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Misal $f(t) = \cos t$

$F(s) = \frac{s}{a^2+s^2}$ ; Domain $F(s)$ : $s > 0$

Buktikan !

$L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\ dt$

$L\{\cos at\} = \int_0^\infty e^{-st} \cdot \cos at\ dt$

$= \lim_{p\to\infty} \int_0^p e^{-st} \cdot \cos at\ dt$

Misal $u = e^{-st}$ $\quad dv = \cos at\ dt$

$du = -se^{-st}\ dt$ $\quad v = \int \cos at\ dt$

$v = \frac{1}{a} \cdot \sin at$

$= \lim_{p\to\infty} \left[ \left(e^{-st} \cdot \frac{1}{a} \cdot \sin at\right) - \left(\int_0^\infty \frac{1}{a} \cdot \sin at \cdot -se^{-st}\ dt\right)\right]_0^p$

$= \lim_{p\to\infty} \left[ \left(e^{-st} \cdot \frac{1}{a} \cdot \sin at\right) - \left(-\frac{s}{a} \cdot \int_0^p e^{-st} \cdot \sin at\ dt\right)\right]_0^p$

$= \lim_{p\to\infty} \left[ \left(e^{-st} \cdot \frac{1}{a} \cdot \sin at\right) + \left(\frac{s}{a} \cdot \int_0^p e^{-st} \cdot \sin at\ dt\right)\right]_0^p$

Misal $u = e^{-st}$ $\quad dv = \sin at\ dt$

$du = -se^{-st}\ dt$ $\quad v = \int \sin at\ dt$

$v = -\frac{1}{a} \cdot \cos at$

$= \lim_{p\to\infty} \left[ \left(e^{-st} \cdot \frac{1}{a} \cdot \sin at\right) + \left(\frac{s}{a} \cdot \left(\left(e^{-st} \cdot -\frac{1}{a} \cdot \cos at\right) - \left(\int_0^p -\frac{1}{a} \cdot \cos at \cdot -se^{-st}\ dt\right)\right)\right)\right]_0^p$

$= \lim_{p\to\infty} \left[ \left(e^{-st} \cdot \frac{1}{a} \cdot \sin at\right) + \left(\frac{s}{a} \cdot e^{-st} \cdot -\frac{1}{a} \cdot \cos at\right) - \left(\frac{s^2}{a^2} \cdot \int_0^p \cos at \cdot e^{-st}\ dt\right)\right]_0^p$

$= \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-st} \cdot \sin at - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-st} \cdot \cos at\right]_0^p - \frac{s^2}{a^2} \cdot \lim_{p\to\infty} \int_0^p e^{-st} \cdot \cos at\,dt$

$= \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-st} \cdot \sin at - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-st} \cdot \cos at\right]_0^p - \frac{s^2}{a^2} \cdot \int_0^\infty e^{-st} \cdot \cos at\ dt$

$= \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-st} \cdot \sin at - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-st} \cdot \cos at\right]_0^p - \frac{s^2}{a^2} \left(L\{\cos at\}\right)$

$L\{\cos at\} + \frac{s^2}{a^2}\left(L\{\cos at\}\right) = \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-st} \cdot \sin at - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-st} \cdot \cos at\right]_0^p$

$\left(L\{\cos at\}\right)\left(1+\frac{s^2}{a^2}\right) = \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-st} \cdot \sin at - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-st} \cdot \cos at\right]_0^p$

$\left(L\{\cos at\}\right)\left(\frac{a^2+s^2}{a^2}\right) = \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-sp} \cdot \sin ap - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-sp} \cdot \cos at\right] - \left[\frac{1}{a} \cdot \lim_{p\to\infty} e^{-s.0} \cdot \sin a.0 - \frac{s}{a^2} \cdot \lim_{p\to\infty} e^{-s.0} \cdot \cos a.0\right]$

$\left(L\{\cos at\}\right)\left(\frac{a^2+s^2}{a^2}\right) = \left[\quad 0 \quad - \quad 0 \quad\right] - \left[\quad 0 \quad - \frac{s}{a^2} \cdot 1\right]$

$\left(L\{\cos at\}\right) = \frac{\frac{s}{a^2}}{\frac{a^2+s^2}{a^2}} = \frac{s}{a^2} \cdot \frac{a^2}{a^2+s^2} = \frac{s}{a^2+s^2}$

$\therefore$ Jika $f(t) = \cos t \Rightarrow F(s) = \frac{s}{a^2+s^2}$ //

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1. $f(t) = 4$ Maka $F(s) = \frac{4}{s}$

2. $f(t) = t^2$ Maka $F(s) = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

3. $f(t) = e^{2t}$ Maka $F(s) = \frac{1}{s-2}$

4. $f(t) = e^{-t}$ Maka $F(s) = \frac{1}{s+1}$

5. $f(t) = \sin 2t$ Maka $F(s) = \frac{2}{2^2 + s^2} = \frac{2}{4+s^2}$

6. $f(t) = \cos 4t$ Maka $F(s) = \frac{s}{4^2 + s^2} = \frac{s}{16+s^2}$