» Convolutional Networks

* *ConvNets*, *CNNs* → have driven change in image processing since around 2012
* Usually *Deep learning* = *ConvNets*
* Also increasingly being used for text analytics/natural language processing, and other places where input data is a sequence.
* ConvNets consist of input and output layers plus multiple hidden layers, e.g. 10-100 layers. Hence “deep” learning since MLPs (“shallow” networks) usually have just one hidden layer.
* Each layer takes output of previous layer as its input.
  * Main types of layer: *Convolutional*, *pooling*, *fully connected*
* Some good resources online (also plenty of terrible ones) e.g.
  * https://www.coursera.org/learn/convolutional-neural-networks/
  * Stanford CS231 course https://cs231n.github.io/

» Convolutional Layer

- Nodes in a convolutional layer use a *kernel* or *filter*
- Basic primitive: take a matrix as input, apply kernel to it (*convolve* the matrix and kernel) and produce a matrix as output
- Conventional to use * to denote convolution, try not to mix up with multiplication
- Example:

1	2	3	4	5
1	3	2	3	10
3	2	1	4	5
6	1	1	2	2
3	2	1	5	4

Input

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

Output

» Convolutional Layer

1^{1}	2^{0}	3^{-1}	4	5
1^{1}	3^{0}	2^{-1}	3	10
3^{1}	2^{0}	1^{-1}	4	5
6	1	1	2	2
3	2	1	5	4

Input

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

-1		

Output

* $1\times1+1\times1+3\times1+2\times0+3\times0+2\times0+3\times(-1)+2\times(-1)+1\times(-1)=5-6=-1$

» Convolutional Layer

1	2^{1}	3^{0}	4^{-1}	5
1	3^{1}	2^{0}	3^{-1}	10
3	2^{1}	1^{0}	4^{-1}	5
6	1	1	2	2
3	2	1	5	4

Input

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

-1	-4	

Output

* $2\times1+3\times1+2\times1+3\times0+2\times0+1\times0+4\times(-1)+3\times(-1)+4\times(-1)=7-11=-4$

» Convolutional Layer

1	2	3^{1}	4^{0}	5^{-1}
1	3	2^{1}	3^{0}	10^{-1}
3	2	1^{1}	4^{0}	5^{-1}
6	1	1	2	2
3	2	1	5	4

Input

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

-1	-4	-14

Output

* $3\times1+2\times1+1\times1+4\times0+3\times0+4\times0+5\times(-1)+10\times(-1)+5\times(-1) = 6-25 = -14$

» Convolutional Layer

1	2	3	4	5
1 ^{1}	3 ^{0}	2 $^{-1}$	3	10
3 ^{1}	2 ^{0}	1 $^{-1}$	4	5
6 ^{1}	1 ^{0}	1 $^{-1}$	2	2
3	2	1	5	4

Input

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

-1	-4	-14
6		

Output

- $1 \times 1 + 3 \times 1 + 6 \times 1 + 3 \times 0 + 2 \times 0 + 1 \times 0 + 2 \times (-1) + 1 \times (-1) + 1 \times (-1) = 10 - 4 = 6$
- Observe that applying 3×3 kernel to a 5×5 matrix gives a 3×3 matrix

» Convolutional Layer

1	2	3	4	5
1	3	2	3	10
3	2	1	4	5
6	1	1	2	2
3	2	1	5	4

Input

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

-1	-4	-14
6	-3	-13
9	-6	-8

Output

» Example: Edge Detection

* Suppose we want to detect vertical edges in image ...
* Try kernel:

1	0	-1
1	0	-1
1	0	-1

» Example: Edge Detection

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*

1	0	-1
1	0	-1
1	0	-1

=

» Example: Edge Detection

10 (1)	10 (0)	10 (-1)	0	0	0
10 (1)	10 (0)	10 (-1)	0	0	0
10 (1)	10 (0)	10 (-1)	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*

1	0	-1
1	0	-1
1	0	-1

=

0			

* $10 \times 1 + 10 \times 1 + 10 \times 1 + 10 \times 0 + 10 \times 0 + 10 \times 0 + 10 \times (-1) + 10 \times (-1) + 10 \times (-1) = 0$

» Example: Edge Detection

10	10 1	10 0	0 -1	0	0
10	10 1	10 0	0 -1	0	0
10	10 1	10 0	0 -1	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*

1	0	-1
1	0	-1
1	0	-1

=

0	30		

* $10 \times 1 + 10 \times 1 + 10 \times 1 + 10 \times 0 + 10 \times 0 + 10 \times 0 + 10 \times (-1) + 0 \times (-1) + 0 \times (-1) = 30$

» Example: Edge Detection

$$
\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}
$$

* Observe that non-zero values in output highlight the edge

» Example: Edge Detection

Dark→light vs light→dark edges

$$
\begin{array}{|c|c|c|c|c|c|}
\hline
0 & 0 & 0 & 10 & 10 & 10 \\ \hline
0 & 0 & 0 & 10 & 10 & 10 \\ \hline
0 & 0 & 0 & 10 & 10 & 10 \\ \hline
0 & 0 & 0 & 10 & 10 & 10 \\ \hline
0 & 0 & 0 & 10 & 10 & 10 \\ \hline
\end{array}
*
\begin{array}{|c|c|c|}
\hline
1 & 0 & -1 \\ \hline
1 & 0 & -1 \\ \hline
1 & 0 & -1 \\ \hline
\end{array}
=
\begin{array}{|c|c|c|c|}
\hline
0 & -30 & -30 & 0 \\ \hline
0 & -30 & -30 & 0 \\ \hline
0 & -30 & -30 & 0 \\ \hline
\end{array}
$$

- Sign of output depends on whether transition is dark→light or light→dark
- To detect horizontal edges use kernel: $\begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$
- Similarly 45° angled edges, 70° etc
- Can use kernel larger than 3×3, but 3×3 is v common in ConvNets.

» Learning Kernels

- Rather than hand-crafting kernels, *learn* them !

$$\begin{array}{|c|c|c|c|c|c|}\hline 0&0&0&10&10&10\\\hline 0&0&0&10&10&10\\\hline 0&0&0&10&10&10\\\hline 0&0&0&10&10&10\\\hline 0&0&0&10&10&10\\\hline\end{array} * \begin{array}{|c|c|c|}\hline w_1&w_2&w_3\\\hline w_4&w_5&w_6\\\hline w_7&w_8&w_9\\\hline\end{array} = \begin{array}{|c|c|c|c|}\hline x_1&x_2&x_3&x_4\\\hline x_5&x_6&x_7&x_8\\\hline x_9&x_{10}&x_{11}&x_{12}\\\hline\end{array}$$

- Output from convolution node is a matrix, we need to map this to a scalar output/prediction $\rightarrow$ reshape/flatten matrix into a vector $x = [x_1, x_2, \ldots, x_{12}]$, then use this as input to one of our usual models E.g.
  - Linear model $\hat{y} = \theta^T x$
  - NB: Can add more convolution & other layers before map to output $\rightarrow$ will come back go this soon!
- Use training data to learn the unknown (i) output parameters θ and (ii) kernel weights $w = [w_1, w_2, \ldots, w_9]$:
  1. Define cost function
  2. Use gradient descent $\rightarrow$ typically use stochastic gradient descent variant.
- Back in familiar territory: a model mapping from input (matrix of pixel values) to predicted output, model has unknown parameters, learn these using a cost function+training data.

» Padding

- Applying 3×3 kernel to 5×5 matrix gives 3×3 matrix $\rightarrow$ output is *smaller* than input
- Often want to keep output same size as input $\rightarrow$ use padding

0	0	0	0	0	0	0
0	1	2	3	4	5	0
0	1	3	2	3	10	0
0	3	2	1	4	5	0
0	6	1	1	2	2	0
0	3	2	1	5	4	0
0	0	0	0	0	0	0

*

1	0	-1
1	0	-1
1	0	-1

Kernel

=

- Add extra rows and columns of zeros to pad original 5×5 input matrix out to 7×7. Apply kernel to this padded matrix to obtain 5×5 output i.e. output same size as original 5×5 input.
- Some terminology:
  - *Valid* convolution: apply kernel directly to input $\rightarrow$ output is smaller than input
  - *Same* convolution: pad original input then apply kernel $\rightarrow$ output is same size as input

» Strided Convolutions

- In previous examples we moved kernel along by one column/row at each step $\rightarrow$ we used a stride of 1
- Can also use larger strides e.g. stride 2

1	2	3	4	5
1	3	2	3	10
3	2	1	4	5
6	1	1	2	2
3	2	1	5	4

*

1	0	-1
1	0	-1
1	0	-1

=

-1	

1	2	3	4	5
1	3	2	3	10
3	2	1	4	5
6	1	1	2	2
3	2	1	5	4

*

1	0	-1
1	0	-1
1	0	-1

=

-1	-14

1	2	3	4	5
1	3	2	3	10
3	2	1	4	5
6	1	1	2	2
3	2	1	5	4

*

1	0	-1
1	0	-1
1	0	-1

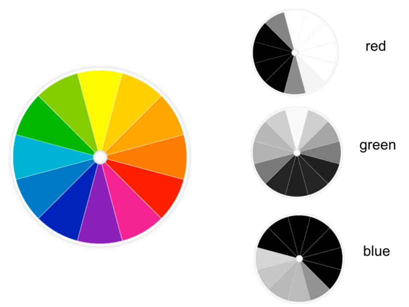
=

-1	-14
9	

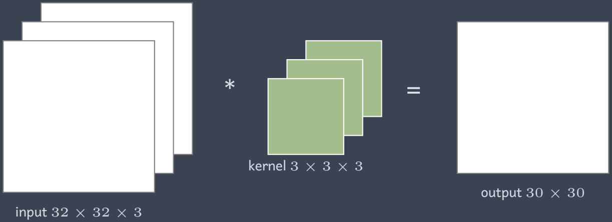
- Observe that increasing stride reduces the size of the output matrix

» Multiple Channels

- Gray-scale images are described by a single matrix, each element of matrix specifying shade of corresponding pixel
- Colour images are described by *three* matrices.
  - E.g. RGB image has one matrix specifying red intensity of each pixel, one specifying green and one specifying blue.

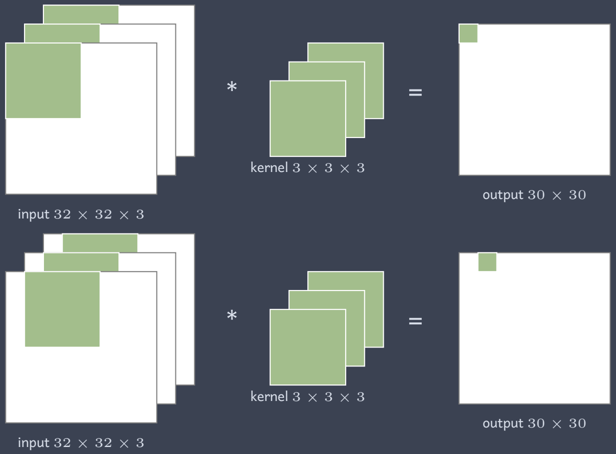


» Multiple Channels



- $32 \times 32 \times 3$ input has three *channels*, each channel is a 32×32 matrix of values
  - The $32 \times 32 \times 3$ stack of three 32×32 matrices is called a *tensor*
- We define a separate 3×3 kernel for each channel, so overall have a $3 \times 3 \times 3$ kernel
- How to calculate output?

» Multiple Channels



- Apply each 3×3 kernel to its corresponding 32×32 input channel. This gives three 30×30 output matrices.
- Now add element (1,1) of each of the three matrices together to get element (1,1) of final output. Repeat for all elements (i, j), $i = 1, \dots, 30$, $j = 1, \dots, 30 \rightarrow$ end result is a single 30×30 matrix as output
- Note: number of channels of kernel *must* match number of input channels. E.g. if have 3 input channels then need 3 kernel channels.

» Multiple Channels

More detailed example:

- Three 3×3 kernels:

channel 1

1	0	-1
1	0	-1
1	0	-1

channel 2

2	0	-2
2	0	-2
2	0	-2

channel 3

3	0	-3
3	0	-3
3	0	-3

- Three input channels:

channel 1

1	2	3	4
1	3	2	3
3	2	1	4
6	1	1	2

channel 2

4	3	1	2
2	6	1	2
1	3	1	2
3	2	1	0

channel 3

7	2	3	4
6	3	2	3
5	2	1	4
1	5	6	4

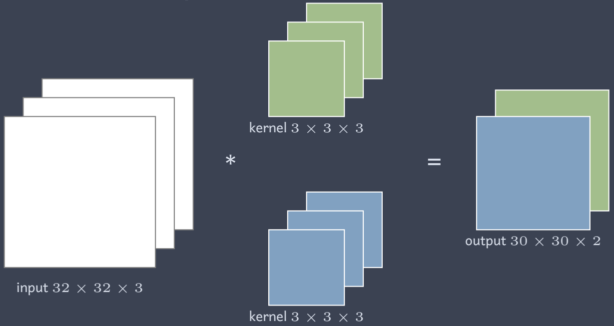
- Output is obtained by applying channel *i* kernel to channel *i* input then summing. E.g. element (1,1) of output is

$$
\begin{aligned}
&1 \times 1 + 1 \times 1 + 3 \times 1 + 2 \times 0 + 3 \times 0 + 2 \times 0 + 3 \times (-1) + 2 \times (-1) + 1 \times (-1) \\
&+4 \times 2 + 2 \times 2 + 1 \times 2 + 3 \times 0 + 6 \times 0 + 3 \times 0 + 1 \times (-2) + 1 \times (-2) + 1 \times (-2) \\
&+7 \times 3 + 6 \times 3 + 5 \times 3 + 2 \times 0 + 3 \times 0 + 2 \times 0 + 3 \times (-3) + 2 \times (-3) + 1 \times (-3) \\
&= -1 + 8 + 36 = 43
\end{aligned}
$$

- Now shift all kernels by one column and repeat to get element (1,2) of output, and so on.

» Multiple Filters

- We can apply several filters to the same input and stack their outputs together
- E.g. Apply two $3 \times 3 \times 3$ kernels to a $32 \times 32 \times 3$ input to get a $30 \times 30 \times 2$ output:



- Note: number of output channels can be larger/smaller/same as number of input channels
- Kernel weights w, input a. After convolution output is $w * a$. Here a and $a * w$ are both tensors i.e. a stack of matrices.
- What is number of parameters w in this setup?
  - Each $3 \times 3 \times 3$ kernel has 27 weights
  - One $3 \times 3 \times 3$ kernel for each output channel, so $27 \times 2 = 54$ weights.