Masalah Syarat Batas / Pertemuan ke-11 / Catatan — Imanuel AS 181114008 — Makassar, 20 April 2021

Contoh Soal

① $B(5,2) = \dfrac{\Gamma(n).\Gamma(m)}{\Gamma(m+n)}$ ($5 \to m$, $2 \to n$)

$= \dfrac{\Gamma(2).\Gamma(5)}{\Gamma(5+2)}$

$= \dfrac{\Gamma(2)\cdot\Gamma(5)}{\Gamma(7)}$

$= \dfrac{1!\cdot 4!}{6!}$

$= \dfrac{1}{6\cdot 5}$

$= \dfrac{1}{30}$ //

② $B\left(\frac{1}{2},3\right) = \dfrac{\Gamma(\frac{1}{2})\cdot\Gamma(3)}{\Gamma(\frac{1}{2}+3)}$

$= \dfrac{\Gamma(\frac{1}{2})\cdot\Gamma(3)}{\Gamma(\frac{7}{2})}$

$= \dfrac{\sqrt{\pi}\cdot 2!}{\frac{15\sqrt{\pi}}{8}}$

$= \sqrt{\pi}\cdot 2\cdot\dfrac{8}{15\sqrt{\pi}}$

$= \dfrac{16}{15}$

$\Gamma(\frac{7}{2}) = \Gamma(\frac{5}{2}+1)$

$= \frac{5}{2}\cdot\Gamma(\frac{5}{2})$

$= \frac{5}{2}\cdot\left[\Gamma(\frac{3}{2}+1)\right]$

$= \frac{5}{2}\cdot\left[\frac{3}{2}\cdot\Gamma(\frac{3}{2})\right]$

$= \frac{5}{2}\cdot\frac{3}{2}\cdot\Gamma(\frac{1}{2}+1)$

$= \frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma(\frac{1}{2})$

$= \frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}$

$= \frac{15}{8}\sqrt{\pi}$

Imanuel AS / 1811141008 ~~Imanuel~~ Manue Makassar, 28 April 2021

(3) $B\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{\Gamma(\frac{1}{2})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}+\frac{1}{2})}$

$= \frac{\sqrt{\pi} \cdot \sqrt{\pi}}{\Gamma(1)}$

$= \frac{\pi}{1}$

$= \pi$ //

(4) $\int_0^1 x^2 (1-x)^5 \, dx = \int_0^1 x^{3-1} \cdot (1-x)^{6-1} \, dx$

$= B(3,6)$

$= \frac{\Gamma(3) \cdot \Gamma(6)}{\Gamma(3+6)}$

$= \frac{2! \cdot 5!}{8!}$

$= \frac{2!}{8.7.6}$

$= \frac{1}{168}$ //

(5) $\int_0^1 (1-x)^4 \, dx = \int_0^1 x^{1-1} \cdot (1-x)^{5-1} \, dx$

$= B(1,5)$

$= \frac{\Gamma(1) \cdot \Gamma(5)}{\Gamma(1+5)}$

$= \frac{\Gamma(1) \cdot \Gamma(5)}{\Gamma(6)}$

$= \frac{0! \cdot 4!}{5!}$

$= \frac{1}{5}$ //

Imanuel AS / 1811141008 ~~Imanuel~~ Imanuel

Makassar, 28 April 2024

(6.) $\int_0^1 x^{\frac{1}{2}} \cdot (1-x)^{\frac{3}{2}}\,dx = \int_0^1 x^{\frac{3}{2}-\frac{2}{2}} \cdot (1-x)^{\frac{5}{2}-\frac{2}{2}}$

$= \int_0^1 x^{\frac{3}{2}-1} \cdot (1-x)^{\frac{5}{2}-1}$

$= B\left(\frac{3}{2}, \frac{5}{2}\right)$

$= \dfrac{\Gamma(\frac{3}{2}) \cdot \Gamma(\frac{5}{2})}{\Gamma(\frac{3}{2}+\frac{5}{2})}$

$= \dfrac{\Gamma(\frac{3}{2}) \cdot \Gamma(\frac{5}{2})}{\Gamma(4)}$

$= \dfrac{\frac{\sqrt{\pi}}{2} \cdot \frac{3\sqrt{\pi}}{4}}{3!}$

$= \frac{3\pi}{8} \cdot \frac{1}{6}$

$= \frac{\pi}{16}$ //

$\Rightarrow \Gamma\left(\frac{3}{2}\right) = \Gamma\left(\frac{1}{2}+1\right)$

$= \frac{1}{2} \cdot \Gamma\left(\frac{1}{2}\right)$

$= \frac{1}{2} \cdot \sqrt{\pi}$

$\Rightarrow \Gamma\left(\frac{5}{2}\right) = \Gamma\left(\frac{3}{2}+1\right)$

$= \frac{3}{2} \cdot \Gamma\left(\frac{3}{2}\right)$

$= \frac{3}{2} \cdot \frac{1}{2}\sqrt{\pi}$

$= \frac{3}{4}\sqrt{\pi}$

~~(7.)~~ $\int_0^1 \frac{x^6}{\sqrt{1-x^3}}\,dx = \int_0^1 x^{\cancel{7-1}}$.

Dipindai dengan CamScanner

Imanuel AS/ 1811141008 Manuel

Makassar, 28 April 2021

(7) $\int_0^1 \frac{x^6}{\sqrt{1-x^3}}\, dx =$ Misal $u = x^3 \Longrightarrow u^{\frac{1}{3}} = x$

$du = 3x^2\, dx$

$\frac{1}{3}\, du = x^2\, dx$

$$= \int_0^1 x^6 \cdot (1-x^3)^{-\frac{1}{2}}\, dx$$

$$= \int_0^1 x^3 \cdot x^3 \cdot (1-x^3)^{-\frac{1}{2}}\, dx$$

$$= \int_0^1 x^3 \cdot x^2 \cdot x\, (1-x^3)^{-\frac{1}{2}}\, dx$$

$$= \int_0^1 x^3 \cdot x \cdot (1-x^3)^{-\frac{1}{2}} \cdot x^2\, dx$$

$$= \int_0^1 x^4 \cdot (1-x^3)^{-\frac{1}{2}} \cdot x^2 \cdot dx$$

$$= \int_0^1 (u^{\frac{1}{3}})^4 \cdot (1-u)^{-\frac{1}{2}} \cdot \frac{1}{3}\, du$$

$$= \frac{1}{3} \cdot \int_0^1 (u)^{\frac{4}{3}} \cdot (1-u)^{-\frac{1}{2}}\, du$$

$$= \frac{1}{3} \cdot \int_0^1 (u)^{\frac{7}{3}-\frac{3}{3}} \cdot (1-u)^{\frac{1}{2}-\frac{2}{2}}\, du$$

$$= \frac{1}{3} \cdot \int_0^1 (u)^{\frac{7}{3}-1} \cdot (1-u)^{\frac{1}{2}-1}\, du$$

$$= \frac{1}{3} \cdot B\left(\frac{7}{3}, \frac{1}{2}\right)$$

$$= \frac{1}{3} \cdot \frac{\Gamma(\frac{7}{3}) \cdot \Gamma(\frac{1}{2})}{\Gamma(\frac{7}{3}+\frac{1}{2})}$$

$$= \frac{1}{3} \cdot \frac{\Gamma(\frac{7}{3}) \cdot \Gamma(\frac{1}{2})}{\Gamma(\frac{17}{6})}$$

$$= \frac{1}{3} \cdot \frac{1{,}18 \cdot \sqrt{\pi}}{1{,}719} = \frac{1180}{5157} \cdot \sqrt{\pi}$$

$$\approx 0{,}228\sqrt{\pi}$$

• $\Gamma(\frac{7}{3}) = \Gamma(\frac{4}{3}+1)$

$= \frac{4}{3} \cdot \Gamma(\frac{4}{3})$

$= \frac{4}{3} \cdot (0{,}89)$

$= 1{,}18$

• $\Gamma(\frac{17}{6}) = 1{,}719$

Nilai $\Gamma(\frac{4}{3})$ dan $\Gamma(\frac{17}{6})$ diambil dari tabel Fungsi gamma : Ebelling, C.E., An Introduction to Reliability and Maintainability Engineering, Mc Graw-Hill, New York, 1997.

Imanuel AS / 1811141008 Makassar, 28 April 2024

8. $\int_0^\infty \frac{x}{(1+x^3)^2}\,dx =$ Misal $u = x^3 \Longrightarrow u = x^3$, $(u)^{\frac{1}{3}} = x$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3x}du = x\,dx$

$\frac{1}{3(u)^{\frac{1}{3}}}du = x\,dx$

$\frac{1}{3}\cdot\frac{1}{(u^{\frac{1}{3}})}du = x\,dx$

$\frac{1}{3u^{\frac{1}{3}}}du = x\,dx$

$$= \int_0^\infty \frac{1}{(1+x^3)^2}\cdot x\,dx$$

$$= \int_0^\infty \frac{1}{(1+u)^2}\cdot\frac{1}{3u^{\frac{1}{3}}}\,du$$

$$= \frac{1}{3}\cdot\int_0^\infty u^{-\frac{1}{3}}\cdot\frac{1}{(1+u)^2}\,du$$

$$= \frac{1}{3}\cdot\int_0^\infty u^{\frac{2}{3}-\frac{3}{3}}\cdot\frac{1}{(1+u)^{\frac{2}{3}+\frac{4}{3}}}\,du$$

$$= \frac{1}{3}\cdot\int_0^\infty u^{\frac{2}{3}-1}\cdot\frac{1}{(1+u)^{\frac{2}{3}+\frac{4}{3}}}\,du$$

$$= \frac{1}{3}\cdot\int_0^\infty \frac{u^{\frac{2}{3}-1}}{(1+u)^{\frac{2}{3}+\frac{4}{3}}}$$

$$= \frac{1}{3}\cdot B\left(\frac{2}{3},\frac{4}{3}\right)$$

$$= \frac{1}{3}\cdot\frac{\Gamma(\frac{2}{3})\cdot\Gamma(\frac{4}{3})}{\Gamma(\frac{2}{3}+\frac{4}{3})}$$

$$= \frac{1}{3}\cdot\frac{(1{,}351)\cdot(0{,}893)}{1{,}18}$$

$$= \frac{1206443}{3540000}$$

$$\approx 0{,}341$$

$\Gamma(\frac{2}{3}) = \frac{\Gamma(\frac{2}{3}+1)}{\frac{2}{3}}$

$= \Gamma(\frac{5}{3})\cdot\frac{3}{2}$

$= 0{,}901\cdot\frac{3}{2}$

$= 1{,}351$

$\Gamma(\frac{4}{3}) = 0{,}893$

$\Gamma(\frac{7}{3}) = 1{,}18$

Based on:
Ebelling, C.E, An Introduction to Reliability and Maintainability Engineering, Mc Graw-Hill, New York, 1997.

Imanuel AS / 1811141008 Imanuel

Makassar, 28 April 2021.

(9.) $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx = \int_0^1 x^2 \cdot (1-x^4)^{-\frac{1}{2}}\,dx$

Misal $u = x^4 \Longrightarrow u^{\frac{1}{4}} = x$

$du = 4x^3\,dx$

$\frac{1}{4}\,du = x^3\,dx$

$\frac{1}{4x}\,du = x^2\,dx$

$\frac{1}{4(u^{\frac{1}{4}})}\,du = x^2\,dx$

$= \int_0^1 (1-x^4)^{-\frac{1}{2}} \cdot x^2\,dx$

$= \int_0^1 (1-u)^{-\frac{1}{2}} \cdot \frac{1}{4(u^{\frac{1}{4}})} \cdot du$

$= \frac{1}{4} \cdot \int_0^1 (u)^{-\frac{1}{4}} \cdot (1-u)^{-\frac{1}{2}}\,du$

$= \frac{1}{4} \cdot \int_0^1 (u)^{\frac{3}{4}-\frac{4}{4}} \cdot (1-u)^{\frac{1}{2}-\frac{2}{2}} \cdot du$

$= \frac{1}{4} \cdot \int_0^1 (u)^{\frac{3}{4}-1} \cdot (1-u)^{\frac{1}{2}-1} \cdot du$

$= \frac{1}{4} \cdot B\left(\frac{3}{4}, \frac{1}{2}\right)$

$= \frac{1}{4} \cdot \frac{\Gamma(\frac{3}{4}) \cdot \Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4}+\frac{1}{2})}$

$= \frac{1}{4} \cdot \frac{(1{,}225) \cdot \sqrt{\pi}}{0{,}906}$

$= \frac{1225}{3624} \cdot \sqrt{\pi}$

$\approx 0{,}338\sqrt{\pi}$

$\cdot\!\!> \Gamma\left(\frac{3}{4}\right) = \frac{\Gamma(\frac{3}{4}+1)}{\frac{3}{4}}$

$= \frac{4}{3} \cdot \Gamma\left(\frac{7}{4}\right)$

$= \frac{4}{3} \cdot 0{,}919$

$= 1{,}225$

$\cdot\!\!> \Gamma\left(\frac{5}{4}\right) = 0{,}906$

Based on :

Ebelling, C. E, An Introduction to Reliability and Maintainability Engineering, Mc Graw-Hill, New York, 1997.

Imanuel AS / 1811141008

Makassar, 28 April 2021

(10.) $\int_0^\infty \frac{x}{(1+x^2)^3}\,dx =$

Misal $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int_0^\infty \frac{1}{(1+x^2)^3} \cdot x\,dx$

$= \int_0^\infty \frac{1}{(1+u)^3} \cdot \frac{1}{2}du$

$= \frac{1}{2} \cdot \int_0^\infty \frac{1}{(1+u)^3}\,du$

$= \frac{1}{2} \cdot \int_0^\infty (u)^{1-1} \cdot \frac{1}{(1+u)^{1+2}} \cdot du$

$= \frac{1}{2} \cdot \int_0^\infty \frac{u^{1-1}}{(1+u)^{1+2}} \cdot du$

$= \frac{1}{2} \cdot B(1,2)$

$= \frac{1}{2} \cdot \frac{\Gamma(1)\cdot\Gamma(2)}{\Gamma(1+3)}$

$= \frac{1}{2} \cdot \frac{0!\cdot 1!}{3!}$

$= \frac{1}{2} \cdot \frac{1}{6}$

$= \frac{1}{12}$ //

Masalah Syarat Batas / Pertemuan ke - 11 / PR Imanuel AS Makassar, 28 April 2021
1811141008
Imanuel

Soal

Tentukanlah rumus dalam bentuk fungsi beta!

$$\int_p^q (q-t)^{m-1} \cdot (t-p)^{n-1}\, dt$$

dengan $q > p$ dan m, n positif.

Penyelesaian :

$$\int_p^q (q-t)^{m-1} \cdot (t-p)^{n-1} \cdot dt =$$

Misal $y = \frac{t-p}{q-p} \Rightarrow t-p = y(q-p)$

$t = yq - yp + p$

$dt = (q-p)\,dy$

Jika $t = p \Rightarrow y = \frac{p-p}{q-p} = \frac{0}{q-p} = 0$

Jika $t = q \Rightarrow y = \frac{q-p}{q-p} = 1$

Maka,

$$= \int_0^1 (q-(yq-yp+p))^{m-1} \cdot ((yq-yp+p)-p)^{n-1} \cdot (q-p)\,dy$$

$$= \int_0^1 (q-qy+py-p)^{m-1} \cdot (qy-py)^{n-1} \cdot (q-p)\,dy$$

$$= \int_0^1 (q(1-y)-p(1-y))^{m-1} \cdot (y(q-p))^{n-1} \cdot (q-p)\,dy$$

$$= \int_0^1 ((q-p)(1-y))^{m-1} \cdot (y(q-p))^{n-1} \cdot (q-p)\,dy$$

$$= \int_0^1 (q-p)^{m-1} \cdot (1-y)^{m-1} \cdot y^{n-1} \cdot (q-p)^{n-1} \cdot (q-p)\,dy$$

$$= \int_0^1 (q-p)^{m-1+n-1+1} \cdot (1-y)^{m-1} \cdot y^{n-1}\,dy$$

$$= \int_0^1 (q-p)^{m-1+n} \cdot (1-y)^{m-1} \cdot y^{n-1}\,dy$$

$$= (q-p)^{m+n-1} \cdot \int_0^1 y^{n-1} \cdot (1-y)^{m-1}\,dy$$

$$= (q-p)^{m+n-1} \cdot B(m,n)$$

//