

①₁) 15^8 possible outcomes no restriction

$15!$ possible outcomes if no student answers more than 1 question

$$\frac{15!}{15^8} = 0.101$$

①₂) Total Combinations: $10^5 = 100,000$

Combinations meeting restrictions:

$$\underbrace{5C_1 \times 4C_1}_{\text{Odd no.s}} \times \underbrace{5C_1 \times 7C_1}_{\text{Even no.}} \times \underbrace{6C_1}_{\substack{\text{Unique random} \\ \text{ints from} \\ \text{rearing} \\ \text{bunch}}} = 4200 \text{ combinations}$$

Probability of one right combo:

$$\frac{4200}{100000} = 0.042$$

Probability of one right after 8 tries:

$$= 0.249 \quad {}^8C_1 \times \underbrace{0.042}_{\text{success}} \times \underbrace{(1-0.042)^7}_{\text{failed}}$$

3) Yes they are independent as one outcome doesn't affect the outcome of the others

4) Total Number of Dealings: $\frac{52!}{47!} = 311875200$

Number of flushes: 4

Number of flush of one suit:

$$\frac{13!}{8!}$$

Number of flush in one deck:

$$4 \times \frac{13!}{8!} = 617760$$

Thus probability of flush is:

$$\frac{617760}{311875200} = 0.00198$$

Expected no. of hands:

$$\frac{1}{0.00198} = 505 \text{ hands}$$

5)

X = Team wins $4/5$ games

S = superstar plays

S' = superstar doesn't play

$$P(S) = 0.75$$

$$P(S') = 0.25$$

$$P(X/S) = {}^5C_4 \times 0.7^4 \times 0.3 = 0.36015$$

$$P(X/S') = {}^5C_4 \times 0.7^4 \times 0.5 = 0.15625$$

$$\text{Total Cases: } \begin{array}{cc} \text{Superstar} & \text{Superstar} \\ \text{plays} & \text{doesn't} \\ \uparrow & \downarrow \end{array} 0.36015 \times 0.75 + 0.15625 \times 0.25$$

Cases where Team wins $4/5$ and
superstar
plays

$$P(S/X) = \frac{0.36015 \times 0.75}{0.36015 \times 0.75 + 0.15625 \times 0.25}$$

$$\approx 0.874$$