10-Aug-2025

Agenda - Distribution → Bernoulli ✓ exponential
Binomial normal
Poisson $\boxed{}$

Binomial D :-

Bernoulli - models 1 trial (1 customer)
out: 0 (failure / no purchase)
: 1 (success / purchase)

Binomial : models n Bernoulli trials (15 customer)
: count of successes
e.g: total purchase across 15 customers.

→ 0 → prob
→ 1 → prob
→ 2 → prob
→ 3 → prob

} PMF

coin
H T
→ 0.7 (1-H)

→ Each call has $p=0.2$, chance of success (sale)
→ $n=15$ independent calls

Q: $p(x=3 \text{ sales})$

PMF (k)
↓
3

$$p(3) = \binom{n}{k} p^k (1-p)^{n-k}$$

$^{n}C_{k} \rightarrow {}^{15}C_{3} = \frac{15!}{3! \cdot 12!} = 455$

$p^{k} \cdot (1-p)^{n-k} \rightarrow (0.2)^{3} \times (0.8)^{12} \approx 0.008 \times 0.0687 \approx 0.00055$

$$^{n}C_{k} \cdot p^{k}(1-p)^{n-k} = 455 \times 0.00055 \approx \underline{\underline{0.25}}$$

$P(3)$ = 25% change of exactly 3 sales.

trial (1) = 15 , 0.2 → 6 ← not with prob, its just simulation from binomial distribution

(2) = — 11 — → 1

(3) — — 11 — → 3

(4)

⋮

(100) — 11 — 4

Call centre

↓

$\boxed{15, 0.2}$

↓

100 agent →

sum( )

↓

estimate

total_calls / total agent → avg calls per agent →

Poisson Distribution

Models count of rare events occurring at a constant rate (λ) in a fixed internal.

Discrete : values are non-negative integers

parameter : λ (average rate of events)

Memoryless : Event occur independently,

web traffic: visit per minute

Custom service: calls per hour

formula: $P(k) = \dfrac{e^{-\lambda} \cdot \lambda^{k}}{k!}$

Example:

A website averages 4 visit per hour $= \lambda = 4$

Question:

what's the prob of getting exactly 6 visit in a hour.

$k = 6$

$P(6) = \dfrac{e^{-4} \cdot 4^{6}}{6!} \approx 0.104$ $\quad e = 2.7182$

: Simulate a website week's traffic : estimate max.

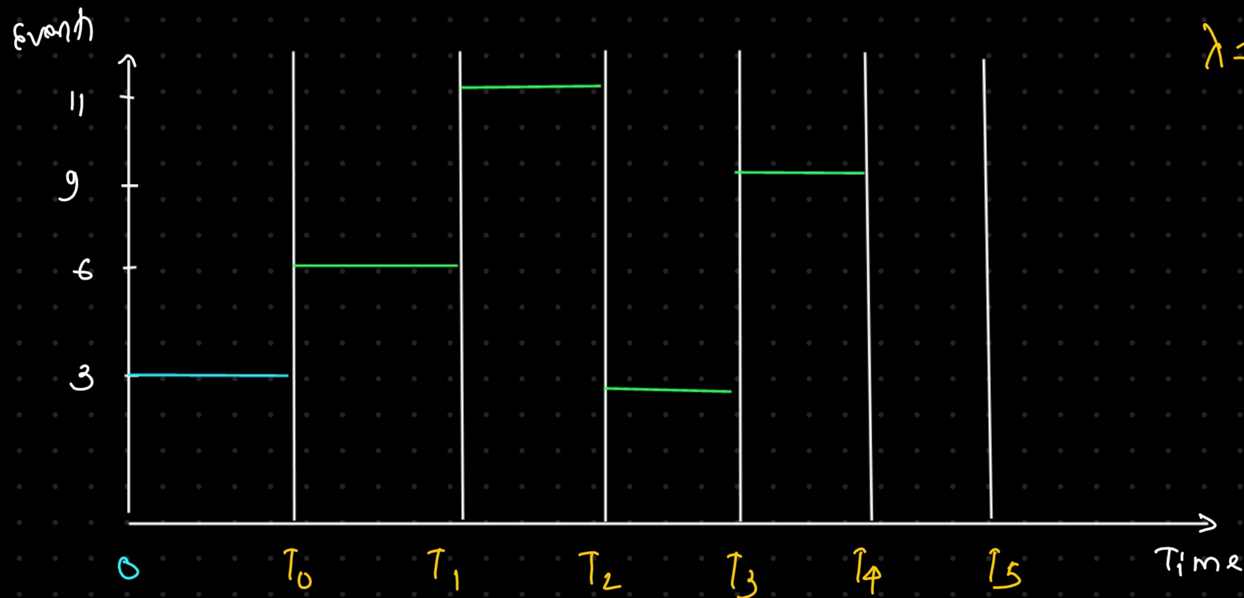
7 days $\times$ 24 hrs $\rightarrow$ 168 hrs

System → AWS

1gb → → 200 mb
→ 1vCPU
→ 10gb disk → 5 calls per hr
↓
estimate
↓
Day - 1

go live → produ → call → $\boxed{5}$ max → 20

$\lambda = 3$



$T_1 - T_0 = 1\ hr$

Exponential distribution:

Models the time between events in a Poisson Distribution.

poisson: How many events happen in a fixed time?

exponential: How much time until the next event?

Examples:

Call centre : Time between two calls,

web server : Time between two user login,

3 requs 4 req/min
5 req/m

Server request

on average, 3 request per minute come in,

Q: what's the prob that the next request comes within 10 seconds?

$$\text{Formula} = P(T \leq t) = 1 - e^{-\lambda t}$$

T = waiting time until the next event

λ = rate

t = time we care about.

λ (poisson rate) = 3 req / minute → rate per second → $\frac{3}{60} = 0.05$

for t = 10 (sec)

second second

$$P(T \leq 10) = 1 - e^{-0.05 \times 10} \approx 0.393$$

~39% chance of getting next request with 10 sec.

expon. pdf → because infinite choice
→ Continuous data,

$scale = 1/\lambda$ (final)

$|\longrightarrow|\longrightarrow|$ 2.0

$\longleftrightarrow \longleftrightarrow$

scale → short value results in compact pdf plot

→ large value results in wider pdf plot

$1-1-1-1+1 \rightarrow 2.0$

server fails : scale is high : failure are more spread out

scale is low: failure are relatively soon.

Question :—

Scenario: A server crashes an average $\lambda = 0.1$ times per day.

Q: what the prob it runs > 10 days without crashing,

$$p(T>t) = 1 - e^{-\lambda t}$$

$$p(x>10) = 1 - e^{-0.1 \cdot 10} \approx \underline{0.387}.$$

exponent

$0.2e+01 \rightarrow 0.2 \times 10^1$

$0.2 \times 10 = 2$

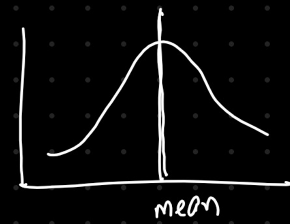
2e+02 2e+01

2×10^2 2×10^1

$2e+03 \rightarrow 2 \times 10^3$

Normal distribution

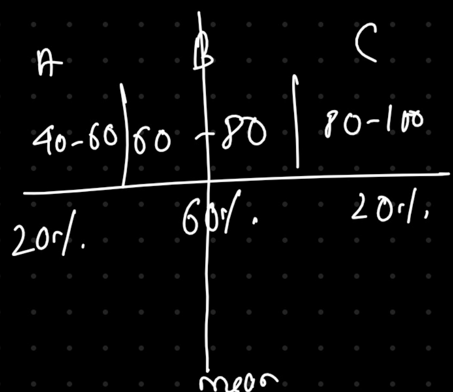
Models continuous data that clusters around the mean



Normal

→ Marks of student → 100

→ Total marks → 100



height : $\mu = 170$ cm

$\sigma = 10$cm

68% population : $\mu \pm \sigma$

→ $170 + 10 = \boxed{180}$

$170 - 10 = \boxed{160}$

95% population → $\mu \pm 2\sigma$

$170 + 2 \times 10 = \boxed{190}$ 95%

$170 - 2 \times 10 = \boxed{150}$

→ Height , marks , exam results

99.7% within $\mu \pm 3\sigma$