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Teori Modul – Latihan-latihan sekedar

SOAL

(1) Diketahui $\mathbb{Z}_{18}$ adalah modul atas ring $\mathbb{Z}_{18}$.

$K = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}$

$J = \{\bar{0}, \bar{6}, \overline{12}\}$

Masing-masing adalah submodul di $\mathbb{Z}_{18}$.

Tentukan elemen-elemen $\dfrac{M/J}{K/J}$ dan $\dfrac{M}{K}$.

Penyelesaian : $\mathbb{Z}_{18} = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}, \overline{16}, \overline{17}\}$

Diketahui : $M = \mathbb{Z}_{18}$ adalah $\mathbb{Z}_{18}$-modul.

K dan J adalah submodul di $M = \mathbb{Z}_{18}$.

- Akan ditunjukkan elemen M/J.

$M/J = \{\bar{a} + J \mid \bar{a} \in M\}$

$= \{\bar{0}+J, \bar{1}+J, \bar{2}+J, \bar{3}+J, \ldots, \overline{17}+J\}$

Dimana,

$\bar{0}+J = \{\bar{0}, \bar{6}, \overline{12}\} = J$

$\bar{1}+J = \{\bar{1}, \bar{7}, \overline{13}\}$

$\bar{2}+J = \{\bar{2}, \bar{8}, \overline{14}\}$

$\bar{3}+J = \{3, \bar{9}, \overline{15}\}$

$\bar{4}+J = \{\bar{4}, \overline{10}, \overline{16}\}$

$\bar{5}+J = \{\bar{5}, \overline{11}, \overline{17}\}$

$\bar{6}+J = \{\bar{6}, \overline{12}, \overline{18} = \bar{0}\} = J$

$\bar{7}+J = \{\bar{7}, \overline{13}, \overline{19} = \bar{1}\} = \bar{1}+J$

$\bar{8}+J = \{\bar{8}, \overline{14}, \overline{20} = \bar{2}\} = \bar{2}+J$

$\bar{9}+J = \{\bar{9}, \overline{15}, \overline{21} = \bar{3}\} = \bar{3}+J$

$\overline{10}+J = \{\overline{10}, \overline{16}, \overline{22} = \bar{4}\} = \bar{4}+J$

$\overline{11}+J = \{\overline{11}, \overline{17}, \overline{23} = \bar{5}\} = \bar{5}+J$

$\overline{12}+J = \{\overline{12}, \overline{18}, \overline{24}\} = \{\overline{12}, \bar{0}, \bar{6}\} = \bar{6}+J = J$

$\overline{13}+J = \{\overline{13}, \overline{19}, \overline{25}\} = \{\overline{13}, \bar{1}, \bar{7}\} = \bar{7}+J = \bar{1}+J$

$\overline{14}+J = \{\overline{14}, \overline{20}, \overline{26}\} = \{\overline{14}, \bar{2}, \bar{8}\} = \bar{8}+J = \bar{2}+J$

$\overline{15}+J = \{\overline{15}, \overline{21}, \overline{27}\} = \{\overline{15}, \bar{3}, \bar{9}\} = \bar{9}+J = \bar{3}+J$

$\overline{16}+J = \{\overline{16}, \overline{22}, \overline{28}\} = \{\overline{16}, \bar{4}, \overline{10}\} = \overline{10}+J = \bar{4}+J$

$\overline{17}+J = \{\overline{17}, \overline{23}, \overline{29}\} = \{\overline{17}, \bar{5}, \overline{11}\} = \overline{11}+J = \bar{5}+J$.

Lebih jauh, M/J adalah Modul Faktor jika berlaku :

- $M = \mathbb{Z}_{18}$ adalah $\mathbb{Z}_{18}$-modul
- J submodul di $M = \mathbb{Z}_{18}$
- $\mathbb{Z}_{18}$ ring dengan unsur kesatuan.

$\therefore M/J = \{J, \bar{1}+J, \bar{2}+J, \bar{3}+J, \bar{4}+J, \bar{5}+J\}$

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- Akan ditunjukkan elemen K/J.

$K/J = \{ \bar{a} + J \mid \bar{a} \in K \}$

$= \{ \bar{0} + J,\ \bar{3} + J,\ \bar{6} + J,\ \bar{9} + J,\ \overline{12} + J,\ \overline{15} + J \}$

Dimana,

$\bar{0} + J = \{ \bar{0}, \bar{6}, \overline{12} \} = J$	$\bar{9} + J = \{ \bar{9}, \overline{15}, \overline{21} = \bar{3} \} = \bar{3} + J$
$\bar{3} + J = \{ \bar{3}, \bar{9}, \overline{15} \}$	$\overline{12} + J = \{ \overline{12}, \overline{18}, \overline{24} = \bar{6} \} = \bar{6} + J = J$
$\bar{6} + J = \{ \bar{6}, \overline{12}, \overline{18} = \bar{0} \} = J$	$\overline{15} + J = \{ \overline{15}, \overline{21}, \overline{27} = \bar{9} \} = \bar{9} + J = \bar{3} + J$

Lebih jauh, K/J adalah Modul Faktor jika berlaku :

- K adalah $\mathbb{Z}_{18}$ - modul.
- J submodul dari K.
- $\mathbb{Z}_{18}$ ring dengan unsur kesatuan

$\therefore K/J = \{ J,\ \bar{3} + J \}$

- Akan ditunjukkan elemen $\dfrac{M/J}{K/J}$.

$M/J \big/ K/J = \{ \bar{a} + K/J \mid \bar{a} \in M/J \}$

$= \{ J + K/J,\ (\bar{1}+J) + K/J,\ (\bar{2}+J) + K/J,\ (\bar{3}+J) + K/J,\ (\bar{4}+J) + K/J,\ (\bar{5}+J) + K/J \}$.

Dimana,

$J + K/J = \{ \bar{0}, \bar{6}, \overline{12} \} + \{ J,\ \bar{3} + J \}$

$= \{ \bar{0}, \bar{6}, \overline{12} \} + \{ \bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15} \}$

$= \{ \bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15} \},\ \{ \bar{6}, \bar{9}, \overline{12}, \overline{15}, \bar{3} \},\ \{ \overline{12}, \overline{15}, \bar{0}, \bar{3}, \bar{9} \}$

$= \{ \bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15} \}$

$(\bar{1}+J) + K/J = \{ \bar{1}, \bar{7}, \overline{13} \} + \{ J,\ \bar{3} + J \}$

$= \{ \bar{1}, \bar{7}, \overline{13} \} + \{ \bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15} \}$

$= \{ \bar{1}, \bar{4}, \bar{7}, \overline{10}, \overline{13}, \overline{16} \},\ \{ \bar{7}, \overline{10}, \overline{13}, \overline{16}, \bar{1}, \overline{10} \},\ \{ \overline{13}, \overline{16}, \bar{1}, \bar{4}, \bar{7}, \overline{10} \}$

$= \{ \bar{1}, \bar{4}, \bar{7}, \overline{10}, \overline{13}, \overline{16} \}$

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$(\bar{2}+J)+K/J = \{\bar{2},\bar{8},\overline{14}\} + \{J, \bar{3}+J\}$

$= \{\bar{2},\bar{8},\overline{14}\} + \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12},\overline{15}\}$

$= \{\bar{2},\bar{5},\bar{8},\overline{11},\overline{14},\overline{17}\}, \{\bar{8},\overline{11},\overline{14},\overline{17},\bar{2},\bar{5}\}, \{\overline{14},\overline{17},\bar{2},\bar{5},\bar{8},\overline{11}\}$

$= \{\bar{2},\bar{5},\bar{8},\overline{11},\overline{14},\overline{17}\}$

$(\bar{3}+J)+K/J = \{\bar{3},\bar{9},\overline{15}\} + \{J, \bar{3}+J\}$

$= \{\bar{3},\bar{9},\overline{15}\} + \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12},\overline{15}\}$

$= \{\bar{3},\bar{6},\bar{9},\overline{12},\overline{15},\bar{0}\}, \{\bar{9},\overline{12},\overline{15},\bar{0},\bar{3},\bar{6}\}, \{\overline{15},\bar{0},\bar{3},\bar{6},\bar{9},\overline{12}\}$

$= \{\bar{3},\bar{6},\bar{9},\overline{12},\overline{15}\} = J + K/J$

$(\bar{4}+J)+K/J = \{\bar{4},\overline{10},\overline{16}\} + \{J, \bar{3}+J\}$

$= \{\bar{4},\overline{10},\overline{16}\} + \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12},\overline{15}\}$

$= \{\bar{4},\bar{7},\overline{10},\overline{13},\overline{16},\bar{1}\}, \{\overline{10},\overline{13},\overline{16},\bar{1},\bar{4},\bar{7}\}, \{\overline{16},\bar{1},\bar{4},\bar{7},\overline{10},\overline{13}\}$

$= \{\bar{4},\bar{7},\overline{10},\overline{13},\overline{16},\bar{1}\} = (\bar{1}+J)+K/J$

$(\bar{5}+J)+K/J = \{\bar{5},\overline{11},\overline{17}\} + \{J, \bar{3}+J\}$

$= \{\bar{5},\overline{11},\overline{17}\} + \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12},\overline{15}\}$

$= \{\bar{5},\bar{8},\overline{11},\overline{14},\overline{17},\bar{2}\}, \{\overline{11},\overline{14},\overline{17},\bar{2},\bar{5},\bar{8}\}, \{\overline{11},\overline{14},\overline{17},\bar{2},\bar{5},\bar{8}\}$

$= \{\bar{5},\bar{8},\overline{11},\overline{14},\overline{17},\bar{2}\}$

$= (\bar{2}+J)+K/J$

$\therefore$ Elemen-elemen $\dfrac{M/J}{K/J}$ adalah

$$\frac{M/J}{K/J} = \{\bar{a}+K/J \mid \bar{a} \in M/J\}$$

$$= \{J+K/J, (\bar{1}+J)+K/J, (\bar{2}+J)+K/J, (\bar{3}+J)+K/J, (\bar{4}+J)+K/J, (\bar{5}+J)+K/J\}$$

$$= \{J+K/J, (\bar{1}+J)+K/J, (\bar{2}+J)+K/J\}$$

$$= \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6},\bar{7},\bar{8},\bar{9},\overline{10},\overline{11},\overline{12},\overline{13},\overline{14},\overline{15},\overline{16},\overline{17}\}$$

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• Akan ditunjukkan elemen $\frac{M}{K}$

$$\frac{M}{K} = \{\overline{a} + K \mid \overline{a} \in M\}$$
$$= \{\overline{0}+K, \overline{1}+K, \overline{2}+K, \overline{3}+K, \dots, \overline{17}+K\}$$

Dimana,

$\overline{0}+K = \{\overline{0}, \overline{3}, \overline{6}, \overline{9}, \overline{12}, \overline{15}\} = K$

$\overline{1}+K = \{\overline{1}, \overline{4}, \overline{7}, \overline{10}, \overline{13}, \overline{16}\}$

$\overline{2}+K = \{\overline{2}, \overline{5}, \overline{8}, \overline{11}, \overline{14}, \overline{17}\}$

$\overline{3}+K = \{\overline{3}, \overline{6}, \overline{9}, \overline{12}, \overline{15}, \overline{18}=\overline{0}\} = K$

$\overline{4}+K = \{\overline{4}, \overline{7}, \overline{10}, \overline{13}, \overline{16}, \overline{19}=\overline{1}\} = \overline{1}+K$

$\overline{5}+K = \{\overline{5}, \overline{8}, \overline{11}, \overline{14}, \overline{17}, \overline{20}=\overline{2}\} = \overline{2}+K$

$\overline{6}+K = \{\overline{6}, \overline{9}, \overline{12}, \overline{15}, \overline{18}=\overline{0}, \overline{21}=\overline{3}\} = K$

$\overline{7}+K = \{\overline{7}, \overline{10}, \overline{13}, \overline{16}, \overline{19}=\overline{1}, \overline{22}=\overline{4}\} = \overline{1}+K$

$\overline{8}+K = \{\overline{8}, \overline{11}, \overline{14}, \overline{17}, \overline{20}=\overline{2}, \overline{23}=\overline{5}\} = \overline{2}+K$

$\overline{9}+K = \{\overline{9}, \overline{12}, \overline{15}, \overline{18}=\overline{0}, \overline{21}=\overline{3}, \overline{24}=\overline{6}\} = K$

$\overline{10}+K = \{\overline{10}, \overline{13}, \overline{16}, \overline{19}=\overline{1}, \overline{22}=\overline{4}, \overline{25}=\overline{7}\} = \overline{1}+K$

$\overline{11}+K = \{\overline{11}, \overline{14}, \overline{17}, \overline{20}=\overline{2}, \overline{23}=\overline{5}, \overline{26}=\overline{8}\} = \overline{2}+K$

$\overline{12}+K = \{\overline{12}, \overline{15}, \overline{18}=\overline{0}, \overline{21}=\overline{3}, \overline{24}=\overline{6}, \overline{27}=\overline{9}\} = K$

$\overline{13}+K = \{\overline{13}, \overline{16}, \overline{19}=\overline{1}, \overline{22}=\overline{4}, \overline{25}=\overline{7}, \overline{28}=\overline{10}\} = \overline{1}+K$

$\overline{14}+K = \{\overline{14}, \overline{17}, \overline{20}=\overline{2}, \overline{23}=\overline{5}, \overline{26}=\overline{8}, \overline{29}=\overline{11}\} = \overline{2}+K$

$\overline{15}+K = \{\overline{15}, \overline{18}=\overline{0}, \overline{21}=\overline{3}, \overline{24}=\overline{6}, \overline{27}=\overline{9}, \overline{30}=\overline{12}\} = K$

$\overline{16}+K = \{\overline{16}, \overline{19}=\overline{1}, \overline{22}=\overline{4}, \overline{25}=\overline{7}, \overline{28}=\overline{10}, \overline{31}=\overline{13}\} = \overline{1}+K$

$\overline{17}+K = \{\overline{17}, \overline{20}=\overline{2}, \overline{23}=\overline{5}, \overline{26}=\overline{8}, \overline{29}=\overline{11}, \overline{32}=\overline{14}\} = \overline{2}+K$

∴ Elemen - elemen $\frac{M}{K}$ adalah

$$\frac{M}{K} = \{\overline{a} + K \mid \overline{a} \in M\}$$
$$= \{\overline{0}+K, \overline{1}+K, \overline{2}+K, \overline{3}+K, \dots, \overline{17}+K\}$$
$$= \{K, \overline{1}+K, \overline{2}+K\}$$
$$= \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}, \overline{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}, \overline{16}, \overline{17}\}$$

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② $\mathbb{Z}_{18}$ modul atas ring $\mathbb{Z}_{18}$.

$S = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}, \overline{16}\}$

$J = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}$

Tunjukkan bahwa order$\left[\frac{S}{S \cap J}\right]$ = order$\left[\frac{S+J}{J}\right]$.

Penyelesaian :

→ Akan ditunjukkan order $\left[\frac{S}{S \cap J}\right]$.

Diketahui : $S = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}, \overline{16}\}$

$S \cap J = \{\bar{0}, \bar{6}, \overline{12}\}$

Perhatikan bahwa,

$$\frac{S}{S \cap J} = \{\bar{a} + (S \cap J) \mid \bar{a} \in S\}$$
$$= \{\bar{a} + \{\bar{0}, \bar{6}, \overline{12}\} \mid \bar{a} \in S\}$$
$$= \{\bar{0} + \{\bar{0}, \bar{6}, \overline{12}\}, \bar{2} + \{\bar{0}, \bar{6}, \overline{12}\}, \dots, \overline{16} + \{\bar{0}, \bar{6}, \overline{12}\}\}$$

Dimana,

$$\bar{0} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\bar{0}, \bar{6}, \overline{12}\} = (S \cap J)$$
$$\bar{2} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\bar{2}, \bar{8}, \overline{14}\} = \bar{2} + (S \cap J)$$
$$\bar{4} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\bar{4}, \overline{10}, \overline{16}\} = \bar{4} + (S \cap J)$$
$$\bar{6} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\bar{6}, \overline{12}, \bar{0}\} = (S \cap J)$$
$$\bar{8} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\bar{8}, \overline{14}, \bar{2}\} = \bar{2} + (S \cap J)$$
$$\overline{10} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\overline{10}, \overline{16}, \bar{4}\} = \bar{4} + (S \cap J)$$
$$\overline{12} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\overline{12}, \bar{0}, \bar{6}\} = (S \cap J)$$
$$\overline{14} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\overline{14}, \bar{2}, \bar{8}\} = \bar{2} + (S \cap J)$$
$$\overline{16} + \{\bar{0}, \bar{6}, \overline{12}\} = \{\overline{16}, \bar{4}, \overline{10}\} = \bar{4} + (S \cap J)$$

Maka, $\frac{S}{S \cap J} = \{(S \cap J), \bar{2} + (S \cap J), \bar{4} + (S \cap J)\}$

∴ Order $\left[\frac{S}{S \cap J}\right] = 3$

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a) Akan ditunjukkan order $\left[\frac{S+J}{J}\right]$.

Diketahui: $S = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}, \overline{12}, \overline{14}, \overline{16}\}$

$J = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}$

$S+J = \{\bar{a}+\bar{b} \mid \bar{a} \in S, \bar{b} \in J\}$

$= \{\bar{0}+\bar{b}, \bar{2}+\bar{b}, \bar{4}+\bar{b}, \dots, \overline{16}+\bar{b}\}$

Dimana,

$\bar{0}+\bar{b} = \bar{0}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = J$

$\bar{2}+\bar{b} = \bar{2}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{2}, \bar{5}, \bar{8}, \overline{11}, \overline{14}, \overline{17}\} = \bar{2}+J$

$\bar{4}+\bar{b} = \bar{4}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{4}, \bar{7}, \overline{10}, \overline{13}, \overline{16}, \bar{1}\} = \bar{4}+J$

$\bar{6}+\bar{b} = \bar{6}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{6}, \bar{9}, \overline{12}, \overline{15}, \bar{0}, \bar{3}\} = J$

$\bar{8}+\bar{b} = \bar{8}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{8}, \overline{11}, \overline{14}, \overline{17}, \bar{2}, \bar{5}\} = \bar{2}+J$

$\overline{10}+\bar{b} = \overline{10}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{10}, \overline{13}, \overline{16}, \bar{1}, \bar{4}, \bar{7}\} = \bar{4}+J$

$\overline{12}+\bar{b} = \overline{12}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{12}, \overline{15}, \bar{0}, \bar{3}, \bar{6}, \bar{9}\} = J$

$\overline{14}+\bar{b} = \overline{14}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{14}, \overline{17}, \bar{2}, \bar{5}, \bar{8}, \overline{11}\} = \bar{2}+J$

$\overline{16}+\bar{b} = \overline{16}+\{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{16}, \bar{1}, \bar{4}, \bar{7}, \overline{10}, \overline{13}\} = \bar{4}+J$

Maka, $S+J = \{J, \bar{2}+J, \bar{4}+J\}$

$= \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}, \overline{16}, \overline{17}\}$

$\therefore$ ~~Order~~ ,

$\longrightarrow$ Lanjut . . .

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Selanjutnya, akan dicari elemen dari $\left[\frac{S+J}{J}\right]$.

Perhatikan bahwa

$$\frac{S+J}{J} = \{ \bar{a} + J \mid \bar{a} \in (S+J) \}$$

$$= \{ \bar{a} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15} \mid \bar{a} \in (S+J) \}$$

$$= \{ \bar{0} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}, \bar{1} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}, \ldots, \overline{17} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}\}$$

Dimana,

$$\bar{0} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = J$$

$$\bar{1} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{1}, \bar{4}, \bar{7}, \overline{10}, \overline{13}, \overline{16}\} = \bar{1} + J$$

$$\bar{2} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{2}, \bar{5}, \bar{8}, \overline{11}, \overline{14}, \overline{17}\} = \bar{2} + J$$

$$\bar{3} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}, \bar{0}\} = J$$

$$\bar{4} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{4}, \bar{7}, \overline{10}, \overline{13}, \overline{16}, \bar{1}\} = \bar{1} + J$$

$$\bar{5} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{5}, \bar{8}, \overline{11}, \overline{14}, \overline{17}, \bar{2}\} = \bar{2} + J$$

$$\bar{6} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{6}, \bar{9}, \overline{12}, \overline{15}, \bar{0}, \bar{3}\} = J$$

$$\bar{7} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{7}, \overline{10}, \overline{13}, \overline{16}, \bar{1}, \bar{4}\} = \bar{1} + J$$

$$\bar{8} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{8}, \overline{11}, \overline{14}, \overline{17}, \bar{2}, \bar{5}\} = \bar{2} + J$$

$$\bar{9} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\bar{9}, \overline{12}, \overline{15}, \bar{0}, \bar{3}, \bar{6}\} = J$$

$$\overline{10} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{10}, \overline{13}, \overline{16}, \bar{1}, \bar{4}, \bar{7}\} = \bar{1} + J$$

$$\overline{11} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{11}, \overline{14}, \overline{17}, \bar{2}, \bar{5}, \bar{8}\} = \bar{2} + J$$

$$\overline{12} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{12}, \overline{15}, \bar{0}, \bar{3}, \bar{6}, \bar{9}\} = J$$

$$\overline{13} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{13}, \overline{16}, \bar{1}, \bar{4}, \bar{7}, \overline{10}\} = \bar{1} + J$$

$$\overline{14} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{14}, \overline{17}, \bar{2}, \bar{5}, \bar{8}, \overline{11}\} = \bar{2} + J$$

$$\overline{15} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{15}, \bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}\} = J$$

$$\overline{16} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{16}, \bar{1}, \bar{4}, \bar{7}, \overline{10}, \overline{13}\} = \bar{1} + J$$

$$\overline{17} + \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\} = \{\overline{17}, \bar{2}, \bar{5}, \bar{8}, \overline{11}, \overline{14}\} = \bar{2} + J$$

Maka, $\frac{S+J}{J} = \{ J, \bar{1} + J, \bar{2} + J \}$

$\therefore$ Order $\left[\frac{S+J}{J}\right] = 3$

$\therefore$ Order $\left[\frac{S}{S \cap J}\right] = $ order $\left[\frac{S+J}{J}\right] = 3$ //