Makassar, 2 Juni 2024

# Analisis Kompleks

Pertemuan ke - 12

Nama : Imanuel Agung Sembe

NIM : 181114100 8

Imanuel

Analisis Kompleks / Pertemuan ke-12 / Catatan Makassar, 28 Mei 2021

Imanuel Agung Sembe

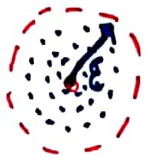
1811141008

# Persekitaran / Lingkungan (Repeat Please)

* Persekitaran $-\varepsilon$ dari $z_0 \in \mathbb{C}$ ditulis

$$V_\varepsilon(z_0) = \{ z \in \mathbb{C} \mid |z - z_0| < \varepsilon \}$$

$|z - z_0| < \varepsilon$




* Lingkungan tanpa pusat

$$V_\varepsilon^*(z_0) = V_\varepsilon(z_0) \setminus \{z_0\}$$


$\boxed{E}$

Misal $z_0 = 1 + 2i$, $\varepsilon = 2$, $V_\varepsilon(z_0) = \dots ?$

$$V_\varepsilon(z_0) = \{ z \in \mathbb{C} \mid |z - z_0| < \varepsilon \}$$

$$V_2(1+2i) = \{ z \in \mathbb{C} \mid |z - (1+2i)| < 2 \}$$

Misal $z = a + bi = (a, b)$
$\underset{Re}{a}$, $\underset{Im}{b}$

$$|z - (1+2i)| < 2$$

$$\Rightarrow |(a+bi) - (1+2i)| < 2$$

$$\Rightarrow |(a-1) + (b-2)i| < 2$$

$$\Rightarrow \sqrt{(a-1)^2 + (b-2)^2} < 2$$

$$\Rightarrow (a-1)^2 + (b-2)^2 < 4$$

Imanuel AS / 1811141008 Imanuel Makassar, 20 Mei 2021

# Limit Fungsi Kompleks

## Definisi

Fungsi $f : V_r^*(z_0) \subseteq \mathbb{C} \longrightarrow \mathbb{C}$, notasi

$\lim_{z \to z_0} f(z) = w_0$ didefinisikan sebagai

$\forall \varepsilon > 0 \ \exists \delta > 0$ sehingga

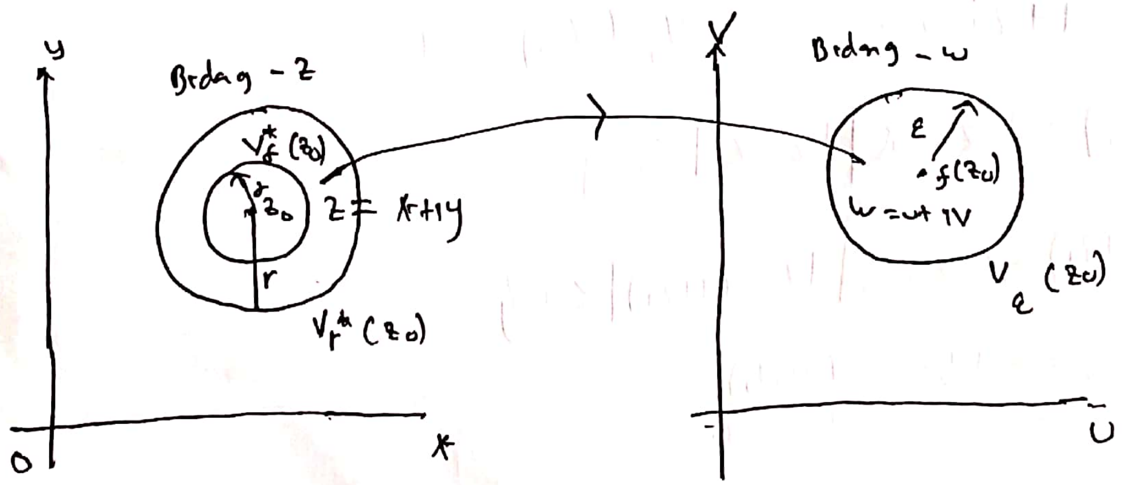
Jika $0 < |z - z_0| < \delta$ maka

$|f(z) - w_0| < \varepsilon$

disini:

$z_0$ = Pusat $V_r^*(z_0)$

$f(z_0) \in V_\varepsilon(w_0) \Leftrightarrow z \in V_r^*(z_0) \cap V_\delta^*(z_0)$



Imanuel AS / 1811141008 Imanuel

Makassar, 20 Mei 2021

Contoh

Buktikan $\lim_{z \to z_0} iz^2 = iz_0^2$

Bukti:

Analisis Pendahuluan

Ambil $\varepsilon > 0$ sebarang akan dicari $\delta > 0$ sehingga jika $0 < |z - z_0| < \delta$ maka $|iz^2 - iz_0^2| < \varepsilon$

Perhatikan bahwa

$$|(z^2 - iz_0^2)| = |i|\,|z^2 - z_0^2|$$
$$= |(z - z_0)(z + z_0)|$$
$$= |z - z_0|\,|z + z_0| \quad \ldots\ldots (*)$$

Misal $|z - z_0| < \delta \leq 1$ akibatnya

$$|z + z_0| = |z - z_0 + 2z_0|$$
$$\leq |z - z_0| + |2z_0|$$
$$= |z - z_0| + 2|z_0|$$
$$< 1 + 2|z_0| \quad \text{, sehingga}$$

berdasarkan (*) diperoleh

$$|iz^2 - iz_0^2| = |z - z_0|\,|z + z_0|$$
$$< |z - z_0|\,(1 + 2|z_0|) < \varepsilon$$

Lp jika $\delta = \dfrac{\varepsilon}{1 + 2|z_0|}$,

Pilih $\delta \leq \min\left\{1, \dfrac{\varepsilon}{1 + 2|z_0|}\right\}$.

Imanuel AS / 1811141008 Imanuel Makassar, 28 Mei 2024.

<u>Bukti Formal</u>

Diberikan $\varepsilon > 0$ sebarang, pilih

$\delta = \min \left\{ 1, \frac{\varepsilon}{1+2|z_0|} \right\}$ sehingga

untuk $|z - z_0| < \delta$ maka

$$
\begin{aligned}
|iz^2 - iz_0^2| &= |i|\,|z^2 - z_0^2| \\
&= |z - z_0|\,|z + z_0| \\
&< \delta\,(1 + 2|z_0|) \\
&< \frac{\varepsilon}{(1+2|z_0|)} \cdot (1 + 2|z_0|) = \varepsilon
\end{aligned}
$$

$\therefore \lim_{z \to z_0} iz^2 = iz_0^2$ //

Analisis Kompleks / Pertemuan ke-12 / PR Makassar, 10 Mei 2021

Imanuel Agung Sembe
1811141008

PR

Buktikan bahwa :

1. $\lim_{z \to z_0} (2z+1) = 2z_0+1$

Penyelesaian :

Bukti :

- Analisis Pendahuluan :

Ambil $\epsilon > 0$ sebarang

Akan dicari $\delta > 0 \ni 0 < |z - z_0| < \delta \Rightarrow |2z+1-(2z_0+1)| < \epsilon$

Perhatikan bahwa,

$$\begin{aligned} |2z+1-(2z_0+1)| &= |2z+1-2z_0-1| \\ &= |2z-2z_0| \\ &= |2|\,|z-z_0| \\ &= 2.\,|z-z_0| \quad \text{.........} (*) \end{aligned}$$

Misal $|z-z_0| < \delta \leq 1$, maka

Berdasarkan (*) diperoleh

$$\begin{aligned} |2z+1-(2z_0+1)| &= 2\,|z-z_0| \\ &< 2\cdot\underbrace{(1)}_{\delta} \\ &< 2 \quad < \epsilon \end{aligned}$$

bila $\delta = \frac{\epsilon}{2}$

Pilih $\delta \leq \min\{1, \frac{\epsilon}{2}\}$

- Bukti Formal

Diberikan $\epsilon > 0$ sebarang

Pilih $\delta = \min\{1, \frac{\epsilon}{2}\}$

Sehingga, jika untuk $|z-z_0| < \delta$ maka

$$\begin{aligned} |2z+1-(2z_0+1)| &= 2\cdot|z-z_0| \\ &< 2\cdot\delta \\ &< 2\cdot\frac{\epsilon}{2} = \epsilon \end{aligned}$$

$\therefore \lim_{z \to z_0} (2z+1) = 2z_0+1$ ▨

Imanuel Agung Sembe / 181141008 Imanuel

Makassar, 10 Mei 2021

(2.) $\lim_{z \to z_0} (2z^2-2) = 2z_0^2 - 2$

Penyelesaian :

Bukti :

-> Analisis Pendahuluan :

Ambil $\epsilon > 0$ sebarang

Akan dicari $\delta > 0 \ni 0 < |z-z_0| < \delta \Rightarrow |2z^2-2-(2z_0^2-2)| < \epsilon$

Perhatikan bahwa

$$
\begin{aligned}
|2z^2-2-(2z_0^2-2)| &= |2z^2-2-2z_0^2+2| \\
&= |2z^2-2z_0^2| \\
&= |2||z^2-z_0^2| \\
&= 2\cdot|z^2-z_0^2| \\
&= 2\,|(z-z_0)(z+z_0)| \\
&= 2\,|z-z_0|\,|z+z_0| \quad \ldots (*)
\end{aligned}
$$

Misal, $|z-z_0| < \delta \leq 1$, akibatnya

$$
\begin{aligned}
|z+z_0| &= |z+z_0-z_0+z_0| \\
&= |z-z_0+2z_0| \\
&\leq |z-z_0| + |2z_0| \qquad [\text{Ketaksamaan } \Delta] \\
&= |z-z_0| + 2|z_0| \\
&< 1 + 2|z_0|
\end{aligned}
$$

sehingga berdasarkan $(*)$ diperoleh

$$
\begin{aligned}
|2z^2-2-(2z_0^2-2)| &= 2\cdot|z-z_0|\cdot|z+z_0| \\
&< 2\cdot|z-z_0|\cdot(1+2|z_0|) < \epsilon
\end{aligned}
$$

$\cancel{\delta\epsilon}$ Bila $\delta = \dfrac{\epsilon}{2(1+2|z_0|)}$

Pilih $\delta \leq \min\left\{1, \dfrac{\epsilon}{2(1+2|z_0|)}\right\}$

-> Bukti Formal

Diberikan $\epsilon > 0$ sebarang, Pilih $\delta = \min\left\{1, \dfrac{\epsilon}{2(1+2|z_0|)}\right\}$, sehingga untuk $|z-z_0| < \delta$ berlaku

$$
\begin{aligned}
|2z^2-2-(2z_0^2-2)| &= 2\,|z-z_0|\,|z+z_0| \\
&< 2\,\delta\cdot(1+2|z_0|) \\
&< 2\cdot\frac{\epsilon}{2(1+2|z_0|)}\cdot(1+2|z_0|) = \epsilon
\end{aligned}
$$

$\therefore \lim_{z \to z_0} 2z^2-2 = 2z_0^2-2$ ∎