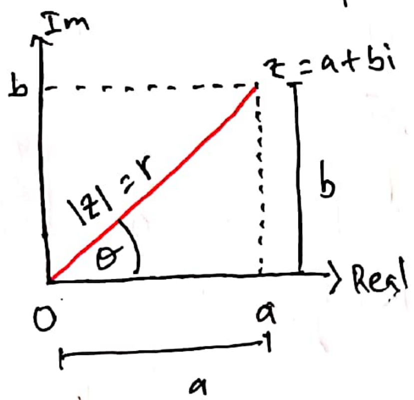
Analisis Kompleks / Pertemuan ke-4 / Catatan

Imanuel AS
1811141008

Makassar, 7 Maret 2021

Bentuk Polar

Argumen bilangan kompleks



$|z| = \sqrt{a^2+b^2}$

Argumen dari $z \in \mathbb{C}$, $z \neq 0$ ditulis $\arg(z)$

Misal $z = a + bi \neq 0$, $\theta = \arg(z)$

$r = |z| = \sqrt{a^2+b^2}$

$\boxed{N}$

$\arg(z)$ tidak tunggal, $\theta + 2n\pi$, $n \in \mathbb{Z}$

nilai tunggal dari $\arg(z)$ ditulis

$\mathrm{Arg}(z)$ dengan $-\pi < \mathrm{Arg} < \pi$

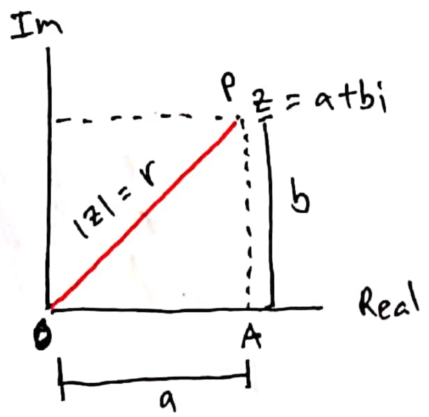
Dipindai dengan CamScanner

Imanuel AS / 1811141008 Makassar, 9 Maret 2021.

Imanuel

hubungan arg(z) dengan Arg(z) adalah

$$\boxed{\arg(z) = \operatorname{Arg}(z) + 2n\pi \quad , n \in \mathbb{Z}}$$



$$|z| = r = \sqrt{a^2+b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \operatorname{arc}\left(\tan\frac{b}{a}\right)$$

$$\sin\theta = \frac{b}{r} \Rightarrow b = r\sin\theta$$

$$\cos\theta = \frac{a}{r} \Rightarrow a = r\cos\theta$$

$$\begin{aligned} z = a+bi &= r\cos\theta + i(r\sin\theta) \\ &= r(\cos\theta + i.\sin\theta) \\ &= r.\operatorname{cis}\theta \end{aligned}$$

$$\operatorname{cis}\theta = \cos\theta + i.\sin\theta$$

Imanuel AS / 1811141008 Manuel   Makassar, 7 Maret 2021

$z = a + bi$

$\Downarrow$

$z = r \text{ cis } \theta = r(\cos\theta + i \cdot \sin\theta)$

(Bentuk Polar)

$\boxed{E}$

Nyatakan dalam bentuk polar

(1) $z = 1 + i$

Jawab :

$z = 1 + i \left< \begin{matrix} a = 1 \\ b = 1 \end{matrix} \right.$ $r = |z| = \sqrt{a^2+b^2} = \sqrt{2}$

$\theta = \text{Arg}(z) = \text{arc}\left(\tan\left(\frac{1}{1}\right)\right) = \pi/4$

diperoleh

$z = r \text{ cis } \theta = \sqrt{2} \cdot \text{cis}\left(\pi/4\right) = \sqrt{2} \cdot \left(\cos(\pi/4) + i \cdot \sin(\pi/4)\right)$

(2) Nyatakan bentuk polar berikut

$z = \sqrt{8} \text{ cis}(\pi/4)$ dalam bentuk $z = a + bi$.

Jawab :

$z = \sqrt{8} \cos(\pi/4)$

$= \sqrt{8}\left(\cos(\pi/4) + i \cdot \sin(\pi/4)\right)$

$= \sqrt{8}\left(\frac{1}{2}\sqrt{2} + i \cdot \left(\frac{1}{2}\sqrt{2}\right)\right)$

$= 2 + 2i$

Misal

$z_1 = r_1 \operatorname{cis}(\theta_1)$

$z_2 = r_2 \operatorname{cis}(\theta_2)$

$$\boxed{\begin{array}{l} z_1 = a_1 + b_1 i \quad , \; z_2 = a_2 + b_2 i \\ z_1 = z_2 \iff a_1 = a_2 \; , \; b_1 = b_2 \end{array}}$$

$$z_1 = z_2 \iff r_1 \cdot \operatorname{cis}(\theta_1) = r_2 \cdot \operatorname{cis}(\theta_2)$$

$$\iff r_1 = r_2 \; , \; \theta_1 = \theta_2 + 2n\pi \; , n \in \mathbb{Z}$$

Konjugat $z = r \cdot \operatorname{cis} \theta$ adalah

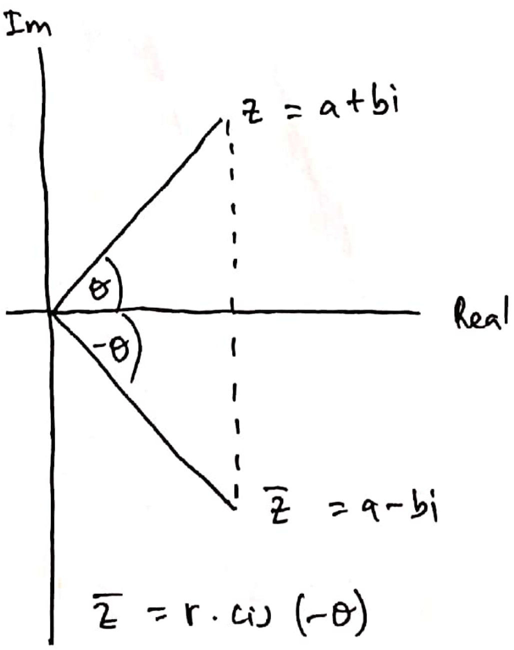
$$\boxed{\bar{z} = r \operatorname{cis}(-\theta)}$$

(1) $z = r \cdot \operatorname{cis}(\theta) \Rightarrow \bar{z} = r \operatorname{cis}(-\theta)$

Bukti

Misal $z = a + bi \Rightarrow \bar{z} = a - bi$

$r = \sqrt{a^2 + (-b)^2} = \sqrt{a^2 + b^2}$



$\bar{z} = r \cdot \operatorname{cis}(-\theta)$

Imanuel AS / 1811141008 Imanuel Makassar, 8 Maret 2021

(2) Jika $z_1 = r_1 \cdot cis(\theta_1)$ dan $z_2 = r_2 \cdot cis(\theta_2)$

Maka

(a) $z_1 \cdot z_2 = r_1 \cdot r_2 \cdot cis(\theta_1 + \theta_2)$

[KUIS] (b) $\frac{z_1}{z_2} = \frac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2)$, $z_2 \neq 0$

Bukti

(a) $z_1 \cdot z_2 = [r_1 \cdot cis(\theta_1)] \cdot [r_2 \cdot cis(\theta_2)]$

$= [r_1 \cdot (\cos\theta_1 + i \cdot \sin\theta_1)][r_2 (\cos\theta_2 + i \cdot \sin\theta_2)]$

$= r_1 \cdot r_2 (\cos\theta_1 + i \cdot \sin\theta_1)(\cos\theta_2 + i \cdot \sin\theta_2)$

$= r_1 \cdot r_2 [\cos\theta_1 \cdot \cos\theta_2 + i \cdot \cos\theta_1 \cdot \sin\theta_2 + i \sin\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2]$

$= r_1 \cdot r_2 \cdot [(\cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2) + i(\cos\theta_1 \cdot \sin\theta_2 + \sin\theta_1 \cdot \cos\theta_2)]$

$= r_1 \cdot r_2 [\cos(\theta_1 + \theta_2) + i \cdot \sin(\theta_1 + \theta_2)]$

$= r_1 \cdot r_2 \cdot cis(\theta_1 + \theta_2)$

(b) (Jawaban ada di lembar kuis)

Imanuel AS / 1811141008 Makassar, 7 Maret 2021

Bentuk Eksponen

$z = a + bi \rightarrow z = r \cdot \text{cis}\,\theta$

$r = \sqrt{a^2 + b^2}$

$\theta = \arctan\left(\frac{b}{a}\right)$

$z = r \cdot \text{cis}\,\theta = r(\cos\theta + i \cdot \sin\theta)$

$= re^{i\theta} \quad [e^{i\theta} = \text{cis}\,\theta]$

Bentuk eksponen $z \neq 0$ adalah

$z = r \cdot \text{cis}\,\theta = r(\cos\theta + i \cdot \sin\theta) = re^{i\theta}$

N

(1) Bentuk eksponen z tidak tunggal

$z = re^{i\theta} = re^{i(\theta + 2n\pi)}, \quad n \in \mathbb{Z}$

(2) $z_1 = r_1 e^{i \cdot \theta_1}, \quad z_2 = r_2 e^{i \cdot \theta_2}$

$z_1 = z_2 \Leftrightarrow r_1 = r_2$ dan

$\theta_1 = \theta_2 + 2n\pi, \quad n \in \mathbb{Z}$

(3) Jika $z_1 = r_1 e^{i \cdot \theta_1}$ dan $z_2 = r_2 \cdot e^{i\theta_2}$

maka :

(a) $z_1 \cdot z_2 = r_1 r_2 \cdot e^{i(\theta_1 + \theta_2)}$

(b) $\frac{z_1}{z_2} = \frac{r_1}{r_2} \cdot e^{i(\theta_1 - \theta_2)}$

Imanuel AS / 1811141008 Makassar, 9 Maret 2021

Bukti

(3) (a) $z_1 = r_1 \cdot cis\ \theta_1$

$z_2 = r_2 \cdot cis\ \theta_2$

$$z_1 \cdot z_2 = (r_1 \cdot cis\ \theta_1)(r_2 \cdot cis\ \theta_2)$$
$$= r_1 \cdot r_2 \, (cis\,(\theta_1 + \theta_2))$$
$$= r_1 \cdot r_2 \cdot e^{i(\theta_1 + \theta_2)} \quad //$$

(b) $z_1 = r_1 \cdot cis\,(\theta_1)$

$z_2 = r_2 \cdot cis\,(\theta_2)$, $z \neq 0$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \cdot cis\,(\theta_1 - \theta_2)$$
$$= \frac{r_1}{r_2} \cdot e^{i(\theta_1 - \theta_2)} \quad //$$

(2) $z_1 = z_2 \Leftrightarrow r_1 \cdot cis(\theta_1) = r_2 \cdot cis(\theta_2)$

$\Leftrightarrow r_1 \cdot e^{i\theta_1} = r_2 \cdot e^{i\theta_2}$

$\Leftrightarrow r_1 = r_2$, $\theta_1 = \theta_2 + 2n\pi$, $n \in \mathbb{Z}$

(1) $\theta = \arg(z)$, dan θ tidak tunggal karena $\arg(z)$ tidak tunggal dimana $\theta + 2n\pi$, $n \in \mathbb{Z}$.

Jadi, $z = re^{i\theta} = re^{i(\theta + 2n\pi)}$, $n \in \mathbb{Z}$

Analisis Kompleks / Pertemuan ke - 4 / Kuis

Imanuel AS
1811141008
Imanuel

Makassar, 4 Maret 2021

(2) Jika $z_1 = r_1 \cdot cis(\theta_1)$ dan $z_2 = r_2 \cdot cis(\theta_2)$ maka, tunjukkan bahwa

(b.) $\frac{z_1}{z_2} = \frac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2)$, $z_2 \neq 0$

Penyelesaian:

$$\frac{z_1}{z_2} = \frac{r_1 \cdot cis(\theta_1)}{r_2 \cdot cis(\theta_2)}$$

$$= \frac{r_1(\cos(\theta_1) + i \cdot \sin(\theta_1))}{r_2(\cos(\theta_2) + i \cdot \sin(\theta_2))}$$

$$= \frac{r_1(\cos(\theta_1) + i \cdot \sin(\theta_1))}{r_2(\cos(\theta_2) + i \cdot \sin(\theta_2))} \cdot \frac{\cos(\theta_2) - i \cdot \sin(\theta_2)}{\cos(\theta_2) - i \cdot \sin(\theta_2)}$$

$$= \frac{r_1}{r_2} \cdot \left[\frac{\cos(\theta_1) \cdot \cos(\theta_2) - \cos(\theta_1) \cdot i \cdot \sin(\theta_2) + i \cdot \sin(\theta_1) \cdot \cos(\theta_2) + \sin(\theta_1) \cdot \sin(\theta_2)}{\cos^2(\theta_2) + \sin^2(\theta_2)}\right]$$

$$= \frac{r_1}{r_2} \cdot \left[\frac{\cos(\theta_1) \cdot \cos(\theta_2) + \sin(\theta_1) \cdot \sin(\theta_2) + i(\sin(\theta_1) \cdot \cos(\theta_2) - \sin(\theta_2) \cdot \cos(\theta_1))}{1}\right]$$

$$= \frac{r_1}{r_2}\left[\frac{\cos(\theta_1 - \theta_2) + i(\sin(\theta_1 - \theta_2))}{1}\right]$$

$$= \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i(\sin(\theta_1 - \theta_2))\right]$$

$$= \frac{r_1}{r_2} \cdot \left[cis(\theta_1 - \theta_2)\right] \quad // \quad \blacksquare$$