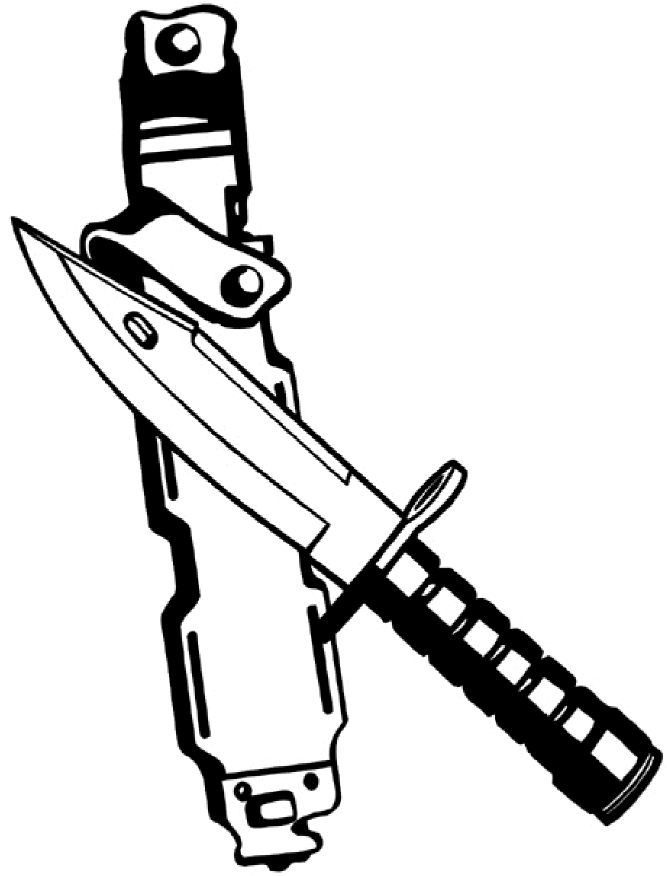
TiZi ACM-ICPC Notebook (2016)

Contents



1 Data Structures

1.1 Union Find Disjoint Set - Kruskal

```
#include <iostream>
#include <vector>
#include <algorithm>
#include <sstream>
#include <string>

using namespace std;

typedef vector<int> vi;

struct DisJointSet {
        vi par, rnk, cnt; int numOfSets;

        DisJointSet(int n = 0){
                par.assign(n,-1); rnk.assign(n,0); cnt.assign(n,1); //par==parent
                numOfSets=n; // if we wanna count number of disjoint sets
        }

        int find(int a){
                int i=a,j=a,tmp;
                while(par[i]!=-1){ i=par[i]; }
                while(par[j]!=-1){ tmp=par[j]; par[j]=i; j=tmp; } //path compression
                return i;
        }
        int uni(int a, int b){
                int A=find(a),B=find(b);
                if(A!=B){
                        if(rnk[A]<rnk[B]) swap(A,B); // union using rank
                        if(rnk[A]==rnk[B]) rnk[A]++;
                        par[B]=A;
                        cnt[A]+=cnt[B]; // if we wanna count each set size
                        numOfSets--; // if we wanna count number of disjoint sets
                }
                return cnt[A]; // if we wanna count each set size
        }
};

struct Edge { int u, v, w;
        Edge(int u=0, int v=0, int w=0):u(u), v(v), w(w){}
        bool operator<(const Edge& b) const { return w < b.w; }
        string toString(){
                stringstream sstr;
                sstr << u << "," << v << "," << w;
                string str; sstr >> str;
                return str;
        } //remove
};

typedef vector<Edge> ve;

struct Kruskal {
        ve edges; vi marked; DisJointSet st;
        Kruskal(int n, ve& edges):edges(edges) { st = DisJointSet(n); }
        int run(){ int result; sort(edges.begin(), edges.end());
                for (int i=0 ; i<edges.size() ; i++) { Edge e = edges[i];
                        if (st.find(e.u) != st.find(e.v)) {
                                st.uni(e.u, e.v); result += e.w; marked.push_back(i);
                        }
                }
                return result;
        }

        void printSelectedEdges(){
                cout << "MST edges:" << endl;
                for(int i=0 ; i<marked.size() ; i++) {
                        Edge e = edges[marked[i]];
                        cout << e.toString() << endl;
                }
        } // remove
};

int main() {
        int n, m;
        cin >> n >> m;

        ve edges;
        for(int i=0 ; i<m ; i++) {
                int u, v, w;
                cin >> u >> v >> w;
                edges.push_back(Edge(u, v, w));
        }

        for(int i=0 ; i<edges.size() ; i++) {
                cout << edges[i].toString() << endl;
        }

        Kruskal kruskal(n, edges);
        cout << kruskal.run() << endl;
        kruskal.printSelectedEdges();
}

/*
IN:
5 6
1 3 5
4 5 0
2 1 3
3 2 1
4 3 4
4 2 2

OUT:
4,5,0
3,2,1
4,2,2
2,1,3

*/
```

1.2 Segment Tree, RSQ, RMQ

```
/******************************************************************************
 * Compilation:  javac SegmentTree.java
 * Execution:    java SegmentTree
 * <p/>
 * A segment tree data structure.
 ******************************************************************************/

import java.util.Arrays;

class SegmentTree {

    private Node[] heap;
    private int[] array;
    private int size;

    public SegmentTree(int[] array) {
        this.array = Arrays.copyOf(array, array.length);
        //The max size of this array is about 2 * 2 log2(n) + 1
        size = (int) (2 * Math.pow(2.0, Math.floor((Math.log((double) array.length) / Math.log(2.0)) +
                 1)));
        heap = new Node[size];
        build(1, 0, array.length);
    }


    public int size() {
        return array.length;
    }

    //Initialize the Nodes of the Segment tree
    private void build(int v, int from, int size) {
        heap[v] = new Node();
        heap[v].from = from;
        heap[v].to = from + size - 1;

        if (size == 1) {
            heap[v].sum = array[from];
            heap[v].min = array[from];
            heap[v].minId = from;
        } else {
            //Build childs
            build(2 * v, from, size / 2);
            build(2 * v + 1, from + size / 2, size - size / 2);

            heap[v].sum = heap[2 * v].sum + heap[2 * v + 1].sum;
            heap[v].min = Math.min(heap[2 * v].min, heap[2 * v + 1].min); //min = min of the children
            heap[v].minId = (heap[2 * v].min < heap[2 * v + 1].min ? heap[2 * v].minId : heap[2 * v +
                 1].minId);
        }
    }

    public int rsq(int from, int to) {
        return rsq(1, from, to);
    }

    private int rsq(int v, int from, int to) {
        Node n = heap[v];

        //If you did a range update that contained this node, you can infer the Sum without going down
                 the tree
```

```java
        if (n.pendingVal != null && contains(n.from, n.to, from, to)) {
            return (to - from + 1) * n.pendingVal;
        }

        if (contains(from, to, n.from, n.to)) {
            return heap[v].sum;
        }

        if (intersects(from, to, n.from, n.to)) {
            propagate(v);
            int leftSum = rsq(2 * v, from, to);
            int rightSum = rsq(2 * v + 1, from, to);

            return leftSum + rightSum;
        }

        return 0;
    }

    public int rMinQ(int from, int to) {
        return rMinQ(1, from, to);
    }

    private int rMinQ(int v, int from, int to) {
        Node n = heap[v];


        //If you did a range update that contained this node, you can infer the Min value without
              going down the tree
        if (n.pendingVal != null && contains(n.from, n.to, from, to)) {
            return n.pendingVal;
        }

        if (contains(from, to, n.from, n.to)) {
            return heap[v].min;
        }

        if (intersects(from, to, n.from, n.to)) {
            propagate(v);
            int leftMin = rMinQ(2 * v, from, to);
            int rightMin = rMinQ(2 * v + 1, from, to);

            return Math.min(leftMin, rightMin);
        }

        return Integer.MAX_VALUE;
    }

    public int rMinIdQ(int from, int to) {
        return rMinIdQ(1, from, to);
    }

    private int rMinIdQ(int v, int from, int to) {
        Node n = heap[v];

        //If you did a range update that contained this node, you can infer the Min value without
              going down the tree
        if (n.pendingVal != null && contains(n.from, n.to, from, to)) {
            return n.pendingVal;
        }

        if (contains(from, to, n.from, n.to)) {
            return heap[v].minId;
        }

        if (intersects(from, to, n.from, n.to)) {
            propagate(v);
            int leftMinId = rMinIdQ(2 * v, from, to);
            int rightMinId = rMinIdQ(2 * v + 1, from, to);
            if (leftMinId == Integer.MAX_VALUE) return rightMinId;
            if (rightMinId == Integer.MAX_VALUE) return leftMinId;

            return (array[leftMinId] < array[rightMinId] ? leftMinId : rightMinId);
        }

        return Integer.MAX_VALUE;
    }

    public void update(int from, int to, int value) {
        update(1, from, to, value);
    }

    private void update(int v, int from, int to, int value) {
        //The Node of the heap tree represents a range of the array with bounds: [n.from, n.to]
        Node n = heap[v];

        if (contains(from, to, n.from, n.to)) {
            change(n, value);
        }

        if (n.size() == 1) return;

        if (intersects(from, to, n.from, n.to)) {
            propagate(v);

            update(2 * v, from, to, value);
            update(2 * v + 1, from, to, value);

            n.sum = heap[2 * v].sum + heap[2 * v + 1].sum;
            n.min = Math.min(heap[2 * v].min, heap[2 * v + 1].min);
            n.minId = (heap[2 * v].min < heap[2 * v + 1].min ? heap[2 * v].minId : heap[2 * v + 1].
                  minId);
        }
    }

    //Propagate temporal values to children
    private void propagate(int v) {
        Node n = heap[v];

        if (n.pendingVal != null) {
            change(heap[2 * v], n.pendingVal);
            change(heap[2 * v + 1], n.pendingVal);
            n.pendingVal = null; //unset the pending propagation value
        }
    }

    //Save the temporal values that will be propagated lazily
    private void change(Node n, int value) {
        n.pendingVal = value;
        n.sum = n.size() * value;
        n.min = value;
        n.minId = n.from;
        array[n.from] = value;
    }

    //Test if the range1 contains range2
    private boolean contains(int from1, int to1, int from2, int to2) {
        return from2 >= from1 && to2 <= to1;
    }

    //check inclusive intersection, test if range1[from1, to1] intersects range2[from2, to2]
    private boolean intersects(int from1, int to1, int from2, int to2) {
        return from1 <= from2 && to1 >= from2   //  (.[..)..] or (.[...]..)
                || from1 >= from2 && from1 <= to2; // [.(..]..) or [..(..)..
    }

    //The Node class represents a partition range of the array.
    static class Node {
        int sum;
        int min;
        int minId;
        //Here We store the value that will be propagated lazily
        Integer pendingVal = null;
        int from;
        int to;

        int size() {
            return to - from + 1;
        }

    }

}

public class Main {
    public static void main(String[] args) {
        int[] a = new int[]{2, 3, 5, 1, 8, 4, 10};

        SegmentTree segmentTree = new SegmentTree(a);
        System.out.println(segmentTree.rsq(1, 4));
        System.out.println(segmentTree.rMinQ(1, 4));
        System.out.println(segmentTree.rMinIdQ(1, 4));

        segmentTree.update(3,3,4);
        System.out.println(segmentTree.rsq(1, 4));
        System.out.println(segmentTree.rMinQ(1, 4));
        System.out.println(segmentTree.rMinIdQ(1, 4));
    }
}
```

1.3 Segment Tree, 2D-RMQ

```java
import java.util.*;

public class SegmentTree2D {
  public static int max(int[][] t, int x1, int y1, int x2, int y2) {
    int n = t.length >> 1;
    x1 += n;
    x2 += n;
```

```
    int m = t[0].length >> 1;
    y1 += m;
    y2 += m;
    int res = Integer.MIN_VALUE;
    for (int lx = x1, rx = x2; lx <= rx; lx = (lx + 1) >> 1, rx = (rx - 1) >> 1)
      for (int ly = y1, ry = y2; ly <= ry; ly = (ly + 1) >> 1, ry = (ry - 1) >> 1) {
        if ((lx & 1) != 0 && (ly & 1) != 0) res = Math.max(res, t[lx][ly]);
        if ((lx & 1) != 0 && (ry & 1) == 0) res = Math.max(res, t[lx][ry]);
        if ((rx & 1) == 0 && (ly & 1) != 0) res = Math.max(res, t[rx][ly]);
        if ((rx & 1) == 0 && (ry & 1) == 0) res = Math.max(res, t[rx][ry]);
      }
    return res;
  }

  public static void add(int[][] t, int x, int y, int value) {
    x += t.length >> 1;
    y += t[0].length >> 1;
    t[x][y] += value;
    for (int tx = x; tx > 0; tx >>= 1)
      for (int ty = y; ty > 0; ty >>= 1) {
        if (tx > 1) t[tx >> 1][ty] = Math.max(t[tx][ty], t[tx ^ 1][ty]);
        if (ty > 1) t[tx][ty >> 1] = Math.max(t[tx][ty], t[tx][ty ^ 1]);
      }
  }

  public static void main(String[] args) {

    int[][] t = new int[sx * 2][sy * 2];
    add(t, x, y, v);//tree-x-y-value
    int res1 = max(t, x1, y1, x2, y2);//t-[x1,y1]*[x2,y2]
  }
}
```

1.4 Static RMQ, Lookup Table

```
/ keep code simple.
int lookup[MAX][LOGMAX];

struct Query
{
  int L, R;
};

void preprocess(int arr[], int n)
{
  // Initialize M for the intervals with length 1
  for (int i = 0; i < n; i++)
    lookup[i][0] = i;

  for (int j = 1; (1 << j) <= n; j++)
  {
    for (int i = 0; (i + (1 << j) - 1) < n; i++)
    {
      if (arr[lookup[i][j - 1]] < arr[lookup[i + (1 << (j - 1))][j - 1]])
        lookup[i][j] = lookup[i][j - 1];
      else
        lookup[i][j] = lookup[i + (1 << (j - 1))][j - 1];
    }
  }
}

// Returns minimum of arr[L..R]
int query(int arr[], int L, int R)
{

  int j = (int)log2(R - L + 1);

  if (arr[lookup[L][j]] <= arr[lookup[R - (int)pow(2, j) + 1][j]])
    return arr[lookup[L][j]];

  else return arr[lookup[R - (int)pow(2, j) + 1][j]];
}


void RMQ(int arr[], int n, Query q[], int m)
{
  // Fills table lookup[n][Log n]
  preprocess(arr, n);

  for (int i = 0; i<m; i++)
  {
    // Left and right boundaries of current range
    int L = q[i].L, R = q[i].R;
    // Print sum of current query range
    cout << "Minimum of [" << L << ", "
      << R << "] is " << query(arr, L, R) << endl;
  }
}

int main()
{
  int a[] = { 7, 2, 3, 0, 5, 10, 3, 12, 18 };
  int n = sizeof(a) / sizeof(a[0]);
  Query q[] = { { 0, 4 },{ 4, 7 },{ 7, 8 } };
  int m = sizeof(q) / sizeof(q[0]);
  RMQ(a, n, q, m);
  return 0;
}
```

1.5 Fenwick Tree, Inversions

```
#include <iostream>
#include <vector>
#include <algorithm>
#include <cstdio>

using namespace std;

typedef long long int64;
typedef vector<int64> vi;

// vector (vi), iostream, algo,
#define LSOne(i) (i & (-i))
struct FenwickTree {
        vi ft; FenwickTree() {}
        FenwickTree(int n) { ft.assign(n + 1, 0); } // init n + 1 zeroes
        int rsq(int b) { // returns RSQ(1, b), pass b >= 1
                int sum = 0; for (; b; b -= LSOne(b)) sum += ft[b];
                return sum;
        }
        int rsq(int a, int b) { // returns RSQ(a, b)
                return rsq(b) - (a == 1 ? 0 : rsq(a - 1));
        }
        // adjusts value of the k-th element by v (v can be +ve/inc or -ve/dec)
        void update(int k, int v) { // note: n = ft.size() - 1
                for (; k < (int)ft.size(); k += LSOne(k)) ft[k] += v;
        }
};

int main() {
        int f[] = { 2,4,5,5,6,6,6,7,7,8,9 }; // m = 11 scores
        FenwickTree ft(10); // declare a Fenwick Tree for range [1..10]
                                                  // insert these scores manually one by one into an
                                                        empty Fenwick Tree
        for (int i = 0; i < 11; i++) ft.update(f[i], 1); // this is O(k log n)
        printf("%d\n", ft.rsq(1, 1)); // 0 => ft[1] = 0
        printf("%d\n", ft.rsq(1, 2)); // 1 => ft[2] = 1
        printf("%d\n", ft.rsq(1, 6)); // 7 => ft[6] + ft[4] = 5 + 2 = 7
        printf("%d\n", ft.rsq(1, 10)); // 11 => ft[10] + ft[8] = 1 + 10 = 11
        printf("%d\n", ft.rsq(3, 6)); // 6 => rsq(1, 6) - rsq(1, 2) = 7 - 1
        ft.update(5, 2); // update demo
        printf("%d\n", ft.rsq(1, 10)); // now 13
} // return 0;

/* extera

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {// ***Change Needed***
  int idx = 0, mask = TREE_SIZE; //(must be a power of 2)
  while(mask && idx < TREE_SIZE) {
    int t = idx + mask;
    if(x >= tree[t]) {idx = t; x -= tree[t]; }
    mask >>= 1;
  }
  return idx;
}

// how to count inversions
int main(){
        vi a;
        while(cin >> n){ // count inversions, (Change Needed)
                a.assign(n,0); b.assign(n,0); tree.assign(n,0);
                for(int i=0 ; i<n; i++){
                        cin >> a[i]; b[i]=a[i];
                }
                sort(b.begin(),b.end());
                for(int i=0 ; i<n ; i++){
                        int rank=(int)(lower_bound(b.begin(),b.end(),a[i])-b.begin());
                        a[i]=rank+1;
                }
                int64 invs=0;//num of inversions
                for(int i=n-1 ; i>=0 ; i--){
                        invs+=read(a[i]-1);
```

```
                    update(a[i],1);
                }
                cout << invs << endl;
            }
    }

    */
```

1.6 Fenwick Tree, 2D

```java
public class FenwickTree2D {

  public static void add(int[][] t, int r, int c, int value) {
    for (int i = r; i < t.length; i |= i + 1)
      for (int j = c; j < t[0].length; j |= j + 1)
        t[i][j] += value;
  }

  // sum[(0, 0), (r, c)]
  public static int sum(int[][] t, int r, int c) {
    int res = 0;
    for (int i = r; i >= 0; i = (i & (i + 1)) - 1)
      for (int j = c; j >= 0; j = (j & (j + 1)) - 1)
        res += t[i][j];
    return res;
  }

  // sum[(r1, c1), (r2, c2)]
  public static int sum(int[][] t, int r1, int c1, int r2, int c2) {
    return sum(t, r2, c2) - sum(t, r1 - 1, c2) - sum(t, r2, c1 - 1) + sum(t, r1 - 1, c1 - 1);
  }

  public static int get(int[][] t, int r, int c) {
    return sum(t, r, c, r, c);
  }

  public static void set(int[][] t, int r, int c, int value) {
    add(t, r, c, -get(t, r, c) + value);
  }

  // Usage example
  public static void main(String[] args) {
    int[][] t = new int[10][20];
    add(t, 0, 0, 1);
    add(t, 9, 19, -2);
    System.out.println(-1 == sum(t, 0, 0, 9, 19));
  }
}
```

1.7 Fenwick Tree, Extended

```java
public class FenwickTreeExtended {

  // T[i] += value
  public static void add(int[] t, int i, int value) {
    for (; i < t.length; i |= i + 1)
      t[i] += value;
  }

  // sum[0..i]
  public static int sum(int[] t, int i) {
    int res = 0;
    for (; i >= 0; i = (i & (i + 1)) - 1)
      res += t[i];
    return res;
  }

  public static int[] createTreeFromArray(int[] a) {
    int[] res = new int[a.length];
    for (int i = 0; i < a.length; i++) {
      res[i] += a[i];
      int j = i | (i + 1);
      if (j < a.length)
        res[j] += res[i];
    }
    return res;
  }

  // sum[a..b]
  public static int sum(int[] t, int a, int b) {
    return sum(t, b) - sum(t, a - 1);
  }

  public static int get(int[] t, int i) {
    int res = t[i];
    if (i > 0) {
      int lca = (i & (i + 1)) - 1;
      for (--i; i != lca; i = (i & (i + 1)) - 1)
        res -= t[i];
    }
    return res;
  }

  public static void set(int[] t, int i, int value) {
    add(t, i, -get(t, i) + value);
  }

  ///////////////////////////////////////////////////////
  // interval add
  public static void add(int[] t, int a, int b, int value) {
    add(t, a, value);
    add(t, b + 1, -value);
  }

  // point query
  public static int get1(int[] t, int i) {
    return sum(t, i);
  }
  ///////////////////////////////////////////////////////

  ///////////////////////////////////////////////////////
  // interval add
  public static void add(int[] t1, int[] t2, int a, int b, int value) {
    add(t1, a, value);
    add(t1, b, -value);
    add(t2, a, -value * (a - 1));
    add(t2, b, value * b);
  }

  // interval query
  public static int sum(int[] t1, int[] t2, int i) {
    return sum(t1, i) * i + sum(t2, i);
  }
  ///////////////////////////////////////////////////////

  // Returns min(p|sum[0,p]>=sum)
  public static int lower_bound(int[] t, int sum) {
    --sum;
    int pos = -1;
    for (int blockSize = Integer.highestOneBit(t.length); blockSize != 0; blockSize >>= 1) {
      int nextPos = pos + blockSize;
      if (nextPos < t.length && sum >= t[nextPos]) {
        sum -= t[nextPos];
        pos = nextPos;
      }
    }
    return pos + 1;
  }

  // Usage example
  public static void main(String[] args) {
    int[] t = new int[10];
    set(t, 0, 1);
    add(t, 9, -2);
    System.out.println(-1 == sum(t, 0, 9));

    t = createTreeFromArray(new int[] {1, 2, 3, 4, 5, 6});
    for (int i = 0; i < t.length; i++)
      System.out.print(get(t, i) + " ");
    System.out.println();
    t = createTreeFromArray(new int[] {0, 0, 1, 0, 0, 1, 0, 0});
    System.out.println(5 == lower_bound(t, 2));

    int[] t1 = new int[10];
    int[] t2 = new int[10];
    add(t1, t2, 0, 9, 1);
    add(t1, t2, 0, 0, -2);
    System.out.println(sum(t1, t2, 9));
  }
}
```

1.8 KDTree

```java
import java.util.*;

public class KdTreePointQuery {

  public static class Point {
    int x, y;

    public Point(int x, int y) {
```

```java
    this.x = x;
    this.y = y;
  }
}

int[] tx;
int[] ty;

public KdTreePointQuery(Point[] points) {
  int n = points.length;
  tx = new int[n];
  ty = new int[n];
  build(0, n, true, points);
}

void build(int low, int high, boolean divX, Point[] points) {
  if (low >= high)
    return;
  int mid = (low + high) >>> 1;
  nth_element(points, low, high, mid, divX);

  tx[mid] = points[mid].x;
  ty[mid] = points[mid].y;

  build(low, mid, !divX, points);
  build(mid + 1, high, !divX, points);
}

static void nth_element(Point[] a, int low, int high, int n, boolean divX) {
  while (true) {
    int k = randomizedPartition(a, low, high, divX);
    if (n < k)
      high = k;
    else if (n > k)
      low = k + 1;
    else
      return;
  }
}

static final Random rnd = new Random();

static int randomizedPartition(Point[] a, int low, int high, boolean divX) {
  swap(a, low + rnd.nextInt(high - low), high - 1);
  int v = divX ? a[high - 1].x : a[high - 1].y;
  int i = low - 1;
  for (int j = low; j < high; j++)
    if (divX ? a[j].x <= v : a[j].y <= v)
      swap(a, ++i, j);
  return i;
}

static void swap(Point[] a, int i, int j) {
  Point t = a[i];
  a[i] = a[j];
  a[j] = t;
}

long bestDist;
int bestNode;

public int findNearestNeighbour(int x, int y) {
  bestDist = Long.MAX_VALUE;
  findNearestNeighbour(0, tx.length, x, y, true);
  return bestNode;
}

void findNearestNeighbour(int low, int high, int x, int y, boolean divX) {
  if (low >= high)
    return;
  int mid = (low + high) >>> 1;
  long dx = x - tx[mid];
  long dy = y - ty[mid];
  long dist = dx * dx + dy * dy;
  if (bestDist > dist) {
    bestDist = dist;
    bestNode = mid;
  }
  long delta = divX ? dx : dy;
  long delta2 = delta * delta;

  if (delta <= 0) {
    findNearestNeighbour(low, mid, x, y, !divX);
    if (delta2 < bestDist)
      findNearestNeighbour(mid + 1, high, x, y, !divX);
  }
  else {
    findNearestNeighbour(mid + 1, high, x, y, !divX);
    if (delta2 < bestDist)
      findNearestNeighbour(low, mid, x, y, !divX);
  }
}

  public static void main(String[] args) {
    Point[] points = new Point[n];
    //fill points
    //build tree
    KdTreePointQuery kdTree = new KdTreePointQuery(points);

    int index = kdTree.findNearestNeighbour(qx, qy);
    Point p = points[index];
  }
}
```

2 Dynamic Programming

2.1 Classic DP problems

```cpp
// ----- Maximum Subrectangle Sum
int main(){
        for(int i=1 ; i<n ; i++)//preprocess
                                for(int j=0 ; j<n ; j++)
                                        a[i][j]+=a[i-1][j];

        int Max=0, ans=0;
        for(int k=0 ; k<n ; k++){//calc
                for(int i=0 ; i<n-k ; i++){ Max=0;
                        for(int j=0 ; j<n ; j++){
                                if(Max<0) Max=a[i+k][j]-a[i][j];
                                else Max+=a[i+k][j]-a[i][j];
                                if(Max>ans) ans=Max;
        } } }

        //sub array, finsh and start point p=(val, startidx, finishidx)
        p ans=p(-1,0,0); int sum=0,id=1;
        for(int i=1 ; i<n ; i++){
                                if(sum<0){sum=0; id=i;}
                                sum+=a[i];
                                p tmp=p(sum,id,i+1); ans=Max(ans,tmp);
        }
}


// ----- Optimal Array Multiplication Sequence (Print Path)
int n,a[10+5],p[10+5][10+5],dp[10+5][10+5];

int solve(int L, int R){
        if(L==R){ return 0; }
        if(dp[L][R]!=-1) return dp[L][R];
        int Min=INF;
        for(int i=L ; i<R ; i++){
                int slv=solve(L,i)+solve(i+1,R)+a[(L-1)]*a[i]*a[R];
                if(Min>slv)     Min=slv; p[L][R]=i;
        }
        return dp[L][R]=Min;
}
//prints like this => (A1 x (A2 x A3))
void print(int L, int R){
        if(L==R){ cout << "A" <<L; return; }
        cout << "("; print(L,p[L][R]);
        cout << " x ";
        print(p[L][R]+1,R); cout << ")";
}

int main(){     int t=1;
        while(cin >> n && n){
                for(int i=1 ; i<=n ; i++)cin >> a[i-1] >> a[i];
                memset(dp,-1,sizeof dp);
                solve(1,n);//cout << solve(1,n) << endl;
                printf("Case %d: ",t++); print(1,n); printf("\n");
        }
        return 0;
}

// ----- LIS
int main(){
        vector<int> v;
        v.push_back(inf);
        for (int i = 0; i<n; i++) {
                int x = dolls[i].w; // array element
                int id = lower_bound(v.begin(), v.end(), x + 1) - v.begin();

                if (id == v.size() - 1) v.push_back(inf); v[id] = x;
        }
        cout << v.size() - 1 << endl;
}
```

```
// -----  LCS
int main(){
        dp[MAX][MAX]={0};
        for(int i=1 ; i<=n ; i++){
                                for(int j=1 ; j<=n ; j++){
                                        if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
                                        else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
        }}
        cout << dp[n][n] << endl;
}

// ----- TSP
p a[15]; int n, dp[15][1<<15];

int solve(int pos, int bitset){
        int& dpp=dp[pos][bitset]; //dpp = dp poniter
        if(bitset==(1<<n)-1) return dist(a[pos],a[0]);
        if(dpp!=-1) return dpp;
        dpp=INF;
        for(int i=0 ; i<n ; i++){
                if(!(bitset&(1<<i))) dpp=min(dpp,solve(i,bitset|(1<<i))+dist(a[pos],a[i]));
        }
        return dpp;
}

int main(){
        int tc; cin >> tc;
        while(tc--){
                cin >> a[0].X >> a[0].Y;  cin >> n; n++;
                for(int i=1 ; i<n ; i++) cin >> a[i].X >> a[i].Y;
                memset(dp, -1, sizeof dp);
                cout << solve(0,1) << endl;
        }
        return 0;
}
```

3 Graph Algorithms

3.1 Articulation Points, Bridges

```
int n, lev, dfsRoot, rootChilds;
int dfsLow[MAX], dfsNum[MAX], parent[MAX];
vvi adj; set<pii> bridges; set<int> artPoints;

void dfs(int u) {
        dfsLow[u] = dfsNum[u] = lev++;
        for (int i = 0; i<adj[u].size(); i++) {
                int v = adj[u][i];
                if (dfsNum[v] == 0) {
                        if (u == dfsRoot) rootChilds++;
                        parent[v] = u; dfs(v);

                        if (dfsLow[v] >= dfsNum[u] && u != dfsRoot)//u is articulation point
                                artPoints.insert(u);

                        if (dfsLow[v] > dfsNum[u]) {
                                bridges.insert(pii(v, u));
                                bridges.insert(pii(u, v));
                        }

                        dfsLow[u] = min(dfsLow[u], dfsLow[v]);
                }
                else if (parent[u] != v)
                        dfsLow[u] = min(dfsLow[u], dfsNum[v]);
        }
}

int main() {
        while (cin >> n) {
                adj.assign(n, vi()); //initialization
                memset(dfsLow, 0, sizeof dfsLow);
                memset(dfsNum, 0, sizeof dfsNum);
                memset(parent, 0, sizeof parent);
                bridges.clear(); artPoints.clear();
                lev = 1; int tmp, u, m;
                for (int i = 0; i<n; i++) { // construct the graph
                        scanf("%d (%d", &u, &m); cin.ignore();
                        for (int i = 0; i<m; i++) {
                                cin >> tmp; adj[u].push_back(tmp);
                        }
                }
                for (int i = 0; i<n; i++) {
                        if (dfsNum[i] == 0) {
                                dfsRoot = i; rootChilds = 0; dfs(i);
                                if (rootChilds >= 2) artPoints.insert(dfsRoot);
                        }
                }
                printf("%d critical links\n", bridges.size());
                set<pii>::iterator itr; // print answer
                for (itr = bridges.begin(); itr != bridges.end(); itr++)
                        printf("%d - %d\n", itr->first, itr->second);
                cout << endl;
        }
        return 0;}
```

3.2 Strongly Connected Component, Kosaraju

```
// Doesn't run properly

vvi adjOrg, adjRev;  vi vis, ord, col;

void dfsOrg(int u){
        if (vis[u]) return; vis[u] = true;
        for (int i = 0; i < adjOrg[u].size(); i++){
                dfsOrg(adjOrg[u][i]);
        }
        ord.push_back(u);
}

int dfsRev(int u, int color){
        if (col[u]) return 0; col[u] = color;

        int ret = 1;
        for (int i = 0; i < adjRev[u].size(); i++){
                ret += dfsRev(adjRev[u][i], color);
        }
        return ret;
}

int main(){
        while (cin >> n && n){
                int u, v; string line;
                adjOrg.assign(n, vi());
                adjRev.assign(n, vi());

                for (int i = 0; i < n; i++){
                        stringstream sstr(line);
                        sstr >> u;
                        while (sstr >> v){
                                adjOrg[u].push_back(v);
                                adjRev[v].push_back(u);
                        }
                }

                ord.clear();
                vis.assign(n, 0);
                for (int u = 0; u < n; u++){
                        if (!vis[u]) dfsOrg(u);
                }
                int color = 1;
                col.assign(n, 0);
                while (!ord.empty()){
                        int u = ord.back();
                        if (!col[u]){
                                int size = dfsRev(u, color); // SCC Size
                                if (size > 1){
                                        for (int v = 0; v < n; v++){
                                                if (col[v] == color); //inSame SCC;

                                        }
                                }
                                color++;
                        }
                        ord.pop_back();
                }
        }
}
```

3.3 Strongly Connected Component, Tarjan

```
#define MAX 100000

using namespace std;

int dfsNum[MAX+10],dfsLow[MAX+10],vis[MAX+10],in[MAX+10],n,m,lev,ans; vector<int> SCC,adj[MAX+10];

void dfs(int u){
        dfsLow[u]=dfsNum[u]=lev++; vis[u]=1; SCC.push_back(u);
        for(int i=0 ; i<adj[u].size() ; i++){
```

```
            int v=adj[u][i];
            if(dfsNum[v]==0) dfs(v);
            if(vis[v]) dfsLow[u]=min(dfsLow[u], dfsLow[v]), in[v]--;
        }
        if(dfsLow[u]==dfsNum[u]){
        // this prints all vertices v blong to SCC with dfsLow[v] == dfsLow[u]
            bool flag=true;
            for(int i=0, v ; !SCC.empty() ; i++){
                v=SCC.back(); SCC.pop_back(); vis[v]=0;
                printf("%d ", v);
                if(in[v]) flag=false;
                if(v==u) break;
            }
            printf("\n");
            if(flag) ans++;
        // counts number of SCCs without indegree outside of other SCCs
        }
}

int main(){
        int tc; scanf("%d", &tc);; int x,y;
        while(tc--){
            scanf("%d %d", &n, &m);
            memset(dfsNum,0,sizeof dfsNum); // memset(adj,0,sizeof adj);
            memset(dfsLow,0,sizeof dfsLow); memset(vis,0,sizeof vis);
            memset(in,0,sizeof in); lev=1; ans=0;

            for(int i=0, j=0 ; i<m ; i++){
                scanf("%d %d", &x, &y); x--; y--;
                adj[x].push_back(y); in[y]++;
            }
            for(int i=0 ; i<n ; i++){
                if(dfsNum[i]==0) dfs(i);
            }
            cout << ans << endl;
        }
        return 0;
}
```

3.4 Shortest Path, Dijkstra

```
   #include <iostream>
#include <vector>
#include <queue>

using namespace std;

const int INF = 1e8;

struct ToNode{
        int v, w;
        ToNode(int v, int w)
                :v(v), w(w){}
};

struct QEntry{
        int node, cost;
        QEntry(int node, int cost)
                :node(node), cost(cost){}
        bool operator<(const QEntry& op) const {
                return cost < op.cost;
        }
};

typedef vector<int>     vi;
typedef vector<ToNode> vtn;
typedef vector<vtn   > vvtn;

int n, m; vvtn adj;

int dijkstra(int s, int t, vi& dist){
        dist.assign(n, INF);
        priority_queue<QEntry> q;
        q.push(QEntry(s, 0)); dist[s] = 0;

        while (!q.empty()){
                QEntry u = q.top(); q.pop();
                if (u.node == t) return u.cost;
                if (u.cost > dist[u.node]) continue;
                for (int i = 0; i < adj[u.node].size(); i++){
                        QEntry v(adj[u.node][i].v, u.cost + adj[u.node][i].w);
                        if (dist[v.node] > v.cost){
                                dist[v.node] = v.cost; q.push(v);
                        }
                }
        }
        return INF;
}
```

3.5 LCA Tree Distance

```
const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes];  // children[i] contains the children of node i
int A[max_nodes][log_max_nodes + 1];  // A[i][j] is the 2^j-th ancestor of node i, or -1 if that
      ancestor does not exist
int L[max_nodes];     // L[i] is the distance between node i and the root

              // floor of the binary logarithm of n
int lb(unsigned int n)
{
  if (n == 0)
    return -1;
  int p = 0;
  if (n >= 1 << 16) { n >>= 16; p += 16; }
  if (n >= 1 << 8) { n >>= 8; p += 8; }
  if (n >= 1 << 4) { n >>= 4; p += 4; }
  if (n >= 1 << 2) { n >>= 2; p += 2; }
  if (n >= 1 << 1) { p += 1; }
  return p;
}

void DFS(int i, int l)
{
  L[i] = l;
  for (int j = 0; j < children[i].size(); j++)
    DFS(children[i][j], l + 1);
}

int LCA(int p, int q)
{
  // ensure node p is at least as deep as node q
  if (L[p] < L[q])
    swap(p, q);

  // "binary search" for the ancestor of node p situated on the same level as q
  for (int i = log_num_nodes; i >= 0; i--)
    if (L[p] - (1 << i) >= L[q])
      p = A[p][i];

  if (p == q)
    return p;

  // "binary search" for the LCA
  for (int i = log_num_nodes; i >= 0; i--)
    if (A[p][i] != -1 && A[p][i] != A[q][i])
    {
      p = A[p][i];
      q = A[q][i];
    }

  return A[p][0];
}

int main(int argc, char* argv[])
{
  // read num_nodes, the total number of nodes
  log_num_nodes = lb(num_nodes);

  for (int i = 0; i < num_nodes; i++)
  {
    int p;
    // read p, the parent of node i or -1 if node i is the root

    A[i][0] = p;
    if (p != -1)
      children[p].push_back(i);
    else
      root = i;
  }

  // precompute A using dynamic programming
  for (int j = 1; j <= log_num_nodes; j++)
    for (int i = 0; i < num_nodes; i++)
      if (A[i][j - 1] != -1)
        A[i][j] = A[A[i][j - 1]][j - 1];
      else
        A[i][j] = -1;

  // precompute L
  DFS(root, 0);


  return 0;
```

```
}
```

3.6 Graphic Sequence

```
// given a sequence of integers see if it s a sequence of degrees of graph or not.
int a[10010]; long long sum,Min;;

int main(){
    int n;
    while(cin >> n && n){
        for(int i=0 ; i<n ; i++) scanf("%d",&a[i]);
        sort(a,a+n, ::greater<int>() );
        bool possible=true; sum=0;
        for(int i=0 ; i<n ; i++){
            sum+=a[i]; Min=0;
            for(int j=i+1 ; j<n; j++) Min+=min(a[j],i+1);
            if(sum>i*(i+1)+Min){
                possible=false;
                break;
            }
        }
        if(!possible || sum%2) cout << "Not possible" << endl;
        else cout << "Possible" << endl;
    }
    return 0;}
```

3.7 Floyd Warshal, Print Path

```
#define MAX (100+10)

int adj[MAX][MAX],path[MAX][MAX]; int n;

void print(int i,int j){
        if(i!=j){
                printf(" %d",i );
                print(path[i][j],j);
        }
}

int main(){
        int tc; cin >> tc;
        while(tc--){
                cin >> n;
                for(int i=0 ; i<n ; i++){
                        for(int j=0 ; j<n ; j++){
                                adj[i][j]=1e9; if(i==j) adj[i][j]=0;
                                path[i][j]=j;//initial parent
                        }
                }

                for(int k=0 ; k<n ; k++){
                        for(int i=0 ; i<n ; i++){
                                for(int j=0 ; j<n ; j++){
                                        if(adj[i][j]>adj[i][k]+adj[k][j]){
                                                adj[i][j]=adj[i][k]+adj[k][j];
                                                path[i][j]=path[i][k];//set parent
                                        }
                                }
                        }
                }
                int s,d;
                cin >> s >> d;
                printf("%d euros\n",adj[s][d]);

                //this prints the path even if source and distinaion are same
                printf("%d",s); print(path[s][d],d); printf(" %d\n",d);

        }
        return 0;
}
```

3.8 Max Flow, Edmonds Karp

```
//UVa 820 - Internet Bandwidth
#define INF (int)1e9
#define MAX 100+10

using namespace std;

int res[MAX][MAX],mf,f,s,t,n,m,par[MAX]; vector<int> dist,adj[MAX];

void agument(int v, int minEdge){
    if(v==s) f=minEdge;
    else if(par[v]!=-1){
        agument(par[v],min(minEdge,res[par[v]][v]));
        res[par[v]][v]-=f; res[v][par[v]]+=f;
    }
}

int main(){
    int tc=1;
    while(cin >> n && n){
        mf=0; memset(res,0,sizeof res); for(int i=0 ; i<n ; i++) adj[i].clear();
        cin >> s >> t >> m; s--; t--;
        int u,v,c;
        while(m--){
            cin >> u >> v >> c; u--; v--;
            res[u][v]+=c; res[v][u]+=c;
            adj[u].push_back(v); adj[v].push_back(u);
        }
        while(1){
            f=0; memset(par,-1,sizeof par); dist.assign(n,INF);
            dist[s]=0; queue<int> q; q.push(s);
            while(!q.empty()){
                int u=q.front(); q.pop();
                if(u==t) break;
                for(int i=0 ; i<adj[u].size(); i++){
                    int v=adj[u][i];
                    if(res[u][v]>0 && dist[v]==INF){
                        dist[v]=dist[u]+1; q.push(v); par[v]=u;
                    }
                }
            }
            agument(t,INF);
            if(f==0) break;
            mf+=f;
        }
        printf("Network %d\n", tc++);
        printf("The bandwidth is %d.\n\n", mf);
    }
    return 0;}
```

3.9 Max Flow, Dinic

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
// Running time: O(|V|^2 |E|)
// INPUT:
//     - graph, constructed using AddEdge() - source - sink
// OUTPUT:
//     - maximum flow value
//     - To obtain the actual flow values, look at all edges with
//       capacity > 0 (zero capacity edges are residual edges).
using namespace std;
const int INF = 2000000000;
struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};
struct Dinic {
    int N; vector<vector<Edge> > G;
    vector<Edge *> dad; vector<int> Q;
    Dinic(int N) : N(N), G(N), dad(N), Q(N) {}
    void AddEdge(int from, int to, int cap) {
        G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
        if (from == to) G[from].back().index++;
        G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
    }
    long long BlockingFlow(int s, int t) {
        fill(dad.begin(), dad.end(), (Edge *)NULL);
        dad[s] = &G[0][0] - 1;
        int head = 0, tail = 0;
        Q[tail++] = s;
        while (head < tail) {
            int x = Q[head++];
            for (int i = 0; i < G[x].size(); i++) {
                Edge &e = G[x][i];
                if (!dad[e.to] && e.cap - e.flow > 0) {
                    dad[e.to] = &G[x][i];
                    Q[tail++] = e.to;
                }}}
        if (!dad[t]) return 0;
        long long totflow = 0;
        for (int i = 0; i < G[t].size(); i++) {
```

```
            Edge *start = &G[G[t][i].to][G[t][i].index];
            int amt = INF;
            for (Edge *e = start; amt && e != dad[s]; e = dad[e->from]) {
                if (!e) { amt = 0; break; }
                amt = min(amt, e->cap - e->flow);
            }
            if (amt == 0) continue;
            for (Edge *e = start; amt && e != dad[s]; e = dad[e->from]) {
                e->flow += amt;
                G[e->to][e->index].flow -= amt;
            }
            totflow += amt;
        }
        return totflow;
    }
    long long GetMaxFlow(int s, int t) {
        long long totflow = 0;
        while (long long flow = BlockingFlow(s, t))
            totflow += flow;
        return totflow;
    }
};
```

3.10 Min Cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time: O(|V|^3)
// INPUT: graph, constructed using AddEdge()
// OUTPUT: (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {
          best_cut = cut;
          best_weight = w[last];
        }
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}

// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and Conquer
int main() {
  int N;
  cin >> N;
  for (int i = 0; i < N; i++) {
    int n, m;
    cin >> n >> m;
    VVI weights(n, VI(n));
    for (int j = 0; j < m; j++) {
      int a, b, c;
      cin >> a >> b >> c;
      weights[a-1][b-1] = weights[b-1][a-1] = c;
    }
    pair<int, VI> res = GetMinCut(weights);
    cout << "Case #" << i+1 << ": " << res.first << endl;
  }
}
// END CUT
```

3.11 MaxCardinalityBipartiteMatching, Alternating path

```
#include <iostream>
#include <algorithm>
#include <vector>

using namespace std;

typedef vector<int> vi;
typedef vector<vi > vvi;

vvi adj; vi owner, vis; int n,b;

int altpath(int u){
        if(vis[u]) return 0; vis[u]=1;
        for(int i=0 ; i<adj[u].size() ; i++){
                int v=adj[u][i];
                if(owner[v]==-1 || altpath(owner[v])){
                        owner[v]=u; return 1;
                }
        }
        return 0;
}

int main(){
        int tmp,tc,t=1; cin >> tc;
        while(tc--){
                cin >> n >> b; adj.assign(n+b,vi());
                for(int i=0 ; i<n ; i++){
                        for(int j=0 ; j<b ; j++){
                                // if there is an edge from n group to b group
                                cin >> tmp; if(tmp==1) adj[i].push_back(j+n);
                        }
                }
                int ans=0; owner.assign(n+b,-1);
                for(int u=0 ; u<n ; u++){
                        vis.assign(n,0); ans+=altpath(u);
                }
                printf("Case %d: a maximum of %d matched\n", t++, ans);
        }
        return 0;
}
```

3.12 Min Cost Max Flow

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972).  This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]).  For a regular max flow, set all edge costs to 0.
//
// Running time, O(|V|^2) cost per augmentation
//     max flow:           O(|V|^3) augmentations
//     min cost max flow:  O(|V|^4 * MAX_EDGE_COST) augmentations
// INPUT:
//     - graph, constructed using AddEdge(), source, sink
// OUTPUT:
//     - (maximum flow value, minimum cost value)
//     - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
```

```
  int N;
  VVL cap, flow, cost;
  VI found;
  VL dist, pi, width;
  VPII dad;

  MinCostMaxFlow(int N) :
    N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
    found(N), dist(N), pi(N), width(N), dad(N) {}

  void AddEdge(int from, int to, L cap, L cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
  }

  void Relax(int s, int k, L cap, L cost, int dir) {
    L val = dist[s] + pi[s] - pi[k] + cost;
    if (cap && val < dist[k]) {
      dist[k] = val;
      dad[k] = make_pair(s, dir);
      width[k] = min(cap, width[s]);
    }
  }

  L Dijkstra(int s, int t) {
    fill(found.begin(), found.end(), false);
    fill(dist.begin(), dist.end(), INF);
    fill(width.begin(), width.end(), 0);
    dist[s] = 0;
    width[s] = INF;

    while (s != -1) {
      int best = -1;
      found[s] = true;
      for (int k = 0; k < N; k++) {
        if (found[k]) continue;
        Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
        Relax(s, k, flow[k][s], -cost[k][s], -1);
        if (best == -1 || dist[k] < dist[best]) best = k;
      }
      s = best;
    }

    for (int k = 0; k < N; k++)
      pi[k] = min(pi[k] + dist[k], INF);
    return width[t];
  }

  pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
      totflow += amt;
      for (int x = t; x != s; x = dad[x].first) {
        if (dad[x].second == 1) {
          flow[dad[x].first][x] += amt;
          totcost += amt * cost[dad[x].first][x];
        } else {
          flow[x][dad[x].first] -= amt;
          totcost -= amt * cost[x][dad[x].first];
        }
      }
    }
    return make_pair(totflow, totcost);
  }
};

// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow

int main() {
  int N, M;

  while (scanf("%d%d", &N, &M) == 2) {
    VVL v(M, VL(3));
    for (int i = 0; i < M; i++)
      scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
    L D, K;
    scanf("%Ld%Ld", &D, &K);

    MinCostMaxFlow mcmf(N+1);
    for (int i = 0; i < M; i++) {
      mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
      mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
    }
    mcmf.AddEdge(0, 1, D, 0);

    pair<L, L> res = mcmf.GetMaxFlow(0, N);

    if (res.first == D) {
      printf("%Ld\n", res.second);
    } else {
      printf("Impossible.\n");
    }
  }

  return 0;
}

// END CUT
```

3.13 Min Cost Matching

```
///////////////////////////////////////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs.  In practice, it solves 1000x1000 problems in around 1
// second.
//
//   cost[i][j] = cost for pairing left node i with right node j
//   Lmate[i] = index of right node that left node i pairs with
//   Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative.  To perform
// maximization, simply negate the cost[][] matrix.
///////////////////////////////////////////////////////////////////////

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
  int n = int(cost.size());

  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
      }
    }
  }

  VD dist(n);
  VI dad(n);
  VI seen(n);

  // repeat until primal solution is feasible
  while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {
```

```
      // find closest
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;

      // termination condition
      if (Rmate[j] == -1) break;

      // relax neighbors
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k] - dist[j];
      u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j] = Rmate[d];
      Lmate[Rmate[j]] = j;
      j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
  }

  double value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

  return value;
}
```

4 Mathematics and Geometry

4.1 Prime Numbers, Factoring

```
#include <iostream>
#include <cstdio>
#include <vector>
#include <bitset>

using namespace std;

typedef long long int64;
const int64 MAX = 1e6 + 100;

bitset<MAX> isp;// isprime
vector<int64> primes, pfs, pws; //pfs = prime factors, pws = prime powers

void genprime(){
    isp.set(); isp[0]=isp[1]=0;
    for(int64 i=2 ; i<MAX; i++){
        if(isp[i]){ primes.push_back(i);
            for(int64 j=i*i ; j<MAX ; j+=i) isp[j]=0;
        }
    }
}

bool isprime(int n){
    if(n<MAX) return isp[n];
    for(int i=0 ; i<primes.size() && primes[i]*primes[i]<=n ; i++){
        if(n%primes[i]==0) return 0;
    }
    return 1;
}
//generation prime factors of a number
int main(){
    int64 n; genprime();
    while(cin >> n){                      // cop = euler Phi funcion
        int64 tmp=n, cnt=0, cop=n, div=1;// cop = coprimes =  all m (m<n && gcd(m,n)==1)
                                          // div = divisors  =  all m (m<n && gcd(m,n)==m)
        for(int i=0, pf=2 ; pf*pf<=n; i++, pf=primes[i]){
            int pow=0;
            while(tmp%pf==0){
                tmp/=pf; pow++;
            }
            if(pow) {
                pfs.push_back(pf), pws.push_back(pow);
                cop -= cop/pf; div *= (pow+1);
            }
        }
        if(tmp>1) cop -= cop/tmp, div *= (1+1);
        cout << cop + div + 1 << endl;
    }
}
```

4.2 Extended Euclid

```
import java.util.Scanner;

public class Main {
    public static class ExtendedEuclid {
        public static int x0;
        public static int y0;
        public static int c;
        public static int d;

        public static int gcd(int a, int b) {
            if (b == 0) return a;
            return gcd(b, a % b);
        }

        public static void calculate(int a, int b) {
            if (b == 0) {
                x0 = 1;
                y0 = 0;
                d = a;
                return;
            }
            calculate(b, a % b);

            int x1 = y0;
            int y1 = x0 - (a / b) * y0;

            x0 = x1;
            y0 = y1;
        }


        public static int howManyPositiveSolutions(int a, int b, int c) {
            if (c % gcd(a, b) != 0) return 0; // no solution even negatives

            x0 *= c / gcd(a, b);
            y0 *= c / gcd(a, b); // x = x0 + (b/d)n, y = y0 - (a/d)n
            double lowerBoundForN = (double) (-x0 + 0.5) / (b / d); // for x>0
            double upperBoundForN = (double) (+y0 - 0.5) / (a / d); // for y>0
            return (int) Math.max(0, Math.ceil(upperBoundForN) - Math.floor(lowerBoundForN) - 1); //
                how many int between
        }

    }

    public static void main(String[] args) {
        ExtendedEuclid.calculate(25, 18); // Copetitive Programming 2 Example
    }

}
```

4.3 Number Theory General

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
```

```cpp
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
  return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
  int tmp;
  while (b) { a %= b; tmp = a; a = b; b = tmp; }
  return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
  return a / gcd(a, b)*b;
}

// returns d = gcd(a,b); finds x,y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
  int xx = y = 0;
  int yy = x = 1;
  while (b) {
    int q = a / b;
    int t = b; b = a%b; a = t;
    t = xx; xx = x - q*xx; x = t;
    t = yy; yy = y - q*yy; y = t;
  }
  return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
  int x, y;
  VI solutions;
  int d = extended_euclid(a, n, x, y);
  if (!(b%d)) {
    x = mod(x*(b / d), n);
    for (int i = 0; i < d; i++)
      solutions.push_back(mod(x + i*(n / d), n));
  }
  return solutions;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
  int x, y;
  int d = extended_euclid(a, n, x, y);
  if (d > 1) return -1;
  return mod(x, n);
}


// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b.  Here, z is unique modulo M = lcm(x,y).
// Return (z,M).  On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
  int s, t;
  int d = extended_euclid(x, y, s, t);
  if (a%d != b%d) return make_pair(0, -1);
  return make_pair(mod(s*b*x + t*a*y, x*y) / d, x*y / d);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i.  Note that the solution is
// unique modulo M = lcm_i (x[i]).  Return (z,M).  On
// failure, M = -1.  Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
  PII ret = make_pair(a[0], x[0]);
  for (int i = 1; i < x.size(); i++) {
    ret = chinese_remainder_theorem(ret.second, ret.first, x[i], a[i]);
    if (ret.second == -1) break;
  }
  return ret;
}

// computes x and y such that ax + by = c; on failure, x = y =-1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
  int d = gcd(a, b);
  if (c%d) {
    x = y = -1;
  }
  else {
    x = c / d * mod_inverse(a / d, b / d);
    y = (c - a*x) / b;
  }
}
long conquer_fibonacci_lgN(long n) {
  long i, h, j, k, t;
  i = h = 1;
  j = k = 0;
  while (n > 0) {
    if (n % 2 == 1) {
      t = j * h;
      j = i * h + j * k + t;
      i = i * k + t;
    }
    t = h * h;
    h = 2 * k * h + t;
    k = k * k + t;
    n = (long)n / 2;
  }
  return j;}
```

4.4 Gauss-Jordan Elimination

```cpp
// Gauss-Jordan elimination with full pivoting.
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
// Running time: O(n^3)
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}

int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
```

```
    b[i] = VT(B[i], B[i] + m);
  }

  double det = GaussJordan(a, b);

  // expected: 60
  cout << "Determinant: " << det << endl;

  // expected: -0.233333 0.166667 0.133333 0.0666667
  //           0.166667 0.166667 0.333333 -0.333333
  //           0.233333 0.833333 -0.133333 -0.0666667
  //           0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }

  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

4.5 BigInteger Square in java

```
import java.math.BigInteger;
import java.util.Scanner;
//import java.util.

public class Main {
    // https://en.wikipedia.org/wiki/Integer_square_root
    public static BigInteger sqrt(BigInteger n) {
        BigInteger cur = null; // X(k)
        BigInteger nxt = n; // X(k+1)
        while(true) {
            cur = nxt;
            nxt = cur.add(n.divide(cur)).divide(BigInteger.valueOf(2));
            if(nxt.equals(cur)) break;
        }
        if(cur.multiply(cur).equals(n)) return cur;
        else return null;
    }
    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);
        int tc = Integer.parseInt(sc.nextLine());
        while(tc-- > 0) {
            sc.nextLine();
            BigInteger y = new BigInteger(sc.nextLine());
            if(y.equals(BigInteger.ZERO)) System.out.println(0);
            else                          System.out.println(sqrt(y));
            if(tc>0) System.out.println();
        }
    }
}
```

4.6 Geometry 1

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
```

```
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion.  Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

int main() {

  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;

  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;

  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

  vector<PT> v;
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push_back(PT(0,5));

  // expected: 1 1 1 0 0
  cerr << PointInPolygon(v, PT(2,2)) << " "
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;

  // expected: 0 1 1 1 1
  cerr << PointOnPolygon(v, PT(2,2)) << " "
       << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;

  // expected: (1,6)
  //           (5,4) (4,5)
  //           blank line
  //           (4,5) (5,4)
  //           blank line
  //           (4,5) (5,4)
  vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

  // area should be 5.0
  // centroid should be (1.1666666, 1.166666)
  PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
```

```
  vector<PT> p(pa, pa+4);
  PT c = ComputeCentroid(p);
  cerr << "Area: " << ComputeArea(p) << endl;
  cerr << "Centroid: " << c << endl;

  return 0;
}
```

4.7 Geometry 2

```
const double eps = 1e-8;
const double PI = acos(-1.0);

struct Point
{
  double x, y;
  Point(double x = 0, double y = 0) : x(x), y(y) { }
  bool operator < (const Point& a) const
  {
    if (a.x != x) return x < a.x;
    return y < a.y;
  }
};

typedef Point Vector;

struct Line
{
  Point P;
  Vector v;
  double ang;
  Line() {}
  Line(Point P, Vector v) : P(P), v(v) { ang = atan2(v.y, v.x); }
  bool operator < (const Line& L) const
  {
    return ang < L.ang;
  }
};

Vector operator + (Vector A, Vector B) { return Vector(A.x + B.x, A.y + B.y); }

Vector operator - (Point A, Point B) { return Vector(A.x - B.x, A.y - B.y); }

Vector operator * (Vector A, double p) { return Vector(A.x*p, A.y*p); }

Vector operator / (Vector A, double p) { return Vector(A.x / p, A.y / p); }

int dcmp(double x)
{
  if (fabs(x) < eps) return 0; else return x < 0 ? -1 : 1;
}

bool operator == (const Point& a, const Point &b)
{
  return dcmp(a.x - b.x) == 0 && dcmp(a.y - b.y) == 0;
}

double Dot(Vector A, Vector B) { return A.x*B.x + A.y*B.y; }

double Length(Vector A) { return sqrt(Dot(A, A)); }

double Angle(Vector A, Vector B) { return acos(Dot(A, B) / Length(A) / Length(B)); }

double Cross(Vector A, Vector B) { return A.x*B.y - A.y*B.x; }
double Area2(Point A, Point B, Point C) { return fabs(Cross(B - A, C - A)) / 2; }

Vector Rotate(Vector A, double rad)
{
  return Vector(A.x*cos(rad) - A.y*sin(rad), A.x*sin(rad) + A.y*cos(rad));
}

Point GetLineIntersection(Point P, Vector v, Point Q, Vector w)
{
  Vector u = P - Q;
  double t = Cross(w, u) / Cross(v, w);
  return P + v*t;
}

bool SegmentProperIntersection(Point a1, Point a2, Point b1, Point b2)
{
  double c1 = Cross(a2 - a1, b1 - a1), c2 = Cross(a2 - a1, b2 - a1);
  double c3 = Cross(b2 - b1, a1 - b1), c4 = Cross(b2 - b1, a2 - b1);
  return dcmp(c1) * dcmp(c2) < 0 && dcmp(c3) * dcmp(c4) < 0;
}

bool OnSegment(Point p, Point a1, Point a2)
{
  return dcmp(Cross(a1 - p, a2 - p)) == 0 && dcmp(Dot(a1 - p, a2 - p)) < 0;
}

double PolygonArea(Point* p, int n)
{
  double area = 0;
  for (int i = 1; i < n - 1; i++)
    area += Cross(p[i] - p[0], p[i + 1] - p[0]);
  return area / 2;
}

double PointDistanceToLine(Point P, Point A, Point B)
{
  Vector v1 = B - A, v2 = P - A;
  return fabs(Cross(v1, v2)) / Length(v1);
}

double PointDistanceToSegment(Point P, Point A, Point B)
{
  if (A == B) return Length(P - A);
  Vector v1 = B - A, v2 = P - A, v3 = P - B;
  if (dcmp(Dot(v1, v2) < 0)) return Length(v2);
  else if (dcmp(Dot(v1, v3) > 0)) return Length(v3);
  else return fabs(Cross(v1, v2)) / Length(v1);
}

int isPointInPolygon(Point p, Point *poly, int n)
{
  int wn = 0;
  for (int i = 0; i < n; i++)
  {
    const Point& p1 = poly[i], p2 = poly[(i + 1) % n];
    if (p == p1 || p == p2 || OnSegment(p, p1, p2)) return -1;
    int k = dcmp(Cross(p2 - p1, p - p1));
    int d1 = dcmp(p1.y - p.y);
    int d2 = dcmp(p2.y - p.y);
    if (k > 0 && d1 <= 0 && d2 > 0) wn++;
    if (k < 0 && d2 <= 0 && d1 > 0) wn--;
  }
  if (wn != 0) return 1;
  return 0;
}

Vector Normal(Vector A)
{
  double L = Length(A);
  return Vector(-A.y / L, A.x / L);
}

double Dist2(Point p1, Point p2)
{
  return (p1.x - p2.x)*(p1.x - p2.x) + (p1.y - p2.y)*(p1.y - p2.y);
}

double RotatingCalipers(Point *P, int n)
{
  if (n == 1) return 0;
  if (n == 2) return Dist2(P[0], P[1]);
  P[n] = P[0];
  double ans = 0;
  for (int u = 0, v = 1; u < n; u++)
  {
    for (;;)
    {
      double diff = Cross(P[u + 1] - P[u], P[v + 1] - P[v]);
      if (diff <= 0)
      {
        ans = max(ans, Dist2(P[u], P[v]));
        if (diff == 0) ans = max(ans, Dist2(P[u], P[v + 1]));
        break;
      }
      v = (v + 1) % n;
    }
  }
  return ans;
}

bool OnLeft(Line L, Point p)
{
  return Cross(L.v, p - L.P) > 0;
}

Point GetLineIntersection2(const Line &a, const Line &b)
{
  Vector u = a.P - b.P;
  double t = Cross(b.v, u) / Cross(a.v, b.v);
  return a.P + a.v*t;
}

int HalfPlaneIntersection(Line* L, int n, Point* poly)
{
  sort(L, L + n);
```

```cpp
  int first, last;
  Point *p = new Point[n];
  Line* q = new Line[n];
  q[first = last = 0] = L[0];
  for (int i = 1; i < n; i++)
  {
    while (first < last && !OnLeft(L[i], p[last - 1])) last--;
    while (first < last && !OnLeft(L[i], p[first])) first++;
    q[++last] = L[i];
    if (fabs(Cross(q[last].v, q[last - 1].v)) < eps)
    {
      last--;
      if (OnLeft(q[last], L[i].P)) q[last] = L[i];
    }
    if (first < last) p[last - 1] = GetLineIntersection2(q[last - 1], q[last]);
  }
  while (first < last && !OnLeft(q[first], p[last - 1])) last--;
  if (last - first <= 1) return 0;
  p[last] = GetLineIntersection2(q[last], q[first]);

  int m = 0;
  for (int i = first; i <= last; i++) poly[m++] = p[i];
  return m;
}

vector<Point> CutPolygon(const vector<Point> &poly, Point A, Point B)
{
  vector<Point> newpoly;
  int n = poly.size();
  for (int i = 0; i < n; i++)
  {
    Point C = poly[i], D = poly[(i + 1) % n];
    if (dcmp(Cross(B - A, C - A)) >= 0) newpoly.push_back(C);
    if (dcmp(Cross(B - A, C - D)) != 0)
    {
      Point ip = GetLineIntersection(A, B - A, C, D - C);
      if (OnSegment(ip, C, D)) newpoly.push_back(ip);
    }
  }
  return newpoly;
}
```

4.8 Convex Hull

```cpp
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm.  Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
// Running time: O(n log n)
//   INPUT:   a vector of input points, unordered.
//   OUTPUT:  a vector of points in the convex hull, counterclockwise, starting
//            with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
#include <map>

using namespace std;

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}

// The following code solves SPOJ problem #26: Build the Fence (BSHEEP)
int main() {
  int t;
  scanf("%d", &t);
  for (int caseno = 0; caseno < t; caseno++) {
    int n;
    scanf("%d", &n);
    vector<PT> v(n);
    for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
    vector<PT> h(v);
    map<PT,int> index;
    for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
    ConvexHull(h);

    double len = 0;
    for (int i = 0; i < h.size(); i++) {
      double dx = h[i].x - h[(i+1)%h.size()].x;
      double dy = h[i].y - h[(i+1)%h.size()].y;
      len += sqrt(dx*dx+dy*dy);
    }

    if (caseno > 0) printf("\n");
    printf("%.2f\n", len);
    for (int i = 0; i < h.size(); i++) {
      if (i > 0) printf(" ");
      printf("%d", index[h[i]]);
    }
    printf("\n");
  }
}
```

4.9 Convex Hull Diameter

```cpp
typedef pair<double, double> point;

bool cw(const point &a, const point &b, const point &c) {
  return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first) <
        0;
}

vector<point> convexHull(vector<point> p) {
  int n = p.size();
  if (n <= 1)
    return p;
  int k = 0;
  sort(p.begin(), p.end());
  vector<point> q(n * 2);
  for (int i = 0; i < n; q[k++] = p[i++])
    for (; k >= 2 && !cw(q[k - 2], q[k - 1], p[i]); --k)
      ;
  for (int i = n - 2, t = k; i >= 0; q[k++] = p[i--])
    for (; k > t && !cw(q[k - 2], q[k - 1], p[i]); --k)
      ;
  q.resize(k - 1 - (q[0] == q[1]));
  return q;
}

double area(const point &a, const point &b, const point &c) {
  return abs((b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first)
        );
}

double dist(const point &a, const point &b) {
  return hypot(a.first - b.first, a.second - b.second);
```

```
}

double diameter(const vector<point> &p) {
  vector<point> h = convexHull(p);
  int m = h.size();
  if (m == 1)
    return 0;
  if (m == 2)
    return dist(h[0], h[1]);
  int k = 1;
  while (area(h[m - 1], h[0], h[(k + 1) % m]) > area(h[m - 1], h[0], h[k]))
    ++k;
  double res = 0;
  for (int i = 0, j = k; i <= k && j < m; i++) {
    res = max(res, dist(h[i], h[j]));
    while (j < m && area(h[i], h[(i + 1) % m], h[(j + 1) % m]) > area(h[i], h[(i + 1) % m], h[j])) {
      res = max(res, dist(h[i], h[(j + 1) % m]));
      ++j;
    }
  }
  return res;
}

int main() {
  vector<point> points(4);
  points[0] = point(0, 0);
  points[1] = point(3, 0);
  points[2] = point(0, 3);
  points[3] = point(1, 1);
  double d = diameter(points);
  cout << d << endl;}
```

4.10 Great Circle Distance

```
struct PT{double lat, lon; PT() {}
  PT(double lat, double lon) : lat(lat), lon(lon) {}
  PT operator * (double c  ) const { return PT(lat*c, lon *c); }
}pts[1000+10];

const double eps = 1e-9;
const double PI = 3.141592653589793;
const double R = 6378.00 ; // radius of earth

double GCDist(PT p1, PT p2) {
          p1 = p1*(PI/180.); p2 = p2*(PI/180.);
       double dlon = p2.lon - p1.lon;
       double dlat = p2.lat - p1.lat;
       double a = pow((sin(dlat/2)),2) + cos(p1.lat) * cos(p2.lat) * pow(sin(dlon/2), 2);
       double c = 2 * atan2(sqrt(a), sqrt(1-a));
       double d = R * c;
       return d+eps;
}
```

5 String Algorithms

5.1 Suffix Array 1

```
// Suffix array construction in O(L log^2 L) time.  Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
// INPUT:   string s
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.
//          That is, if we take the inverse of the permutation suffix[],
//          we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<pair<int,int>,int> > M;

  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
    }
  }

  vector<int> GetSuffixArray() { return P.back(); }

  // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
  int T;
  cin >> T;
  for (int caseno = 0; caseno < T; caseno++) {
    string s;
    cin >> s;
    SuffixArray array(s);
    vector<int> v = array.GetSuffixArray();
    int bestlen = -1, bestpos = -1, bestcount = 0;
    for (int i = 0; i < s.length(); i++) {
      int len = 0, count = 0;
      for (int j = i+1; j < s.length(); j++) {
        int l = array.LongestCommonPrefix(i, j);
        if (l >= len) {
          if (l > len) count = 2; else count++;
          len = l;
        }
      }
      if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
        bestlen = len;
        bestcount = count;
        bestpos = i;
      }
    }
    if (bestlen == 0) {
      cout << "No repetitions found!" << endl;
    } else {
      cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
    }
  }
}

#else
// END CUT
int main() {

  // bobocel is the 0'th suffix
  //  obocel is the 5'th suffix
  //   bocel is the 1'st suffix
  //    ocel is the 6'th suffix
  //     cel is the 2'nd suffix
  //      el is the 3'rd suffix
  //       l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();

  // Expected output: 0 5 1 6 2 3 4
  //                  2
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl;
  cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
// BEGIN CUT
#endif
// END CUT
```

5.2 Suffix Array 2

```
/*
Suffix array O(n lg^2 n)
LCP table O(n)
*/
#include <cstdio>
#include <algorithm>
#include <cstring>

using namespace std;

#define REP(i, n) for (int i = 0; i < (int)(n); ++i)

const int MAXN = 1 << 21;
char * S;
int N, gap;
int sa[MAXN], pos[MAXN], tmp[MAXN], lcp[MAXN];

bool sufCmp(int i, int j)
{
  if (pos[i] != pos[j])
    return pos[i] < pos[j];
  i += gap;
  j += gap;
  return (i < N && j < N) ? pos[i] < pos[j] : i > j;
}

void buildSA()
{
  N = strlen(S);
  REP(i, N) sa[i] = i, pos[i] = S[i];
  for (gap = 1;; gap *= 2)
  {
    sort(sa, sa + N, sufCmp);
    REP(i, N - 1) tmp[i + 1] = tmp[i] + sufCmp(sa[i], sa[i + 1]);
    REP(i, N) pos[sa[i]] = tmp[i];
    if (tmp[N - 1] == N - 1) break;
  }
}

void buildLCP()
{
  for (int i = 0, k = 0; i < N; ++i) if (pos[i] != N - 1)
  {
    for (int j = sa[pos[i] + 1]; S[i + k] == S[j + k];)
      ++k;
    lcp[pos[i]] = k;
    if (k)--k;
  }
}
```

5.3 Prefix Function

```
std::vector<int> prefix_function(const std::string& str) {
  std::vector<int> prefs(str.size(), 0);
  for (int i = 1; i < str.size(); ++i) {
    int pref = prefs[i - 1];
    while (pref > 0 && str[i] != str[pref]) {
      pref = prefs[pref - 1];
    }
    if (str[i] == str[pref]) {
      ++pref;
    }
    prefs[i] = pref;
  }
  return prefs;
}

std::vector<int> z_function(const std::string& str) {
  std::vector<int> zfunc(str.size(), 0);
  zfunc[0] = str.size();
  for (int i = 1, left = 0, right = 0; i < str.size(); ++i) {
    if (i <= right) {
      zfunc[i] = std::min(right - i + 1, zfunc[i - left]);
    }
    while (i + zfunc[i] < str.size() && str[zfunc[i]] == str[i + zfunc[i]]) {
      ++zfunc[i];
    }
    if (i + zfunc[i] - 1 > right) {
      left = i;
      right = i + zfunc[i] - 1;
    }
  }
  return zfunc;
}

std::string from_prefix_function(const std::vector<int>& prefs) {
  std::string str(prefs.size(), '.');
  char current_symbol = 'a';
  for (int i = 0; i < prefs.size(); ++i) {
    if (prefs[i] > 0) {
      str[i] = str[prefs[i] - 1];
    }
    else {
      str[i] = current_symbol++;
    }
  }
  return str;
}

std::vector<int> prefix_to_z(const std::vector<int>& prefs) {
  return z_function(from_prefix_function(prefs));
}

std::vector<int> z_to_prefix(const std::vector<int>& z_func) {
  std::vector<int> prefs(z_func.size(), 0);
  for (int i = 1; i < z_func.size(); ++i) {
    prefs[i + z_func[i] - 1] = std::max(prefs[i + z_func[i] - 1], z_func[i]);
  }
  for (int i = z_func.size() - 2; i >= 0; --i) {
    prefs[i] = std::max(prefs[i + 1] - 1, prefs[i]);
  }
  return prefs;
}
```

5.4 Infix to Postfix

```
import java.util.Stack;

public class ShuntingYard {

  public static void main(String[] args) {
    String infix = "3 + 4 * 2 / ( 1 - 5 ) ^ 2 ^ 3";
    System.out.printf("infix:   %s%n", infix);
    System.out.printf("postfix: %s%n", infixToPostfix(infix));
  }

  static String infixToPostfix(String infix) {
    final String ops = "-+/*^";
    StringBuilder sb = new StringBuilder();
    Stack<Integer> s = new Stack<>();

    for (String token : infix.split("\\s")) {
      if (token.isEmpty())
        continue;
      char c = token.charAt(0);
      int idx = ops.indexOf(c);

      // check for operator
      if (idx != -1) {
        if (s.isEmpty())
          s.push(idx);

        else {
          while (!s.isEmpty()) {
            int prec2 = s.peek() / 2;
            int prec1 = idx / 2;
            if (prec2 > prec1 || (prec2 == prec1 && c != '^'))
              sb.append(ops.charAt(s.pop())).append(' ');
            else break;
          }
          s.push(idx);
        }
      }
      else if (c == '(') {
        s.push(-2); // -2 stands for '('
      }
      else if (c == ')') {
        // until '(' on stack, pop operators.
        while (s.peek() != -2)
          sb.append(ops.charAt(s.pop())).append(' ');
        s.pop();
      }
      else {
        sb.append(token).append(' ');
      }
    }
    while (!s.isEmpty())
      sb.append(ops.charAt(s.pop())).append(' ');
    return sb.toString();
  }
```

```
}
```

5.5 Simple Parser

```
const char * expressionToParse = "3*2+4*1+(4+9)*6";
char peek(){
  return *expressionToParse;
}
char get(){
  return *expressionToParse++;
}
int expression();
int number(){
  int result = get() - '0';
  while (peek() >= '0' && peek() <= '9'){
    result = 10 * result + get() - '0';
  }
  return result;
}

int factor(){
  if (peek() >= '0' && peek() <= '9')
    return number();
  else if (peek() == '('){
    get(); // '('
    int result = expression();
    get(); // ')'
    return result;
  }
  else if (peek() == '-'){
    get();
    return -factor();
  }
  return 0; // error
}

int term(){
  int result = factor();
  while (peek() == '*' || peek() == '/')
    if (get() == '*')
      result *= factor();
    else
      result /= factor();
  return result;
}

int expression(){
  int result = term();
  while (peek() == '+' || peek() == '-')
    if (get() == '+')
      result += term();
    else
      result -= term();
  return result;
}
int _tmain(int argc, _TCHAR* argv[]){
  int result = expression();
  return 0;
}
```

5.6 Longest Palindrome

```
using namespace std;
template <class RAI1, class RAI2>
void fastLongestPalindromes(RAI1 seq, RAI1 seqEnd, RAI2 out)
{
  int seqLen = seqEnd - seq;
  int i = 0, j, d, s, e, lLen, k = 0;
  int palLen = 0;
  while (i<seqLen)
  {
    if (i>palLen && seq[i - palLen - 1] == seq[i])
    {
      palLen += 2;
      i++;
      continue;
    }
    out[k++] = palLen;
    s = k - 2;
    e = s - palLen;
    bool b = true;
    for (j = s; j>e; j--)
    {
      d = j - e - 1;
      if (out[j] == d){
        palLen = d;
        b = false;
        break;
      }
      out[k++] = min(d, out[j]);
    }
    if (b)
    {
      palLen = 1;
      i++;
    }
  }
  out[k++] = palLen;
  lLen = k;
  s = lLen - 2;
  e = s - (2 * seqLen + 1 - lLen);
  for (i = s; i>e; i--)
  {
    d = i - e - 1;
    out[k++] = min(d, out[i]);
  }
}

//Example
//opposes
//[0, 1, 0, 1, 4, 1, 0, 1, 0, 1, 0, 3, 0, 1, 0]
//Longest palindrome has length 4
int main()
{
  string s; cin >> s;
  vector<int> V(2 * s.length() + 1);
  fastLongestPalindromes(s.begin(), s.end(), V.begin());
  int best = 0;
  cout << "[";
  for (int i = 0; i<V.size(); i++)
  {
    if (i>0) cout << ", ";
    cout << V[i];
    best = max(best, V[i]);
  }
  cout << "]" << endl << "Longest palindrome has length " << best << endl;
  return 0;
}
```

6 Miscellaneous

6.1 Notes

```
/* ------ Bitmask

bit&(1<<i) // bit i is 0 or 1
(bit>>j)&1// bit i is 0 or 1 // use this & multiplication to avoid TLE
bit|(1<<i) // set bit i to 1
bit^(1<<i) // toggle bit i
x & ( x     1) // check if x is a power of 2
string stmp; bitset<12> tmp; //Debuging
tmp=bit; stmp=tmp.to_string();

/* ------ Data Structure Ideas
-Hash Table + Lookup
- Sparse Table
- SQRT Decomposition
- Bucketing
- Interger Arrays as matrices
- Recursive Tree Building
- Shortest Cycles
- Problem DAG
```