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Teori Modul / Pertemuan ke - 7 / Catatan

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Modul Kanan Atas Ring

(D)

Misalkan M himpunan tak kosong dan R ring dan diberikan operasi penggandaan skalar

$$* : M \times R \longrightarrow M$$
$$(\alpha, a) \longmapsto \alpha a$$

Himpunan M disebut modul kanan atas R

(1) $(M, +)$ Grup Abelian

(a) $\forall a, b \in M \Rightarrow a+b \in M$

(b) $\forall a, b, c \in M \Rightarrow a+(b+c) = (a+b)+c$

(c) $\exists 0_M \in M, \forall a \in M \ni 0_M + a = a + 0_M = a$

(d) $\forall a \in M, \exists -a \in M \ni a+(-a) = (-a)+a = 0_M$

(e) $\forall a, b \in M \Rightarrow a+b = b+a$

(2) Terhadap operasi penggandaan skalar $*$ memenuhi

(a) $a * \alpha \in M \quad \forall a \in M, \alpha \in R$

(b) $(a+b) * \alpha = (a*\alpha)+(b*\alpha) \quad \forall a, b \in M, \alpha \in R$

(c) $a*(\alpha+\beta) = (a*\alpha)+(a*\beta) \quad \forall a \in M, \alpha, \beta \in R$

(d) $a*(\alpha\beta) = (a*\alpha)*\beta \quad \forall a \in M, \alpha, \beta \in R$

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$\boxed{E}$

(1) $M_2(F) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a,b,c,d \in F \right\} \leftarrow$ Ring (R)

$F^2 = \{ (x, y) \mid x, y \in F \} \quad \leftarrow M$

didefinisikan

$* : F^2 \times M_2(F) \longrightarrow F^2$

$\left((x,y), \begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) \longmapsto (ax + cy \quad bx + yd)$

Buktikan F^2 modul kanan atas $M_2(F)$

<u>Bukti :</u>

Ambil $A, B, C \in F^2$, $\alpha \in M_2(R)$ sebarang

Tulis

$A = (x_1 \ \ y_1)$ u/ suatu $x_1, y_1 \in F$

$B = (x_2 \ \ y_2)$ u/ suatu $x_2, y_2 \in F$

$C = (x_3 \ \ y_3)$ u/ suatu $x_3, y_3 \in F$

$\alpha = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ u/ suatu $a, b, c, d \in F$

$\beta = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ u/ suatu $e, f, g, h \in F$

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(1) Adb. $(F^2, +)$ Abelian

(a) Adb. $A+B \in F^2$

$A+B = (x_1 \;\; y_1) + (x_2 \;\; y_2) = (x_1+x_2 \;\; y_1+y_2)$

(b) Adb. $(A+B)+C = A+(B+C)$

Perhatikan bahwa

$(A+B)+C = [(x_1 \;\; y_1)+(x_2 \;\; y_2)] + (x_3 \;\; y_3)$

$= [(x_1+x_2) \;\; (y_1+y_2)] + (x_3 \;\; y_3)$

$= (x_1+(x_2+x_3) \;\; y_1+(y_2+y_3))$

$= (x_1 \;\; y_1) + [(x_2+x_3) \;\; (y_2+y_3)]$

$= (x_1 \;\; y_1) + [(x_2 \;\; y_2) \;\; (x_1 \;\; y_1)]$

$= A + (B+C)$

(c) Terdapat $0 = (0_F \;\; 0_F) \in F^2$ sehingga untuk setiap $A = (x_1 \;\; y_1) \in F^2$ Berlaku

$0+A = (0_F \;\; 0_F) + (x_1 \;\; y_1)$

$= (0_F+x_1 \;\; 0_F+y_1) = (x_1, y_1) = A$ ....(1)

$A+0 = (x_1 \;\; y_1) + (0_F \;\; 0_F)$

$= (x_1+0_F \;\; y_1+0_F) = (x_1 \;\; y_1) = A$ ....(2)

Jadi dari (1) dan (2) diperoleh

$0+A = A+0 = A$

(d) Untuk setiap $A = (x_1 \;\; y_1) \in F^2$, Pilih $-A = (-x_1 \;\; -y_1) \in F^2$ sehingga

$A+(-A) = (x_1 \;\; y_1) + (-x_1 \;\; -y_1)$

$= (x_1-x_1 \;\; y_1-y_1) = (0_F \;\; 0_F) = 0$

$-A+A = (-x_1 \;\; -y_1) + (x_1 \;\; y_1)$

$= (-x_1+x_1 \;\; -y_1+y_1) = (0_F \;\; 0_F) = 0$

Jadi

$A+(-A) = (-A)+A = 0$

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(e) Adb. $A+B = B+A$

$A+B = (x_1 \quad y_1) + (x_2 \quad y_2)$

$= (x_1+x_2 \quad y_1+y_2)$

$= (x_2+x_1 \quad y_2+y_1) = (x_2 \quad y_2) + (x_1 \quad y_1) = B+A$

(2) (a) Adb. $A\alpha \in F^2$

$A\alpha = (x \ y) \begin{pmatrix} a & b \\ c & d \end{pmatrix} = (xa + yc \quad xb + yd) \in F^2$

(b) Adb. $(A+B)\alpha = A\alpha + B\alpha$

$(A+B)\alpha = [(x_1 \quad y_1) + (x_2 \quad y_2)] \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= (x_1+x_2 \quad y_1+y_2) \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= ((x_1+x_2)a + (y_1+y_2)c \quad (x_1+x_2)b + (y_1+y_2)d)$

$= (x_1a + x_2a + y_1c + y_2c \quad x_1b + x_2b + y_1d + y_2d)$

$= (x_1a + y_1c + x_2a + y_2c \quad x_1b + y_1d + x_2b + y_2d)$

$= (x_1a + y_1c \quad x_1b + y_1d) + (x_2a + y_2c \quad x_2b + y_2d)$

$= \left[(x_1 \quad y_1)\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] + \left[(x_2 \quad y_2)\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]$

$= A\alpha + B\alpha$

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(c) Adb. $A(\alpha+\beta) = A\alpha + A\beta$

Perhatikan bahwa

$$A(\alpha+\beta) = (x_1 \;\; y_1) \cdot \left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix}\right]$$

$$= (x_1 \;\; y_1) \cdot \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$= \big(x_1(a+e) + y_1(c+g) \quad x_1(b+f) + y_1(d+h)\big)$$

$$= \big((x_1a + y_1c) + (x_1e + y_1g) \quad (x_1b + y_1d) + (x_1f + y_1h)\big)$$

$$= (x_1a + y_1c \quad x_1b + y_1d) + (x_1e + y_1g \quad x_1f + y_1h)$$

$$= \left[(x_1 \;\; y_1)\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] + \left[(x_2 \;\; y_2)\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right]$$

$$= A\alpha + A\beta$$

(d) Adb. $A(\alpha\beta) = (A\alpha)\beta$

Perhatikan bahwa

$$A(\alpha\beta) = (x_1 \;\; y_1)\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right]$$

$$= (x_1 \;\; y_1)\begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$= \big(x_1(ae+bg) + y_1(ce+dg) \quad x_1(af+bh) + y_1(cf+dh)\big)$$

$$= (x_1a + y_1c \quad x_1b + y_1d)\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$= (A\alpha)\beta$$

$\therefore$ Jadi F^2 modul kanan atas $M_2(F)$