MSB / Pertemuan ke - 9 / Latihan soal

Imanel AS
181114108

Makassar, 21 April 2021

Selesaikanlah soal berikut

① $\Gamma(100) = \int_0^\infty x^{n-1} \cdot e^{-x}\, dx$

$= \int_0^\infty x^{100-1} \cdot e^{-x}\, dx$

$= \int_0^\infty x^{99} \cdot e^{-x}\, dx \quad \leadsto \int_0^\infty x^n \cdot e^{-x}\, dx = n!$

$= 99!$

② $\Gamma\left(\frac{5}{2}\right) = \Gamma\left(\frac{3}{2}+1\right)$

$= \frac{3}{2} \cdot \left[\Gamma\left(\frac{1}{2}+1\right)\right]$

$= \frac{3}{2}\left[\frac{1}{2} \cdot \Gamma\left(\frac{1}{2}\right)\right]$

$= \frac{3}{2}\left[\frac{1}{2} \cdot \sqrt{\pi}\right]$

$= \frac{3}{4}\sqrt{\pi}.$

③ $\Gamma\left(-\frac{1}{2}\right) = \Gamma\left(-\frac{3}{2}+1\right)$

$= -\frac{3}{2} \cdot \Gamma\left(-\frac{3}{2}\right)$

$= -\frac{3}{2} \cdot \left[\Gamma\left(-\frac{5}{2}+1\right)\right]$

$= -\frac{3}{2} \cdot \left[-\frac{5}{2} \cdot \Gamma\left(-\frac{5}{2}\right)\right]$

$= -\frac{3}{2} \cdot \left[-\frac{5}{2} \cdot \left(\Gamma\left(-\frac{7}{2}+1\right)\right)\right]$

Tdk efektif

$= \frac{\Gamma\left(-\frac{1}{2}+1\right)}{-\frac{1}{2}}$

$= \Gamma\left(\frac{1}{2}\right) \cdot (-2)$

$= \sqrt{\pi} \cdot (-2)$

$= -2\sqrt{\pi}$

lebih efektif

Imanuel AS / 1811141008 Imanuel

Makassar, 21 April 2021

(4.) $\Gamma\left(-\frac{3}{2}\right) = \dfrac{\Gamma\left(-\frac{3}{2}+1\right)}{-\frac{3}{2}}$

$= \Gamma\left(-\frac{1}{2}\right) \cdot \left(-\frac{2}{3}\right)$

$= \left[\dfrac{\Gamma\left(-\frac{1}{2}+1\right)}{-\frac{1}{2}}\right] \cdot \left(-\frac{2}{3}\right)$

$= \left[\Gamma\left(\frac{1}{2}\right) \cdot (-2)\right] \cdot \left(-\frac{2}{3}\right)$

$= \left[\sqrt{\pi} \cdot (-2)\right] \cdot \left(-\frac{2}{3}\right)$

$= \frac{4}{3}\sqrt{\pi}$ //

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Hitung dengan menggunakan definisi fungsi gamma

(1.) $\int_0^\infty x^3 \cdot e^{-x}\, dx = 3! = 3 \times 2 \times 1 = 6$

(2) $\int_0^\infty x^6 \cdot e^{-2x}\, dx = \dfrac{6!}{2^{6+1}} = \dfrac{6!}{2^7} = \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{128} = \dfrac{720}{128} = 5{,}63$

(3) $\int_0^\infty \sqrt{x} \cdot e^{-x^3}\, dx = \int_0^\infty e^{-x^3} \cdot x^{\frac{1}{2}}\, dx$

$= \int_0^\infty e^{-x^3} \cdot x^{2-\frac{3}{2}}\, dx$

$= \int_0^\infty e^{-x^3} \cdot x^2 \cdot x^{-\frac{3}{2}}\, dx$

Misal $y = x^3 \longrightarrow \boxed{y^{\frac{1}{3}} = x}$

$dy = 3x^2\, dx$

$\frac{1}{3} dy = x^2\, dx$

$= \int_0^\infty e^{-y} \cdot \left(y^{\frac{1}{3}}\right)^{-\frac{3}{2}} \cdot \left(\frac{1}{3}\, dy\right)$

$= \int_0^\infty e^{-y} \cdot (y)^{-\frac{1}{2}} \cdot \left(\frac{1}{3}\right) dy$

$= \frac{1}{3} \cdot \int_0^\infty e^{-y} \cdot (y)^{-\frac{1}{2}}\, dy \rightsquigarrow \int_0^\infty x^{n-1} \cdot e^{-x}\, dx$

$= \frac{1}{3} \cdot \int_0^\infty (y)^{\frac{1}{2}-1} \cdot e^{-y}\, dy$

$= \frac{1}{3} \cdot \left[\Gamma\left(\frac{1}{2}\right)\right]$

$= \frac{1}{3} \cdot \sqrt{\pi}$ //

(4.) $\int_0^\infty \sqrt[4]{x} \cdot e^{-\sqrt{x}}\, dx = \int_0^\infty (x)^{\frac{1}{4}} \cdot e^{-(x^{\frac{1}{2}})}\, dx$

$= \int_0^\infty e^{-(x^{\frac{1}{2}})} \cdot (x)^{\frac{1}{4}}\, dx$

$= \int_0^\infty e^{-(x^{\frac{1}{2}})} \cdot (x)^{\frac{3}{4}-\frac{2}{4}}\, dx$

$= \int_0^\infty e^{-(x^{\frac{1}{2}})} \cdot (x)^{\frac{3}{4}} \cdot (x)^{-\frac{2}{4}}\, dx$

$= \int_0^\infty e^{-(x^{\frac{1}{2}})} \cdot (x)^{\frac{3}{4}} \cdot (x)^{-\frac{1}{2}}\, dx$

= Misal $y = x^{\frac{1}{2}} \longrightarrow y = \sqrt{x}$

$dy = \frac{1}{2} x^{-\frac{1}{2}}\, dx \qquad y^2 = x$

$2\, dy = x^{-\frac{1}{2}}\, dx$

$\int_0^\infty e^{-y} \cdot (y^2)^{\frac{3}{4}} \cdot 2\, dy$

$= \int_0^\infty e^{-y} \cdot (y)^{\frac{3}{2}} \cdot 2\, dy$

$= 2\int_0^\infty e^{-y} \cdot y^{\frac{3}{2}}\, dy$

$= 2\int_0^\infty y^{\frac{3}{2}} \cdot e^{-y}\, dy \rightsquigarrow \int_0^\infty x^{n-1} \cdot e^{-x}\, dx$

$= 2\int_0^\infty y^{\frac{5}{2}-1} \cdot e^{-y}\, dy$

$= 2 \cdot \Gamma\left(\frac{5}{2}\right)$

$= 2 \cdot \left[\Gamma\left(\frac{3}{2}+1\right)\right]$

$= 2 \cdot \left[\frac{3}{2} \cdot \Gamma\left(\frac{3}{2}\right)\right]$

$= 2 \cdot \left[\frac{3}{2} \cdot \left(\Gamma\left(\frac{1}{2}+1\right)\right)\right]$

$= 2 \cdot \left[\frac{3}{2} \cdot \left(\frac{1}{2} \cdot \Gamma\left(\frac{1}{2}\right)\right)\right]$

$= 2 \cdot \left[\frac{3}{2} \cdot \left(\frac{1}{2} \cdot \sqrt{\pi}\right)\right]$

$= \frac{3}{2}\sqrt{\pi}$