M}\right|$$

$$\leq \left|f(x) \cdot \frac{1}{g(x)} - L \cdot \frac{1}{g(x)}\right| + \left|L \cdot \frac{1}{g(x)} - L \cdot \frac{1}{M}\right|$$

$$< \left|\frac{1}{g(x)}\right| \cdot |f(x) - L| + |L| \cdot \left|\frac{1}{g(x)} - \frac{1}{M}\right|$$

$$< \left|\frac{1}{M}\right| \cdot \frac{\varepsilon M}{2} + |L| \cdot \frac{\varepsilon}{2L}$$

$$= \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$

$$= \varepsilon$$

$\blacksquare$

(Terbukti).