Modeling the Yield Curve in a New-Keynesian Three-Equation Framework

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INTRODUCTION

In this project we aim to build a system to predict future interest rates and construct the yield curve using a New-Keynesian three-equation (3EQ) model. Our main goal is to understand the yield curve as the result of macroeconomic forces, not just as a financial indicator. In particular, we focus on how inflation, economic activity, expectations and monetary policy interact to shape interest rates across different maturities.

We start from the standard 3EQ framework, which includes the IS equation, the New-Keynesian Phillips Curve and a monetary policy rule. We extend this model to explicitly include the term structure of interest rates so that long-term yields are derived from expectations about future short-term policy rates. In this way, the yield curve is generated endogenously by the model and reflects expected monetary policy decisions.

An important idea in our work is that changes in the yield curve, such as flattening or inversion, can be explained by the central bank's response to macroeconomic shocks and the transmission mechanism between output and inflation.

Overall, our objective is to build a yield curve that is consistent with macroeconomic theory, central bank behavior, and financial market literature, and that allows us to interpret interest rate movements in terms of underlying economic dynamics rather than purely statistical patterns.

ASSUMPTIONS

In order to carry out this project the following assumptions have been made:

Usage of the New-Keynesian 3EQ structure

We assume the economy follows a New-Keynesian three-equation model as it captures the main macroeconomic relationships used in modern monetary economics: the IS equation explains how interest rates affect aggregate demand, the Phillips Curve explains inflation dynamics and the monetary rule describes how the central bank reacts to economic conditions.

A rule-based monetary policy

The central bank is assumed to follow a systematic rule when setting interest rates. This reflects the idea that modern central banks aim to be predictable and transparent, which helps stabilize expectations and reduce uncertainty in financial markets.

Inflation Expectations matter

Expectations about future inflation and policy rates influence both current inflation and long-term interest rates. We assume expectations to be backwards looking, meaning that inflation has a well-defined and structural persistence.

Expectations theory of the yield curve

Long-term interest rates are assumed to be driven mainly by expectations of future short-term rates. This allows us to directly link the yield curve to the expected path of monetary policy derived from the 3EQ model.

Rational expectations

We assume that agents understand the structure of the economy and anticipate how the central bank will react to shocks. This makes financial markets forward-looking and ensures consistency between macroeconomic dynamics and interest rate expectations.

Central bank controls the short-term rate

The short-term nominal interest rate is assumed to be fully controlled by the central bank. Through communication and policy actions, the central bank can influence expectations and therefore affect longer-term interest rates.

Sticky prices in the short run

Prices and wages do not adjust immediately, which allows monetary policy to have real effects in the short run. Without price rigidities, changes in interest rates would only affect inflation, not real output.

Single representative economy

We abstract from international trade and capital flows and treat the economy as closed. This simplifies the analysis and allows us to focus on the core relationship between monetary policy, expectations, and the yield curve.

Stable structural parameters

We assume that key parameters such as the inflation target or the sensitivity of output to interest rates remain stable over time. This allows the model to generate clear predictions without frequent structural breaks.

No financial frictions beyond the yield curve

We do not explicitly model banks or credit constraints. Financial markets affect the economy mainly through interest rates and the yield curve, which keeps the model focused on monetary transmission rather than financial instability.

FORMULAE

In this section we present and formally derive the algebraic procedure used to obtain the expression governing the evolution of the interest rate set by the central bank (CB) within a closed, deterministic New-Keynesian three-equation (3EQ) model.

1. The New-Keynesian 3EQ Model

We begin by defining the three core equations of the model, which jointly determine the dynamics of the three endogenous variables:

Equation (1): IS Curve

$$y_t = A_t - ar_{t-1}$$

Equation (2): Phillips Curve

$$\pi_t = \pi_{t-1} + \alpha(y_t - y^*)$$

Equation (3): Monetary rule

$$(y_t - y^*) = -\alpha\beta(\pi_t - \pi^T)$$

By combining Equations (1)-(3) and interpreting the monetary rules as the central bank's policy decision rule, we obtain the real interest rate determination equation:

Equation (4): Real Interest Rate Determination

$$r_t = r_t^s + \frac{\alpha\beta}{a(1+\alpha^2\beta)}(\pi_t - \pi^T)$$

Throughout the analysis, we assume a constant natural rate of interest r*, an assumption that will be justified later.

2. Auxiliary Variables and Shock Representation

The next step is to introduce three auxiliary variables that fully characterize the dynamics of the 3EQ model:

$$\tilde{r}_t = r_t - r^* \ \tilde{\pi}_t = \pi_t - \pi^T \ \tilde{A}_t = A_t - y^*$$

One of these variables captures the deviation between autonomous consumption and equilibrium output. This variable serves as a proxy for exogeneity and indicates the presence or absence of aggregate demand shocks.

In equilibrium, this variable satisfies:

Equation (5): No-Shock Condition

$$\tilde{A}_t = ar*$$

Thus, whenever no exogenous shocks are present, this relationship must hold. Since our analysis is conducted in a no-shock environment, this condition can be imposed without loss of generality.

Using this assumption we combine the three auxiliary variables into the real interest determination equation by first incorporating them into a modified Phillips curve.

Equation (6): Modified Phillips Curve

$$\tilde{\pi}_t = \tilde{\pi}_{t-1} - a\tilde{r}_{t-1}\alpha$$

3. Inflation Dynamics and Interest Rate Recursion

To characterize the dynamic behavior of inflation, we invert the monetary policy rule to express inflation in terms of interest rates.

Equation (7): Inflation as a Function of Interest Rates

$$\tilde{\pi}_{t-1} = \frac{a(1+\alpha^2\beta)}{\alpha\beta}\tilde{r}_{t-1}$$

We now introduce our final and most fundamental assumption.

4. No-Shock Assumption and Recursive Interest Rate Dynamics

We assume that no exogenous shocks occur after period t=0. In other words, the model describes the evolution of interest rates over a horizon in which shocks are absent.

Formally, for all t>0, the exogeneity proxy satisfies equation (5). Under this assumption, the real interest rate setting equation for t>0 becomes:

Equation (8): Interest Rate Decision Rule

$$\tilde{r}_t = \frac{\alpha\beta}{a(1+\alpha^2\beta)}\tilde{\pi}_t$$

Substituting Equation (7) and using condition (5), we reorganize terms to obtain the final expression for the central bank's real interest rate decision rule:

Equation (9): Recursive Real Interest Rate Rule

$$r_t = r^* + (\frac{1}{1+\alpha^2\beta})(r_{t-1} - r^*)$$

This expression is recursive: The current real interest rate depends on its lagged variable value and a constant term. Given an initial condition r0, the entire path of the real interest rate can be determined.

Equation (10): General Solution

$$r_n = r^* + (r_0 - r^*)(\frac{1}{1+\alpha^2\beta})^n \ \lim_{n\to\infty} r_n = r^*$$

This result is consistent with standard macroeconomic theory and implies that real interest rates converge in the long run to their equilibrium level.

5. Extension to Nominal Interest Rates

The same recursive structure applies to nominal interest rates, which are the relevant rates observed along the yield curve. Real and nominal interest rates follow the same geometric progression; the only difference is that nominal rates are shifted by the inflation rate.

To derive nominal interest rate dynamics, we start from the inflation equation introduced earlier:

Equation (6): Inflation Dynamics

$$\tilde{\pi}_t = \tilde{\pi}_{t-1} - a\tilde{r}_{t-1}\alpha$$

Using the recursive expression for rt, we obtain the linear equation to express inflation in recursive terms.

Equation (11): Recursive Inflation Equation

$$\tilde{\pi}_t = \tilde{\pi}_{t-1} - a(r_0 - r^*)\alpha\lambda^{t-1}, \lambda = (\frac{1}{1+\alpha^2\beta})^{t-1}$$

We iterate this linear equation to express inflation in terms of initial conditions and apply the standard formula for a convergent geometric series:

Equation (12): Inflation Closed Form

$$\tilde{\pi}_t = \tilde{\pi}_0 - a(r_0 - r^*)\alpha \sum_{j=0}^{t-1} \lambda_j$$

Finally, we combine the inflation dynamics with Fisher's equation:

Equation (13): Fisher's Equation

$$i_t = r_t + \pi_t$$

Rearranging term yields:

Equation (14): Nominal Interest Rate Dynamics

$$i_t = r^* + \pi^T + \tilde{\pi}_0 - \frac{a\alpha(r_0 - r^*)}{1-\lambda} + (r_0 - r^*)(\lambda^t + \frac{a\alpha}{1-\lambda}\lambda^t)$$

$$i_t = i^* + C + D\lambda^t$$

Where i* denotes the equilibrium interest rate. Thus, nominal interest rates follow the same geometric structure as real rates, displaced by inflation and adjusted by initial conditions.

6. Summary of Assumptions

The assumptions underlying the algebraic derivations can be summarized as follows:

1. No-Shock Assumption
   To characterize the central bank interest rate setting as a geometric sequence, the model assumes the absence of exogenous shocks over the analyzed horizon.

2. Perfect Information
   All economic agents possess perfect information. When a shock occurs at t=0, agents can immediately identify whether it is permanent or temporary. This assumption allows the equilibrium interest rate to be treated as constant.

Importantly, the second assumption does not impose additional restrictions. Perfect information for the central bank is already implicit in the standard New-Keynesian 3EQ framework

7. Yield Curve Modeling

Having determined the evolution of real and nominal interest rates under clearly stated assumptions, we can now construct a yield curve model based on the no-arbitrage principle.

The no-arbitrage condition implies that there are no profit opportunities from investing in bonds of identical risk but different maturities. Formally:

Equation (15): No-Arbitrage Condition

$$(1 + i_{l/r,t})^2 = (1 + i_{s/r,t})(1 + i^e_{s/r,t+1})$$

For simplicity, we take logarithms and apply a first-order approximation:

Equation (16): Log-Linear Approximation

$$i_{l/r,t} = \frac{i_{s/r,t} + i^e_{s/r,t+1}}{2}$$

Generalizing to an n-period bond:

Equation (17): Yield Curve Equation

$$i_{l/r,t} = \frac{i_{s/r,t} + \sum_{t+1}^{t+N-1} i^e_{s/r,t}}{N}$$

The second term represents the sum of expected future nominal interest rates. Under perfect information, these expectations are formed without error.

Substituting our recursive interest rate model into Equation (19):

Equation (18): Yield Curve with Recursive Rates

$$i_{l/r,t} = \frac{1}{N}\sum_{t=0}^{N} i^* + C + D\lambda^t$$

$$i_{l/r,t} = i^* + C + \frac{D}{N}\frac{(1-\lambda^N)}{1-\lambda}$$

Applying the geometric series formula yields a closed-form solution for yields at any maturity.

8. Term Premium and Model Extension

To enhance realism without altering the core conclusions, we relax the strict no-shock assumption by introducing a term premium

Equation (19): Yield Curve with Term Premium

$$i_{l/r,t} = i^* + C + \frac{D}{N}\frac{(1-\lambda^N)}{1-\lambda} + TP$$

The term premium captures compensation for unexpected shocks and deviations from the no-arbitrage condition. This addition aligns the model with financial market reality while preserving its structural consistency.

9. Concluding Remarks

This framework demonstrates how the recursive structure of the closed and deterministic New-Keynesian 3EQ model can be extended to financial markets. By integrating macroeconomic dynamics with no-arbitrage bond pricing, we expand the scope of the model without imposing additional restrictive assumptions.

In this way, the recursiveness of New-Keynesian monetary policy is naturally linked to yield curve modeling, bridging macroeconomic theory and financial literature.

PRACTICAL CASES

We now apply the extended model to two illustrative scenarios. Both cases fall under the core assumptions that allow us to characterize interest rate dynamics in the New-Keynesian 3EQ framework as recursive and to derive the associated yield curves under the no-arbitrage condition.

The first scenario considers a positive permanent aggregate demand (AD) shock, with no further exogenous disturbances occurring over the maturities of the bonds considered. This case illustrates the implications of the perfect information assumption, under which economic agents immediately identify the permanent nature of the shock.

The second scenario examines a negative permanent aggregate demand shock. The change in analytical structure and the sign of the shock leads to qualitatively different interest rate dynamics and yield curve shapes. We relate these results to standard financial theory.

In both cases, the yield curve is derived using the deterministic yield curve equation obtained from the New-Keynesian 3EQ model:

Equation (18): Yield Curve with Recursive Rates

$$i_{l/r,t} = i^* + C + \frac{D}{N}\frac{(1-\lambda^N)}{1-\lambda}$$

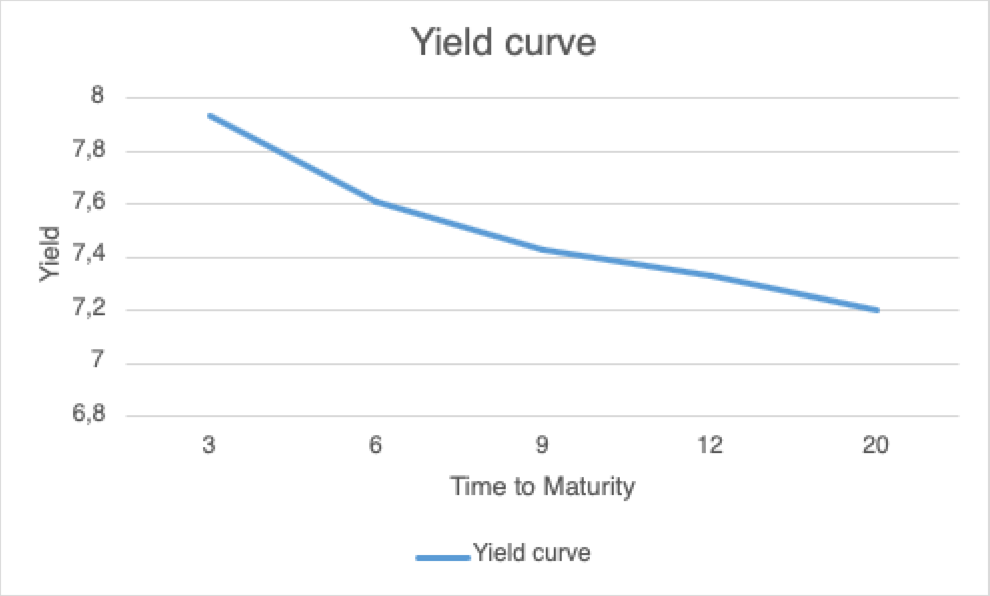
where expectations are formed under perfect information and no-arbitrage conditions.

First Simulation: Positive Permanent AD Shock

In the first simulation the economy is subject to a positive permanent aggregate demand shock at period t=0. The structural parameters and target values governing the 3EQ model are set as follows:

r0	r*	i*	α	β	a	C	D	λ
5,3333	5	7	0,5	2	2	1/1000	1,3332	2/3

Given the permanent nature of the shock and the absence of subsequent disturbances, the central bank raises the policy rate above its equilibrium level in order to counteract inflationary pressures.



Results and Interpretation

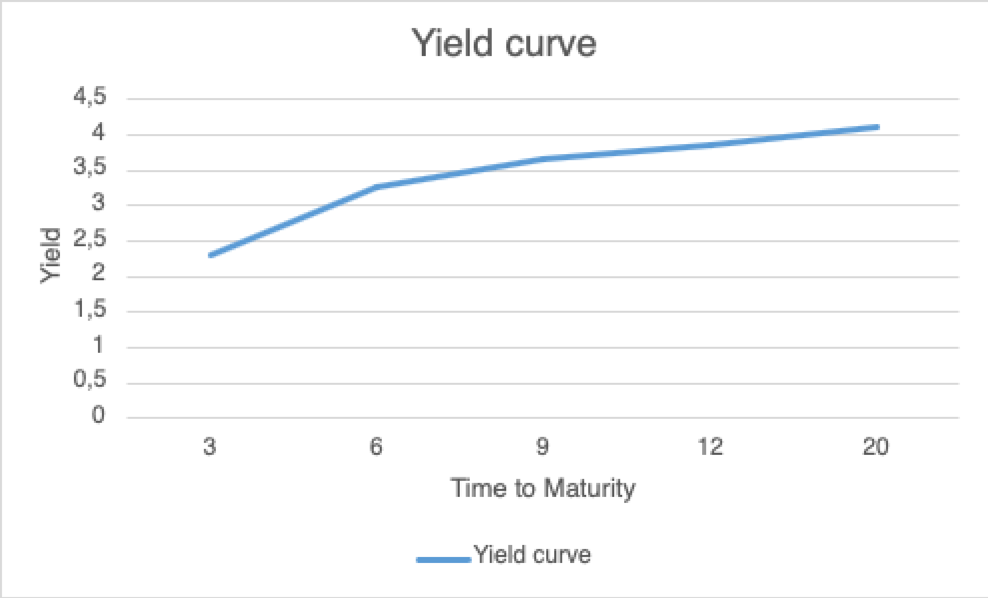
The resulting yield curve captures the full dynamic adjustment of nominal interest rates implied by the central bank's policy response. In this case, the highest yields are observed at shorter maturities, while yields decline as maturity increases.

This produces an inverted yield curve, reflecting expectations of declining future short-term interest rates as monetary policy gradually converges back to its long-run equilibrium. Such a pattern is consistent with financial and macroeconomic literature, where inverted yield curves are commonly interpreted as signals of future economic slowdown following periods of monetary tightening.

Second Simulation: Negative Permanent AD Shock

In the second simulation, the economy experiences a negative permanent aggregate demand shock at period t=0. Different structural parameters are used, allowing that differences in outcomes arise from the sign of the shock and the structure of the economy.

r0	r*	i*	α	β	a	C	D	λ
1,75	2,5	4,5	1	1	2	0	-3,75	1/2



Results and Interpretation

In response to the negative AD shock, the central bank lowers the policy rate below its equilibrium level to stimulate economic activity. As a result, the lowest yields are observed at shorter maturities, while yields increase with maturity.

The resulting yield curve exhibits a positive (non-inverted) slope, with longer-maturity bonds offering higher yields. This shape reflects market expectations of a gradual recovery in economic activity and rising future interest rates, in line with standard interpretations in financial economics.

Concluding Remarks

As emphasized throughout the theoretical section, the deterministic applicability of the model requires that no exogenous shocks occur over the maturity horizon of the bonds considered. This condition ensures that the no-arbitrage principle holds and that expectations are formed without error.

The perfect information assumption is also evident in these simulations. In both cases, the shock induces a permanent shift in the economy toward a new equilibrium level of real and nominal interest rates. This is reflected in the yield curves, which converge toward the new equilibrium nominal rate as maturity increases.

The slope of the yield curve emerges endogenously from the persistence parameter λ embedded in the interest rate dynamics. Higher values of λ ,as the one of the first case, imply slower convergence of short-term interest rates toward equilibrium, resulting in flatter yield curves. Conversely, lower values of λ generate steeper curves due to faster adjustment. This mechanism captures the diminishing marginal effect of maturity on yields as equilibrium is approached.

An additional key property is the inversion or non-inversion of the yield curve, which is particularly relevant in the short run. Since shocks are neutralized in the long run within the 3EQ framework, yield curve shape primarily reflects short- and medium-term expectations. In the first simulation, yield curve inversion signals anticipated economic deceleration, consistent with empirical findings that associate inverted curves with recessions. In the second simulation, the positive slope reflects expectations of economic recovery and output expansion.

Finally, the direction of the yield curve has implications for investment incentives across maturities. Beyond the strict confine of the model, in a real-world environment where perfect information and strict no-arbitrage conditions may not hold, longer maturities entail greater risk. An inverted yield curve may therefore discourage long-term investment and increase governments' financing costs through higher term premia. By contrast, the positively sloped curve observed in the second simulation aligns with the conventional structure of yield curves and is more conducive to long-term financing.

Overall, these practical cases illustrate how the recursive structure of the New-Keynesian 3EQ model can be meaningfully extended to financial markets, allowing macroeconomic dynamics to inform yield curve behavior in a theoretically consistent manner.

GENERAL CONCLUSIONS

In this project, we have shown how the standard New-Keynesian three-equation (3EQ) model can be used to construct and explain the yield curve in a clear and consistent way. By adding the term structure of interest rates to the 3EQ framework, we are able to connect macroeconomic variables such as inflation and output with financial market interest rates.

One of the main results of our analysis is that interest rates in the model follow a recursive path and gradually converge to a long-run equilibrium level. Under our assumptions, this allows us to derive explicit expressions for real and nominal interest rates, and then use the no-arbitrage condition to obtain yields for different maturities. As a result, the yield curve is generated endogenously by the model rather than being imposed from outside.

Our simulations show that the shape of the yield curve depends on expectations about future monetary policy and how fast interest rates return to equilibrium. In the case of a positive aggregate demand shock, the central bank raises interest rates, which leads to an inverted yield curve. This reflects expectations that short-term rates will fall in the future as policy becomes less restrictive. On the other hand, a negative demand shock leads to lower short-term rates and a positively sloped yield curve, which is consistent with expectations of economic recovery.

These results help explain why yield curve inversions are often associated with future economic slowdowns, but also show that inversion does not automatically mean a recession will occur. Instead, the yield curve mainly reflects how markets expect the central bank to react over time.

Although our model is based on strong assumptions, such as perfect information and no further shocks, it provides a useful and simple framework to understand the link between monetary policy and the yield curve. Overall, this project shows that the New-Keynesian 3EQ model can be extended to financial markets in a meaningful way and helps explain yield curve behavior using standard macroeconomic theory.

Despite its strengths, this framework is subject to several limitations. The model relies on strong assumptions, such as perfect information, the absence of additional shocks and a deterministic environment. These assumptions simplify the analysis and allow for closed-form solutions, but they reduce realism, especially in a financial context where uncertainty and risk premia are important. In addition, we abstract from financial frictions, open-economy effects and time-varying structural parameters. These features could be incorporated in future extensions of the model to better match real-world yield curve behavior. Nevertheless, within its assumptions, the model provides a clear and consistent link between monetary policy, macroeconomic dynamics and the yield curve.

REFERENCES

Carlin, W. and Soskice, D. (2024): "Macroeconomics: Institutions, Instability, and Inequality", Oxford Uni. Press

Leila Davis & Thomas R. Michl (2024): The inverted yield curve in a 3-equation model Leila Davis & Thomas R. Michl*

Nicola Anderson, Francis Breedon, Mark Deacon, Andrew Derry, Gareth Murphy (1996): ECB yield curve methodology
https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/shared/pdf/technical_notes.pdf