Università degli Studi di Messina

# Computer system analysis

## Discrete event simulation

# Simulation

- Software program emulating how a system works
  - Existing systems
  - Systems in designing phase
- System behavior is described by its state

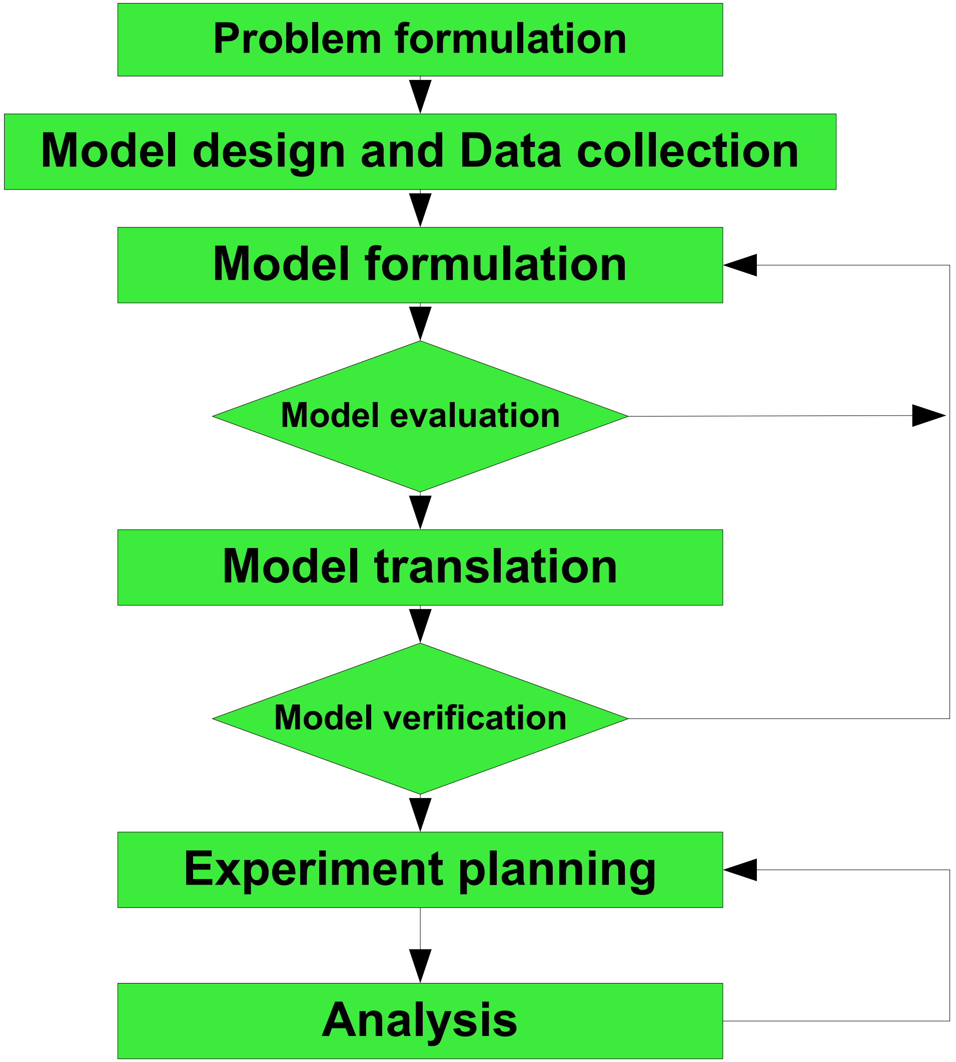
# Applications

- System components interactions
- Evaluation on how the system behavior changes when subject to modifications
- Performances in different functional modes (design phase)
- Analytical models validation

# Typical phases

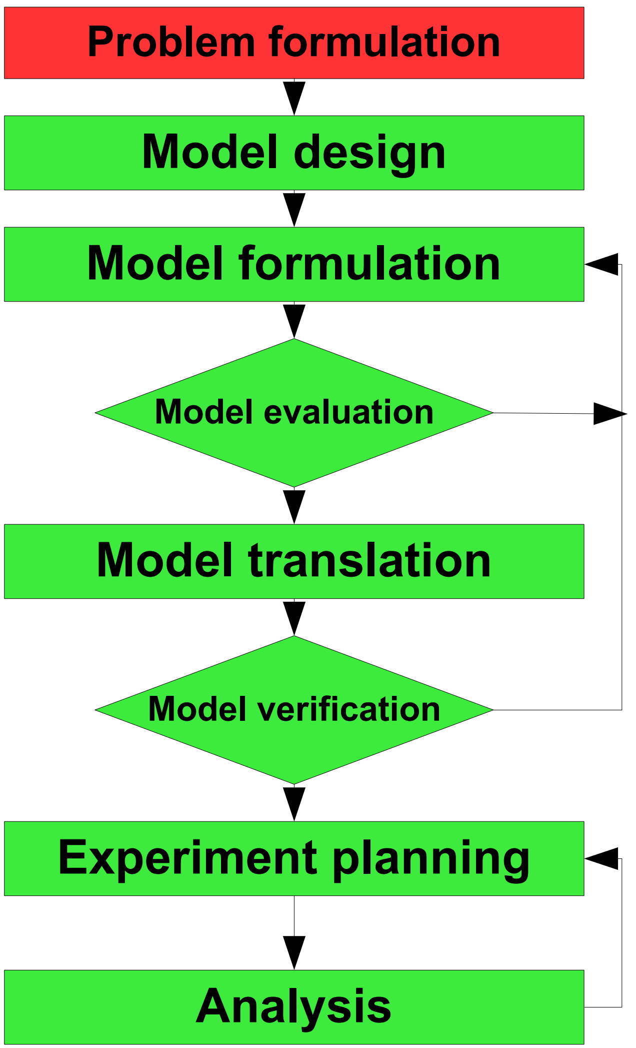
1) Problem formulation
2) Data collection
3) Model formulation
4) Model evaluation
5) Model translation
6) Model verification
7) Experiments planning
8) Analysis

# Phases diagram



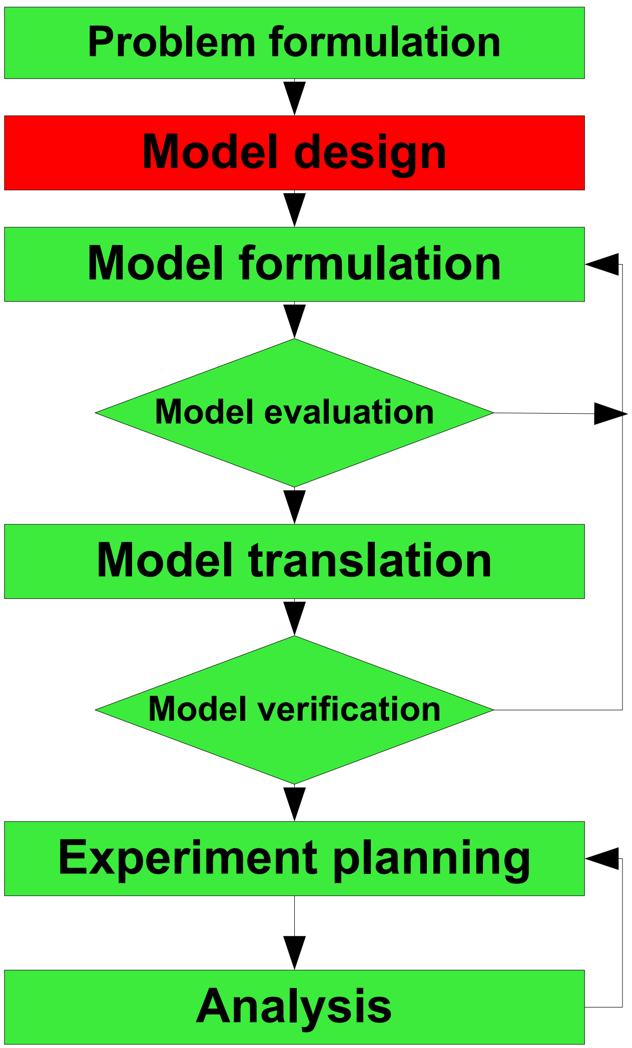
# Problem formulation

- Goals
  - performances
  - input/output relations
  - optimization
  - comparison
- Measures of interest
- Input parameters



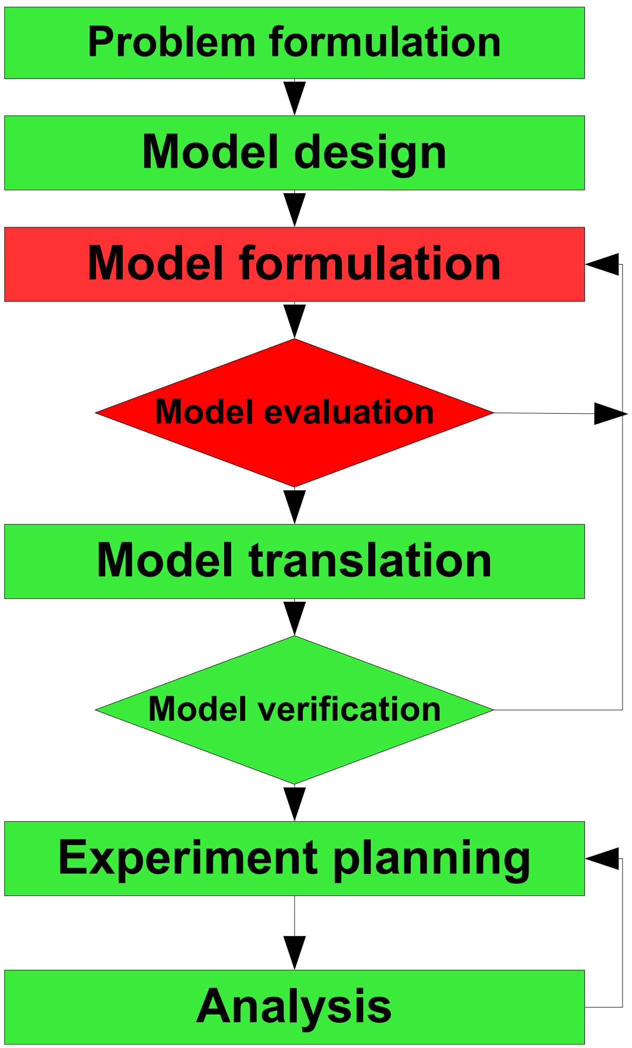
# Model design and data collection

- Choice on what to represent
- Inputs
  - Service times
  - Inter-arrival times
- Experimental data
- Realistic hypothesis



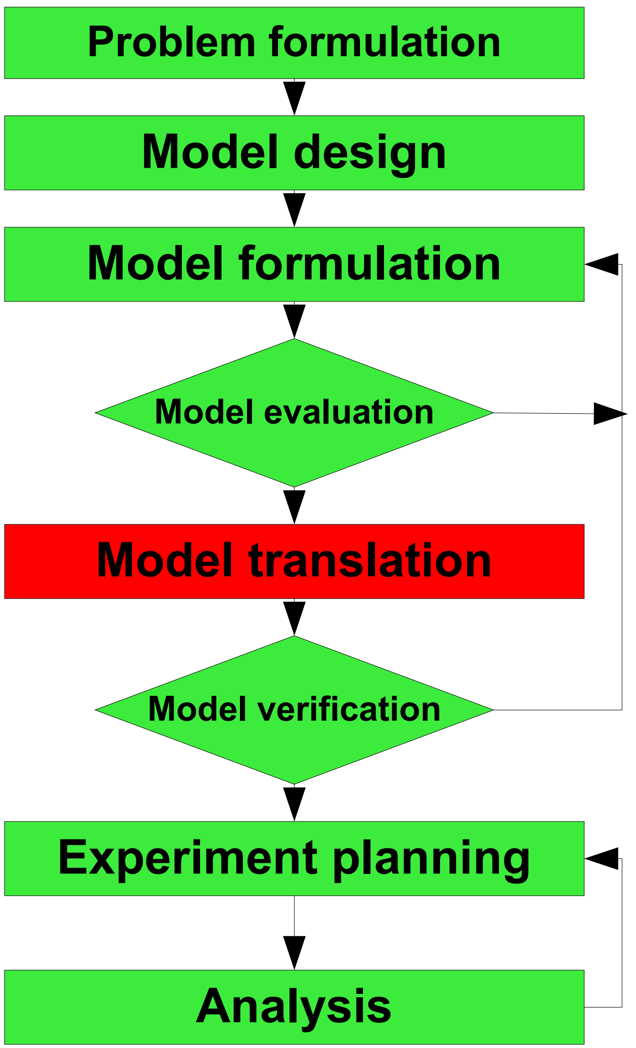
# Model formulation and evaluation

- Model simplification
- Refinement
- Abstraction of main characteristics



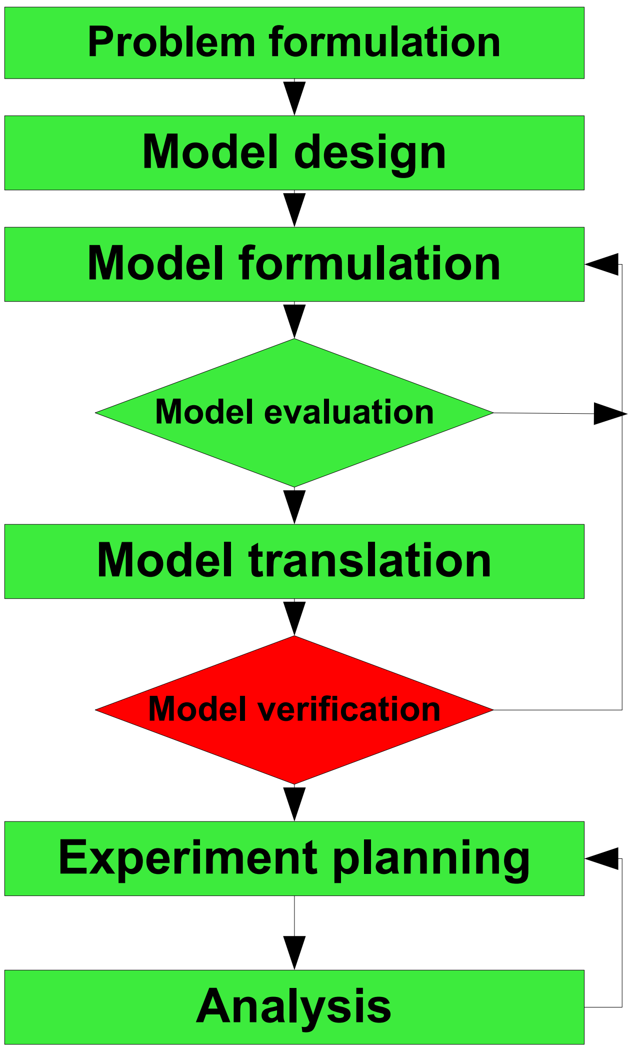
# Model translation

- Programming language
  - General purpose (SIMSCRIPT, MODSIM, ...)
  - Simulation oriented



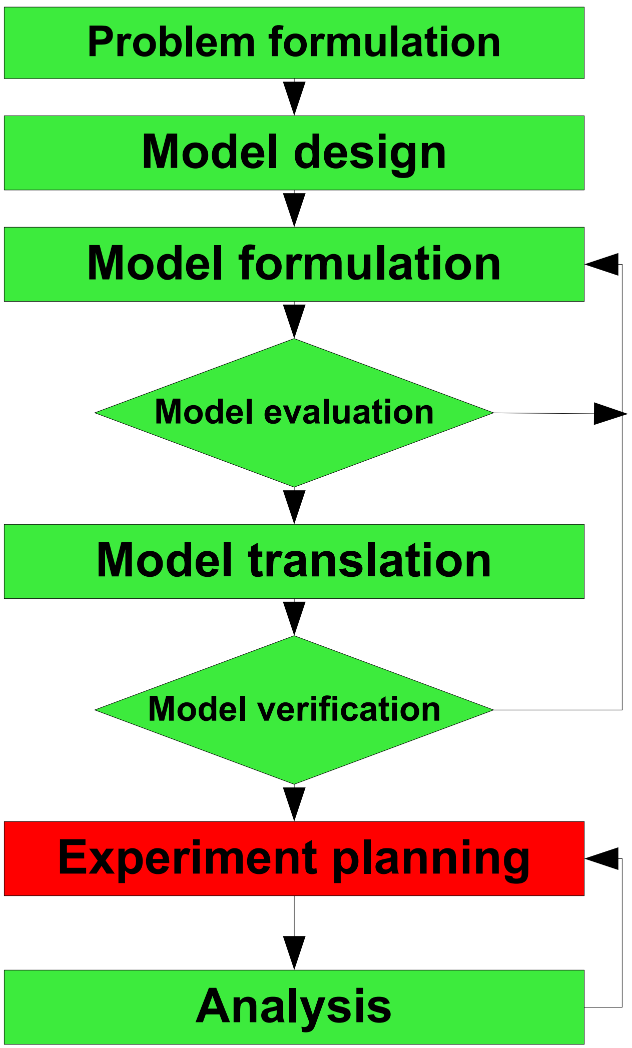
# Model verification

- Correctness
  - Logic structure
  - Input and output interface modules



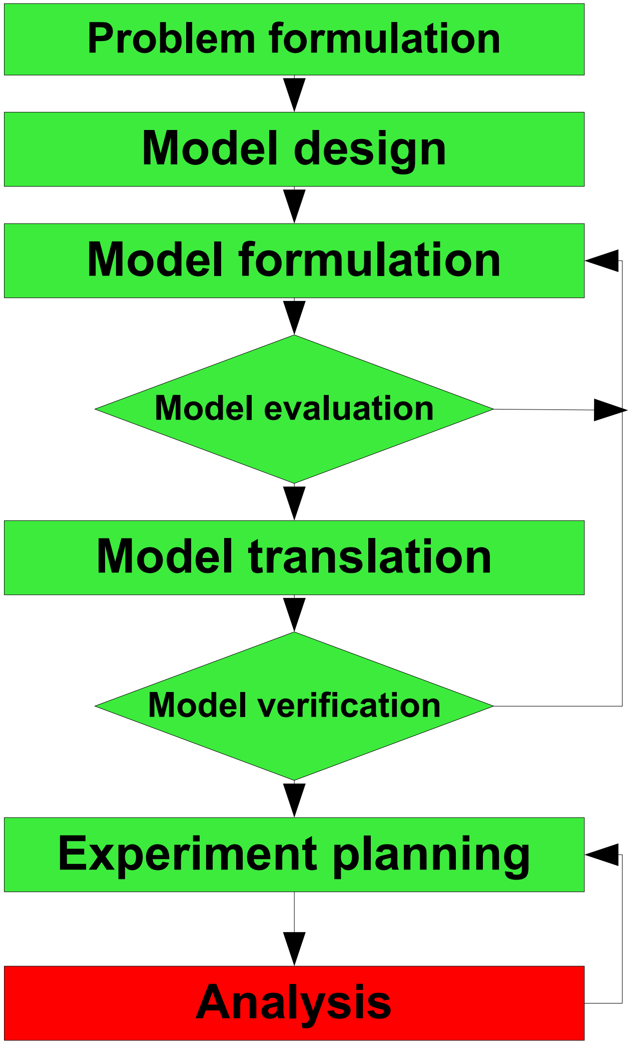
# Experiment planning

- In this phase should be choosen
  - Initialization period
  - Simulation time
  - Number of simulations



# Result Analysis

- Results are collected
- Goodness of results
  - More simulations can be needed



# Some definitions

- *System* : set of entities interacting to achieve a goal
- *Model*: abstract representation of logical system connections; it describes the system through state, activities and their attributes

# Some definitions (contd.)

- *System state*: the set of variables that identify the system at any time instant
- *Entity*: each system component we want to represent
- *Attributes*: characteristic of an entity
- *Events*: a fact able to change system state
  - conditioned (or dependent) event
  - primary (or independent) event

# Some definitions (contd.)

- *Event calendar*: list of the happening events, ordered over the time
- *Activity*: any well identified time period
- *Delay*: any undefined time period (the length is unknown)
- *Clock*: a variable accounting for the simulated time

# Events

- System state changes in a discrete way
- Type of events must be identified
- *Longitudinal* analysis
  - entities evolve in parallel
  - different objects in movement

# Events

- Given an event, how it influences the system state and the attributes
- Based on the attributes at a given simulated time, future events are identified

# Discrete event simulation

- A sequence of *system snapshots* over the time (simulated)
- A snapshot is constituted by:
  - System state
  - Active activities (a list)
  - Entity states
  - Cumulative statistics and counters
- Final state

# Events

- When an event happens
  - The clock advances
  - Future events list is modified (one event is removed)
- System state updating
  - Data for statistics
  - Entity set
  - System state

# Notation

- $\tau$: simulation time
- $x_i$: system state at $i$-th step
- $e$: generic event
- $\delta(x_i,e)$: transition function state

# Initialization

- Future event calendar is set with the happening events at the time 0 (primary events)
- Implementation
  - Chained list ordered according to the scheduling time (the time when an event happens)

# Future event list

- It is an important data structure
  - It is always accessed by read and insert operations
  - efficiency
- The head is removed (next event)
- Sets of event are modified

# Simulation running

Time instant $\tau_k$, states $x_k$

1) Get ($e_k$, $\tau_k$) by the future event list

2) $\tau = \tau + \tau_k$

3) $x_{k+1} = \delta(x_k, e_k)$

   - Update statistics
   - impossible events in $x_{k+1}$ are removed
   - New possible events, due to the happening of $e_k$, are added

4) Sort the list

# An example: a M/M/1 queue

- Costumers arrive at random inter-arrival time (Poissonian process)
- If the server is busy, costumer waits into the queue
- When the service completes, costumer under service leaves the system
- The server can be into one of two states: *busy* and *free*

# Basic elements

- *System state*: number of costumers in the queue, server state
- *Entities*: costumer generator, costumers, queue
- *Events*: costumer arrival, service starting (dependent event), costumer departure, simulation end
- *Activities*: inter-arrival and service times
- *Delays*: costumer waiting times

# Event: *costumer arrival*

- Actions
  - next costumer arrival is planned (primary event)
  - free server
    - Server state changes to busy and the end of service event is planned
  - busy server
    - number of queued client increases

# Event: *costumer departure*

- Actions
  - the server becomes free
  - if the queue is not empty
    - the number of queued costumers decreases
    - server becomes busy
    - a departure is planned

# Event: *simulation end*

- Action
  - none (system state remain the same)

- Simulation ends and all the statistics are computed

# Simulation run

- Simulation time?
  - Time to serve the first five costumers
    - end simulation event
  - Unknown
- Measures
  - Expected waiting time
  - Server utilization
  - Expected queue length

# Expected waiting time

- Each client has a different waiting time ($\theta_i$) in the queue
- $N$ costumers
- A possible estimation is

$$\hat{\theta} = \frac{1}{N} \sum_{k=1}^{N} \theta_k$$

# Utilization

- $T_N = \tau_f - \tau_0$
- $T(i)$: overall time during which $i$ costumers are inside the system

$$\hat{v} = \frac{1}{T_N} \sum_{i=1}^{\infty} T(i) = 1 - \frac{T(0)}{T_N}$$

# Expected queue length

- $p_N(i)$: probability that the queue length is $i$

$$\bar{x} = \sum_{i=1}^{\infty} i \, p_N(i)$$

$$\hat{x} = \sum_{i=1}^{\infty} i \, \hat{p}_N(i) = \frac{1}{T_N} \sum_{i=0}^{\infty} i \, T(i)$$

# Events: occurrences

time incurred between two
customer arrivals in the system

| Costumer | Inter-arrival times | Arrival times |
|---|---|---|
| 1 | | 0 |
| 2 | 2 | 2 |
| 3 | 4 | 6 |
| 4 | 1 | 7 |
| 5 | 1 | 8 |

| Costumers | Service time |
|---|---|
| 1 | 3 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 2 |

# Calendar

| Simulated time | Event type | Client number |
|---|---|---|
| 0 | arrival | 1 |
| 2 | arrival | 2 |
| 3 | depature | 1 |
| 5 | depature | 2 |
| 6 | arrival | 3 |
| 7 | arrival | 4 |
| 8 | arrival | 5 |
| 9 | depature | 3 |
| 13 | depature | 4 |
| 15 | depature | 5 |

# Simulation run

- $\tau=0$: simulation starts
- Server state: busy
- Queue lenght: 1
- Future events
  - $\tau=2$ : arrival (2)
  - $\tau=3$ : departure (1)
  - $\tau=\tau_f$ : simulation end
- Statistics
  - $N = 0$

# Simulation run

- $\tau=2$: costumer 2 arrives
- Server state: busy
- Queue length: 2
- Future events
  - $\tau=3$ : partenza (1)
  - $\tau=6$ : arrivo (3)
  - $\tau=\tau_f$ : simulation end

- Statistics
  - N = 0
  - T(1) = 2

# Simulation run

- $\tau=3$: client 1 departs
- Server state: busy
- Queue length: 1
- Future events
  - $\tau=5$ : departure (2)
  - $\tau=6$ : arrival (3)
  - $\tau=\tau_f$ : simulation end

- Statistics
  - $N = 1$
  - $\theta_1 = 3$
  - $T(1) = 2$
  - $T(2) = 1$

# Simulation run

- $\tau=5$: client 2 departs
- Server state: free
- Queue length: 0
- Future events
  - $\tau=6$ : arrival (3)
  - $\tau=\tau_f$ : simulation end
- Statistics
  - $N = 2$
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $T(1) = 4$
  - $T(2) = 1$

# Simulation run

- $\tau=6$: client 3 arrives
- Server state: busy
- Queue length: 1
- Future events
  - $\tau=7$ : arrival (4)
  - $\tau=9$ : departure (3)
  - $\tau=\tau_f$: simulation end

- Statistics
  - $N = 2$
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $T(0) = 1$
  - $T(1) = 4$
  - $T(2) = 1$

# Simulation run

- $\tau=7$: client 4 arrives
- Server state: busy
- Queue length: 2
- Future events
  - $\tau=8$ : arrival (5)
  - $\tau=9$ : departure (3)
  - $\tau=\tau_f$: simulation end

- Statistics
  - $N = 2$
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $T(0) = 1$
  - $T(1) = 5$
  - $T(2) = 1$

# Simulation run

- $\tau=8$: client 5 arrives
- Server state: busy
- Queue length: 3
- Future events
  - $\tau=9$ : departure (3)
  - $\tau=\tau_f$ : simulation end

- Statistics
  - $N = 2$
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $T(0) = 1$
  - $T(1) = 5$
  - $T(2) = 2$

# Simulation run

- $\tau=9$: client 3 departs
- Server state: busy
- Queue length: 2
- Future events
  - $\tau=13$ : departure (4)
  - $\tau=\tau_f$ : simulation end

- Statistics
  - $N = 3$
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $\theta_3 = 3$
  - $T(0) = 1$
  - $T(1) = 5$
  - $T(2) = 2$
  - $T(3) = 1$

# Simulation run

- $\tau$=13: client 4 departs
- Server state: busy
- Queue length: 1
- Future events
  - $\tau$=15 : departure (5)
  - $\tau=\tau_f$ : simulation end
- Statistics
  - N = 4
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $\theta_3 = 3$
  - $\theta_4 = 6$
  - T(0) = 1
  - T(1) = 5
  - T(2) = 6
  - T(3) = 1

# Simulation run

- $\tau=15$: client 5 departs
- Server state: free
- Queue length: 0
- Future events

- Statistics
  - $N = 5$
  - $\theta_1 = 3$
  - $\theta_2 = 3$
  - $\theta_3 = 3$
  - $\theta_4 = 6$
  - $\theta_5 = 7$
  - $T(0) = 1$
  - $T(1) = 7$
  - $T(2) = 6$
  - $T(3) = 1$

# Measures

- Simulation ends at time 15

$$\hat{\theta}=\frac{3+3+3+6+7}{5}=4{,}4$$

$$\hat{v}=1-\frac{T(0)}{15}=1-\frac{1}{15}\simeq 0.933$$

$$\hat{x}=\frac{1}{15}\sum_{i=0}^{\infty} i\,T(i)=\frac{(0\times 1)+(1\times 7)+(2\times 6)+(3\times 1)}{15}\simeq 1{,}467$$