Makassar, 30 November 2020

STRUKTUR ALJABAR II

— Pertemuan XV —

(Catatan)

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Struktur Aljabar II / Pertemuan ke - 15 / Catatan

Makassar, 25 November 20..

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Struktur Aljabar II : Catatan Pertemuan ke - 15.

Gelanggang Polinom

Misal, R ring himpunan pasangan terurut tak hingga $(a_0, a_1, a_2, \ldots, a_n, \ldots)$, $a_i \in R$, $\forall i$ yang bernilai nol kecuali disejumlah hingga disebut polinom atas R.

Polinom ini ditulis

$$f(x) = a_0 x^0 + a_1 x + a_2 x^2 + \ldots + a_n x^n + 0 x^{n+1} + 0 x^{n+2} + \ldots$$
$$= \sum_{i=0}^{\infty} a_i x^i$$

$x \notin R$ disebut Indeterminate atas ring R.

Kemudian, $a_0 x^0, a_1 x, a_2 x^2, \ldots, a_n x^n, \ldots$

disebut suku dari polinomial $f(x)$

dan $a_0, a_1, a_2, \ldots, a_n, \ldots$ disebut koefisien dari suatu polinomial $f(x)$.

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Makassar, 25 November 2020

Derajat dari Polinomial

Polinom $f(x) = a_0 x^0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$

$a_n \neq 0$, $a_i = 0$, $i > n$ disebut

polinomial dengan derajat n, ditulis

$$der(f(x)) = n$$

disini :

$a_n x^n \longrightarrow$ disebut "leading term"

$a_n \longrightarrow$ disebut "leading coefficient"

misal

$$f(x) = \sum_{i=1}^{n} a_i x^i = a_0 x^0 + a_1 x + \dots + a_n x^n$$

Jika $a_i = 0$, $\forall i = 1, 2, 3, \dots, n \Longrightarrow f(x)$ disebut polinom nol

dan $f(x) = a_0 x^0 \Longrightarrow$ disebut polinom konstan.

Polinom $f(x)$ atas ring R disebut monic jika leading coefficient-nya adalah 1_R. (Maksdnya adalah: Ring dengan unsur kesatuan 1_R)

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Makassar, 25 Novemb 2020

Kesamaan Polinom

Polinom

$$f(x) = a_0x^0 + a_1x^1 + a_2x^2 + \dots + a_nx^n + \dots$$
$$h(x) = b_0x^0 + b_1x^1 + b_2x^2 + \dots + b_nx^n + \dots$$

atas R disebut sama jika

$$a_i = b_i \quad \forall i$$

himpunan semua polinomial atas R ditulis

$$R[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i , a_i \in R , a_i = 0 \text{ kecuali dixejumlah hingga} \right\}.$$

Misal $f, h \in R[x]$

$$f(x) = a_0x^0 + a_1x + \dots + a_nx^n + \dots = \sum_{i=0}^{\infty} a_i x^i$$
$$h(x) = b_0x^0 + b_1x + \dots + b_nx^n + \dots = \sum_{i=0}^{\infty} b_i x^i$$

Penjumlahan

$$\to f(x) + h(x) = \sum_{i=0}^{\infty} a_i x^i + \sum_{i=0}^{\infty} b_i x^i$$
$$= \sum_{i=0}^{\infty} (a_i + b_i) x^i \in R[x]$$

dimana

$$\operatorname{der}(f(x) + h(x)) \leq \text{maks}(\operatorname{der}(f(x)), \operatorname{der}(h(x)))$$

Misal

$$f(x) = a_0x^0 + a_1x + a_2x^2 + \dots + a_mx^m \quad , a_m \neq 0$$
$$h(x) = b_0x^0 + b_1x + b_2x^2 + \dots + b_nx^n \quad , b_n \neq 0$$

diperoleh:

$$\operatorname{der}(f(x) + h(x)) = \begin{cases} \text{maks}(m,n) & , m \neq n \\ m & , \text{jika } m = n, \ a_m + b_n \neq 0 \\ < m & , \text{jika } m = n, \ a_m + b_n = 0 \end{cases}$$

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Makassar, 25 Nov 2020

$\boxed{E}$

$\mathbb{Z}[x]$

$f(x) = 5 + 2x^2 + 3x^5 + \dots + 7x^{2020}$

$h(x) = 7x + 5x^2 + \dots + 8x^{1980}$

$$\text{der}(f(x) + h(x)) = \text{maks}(\text{der}(f(x)), \text{der}(h(x)))$$
$$= \text{maks}(2020, 1980)$$
$$= 2020$$

Pertalian

$f(x) \cdot h(x) = c_0 x^0 + c_1 x + c_2 x^2 + \dots + c_i x^i + \dots$

dimana

$$c_i = \sum_{(j+k)=i} a_j b_k \quad , j = 0, 1, 2, \dots$$

yaitu

$$c_0 = \sum_{(j+k)=0} a_j b_k = a_0 b_0$$

$$c_1 = \sum_{(j+k)=1} a_j b_k = a_0 b_1 + a_1 b_0$$

$$c_2 = \sum_{(j+k)=2} a_j b_k = a_0 b_2 + a_1 b_1 + a_2 b_0$$

$$\vdots \qquad \vdots$$

$$c_i = \sum_{(j+k)=i} a_j b_k = a_0 b_i + a_i b_{i-1} + a_2 b_{i-2} + \dots + a_i b_0$$

Imanuel AS / 1811141008 / Tugas Makalah

Makassar, 25 Agustus 2020

$\boxed{E}$

$\mathbb{Z}[x]$, misal $f(x), h(x) \in \mathbb{Z}[x]$

$f(x) = 2x^0 + 3x + 5x^2 = 2 + 3x + 5x^2$

$h(x) = 3x^0 - 5x + 4x^2 - 9x^4 = 3 + 5x + 4x^2 - 9x^3$

Jelas $a_0 = 2$, $a_1 = 3$, $a_2 = 5$, $a_3 = 0$

$b_0 = 3$, $b_1 = -5$, $b_2 = 4$, $b_3 = -9$

$f(x) + h(x) = (2+3)x^0 + (3-5)x + (5+4)x^2 + (0+(-9))x^3$

$= 5x^0 - 2x + 9x^2 - 9x^3$

$= 5 - 2x + 9x^2 - 9x^3$

$f(x) \cdot g(x) = c_0x^0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5$

dimana,

$c_0 = \sum_{(j+k)=0} a_j b_k = a_0 b_0 = 2 \times 3 = 6$

$c_1 = \sum_{(j+k)=1} a_j b_k = a_0 b_1 + a_1 b_0$

$= [2 \times (-5)] + (3)(3)$

$= -1$

$c_2 = \sum_{(j+k)=2} a_j b_k = a_0 b_2 + a_1 b_1 + a_2 b_0$

$= (2)(4) + (3)(-5) + (5)(3) = 8$

$c_3 = \sum_{(j+k)=3} a_j b_k = a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0$

$= (2)(-9) + (3)(4) + (5)(-5) + (0)(3)$

$= -31$

$c_4 = \sum_{(j+k)=4} a_j b_k = a_1 b_3 + a_2 b_2 + a_3 b_1$

$= (3)(-9) + (5)(4) + (0)(-5) = -7$

$c_5 = \sum_{(j+k)=5} a_j b_k = a_2 b_3 + a_3 b_2 = 5(-9) + (0)(4) = -45$

Jadi,

$f(x) \cdot h(x) = c_0x^0 + c_1x^1 + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5$

$= 6x^0 - x^1 + 8x^2 - 31x^3 - 7x^4 - 45x^5$

$= 6 - x + 8x^2 - 31x^3 - 7x^4 - 45x^5$.

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Makassar, 25 Nove 2020

$\boxed{N}$ $\boxed{D_1}$

(1) R ring
$(R[x], +, \times) \rightsquigarrow$ Ring (Buktikan)

(2) R daerah integral
$(R[x], +, \times) \rightsquigarrow$ Daerah Integral (Buktikan)

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Makassar, 30 Novel 2020

N D1

① R ring

$(R[x], +, \times) \rightsquigarrow$ Ring (Buktikan)

Penyelesaian:

Akan dibuktikan : $(R[x]\ ; +, \times)$ Ring

Akan ditunjukkan : $R[x]$ memenuhi syarat ring.

(1.) Akan ditunjukkan : $\forall a, b \in R[x] \Rightarrow a+b \in R[x]$

Ambil sebarang $a, b \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \quad ; a_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

dan $b = b(x)$

$= b_0 + b_1 x + b_2 x^2 + \ldots + b_n x^n \quad ; b_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa,

$$\begin{aligned} a+b &= a(x) + b(x) \\ &= (a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n) + (b_0 + b_1 x + b_2 x^2 + \ldots + b_n x^n) \\ &= (a_0 + b_0) + (a_1 x + b_1 x) + (a_2 x + b_2 x) + \ldots + (a_n x^n + b_n x^n) \\ &= (a_0 + b_0) + (a_1 + b_1) x + (a_2 + b_2) x + \ldots + (a_n + b_n) x^n \end{aligned}$$

Karena $a_i + b_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Maka $a(x) + b(x) = a + b \in R[x]$

$\therefore$ Sifat Tertutup penjumlahan $R[x]$ terpenuhi.

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(2) Akan ditunjukkan : $\forall a, b, c \in R[x] \Rightarrow (a+b)+c = a+(b+c)$

Ambil sebarang $a, b, c \in R[x]$

Tulis, $a = a(x)$

$= a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n \quad ; a_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$

$= b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n \quad ; b_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$c = c(x)$

$= c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n \quad ; c_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa,

$(a+b)+c = [a(x)+b(x)] + c(x)$

$= [(a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n) + (b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n)] + (c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n)$

$= [(a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + \dots + (a_n+b_n)x^n] + (c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n)$

$= (a_0+b_0+c_0) + (a_1+b_1+c_1)x + (a_2+b_2+c_2)x^2 + \dots + (a_n+b_n+c_n)x^n$

$= [a_0+(b_0+c_0)] + [a_1+(b_1+c_1)]x + [a_2+(b_2+c_2)]x^2 + \dots + [a_n+(b_n+c_n)]x^n$

$= (a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n) + [(b_0+c_0) + (b_1+c_1)x + (b_2+c_2)x^2 + \dots + (b_n c_n)x^n]$

$= a(x) + [b(x) + c(x)]$

$= a + (b+c)$

$\therefore$ Sifat Assosiatif terhadap penjumlahan di $R[x]$ terpenuhi.

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(3) Akan ditunjukkan: $\exists\, 0_{R[x]} \in R[x]$, $\forall a \in R[x] \ni a + 0_{R[x]} = 0_{R[x]} + a$

Pilih $0_{R[x]} \in R[x]$

Tulis, $0_{R[x]} = 0 + 0x + 0x^2 + \dots + 0x^n$; $0 \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Ambil sebarang $a \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1x + a_2x^2 + \dots + a_nx^n$; $a_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa,

$$\begin{aligned} a + 0_{R[x]} &= a(x) + 0_{R[x]} \\ &= (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) + (0 + 0x + 0x^2 + \dots + 0x^n) \\ &= (a_0+0) + (a_1+0)x + (a_2+0)x^2 + \dots + (a_n+0)x^n \\ &= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \\ &= a(x) \\ &= a \end{aligned}$$

$$\begin{aligned} 0_{R[x]} + a &= 0_{R[x]} + a(x) \\ &= (0 + 0x + 0x^2 + \dots + 0x^n) + (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) \\ &= (0+a_0) + (0+a_1)x + (0+a_2)x^2 + \dots + (0+a_n)x^n \\ &= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \\ &= a(x) \\ &= a \end{aligned}$$

$\therefore$ Adanya unsur identitas terhadap penjumlahan di $R[x]$ terpenuhi.

Catatan: $0_{R[x]} \in R[x]$ dijamin ada karena untuk setiap $0 \in R$ adalah identitas penjumlahan dari $a_i \in R$

Imanuel AS / 181114008 [illegible]

Makassar 30 Novem 2?

(4) Akan ditunjukkan : $\forall a \in R[x]\ \exists (-a) \in R[x] \ni a+(-a) = (-a)+a = 0_{R[x]}$

Ambil sebarang $a \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \ ; \ a_i \in R \ , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Pilih $(-a) \in R[x]$

Tulis $(-a) = -a(x)$

$= (-a_0) + (-a_1)x + (-a_2)x^2 + \cdots + (-a_n)x^n \ ; \ -a_i \in R \ ; n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa,

$$
\begin{aligned}
a+(-a) &= a(x) + (-a(x)) \\
&= [(a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n)] + [(-a_0) + (-a_1)x + (-a_2)x^2 + \cdots + (-a_n)x^n] \\
&= (a_0 + (-a_0)) + (a_1 + (-a_1))x + (a_2 + (-a_2))x^2 + \cdots + (a_n + (-a_n))x^n \\
&= 0 + 0x + 0x^2 + \cdots + 0x^n \\
&= 0_{R[x]}
\end{aligned}
$$

$$
\begin{aligned}
(-a)+a &= (-a(x)) + a(x) \\
&= [(-a_0) + (-a_1)x + (-a_2)x^2 + \cdots + (-a_n)x^n] + [(a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n)] \\
&= (-a_0 + a_0) + (-a_1 + a_1)x + (-a_2 + a_2)x^2 + \cdots + (-a_n + a_n)x^n \\
&= 0 + 0x + 0x^2 + \cdots + 0x^n \\
&= 0_{R[x]}
\end{aligned}
$$

$\therefore$ Adanya unsur invers terhadap penjumlahan $R[x]$ terpenuhi.

Catatan: $(-a) \in R[x]$ dijamin ada karena untuk setiap $-a_i \in R$ adalah invers dari $a_i \in R$.

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(5) Akan ditunjukkan: $\forall a, b \in R[x] \Rightarrow a+b = b+a$.

Ambil sebarang $a, b \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \quad ; a_i \in R, \ n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$

$= b_0 + b_1x + b_2x^2 + \dots + b_nx^n \quad ; b_i \in R, \ n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa,

$a+b = a(x) + b(x)$

$= (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) + (b_0 + b_1x + b_2x^2 + \dots + b_nx^n)$

$= (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + \dots + (a_n+b_n)x^n$

$= (b_0+a_0) + (b_1+a_1)x + (b_2+a_2)x^2 + \dots + (b_n+a_n)x^n$ [$a_0, b_0 \in R$ ring]

$= (b_0 + b_1x + b_2x^2 + \dots + b_nx^n) + (a_0 + a_1x + a_2x^2 + \dots + a_nx^n)$

$= b(x) + a(x)$

$= b + a$

$\therefore$ Sifat komutatif terhadap penjumlahan di $R[x]$ terpenuhi.

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(6) Akan ditunjukkan : $\forall a, b \in R[x] \Rightarrow (ab) \in R[x]$

Ambil sebarang $a, b \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n \; ; a_i \in R \; , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$

$= b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n \; , b_i \in R \; , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa

$ab = a(x) \cdot b(x)$

$= (a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n) \cdot (b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n)$

$= (a_0 b_0) + (a_0 b_1 + a_1 b_0) x + (a_0 b_2 + a_1 b_1 + a_2 b_0) x^2 + \dots +$
$(a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \dots + a_n b_0) x^n$

Karena $(a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \dots + a_n b_0) \in R$ dan $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$
maka $ab \in R[x]$.

Irmawati AS / 181114008

Makassar, 30 Nov 21

(7) Akan ditunjukkan: $\forall a, b, c \in R[x] \Rightarrow a(bc) = (ab)c$

Ambil sebarang $a, b, c \in R[x]$

Tulis $a = a(x)$

$= (a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n)$; $a_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$

$= (b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n)$; $b_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$c = c(x)$

$= (c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n)$; $c_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa

$$a(bc) = (a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n) \cdot [(b_0 + b_1 x + b_2 x^2 + \dots + b_n x^n) \cdot (c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n)]$$

$$= (a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n) \cdot [(b_0 c_0) + (b_0 c_1 + b_1 c_0) x + \dots + (b_0 c_n + b_1 c_{n-1} + \dots + b_n c_0) x^n]$$

$$= [(a_0 b_0 c_0)] + [a_0 \cdot (b_0 c_1 + b_1 c_0) + a_1 \cdot (b_0 c_0)] x + \dots +$$
$$[a_0 \cdot (b_0 c_n + b_1 c_{n-1} + \dots + b_n c_0) + a_1 (b_0 c_{n-1} + b_1 c_{n-2} + \dots + b_{n-1} c_0) \dots +$$
$$a_n (b_0 c_0)] x^n$$

$$= [(a_0 b_0) c_0] + [(a_0 b_0) c_1 + (a_0 b_1) c_0 + (a_1 b_0) c_0] x + \dots +$$
$$[((a_0 b_0) c_n + (a_0 b_1) c_{n-1} + \dots + (a_0 b_n) c_0) + ((a_1 b_0) c_{n-1} + (a_1 b_1) c_{n-2} + \dots + (a_1 b_{n-1}) c_0)$$
$$+ \dots + (a_n b_0) c_0] x^n$$

$$= [(a_0 b_0) c_0] + [(a_0 b_0) c_1 + ((a_0 b_1) + (a_1 b_0)) c_0] x + \dots +$$
$$[(a_0 b_0) c_n + ((a_0 b_1) + (a_1 b_0)) c_{n-1} + \dots + ((a_0 b_0) + (a_1 b_{n-1}) + \dots + (a_n b_n)) c_0]$$
$$((a_0 b_0) + (a_1 b_{n-1}) + \dots + (a_n b_n)) c_0] x^n$$

$\dots\dots\dots\dots (*)$

Dipindai dengan CamScanner

Imanuel AS / 1811141008 ~~Imanuel~~ Makassar, 30 Nov 2020

$$(ab)c = \left[(a_0 + a_1x + a_2x^2 + \dots + a_nx^n)\cdot(b_0 + b_1x + b_2x^2 + \dots + b_nx^n)\right]\cdot(c_0 + c_1x + \dots + c_nx^n)$$

$$= \left[(a_0b_0) + (a_0b_1 + a_1b_0)x + \dots + (a_0b_n + a_1b_{n-1} + \dots + a_nb_0)x^n\right]\cdot(c_0 + c_1x + \dots + c_nx^n)$$

$$= \left[(a_0b_0)c_0\right] + \left[(a_0b_0)c_1 + (a_0b_1 + a_1b_0)c_0\right]x + \dots +$$

$$\left[(a_0b_0)c_n + (a_0b_1 + a_1b_0)\cdot c_{n-1} + \dots + (a_0b_n + a_1b_{n-1} + \dots + a_nb_0)c_0\right]x^n \quad \dots\dots (**)$$

$\therefore$ Karena $(*) = (**)$ maka $a(bc) = (ab)c$

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Mekassar, 30 November 2020

(8) Akan ditunjukkan: $\forall a,b,c \in R[x] \Rightarrow a(b+c) = ab + ac$

Ambil sebarang $a,b,c \in R[x]$

Tulis, $a = a(x)$

$= a_0 + a_1x + a_2x^2 + \dots + a_nx^n$; $a_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$

$= b_0 + b_1x + b_2x^2 + \dots + b_nx^n$; $b_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$c = c(x)$

$= c_0 + c_1x + c_2x^2 + \dots + c_nx^n$; $c_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa

$$\begin{aligned}
a(b+c) &= a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \left[(b_0 + b_1x + \dots + b_nx^n) + (c_0 + c_1x + \dots + c_nx^n)\right] \\
&= a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \left[(b_0+c_0) + (b_1+c_1)x + \dots + (b_n+c_n)x^n\right] \\
&= a_0(b_0+c_0) + \left[a_0(b_1+c_1) + a_1(b_0+c_0)\right]x + \dots + \\
&\quad \left[a_0(b_n+c_n) + a_1(b_{n-1}+c_{n-1}) + \dots + a_n(b_0+c_0)\right]x^n \\
&= \left[a_0b_0 + (a_0b_1 + a_1b_0)x + \dots + (a_0b_n + a_1b_{n-1} + \dots + a_nb_0)x^n\right] \\
&\quad \left[a_0c_0 + (a_0c_1 + a_1c_0)x + \dots + (a_0c_n + a_1c_{n-1} + \dots + a_nc_0)x^n\right] \\
&= \left[(a_0 + a_1x + a_2x^2 + \dots + a_nx^n) \cdot (b_0 + b_1x + b_2x^2 + \dots + b_nx^n)\right] + \\
&\quad \left[(a_0 + a_1x + a_2x^2 + \dots + a_nx^n) \cdot (c_0 + c_1x + c_2x^2 + \dots + c_nx^n)\right] \\
&= \left[a(x) \cdot b(x)\right] + \left[a(x) \cdot c(x)\right] \\
&= ab + ac
\end{aligned}$$

$\therefore a(b+c) = ab + ac$

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Makassar, 30 Nov 2021

(9) Akan ditunjukkan: $\forall a,b,c \in R[x] \Rightarrow (a+b)c = ac+bc$

Ambil sebarang $a,b,c \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \quad ; a_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$

$= b_0 + b_1x + b_2x^2 + \dots + b_nx^n \quad ; b_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$c = c(x)$

$= c_0 + c_1x + c_2x^2 + \dots + c_nx^n \quad ; c_i \in R \quad , n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa

$(a+b)c = [(a_0 + a_1x + a_2x^2 + \dots + a_nx^n) + (b_0 + b_1x + b_2x^2 + \dots + b_nx^n)] \cdot (c_0 + c_1x + \dots + c_nx^n)$

$= [(a_0+b_0) + (a_1+b_1)x + \dots + (a_n+b_n)x^n] \cdot (c_0 + c_1x + \dots + c_nx^n)$

$= [(a_0+b_0)\cdot c_0] + [(a_0+b_0)\cdot c_1 + (a_1+b_1)c_0]x + \dots +$

$[(a_0+b_0)c_n + (a_1+b_1)c_{n-1} + \dots + (a_n+b_n)c_0]x^n$

$= [(a_0c_0) + (a_0c_1 + a_1c_0)x + \dots + (a_0c_n + a_1c_{n-1} + \dots + a_nc_0)x^n] +$

$[(b_0c_0) + (b_0c_1 + b_1c_0)x + \dots + (b_0c_n + b_1c_{n-1} + \dots + b_nc_0)x^n]$

$= [a(x)\cdot c(x)] + [b(x)\cdot c(x)]$

$= ac + bc$

$\therefore (a+b)c = ac+bc$

$\therefore$ Karena $R[x]$ memenuhi seluruh syarat ring maka $(R[x], +, \times)$ Ring. ▨

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Makassar, 30 Nov 2020

N D1

② R Daerah Integral

$(R[x];+,\times) \rightarrow$ Daerah Integral (Buktikan)

Penyelesaian :

Akan dibuktikan : $(R[x],+,\times)$ Daerah Integral

Akan ditunjukkan : (1) $R[x]$ ring abelian.

(2) $R[x]$ ring dengan unsur kesatuan.

(3) $R[x]$ tidak memuat pembagi nol.

(1). Adit. $R[x]$ ring abelian

Diketahui, karena R Daerah Integral maka R Ring Abelian,

Maka untuk setiap dua unsur sebarang $a,b \in R$ berlaku $ab=ba$.

Diketahui juga R daerah integral maka $(R,+)$ ring abel, $\forall a b \in R \Rightarrow a+b=b+a$.

Ambil sebarang $a,b \in R[x]$

Tulis $a = a(x)$

$= a_0+a_1x+a_2x^2+\dots+a_nx^n \quad ; a_i \in R \ , n \in \mathbb{Z}\setminus\{\mathbb{Z}^-\}$

$b = b(x)$

$= b_0+b_1x+b_2x^2+\dots+b_nx^n \quad ; b_i \in R \ , n \in \mathbb{Z}\setminus\{\mathbb{Z}^-\}$

Perhatikan bahwa

$$\begin{aligned}
ab &= (a_0+a_1x+a_2x^2+\dots+a_nx^n)\cdot(b_0+b_1x+b_2x^2+\dots+b_nx^n) \\
&= (a_0b_0)+(a_0b_1+a_1b_0)x+\dots+(a_0b_n+a_1b_{n-1}+a_2+b_{n-2}+\dots+a_nb_0)x^n \\
&= (b_0a_0)+(b_0a_1+a_0b_1)x+\dots+(b_0a_n+b_1a_{n-1}+b_2+a_{n-2}+\dots+b_na_0)x^n \\
&= (b_0+b_1x+b_2x^2+\dots+b_nx^n)\cdot(a_0+a_1x+a_2x^2+\dots+a_nx^n) \\
&= b(x) \cdot a(x) \\
&= b \cdot a
\end{aligned}$$

$\therefore$ Karena $ab=ba$ maka $R[x]$ ring abelian.

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Makassar, 30 November 2020

(2) Adit. $R[x]$ ring dengan unsur kesatuan.

Diketahui, karena R suatu daerah integral, maka jelas R memiliki unsur kesatuan, yakni $1_R \in R \ni a \cdot 1_R = 1_R \cdot a = a; \forall a \in R$.

Jadi dapat dibentuk polinomial $e = 1_R + 0x + 0x^2 + \dots + 0x^n \in R[x]$ dimana $1_R \in R$.

Ambil sebarang $a \in R[x]$

Tulis $a = a(x)$

$= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \; ; a_i \in R, \; n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Perhatikan bahwa

$$
\begin{aligned}
a \cdot e &= (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) \cdot (1_R + 0x + 0x^2 + \dots + 0x^n) \\
&= (a_0 \cdot 1_R) + (a_0 \cdot 0 + a_1 \cdot 1_R)x + (a_0 \cdot 0 + a_1 \cdot 0 + a_2 \cdot 1_R)x^2 + \dots + \\
&\quad (a_0 \cdot 0 + a_1 \cdot 0 + a_2 \cdot 0 + \dots + a_n \cdot 1_R) \\
&= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \\
&= a
\end{aligned}
$$

$$
\begin{aligned}
e \cdot a &= (1_R + 0x + 0x^2 + \dots + 0x^n) \cdot (a_0 + a_1x + a_2x^2 + \dots + a_nx^n) \\
&= (1_R \cdot a_0) + (1_R \cdot a_1 + 0 \cdot a_0)x + (1_R \cdot a_2 + 0 \cdot a_1 + 0 \cdot a_0)x^2 + \dots + \\
&\quad (1_R \cdot a_n + 0 \cdot a_{n-1} + 0 \cdot a_{n-2} + \dots + 0 \cdot a_0)x^n \\
&= a_0 + a_1x + a_2x^2 + \dots + a_nx^n \\
&= a
\end{aligned}
$$

$\therefore$ karena $ae = ea = a$ maka $R[x]$ ring dengan unsur kesatuan.

Imanuel AS / 1811141008 4 TTT Imanuel   Makassar, 30 November 2020

(3) Adit. $R[x]$ Tidak memuat pembagi nol. ($R[x]$ adalah RTPN)

Ambil sebarang $a, b \in R[x]$ yaitu:

$a = a(x)$
$= a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$; $a_n \neq 0, a_i \in R$, $n \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

$b = b(x)$
$= b_0 + b_1 x + b_2 x^2 + \dots + b_m x^m$; $b_m \neq 0$, $b_i \in R$, $m \in \mathbb{Z} \setminus \{\mathbb{Z}^-\}$

Karena $a_n \neq 0$ dan $b_m \neq 0$ maka perkalian polynomial menghasilkan
$a.b = a(x) \cdot b(x) \neq 0$, hal ini disebabkan oleh $a_n b_m \neq 0$.
Ini berarti bahwa $ab = a(x) . b(x) = 0$ dipenuhi hanya bila
$a = a(x) = 0$ atau $b = b(x) = 0$.

Sehingga $R[x]$ disebut Tidak memuat pembagi nol. ($R[x]$ adalah RTPN)
$\therefore$ $R[x]$ tidak memuat pembagi nol. ($R[x]$ adalah RTPN)

$\therefore$ Karena $R[x]$ ring abelian dengan unsur kesatuan dan tidak memuat pembagi nol (RTPN) maka disebut
($R[x], +, \times$) Daerah Integral.

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$\boxed{N}$

- Polinom nol

  $\hookrightarrow 0x^0 + 0x^1 + 0x^2 + \dots + 0x^n + \dots = 0$

- Polinom konstan

  $\hookrightarrow ax^0 + 0x^1 + 0x^2 + \dots + 0x^n + \dots = a$

- Perkalian polinom konstan $a, f(x) \in R[x]$ adalah

  $a \cdot f(x)$ "

Pandang

$F[x]$ ← Polinom atas lapangan

↑

Algoritma pembagian berlaku.

Fakta.

(1) $F[x]$ merupakan daerah euclid.

(2) $F[x]$ merupakan principal ideal ring.