Makassar, 2 September 2020

Struktur Aljabar II / Pertemuan III / Ring / PR

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Struktur Aljabar II

PR

1. Buktikan secara lengkap, himpunan

$$\mathbb{M}_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\}$$

merupakan Ring.

Penyelesaian :

Akan ditunjukkan : 1) $M_2(\mathbb{R}) \neq \phi$
2) $(M_2(\mathbb{R}), +)$ Grup Abel
3) $(M_2(\mathbb{R}), \times)$ Semigrup
4) $(\mathbb{M}_2(\mathbb{R}), +, \times)$ Distributif

Note that,

1) $\mathbb{M}_2(\mathbb{R}) \neq \phi$ sebab $\exists \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in \mathbb{M}_2(\mathbb{R})$ .... $\blacksquare$

2) Akan ditunjukkan : $(\mathbb{M}_2(\mathbb{R}), +)$ memenuhi sifat tutup, assosiatif, $\exists$ identitas, $\exists$ invers dan komutatif

• Tutup, $\forall a, b \in \mathbb{M}_2(\mathbb{R}) \Rightarrow a + b \in \mathbb{M}_2(\mathbb{R})$

Ambil sebarang $a, b \in \mathbb{M}_2(\mathbb{R})$

Tulis $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2, b_2, c_2, d_2 \in \mathbb{R}$

Note that, $a + b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$

$= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix} \in \mathbb{M}_2(\mathbb{R})$ .... $\blacksquare$

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Makassar, 3 September 2020

•) Assosiatif, $\forall a,b,c \in M_2(\mathbb{R}) \Rightarrow (a+b)+c = a+(b+c)$

Ambil sebarang $a,b,c \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2, b_2, c_2, d_2 \in \mathbb{R}$

$c = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$, $a_3, b_3, c_3, d_3 \in \mathbb{R}$

Note that,

$$(a+b)+c = \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right] + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} (a_1+a_2)+a_3 & (b_1+b_2)+b_3 \\ (c_1+c_2)+c_3 & (d_1+d_2)+d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1+(a_2+a_3) & b_1+(b_2+b_3) \\ c_1+(c_2+c_3) & d_1+(d_2+d_3) \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= a + (b+c) \quad \ldots\ldots\ldots\ldots \blacksquare$$

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Makassar, 3 September 20

- $\exists$ identitas, yaitu $\forall a \in M_2(\mathbb{R})$, $\exists\, b \in M_2(\mathbb{R}) \ni a+b = b+a = a$

Ambil sebarang $a \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Pilih $b = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in M_2(\mathbb{R})$

Note that,

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a_1+0 & b_1+0 \\ c_1+0 & d_1+0 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = a \quad \ldots\ldots (*)$$

$$b+a = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} 0+a_1 & 0+b_1 \\ 0+c_1 & 0+d_1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = a \quad \ldots\ldots (**)$$

Karena $(*) = (**) \Rightarrow a+b = b+a = a$, maka adanya identitas penjumlahan terbukti ...... $\blacksquare$

- $\exists$ invers, yaitu $\forall a \in M_2(\mathbb{R})$, $\exists\, b \in M_2(\mathbb{R}) \ni a+b = b+a = \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Ambil sebarang $a \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

Pilih $b = \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} \in M_2(\mathbb{R})$, $-a_1, -b_1, -c_1, -d_1 \in \mathbb{R}$

Note that,

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} = \begin{pmatrix} a_1+(-a_1) & b_1+(-b_1) \\ c_1+(-c_1) & d_1+(-d_1) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{0} \quad \ldots\ldots (*)$$

$$b+a = \begin{pmatrix} -a_1 & -b_1 \\ -c_1 & -d_1 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} -a_1+a_1 & -b_1+b_1 \\ -c_1+c_1 & -d_1+d_1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{0} \quad \ldots\ldots (**)$$

Karena $(*) = (**) \Rightarrow a+b = b+a = \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, maka adanya invers penjumlahan terbukti. ...... $\blacksquare$

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Makassar, 3 Sept 2020

- Komutatif, $\forall\ a,b \in M_2(\mathbb{R}) \Rightarrow (a+b) = (b+a)$

Ambil sebarang $a,b \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2, b_2, c_2, d_2 \in \mathbb{R}$

Note that,

$$a+b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 + a_1 & b_2 + b_1 \\ c_2 + c_1 & d_2 + d_1 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

$$= b+a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \blacksquare$$

$\therefore$ $(M_2(\mathbb{R}), +)$ adalah grup abelian.

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Makassar, 3 Septembe 2020

3.) Akan ditunjukkan : $(M_2(\mathbb{R}), \times)$ memenuhi sifat tutup dan assosiatif

•) Tutup, $\forall a, b \in M_2(\mathbb{R}) \Rightarrow a \times b \in M_2(\mathbb{R})$

Ambil sebarang, $a, b \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2, b_2, c_2, d_2 \in \mathbb{R}$

Note that

$$a \times b = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$$

$$= \begin{pmatrix} (a_1 . a_2) + (b_1 . c_2) & (a_1 . b_2) + (b_1 . d_2) \\ (c_1 . a_2) + (d_1 . d_2) & (c_1 . b_2) + (c_2 . d_2) \end{pmatrix} \in M_2(\mathbb{R})$$

Karena $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2 \in \mathbb{R}$ maka berlaku sifat tutup pada perkalian dan penjumlahan bilangan Real, sedemikian sehingga $a \times b \in M_2(\mathbb{R})$

Emanuel AS / 1811141008 Makassar, 3 September 2020

•) Assosiatif, $\forall a,b,c \in M_2(\mathbb{R}) \Rightarrow a\times(b\times c) = (a\times b)\times c$

Ambil sebarang $a,b,c \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2, b_2, c_2, d_2 \in \mathbb{R}$

$c = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$, $a_3, b_3, c_3, d_3 \in \mathbb{R}$

Note that,

$$a\times(b\times c) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} (a_2.a_3)+(b_2.c_3) & (a_2.b_3)+(b_2.d_3) \\ (c_2.a_3)+(d_2.c_3) & (c_2.b_3)+(d_2 d_3) \end{pmatrix}$$

$$= \begin{pmatrix} a_1.[(a_2.a_3)+(b_2.c_3)]+b_1.[(c_2.a_3)+(d_2.c_3)] & a_1.[(a_2.b_3)+(b_2 d_3)]+b_1.[(c_2.b_3)+(d_2.d_3)] \\ c_1.[(a_2 a_3)+(b_2.c_3)]+d_1.[(c_2.a_3)+(d_2.c_3)] & c_1.[(a_2.b_3)+(b_2 d_3)]+d_1.[(c_2.b_3)+(d_2.d_3)] \end{pmatrix}$$

$$= \begin{pmatrix} a_1.a_2.a_3+a_1.b_2.c_3+b_1.c_2.a_3+b_1.d_2 c_3 & a_1.a_2.b_3+a_1.b_2.d_3+b_1.c_2.b_3+b_1.d_2.d_3 \\ c_1.a_2.a_3+c_1.b_2.c_3+d_1.c_2.a_3+d_1.d_2.c_3 & c_1.a_2.b_3+c_1.b_2.d_3+d_1.c_2.b_3+d_1.d_2.d_3 \end{pmatrix} \dots (*)$$

$$(a\times b)\times c = \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right] \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} (a_1.a_2)+(b_1.c_2) & (a_1.b_2)+(b_1.d_2) \\ (c_1.a_2)+(d_1.c_2) & (c_1.b_2)+(d_1.d_2) \end{pmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} [(a_1.a_2)+(b_1.c_2)].a_3+[(a_1.b_2)+(b_1.d_2)].c_3 & [(a_1.a_2)+(b_1.c_2)].b_3+[(a_1.b_2)+(b_1.d_2)]d_3 \\ [(c_1.a_2)+(d_1.c_2)].a_3+[(c_1.b_2)+(d_1.d_2)].c_3 & [(c_1.a_2)+(d_1.c_2)].b_3+[(c_1.b_2)+(d_1.d_2)].d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_3.a_1.a_2+a_3.b_1.c_2+c_3.a_1.b_2+c_3 b_1.d_2 & b_3.a_1.a_2+b_3.b_1.c_2+d_3.a_1.b_2+d_3.b_1.d_2 \\ a_3.c_1.a_2+a_3 d_1.c_2+c_3.c_1.b_2+c_3.d_1.d_2 & b_3.c_1.a_2+b_3.d_1.c_2+d_3.c_1.b_2+d_3.d_1.d_2 \end{pmatrix}$$

$$= \begin{pmatrix} a_1.a_2.a_3+a_1.b_2.c_3+b_1.c_2.a_3+d_1.d_2.c_3 & a_1.a_2.b_3+a_1.b_2.d_3+b_1.c_2 b_3+b_1.d_2.d_3 \\ c_1.a_2.a_3+c_1.b_2.c_3+d_1.c_2.a_3+d_1.d_2.c_3 & c_1.a_2.b_3+c_1.b_2 d_3+d_1.c_2.b_3+d_1.d_2.d_3 \end{pmatrix} (**)$$

Karena $(*) = (**) \Rightarrow a\times(b\times c) = (a\times b)\times c$, maka sifat assosiatif perkalian terbukti. $\boxed{//}$

$\therefore$ $(M_2(\mathbb{R}), \times)$ adalah semigrup.

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Makassar 3 Septem 2020

4) Akan ditunjukkan : $\forall a,b,c \in M_2(\mathbb{R})$ berlaku:

•) $a \times (b+c) = ab + ac$

•) $(a+b) \times c = ac + bc$

Penyelesaian:

•) Ambil sebarang $a,b,c \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1,b_1,c_1,d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2,b_2,c_2,d_2 \in \mathbb{R}$

$c = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$, $a_3,b_3,c_3,d_3 \in \mathbb{R}$

Note that,

$$a \times (b+c) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \times \begin{pmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1.(a_2+a_3) + b_1(c_2+c_3) & a_1.(b_2+b_3) + b_1.(d_2+d_3) \\ c_1.(a_2+a_3) + d_1(c_2+c_3) & c_1(b_2+b_3) + d_1.(d_2+d_3) \end{pmatrix}$$

$$= \begin{pmatrix} a_1.a_2 + a_1.a_3 + b_1.c_2 + b_1c_3 & a_1.b_2 + a_1.b_3 + b_1.d_2 + b_1.d_3 \\ c_1.a_2 + c_1.a_3 + d_1.c_2 + d_1.c_3 & c_1.b_2 + c_1.b_3 + d_1.d_2 + d_1.d_3 \end{pmatrix} \text{....}(*)$$

$$ab + ac = \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right] + \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= \begin{pmatrix} (a_1.a_2)+(b_1.c_2) & (a_1.b_2)+(b_1.d_2) \\ (c_1.a_2)+(d_1.c_2) & (c_1.b_2)+(d_1.d_2) \end{pmatrix} + \begin{pmatrix} (a_1.a_3)+(b_1.c_3) & (a_1.b_3)+(b_1.d_3) \\ (c_1.a_3)+(d_1.c_3) & (c_1.b_3)+(d_1.d_3) \end{pmatrix}$$

$$= \begin{pmatrix} a_1.a_2 + a_1.a_3 + b_1.c_2 + b_1.c_3 & a_1.b_2 + b_1.d_2 + a_1.b_3 + b_1.d_3 \\ c_1.a_2 + d_1.c_2 + c_1a_3 + d_1.c_3 & c_1.b_2 + d_1.d_2 + c_1.b_3 + d_1.d_3 \end{pmatrix} \text{...}(**)$$

Karena $(*) = (**)$ $\Rightarrow$ $a \times (b+c) = ab + ac$ .... $\blacksquare$

Dipindai dengan CamScanner

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Makassar, 3 September 2020

$\Rightarrow$ Ambil Sebarang, $a, b, c \in M_2(\mathbb{R})$

Tulis, $a = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $a_1, b_1, c_1, d_1 \in \mathbb{R}$

$b = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $a_2, b_2, c_2, d_2 \in \mathbb{R}$

$c = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$, $a_3, b_3, c_3, d_3 \in \mathbb{R}$

Note that,

$$(a+b) \times c = \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right] \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix} \times \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

$$= \begin{pmatrix} [(a_1+a_2) \cdot a_3] + [(b_1+b_2) \cdot c_3] & [(a_1+a_2) \cdot b_3] + [(b_1+b_2) \cdot d_3] \\ [(c_1+c_2) \cdot a_3] + [(d_1+d_2) \cdot c_3] & [(c_1+c_2) \cdot b_3] + [(d_1+d_2) \cdot d_3] \end{pmatrix}$$

$$= \begin{pmatrix} a_1 \cdot a_3 + a_2 \cdot a_3 + b_1 \cdot c_3 + b_2 \cdot c_3 & a_1 \cdot b_3 + a_2 b_3 + b_1 \cdot d_3 + b_2 \cdot d_3 \\ c_1 \cdot a_3 + c_2 \cdot a_3 + d_1 \cdot c_3 + d_2 \cdot c_3 & c_1 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \ldots (*)$$

$$ac + bc = \left[\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right] + \left[\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \cdot \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}\right]$$

$$= \begin{pmatrix} (a_1 \cdot a_3) + (b_1 \cdot c_3) & (a_1 \cdot b_3) + (b_1 \cdot d_3) \\ (c_1 \cdot a_3) + (d_1 \cdot c_3) & (c_1 \cdot b_3) + (d_1 d_3) \end{pmatrix} + \begin{pmatrix} (a_2 \cdot a_3) + (b_2 \cdot c_3) & (a_2 \cdot b_3) + (b_2 \cdot d_3) \\ (c_2 \cdot a_3) + (d_2 \cdot c_3) & (c_2 \cdot b_3) + (d_2 \cdot d_3) \end{pmatrix}$$

$$= \begin{pmatrix} a_1 \cdot a_3 + a_2 \cdot a_3 + b_1 \cdot c_3 + b_2 \cdot c_3 & a_1 \cdot b_3 + a_2 \cdot b_3 + b_1 \cdot d_3 + b_2 \cdot d_3 \\ c_1 \cdot a_3 + c_2 \cdot a_3 + d_1 \cdot c_3 + d_2 \cdot c_3 & c_1 \cdot b_3 + c_2 \cdot b_3 + d_1 \cdot d_3 + d_2 \cdot d_3 \end{pmatrix} \ldots (**)$$

Karena $(*) = (**) \Rightarrow (a+b) \times c = ac + bc$ .... ▨

$\therefore$ $(M_2(\mathbb{R}), +, \times)$ adalah Distributif.

$\therefore$ Karena $M_2(\mathbb{R}) \neq \emptyset$, $(M_2(\mathbb{R}), +)$ Grup abelian, $(M_2(\mathbb{R}), \times)$ Semigrup, $(M_2(\mathbb{R}), +, \times)$ Distributif, maka $(M_2(\mathbb{R}), +, \times)$ adalah Ring.

(Terbukti) ▨

8

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Makassar, 3 September 2020

② Misalkan R Ring dan memenuhi $a^2 = a \ \forall a \in R$. Buktikan bahwa R ring komutatif.

<u>Penyelesaian</u>:

Ambil $a, b \in R$ sebarang

Akan ditunjukkan : $ab = ba$

(i) Karena R ring $\Rightarrow a+b \in R$

Berdasarkan yang diketahui, maka diperoleh:

$$(a+b)^2 = a+b \qquad [\text{Definisi}]$$
$$(a+b)(a+b) = a+b$$
$$(a+b)a + (a+b)b = a+b \qquad [\text{Distributif kiri}]$$
$$a^2 + ba + ab + b^2 = a+b \qquad [\text{Distributif kanan}]$$
$$\cancel{a} + ba + ab + \cancel{b} = \cancel{a} + \cancel{b} \qquad [\text{Definisi}]$$
$$ba + ab = 0$$
$$\boxed{ab = -ba}$$

(ii) Akan ditunjukkan : $-ba = ba$

Karena R ring $\Rightarrow ba \in R$, $\forall b, a \in R \ni (ba + ba) \in R$

Berdasarkan yang diketahui, maka diperoleh:

$$(ba+ba)^2 = ba + ba \qquad [\text{Definisi}]$$
$$(ba+ba)(ba+ba) = ba + ba$$
$$(ba+ba)ba + (ba+ba)ba = ba + ba \qquad [\text{Distributif kiri}]$$
$$(ba)^2 + (ba)^2 + (ba)^2 + (ba)^2 = ba + ba \qquad [\text{Distributif kanan}]$$
$$\cancel{ba} + ba + ba + \cancel{ba} = \cancel{ba} + \cancel{ba} \qquad [\text{Definisi}]$$
$$ba + ba = 0$$
$$\boxed{ba = -ba}$$

∴ Karena dari (i) dan (ii) diperoleh $ab = -ba = ba$, maka R ring komutatif.

(Terbukti) ∎