Answer all eight questions - Total marks 40

For all questions show all working and carefully state any facts or properties you use. 4 significant figures are acceptable for decimal answers.

1. (**5 marks**) For a cable which is subjected to a horizontally distributed uniform load, the horizontal distance x from the lowest point of the cable to one of the supports is given by

$$x = \frac{L}{1 + \sqrt{\frac{S-h}{S}}},$$

where L is the length of the cable, S the maximum sag, and h the vertical distance between the supports. Solve the equation for S.

2. (**5 marks**) A team of Civil Engineers is looking to calculate the amount of cement needed to fill a structure obtained from the function

$$f(x) = \frac{2x-1}{(x+2)(x^2+2x+5)}.$$

Luckily for them, the function $f(x)$ can be written in a simpler form after applying partial fraction decomposition. Help the team of Civil Engineers by expressing $f(x)$ in terms of partial fractions.

3. (**5 marks**) Let $3^{x+1}5^{1-2x} = 1$. Solve the equation for x, without evaluating your answer. Then evaluate your answer and write it as a decimal.

4. (**5 marks**) Reduce to a single logarithmic term and write in its simplest form the expression: $\ln(x^2-4) - \ln(x+2) - 3\ln(x^2y) + 2\ln(y^2)$.

5. (**5 marks**) Sketch the graph of $y = 9 - 2e^{-3x}$ for $x \geq 0$. Express x in terms of y.

6. (**5 marks**) In 1834 John Scott Russell observed a special type of travelling wave, nowadays known as a soliton, at the Union Canal. A soliton with velocity 1 is given by the expression

$$u = \frac{1}{2}\operatorname{sech}^2(x-t),$$

with x, t denoting the position and time respectively, and $\operatorname{sech}(x) = \dfrac{1}{\cosh(x)}$. Show that u can also be written as

$$u = \frac{2e^{-(x-t)}}{e^{x-t} + e^{-3(x-t)} + 2e^{-(x-t)}}.$$

7. (**5 marks**) The atmospheric pressure $P(h)$ (measured in psi) at height h (in km) above the surface of a planet is given by

$$P(h) = P_0 e^{-nh},$$

with n a constant that depends on the planet. The atmospheric pressure on the surface of Venus (at $h = 0$) is $P(0) = 1350$ psi, while the pressure at 60 km above Venus' surface is 14.7 psi. First find the constant n and hence find the pressure at $h = 100$ km above the surface of Venus.

8. (**5 marks**) As shown below, a set of experimental results gives a straight line graph when $\ln y$ is plotted against $\ln x$. The straight line has a gradient $1/5$ and passes through $(0, -1)$. Express y in terms of x.