Makassar, 4 Juli 2021

Analisis Kompleks

Pertemuan ke - 15

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Analisis Kompleks / Pertemuan ke-15 / Catatan

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Kekontinuan Fungsi Kompleks

Definisi:

Misalkan z_0 titik limit dari D_f

$f : D_f \to \mathbb{C}$ kontinu di $z_0 \in D_f$

Jika $\boxed{\lim_{z \to z_0} f(z) = f(z_0)}$

$\downarrow$

(1) $\lim_{z \to z_0} f(z)$ ada

(2) $f(z_0)$ ada

(3) $\lim_{z \to z_0} f(z) = f(z_0)$

$\lim_{z \to z_0} f(z) = f(z_0) \iff \forall \varepsilon > 0 \ \exists \delta > 0$ sehingga

jika $|z - z_0| < \delta$ maka

$|f(z) - f(z_0)| < \varepsilon$

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Sifat - sifatnya:

(1) Jika f dan g kontinu di $z_0 \in D_f \cap D_g$ maka $f+g$, $f-g$, αf (α konstanta kompleks), $f \cdot g$, $\frac{f}{g}$ $(g(z_0) \neq 0)$ kontinu di z_0.

(2) Jika f kontinu di z_0 dan g kontinu $f(z_0)$ maka $g \circ f$ kontinu di z_0.

(3) Jika $z = x + yi$, $f(z) = u(x,y) + v(x,y)i$ dan $z_0 = x_0 + y_0 i$ maka f kontinu di z_0 jika dan hanya jika $u = u(x,y)$ dan $v = v(x,y)$ keduanya kontinu di (x_0, y_0).

(4) Jika $z = r \cdot cis(\theta)$, $f(z) = u(r,\theta) + v(r,\theta)i$ dan $z_0 = r_0 \cdot cis(\theta_0)$ maka f kontinu di z_0 jika dan hanya jika $u = u(r,\theta)$ dan $v = v(r,\theta)$ kontinu di (r_0, θ_0).

(5.) Jika f kontinu di z_0 dan $f(z_0) \neq 0$ maka $\exists r > 0$ sehingga $|f(z)| > 0$ pada $V_r(z_0)$.

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f kontinu di z_0 jika $\boxed{\lim_{z \to z_0} f(z) = f(z_0)}$

$\Updownarrow$

(1) $\lim_{z \to z_0} f(z)$ = ada

(2) $f(z_0)$ ada

(3) $\lim_{z \to z_0} f(z) = f(z_0)$

E (1) Buktikan bahwa $f(z) = z^3$ kontinu di $z_0 = i$

Solusi :

Adb. $\lim_{z \to z_0} f(z) = f(z_0)$

Perhatikan bahwa

(1) $\lim_{z \to z_0} f(z) = \lim_{z \to i} z^3 = (i)^3 = -i$

(2) $f(z_0) = f(i) = (i)^3 = -i$

Dari (1) dan (2) diperoleh

$\lim_{z \to z_0} f(z) = f(z_0)$.

Jadi, $f(z) = z^3$ kontinu di $z_0 = i$.

(Kalau tidak kontinu, berarti dia tidak memenuhi salah satu dari ketiga syarat di atas).

(2) Misal

$$f(z) = \begin{cases} z^2 & ; z \neq i \\ 0 & ; z = i \end{cases}$$

Tidak kontinu di $z = i$, sebab

$\lim_{z \to i} f(z) = \lim_{z \to i} z^2 = (i)^2 = -1$.

Di lain pihak $f(i) = 0$

Jadi, $\lim_{z \to i} f(z) \neq f(i)$

$\therefore f(z)$ tidak kontinu.

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Turunan Fungsi Kompleks

Untuk fungsi $w = f(z)$ terdefinisi pada $V_\varepsilon(z_0)$.
Turunan f di z_0 ditulis $f'(z_0)$ dituliskan sebagai

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

Asalkan Limit ini ada.
Fungsi $w = f(z)$ disebut terdiferensialkan di z_0 jika $f'(z_0)$ ada.
Substitusi $\Delta z = z - z_0$ diperoleh

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z}$$

Apa hubungan antara kekontinuan dan turunan pada fungsi kompleks?
Jika dia punya turunan berarti dia kontinu.

Teorema :

Jika $w = f(z)$ punya turunan di z_0 maka f kontinu di z_0.

Bukti: $\lim_{z \to z_0} f(z) = f(z_0)$

Note that, $f(z) = \frac{f(z) - f(z_0)}{z - z_0} \cdot (z - z_0) + f(z_0)$

$$\lim_{z \to z_0} f(z) = \lim_{z \to z_0} \left(\frac{f(z) - f(z_0)}{z - z_0} \cdot (z - z_0) \right) + f(z_0)$$

$$= \left(\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \right) \cdot \lim_{z \to z_0} (z - z_0) + f(z_0)$$

$$= f'(z_0) \cdot 0 + f(z_0)$$

$$\lim_{z \to z_0} f(z) = f(z_0)$$

$\therefore$ f kontinu di z_0. //

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Turunan fungsi $w = f(z)$ pada daerah D ditulis $f'(z)$ atau $\frac{dw}{dz}$ didefinisikan $f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$, asalkan limitnya ada.

E Tentukan turunan dari $f(z) = z^2$ pada $\mathbb{C}$

Solusi :

$$
\begin{aligned}
f'(z) &= \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z} \\
&= \lim_{\Delta z \to 0} \frac{(z+\Delta z)^2 - z^2}{\Delta z} \\
&= \lim_{\Delta z \to 0} \frac{\cancel{z^2} + 2z.\Delta z + \Delta z^2 - \cancel{z^2}}{\Delta z} \\
&= \lim_{\Delta z \to 0} \frac{\Delta z\,(2z + \Delta z)}{\Delta z} \\
&= \lim_{\Delta z \to 0} (2z + \Delta z) \\
&= 2z \,//
\end{aligned}
$$

L Tentukan $f'(z)$ dari

(1) $f(z) = 3z^2 - 2z + 4$

(2) $f(z) = (1 - 4z^2)^3$

(3) $f(z) = \frac{z-1}{2z+1}$, $z \neq -\frac{1}{2}$

(4) $f(z) = \frac{(1+z^2)^4}{z^2}$, $z \neq 0$

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Penyelesaian :

(1) $f(z) = 3z^2 - 2z + 4$.

Penyelesaian :

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{[3(z+\Delta z)^2 - 2(z+\Delta z) + 4] - [3z^2 - 2z + 4]}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{[3(z^2 + 2z \cdot \Delta z + \Delta^2 \cdot z^2) - 2(z+\Delta z) + 4] - [3z^2 - 2z + 4]}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\cancel{3z^2} + 6z \cdot \Delta z + 3 \cdot \Delta^2 \cdot z^2 - \cancel{2z} - 2 \cdot \Delta z \cancel{+4} - \cancel{3z^2} + \cancel{2z} \cancel{-4}}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{6z \cdot \Delta z + 3 \cdot \Delta^2 \cdot z^2 - 2 \cdot \Delta z}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\cancel{\Delta z}(6z + \Delta z - 2)}{\cancel{\Delta z}}$$

$$= \lim_{\Delta z \to 0} (6z + \Delta z - 2)$$

$$= 6z - 2 \;//$$

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(2) $f(z) = (1-4z^2)^3$

Penyelesaian :

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{(1-4(z+\Delta z)^2)^3 - (1-4z^2)^3}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\cancel{1} - 12(z+\Delta z)^2 + 48(z+\Delta z)^4 - 64(z+\Delta z)^6 - \cancel{1} + 12z^2 - 48z^4 + 64z^6}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\cancel{-12z^2} - 24\Delta z \cdot z - 12(\Delta z)^2 + \cancel{48z^4} + 192z^3\Delta z + 288z^2(\Delta z)^2 + 192z(\Delta z)^3}{\Delta z}$$

$$\frac{+48(\Delta z)^4 - \cancel{64z^6} - 384z^5 \cdot \Delta z - 960z^4(\Delta z)^2 - 1280z^3(\Delta z)^3 - 960z^2(\Delta z)^4}{}$$

$$\frac{-384z(\Delta z)^5 - 64(\Delta z)^6 + \cancel{12z^2} - \cancel{48z^4} + \cancel{64z^6}}{}$$

$$= \lim_{\Delta z \to 0} \frac{\cancel{\Delta z}(-24z - 12\Delta z + 192z^3 + 288z^2\Delta z + 192z(\Delta z)^2 + 48(\Delta z)^3 - 384z^5}{\cancel{\Delta z}}$$

$$\frac{-960z^4\Delta z - 1280z^3(\Delta z)^2 - 960z^2(\Delta z)^3 - 384z(\Delta z)^4 - 64(\Delta z)^5)}{}$$

$$= \lim_{\Delta z \to 0} -24z - 12\Delta z + 192z^3 + 288\Delta z + 192z(\Delta z)^2 + 48(\Delta z^3) - 384z^5$$

$$- 960z^4\Delta z - 1280z^3(\Delta z)^2 - 960z^2(\Delta z)^3 - 384z(\Delta z)^4 - 64(\Delta z)^3$$

$$= -24z + 192z^3 - 384z^5 \; //$$

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Makassar, 4 Juli 2019

(3) $f(z) = \dfrac{z-1}{2z+1}$; $z \neq -\frac{1}{2}$

Penyelesaian :

$$f'(z) = \lim_{x \to z} \frac{f(x) - f(z)}{x - z}$$ [Lihat buku kalkulus I hal 101]

$$= \lim_{x \to z} \frac{\frac{x-1}{2x+1} - \frac{z-1}{2z+1}}{x - z}$$

$$= \lim_{x \to z} \frac{(x-1)(2z+1) - (z-1)(2x+1)}{(2x+1)(2z+1)} \cdot \frac{1}{x-z}$$

$$= \lim_{x \to z} \frac{(2xz + x - 2z - 1) - (2xz + z - 2x - 1)}{(2x+1)(2z+1)} \cdot \frac{1}{x-z}$$

$$= \lim_{x \to z} \frac{x - 2z - z + 2x}{(2x+1)(2z+1)} \cdot \frac{1}{x-z}$$

$$= \lim_{x \to z} \frac{3(x-z)}{(2x+1)(2z+1)} \cdot \frac{1}{(x-z)}$$

$$= \lim_{x \to z} \frac{3}{(2x+1)(2z+1)}$$

$$= \frac{3}{(2z+1)(2z+1)}$$

$$= \frac{3}{(2z+1)^2}$$ //

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Cara bapak (Normal)

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \left[\left(\frac{(z+\Delta z)-1}{2(z+\Delta z)+1} - \frac{z-1}{2z+1} \right) \cdot \frac{1}{\Delta z} \right]$$

$$= \lim_{\Delta z \to 0} \left[\left(\frac{(z+\Delta z-1)(2z+1) - (z-1)(2z+2\Delta z+1)}{(2z+2\Delta z+1)(2z+1)} \right) \cdot \frac{1}{\Delta z} \right]$$

$$= \lim_{\Delta z \to 0} \left[\left(\frac{(\cancel{2z^2} + \cancel{2z\cdot\Delta z} - \cancel{2z} + \cancel{z} + \Delta z - \cancel{1}) - (\cancel{2z^2} + \cancel{2z\cdot\Delta z} + \cancel{z} - \cancel{2z} - 2\Delta z - \cancel{1})}{(2z+2\Delta z+1)(2z+1)} \right) \frac{1}{\Delta z} \right]$$

$$= \lim_{\Delta z \to 0} \left[\frac{\Delta z + 2\Delta z}{(2z+2\Delta z+1)(2z+1)} \cdot \frac{1}{\Delta z} \right]$$

$$= \lim_{\Delta z \to 0} \left[\frac{3\cancel{\Delta z}}{(2z+2\Delta z+1)(2z+1)} \cdot \frac{1}{\cancel{\Delta z}} \right]$$

$$= \lim_{\Delta z \to 0} \left[\frac{3}{(2z+2\Delta z+1)(2z+1)} \right]$$

$$= \frac{3}{(2z+1)(2z+1)}$$

$$= \frac{3}{(2z+1)^2} \quad //$$

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Makassar, 4 Juni 2017

(4) $f(z) = \dfrac{(1+z^2)^4}{z^2}$

Penyelesaian :

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \left[\frac{(1+(z+\Delta z)^2)^4}{(z+\Delta z)^2} - \frac{(1+z^2)^4}{z^2} \right] \cdot \frac{1}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \left[\frac{(1+4(z+\Delta z)^2 + 6(z+\Delta z)^4 + 4(z+\Delta z)^6 + (z+\Delta z))\, z^2 - (1+4z^2+6z^4+4z^6+z^8)}{(z+\Delta z)\, z^2} \right] \cdot \frac{1}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \left[\frac{(1+4z^2+8z\cdot\Delta z+4(\Delta z)^2+6z^4+24z^3\Delta z+36z^2(\Delta z)^2+24z(\Delta z)^3}{(z+\Delta z)^2 \cdot z^2} \right.$$
$$+6(\Delta z)^4+4z^6+24z^5\Delta z+60z^4(\Delta z)^2+80z^3(\Delta z)^3+60z^2(\Delta z)^4$$
$$+24z(\Delta z)^5+4(\Delta z)^6+z^8+8z^7\cdot\Delta z+28z^6(\Delta z)^2+56z^5(\Delta z)^3$$
$$+70z^4(\Delta z)^4+56z^3(\Delta z)^5+28z^2(\Delta z)^6+8z(\Delta z)^7+(\Delta z)^8)\,z^2$$
$$\left. -(1+4z^2+6z^4+4z^6+z^8)(z^2+2z\cdot\Delta z+(\Delta z)^2) \right] \cdot \frac{1}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \left[\frac{z^2+4z^4+8z^3\Delta z+4z^2(\Delta z)^2+6z^6+24z^5\cdot\Delta z+36z^4(\Delta z)^2+24z^3(\Delta z)^3}{(z+\Delta z)^2 \cdot z^2} \right.$$
$$+6z^2(\Delta z)^4+4z^8+24z^7\cdot\Delta z+60z^6(\Delta z)^2+80z^5(\Delta z)^3+60z^4(\Delta z)^4$$
$$+24z^3(\Delta z)^5+4z^2(\Delta z)^6+z^{10}+8z^9\Delta z+28z^8(\Delta z)^2+56z^7(\Delta z)^3$$
$$+70z^6(\Delta z)^4+56z^5(\Delta z)^5+28z^4(\Delta z)^6+8z^3(\Delta z)^7+z^2(\Delta z)^8$$
$$-z^2-2z\cdot\Delta z-(\Delta z)^2-4z^4-8z^3\Delta z-4z^2(\Delta z)-6z^6-12z^5\Delta z-6z^4(\Delta z)^2$$
$$\left. -4z^8-8z^7\cdot\Delta z-4z^6(\Delta z)^2-z^{10}-2z^9\Delta z-z^8(\Delta z)^2 \right] \cdot \frac{1}{\Delta z}$$

$$= \frac{12z^5+16z^7+6z^9-2z}{z^2 \cdot z^2}$$

$$= \frac{12z^5+16z^7+6z^9-2z}{z^4} \quad //$$

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Makassar, 4 Juni 2020

E

Seperti Teorema sebelumnya (hal 4) bahwa teorema tersebut tidak berlaku sebaliknya. Berikut contohnya:

$f(z) = |z|^2$ kontinu di pada $\mathbb{C}$ tetapi dia hanya punya turunan di $z=0$.

Buktinya :

① Adb. $f(z) = |z|^2$ kontinu pada $\mathbb{C}$

Solusi :

Adb. $\lim_{z\to 0} f(z) = 0$

Perhatikan bahwa:

(a) $\lim_{z\to 0} f(z) = \lim_{z\to 0} |z|^2 = 0$

(b.) $f(0) = 0$

$\lim_{z\to 0} f(z) = f(0)$

Artinya f kontinu di $z=0$

Adb : $f(z) = |z|^2$ kontinu pada $\mathbb{C}$

Karena $f(z) = |z|^2 = x^2 + y^2$, $z = x+yi$

$u(x,y) = x^2+y^2$ ← Real $v(x,y) = 0$ ← Im

u dan v kontinu diseluruh bidang datar (Fungsi polinom pasti kontinu).

② Adb. $f(z) = |z|^2$ hanya punya turunan di $z=0$

Perhatikan bahwa, misalkan $z_0 \in D_f = \mathbb{C}$ diperoleh

$$f'(z_0) = \lim_{z\to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$= \lim_{z\to z_0} \frac{|z|^2 - |z_0|^2}{z - z_0} \quad \ldots (*)$$

→ Tinjau $z_0 = 0$, diperoleh

$$f'(0) = \lim_{z\to 0} \frac{|z|^2}{z} = \lim_{z\to 0} \frac{z\bar{z}}{z}$$

$$= \lim_{z\to 0} \bar{z}$$

$$= 0 \;//$$

→ Tinjau $z_0 \neq 0$

Misal $z_0 = x_0 + y_0 i$ akibatnya diperoleh

$$f'(z_0) = \lim_{z\to z_0} \frac{|z|^2 - |z_0|^2}{z - z_0}$$

$$f'(z_0) = \lim_{(x,y)\to(x_0,y_0)} \frac{x^2+y^2-(x_0^2+y_0^2)}{(x-x_0)+i(y-y_0)}$$

Misal $y = y_0$ diperoleh

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② Adb. $f(z) = |z|^2$ hanya punya turunan di $z=0$.

Perhatikan bahwa, misalkan

$z_0 \in D_f = \mathbb{C}$ diperoleh

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$$

$$= \lim_{z \to z_0} \frac{|z|^2 - |z_0|^2}{z - z_0} \quad \ldots\ldots (*)$$

•) Tinjau $z_0 = 0$, diperoleh

$$f'(0) = \lim_{z \to 0} \frac{|z|^2}{z}$$

$$= \lim_{z \to 0} \frac{\cancel{z} \cdot \bar{z}}{\cancel{z}}$$

$$= \lim_{z \to 0} \bar{z}$$

$$= 0_{//}$$

•) Tinjau $z_0 \neq 0$

Misal $z_0 = x_0 + y_0 i$ akibatnya diperoleh

$$f'(z_0) = \lim_{z \to z_0} \frac{|z|^2 - |z_0|^2}{z - z_0}$$

$$f'(z_0) = \lim_{(x,y) \to (x_0,y_0)} \frac{x^2+y^2-(x_0^2+y_0^2)}{(x-x_0)+i(y-y_0)}$$

Misal $y = y_0$ diperoleh

$$= \lim_{(x,y)\to(x_0,y_0)} \frac{x^2+y^2-x_0^2-y_0^2}{(x-x_0)+i(y-y_0)}$$

$$= \lim_{x \to x_0} \frac{x^2+\cancel{y_0^2}-x_0^2-\cancel{y_0^2}}{x-x_0+i(\cancel{y_0-y_0})}$$

$$= \lim_{x \to x_0} \frac{(x-x_0)(x+x_0)}{(x-x_0)}$$

$$= \lim_{x \to x_0} (x+x_0)$$

$$= 2x_0 \quad \ldots\ldots\ldots\ldots (**)$$

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Di lain pihak,

Misal $x = x_0$ diperoleh

$$= \lim_{(x,y)\to(x_0,y_0)} \frac{x^2+y^2-(x_0^2+y_0^2)}{(x-x_0)+i(y-y_0)}$$

$$= \lim_{y\to y_0} \frac{\cancel{x_0^2}+y^2-\cancel{x_0^2}-y_0^2}{\cancel{(x_0-x_0)}+i(y-y_0)}$$

$$= \lim_{y\to y_0} \frac{y^2-y_0^2}{i(y-y_0)}$$

$$= \lim_{y\to y_0} \frac{(y+y_0)\cancel{(y-y_0)}}{i\cancel{(y-y_0)}}$$

$$= \lim_{y\to y_0} \frac{y+y_0}{i} \quad \to \frac{y+y_0}{i}\cdot\frac{i}{i} = -i(y+y_0)$$

$$= \lim_{y\to y_0} -i(y+y_0)$$

$$= -2iy_0 \quad \ldots\ldots\ldots\ldots (***)$$

Perhatikan $(***) \neq (**)$

Jadi $f'(z_0)$ tidak ada untuk $z_0 \neq 0$

$\therefore$ f kontinu di seluruh $\mathbb{C}$ tetapi

f hanya punya turunan di $z = 0$.

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Aturan Turunan

Jika f dan g punya turunan di $z \in \mathbb{C}$ maka

$f+g$, $f-g$, αf (α konstanta kompleks), fg, $\frac{f}{g}$ ($g(z) \neq 0$)

punya turunan di $z \in \mathbb{C}$ dengan

(1) $(f+g)'(z) = f'(z) + g'(z)$

(2) $(f-g)'(z) = f'(z) - g'(z)$

(3) $(\alpha f)'(z) = \alpha \cdot f'(z)$

(4) $(f \cdot g)'(z) = f(z) \cdot g'(z) + f'(z) \cdot g(z)$

(5) $$\left(\frac{f}{g}\right)'(z) = \frac{g(z) \cdot f'(z) - f(z) \cdot g'(z)}{(g(z))^2}$$

Aturan Rantai

Jika fungsi f punya turunan di $z \in \mathbb{C}$ dan g punya turunan di $f(z)$ maka $g \circ f$ punya turunan di z dengan

$$(g \circ f)'(z) = g'(f(z)) \cdot f'(z)$$

Notasi Leibniz

Jika $s = g(w)$, $w = f(z)$ $\quad s = g(f(z))$

maka $\dfrac{ds}{dz} = \dfrac{ds}{dw} \cdot \dfrac{dw}{dz}$.

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Teorema

Misal f punya turunan pada $\mathbb{C}$

(1) Jika $f(z)=k$ $\forall z \in \mathbb{C}$ maka $f'(z)=0$

(2) Jika $f(z)=z$ $\forall z \in \mathbb{C}$ maka $f'(z)=1$

(3) Jika $f(z)=z^n$ $\forall z \in \mathbb{C}$, $n \in \mathbb{Z}$ maka $f'(z)=n \cdot z^{n-1}$

$\boxed{E}$

$$f(z)=z^{2021} \rightarrow f'(z)=(2021)\, z^{2020}$$

$\boxed{E}$ $f(z)=z^2+3z \rightarrow f'(z)=2z+3$

Bisa pakai cara: $f'(z)=\lim\limits_{z\to z_0} \dfrac{f(z)-f(z_0)}{z-z_0}$

atau $f'(z)=\lim\limits_{\Delta z\to 0} \dfrac{f(z+\Delta z)-f(z)}{\Delta z)}$

Contoh pakai cara yang dibawah:

$$f'(z)=\lim_{\Delta z\to 0} \frac{[(z+\Delta z)^2+3(z+\Delta z)]-(z^2+3z)}{\Delta z}$$

$$=\lim_{\Delta z\to 0} \frac{\cancel{z^2}+2z\Delta z+\Delta z^2+\cancel{3z}+3\Delta z-\cancel{z^2}-\cancel{3z}}{\Delta z}$$

$$=\lim_{\Delta z\to 0} \frac{\cancel{\Delta z}(2z+\Delta z+3)}{\cancel{\Delta z}}$$

$$=\lim_{\Delta z\to 0} 2z+\Delta z+3$$

$$=2z+3 \;//$$