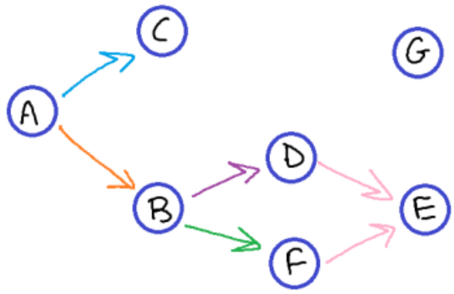
homework 03 solutions

Exercise 1

First solution



Homework 3

Exercise 1

$P(A, B, C, D, E, F, G) = P(A)P(B|A)P(F|B)P(C|A)P(D|B)P(E|D, F)P(G)$

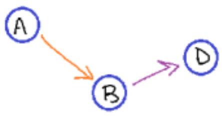
consider

… $a \leftarrow c \rightarrow b$ … TT $a \not\perp\!\!\!\perp b$ $a \perp\!\!\!\perp b \mid c$

… $a \rightarrow c \rightarrow b$ … HT $a \not\perp\!\!\!\perp b$ $a \perp\!\!\!\perp b \mid c$

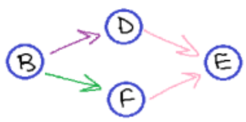
… $a \rightarrow c \leftarrow$ … HH $a \perp\!\!\!\perp b$ $a \not\perp\!\!\!\perp b \mid c$

(a) $A \perp\!\!\!\perp D$ False!



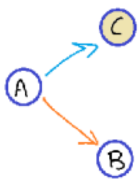
not independent, case HT with unobserved c (here, B)

(b) $F \perp\!\!\!\perp D$ False!



D and F have the same parent B which is not obseved (TT case with c unobserved)

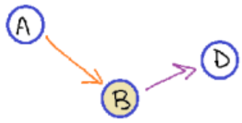
(c) $A \perp\!\!\!\perp B \mid C$ False!



The path $A \rightarrow B$ is not blocked by C in any way, they remain dependent

(d) $A \perp\!\!\!\perp D \mid B$

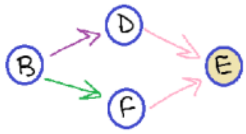
True!



The path passing through B is the only one from A to D, and B is observed: HT case

(e) $D \perp\!\!\!\perp F \mid E$

False!

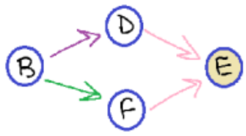


No blocked paths leading to conditional independence

- HH at OBSERVED E
- TT at OBSERVED B

(f) $B \perp\!\!\!\perp F \mid E$

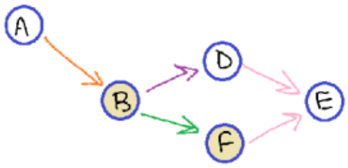
False!



Observing E does not make F independent from B: it's its child, can't block B → F

(g) $A \perp\!\!\!\perp D \mid \{B, F\}$

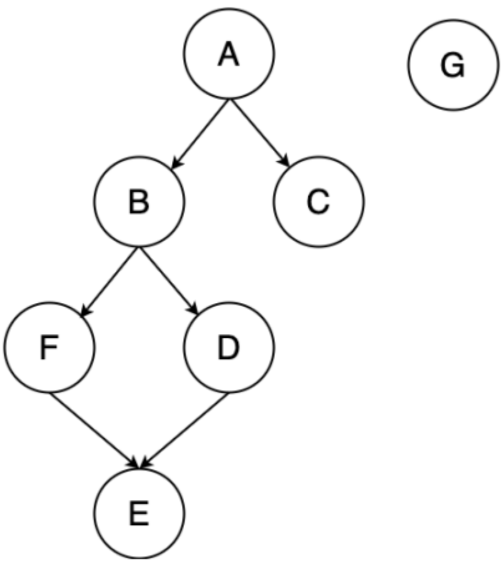
True!



All the paths from A to D pass through B, which blocks (HT case)

Second solution

The Bayesian network representing it is the following:



a) $A \perp\!\!\!\perp D$: FALSE

1. $P(A, D) = \sum_B P(D|B)P(B|A)P(A) = P(D|A)P(A) \neq P(D)P(A)$.
2. Graphically, this is a Head-to-Tail case with B not blocking.

b) $F \perp\!\!\!\perp D$: FALSE

1. $P(D, F) = \sum_B P(F, B, D) = \sum_B P(F|B)P(D|B)P(B) \neq P(F)P(D)$
2. Graphically, this is a Tail-to-Tail case with B not blocking.

c) $A \perp\!\!\!\perp B|C$: FALSE

1. $P(A, B|C) = \frac{P(A,B,C)}{P(C)} = \frac{P(C|A)P(B|A)P(A)}{P(C)} = P(A|C)P(B|A) \neq P(A|C)P(B|C)$
2. Graphically, it is obvius that observing C doesn't make A, B independent.

d) $A \perp\!\!\!\perp D|B$: TRUE

1. $P(A, D|B) = \frac{P(A,B,D)}{P(B)} = \frac{P(D|B)P(B|A)P(A)}{P(B)} = P(D|B)P(A|B)$
2. Graphically, this is a Tail-to-Tail case with B blocking.

e) $D \perp\!\!\!\perp F|E$: FALSE

1. $P(D, F|E) = \frac{P(D,F,E)}{P(E)} = \frac{P(E|F,D)P(F)P(D)}{P(E)} = P(D|E)P(F|E)$
2. Graphically, this is a Head-to-Head case with E blocking.

f) $B \perp\!\!\!\perp F|E$: FALSE

1. $P(B, F|E) = \frac{P(B,F,E)}{P(E)} = \frac{P(E|F)P(F|B)P(B)}{P(E)} \neq P(B|E)P(F|E)$
2. Graphically, it is obvius that observing E doesn't make B, F independent.

g) $A \perp\!\!\!\perp D|\{B, F\}$: TRUE

1. $P(A, D|B, F) = \frac{P(A,B,F,D)}{P(B,F)} = P(D|B)P(B|A)\frac{P(A)}{P(B)} = P(D|B)P(A|B) = P(D|B, F)P($
2. Graphically, this is a Head-to-Tail case with B blocking A and D.

Exercise 2

The joint distribution factorizes as:

$$p(p, r, u, \pi|\alpha_p, \beta_p, \alpha_\pi, \beta_\pi) = p(p|\alpha_p, \beta_p) \prod_{i=1}^{S} p(r_i|p) \prod_{j=1}^{N} p(\pi_j|\alpha_\pi, \beta_\pi) \prod_{i=1}^{S} \prod_{j=1}^{N} p(u_{ij}|r_i, \pi_j)$$

```
# hyperparameters
alpha_p = 1
beta_p = 1
alpha_pi = 1
beta_pi = 5

def model(data, verbose=False):
    S = data.shape[0]
    N = data.shape[1]

    # Global variables
    p = pyro.sample('p', dist.Beta(alpha_p,beta_p))

    r_plate = pyro.plate('components', S, dim=-2)
    pi_plate = pyro.plate('data', N, dim=-1)

    with r_plate:
        r = pyro.sample('r', dist.Bernoulli(p))

    with pi_plate:
        pi = pyro.sample('pi', dist.Beta(alpha_pi, beta_pi))

    with r_plate,pi_plate:
        u = pyro.sample('u', dist.Bernoulli(r*(1 - pi)+(1-r)*pi), obs=data)
```

```
data = dist.Bernoulli(0.6).sample((12,6))
model(data,True)
```

```
p = tensor(0.7678)
r = tensor([1., 1., 1., 1., 0., 1., 1., 0., 1., 0., 1., 1.])    shape =  12
pi = tensor([0.2320, 0.1038, 0.1691, 0.2006, 0.1639, 0.0195])   shape =  6
u = tensor([[0., 1., 1., 0., 1., 0.],
        [1., 0., 1., 1., 0., 1.],
        [0., 1., 1., 1., 1., 1.],
        [0., 0., 1., 1., 1., 1.],
        [1., 1., 0., 0., 1., 0.],
        [0., 0., 0., 1., 1., 1.],
        [0., 1., 0., 1., 0., 1.],
        [0., 1., 1., 0., 1., 0.],
        [1., 1., 1., 1., 0., 1.],
        [0., 0., 1., 1., 0., 0.],
        [1., 0., 0., 0., 1., 1.],
        [1., 1., 1., 1., 0., 1.]])      shape = (12,6)
```