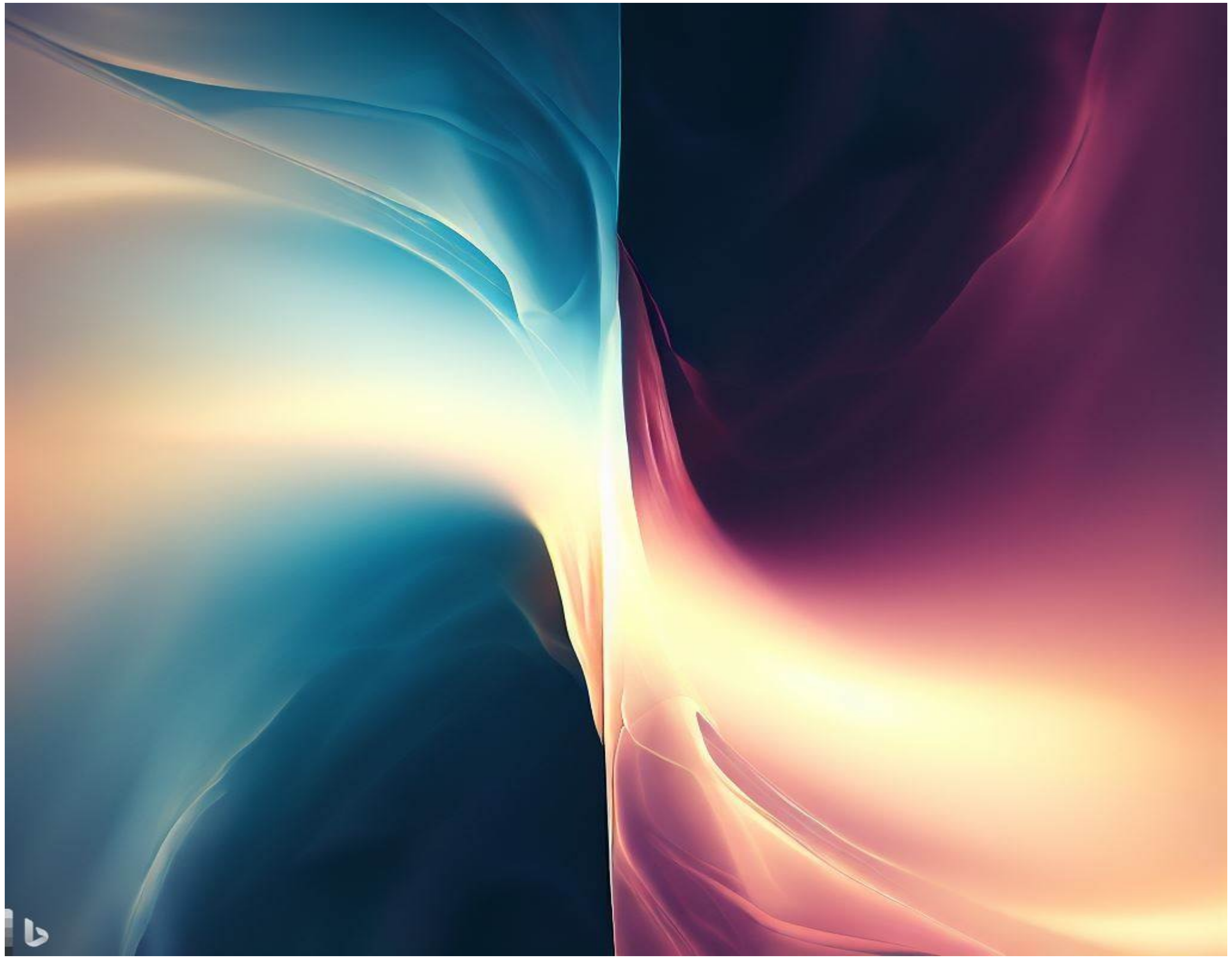
Supplemental Slides of Ch26

Sungjin Im

4/27/2023

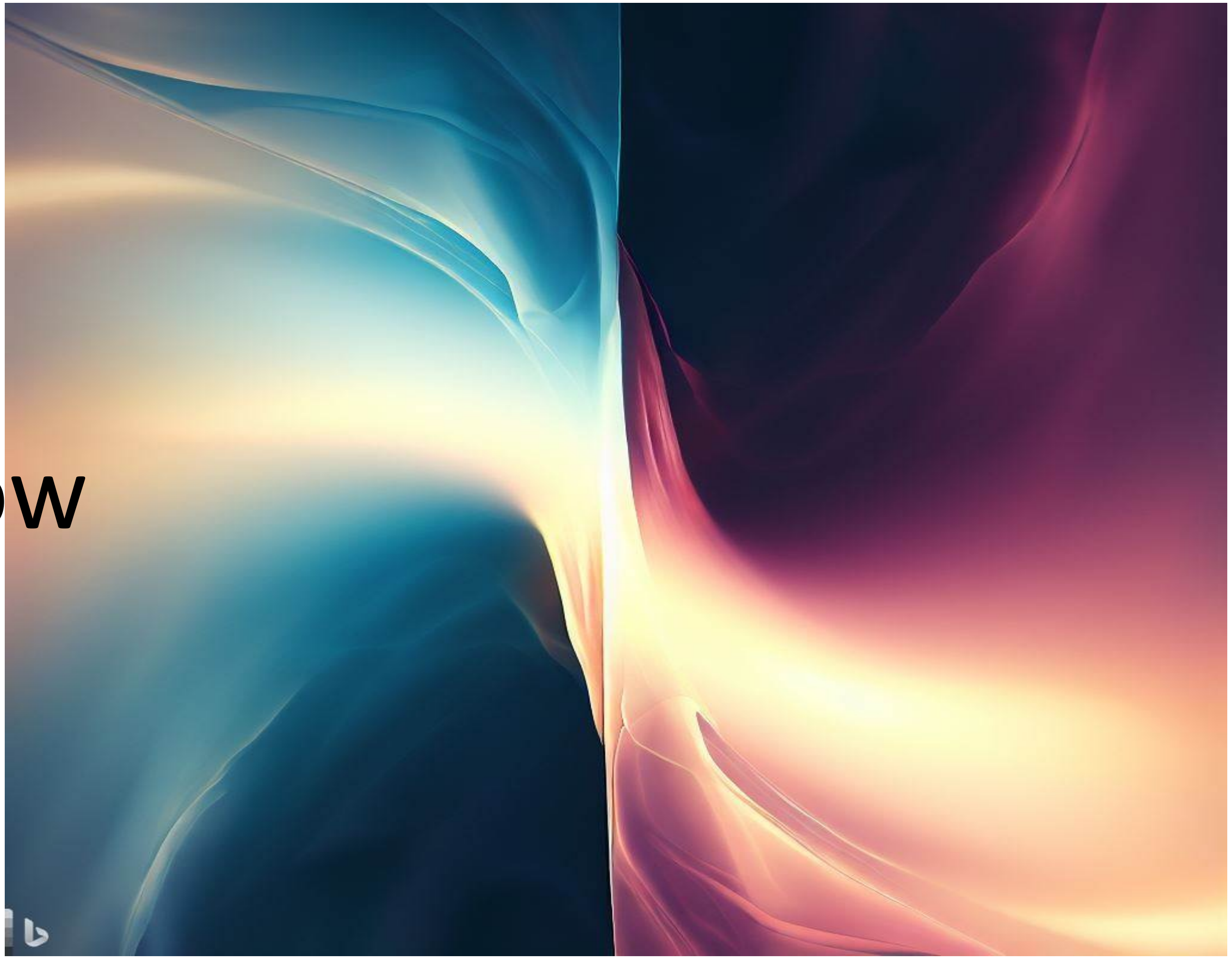
A Story of Dual

Max Flow



Min Cut

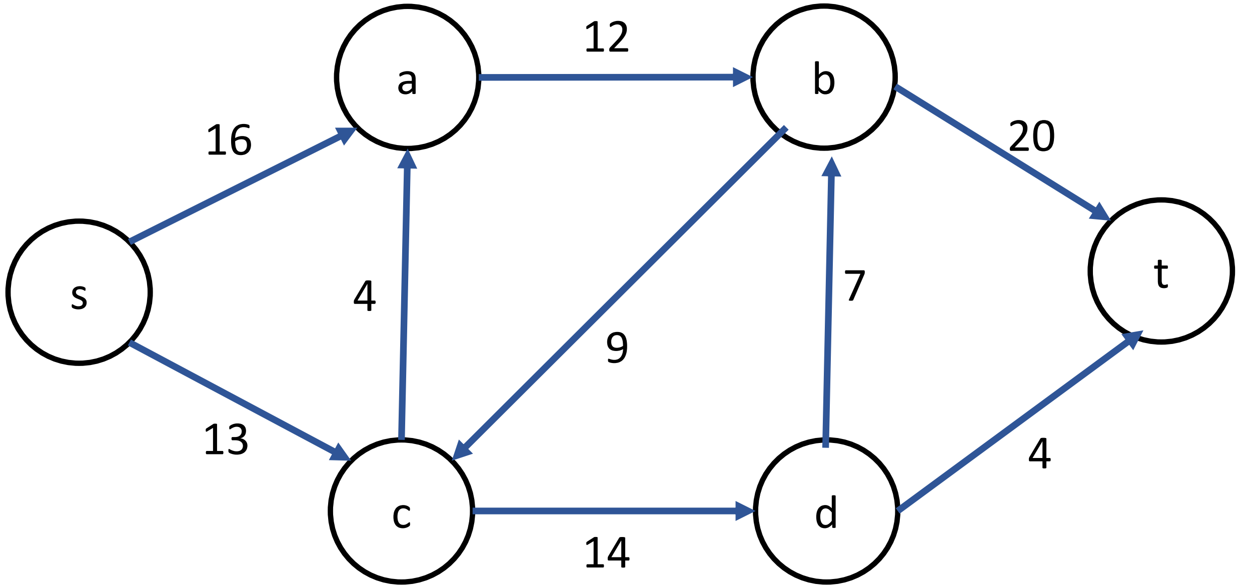
Max Flow



Finding Max Flow: Input

s: source, t: sink
Each edge (u, v) has a capacity c(u,v)

Want to send out as much flow as possible from s to t



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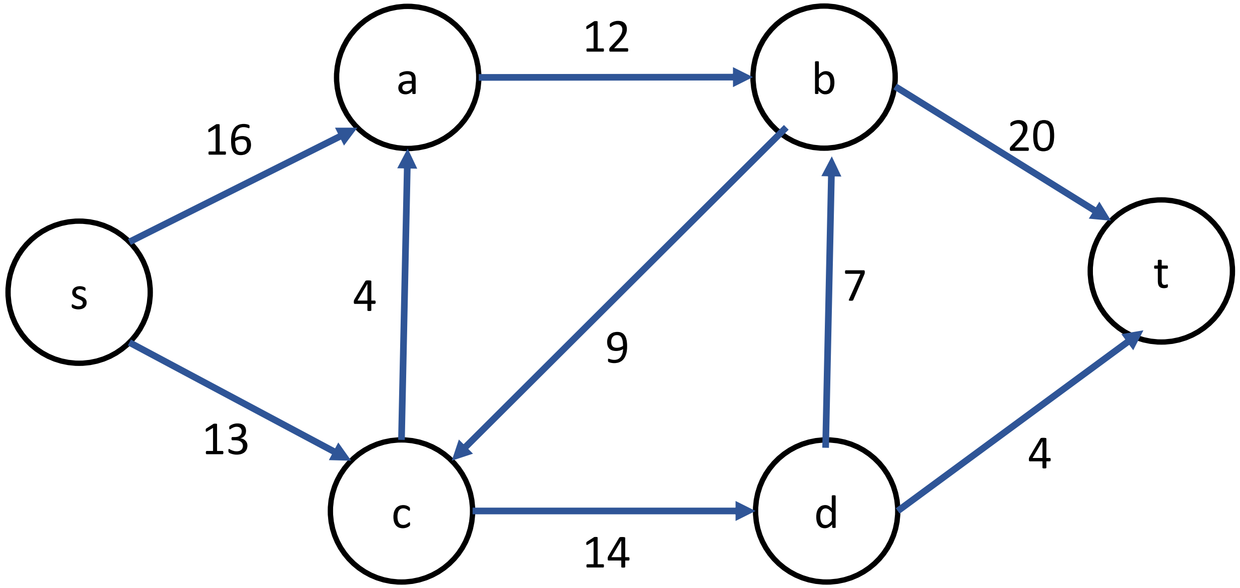


s: source, t: sink

Each edge (u, v) has a capacity c(u,v)

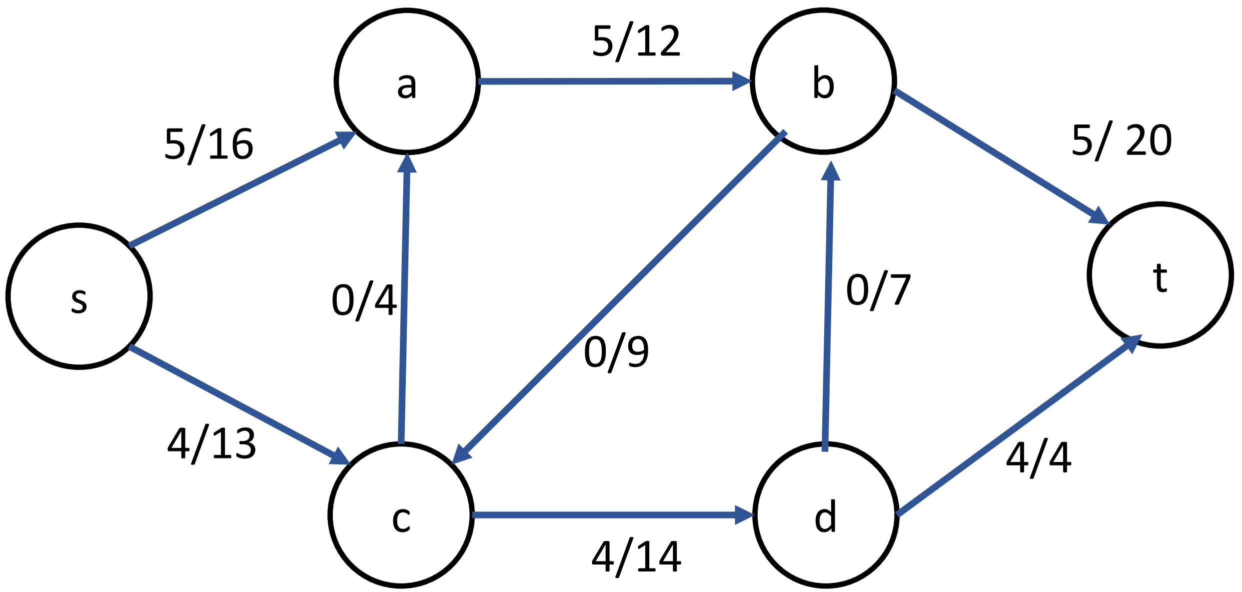
Want to send out as much flow as possible from s to t

Flow value $|f| = \Sigma_v f(s, v) - \Sigma_v f(v, s)$

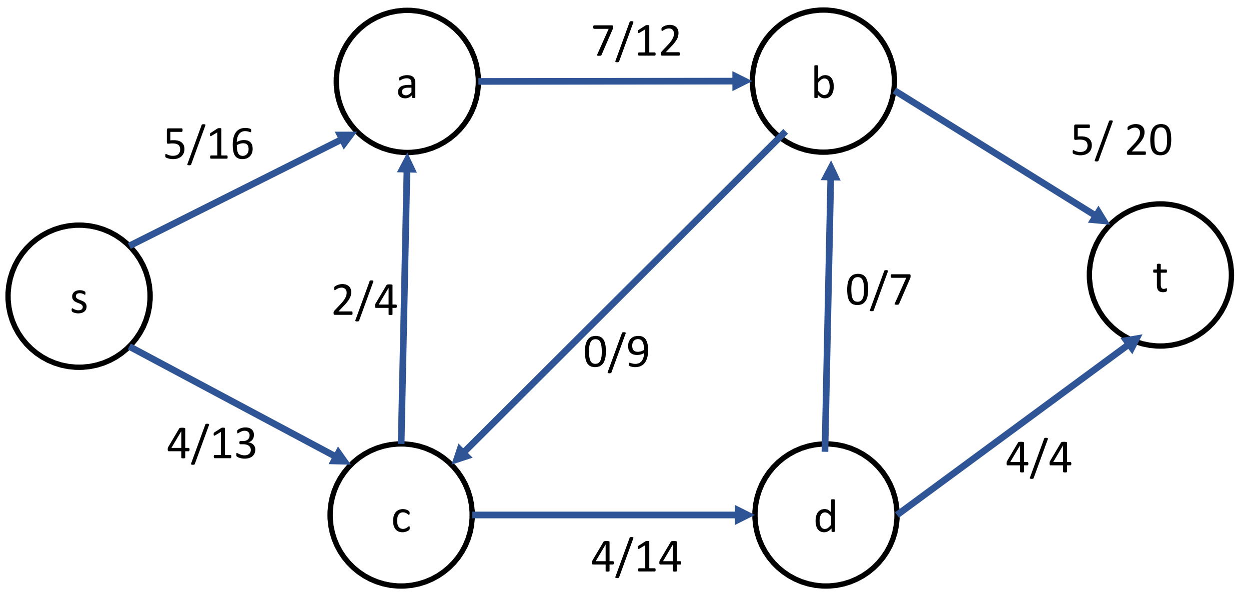
- Capacity constraint
  $f(u,v) <= c(u,v)$ for all edges (u,v)
- Flow conservation
  $$\Sigma_v f(v, u) = \Sigma_v f(u, v)$$
  for all nodes u except, s, t

Finding Max Flow: A Feasible Flow

Flow value is 9

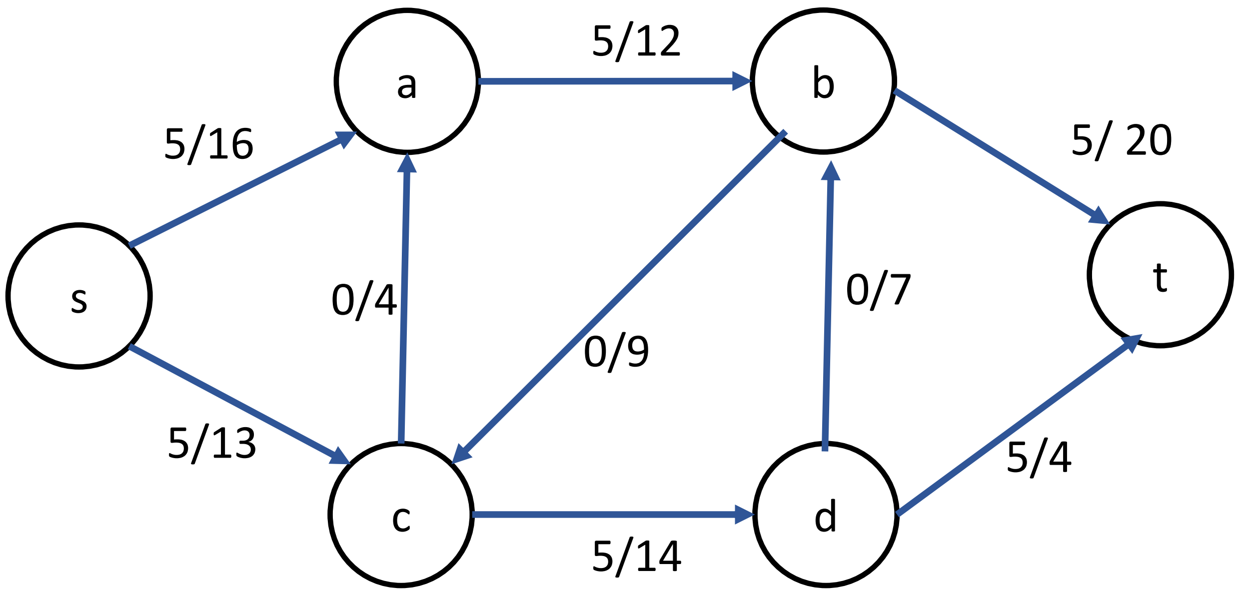


Finding Max Flow: An Infeasible Flow



Do you see why?

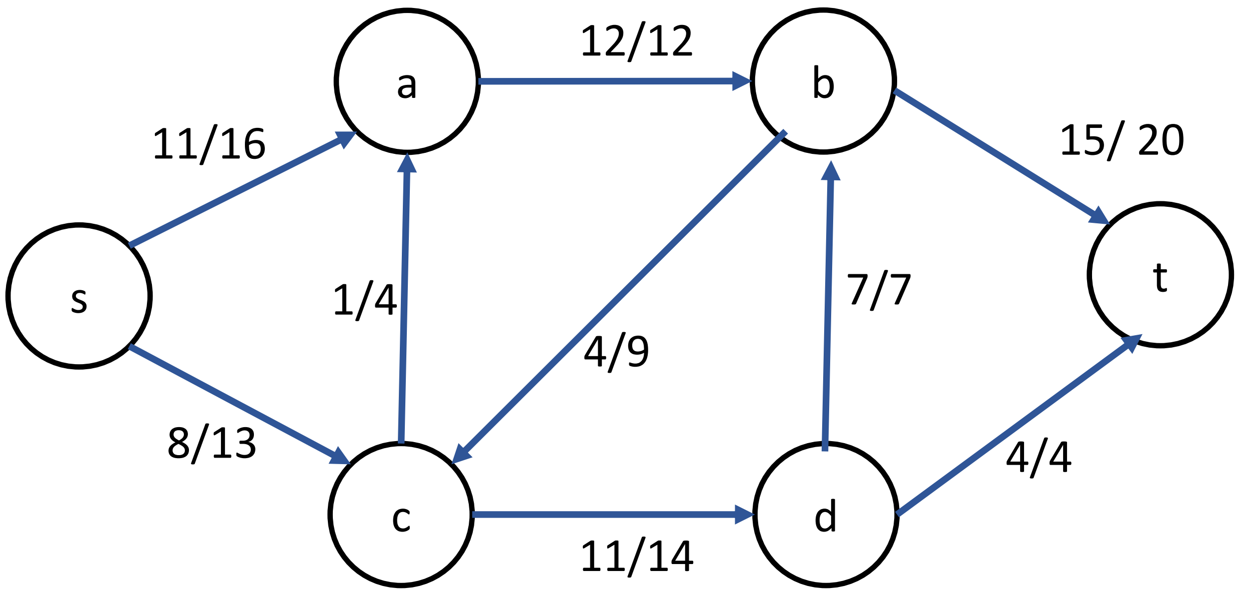
Finding Max Flow: Another Infeasible Flow

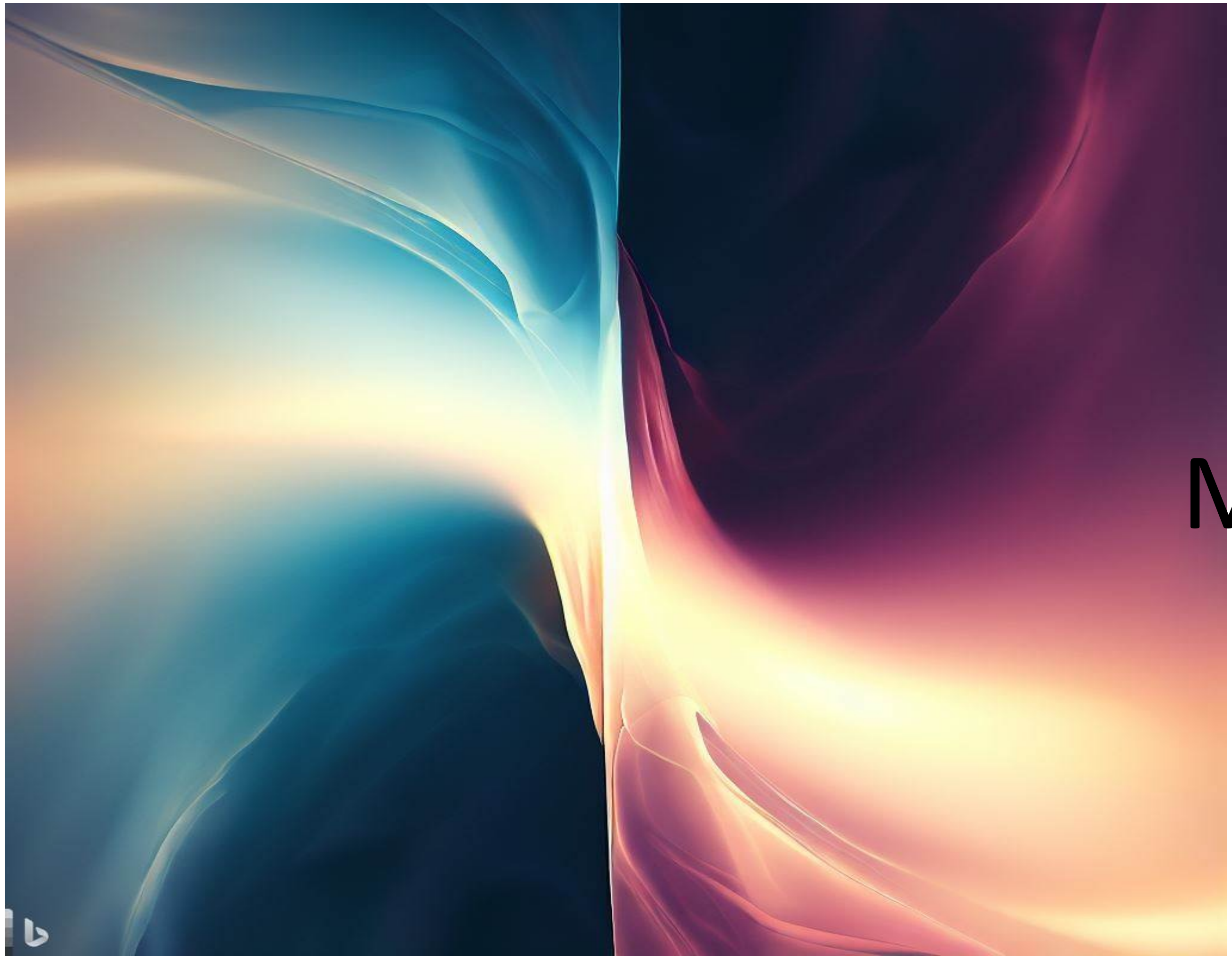


Do you see why?

Finding Max Flow: A Desired Output

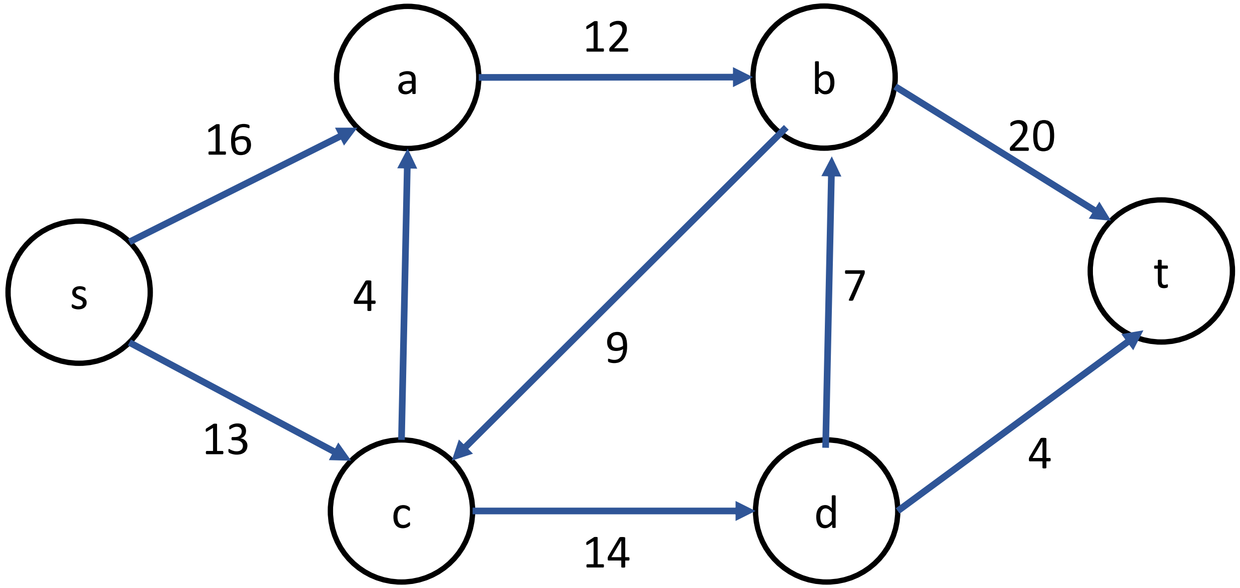
A feasible flow of value





Min Cut

s-t Min Cut



Find a min s-t cut, i.e.
s-t cut (S, T) with the max capacity

s-t min cut of G = (V, E):
(S, T = V \ S) such that
s in S and t in T

Capacity of cut (S,T):
$\Sigma_{u \text{ in } S} \Sigma_{v \text{ in } T} c(u, v)$

s-t Min Cut

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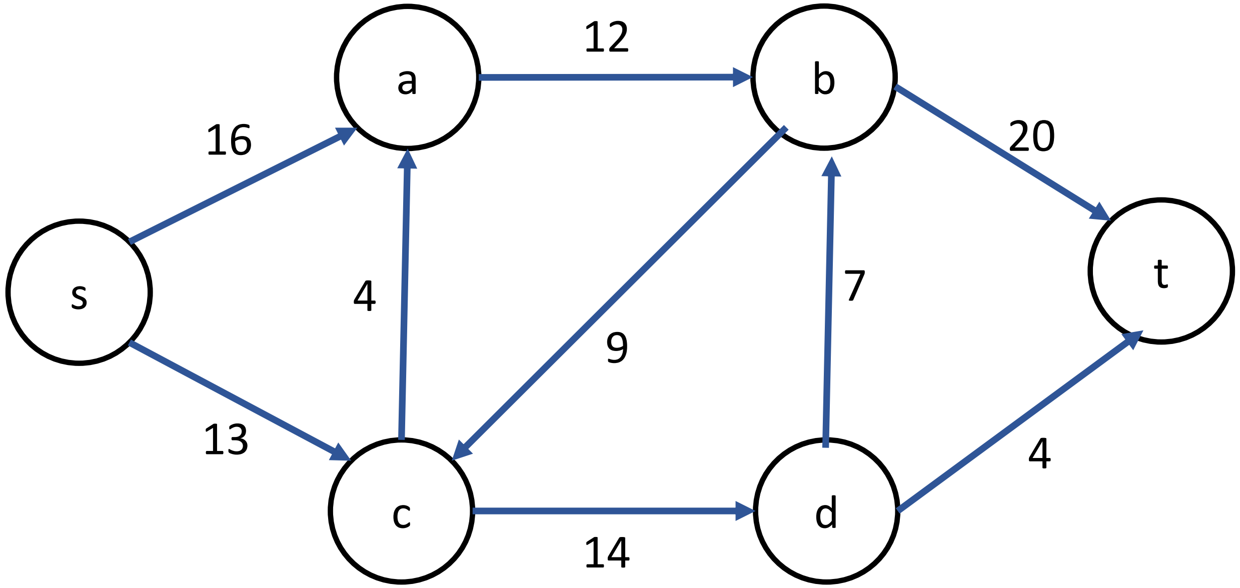

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({s, a, c}, {b, d, t}) is an s-t cut of capacity

s-t Min Cut

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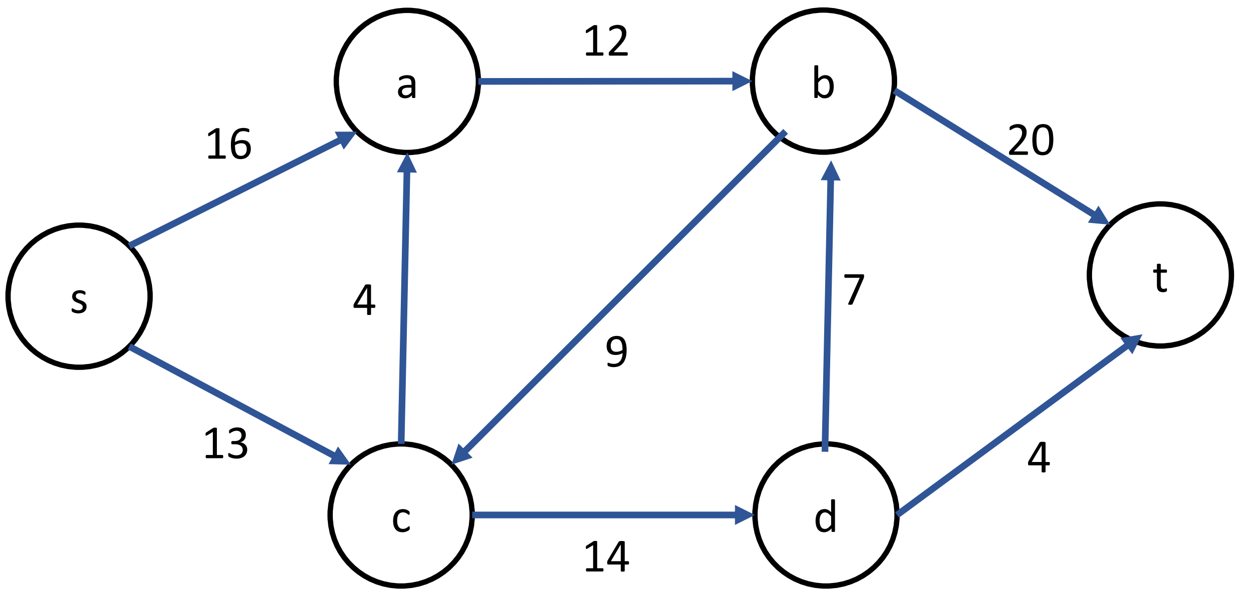

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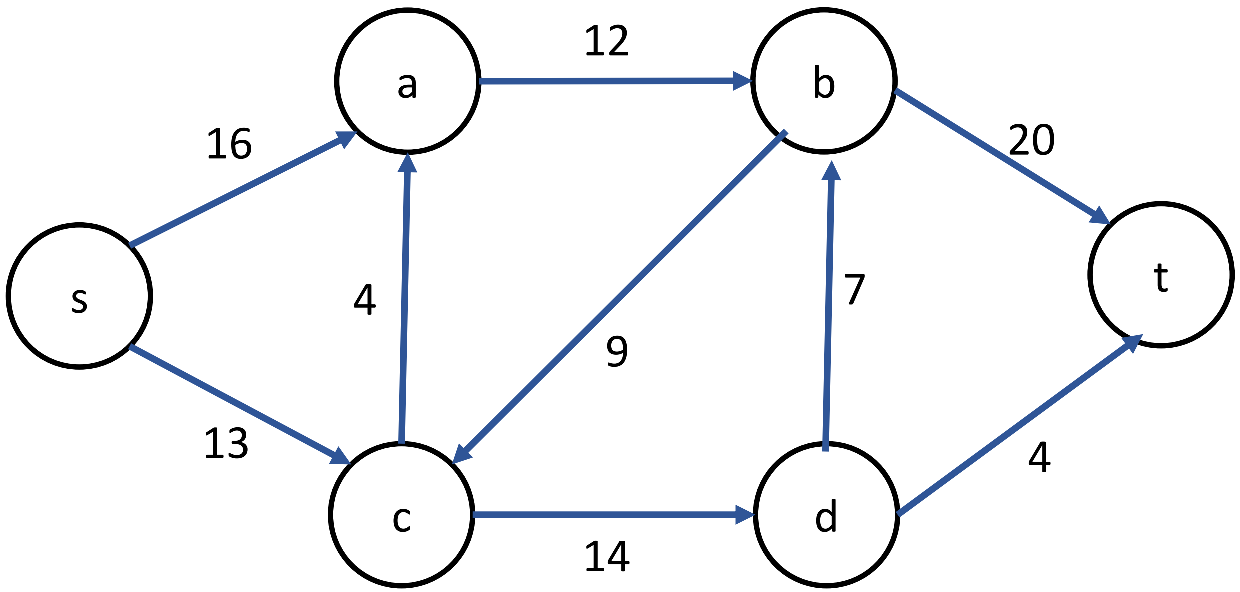
({s, a, b, t}, {c, d}) is not an s-t cut

s-t Min Cut



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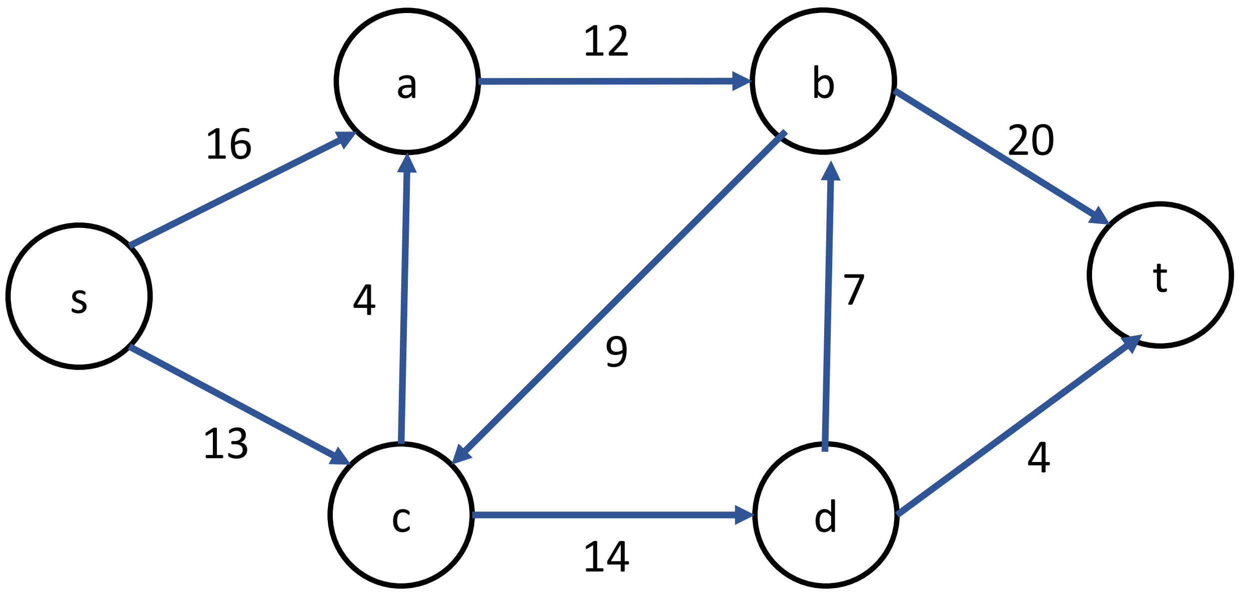

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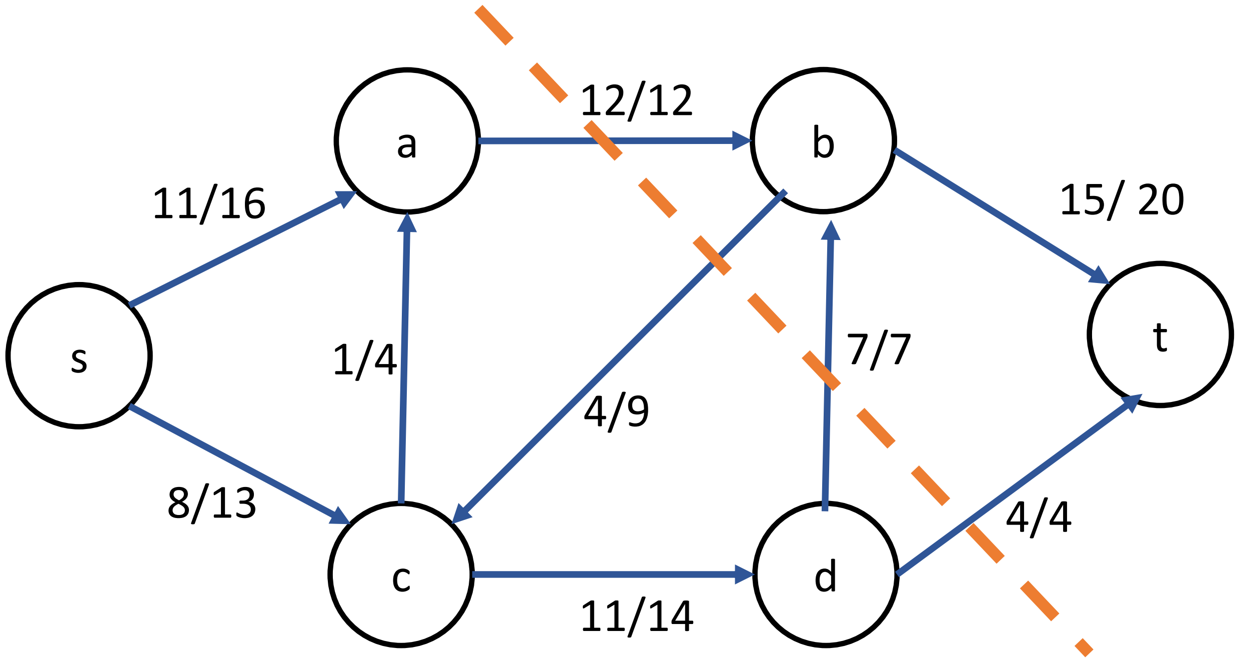
({s, a, c, d}, {b, t}) is is a min s-t cut

How do we know if we have found a max flow and a min s-t cut?

Theorem: Max Flow Value = Min Cut Capacity.
Formally, if f is an s-t max flow, then

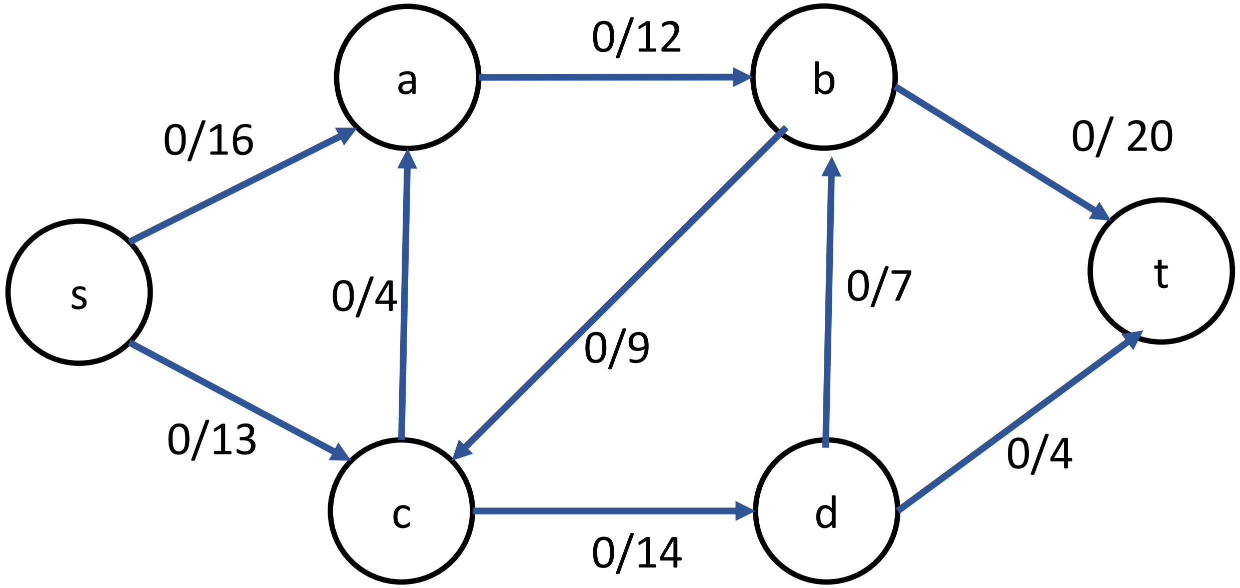
$|f| = c(S, T)$ for some s-t cut (S, T)

Max Flow Value = Min Cut Capacity



Find a Max Flow (Ford-Fulkerson-Method)

Start with flow f = 0



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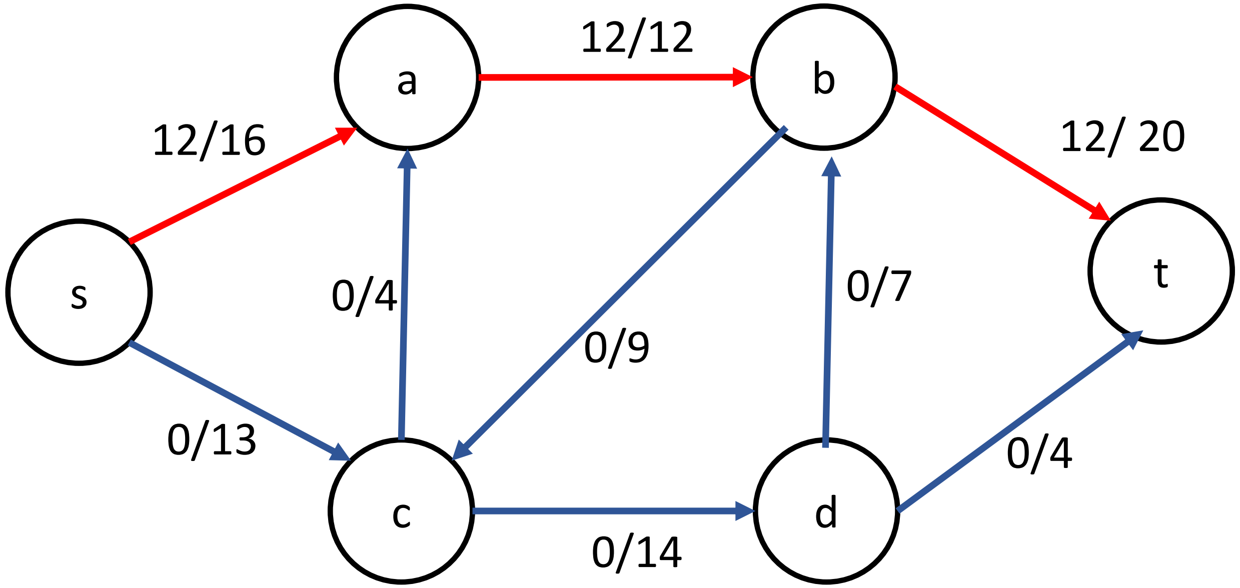

Find an "augmenting path" p in residual graph G_f and augment flow f along p

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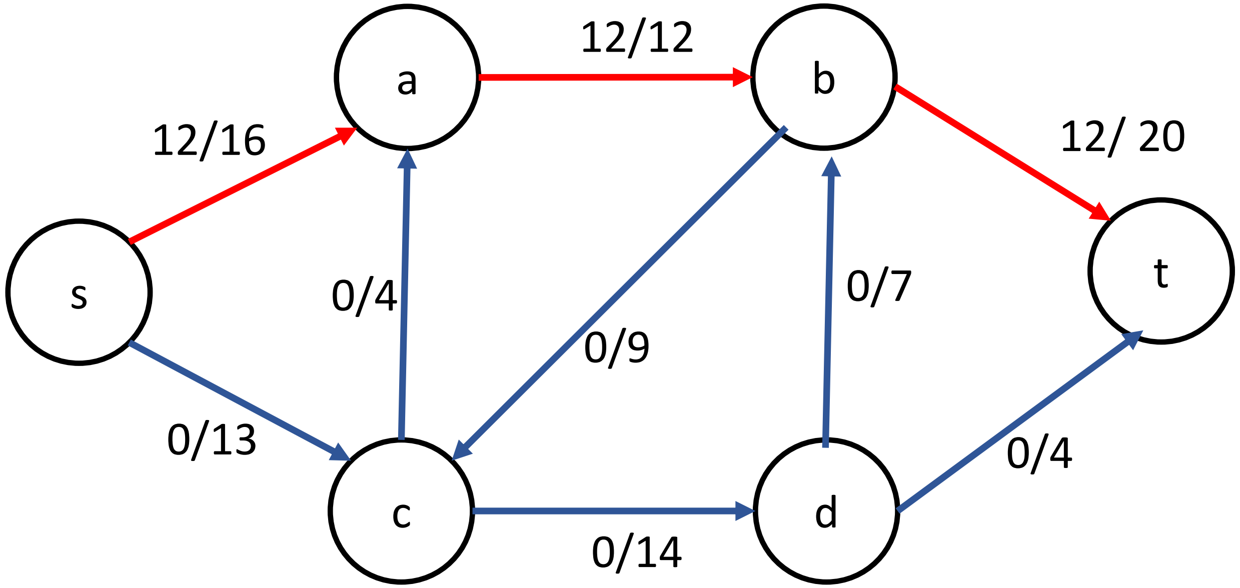

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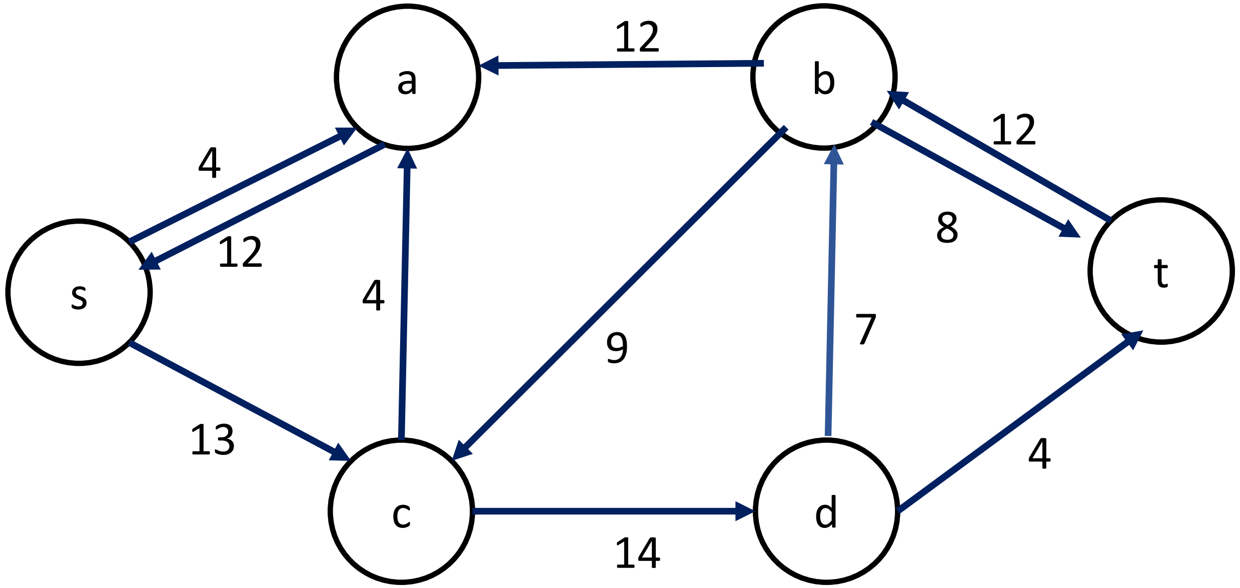

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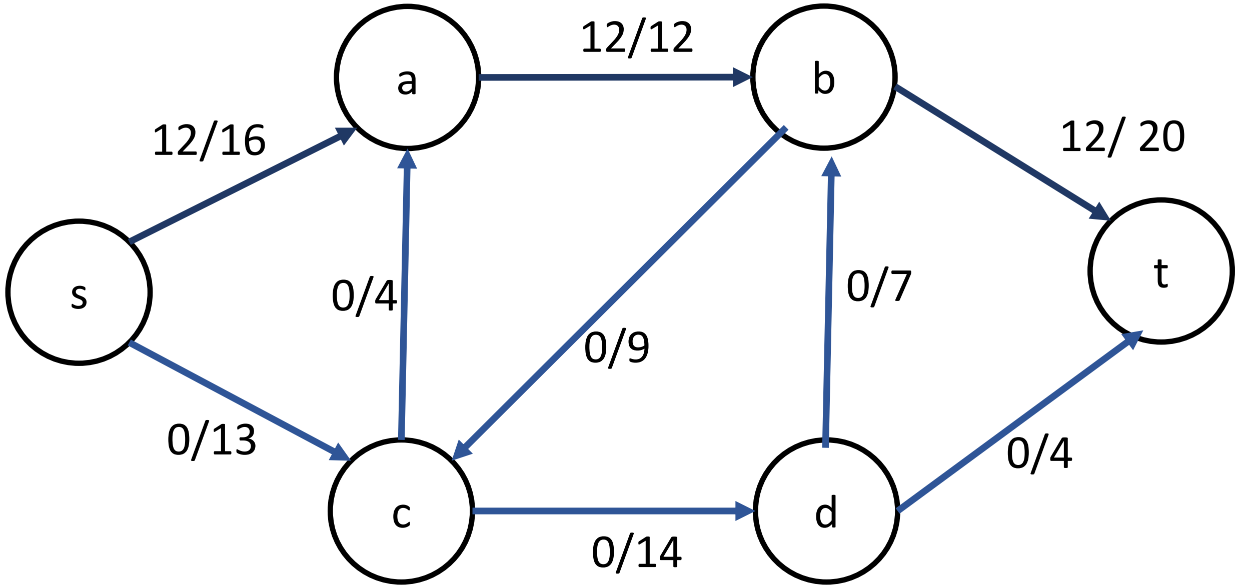

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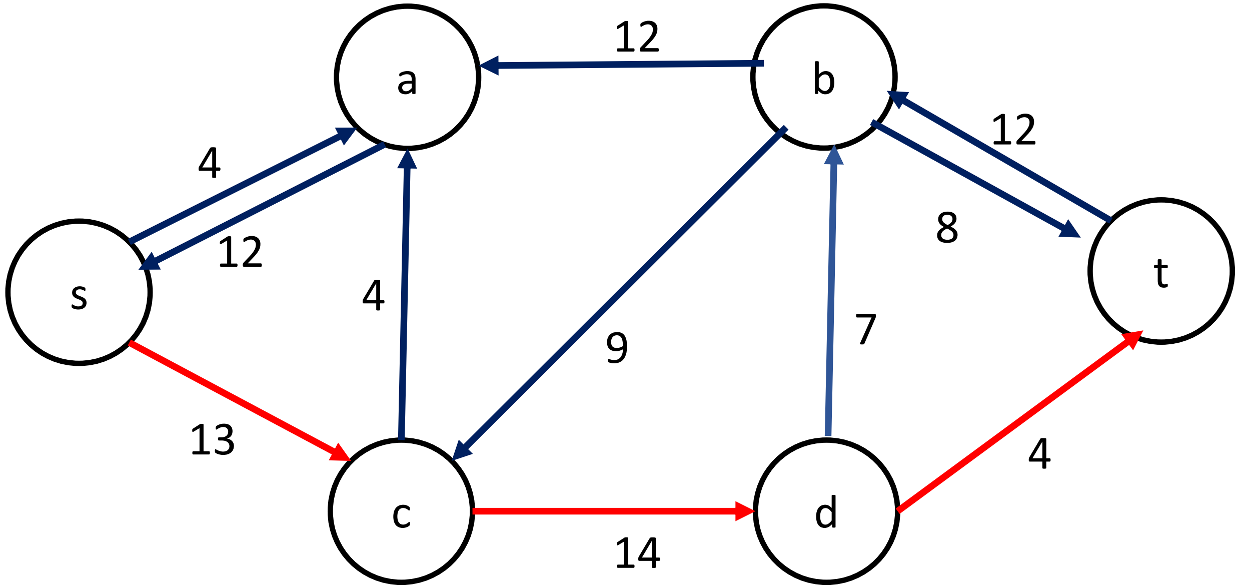

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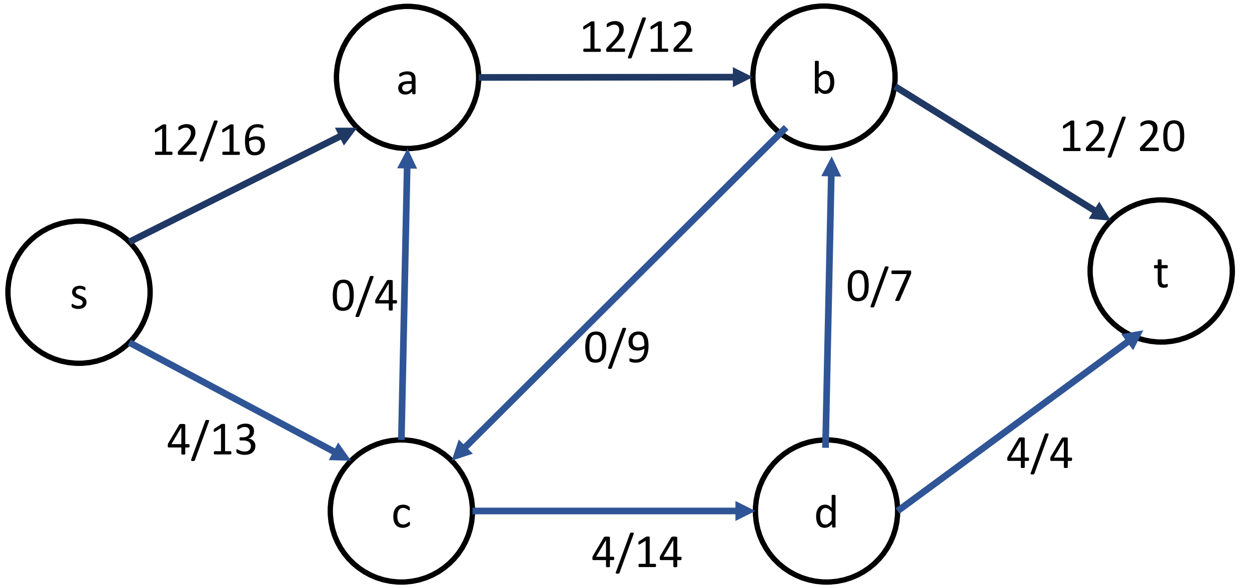

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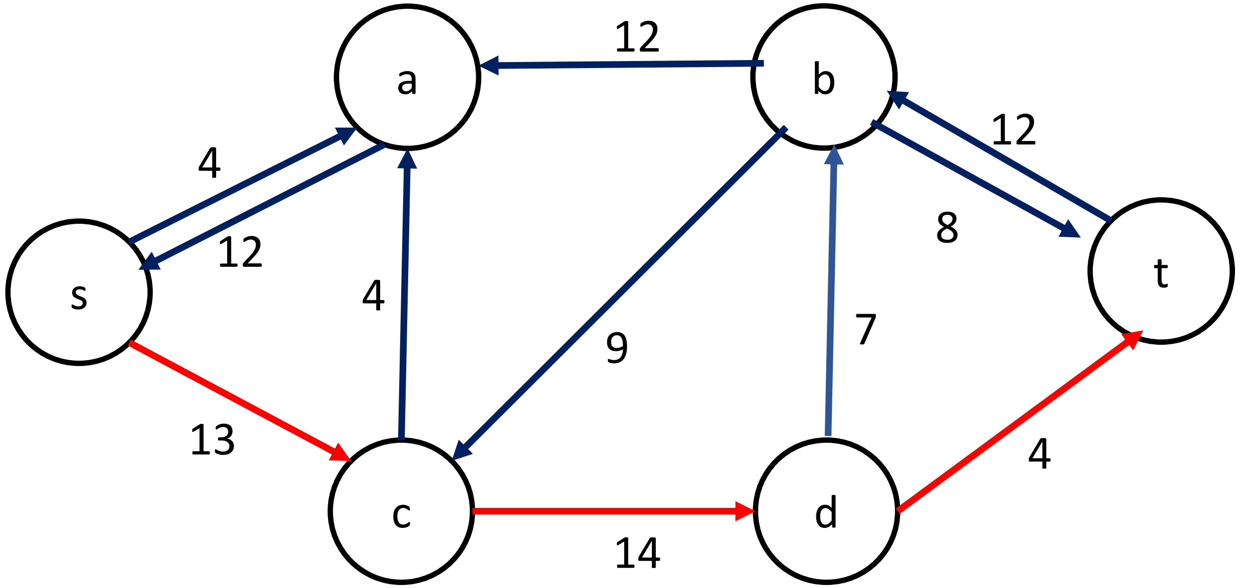

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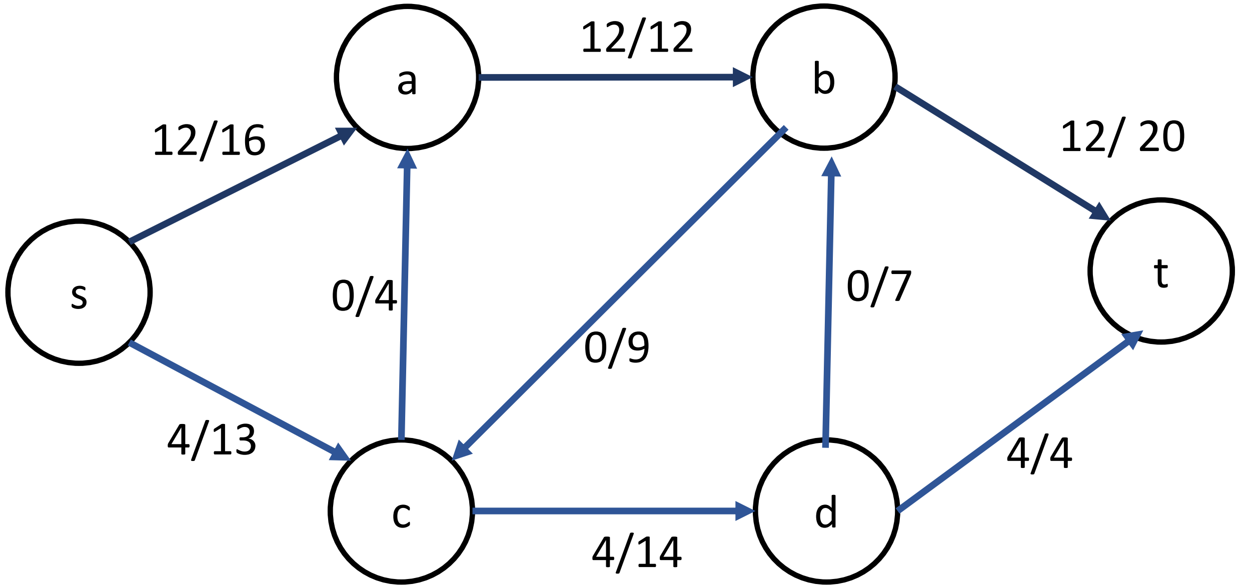

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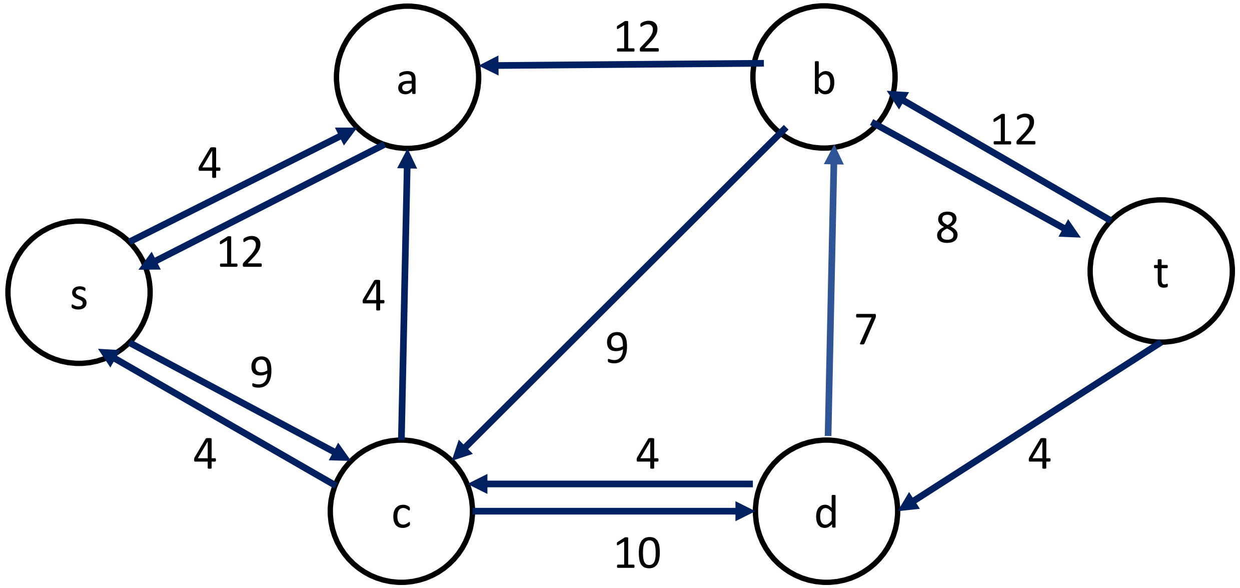

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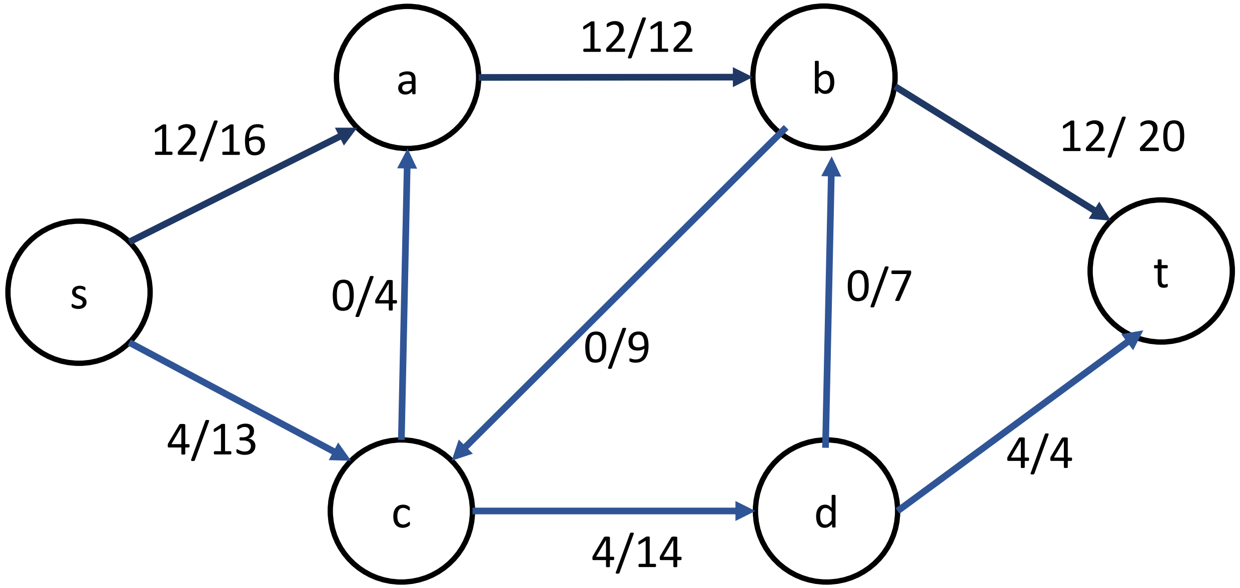

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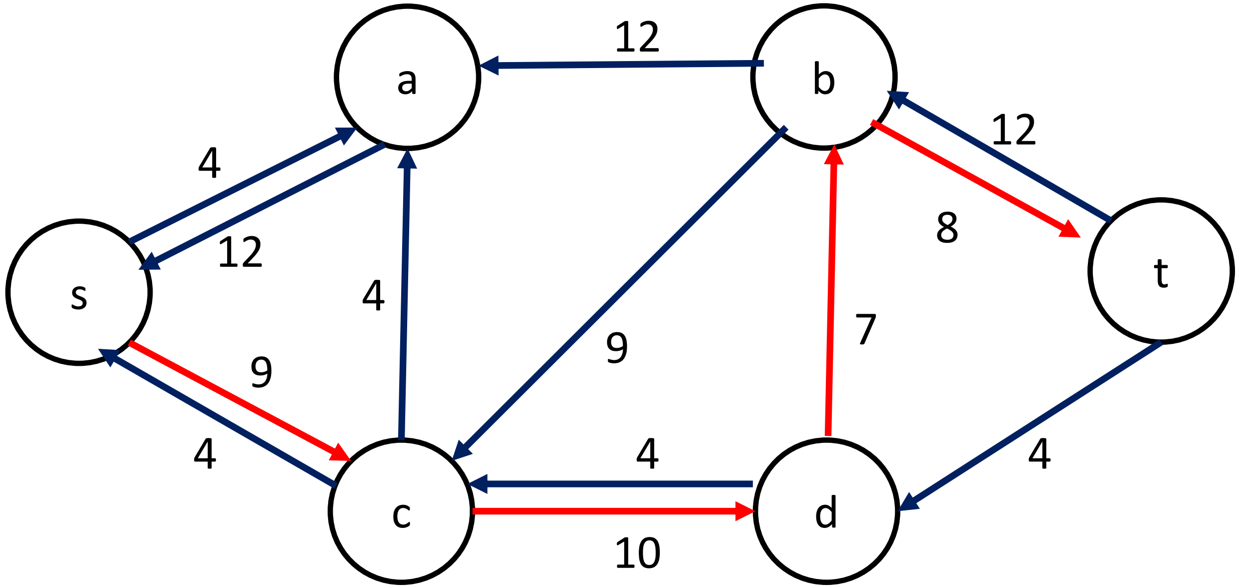

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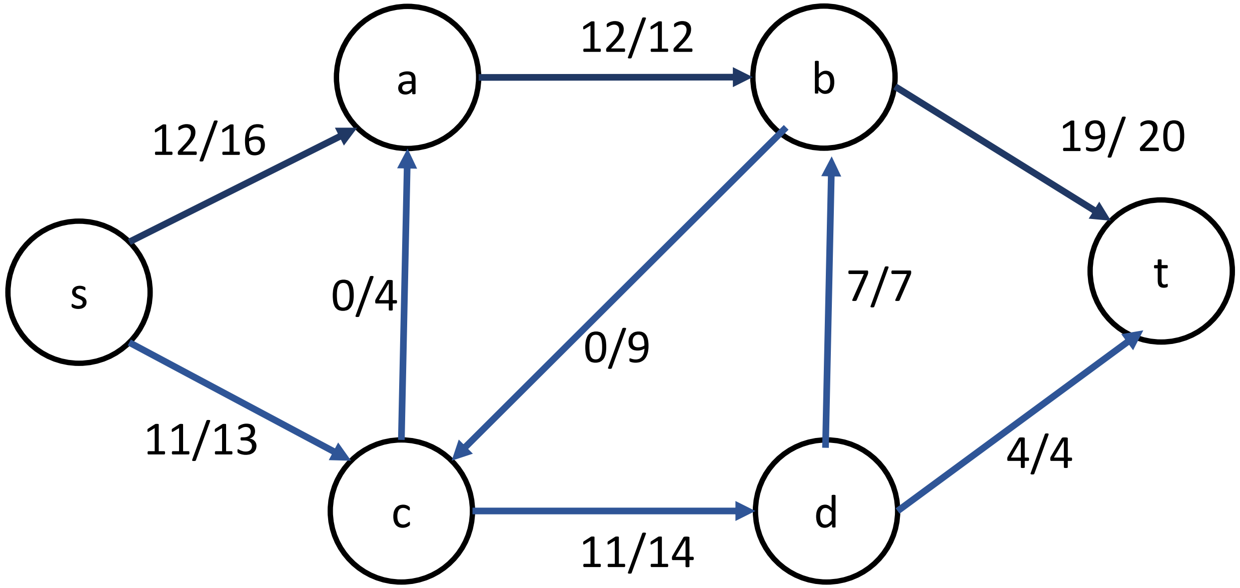

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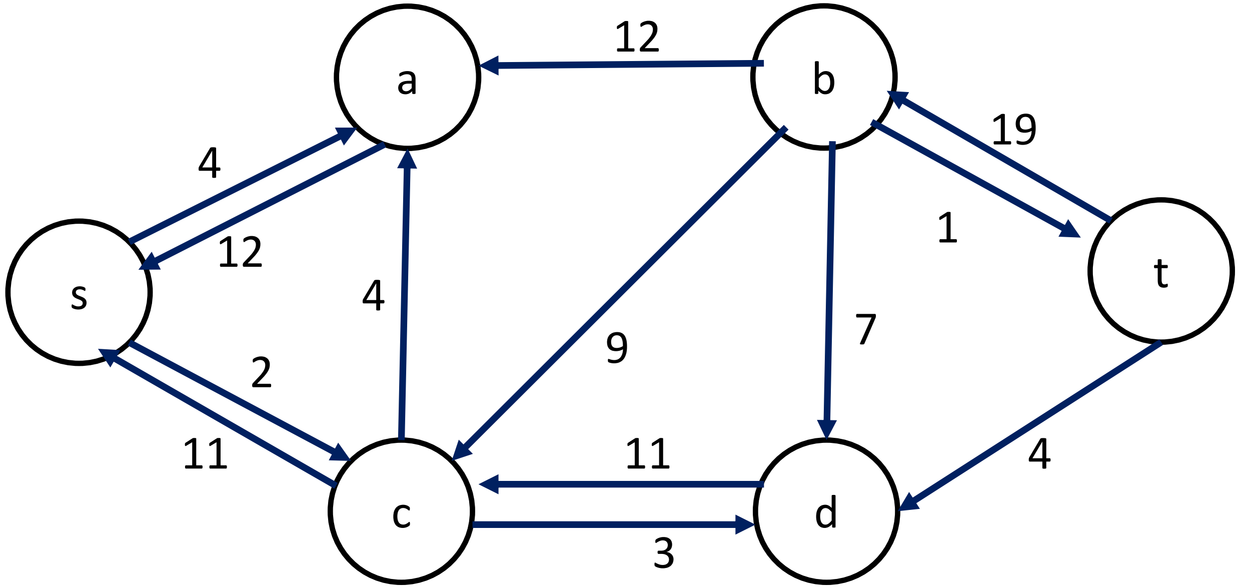

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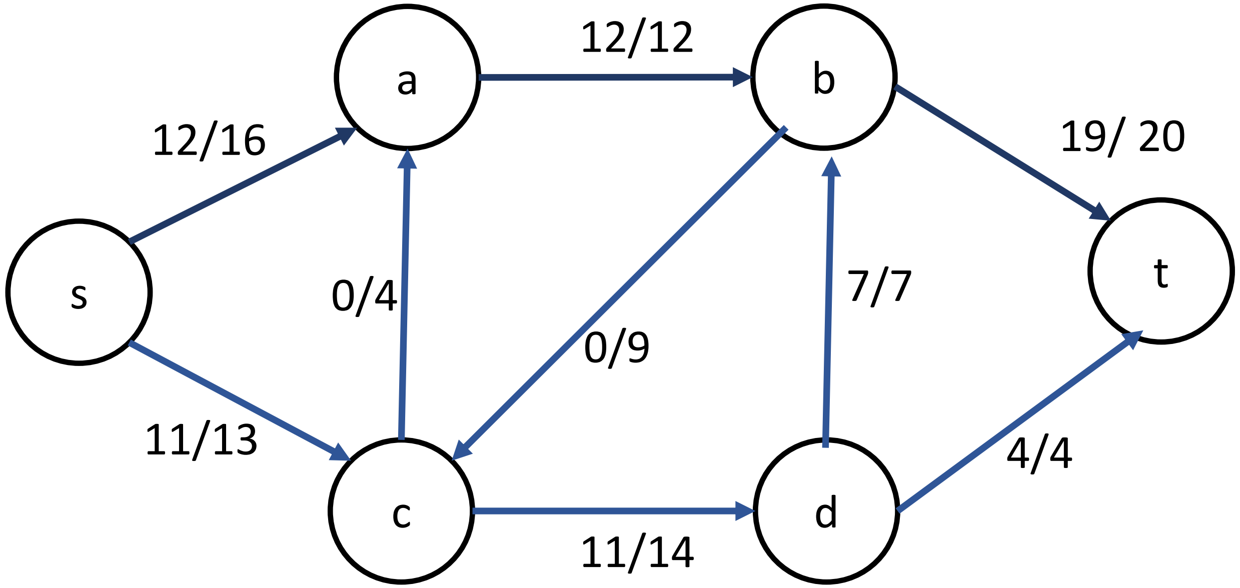
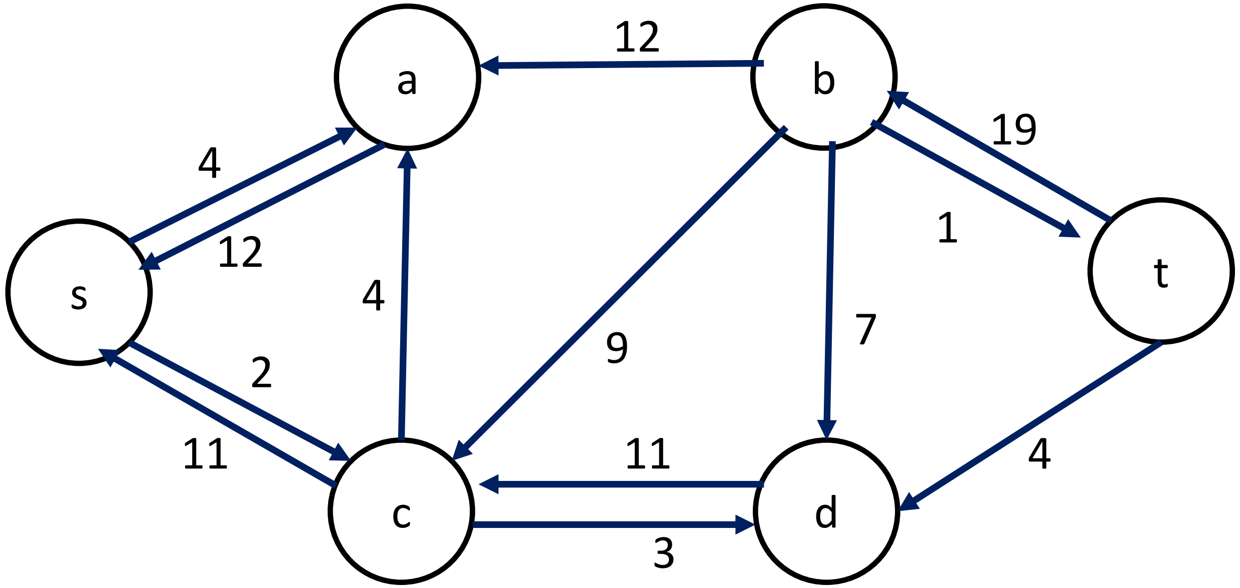


No path from s to t in residual graph G_f
We stop and we have found a max flow.
(But, why?)



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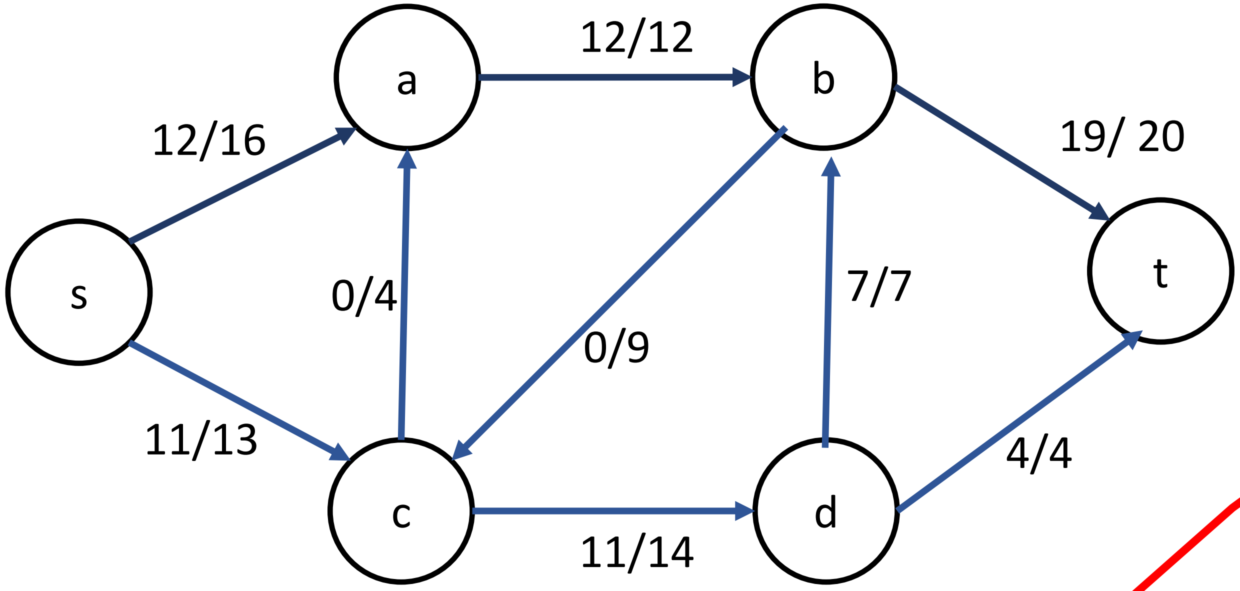
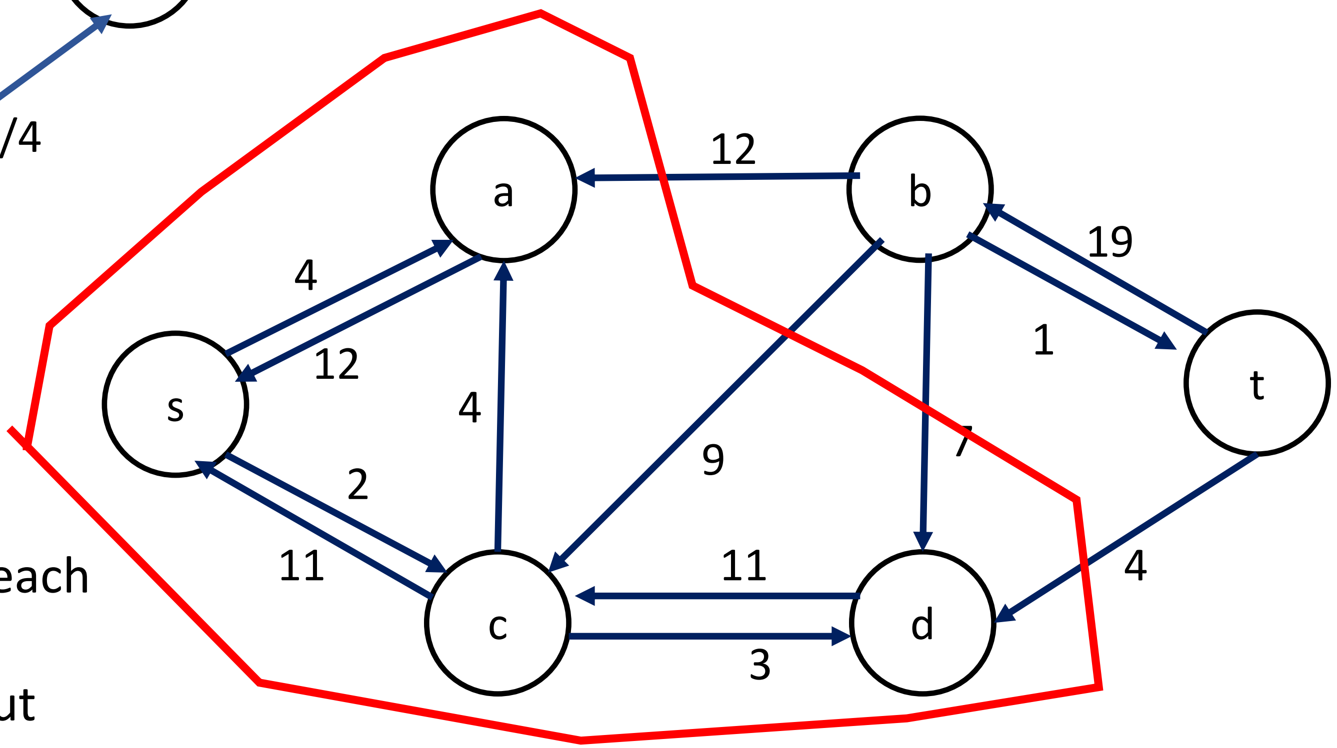


Why?

(Nodes reachable from s, others) is a s-t cut whose capacity matches the flow value



Max Flow <= Min Cut

And we have found flow and cut that match each other.

So, we have found s-t max flow and min cut