Teori Modul / Pertemuan ke - 2 / Catatan

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Teori Modul : Catatan Pertemuan ke - 2

Definition A Field is a set F, containing at least two elements, together with two binary operations, called addition (denoted by : +) and multiplication (denoted by : justaposition) for which the following hold:

1.) F is an abelian grup under addition

2) The set F of all nonzero elements in F is an abelian group under multiplication.

3) (Distributivity) For all $a, b, c \in F$

$$(a+b)c = ac + bc \text{ and } c(a+b) = ca + cb \quad \square$$

Lapangan

$\boxed{D}$

Misalkan F himpunan, $F \neq \phi$, $(F, +, \times)$ disebut lapangan jika

(1). $\forall a, b \in F \Rightarrow a + b \in F$

(2). $\forall a, b, c \in F \Rightarrow a + (b+c) = (a+b) + c$

(3). $\exists 0_F \in F, \forall a \in F \Rightarrow a + 0_F = 0_F + a = a$

(4). $\forall a \in F, \exists (-a) \in F \ni a + (-a) = (-a) + a = 0_F$

(5). $\forall a, b \in F \Rightarrow a + b = b + a$

$(F, +)$ Grup Abelian

(6). $\forall a, b \in F \Rightarrow a, b \in F$

(7). $\forall a, b, c \in F \Rightarrow a(bc) = (ab)c$

(8). $\exists 1_F \in F, \forall a \in F \Rightarrow a \cdot 1_F = 1_F \cdot a = a$

(9). $\forall a \in F, a \neq 0, \exists a' \in F \ni a \cdot a' = a' \cdot a = 1_F$

(10). $\forall a, b \in F \Rightarrow ab = ba$

$(F \setminus \{0\}, \times)$ Grup Abelian

$\boxed{\text{Notasi : } F = \text{Lapangan}}$

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(1)

[Jelas]

(2) Periksa apakah $\mathbb{Z}$ lapangan?

Jawab:

Jelas $1 \in \mathbb{Z}$ unsur identitas terhadap operasi kali karena $\forall a \in \mathbb{Z}$. Berlaku $\boxed{a.1 = 1.a = a}$ misal $2 \in \mathbb{Z}$ tidak ada $b \in \mathbb{Z}$ sehingga $2.b = b.2 = 1$. Jadi 2 tidak memiliki invers terhadap operasi perkalian, dengan demikian $\mathbb{Z}$ bukan lapangan.

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Teori Modul

Tugas Pertemuan ke-2

Misalkan $\mathbb{R}^2 = \{(a,b) \mid a,b \in \mathbb{R}\}$ dengan operasi penjumlahan dan perkaliannya,

$x = (a_1, b_1) \in \mathbb{R}^2 \quad , a_1, b_1 \in \mathbb{R}$

$y = (a_2, b_2) \in \mathbb{R}^2 \quad , a_2, b_2 \in \mathbb{R}$

yaitu

$$x + y = (a_1, b_1) + (a_2, b_2) = (a_1 + a_2 , b_1 + b_2)$$

$$x.y = (a_1, b_1) \cdot (a_2, b_2) = (a_1 a_2 , b_1 b_2)$$

Periksa apakah $\mathbb{R}^2$ merupakan lapangan!

Penyelesaian:

Akan ditunjukkan : $(\mathbb{R}^2, +, \times)$. adalah bukan Lapangan

(1.) $\forall x, y \in \mathbb{R}^2 \Rightarrow x+y \in \mathbb{R}^2$

Ambil sebarang $x, y \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1) \quad , a_1, b_1 \in \mathbb{R}$

$y = (a_2, b_2) \quad , a_2, b_2 \in \mathbb{R}$

Note that,

$x + y = (a_1, b_1) + (a_2, b_2)$

$= (a_1 + a_2 , b_1 + b_2) \in \mathbb{R}^2$ ∎

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(2) $\forall x, y, z \in \mathbb{R}^2 \Rightarrow x + (y+z) = (x+y) + z$

Ambil sebarang $x, y, z \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1)$, $a_1, b_1 \in \mathbb{R}$

$y = (a_2, b_2)$, $a_2, b_2 \in \mathbb{R}$

$z = (a_3, b_3)$, $a_3, b_3 \in \mathbb{R}$

Note that,

$$
\begin{aligned}
x+(y+z) &= (a_1, b_1) + [(a_2, b_2) + (a_3, b_3)] \\
&= (a_1, b_1) + (a_2+a_3, b_2+b_3) \\
&= (a_1+(a_2+a_3), b_1+(b_2+b_3)) \\
&= ((a_1+a_2)+a_3, (b_1+b_2)+b_3) \\
&= (a_1+a_2, b_1+b_2) + (a_3, b_3) \\
&= (x+y)+z
\end{aligned}
$$

$\blacksquare$

(3) $\exists 0_{\mathbb{R}^2} \in \mathbb{R}^2, \forall x \in \mathbb{R}^2 \Rightarrow x + 0_{\mathbb{R}^2} = 0_{\mathbb{R}^2} + x = x$

Ambil sebarang $x \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1)$; $a_1, b_1 \in \mathbb{R}$

Note that,

$\exists 0_{\mathbb{R}^2} = (0,0) \in \mathbb{R}^2 \ni$

$$
\begin{aligned}
x + 0_{\mathbb{R}^2} &= (a_1, b_1) + (0,0) \\
&= (a_1+0, b_1+0) \\
&= (a_1, b_1) \\
&= x \quad \text{-----} (*)
\end{aligned}
$$

dan,

$$
\begin{aligned}
0_{\mathbb{R}^2} + x &= (0,0) + (a_1, b_1) \\
&= (0+a_1, 0+b_1) \\
&= (a_1, b_1) \\
&= x \quad \text{----} (**)
\end{aligned}
$$

Karena $(*) = (**)$ maka identitas penjumlahan ada (terbukti) $\blacksquare$

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(4) $\forall x \in \mathbb{R}^2, \exists (-x) \in \mathbb{R}^2 \ni x+(-x) = (-x)+x = 0_{\mathbb{R}^2}$

Ambil sebarang $x \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1)$, $a_1, b_1 \in \mathbb{R}$

Note that,

$$\exists (-x) = (-a_1, -b_1) \in \mathbb{R}^2 \ni x+(-x) = (a_1, b_1) + (-a_1, -b_1)$$
$$= (a_1 + (-a_1), b_1 + (-b_1))$$
$$= (a_1 - a_1, b_1 - b_1)$$
$$= (0, 0)$$
$$= 0_{\mathbb{R}^2} \quad \ldots\ldots\ldots (*)$$

dan

$$(-x)+x = (-a_1, -b_1) + (a_1, b_1)$$
$$= (-a_1 + a_1, -b_1 + b_1)$$
$$= (0, 0)$$
$$= 0_{\mathbb{R}^2} \quad \ldots\ldots\ldots (**)$$

Karena $(*) = (**)$ maka adanya invers penjumlahan, terbukti. ▨

(5) $\forall x, y \in \mathbb{R}^2 \Rightarrow x+y = y+x$

Ambil sebarang $x, y \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1)$, $a_1, b_1 \in \mathbb{R}$

$y = (a_2, b_2)$, $a_2, b_2 \in \mathbb{R}$

Note that,

$$x+y = (a_1, b_1) + (a_2, b_2)$$
$$= (a_1 + a_2, b_1 + b_2)$$
$$= (a_2 + a_1, b_2 + b_1)$$
$$= (a_2, b_2) + (a_1, b_1)$$
$$= y + x$$

▨

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(6) $\forall\ x, y \in \mathbb{R}^2 \Rightarrow x \cdot y \in \mathbb{R}^2$

Ambil sebarang $x, y \in \mathbb{R}^2$

Tulis $x = (a_1, b_1)$, $a_1, b_1 \in \mathbb{R}$

$y = (a_2, b_2)$, $a_2, b_2 \in \mathbb{R}$

Note that,

$$x \cdot y = (a_1, b_1) \cdot (a_2 \cdot b_2)$$
$$= (a_1 \cdot a_2\ ,\ b_1 \cdot b_2) \in \mathbb{R}^2$$

(7) $\forall\ x, y, z \in \mathbb{R}^2 \Rightarrow x(y \cdot z) = (x \cdot y) \cdot z$

Ambil sebarang $x, y, z \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1)$, $a_1, b_1 \in \mathbb{R}$

$y = (a_2, b_2)$, $a_2, b_2 \in \mathbb{R}$

$z = (a_3, b_3)$, $a_3, b_3 \in \mathbb{R}$

Note that,

$$x \cdot (y \cdot z) = (a_1, b_1) \cdot ((a_2, b_2) \cdot (a_3, b_3))$$
$$= (a_1, b_1) \cdot (a_2 \cdot a_3\ ,\ b_2 \cdot b_3)$$
$$= (a_1 \cdot (a_2 \cdot a_3)\ ,\ b_1 \cdot (b_2 \cdot b_3))$$
$$= ((a_1 \cdot a_2) \cdot a_3\ ,\ (b_1 \cdot b_2) \cdot b_3)$$
$$= (a_1 \cdot a_2\ ,\ b_1 \cdot b_2) \cdot (a_3, b_3)$$
$$= ((a_1, b_1) \cdot (a_2, b_2)) \cdot (a_3, b_3)$$
$$= (x \cdot y) \cdot z$$

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(8) $\exists 1_{\mathbb{R}^2} \in \mathbb{R}^2, \forall x \in \mathbb{R} \Rightarrow x \cdot 1_{\mathbb{R}^2} = 1_{\mathbb{R}^2} \cdot x = x$

Ambil sebarang $x \in \mathbb{R}^2$

Tulis, $x = (a_1, b_1)$; $a_1, b_1 \in \mathbb{R}$

Note that,

$$\exists 1_{\mathbb{R}^2} = (1,1) \in \mathbb{R}^2 \ni x \cdot 1_{\mathbb{R}^2} = (a_1, b_1) \cdot (1,1)$$
$$= (a_1 \cdot 1 \, , \, b_1 \cdot 1)$$
$$= (a_1 \, , \, b_1)$$
$$= x \quad \ldots\ldots\ldots\ldots (*)$$

dan

$$1_{\mathbb{R}^2} \cdot x = (1,1) \cdot (a_1, b_1)$$
$$= (1 \cdot a_1 \, , \, 1 \cdot b_1)$$
$$= (a_1, b_1)$$
$$= x \quad \ldots\ldots\ldots\ldots (**)$$

Karena (*) = (**), maka adanya identitas perkalian terbukti $\therefore \blacksquare$

(9) $\forall x \in \mathbb{R}^2, x \neq 0, \exists x' \in \mathbb{R}^2 \ni x \cdot x' = x' \cdot x = 1_{\mathbb{R}^2}$

Ambil sebarang $x \in \mathbb{R}^2$

Misalkan $x = (2,0)$; $x \neq 0$ karena $x = (1,0) \neq (0,0)$

Note that,

$$\nexists x' = \left(\frac{1}{2}, \frac{1}{b_1}\right) \in \mathbb{R} ; b_1 \in \mathbb{R}, b_1 \neq 0 \ni x \cdot x' = (2,0) \cdot \left(\frac{1}{2}, \frac{1}{b_1}\right)$$
$$= \left(2 \cdot \frac{1}{2} \, , \, 0 \cdot \frac{1}{b_1}\right)$$
$$= (1 \, , \, 1)$$
$$= 1_{\mathbb{R}^2}$$

Karena $x = (2,0) \in \mathbb{R}^2$ tidak memiliki invers perkalian, maka adanya invers perkalian tidak terbukti.

$\therefore$ Karena Aksioma ke-9 Definisi Lapangan tidak dipenuhi, maka $(\mathbb{R}^2, +, \times)$ adalah BUKAN Lapangan.

$\blacksquare$

(terbukti)

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KUD MIPA LEARN / Pertemuan ke-2

Teori Modul

Periksa apakah $M_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{R} \right\}$, $(M_2(\mathbb{R}), +, \times)$ merupakan lapangan?

Penyelesaian :

Akan dibuktikan : $M_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{R} \right\}$, $(M_2(\mathbb{R}), +, \times)$ bukan lapangan

Akan ditunjukkan : $M_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{R} \right\}$, $(M_2(\mathbb{R}), +, \times)$ tidak memenuhi aksioma ke-9 (invers perkalian) Definisi lapangan.

Adb. $\exists A \in M_2(\mathbb{R})$, $A \neq 0$, $\nexists A' \in M_2(\mathbb{R}) \ni A \cdot A' = A' \cdot A = I$

Pilih $A = \begin{pmatrix} 5 & 2 \\ 5 & 2 \end{pmatrix} \in M_2(\mathbb{R})$

Note that,

$$\det(A) = (5 \cdot 2) - (2 \cdot 5) = 10 - 10 = 0$$

Berdasarkan Teorema 2.3.3 Anton Rorres, Edisi Kedelapan • Jilid 1 (hal 108)
"Suatu matriks bujur sangkar A dapat dibalik, jika dan hanya jika $\det(A) \neq 0$"

$\Rightarrow$ Matriks A tidak dapat dibalik

Berdasarkan teorema 2.3.6 Anton Rorres, Edisi Kedelapan • Jilid 1 (hal 112) bagian (a) dan (c),
Karena A tidak dapat dibalik maka bentuk eselon baris tereduksi dari A adalah BUKAN Matriks Identitas sedemikian sehingga aksioma ke-9 Definisi Lapangan tidak terbukti.

$\therefore \mathbb{R}^2$ BUKAN Lapangan. $\blacksquare$

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