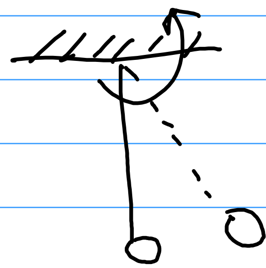
Session 3: Non-Linear ODEs

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad x(t) = A\sin(\omega t + \phi)$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta)$$

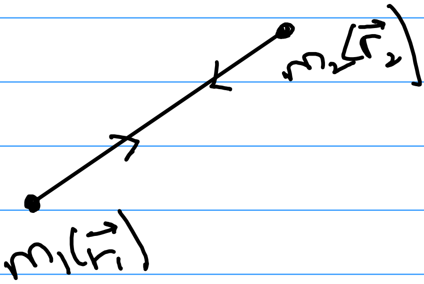


$$\sin\theta \approx \theta \qquad \theta << 1$$

$$\sin\theta = \theta - \cancel{\frac{\theta^3}{3!}} + \cancel{\frac{\theta^5}{5!}} - \dots$$

$\theta = 0 \longrightarrow$ singularity

Gravitation (Newtonian)



$$F_{12} = -\frac{Gm_1m_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2) = m_1\frac{d^2\vec{r}_1}{dt^2} \quad (1)$$

$$F_{21} = -\frac{Gm_1m_2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1) = m_2\frac{d^2\vec{r}_2}{dt^2} \quad (2)$$

$$\vec{F} = \frac{Gm_1m_2}{|r|^2}\hat{r}\frac{|r|}{|r|} \longrightarrow \vec{r}$$

(2) − (1)

$$\frac{d^2(\vec{r}_2-\vec{r}_1)}{dt^2} = -\frac{G}{|\vec{r}_2-\vec{r}_1|^3}\left[m_1(\vec{r}_2-\vec{r}_1) - m_2(\vec{r}_1-\vec{r}_2)\right]$$

$\vec{r}_2 - \vec{r}_1 = \vec{r} \longrightarrow$ separation vector

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G}{|\vec{r}_2 - \vec{r}_1|^3}(m_1+m_2)\vec{r} = -\frac{GM}{|\vec{r}|^3}\vec{r}$$

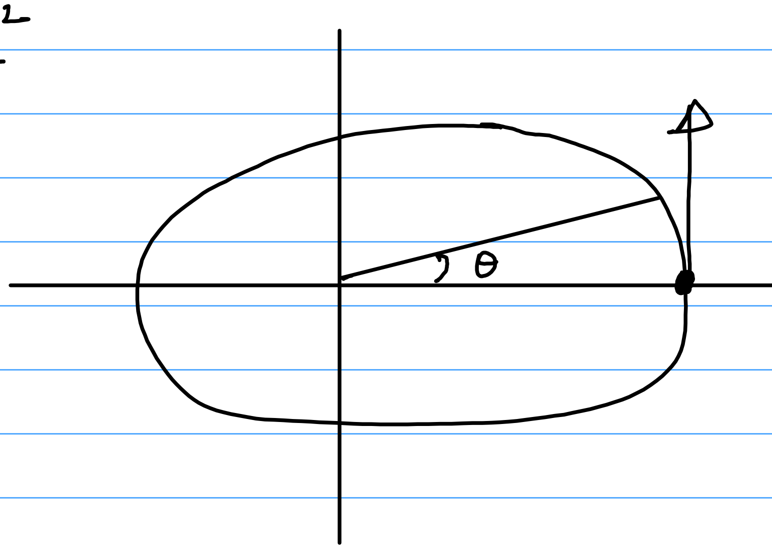
$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^2}\hat{r} \qquad \vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left[\begin{array}{l} \dfrac{d\vec{r}}{dt} = \dot{\vec{r}} \\[2ex] \dfrac{d\dot{\vec{r}}}{dt} = -\dfrac{GM}{r^2}\hat{r} = -\underbrace{\dfrac{GM}{r^3}}_{K}\vec{r} \end{array}\right.$$

$$\frac{d}{dt}\begin{bmatrix} \vec{r} \\ \dot{\vec{r}} \end{bmatrix} = \begin{bmatrix} \dot{\vec{r}} \\ -\frac{GM}{r^3}\vec{r} \end{bmatrix} \qquad A(r)$$

$$|\vec{r}| = \sqrt{r[0]^2 + r[1]^2} = \sqrt{x^2+y^2}$$



$$r(\theta) = \frac{p}{1 + e\cos(\theta)}$$