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Analisis Real II / Pertemuan ke-2 / Tugas

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5.1.4 Prove the validity of the limit $\lim_{x \to x_0} x^2 = x_0^2$

Penyelesaian :

Analisis Pendahuluan :

Adb. $\forall \varepsilon > 0, \exists \delta > 0 \ni 0 < |x - x_0| < \delta \Rightarrow |x^2 - x_0^2| < \varepsilon$

Mencari nilai δ

(1) $\delta = 1$

Note that,

$$|x - x_0| < 1 \Rightarrow -1 < x - x_0 < 1$$

$$-1 + 2x_0 < x - x_0 + 2x_0 < 1 + 2x_0$$

$$-1 + 2x_0 < x + x_0 < 1 + 2x_0$$

$$|x + x_0| < 1 + 2x_0$$

Selanjutnya,

$$|x - x_0| < \delta \Rightarrow |x^2 - x_0^2| = |(x - x_0).(x + x_0)|$$

$$= |x - x_0| . |x + x_0|$$

$$< 1 . (1 + 2x_0)$$

$$< 1 + 2x_0 = \varepsilon$$

Artinya, $\delta = 1$, $\varepsilon \geq 1 + 2x_0$

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(2) $\delta = \dots ?$ (untuk $\varepsilon < 1 + 2x_0$)

Note that,

$$|x - x_0| < \delta \Rightarrow |x^2 - x_0^2| = |(x - x_0).(x + x_0)|$$
$$= |x - x_0| . |x + x_0|$$
$$< \delta . (1 + 2x_0) = \varepsilon$$

Artinya, $\delta = \varepsilon/(1 + 2x_0)$; $\varepsilon < 1 + 2x_0$

Dari (1) dan (2) diperoleh,

$$\delta = \begin{cases} 1 & , \varepsilon \geq 1 + 2x_0 \\ \varepsilon/(1 + 2x_0) & , \varepsilon < 1 + 2x_0 \end{cases}$$

Bukti Formal :

Diambil $\varepsilon > 0$ sebarang

Dipilih $\delta = \min \{ 1, \varepsilon/(1 + 2x_0) \}$

Maka $\forall 0 < |x - x_0| < \delta$ diperoleh

$$|x^2 - x_0^2| = |(x - x_0).(x + x_0)|$$
$$= |x - x_0| . |x + x_0|$$
$$\leq \delta . (1 + 2x_0) = \varepsilon$$

$\therefore \lim_{x \to x_0} x^2 = x_0^2$. (Terbukti) $\blacksquare$

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Catatan :

Misal, $x_0 = 3 \Rightarrow 1 + 2x_0 = 1 + (2 \cdot 3) = 1 + 6 = 7$

•) u/ $\varepsilon = 7 \Rightarrow \delta = 1$ (krn $\varepsilon \geq 1 + 2x_0$)

Note that,

$$
\begin{aligned}
|x^2 - x_0^2| &= |(x - x_0) \cdot (x + x_0)| \\
&= |x - x_0| \cdot |x + x_0| \\
&\leq \delta \cdot (1 + 2x_0) \\
&\leq \delta \cdot 7 \\
&\leq 1 \cdot 7 \\
&\leq 7 \qquad = \varepsilon
\end{aligned}
$$

(Memenuhi) ✓

•) u/ $\varepsilon = 10 \Rightarrow \delta = 1$ (krn $\varepsilon \geq 1 + 2x_0$)

Note that,

$$
\begin{aligned}
|x^2 - x_0^2| &= |(x - x_0) \cdot (x + x_0)| \\
&= |x - x_0| \cdot |x + x_0| \\
&\leq \delta \cdot (1 + 2x_0) \\
&\leq \delta \cdot 7 \\
&\leq 1 \cdot 7 \\
&\leq 7 \\
&\leq 10 \qquad = \varepsilon
\end{aligned}
$$

(Memenuhi) ✓

•) u/ $\varepsilon = 5 \Rightarrow \delta = \frac{\varepsilon}{1 + 2x_0} = \frac{5}{1 + (2 \cdot 3)} = \frac{5}{1 + 6} = \frac{5}{7}$ (krn $\varepsilon < 1 + 2x_0$)

Note that,

$$
\begin{aligned}
|x^2 - x_0^2| &= |(x - x_0) \cdot (x + x_0)| \\
&= |x - x_0| \cdot |x + x_0| \\
&\leq \delta \cdot (1 + 2x_0) \\
&\leq \delta \cdot 7 \\
&\leq \frac{5}{7} \cdot 7 \\
&\leq 5 \qquad = \varepsilon
\end{aligned}
$$

(Memenuhi) ✓

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5-1-3 Prove the existence of the limit of $\lim_{x \to -4} x^2$

Penyelesaian :

Analisis Pendahuluan :

Adb $\forall \epsilon > 0, \exists \delta > 0 \ni 0 < |x-(-4)| < \delta \Rightarrow |x^2-(-4)^2| < \epsilon$

$\forall \epsilon > 0, \exists \delta > 0 \ni 0 < |x+4| < \delta \Rightarrow |x^2-16| < \epsilon$

Mencari nilai δ:

(1) $\delta = 1$

Note that,

$|x+4| < 1 \Rightarrow -1 < x+4 < 1$

$-1-8 < x+4-8 < 1-8$

$-9 < x-4 < -7$

$|x-4| < 9$ [Pilih yg besar 0]

Selanjutnya,

$|x+4| < \delta \Rightarrow |x^2-16| = |(x+4)(x-4)|$

$= |x+4| \cdot |x-4|$

$< 1 \cdot (9)$

$< 9 = \epsilon$

Artinya, $\delta = 1, \epsilon \geq 9$

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(2) $\delta = \ldots ?$ (untuk $\varepsilon < 9$)

Note that,

$$|x+4| < \delta \Rightarrow |x^2-16| = |(x+4)(x-4)|$$
$$= |x+4| \cdot |x-4|$$
$$< \delta \cdot (9) = \varepsilon$$

Artinya, $\delta = \varepsilon/9$, $\varepsilon < 9$

Dari (1) dan (2) diperoleh,

$$\delta = \begin{cases} 1 & , \varepsilon \geq 9 \\ \varepsilon/9 & , \varepsilon < 9 \end{cases}$$

Bukti Formal :

Diambil $\varepsilon > 0$

Dipilih $\delta = \min\{1, \varepsilon/9\}$

Maka u/ $0 < |x+4| < \delta$ diperoleh

$$|x^2-16| = |(x+4)\cdot(x-4)|$$
$$= |x+4| \cdot |x-4|$$
$$\leq \delta \cdot (9) = \varepsilon$$

$\therefore \lim_{x \to -4} x^2$ ada

Dipindai dengan CamScanner

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Catatan :

Misal, $x = -4$

- $\forall \varepsilon = 9 \Rightarrow \delta = 1$ (krn $\varepsilon \geq 9$)

  Note that,

  $$|x^2 - 16| = |(x+4).(x-4)|$$
  $$= |x+4| . |x-4|$$
  $$\leq \delta \quad . (9)$$
  $$\leq 1 \quad . 9$$
  $$\leq 9 \qquad = \varepsilon \quad \text{(Memenuhi)} \checkmark$$

- $\forall \varepsilon = 11 \Rightarrow \delta = 1$ (krn $\varepsilon \geq 9$)

  Note that,

  $$|x^2 - 16| = |(x+4).(x-4)|$$
  $$= |x+4| . |x-4|$$
  $$\leq \delta \quad . 9$$
  $$\leq 1 \quad . 9$$
  $$\leq 9$$
  $$< 11 \qquad = \varepsilon \quad \text{(Memenuhi)} \checkmark$$

- $\forall \varepsilon = 5 \Rightarrow \delta = \frac{\varepsilon}{9} = \frac{5}{9}$ (krn $\varepsilon < 9$)

  Note that,

  $$|x^2 - 16| = |(x+4).(x-4)|$$
  $$= |x+4| . |x-4|$$
  $$\leq \delta \quad . 9$$
  $$\leq \frac{5}{9} . 9$$
  $$\leq 5 \qquad = \varepsilon \quad \text{(Memenuhi)} \checkmark$$

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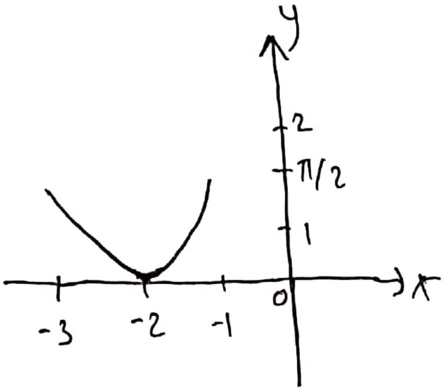
5.1.7 Which values of x_0 would be excluded from consideration in limit

$$\lim_{x \to x_0} \arcsin |x+2| \; ?$$

Penyelesaian



Grafik dari $y = \sin^{-1}|x+2|$ memiliki daerah asal $[-3,-1]$ sehingga $\lim_{x \to x_0} \sin^{-1}|x+2|$ ada jika $x_0 \in [-3,-1]$ maka dari itu nilai x_0 yang harus dikecualikan agar $\lim_{x \to x_0} \sin^{-1}|x+2|$ ada adalah $x_0 < -3$ atau $x_0 > -1$.

③ Misalkan $f: R \to R$ didefinisikan dengan

$$f(x) = \begin{cases} x, & x \text{ rasional} \\ 0, & x \text{ irrasional} \end{cases}$$

a.) Tunjukkan f mempunyai limit di $x = 0$

b.) Gunakan kriteria barisan untuk menunjukkan jika $c \neq 0$ maka f tidak mempunyai limit di c.

Jawab:

a.) Tunjukkan f mempunyai limit di $x = 0$

MIS : $\lim_{x \to 0} f(x) = 0$

Analisis Pendahuluan :

Adb. $\forall \varepsilon > 0, \exists \delta > 0 \ni 0 < |x - 0| < \delta \Rightarrow |f(x) - 0| < \varepsilon$

$\forall \varepsilon > 0, \exists \delta > 0 \ni 0 < |x| < \delta \Rightarrow |f(x)| < \varepsilon$

Note that

$$|x - 0| < \delta \Rightarrow |f(x) - 0| = |f(x)| = |x| < \delta = \varepsilon$$

Bukti Formal :

Diambil $\varepsilon > 0$

Dipilih $\delta = \varepsilon$

Maka untuk $0 < |x - 0| < \delta$ diperoleh

$$|f(x) - 0| = |f(x)| = |x| < \delta = \varepsilon$$

$\therefore$ Karena $\forall \varepsilon > 0, \exists \delta > 0 \ni 0 < |x - 0| < \delta \Rightarrow |f(x) - 0| < \varepsilon$ terbukti, maka $\lim_{x \to 0} f(x)$ ada.

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b.) Gunakan kriteria barisan untuk menunjukkan jika $c \neq 0$ maka f tidak mempunyai limit di c.

Misal : $L \in \mathbb{R}$, limit dari F pada $c \neq 0$

Pilih, $\varepsilon_0 = \frac{1}{2} \cdot |L|$ sehingga, untuk setiap $\delta > 0$

terdapat $x \in \mathbb{R} \setminus \{\mathbb{Q}\}$ sehingga $0 < |x - c| < \delta$, $x \neq c$.
maka : $|f(x) - L| < \varepsilon$.

Karena, $x \in \mathbb{R} \setminus \{\mathbb{Q}\}$ maka $f(x) = 0$ sehingga
$$|f(x) - L| = |0 - L| = L > \frac{1}{2}|L| = \varepsilon_0$$

$\therefore$ Dari hal ini jika $c \neq 0$ maka f tidak memiliki limit di c.