homework 01 solutions

Exercise 1

Let y indicate the cost of a trip. Since x is a random variable, from the above relation y is a random variable as well. Hence, we can compute the expected cost $E[y]$ as follow:

$$E[y] = E[8 + 12x] = 8 + 12\,E[x].$$

Since x is a continous random variable, the expected cost for x is:

$$E[x] = \int_0^2 x f(x)\,dx = \int_0^2 x(1 - \frac{x}{2})\,dx = \frac{2}{3}.$$

Substituting in the first equation we obtain the expected cost $E[y] = 16€$.

Exercise 2

No, two independent random variables X and Y cannot be correlated.

Given two independent variables X and Y, the covariance is

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$$

Then because the variables are independent we have:

$$E[XY] = E[X][Y] \Rightarrow Cov(X, Y) = 0$$

Exercise 3

If a is the event of *laptop breaking in the* 3^{rd} *year* and b is the event of *laptop lasting two years* then

$$P(a) = P(T \geq 2 \cup T \leq 3) = P(T \geq 2) - P(T \geq 3) = e^{-\frac{2}{5}} - e^{-\frac{3}{5}} = 0.12$$

and

$$P(b) = P(T \geq 2) = e^{-\frac{2}{5}} = 0.67$$

The answer, considering that in this case P(a ∩ b) = P(a), is given by

$$P(a|b) = \frac{P(a)}{P(b)} = 0.18$$

and it can be observed that the condition of *given* b raises the probability of the laptop breaking.

Exercise 4

Take a circle in the $\mathbb{R}^2$ plane. For any positive integer n, place n points on the circle's circumference. Any subset of the n points are vertices of a convex polytope in $\mathbb{R}^2$. Said polytope won't contain any of the points that aren't in the subset. Since infinite points can lie on the circle's circumference, the VC dimension of H is infinite. Below, an example with four points with some of the configurations, just to give an idea of the concept explained: here, the subset of the chosen points are marked with a small circle, otherwise a cross is used.