Statur Aljaba II / Pertenen III / Ring /PR

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Structur Aljabar II

PR

(1) Buktikan secara lengkap, himpunan $|M_2UR| = \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a_1b_1 c_1d \neq |R|^2$ merupakan Ring.

Penyelevalon:

Akan ditungukkan: 1) M₂(R) f \$\phi\$
2) (M₂(R), +) Grup AbeL
3) (M₂(R), x) Semigrup
4) (M₂(R), +, x) Distribution

Note that, $f(R) \neq \emptyset$ sebab $f(R) \neq \emptyset$

Tutup, $\forall a,b \in M_2(IR) \Rightarrow a+b \in M_2(IR)$ Ambil sebarang $a,b \in M_2(IR)$ Tulis $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1,b_1,c_1,d_1 \in IR$ $b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2,b_2,c_2,d_2 \in IR$ Note that, $a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ $= \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix} \in M_2(IR)$

* Assosiatiffer ,
$$\forall a_1b_1, c \in M_2(\mathbb{R}) \Rightarrow (a+b)+c = a+(b+c)$$

Ambil scharang $a_1b_1, c \in M_2(\mathbb{R})$

Titu, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$
 $b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_1 \end{pmatrix}$, $a_2, b_2, c_2, d_1 \in \mathbb{R}$
 $c = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$, $a_3, b_3, c_3, d_3 \in \mathbb{R}$

Note that,
 $(a+b)+c = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$
 $= \begin{pmatrix} (a_1+a_1)+a_3 & (b_1+b_2)+b_3 \\ (c_1+c_2)+c_3 & (d_1+d_1)+d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1+a_2+a_3 & b_1+(b_2+b_3) \\ c_1+(c_2+c_3) & d_1+(d_2+d_3) \end{pmatrix}$
 $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$
 $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$

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> Fidentitas, yaitu ta f M2(R), I b f M2(R) + atb = b+a = a

Note that,

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} a_1+0 & b_1+0 \\ c_1+0 & d_1+0 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$b+a=\begin{pmatrix}0&0\\0&0\end{pmatrix}+\begin{pmatrix}a_1&b_1\\c_1&d_1\end{pmatrix}$$

$$= \begin{pmatrix} 0 + \alpha_1 & 0 + b_1 \\ 0 + c_1 & 0 + d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & ol_1 \end{pmatrix}$$

Karena (*)=(**) => a+b=b+a=a, make adanya identity terbukti.......

-> Finun, yaito + a + M2(R), Fb + M2(R) + atb= bta = 0=(00)

Ambil sebarang a fM2UR)

Pilih b =
$$\begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} \in M_2(\mathbb{R})$$
 , $-a_1$, $-b_1$, $-c_1$, $-d_1 \in \mathbb{R}$

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + (-a_1) & b_1 + (-b_1) \\ c_1 + (-c_1) & d_1 + (-d_1) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Note that
$$r$$

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + (-a_1) & b_1 + (-b_1) \\ & & & \end{pmatrix}$$

$$= \begin{pmatrix} -a_1 + a_1 & -b_1 + b_1 \\ & & & \end{pmatrix}$$

$$= \begin{pmatrix} -a_1 + a_1 & -b_1 + b_1 \\ -c_1 + d_1 & -d_1 + d_1 \end{pmatrix}$$

.) Fountatif, + q,5 + M2(R) => (a+b)=(b+a)

Aubil sebarang $a_1b \in M_2(IR)$ Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in IR$ $b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2 \cdot b_2, c_1 \cdot d_2 \in R$

Nok that,

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 + a_1 & b_2 + b_1 \\ c_2 + c_1 & d_2 + d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

h+a

.. (M2(IF), +) adalah grup abelian.

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3.) Akan diturjukkan: (M2(R), X) memenhi sipat.

tutup dan assosiatif

Tutup, $\forall a_1b \in M_2(\mathbb{R}) \Rightarrow a \times b \in M_1(\mathbb{R})$ And sebarang, $a_1b \in M_2(\mathbb{R})$ Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1 \cdot b_1$, c_1 , $d_1 \in \mathbb{R}$ $b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2 \cdot b_2$, $(a_2, d_2 \in \mathbb{R})$

Note that $a \times b = \begin{pmatrix} a_1 & b_2 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} a_1 & b_2 \\ c_2 & d_2 \end{pmatrix}$ $= \begin{pmatrix} (a_1 \cdot a_2) + (b_1 \cdot c_2) & (a_1 \cdot b_2) + (b_1 \cdot d_2) \\ (c_1 \cdot a_2) + (d_1 \cdot d_2) & (c_1 \cdot b_2) + (c_2 \cdot b_2) \end{pmatrix} \leftarrow (M_2(R)$

Karena a, bi, Ci, di, az, bz, cz, dz ElR maka berlaku SiFat tutup pada perkalian dan penjualahan bilangan Real, Sedemikan sehingga axb (Mz (IR)

mayser, 3 September reed Emanuel AS/1811/41008 > Associatif, Harb, C & M2(IR) => ax(bxc) = (axb) x C Ambil sebaranoj urb, (E Ma (R) Tus, a = (a, b), a, b, (1, d) + R b = (a2 b2) | a2, b2, (2, d2 + 1) c = (a3 b3), a3, b3, (3, d3+1) Note that $a \times (b \times c) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{bmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$ $= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} (a_2.a_3) + (b_2.c_3) & (a_2.b_3) + (b_2.d_3) \\ (.c_2.a_3) + (d_2.c_3) & (.c_2.b_3) + (d_2.d_3) \end{pmatrix}$ ai.az.b3+ ai.bz.d3 + bi.c2.b3+ bi.dz.d3

Ci.az.b3+ ci.bz.d3+ di.cz.b3+di.dz.d3 = (a1.92.03 + a1.62.63 + b1.62.03 + b1.62.03

- (C1.92.93 + C1.62.03 + d1.62.03 + d1.d2.03 $(a \times b) \times C = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \times \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \times \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$ $= \begin{pmatrix} (a_1.a_2) + (b_1.c_2) & (a_1.b_2) + (b_1.d_2) \\ (c_1.a_2) + (d_1.c_2) & (c_1.b_2) + (d_1.d_2) \end{pmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & cl_3 \end{pmatrix}$ $\left\{ \left[(a_1.a_2) + (b_1.c_2) \right] \cdot a_3 + \left[(a_1.b_2) + (b_1.d_2) \right] \cdot c_3 \right.$ $[(a_1.a_2)+(b_1.c_2)].b_3+[(a_1.b_2)+(b_1.d_2)]d_3$ $[(c_1,a_2)+(d_1,c_2)]\cdot a_3+[(c_1,b_2)+(d_1,d_2)]\cdot c_3$ [(C1.92)+(d1.62)]. b3+ [(C1.62)+(d1.d2)].d3/

 $= \begin{pmatrix} a_3 \cdot a_1 \cdot a_2 + a_3 \cdot b_1 \cdot (2 + c_3 \cdot a_1 \cdot b_2 + c_3 \cdot b_1 \cdot d_2 \\ a_3 \cdot c_1 \cdot a_2 + a_3 \cdot d_1 \cdot c_2 + c_3 \cdot t_1 \cdot b_2 + c_3 \cdot d_1 \cdot d_2 \end{pmatrix}$ $= \begin{pmatrix} a_1 \cdot a_2 \cdot a_3 + a_1 \cdot b_2 \cdot c_3 + b_1 \cdot c_2 \cdot a_3 + d_1 \cdot d_2 \cdot c_3 \\ c_1 \cdot a_2 \cdot a_3 + c_1 \cdot b_2 \cdot c_3 + d_1 \cdot c_2 \cdot a_3 + d_1 \cdot d_2 \cdot c_3 \end{pmatrix}$

b3.a1.a2+ b3.b1.62+ d3.a1.b2+d3.b1.d2 b3. C1. a2 + b3. d1. c2 + d3. C1. b2 + d3. d1. d2 91.92.b3 + 91.52.d3 + 61.62.d3 + 51.d2.d3 a-d2-b3 + a-b2 d3 + d, 62-b3+d1.d2.d3

Karena (+) = (**) =) ax(bxc) = (axb)x(, maka signt a))oxiatip perkulian terbolds. In

. (M2(IR), X) adalah senigrup.

Penyelisaran:

Ambl sebarang a, b, c
$$\in$$
 M2UR)

Tdo, $\alpha = \begin{pmatrix} \alpha_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $\alpha_1, b_1, c_1, d_1 \in$ R

$$b = \begin{pmatrix} \alpha_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$
, $\alpha_2, b_2, c_2, d_2 \in$ R

$$c = \begin{pmatrix} \alpha_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$
, $\alpha_3, b_3, c_3, d_3 \in$ R

Note that,

$$a \times (b+c) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{bmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} \end{bmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} a_2 + a_3 & b_2 + b_3 \\ c_2 + c_3 & d_2 + d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 \cdot (a_2 + a_3) + b_1 (c_2 + c_3) & a_1 \cdot (b_2 + c_3) \\ c_1 \cdot (a_2 + a_3) + d_1 (c_2 + c_3) & c_1 \cdot b_2 + c_3 \\ c_1 \cdot a_2 + a_1 \cdot a_3 + b_1 \cdot c_2 + b_1 \cdot c_3 & c_1 \cdot b_2 + c_3 \\ c_1 \cdot a_2 + c_1 \cdot a_3 + d_1 \cdot c_2 + d_1 \cdot c_3 & c_1 \cdot b_2 + c_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_{1} \cdot (a_{2}+a_{3}) + b_{1} \cdot (b_{2}+c_{3}) & a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot (a_{2}+a_{3}) + d_{1} \cdot (c_{2}+c_{3}) & c_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot (d_{2}+d_{3}) \end{pmatrix}$$

$$= \begin{pmatrix} a_{1} \cdot a_{2} + a_{1} \cdot a_{3} + b_{1} \cdot c_{2} + b_{1} \cdot c_{3} \\ c_{1} \cdot a_{2} + a_{1} \cdot a_{3} + b_{1} \cdot c_{2} + b_{1} \cdot c_{3} \end{pmatrix}$$

$$= \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot (d_{2}+d_{3}) \end{pmatrix}$$

$$= \begin{pmatrix} a_{1} \cdot (a_{2}+a_{3}) + d_{1} \cdot (a_{2}+d_{3}) \\ c_{1} \cdot b_{2} + a_{1} \cdot a_{3} + b_{1} \cdot c_{2} + b_{1} \cdot c_{3} \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + a_{1} \cdot a_{3} + b_{1} \cdot c_{2} + b_{1} \cdot c_{3} \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + b_{1} \cdot (d_{2}+d_{3}) \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \\ c_{1} \cdot b_{2} + c_{1} \cdot b_{3} + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \\ c_{1} \cdot b_{2} + d_{1} \cdot d_{2} + d_{1} \cdot d_{2} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot d_{2} + d_{1} \cdot d_{3} \\ c_{1} \cdot b_{2} + d_{1} \cdot d_{2} + d_{1} \cdot d_{2} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot d_{2} + d_{1} \cdot d_{2} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot d_{2} + d_{1} \cdot d_{2} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{1} \cdot d_{2} + d_{1} \cdot d_{2} \end{pmatrix} \dots \begin{pmatrix} a_{1} \cdot (b_{2}+b_{3}) + d_{2}$$

$$ab + ac = \begin{bmatrix} (a_1 & b_1) & (a_2 & b_2) \\ (c_1 & d_1) & (c_2 & d_2) \end{bmatrix} + \begin{bmatrix} (a_1 & b_1) & (a_3 & b_3) \\ (c_1 & d_1) & (c_3 & d_3) \end{bmatrix}$$

$$= \begin{pmatrix} (a_1 \cdot a_2) + (b_1 \cdot c_2) & (a_1 \cdot b_2) + (b_1 \cdot d_2) \\ (c_1 \cdot a_2) + (d_1 \cdot c_2) & (c_1 \cdot b_2) + (d_1 \cdot d_2) \end{pmatrix} + \begin{pmatrix} (a_1 \cdot a_3) + (b_1 \cdot c_3) & (a_1 \cdot b_3) + (b_1 \cdot d_3) \\ (c_1 \cdot a_2) + (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot c_3) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1 \cdot b_1) \cdot (a_2 \cdot b_2) + (d_1 \cdot d_2) \\ (c_1 \cdot a_2) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot d_3) \\ (c_1 \cdot a_2 \cdot d_1 \cdot c_2) & (c_1 \cdot b_3 \cdot d_1 \cdot d_3) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1 \cdot b_1) \cdot (a_2 \cdot b_2) + (b_1 \cdot d_2) \\ (c_1 \cdot a_2) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot d_3) \\ (c_1 \cdot a_2 \cdot d_1 \cdot c_2) & (c_1 \cdot b_3 \cdot d_1 \cdot d_3) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1 \cdot b_1) \cdot (a_2 \cdot b_2) + (b_1 \cdot d_2) \\ (c_1 \cdot a_2) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot d_3) \\ (c_1 \cdot a_2) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot d_3) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1 \cdot b_2) \cdot (a_1 \cdot b_2) + (a_1 \cdot d_2) \\ (c_1 \cdot a_3) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot d_3) \\ (c_1 \cdot a_2) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) \cdot (d_1 \cdot d_3) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1 \cdot b_2) \cdot (a_1 \cdot b_2) + (a_1 \cdot d_2) \\ (c_1 \cdot a_3) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) \cdot (d_1 \cdot d_3) \\ (c_1 \cdot a_3) \cdot (d_1 \cdot c_3) & (c_1 \cdot b_3) \cdot (d_1 \cdot d_3) \end{pmatrix}$$

Note that, $(a+b) \times (= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$ $= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$ $= \begin{pmatrix} [a_1 + a_2) \cdot a_3] + [b_1 + b_1) \cdot c_3 \end{bmatrix} \quad \begin{bmatrix} (a_1 + a_2) \cdot b_3 \end{bmatrix} + \begin{bmatrix} b_1 + b_2 \cdot d_3 \end{bmatrix}$ $= \begin{pmatrix} [c_1 + c_2) \cdot a_3] + [d_1 + d_2) \cdot c_3 \end{bmatrix} \quad \begin{bmatrix} (c_1 + c_2) \cdot b_3 \end{bmatrix} + \begin{bmatrix} (d_1 + d_2) \cdot d_3 \end{bmatrix}$ $= \begin{pmatrix} a_1 \cdot a_3 + a_2 \cdot a_3 + b_1 \cdot c_3 + b_2 \cdot c_3 \\ c_1 \cdot a_3 + c_2 \cdot a_3 + d_1 \cdot c_3 + d_2 \cdot c_3 \end{pmatrix} \cdot a_1 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_2 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 \cdot c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 \cdot c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \cdot a_1 \cdot b_2 \cdot c_3 \cdot b_3 \cdot c_2 \cdot b_3 \cdot d_3 \cdot c_3 \cdot d_3 \cdot d_3 \cdot c_3 \cdot d_3 \cdot$

$$ac + bc = \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} \right] + \left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \cdot \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix} \right]$$

$$= \begin{pmatrix} (a_1 \cdot a_3) + (b_1 \cdot c_3) & (a_1 \cdot b_3) + (b_1 \cdot d_3) \\ (c_1 \cdot a_3) + (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 \cdot d_3) \end{pmatrix} + \begin{pmatrix} (a_2 \cdot a_3) + (b_2 \cdot c_3) & (a_2 \cdot b_3) + (b_2 \cdot d_3) \\ (c_2 \cdot a_3) + (d_2 \cdot c_3) & (c_2 \cdot b_3) + (d_2 \cdot d_3) \end{pmatrix}$$

$$= \begin{pmatrix} a_1 \cdot a_3 + a_2 \cdot a_3 + b_1 \cdot c_3 + b_2 \cdot c_3 & a_1 \cdot b_3 + a_2 \cdot b_3 + b_1 \cdot d_3 + b_2 \cdot d_3 \\ c_1 \cdot a_3 + c_2 \cdot a_3 + d_1 \cdot c_3 + d_2 \cdot c_3 & c_1 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} ... \{b_4\}$$

. (M2(IR),+,x) adalch Brytributy.

* Karenn M2(R) & P , (M2(IR),+) Grup abelran, (M2(IR),X) Semigrap, (M2(IR),+,X) Distiribution, maken (M2(IR),+,X) adalah Ring.

Terbolaj) 🌃

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1 Misalkan R King dan menanh; a2 = a \ a \ R. Buktilon behing k ring tometelf.

Penyelesain:

And a, b & R sebaran Akan ditunjukkan: ab = ba

(i) Karena R Ting => atb ER Berdantan yag diketchi, makadipenleh:

[Definisi |

(a+b)(a+b) = a+b

(atb) a + (atb) b = a +b

[Distributif KIH]

 $a^{2}+b$ $a+ab+b^{2}=a+b$

[Dotabutif Kanan] [Detining

&+ batab+5 = A+8

 $b_{a} + ab = 0$ $\int ab = -ba$

(ii) Atan ditensullern: -ba = ba

Faren Fring => ba ER , + b, a ER + (ba+kia) ER Berdsorton you diketahui, make dipooleh:

(bat ba)2 = ba + ba

[Refind]

(batba) (batba) = ba + ba

(batba) ba + (batba) ba = ba + ba

[outribute Em]

(ba)2+(ba)2+(ba)2 = ba + ba

[00 tributife Icanan]

36 + ba + bn + bn = bn + ba

[pepihioi]

ba - 0
| ba - - ba

io Karenti(i) don(ii) dipodeh ab = -ba = ba , maka R reg foundatife. (Terroulth)