Discrete Structures 1 - Comprehensive Makeup

December 7, 2023

Abstract

The retake has two parts, an "in person" part and a "homework" part. The in person part must be uploaded through blackboard as answer to the exam "Comprehensive Makeup in Person". The homework must be uploaded before Sunday, 10h December, at 23:00 hours as answer to the assignment "Comprehensive Makeup Homework".

For this exam you may use any material you have brought with you. You cannot access the internet, though. Questions are allowed during the test, but the instructors retain the right of not answering if they consider it is not appropriate.

Exercise 1: (first midterm in person) Given sets A, B and C, answer true or false to the following assertions:

1. If $AUC = BUC$ then $A = B$
2. If $A - C = B - C$ then $A = B$
3. $AU(B \cap C) = (AUB) \cap (AUC)$
4. If $\sim A = \sim B$ then $A = B$
5. $(A \cap B)UC = A \cap (BUC)$

Expected time to solve: 15'

Exercise 2: (first midterm in person) For each one of the following functions determine if they are one-to-one or onto.

1. $f : R \to R$, $f(x) = x^3$.
2. $f : R^{>0} \to R$, $f(x) = x^3$.
3. $f : R \to R$, $f(x) = |x^3|$.

Expected time to solve: 15'

Exercise 3: (first midterm in person) For the following propositions determine if they are tautologies. Prove it or give a counterexample.

1. $p1 \vee !p1$
2. $p1 => (p2 \vee p1)$
3. $(p1 ==> p2) \wedge (p1 ==> p3) ==> (p1 ==> p2 \wedge p3)$

Expected time to solve: 15'

Exercise 1: (second midterm in person) Consider the "code" where all n-bit strings are codewords. (That is, $C = \{0,1\}^n$.) What is the rate and minimum distance, and how many errors can it detect or correct?
Expected time to solve: 15'

Exercise 2: (second midterm in person) One of the following statements is true and one is false:

- If $x.y$ and x are both rational, then y is too.
- If $x - y$ and x are both rational, then y is too.

Decide which statement is true and which is false, and give proof/disproof of both.
Expected time to solve: 15'

Exercise 3: (second midterm in person) Prove that for all nonnegative integers

$$\sum_{i=0}^{n} i.(i!) = (n+1)! - 1.$$

Expected time to solve: 15'

Exercise 4: (first midterm homework) Add all the missing paretheses to the following propositions so that the resulting proposition is equivalent to the original one.

1. $p1 \wedge p2 => p3 \vee p2$
2. $p1 \wedge (p2 \vee p3) => p1 => p2 \vee p3$
3. $!p2 => p1 \vee p2$

Expected time to solve: 15'

Exercise 5: (first midterm homework) Translate the following assertions in English into propositions.

1. If you want to change shifts, you must step on the brake and on the clutch.

2. if you do not like horror movies, then you should not like Director M. Night Shyamalan

3. you will pass the midterm only if you study and solve this practice test.

Expected time to solve: 15'

Exercise 6 (first midterm homework) Consider the language $L =< \{0,1\}, \{+_2, *_2, -1\}, \{\geq, ! =\} >$. Consider the adequate structure $Int =< Z, \{0,1\}, \{+, *, ^- 1\}, \{\geq, ! =\} >$. For each of the following predicate determine if they are valid in Int.

1. $(Ex)(0 \leq x \wedge 0! = x \wedge x \leq 1 \wedge x! = 1)$
2. $isPrime(1) => (Vx)(isPrime(x) => isPrime(x+1))$
3. $(Vx)(Vy)(x \leq y => x+1 \leq y+1)$

Expected time to solve: 25'

Exercise 4: (second midterm homework) Let $n \geq 10$ be any positive integer. Prove that the set $\{n, n+1, ..., n+5\}$ contains at most two prime numbers.
Expected time to solve: 15'

Exercise 5: (second midterm homework) False Claim:. $\frac{\sqrt{2}}{4}$ and $\frac{8}{\sqrt{2}}$ are both rational.
Bogus proof. In class we proved that if x and y are rational then $x.y$ is rational too. Here, let $x = \frac{\sqrt{2}}{4}$ and $y = \frac{8}{\sqrt{2}}$. Then $x.y = \frac{\sqrt{2}}{4} = \frac{8}{\sqrt{2}} = 2$. So $x.y = 2$ is rational, and x and y are too.
Expected time to solve: 15'

Exercise 6 (second midterm homework) This is a variant of the definition of lists, where we insist that the elements be in increasing order. Define a nonempty sorted list as one of the following:

- $<x, <>>$; or
- $<x, <y, L>>$ where $x \leq y$ and $<y, L>$ is a nonempty sorted list.

Prove by structural induction that in a nonempty sorted list $<x, L>$, every element z in L satisfies $z \geq x$.
Expected time to solve: 25'