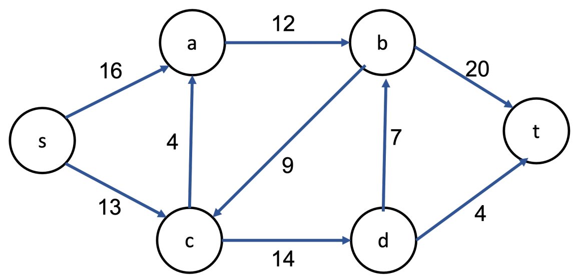
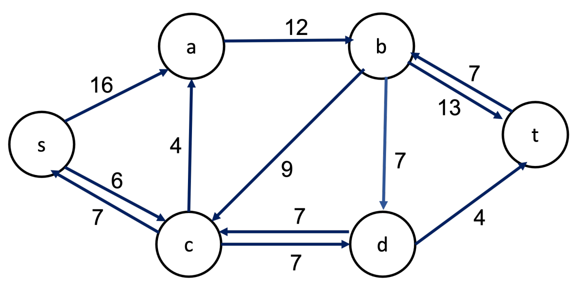
You're expected to work on the discussion problems before coming to the lab. Discussion session is not meant to be a lecture. TA will guide the discussion and correct your solutions if needed. We will not release 'official' solutions. If you're better prepared for discussion, you will learn more. TAs will record names of the students who actively engage in discussion and report them to the instructor; they are also allowed to give some extra points to those students at their discretion. The instructor will factor in participation in final grade.

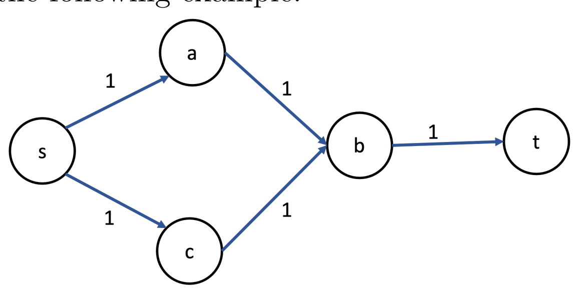
1. (basic) Consider the following input graph to the max flow problem. Suppose you augment a flow of value 7 along the path $\langle s, c, d, b, t \rangle$. Show the resulting residual graph.



Sol.



2. (basic) Technically speaking, a max flow may consist of fractional flows on edges. For example, consider the following example.



In this example, you can send a flow of 0.5 along $\langle s, a, b, t \rangle$, together with another flow of value 0.5 along $\langle s, c, b, t \rangle$. However, we can show that if every edge has an integer capacity, there is an integer max flow, where every edge has an integer-valued flow. Explain why.

Sol. We can show the following loop invariant: In FF, the residual graph has integer capacities. This is because we can assume wlog that we only augment an integer valued flow.

3. (basic) Show an example for which FF takes $|f^*|$ iterations before finding a max flow. Here $|f^*|$ denotes the max flow value.

   Sol. See the lecture slides.

4. (intermediate/advanced) Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professor's house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of his town. Show how to formulate the problem of determining whether both his children can go to the same school as a maximum-flow problem.

5. (basic) Explain how you can find an s-t min-cut using an algorithm for computing an s-t max flow. **Sol.** After finding a max flow, let S be the vertices reachable from s, and T be the other vertices.

6. (basic) In Figure 26.1(b) (see CLRS page 710), what is the flow across the cut $(\{s, v_2, v_4\}, \{v_1, v_3, t\})$? What is the capacity of this cut?
   Sol.
   The flow across the cut $(\{s, v_2, v_4\}, \{v_1, v_3, t\})$ is 19 (23 units going from $(\{s, v_2, v_4\}$ to $\{v_1, v_3, t\}$ and 4 units going back), and the capacity of the cut is 31.

7. (advanced) Suppose that we redefine the residual network to disallow edges into s. Argue that the procedure FORD-FULKERSON still correctly computes a maximum flow.