Imperial College London

Monte Carlo Analysis of Gamma Ray Spectroscopy E3

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The following experiment guide is NOT intended to be a step-by-step manual for the experiment but rather provides an overall introduction to the experiment and outlines the important tasks that need to be performed in order to complete the experiment. Additional sources of documentation may need to be researched and consulted during the experiment as well as for the completion of the report. This additional documentation must be cited in the references of the report.

additional material needed.

3rd Year Lab Module
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Based on E1, Version 2.3 2019
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Monte Carlo Analysis of Gamma Ray Detection

1. Overview

The key objectives for this experiment are:

- Analyse data from a gamma ray spectrometer and Compton scattering experiment.
- Use Monte Carlo methods to simulate the scattering experiments.
- Further investigate applications or characteristics of the detector.

2. Gamma Ray - Electron Scattering

The collision of a photon and an electron is depicted in Figure 1. Due to this interaction, there is a change in the photon's energy, as well as its direction. This phenomenon is known as the Compton effect, and historically, its discovery played an important role in understanding the particle-like characteristics of light. In the present day, the Compton effect remains a fundamental aspect of light-matter interaction studies, and it is integral to many technical and scientific applications of X-rays and gamma rays.

To derive an equation to describe the Compton effect, we consider a collision in which an electron is initially at rest. In this case, the conservation of energy and momentum results in the following relationship between the initial photon energy $E_{\gamma 0}$, final photon energy E_γ, and scattering angle θ:

final photon energy

$$\boxed{E_\gamma(\theta) = \frac{E_{\gamma 0}}{1 + \frac{E_{\gamma 0}}{m_e c^2}(1 - \cos\theta)}} \qquad (1)$$

initial photon energy

angle scattered by photon.

where $m_e c^2$ = 511 keV is the rest energy of the electron.

- Derive Eq. (1) using conservation laws and the relativistic energy-momentum relations.
- Why is the Compton effect typically disregarded when considering visible light?
- When is it justified to approximate an electron as free and at rest, as assumed for Eq. (1)?

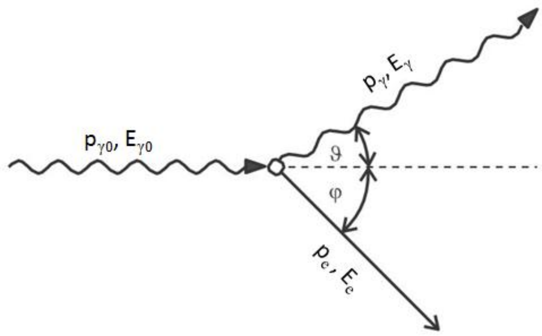


Figure 1. An idealised picture of the collision of a photon with an electron at rest.

3. Instrumentation

An idealised sketch of a typical experimental setup used to observe the Compton effect is shown in Figure 2. The main elements are:

(a) A radioactive sample and sample holder.
(b) Aluminium scattering target (cylindrical rod).
(c) Lead shielding.
(d) Entrance aperture of detector with shielding.
(e) NaI(TI) scintillator and photomultiplier tube.
(f) Power and signal connections.
(g) High voltage power supply.
(h) Multi-channel analyser (MCA) for pulse height analysis.

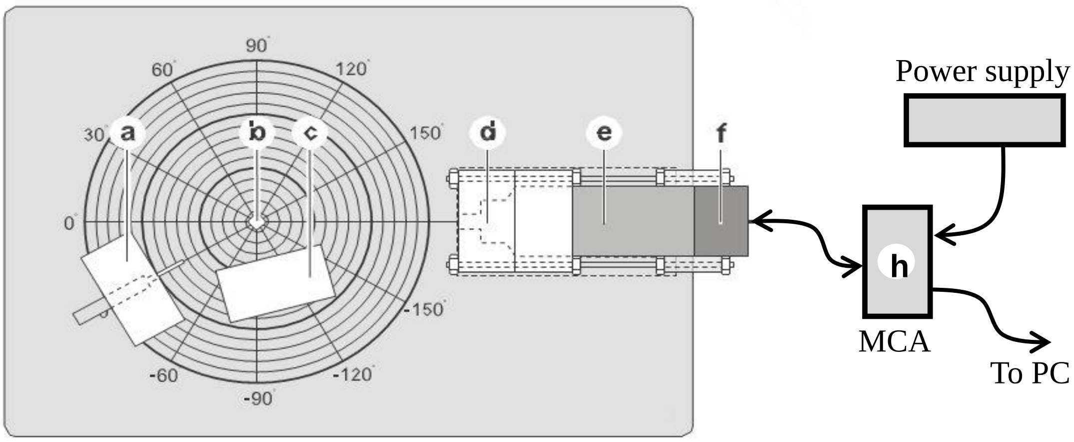


Figure 2. Experimental setup for measuring the Compton effect.

- How do the scintillating crystal and photomultiplier tube work? What does the "Tl" in NaI(Tl) mean and why is it technically important? What are the likely maximum and minimum detectable gamma ray energies with such a system?

- What does the MCA unit do? How do the signals input and output from the MCA differ?

4. Detector calibration

A calibration is needed to express the detector output as an energy. This can be achieved using radioactive samples from which the dominant emission energies are known, for example 662 keV for Cs-137, 122 keV for Co-57, and 59.6 keV for Am-241.

The calibration data provided was recorded with the specified radioactive source placed in the direct line-of-sight of the detector, without any scattering target in place.

- Use the calibration data to determine a means to convert the detector output to units of energy.

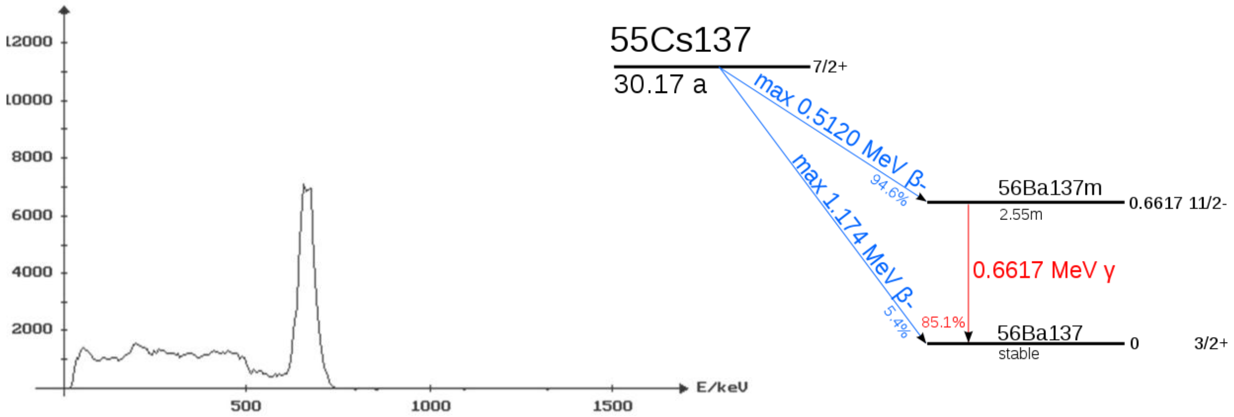


Figure 3. Example spectrum from Cs-137 and an idealised nuclear energy level diagram.

An example spectrum of Cs-137 from a calibrated detector is shown in Figure 3 below to give you a general sense of the "look and feel" of the data. The provided data sets, along with other data examples you might find online, will show a number of features in addition to the central peak, which you may wish to explore further and discuss with a demonstrator.

There are many useful sources online and in the literature for information about gamma emissions from a range of radioactive materials. Here is a useful starting point (including some data taken with research grade NaI scintillator detectors for reference):

http://www.radiochemistry.org/periodictable/gamma_spectra/

5. Analysing Compton scattering

Once the detector is well calibrated, the Compton effect can be verified using an experimental arrangement similar to that shown in Figure 2 to measure the energy of photons scattered at different angles.

The scattering data provided has been recorded at a range of scattering angles, defined by lines between the centres of the source, target, and detector apertures.

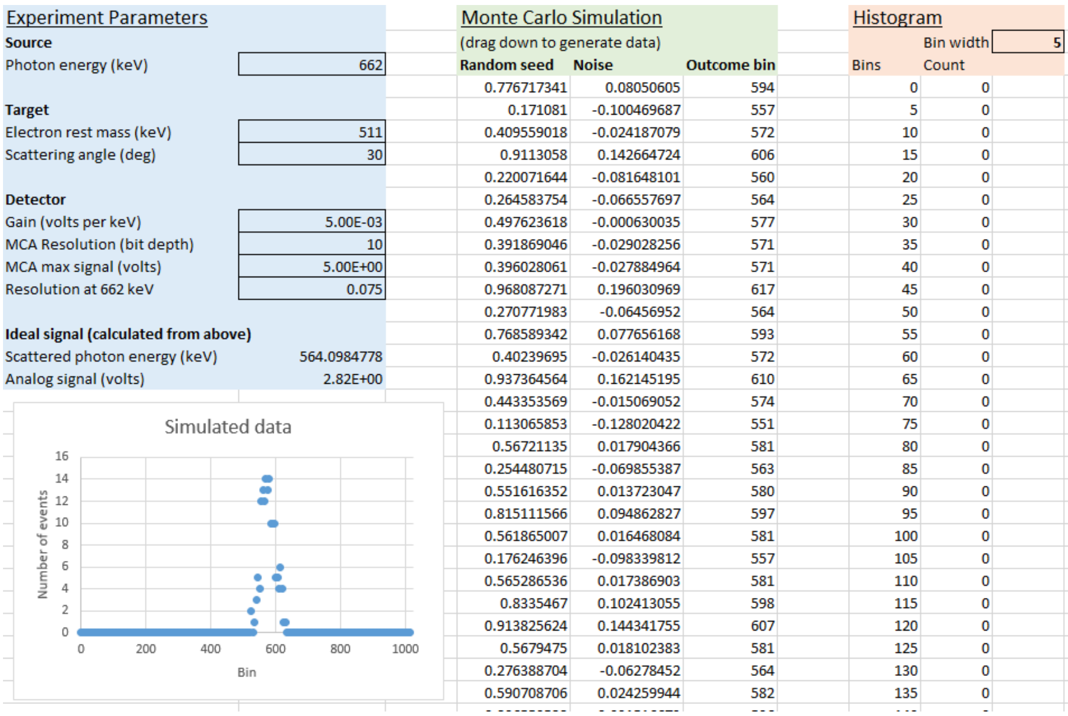
- Use the scattering data to verify the Compton effect described by Eq. (1).

Undertake a full error analysis of the Compton scattering experiment. What uncertainty is there in finding the peak signal position? How does this relate to the background signal and noise? What uncertainties arise due to the calibration methods? There are many topics here you may wish to discuss with a demonstrator.

- How well can you identify and quantify sources of uncertainty that limit the precision with which you are able to verify Compton's formula?

6. Monte Carlo Simulation

Monte Carlo (MC) simulations are a widely used tool in experimental physics to design, model, and analyse the results of complicated apparatus. The Compton scattering



Experiment Parameters

Source	
Photon energy (keV)	662
Target	
Electron rest mass (keV)	511
Scattering angle (deg)	30
Detector	
Gain (volts per keV)	5.00E-03
MCA Resolution (bit depth)	10
MCA max signal (volts)	5.00E+00
Resolution at 662 keV	0.075
Ideal signal (calculated from above)	
Scattered photon energy (keV)	564.0984778
Analog signal (volts)	2.82E+00

Monte Carlo Simulation

(drag down to generate data)

Random seed	Noise	Outcome bin
0.776717341	0.08050605	594
0.171081	-0.100469687	557
0.409559018	-0.024187079	572
0.9113058	0.142664724	606
0.220071644	-0.081648101	560
0.264583754	-0.066557697	564
0.497623618	-0.000630035	577
0.391869046	-0.029028256	571
0.396028061	-0.027884964	571
0.968087271	0.196030969	617
0.270771983	-0.06456952	564
0.768589342	0.077656168	593
0.40239695	-0.026140435	572
0.937364564	0.162145195	610
0.443353569	-0.015069052	574
0.113065853	-0.128020422	551
0.56721135	0.017904366	581
0.254480715	-0.069855387	563
0.551616352	0.013723047	580
0.815111566	0.094862827	597
0.561865007	0.016468084	581
0.176246396	-0.098339812	557
0.565286536	0.017386903	581
0.8335467	0.102413055	598
0.913825624	0.144341755	607
0.5679475	0.018102383	581
0.276388704	-0.06278452	564
0.590708706	0.024259944	582

Histogram

Bin width 5

Bins	Count
0	0
5	0
10	0
15	0
20	0
25	0
30	0
35	0
40	0
45	0
50	0
55	0
60	0
65	0
70	0
75	0
80	0
85	0
90	0
95	0
100	0
105	0
110	0
115	0
120	0
125	0
130	0
135	0

Figure 4. Example Monte Carlo simulation provided

experiment provides a good scenario to try these methods. Use MC methods to simulate scattering data sets and investigate how experimental parameters influence your results.

As a starting point, use the example MC calculation is provided as an Excel spreadsheet, shown in Fig. 4. The experiment parameters (blue box) are used to model a scattering experiment, with inputs entered in the outlined boxes. The simulation outputs (green heading) include the random seed used for each event and the resulting outcome bin. The final section (orange heading) is used to calculate the shown histogram.

Use the program to replicate experimental data, before moving on to explore additional aspects of the experiment and detector. Documentation for Excel is readily found online, but here are a few tips to start: To repeat a calculation (i.e. add more simulated events) use a mouse to select the entries to repeat and drag the bottom right of the selection downwards. In entering formulas, you might use relative references (e.g. F4) or absolute references (e.g. F4).

To go beyond these initial tests, you likely want to write your own simulation code, by either adapting the Excel program or writing code in a different format (e.g. Python). Similarly, you may wish to develop more effective code for analysing and plotting data. Be sure to keep a good record in a lab book, which documents your aims, progress, and outputs. Discuss methods with the demonstrators for guidance on how to develop and use your MC simulations.

Here are few topics you may wish to explore with your simulations:

- How is the precision and accuracy of the Compton scattering test influenced by the number of observed events? The finite size of the target and apertures? Background events?

7. Further investigations and Extensions.

Once the Compton effect is studied and you have gained a good understanding of MC simulations to model experiments, use these for other investigations that go beyond the carefully defined limits of the basic Compton measurement. You are encouraged to further explore the gamma ray spectrometer and its applications.

- What else can be deduced from your data, and how might the instrument be improved? What role does Compton scattering play in the detector itself?
- Can this spectrometer be used to study cosmic rays? Would it be better to use more than one instrument in that case? If so, why and how should they be orientated?
- Could this type of gamma ray spectrometer be used to identify unknown materials, such as radioactive rocks? Can radioactive cascades be used to estimate the age of materials?
- Can Compton scattering provide an approach to non-invasive imaging? For one example of this, see the next section.
- How else might these techniques be extended?

8. Optional extension: Imaging using the Compton Effect

The following guide describes an optional extension developed in Summer 2019 by undergraduate students Michaela Flegrova and Timothy Marley. Modifications for Monte Carlo analysis added in 2020.

Overview

X-ray imaging devices, such as the ones used in hospitals or at airports, rely on measuring the transmission and attenuation of light rays that pass through an object of interest. This however means that you must have access to both sides of the item, which makes the technique unsuitable for some purposes. Since Compton scattering causes some photons to backscatter, it has been suggested that it could be utilised to design imaging devices for items or areas with access restrictions where you can't be on the other side of what you're imaging; those devices could be used for example in geology, for landmine detection, or to simply "look behind a wall".

In this extension, you will use a Monte Carlo analysis to investigate whether it would be feasible to design a Compton effect-based device that could be used to **image the contents of a sealed steel shipping container**.

For an introductory problem, you might investigate how well you can locate a single scattering object within the container. Some questions you might wish to consider:

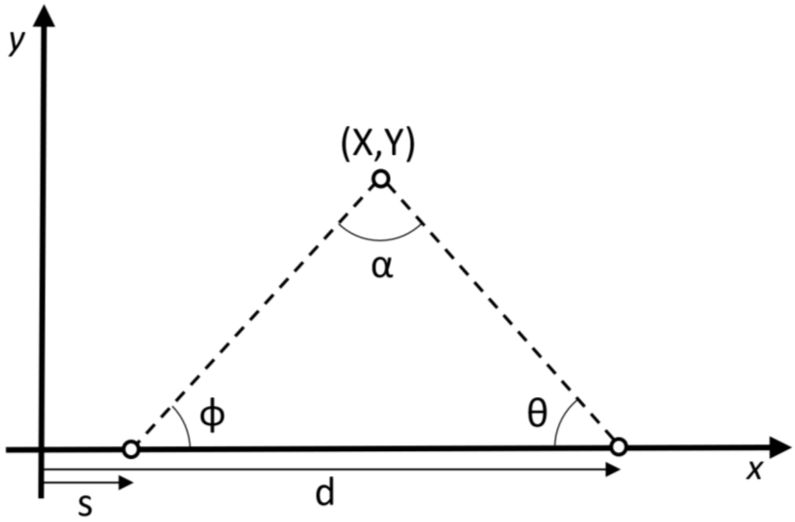
- How precisely can a single object be located?
- What is the optimal position of the source and detector? What is the optimal protocol for using multiple measurement positions?
- For how long do you need to collect data?
- How does the container wall thickness influence the results?
- Can the method be extended to multiple or more complex objects?
- Is this technology practical? In what scenarios? What improvements might be made?

Suggestions for analysis

To be able to determine the position of the scatterer based on the scattering angle, you will need to take at least two different measurements with different source and/or detector positions. The diagram below shows the source, detector and scatterer that are positioned in the x-y plane. The distances of the source and detector from the origin along the x-axis *s* and *d* respectively are known, and the angle α can be determined from the measured scattering angle. The angles θ and φ are then given by

$$\theta = \arctan\left(\frac{Y}{X-s}\right), \qquad \varphi = \arctan\left(\frac{Y}{d-X}\right),$$

where *X*, *Y* are the (unknown) coordinates of the scatterer.



Since the angles α, θ and φ must add up to π, we can write

$$0 = \pi - \alpha_i - \arctan\left(\frac{Y}{X - s_i}\right) - \arctan\left(\frac{Y}{d_i - X}\right).$$

At least two independent measurements of α_i with different s_i and d_i positions are needed to determine the two unknowns *X* and *Y*, giving a minimum of two simultaneous equations. It is difficult to find an analytical solution to this set of equations, but we can find a numerical one. We start by defining a function *f*, a measure of how much a given estimate of *X* and *Y* deviates from the true scatterer position, as

$$f(\alpha_i, d_i, s_i, X, Y) = |\pi - \alpha - \theta(s_i, X, Y) - \phi(d_i, X, Y)|^2.$$

We then define the loss function as

$$L(X, Y) = \sum_{i=1}^{N} f(\alpha_i, d_i, s_i, X, Y),$$

where *N* is the number of measurements taken. With ideal measurements, the correct *X* and *Y* coordinates are those for which the loss function equals zero. With realistic data, the position can be estimated by numerically finding the *X* and *Y* that minimizes the loss function. One approach is to write a computer program that calculates the loss function for many trial positions.