Makassar, 20 Agustus 2020

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S.1.1 Prove the existance of the limit $\lim_{x \to x_0} (4-12x)$

Answear :

Analisis Pendahuluan:

Akan ditunjukkan : $\forall \varepsilon > 0$, $\exists \delta > 0 \ni$

$$0 < |x - x_0| < \delta \Rightarrow |4 - 12x - (4 - 12x_0)| < \varepsilon$$

Note that,

$$|4 - 12x - (4-12x_0)| = |4 - 12x - 4 + 12x_0|$$
$$= |-12x + 12x_0|$$
$$= |-12|\,|x - x_0|$$
$$= 12\,|x - x_0| < 12\,\delta = \varepsilon$$

Bukti Formal :

Diambil $\varepsilon > 0$

Dipilih $\delta = \frac{\varepsilon}{12}$

Maka untuk $0 < |x - x_0| < \delta$ diperoleh

$$|4-12x - (4-12x_0)| = |4 - 12x - 4 + 12x_0|$$
$$= |-12x + 12x_0|$$
$$= |-12|\,|x - x_0|$$
$$= 12\,|x - x_0| < 12 \cdot \delta = 12 \cdot \frac{\varepsilon}{12} = \varepsilon$$

$\therefore \lim_{x \to x_0} (4-12x)$ Ada.

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Nabangsa, 20 Agustus 20..

5.1.2 Prove the validity of the limit $\lim_{x \to x_0} (ax+b) = ax_0+b$

Answer:

Analisis Pendahuluan:

Akan ditunjukkan: $\forall \varepsilon > 0, \exists \delta > 0 \ni$

$$0 < |x - x_0| < \delta \Rightarrow |ax+b-(ax_0+b)| < \varepsilon$$

Note that,

$$|ax+b-(ax_0+b)| = |ax+b-ax_0+b|$$
$$= |ax - ax_0|$$
$$= a|x-x_0| < a\delta = \varepsilon$$

Bukti Formal:

Diambil $\varepsilon > 0$

Dipilih $\delta = \frac{\varepsilon}{a}$

Maka untuk $0 < |x-x_0| < \delta$ diperoleh

$$|ax+b-(ax_0+b)| = |ax+b-ax_0-b|$$
$$= |ax - ax_0|$$
$$= a|x-x_0| < a\delta = a \cdot \frac{\varepsilon}{a} = \varepsilon$$

$\therefore \lim_{x \to x_0} (ax+b) = ax_0+b$

(Terbukti) ▯

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Buktikan $\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1} = 2$

Answer:

Analisis Pendahuluan:

$\forall \varepsilon > 0, \exists \delta > 0, \ni 0 < |x-1| < \delta$

$\Rightarrow \left| \dfrac{x^2-1}{x-1} - 2 \right| < \varepsilon$

Note that,

karena $\lim\limits_{x \to 1} \Rightarrow x \neq 1$, akibatnya $x-1 \neq 0$,

diperoleh,

$$\left| \frac{x^2-1}{x-1} - 2 \right| = \left| \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} - 2 \right|$$

$$= |x-1| < \delta = \varepsilon$$

Bukti Formal:

Diambil $\varepsilon > 0$

Dipilih $\delta = \varepsilon$

Maka untuk $0 < |x-1| < \delta$ diperoleh

$$\left| \frac{x^2-1}{x-1} - 2 \right| = \left| \frac{(x-1)(x+1)}{(x-1)} - 2 \right|$$

$$= |(x+1) - 2|$$

$$= |x-1| < \delta = \varepsilon$$

$\therefore \lim\limits_{x \to 1} \dfrac{x^2-1}{x-1} = 2$ (Terbukti) $\blacksquare$

Dipindai dengan CamScanner