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Imanuel

Tentukan hasil transformasi jika diketahui f(t) seperti berikut.

① $f(t) = 5$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{5}{S}$

② $f(t) = t^3$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{3!}{S^{3+1}} = \frac{6}{S^4}$

③ $f(t) = \sin 6t$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{6}{6^2 + S^2} = \frac{6}{36 + S^2}$

④ $f(t) = e^{-2t}$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{1}{S-(-2)} = \frac{1}{S+2}$

⑤ $f(t) = e^{5t}$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{1}{S-5}$

⑥ $f(t) = \sinh 2t$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{2}{S^2 - 2^2} = \frac{2}{S^2 - 4}$

⑦ $f(t) = \cosh 4t$

$\Rightarrow$ Hasil transformasi $F(S) = \frac{S}{S^2 - 4^2} = \frac{S}{S^2 - 16}$

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Sifat Transformasi Laplace – Invers

(1.) $\mathcal{L}^{-1}\left\{\frac{2}{4+s^2}\right\} = \sin 2t$

(2) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+6s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+1}\right\}$

= Dilain pihak, kita ketahui bahwa

ketika $f(t) = e^{-3t} \cdot \sin t$

diperoleh Sifat Transformasi Laplace – Translasi :

$$\mathcal{L}\{e^{-3t} \cdot \sin t\} = F(s-(-3))$$
$$= F(s+3)$$

$$\mathcal{L}\{\sin at\} = \frac{a}{a^2+s^2}$$

$$\mathcal{L}\{\sin t\} = \frac{1}{1+s^2}$$

$$\mathcal{L}\{e^{-3t} \cdot \sin t\} = F(s+3)$$
$$= \frac{1}{1+(s+3)^2}$$

Maka,

$= e^{-3t} \cdot \sin t$ //

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③ $\mathcal{L}^{-1}\left\{\frac{2}{s-2} - \frac{3}{s^2+5}\right\} = \mathcal{L}^{-1}\left\{2 \cdot \frac{1}{s-2} - 3 \cdot \frac{1}{s^2+5}\right\}$

$= 2 \cdot \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 3 \cdot \mathcal{L}^{-1}\left\{\frac{1}{s^2+5}\right\}$

$= 2 \cdot e^{2t} - \frac{3}{\sqrt{5}} \mathcal{L}^{-1}\left\{\frac{\sqrt{5}}{s^2+(\sqrt{5})^2}\right\}$

$= 2 \cdot e^{2t} - \frac{3}{\sqrt{5}} \cdot \sin \sqrt{5} \cdot t$ //

④ $\mathcal{L}^{-1}\left\{\frac{3s+16}{s^2-s-6}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{s-3} + \frac{-2}{s+2}\right\}$

$= \mathcal{L}^{-1}\left\{5 \cdot \frac{1}{s-3} + (-2) \cdot \frac{1}{s+2}\right\}$

$= 5 \cdot \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} - 2 \cdot \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$

$= 5 \cdot e^{3t} - 2 \cdot e^{-2t}$ //

Pecahan Parsial:

$\frac{3s+16}{s^2-s-6} = \frac{A}{s-3} + \frac{B}{s+2}$

$= \frac{A(s+2) + B(s-3)}{(s-3)(s+2)}$

$= \frac{As + 2A + Bs - 3B}{(s-3)(s+2)}$

$= \frac{(A+B)s + (2A-3B)}{(s-3)(s+2)}$

$A+B = 3 \quad |\times 2|$

$2A - 3B = 16 \quad |\times 1|$

$2A + 2B = 6$

$2A - 3B = 16$ −

$5B = -10$

$\boxed{B = -2}$

$2A - 3B = 16$

$2A - 3(-2) = 16$

$2A + 6 = 16$

$2A = 10$

$\boxed{A = 5}$

$\therefore \frac{3s+16}{s^2-s-6} = \frac{5}{s-3} + \frac{-2}{s+2}$

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(5) $\mathcal{L}^{-1}\left\{\frac{s^2-15s+41}{(s+2)(s-3)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{3}{(s+2)} - \frac{2}{(s-3)} + \frac{1}{(s-3)^2}\right\}$

$= \mathcal{L}^{-1}\left\{3\cdot\frac{1}{(s+2)} - 2\cdot\frac{1}{(s-3)} + \frac{1}{(s-3)^2}\right\}$

$= 3\cdot\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - 2\cdot\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\}$

$= 3\cdot(e^{-2t}) - 2\cdot(e^{3t}) + (e^{3t}\cdot t)$ //

• $\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t}$ [Jelas]

• $\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} = e^{3t}$ [Jelas]

• $\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2}\right\} = \dots ?$

$\Rightarrow \mathcal{L}^{-1}\{e^{3t}\cdot t\} = F(s-3)$

$\mathcal{L}^{-1}\{t^n\} = \frac{n!}{s^{n+1}}$

$\mathcal{L}^{-1}\{t\} = \frac{1!}{s^{1+1}} = \frac{1}{s^2}$

$\therefore \mathcal{L}^{-1}\{e^{3t}\cdot t\} = F(s-3)$

$= \frac{1}{(s-3)^2}$ //

• Pecahan parsial

$\frac{s^2-15s+41}{(s+2)(s-3)^2} = \frac{A}{(s+2)} + \frac{B}{(s-3)} + \frac{C}{(s-3)^2}$

$= \frac{A(s-3)^2 + B(s+2)(s-3) + C(s+2)}{(s+2)(s-3)^2}$

$= \frac{A(s^2-6s+9) + B(s^2-s-6) + C(s+2)}{(s+2)(s-3)^2}$

$\frac{s^2-15s+41}{(s+2)(s-3)^2} = \frac{As^2-6As+9A+Bs^2-Bs-6B+Cs+2C}{(s+2)(s-3)^2}$

$s^2-15s+41 = (A+B)s^2 - (6A+B-C)s + (9A-6B+2C)$

Maka,

$$\left.\begin{aligned} A+B &= 1 \\ 6A+B-C &= 15 \\ 9A-6B+2C &= 41 \end{aligned}\right\} \begin{aligned} A &= 3 \\ B &= -2 \\ C &= 1 \end{aligned}$$

$\Rightarrow \therefore \frac{s^2-15s+41}{(s+2)(s-3)^2} = \frac{A}{(s+2)} + \frac{B}{(s-3)} + \frac{C}{(s-3)^2}$

$= \frac{3}{(s+2)} + \frac{-2}{(s-3)} + \frac{1}{(s-3)^2}$

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6. $$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = \mathcal{L}^{-1}\left\{-\frac{s}{s^2+1}+\frac{1}{s}\right\}$$
$$= -\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$
$$= -\cos t + 1$$

Maaf... spasinya kejauhan ^o^'' he he he...

∘) Pecahan parsial

$$\frac{1}{s(s^2+1)} = \frac{As+B}{(s^2+1)} + \frac{C}{s}$$
$$= \frac{(As+B)(s)+C(s^2+1)}{(s^2+1)\cdot s}$$
$$\frac{1}{s(s^2+1)} = \frac{As^2+Bs+Cs^2+C}{(s^2+1)\cdot s}$$
$$1 = (A+C)s^2+Bs+C$$

Maka

$$\left.\begin{array}{l} A+C=0 \\ B=0 \\ C=1 \end{array}\right\} \begin{array}{l} A=-1 \\ B=0 \\ C=1 \end{array}$$

$$\Rightarrow \frac{1}{s(s^2+1)} = \frac{As+B}{(s^2+1)} + \frac{C}{s}$$
$$= \frac{-s+0}{s^2+1} + \frac{1}{s}$$
$$= \frac{-s}{s^2+1} + \frac{1}{s}$$

Transformasi Laplace pada Turunan

contoh soal (5):

$\mathcal{L}\{t^2\} = \ldots$

$f(t) = t^2$

$f'(t) = 2t$

$f''(t) = 2$

$$\mathcal{L}\{f''(t)\} = s^2 . F(s) - s^{2-1} . f(0) - s^{2-2} f'(0)$$
$$\mathcal{L}\{2\} = s^2 . \mathcal{L}\{f(t)\} - s^1 . f(0) - s^0 . f'(0)$$
$$\mathcal{L}\{2\} = s^2 . \mathcal{L}\{f(t)\} - s . f(0) - f'(0)$$
$$\mathcal{L}\{2\} = s^2 . \mathcal{L}\{t^2\} - s . 0 - 0$$
$$\mathcal{L}\{2\} = s^2 . \mathcal{L}\{t^2\} - 0 - 0$$
$$s^2 . \mathcal{L}\{t^2\} = \mathcal{L}\{2\}$$
$$s^2 . \mathcal{L}\{t^2\} = \frac{2}{s}$$
$$\mathcal{L}\{t^2\} = \frac{2}{s^3} \;//$$

Latihan

contoh soal (6):

$\mathcal{L}\{t^2 . e^{4t}\} =$

Penyelesaian:

$$\mathcal{L}\{t^2 . e^{4t}\} = (-1)^2 . \frac{d^2}{ds^2} . \mathcal{L}\{e^{4t}\}$$
$$= \frac{d^2}{ds^2}\left(\frac{1}{s-4}\right)$$
$$= \frac{d}{ds}\left(\frac{(s-4)(0) - (1).(1)}{(s-4)^2}\right)$$
$$= \frac{d}{ds}\left(\frac{-1}{(s-4)^2}\right)$$
$$= \frac{(s-4)^2(0) - (-1).2(s-4).(1)}{((s-4)^2)^2}$$
$$= \frac{2(s-4)}{(s-4)^4}$$
$$= \frac{2}{(s-4)^3}$$

Contoh soal (7):

$\mathcal{L}\{t . \sin 2t\} =$

Penyelesaian:

$$\mathcal{L}\{t . \sin 2t\} = (-1)^1 . \frac{d}{ds} . \mathcal{L}\{\sin 2t\}$$
$$= -\frac{d}{ds} . \left(\frac{2}{4+s^2}\right)$$
$$= -\frac{(4+s^2).0 - 2.2s}{(4+s^2)^2}$$
$$= \frac{4s}{(4+s^2)^2} \;//$$