You're expected to work on the problems before coming to the lab. Discussion session is not meant to be a one-way lecture. The TA will lead the discussion and correct your solutions if needed. For many problems, we will not release 'official' solutions. If you're better prepared for discussion, you will learn more. The TA is allowed to give some bonus points to students who actively engage in discussion and report them to the instructor. The bonus points earned will be factored in the final grade.

In the following, you can assume that all elements in the input are distinct.

1. (Basic) Using Figure 7.1 as a model, illustrate the operation of Partition on the array $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2\rangle$. Do the same thing when we have $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 11\rangle$.

2. (Basic) Explain the meaning of in-place sorting.

3. (Basic) Is Quick-Sort an in-place sorting algorithm? How about Merge-sort, Heap-sort, Insertion-sort?

4. (Basic) The running time of Randomized-Quicksort that picks the pivot uniformly at random is always $O(n \log n)$ for any input of size n. True of False?

5. (Basic) The *expected* running time of Randomized-Quicksort that picks the pivot uniformly at random is $O(n \log n)$ for any input of size n. True of False?

6. (Intermediate) The running time of (Deterministic) Quicksort is $\Theta(n^2)$. True of False?

7. (Basic) Suppose the input is a random permutation of integers 1 through n. The average-case running time of (Deterministic) Quicksort is $\Theta(n^2)$. True of False?

8. (Intermediate) Solve $T(n) = T(0.9n) + T(0.1n) + \Theta(n)$. What is your answer? No need to give any explanations. Solve $T(n) = T((2/3)n) + T((1/3)n) + \Theta(n)$. What is your answer?

9. (Basic) In (Determinisitc) Quicksort, suppose the input is a sequence of integers $a_1, a_2, \ldots, a_n$. Let $X_{ij} = 1$ if a_i is compared to a_j in the execution; otherwise $X_{ij} = 0$. Consider another input sequence of integers, $b_1, b_2, \ldots, b_n$, where $b_1 = 2a_1, \ldots, b_n = 2a_n$. Let $Y_{ij} = 1$ if and only if b_i is compared to b_j. Is it true that $X_{ij} = Y_{ij}$ for any $1 \leq i < j \leq n$?

10. (Intermediate) In Randomized-Quicksort, explain why it is wlog to assume that the elements are $1, 2, \ldots n$ for the analysis of the running time.

11. (Advanced) In Randomized-Quicksort, explain why the probability that the ith smallest element is compared to jth smallest element is $\frac{2}{j-i+1}$, where $i < j$