Analisis Kompleks / Syam - ok / Pertemuan II   Imanuel AS 1811141008   Makassar, 18 Februari 2021

Pendahuluan

Bilangan kompleks dan satuan imajinernya seperti apa yaa... mm

$$\mathbb{C} = \{a + bi \mid a,b \in \mathbb{R}\}$$

Ingat bahwa bilangan kompleks ini, bisa kita pandang sebagai pasangan terurut. Seperti berikut ini,

$$\underset{\text{}}{z = a + bi \in \mathbb{C}} \xrightarrow{\text{pasangan terurut}} z = (a, b)$$

(part) real   (part) Imaginer

Perhatikan, misalkan kita punya

$$z = (a,b) = \underbrace{(a,0)}_{a} + \underbrace{(b,0)}_{b}\underbrace{(0,1)}_{i}$$

$$i = (0,1) \quad , \quad i = \sqrt{-1} \longrightarrow i^2 = -1$$

$$\begin{aligned} i^2 = i \cdot i &= (0,1)(0,1) \\ &= (0.0 - 1.1 \; , \; 0.1 + 1.0) \quad \text{[Definisi Perkalian Bil. komp]} \\ &= (-1, 0) \\ &= -1 \end{aligned}$$

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Sifat - Sifat Pada Bilangan Kompleks

Bil. Real berlaku
- sifat Aljabar
- sifat Urutan
- sifat Kelengkapan

Di bilangan kompleks, ternyata sifat aljabar juga berlaku.

Sifat Aljabar Bilangan Kompleks

- Penjumlahan (+)

(1) Tertutup

$$\forall z_1, z_2 \in \mathbb{C} \Rightarrow z_1 + z_2 \in \mathbb{C}$$

(2) Asosiatif

$$\forall z_1, z_2, z_3 \in \mathbb{C} \Rightarrow z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$$

(3) Unsur Identitas / Unsur nol

Terdapat $0 \in \mathbb{C}$ sehingga untuk setiap $z \in \mathbb{C}$ berlaku

$$0 + z = z + 0 = z$$

(4) Invers

Untuk setiap $z \in \mathbb{C}$ terdapat $-z \in \mathbb{C}$ sehingga

$$z + (-z) = (-z) + z = 0$$

catatan

$0 = 0 + 0i$, $z = a + bi$, $a, b \in \mathbb{R}$

(5) Komutatif

$\forall z_1, z_2 \in \mathbb{C}$ berlaku

$$z_1 + z_2 = z_2 + z_1$$

catatan

$0 + 0i$, $z = a + bi$, $a, b \in \mathbb{R}$

lebih jauh, kita punya fakta bahwa $(\mathbb{C}, +)$ adalah Grup Abelian.

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- Perkalian $(\times)$

(1) Tertutup

$\forall z_1, z_2 \in \mathbb{C} \Rightarrow z_1 . z_2 \in \mathbb{C}$

(2) Assosiatif

$\forall z_1, z_2, z_3 \in \mathbb{C}$ berlaku $z_1(z_2 z_3) = (z_1 z_2) z_3$

(3) Unsur Identitas / kesatuan

Terdapat $1 \in \mathbb{C}$ sehingga untuk setiap $z \in \mathbb{C}$ berlaku

$$z . 1 = 1 . z = z$$

Catatan : $1 = 1 + 0i$

(4) Unsur Invers

$\forall z \in \mathbb{C}$, $z \neq 0$, terdapat $\frac{1}{z} \in \mathbb{C}$ sehingga

$$z\left(\frac{1}{z}\right) = \left(\frac{1}{z}\right) z = 1$$

Catatan :

$$z^{-1} = \frac{1}{z} = \frac{1}{a+bi}$$

$$= \frac{1}{a+bi} \times \frac{a-bi}{a-bi}$$

$$= \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} . i$$

(5) Komutatif

$\forall z_1, z_2 \in \mathbb{C} \Rightarrow z_1 z_2 = z_2 z_1$

Lebih jauh, $(\mathbb{C} \setminus \{0\}, \times) \rightarrow$ Grup Abel

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$z_1, z_2 \in \mathbb{C}$

(1) $z_1 - z_2 = z_1 + (-z_2)$

(2) $\frac{z_1}{z_2} = z_1 \cdot \left(\frac{1}{z_2}\right)$, $z_2 \neq 0$

dimana $1 = 1 + 0i$

dan $0 = 0 + 0i$

E

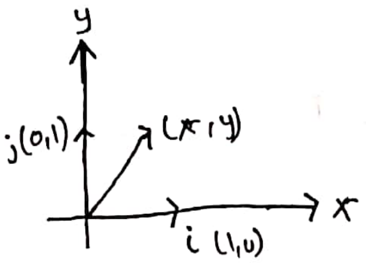
$z = 2 + 3i \rightarrow z^{-1} = \ldots$

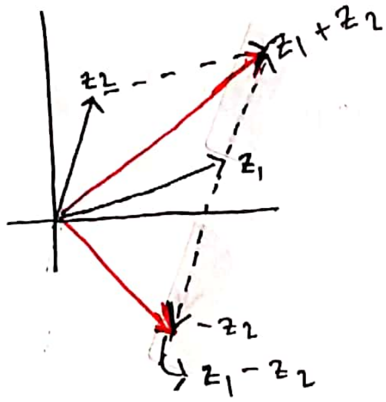
$$z^{-1} = \frac{2}{(2)^2 + (3)^2} - \frac{3}{(2)^2 + (3)^2} \cdot i$$

$$= \frac{4}{13} - \frac{3}{13} i$$

$z \cdot z^{-1} = 1$

Tafsiran Geometri





$z = a + bi$
$z = x + yi$ } Basis $\{1, i\}$

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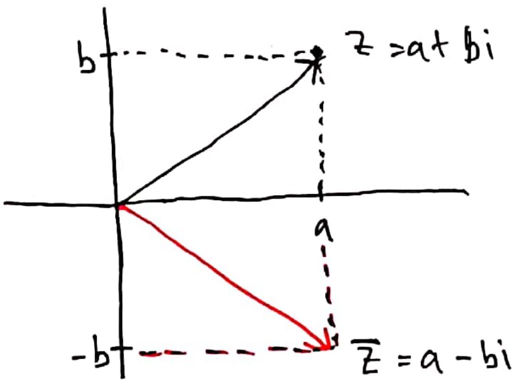
Konjugat Bilangan Kompleks

Misal $z = a + bi \in \mathbb{C}$, konjugat dari z ditulis $\bar{z} = a - bi$



Sifat konjugat bilangan kompleks

$\forall z_1, z_2 \in \mathbb{C}$ berlaku

(1) $\bar{\bar{z}} = z$

(2) $z + \bar{z} = 2 \operatorname{Re}(z)$

(3) $z - \bar{z} = 2i \operatorname{Im}(z)$

(4) $z\bar{z} = (\operatorname{Re}(z))^2 + (\operatorname{Im}(z))^2$

(5) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(6) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(7) $\overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2$

(8) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

(9) $z_1 \bar{z}_2 + \bar{z}_1 z_2 = 2 \operatorname{Re}(z_1 \bar{z}_2)$

$\boxed{N}$ $z = a + bi \rightarrow \operatorname{Re}(z) = a$ dan $\operatorname{Im}(z) = b$

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BUKTI

① Adb. $\overline{\overline{z}} = z$

Ambil $z \in \mathbb{C}$ sebarang, tulis

$z = a + bi$ untuk suatu $a, b \in \mathbb{R}$

$\overline{z} = a - bi$. Jadi

$\overline{\overline{z}} = \overline{a - bi} = a - (-b)i$

$= a + bi = z$ //

③ Adb. $z - \overline{z} = 2i\ \mathrm{Im}(z)$

Ambil $z \in \mathbb{C}$ sebarang,

Tulis, $z = a + bi$ u/ suatu $a, b \in \mathbb{R}$

Perhatikan bahwa,

$z - \overline{z} = (a + bi) - \overline{(a + bi)}$

$= a + bi - (a - bi)$

$= 2bi$

$= 2i\ \mathrm{Im}(z)$ //

PR 1 Buktikan sifat konjugat nomor 1/ sd 9.

② Adb. $z + \overline{z} = 2\,\mathrm{Re}(z)$

Ambil $z \in \mathbb{C}$ sebarang,

Tulis, $z = a + bi$ u/suatu $a, b \in \mathbb{R}$

Note that,

$z + \overline{z} = (a + bi) + \overline{(a + bi)}$

$= (a + bi) + (a - bi)$

$= (a + a) + (b - b)i$

$= 2a + (0)i$

$= 2a$

$= 2\,\mathrm{Re}(z)$

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(4) Adb $z\bar{z} = (Re(z))^2 + (Im(z))^2$

Ambil sebarang $z \in \mathbb{C}$

Tulis, $z = a+bi$ u/ suatu $a,b \in \mathbb{R}$

Note that,

$$
\begin{aligned}
z\bar{z} &= (a+bi) \cdot \overline{(a+bi)} \\
&= (a+bi) \cdot (a-bi) \\
&= (a.a) - (ab)i + (ba)i + b^2 \\
&= a^2 + 0 + b^2 \\
&= (Re(z))^2 + (Im(z))^2 \ _\#
\end{aligned}
$$

(5) Adb $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

Ambil sebarang $z_1, z_2 \in \mathbb{C}$

Tulis, $z_1 = a+bi$ u/ suatu $a,b \in \mathbb{R}$ dan $z_2 = c+di$ u/ suatu $c,d \in \mathbb{R}$

Note that,

$$
\begin{aligned}
\overline{z_1+z_2} &= \overline{(a+bi)+(c+di)} \\
&= \overline{(a+c)+(b+d)i} \\
&= (a+c) - (b+d)i \\
&= (a-bi) + (c-di) \\
&= \overline{z_1} + \overline{z_2}
\end{aligned}
$$

(6) Adb. $\overline{z_1-z_2} = \overline{z_1} - \overline{z_2}$

Ambil sebarang $z_1, z_2 \in \mathbb{C}$

Tulis, $z_1 = a+bi$ u/ suatu $a,b \in \mathbb{R}$

dan $z_2 = c+di$ u/ suatu $c,d \in \mathbb{R}$

Note that

$$
\begin{aligned}
\overline{z_1-z_2} &= \overline{(a+bi)-(c+di)} \\
&= \overline{(a-c)+(b-d)i} \\
&= (a-c) - (b-d)i \\
&= (a-bi) - (c-di) \\
&= \overline{z_1} - \overline{z_2}
\end{aligned}
$$

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(7) Adb. $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$

Ambil sebarang $z_1, z_2 \in \mathbb{C}$

Tulis, $z_1 = a + bi$ u/ suatu $a, b \in \mathbb{R}$

dan $z_2 = c + di$ u/ suatu $c, d \in \mathbb{R}$

Note that

$$\begin{aligned}
\overline{z_1 \cdot z_2} &= \overline{(a+bi)\cdot(c+di)} \\
&= \overline{ac + (ad)i + (bc)i - bd} \\
&= \overline{(ac-bd) + (ad+bc)i} \\
&= (ac-bd) - (ad+bc)i \\
&= ac - (ad)i - (bc)i - bd \\
&= (a-bi)\cdot(c-di) \\
&= \overline{z_1} \cdot \overline{z_2} \quad //
\end{aligned}$$

(8) Adb. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$

Ambil $z_1, z_2 \in \mathbb{C}$ sebarang, dengan $z_2 \neq 0$

tulis, $z_1 = a + bi$ u/ suatu $a, b \in \mathbb{R}$

dan $z_2 = c + di$ u/ suatu $c, d \in \mathbb{R}$

Note that,

$$\begin{aligned}
\overline{\left(\frac{z_1}{z_2}\right)} &= \overline{\left(\frac{a+bi}{c+di}\right)} \\
&= \overline{\left(\frac{a+bi}{c+di} \times \frac{c-di}{c-di}\right)} \\
&= \overline{\left(\frac{ac - (ad)i + (bc)i + bd}{c^2 + d^2}\right)} \\
&= \overline{\left(\left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right) i\right)} \\
&= \left(\frac{ac+bd}{c^2+d^2}\right) - \left(\frac{bc-ad}{c^2+d^2}\right) i
\end{aligned}$$

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$$\hookrightarrow = \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{ad-bc}{c^2+d^2}\right)i$$

$$= \frac{ac+(ad)i-(bc)i+bd}{c^2+d^2}$$

$$= \frac{a-bi}{c-di} \times \frac{c+di}{c+di}$$

$$= \frac{a-bi}{c-di}$$

$$= \frac{\overline{z_1}}{\overline{z_2}} \quad //$$

(9) Adb. $z_1 \cdot \overline{z_2} + \overline{z_1} \cdot z_2 = 2\,\mathrm{Re}\,(z_1 \overline{z_2})$

Ambil sebarang $z_1, z_2 \in \mathbb{C}$

Tulis, $z_1 = a+bi$ u/ suatu $a, b \in \mathbb{R}$

dan $z_2 = c+di$ u/ suatu $c, d \in \mathbb{R}$

Note that,

$$\begin{aligned}
z_1 \cdot \overline{z_2} + \overline{z_1} \cdot z_2 &= \left(a+bi \cdot \overline{c+di}\right) + \left(\overline{a+bi} \cdot c+di\right) \\
&= \left(a+bi \cdot c-di\right) + \left(a-bi \cdot c+di\right) \\
&= \left(ac-(ad)i+(bc)i+bd\right) + \left(ac+(ad)i-(bc)i+bd\right) \\
&= \left[(ac+bd)-(ad-bc)i\right] + \left[(ac+bd)+(ad-bc)i\right] \\
&= \left((ac+bd)+(ac+bd)\right) - \left((ad-bc)-(ad-bc)\right)i \\
&= 2(ac+bd) - (0)i \\
&= 2(ac+bd) \\
&= 2\,\mathrm{Re}\,(z_1 \cdot \overline{z_2}) \quad //
\end{aligned}$$