Makassar, 2 Juni 2024

Analisis Kompleks

Pertemuan ke - 14

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Analisis Kompleks / Pertemuan ke - 14 / Catatan

Makassar, 1 Juni 2021

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Limit Tak Hingga

$\lim\limits_{z \to z_0} = \infty \Leftrightarrow \forall \varepsilon > 0 \quad \exists \delta > 0$ sehingga

$$\underbrace{0 < |z - z_0| < \delta}_{z \in V_\delta^*(z_0)} \quad \text{maka} \quad \underbrace{|f(z)| > \frac{1}{\varepsilon}}_{f(z) \in V_\varepsilon(\infty)}$$

dimana $V_\delta(z_0) = \{ z \in \mathbb{C} \mid |z - z_0| < \delta \}$

dan $V_\delta^*(z_0) = V_\delta(z_0) \setminus \{ z_0 \}$

dimana $V_\varepsilon(\infty) = \{ z \in \mathbb{C} \mid |z| > \frac{1}{\varepsilon} \}$

↑ lingkungan / persekitaran titik tak hingga

$$\left.\begin{aligned} \lim_{z \to z_0} f(z) = \infty &\Leftrightarrow \forall \varepsilon > 0 \ \exists \delta > 0 \ \ni z \in V_\delta^*(z_0) \Rightarrow f(z) \in V_\varepsilon(\infty) \\ &\Leftrightarrow \forall \varepsilon > 0 \ \exists \delta > 0 \ \ni 0 < |z - z_0| < \delta \Rightarrow |f(z)| > \frac{1}{\varepsilon} \\ &\Leftrightarrow \forall \varepsilon > 0 \ \exists \delta > 0 \ \ni 0 < |z - z_0| < \delta \Rightarrow \left|\frac{1}{f(z)}\right| < \varepsilon \end{aligned}\right\} \text{ekivalen}$$

Fakta menarik :

$$\lim_{z \to z_0} f(z) = \infty \Leftrightarrow \lim_{z \to z_0} \frac{1}{f(z)} = 0 .$$

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E. Tunjukkan bahwa

$$\lim_{z \to -1} \frac{iz+3}{z+1} = \infty$$

Bukti :

Misal $f(z) = \dfrac{iz+3}{z+1}$

Perhatikan bahwa,

$$\lim_{z \to -1} \left(\frac{1}{\frac{iz+3}{z+1}} \right) = \lim_{z \to -1} \left(\frac{z+1}{iz+3} \right)$$

$$= \frac{0}{-i+3}$$

$$= 0$$

$\therefore$ Karena $\lim_{z \to -1} \left(\frac{1}{\frac{iz+3}{z+1}} \right) = 0$ akibatnya $\lim_{z \to -1} \left(\frac{iz+3}{z+1} \right) = \infty$.

Latihan :

Tunjukkan bahwa $\lim_{z \to -2} \left(\frac{z+5}{z+2} \right) = \infty$

Penyelesaian :

Misal $f(z) = \dfrac{z+5}{z+2}$

Perhatikan bahwa,

$$\lim_{z \to -2} \left(\frac{1}{\frac{z+5}{z+2}} \right) = \lim_{z \to -2} \left(\frac{z+2}{z+5} \right)$$

$$= \frac{0}{3}$$

$$= 0$$

$\therefore$ Karena $\lim_{z \to -2} \left(\frac{1}{\frac{z+5}{z+2}} \right) = 0$ akibatnya $\lim_{z \to -2} \left(\frac{z+5}{z+2} \right) = \infty$

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Limit di Tak Hingga

$$\boxed{\lim_{z\to\infty} f(z) = w_0} \Leftrightarrow \forall \varepsilon > 0\ \exists \delta > 0 \ni \underbrace{|z| > \frac{1}{\delta}}_{z \in V_\delta(\infty)} \Rightarrow \underbrace{|f(z) - w_0| < \varepsilon}_{f(z) \in V_\varepsilon(w_0)}$$

z diganti $\frac{1}{z} \Rightarrow \forall \varepsilon > 0\ \exists \delta > 0$ sehingga $0 < |z| < \delta \Rightarrow |f(z) - w_0| < \varepsilon$

$$\Updownarrow$$

$$\boxed{\lim_{z\to z_0} f\left(\frac{1}{z_0}\right) = w_0}$$

Berarti $\lim_{z\to\infty} f(z) = w_0 \Leftrightarrow \lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0$

$\boxed{E}$ Tunjukkan bahwa $\lim_{z\to\infty} \frac{2z+i}{z+1} = 2$

Bukti

Misalkan $f(z) = \frac{2z+i}{z+i}$ ~~jadi~~

Jadi $f\left(\frac{1}{z}\right) = \frac{2\left(\frac{1}{z}\right)+i}{\left(\frac{1}{z}\right)+i}$

Sehingga $\lim_{z\to 0} f\left(\frac{1}{z}\right) = \lim_{z\to 0} \left(\frac{2\left(\frac{1}{z}\right)+i}{\frac{1}{z}+i}\right)$

$$= \lim_{z\to 0} \frac{2+iz}{1+iz}$$

$$= 2$$

Akibatnya, $\lim_{z\to\infty} \frac{2z+i}{z+i} = 2$ //

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Latihan.

1. Tunjukkan Bahwa $\lim\limits_{z \to \infty} \dfrac{2z^2+i}{z^2+2i} = 2$

Jawab:

Bukti:

Misal $f(z) = \dfrac{2z^2+i}{z^2+2i}$

Maka, $\lim\limits_{z \to 0} f\left(\frac{1}{z}\right) = \lim\limits_{z \to 0} \dfrac{2\left(\frac{1}{z}\right)^2+i}{\left(\frac{1}{z}\right)^2+2i}$

$= \lim\limits_{z \to 0} \dfrac{2\left(\frac{1}{z^2}\right)+i}{\frac{1}{z^2}+2i}$

$= \lim\limits_{z \to 0} \dfrac{2+iz^2}{1+2z^2 i}$

$= 2$ //

Karena $\lim\limits_{z \to 0} f\left(\frac{1}{z}\right) = 2$ dimana $f(z) = \dfrac{2z^2+i}{z^2+2i}$, ~~maka~~ akibatnya $\lim\limits_{z \to \infty} \dfrac{2z^2+i}{z^2+2i} = 2$. //

2. Tunjukkan bahwa $\lim\limits_{z \to \infty} \dfrac{3z^3-2z^2+z}{3z^3+z} = 1$

Jawab:

Bukti:

Misal $f(z) = \dfrac{3z^3-2z^2+z}{3z^3+z}$

Maka, $\lim\limits_{z \to 0} f\left(\frac{1}{z}\right) = \lim\limits_{z \to 0} \dfrac{3\left(\frac{1}{z}\right)^3-2\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)}{3\left(\frac{1}{z}\right)^3+\left(\frac{1}{z}\right)}$

$= \lim\limits_{z \to 0} \dfrac{3\cdot\frac{1}{z^3}-2\frac{1}{z^2}+\frac{1}{z}}{3\frac{1}{z^3}+\frac{1}{z}}$

$= \lim\limits_{z \to 0} \dfrac{\frac{3-2z+z^2}{z^3}}{\frac{3+z^2}{z^3}}$

$= \lim\limits_{z \to 0} \dfrac{3-2z+z^2}{3+z^2}$

$= \dfrac{3}{3}$

$= 1$ //

Karena $\lim\limits_{z \to 0} f\left(\frac{1}{z}\right) = 1$ dimana $f(z) = \dfrac{3z^3-2z^2+z}{3z^3+z}$, maka berakibat $\lim\limits_{z \to \infty} \dfrac{3z^3-2z^2+z}{3z^3+z} = 1$. //