高等数学习题册-第四章-不定积分参考答案

第一节 不定积分的概念与性质

习题 4.1

1、求下列不定积分：

(1) $\int \frac{dx}{x^3} = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C$

(2) $\int \frac{1}{\sqrt{x}}\mathrm{dx} = \int x^{-1/2}\,dx = 2\sqrt{x} + C$

(3) $\int x^3 \cdot \sqrt[5]{x}dx = \int x^{16/5}\,dx = \frac{5}{21}x^{21/5} + \mathrm{C}$

(4) $\int (x^3+1)^2 dx = \int (x^6 + 2x^3 + 1)dx = \frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(5) $\int \frac{(2-x)^3}{\sqrt{x}}dx = \int \frac{-x^3+6x^2-12x+8}{\sqrt{x}}dx = \int \left(-x^{\frac{5}{2}} + 6x^{\frac{3}{2}} - 12\sqrt{x} + 8x^{-\frac{1}{2}}\right)dx = -\frac{2}{7}x^{7/2} + \frac{12}{5}x^{5/2} - 8x^{\frac{3}{2}} + 16x^{\frac{1}{2}} + C$

(6) $\int \frac{5x^5+5x^3+3}{x^2+1}dx = \int \frac{5x^3(x^2+1)+3}{x^2+1}dx = \int 5x^3dx + \int \frac{3}{x^2+1}dx$

$$= \frac{5x^4}{4} + 3arctanx + C$$

(7) $\int \frac{3x^2}{1+x^2} = \int \frac{3(1+x^2)-3}{1+x^2}dx = \int 3dx - \int \frac{dx}{1+x^2} = 3x - 3arctanx + C$

(8) $\int \left(5e^x - \frac{2}{x}\right)dx = 5\int e^x dx - 2\int \frac{dx}{x} = 5e^x - 2ln|x| + C$

(9) $\int \left(\frac{4}{1+x^2} + \frac{3}{\sqrt{1-x^2}}\right)dx = 4\int \frac{dx}{1+x^2} - 3\int \frac{dx}{\sqrt{1-x^2}}$

$$= 4arctanx - 3arcsinx + C$$

(10) $\int \frac{3\cdot 2^x - 7\cdot 3^x}{2^x}dx = \int 3dx - 7\int (\frac{3}{2})^x dx = 3x - \frac{7}{ln3-ln2}\left(\frac{3}{2}\right)^x + C$

(11) $\int secx(secx + 2tanx)dx = \int sec^2xdx + 2\int secxtanxdx$

$$= tanx + 2secx + C$$

(12) $\int \frac{1}{1+cos2x}dx = \int \frac{1}{2cos^2x}dx = \frac{1}{2}\int sec^2xdx = \frac{1}{2}tanx + C$

(13) $\int \frac{cos2x}{cosx-sinx}dx = \int \frac{cos^2x-sin^2x}{cosx-sinx}dx = \int (cosx + sinx)dx$

$$= sinx - cosx + C$$

(14) $\int \frac{cos2x}{cos^2x\cdot sin^2x}dx = \int \frac{cos^2x-sin^2x}{cos^2x\cdot sin^2x}dx = \int csc^2xdx - \int sec^2xdx$

$$= -cotx - tanx + C$$

2、一曲线通过点$(e^2,3)$，且在任一点处的切线的斜率等于该点横坐标的倒数，求该曲线的方程。

解：令曲线方程为$y=y(x)$,根据题意可得

$$\frac{dy}{dx}=\frac{1}{x}\rightarrow \mathrm{y}=\int\frac{1}{x}dx=ln|x|+C$$

因为曲线通过点$(e^2,3)$，则： $3=lne^2+C\rightarrow C=1$

故，曲线方程为：$\mathrm{y}=\ln|x|+1$

第二节 换元积分法

习题 4.2

1、在下列各等号右边的空白处填入适当的系数，使等式成立（如$\mathrm{d}x=\frac{1}{4}\mathrm{d}(4x+3)$）

（1）$\mathrm{d}x=$ **1/b** $\mathrm{d}(bx)(b\neq 0)$ （2）$x\mathrm{d}x=$ **-1/2** $\mathrm{d}(1-x^2)$

（3）$e^{2x}\mathrm{d}x=$ **1/6** $\mathrm{d}(3e^{2x})$ （4）$\frac{x\mathrm{d}x}{\sqrt{1-x^2}}=$ **-1** $\mathrm{d}(\sqrt{1-x^2})$

（5）$\cos\frac{2}{3}x\mathrm{d}x=$ **3/2** $d(\sin\frac{2x}{3})$ （6）$\frac{\mathrm{d}x}{x}=$ **-1/4** $\mathrm{d}(5-4\ln|x|)$

（7）$\frac{\mathrm{d}x}{1+4x^2}=$ **1/2** $d(\arctan 2x)$ （8）$\frac{\mathrm{d}x}{\sqrt{1-x^2}}=$ **-1** $d(2-\arcsin x)$

2、求下列不定积分

（1）$\int e^{6t}\mathrm{d}t=\frac{1}{6}e^{6t}+C$

（2）$\int(5-3x)^4\mathrm{d}x=-\frac{1}{3}\int(5-3x)^4d(5-3x)=-\frac{1}{15}(5-3x)^5+C$

（3）$\int\frac{\mathrm{d}x}{2-3x}=-\frac{1}{3}\int\frac{d(2-3x)}{2-3x}=-\frac{1}{3}ln|2-3x|+C$

（4）$\int(\cos ax-e^{\frac{x}{b}})\mathrm{d}x=\int cosaxdx-\int e^{\frac{x}{b}}dx=\frac{1}{a}sinax-be^{\frac{x}{b}}+C$

（5）$\int\frac{\cos\sqrt{t}}{\sqrt{t}}\mathrm{d}t$

解：令$u=\sqrt{t}$，则$u^2=t\rightarrow 2udu=dt$

原式=$\int\frac{cosu}{u}2udu=2\int cosudu=2sinu+C=2sin\sqrt{t}+C$

（6）$\int 5\tan^{10}x\cdot\sec^2 x\mathrm{d}x$

解：令$u=tanx\rightarrow du=sec^2xdx$

原式=$5\int u^{10}du=\frac{5}{11}u^{11}+C=\frac{5}{11}tan^{11}x+C$

（7）$\int \tan\sqrt{1+x^2}.\frac{x}{\sqrt{1+x^2}}\mathrm{d}x$

解：令$u=\sqrt{1+x^2}$

原式=$\int tanudu=\int\frac{sinu}{cosu}du=-\int\frac{dcosu}{cosu}=-ln|cosu|+C=-ln\left|cos\sqrt{1+x^2}\right|+C$

（8）$\int\frac{1}{e^x+e^{-x}}\mathrm{d}x$

解：原式=$\int\frac{e^x}{1+(e^x)^2}dx=\int\frac{d(e^x)}{1+(e^x)^2}=\arctan(e^x)+C$

（9）$\int xe^{-2x^2}\mathrm{d}x$

解：令$u=x^2\rightarrow du=2xdx$

原式=$\frac{1}{2}\int e^{-2u}du=\frac{1}{2}\cdot(\frac{-1}{2})e^{-2u}+C=-\frac{1}{4}e^{-2x^2}+C$

（10）$\int\frac{x}{\sqrt{3-2x^2}}\mathrm{d}x$

解：令$u=3-2x^2$

原式=$-\frac{1}{4}\int\frac{d(3-2x^2)}{\sqrt{3-2x^2}}=-\frac{1}{4}\int\frac{du}{\sqrt{u}}=-\frac{1}{2}\sqrt{u}+C=-\frac{1}{2}\sqrt{3-2x^2}+C$

（11）$\int\frac{x^3}{1-x^4}\mathrm{d}x$

解：原式=$-\frac{1}{4}\int\frac{d(1-x^4)}{1-x^4}=-\frac{1}{4}ln|1-x^4|+C$

（12）$\int\frac{2\sin x}{\cos^3 x}\mathrm{d}x$

解：原式=$-2\int\frac{d(cosx)}{cos^3x}=-2\frac{(cosx)^{-2}}{-2}+C=(cosx)^{-2}+C$

（13）$\int\frac{1-x}{\sqrt{4-9x^2}}\mathrm{d}x$

解：原式=$\frac{1}{2}\int\frac{1-x}{\sqrt{1-(\frac{3}{2}x)^2}}dx=\frac{1}{2}\cdot\frac{2}{3}\int\frac{d(\frac{3}{2}x)}{\sqrt{1-(\frac{3}{2}x)^2}}-\frac{1}{2}\int\frac{xdx}{\sqrt{1-(\frac{3}{2}x)^2}}$

$=\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)+\frac{1}{2}\cdot\frac{2}{9}\int\frac{d(1-\frac{9}{4}x^2)}{\sqrt{1-\frac{9}{4}x^2}}$

$=\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)+\frac{1}{9}\cdot 2\sqrt{1-\frac{9}{4}x^2}+\mathrm{C}$

$=\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)+\frac{1}{9}\cdot\sqrt{4-9x^2}+\mathrm{C}$

（14） $\int\frac{x^3}{4+x^2}\mathrm{d}x$

解：原式$=\int\frac{x(4+x^2)-4x}{4+x^2}dx=\int xdx-4\int\frac{xdx}{4+x^2}$

$=\frac{1}{2}x^2-2\int\frac{d(4+x^2)}{4+x^2}$

$=\frac{1}{2}x^2-2\ln(4+x^2)+C$

（15） $\int\frac{\mathrm{d}x}{(x+1)(x-2)}$

解：原式$=\frac{1}{3}\int\left(\frac{1}{x-2}-\frac{1}{x+1}\right)dx=\frac{1}{3}ln\left|\frac{x-2}{x+1}\right|+C$

（16） $\int\cos^3 x\mathrm{d}x$

解：原式$=\int cos^2xd(sinx)=\int(1-sin^2x)d(sinx)=sinx-\frac{1}{3}sin^3x+C$

（17） $\int\sin 3x\cdot\sin 5x\mathrm{d}x$

解：$sin3xsin5x=\frac{1}{2}(cos2x-cos8x)$

原式$=\frac{1}{2}\int cos2xdx-\frac{1}{2}\int cos8xdx=\frac{1}{4}\int cos2xd(2x)-\frac{1}{16}\int cos8xd(8x)$

$=\frac{1}{4}sin2x-\frac{1}{16}sin8x+C$

（18） $\int\frac{2\arctan\sqrt{x}}{\sqrt{x}(1+x)}\mathrm{d}x$

解：令$u=\sqrt{x}\rightarrow x=u^2\rightarrow dx=2udu$

原式$=4\int\frac{arctanu}{u(1+u^2)}udu=4\int\frac{arctanu}{1+u^2}du=4\int arctanud(arctanu)$

$=2(arctanu)^2+\mathrm{C}$

（19）$\int \frac{\mathrm{d}x}{(\arcsin x)^2 \cdot \sqrt{1-x^2}}$

解：原式=$\int \frac{d(arcsinx)}{(arcsinx)^2} = -\frac{1}{arcsinx} + C$

（20）$\int \frac{\mathrm{d}x}{x\sqrt{x^2-1}}$

解：令 $u = \sqrt{x^2-1} \to u^2 = x^2 - 1 \to 2udu = 2xdx$

原式=$\int \frac{xdx}{x^2\sqrt{x^2-1}} = \int \frac{udu}{(1+u^2)u} = \int \frac{du}{1+u^2} = arctanu + C$

$=\arctan\left(\sqrt{x^2-1}\right) + C$

（21）$\int \frac{\mathrm{d}x}{\sqrt{(x^2+1)^3}}$

解：令$x = tant, t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to dx = sec^2tdt$

原式=$\int \frac{sec^2tdt}{sec^3t} = \int \frac{dt}{sect} = \int costdt = sint + C = \frac{x}{\sqrt{1+x^2}} + C$

（22）$\int \frac{\mathrm{d}x}{1+\sqrt{2x}}$

解：令$u = \sqrt{2x} \to u^2 = 2x \to 2udu = 2dx$

原式=$\int \frac{udu}{1+u} = \int \frac{u+1-1}{1+u}du = \int 1du - \int \frac{d(1+u)}{1+u}$

$=\mathrm{u} - \ln|1+u| + C$

$= \sqrt{2x} - \ln(1+\sqrt{2x}) + C$

（23）$\int \frac{\mathrm{d}x}{1+\sqrt{1-x^2}}$

解：令$x = sint, t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to dx = costdt$

原式=$\int \frac{costdt}{1+cost} = \int dt - \int \frac{dt}{2cos^2\frac{t}{2}} = t - \int sec^2\frac{t}{2} d(\frac{t}{2})$

$= t - \tan\left(\frac{t}{2}\right) + C$

$= t - \frac{sint}{1+cost} + C \ = arcsinx - \frac{x}{1+\sqrt{1-x^2}} + C$

第三节　分部积分法

习题 4.3

一、求下列不定积分：

1、$\int x^2 \cos x \mathrm{d}x$

解：原式=$\int x^2 d(sinx) = x^2 sinx - 2\int sinx \cdot xdx$

$$= x^2 sinx + 2\int xd(cosx)$$

$$= x^2 sinx + 2(xcosx - \int cosxdx)$$

$$= x^2 sinx + 2xcosx - 2sinx + C$$

2、$\int x \ln x \mathrm{d}x$

解：原式=$\frac{1}{2}\int lnxd(x^2) = \frac{1}{2}(x^2 lnx - \int x^2 d(lnx))$

$$= \frac{1}{2}(x^2 lnx - \int xdx)$$

$$= \frac{1}{2}(x^2 lnx) - \frac{1}{4}x^2 + \mathrm{C}$$

3、$\int \arccos x \mathrm{d}x$

解：原式=$xarccosx + \int \frac{xdx}{\sqrt{1-x^2}} = xarccosx - \frac{1}{2}\int \frac{d(1-x^2)}{\sqrt{1-x^2}}$

$$= xarccosx - \sqrt{1-x^2} + C$$

4、$\int x^2 e^{-x} \mathrm{d}x$

解：原式=$-\int x^2 d(e^{-x}) = -x^2 e^{-x} + 2\int e^{-x} \cdot xdx$

$$= -x^2 e^{-x} - 2\int xd(e^{-x})$$

$$= -x^2 e^{-x} - 2(xe^{-x}) + 2\int e^{-x} dx$$

$$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

5、$\int x^3 \ln x dx$

解：原式=$\frac{1}{4}\int lnxd(x^4)=\frac{1}{4}x^4lnx-\frac{1}{4}\int x^4\cdot\frac{1}{x}dx$

$$=\frac{1}{4}x^4lnx-\frac{1}{16}x^4+C$$

6、$\int e^{-2x}\sin x dx$

解：原式=$-\int e^{-2x}d(cosx)=-e^{-2x}cosx+\int cosxd(e^{-2x})$

$$=-e^{-2x}cosx-2\int cosx\cdot e^{-2x}dx$$

$$=-e^{-2x}cosx-2\int e^{-2x}d(sinx)$$

$$=-e^{-2x}cosx-2(e^{-2x}sinx+2\int sinx\cdot e^{-2x}dx)$$

$$=-e^{-2x}cosx-2e^{-2x}sinx-4\int sinx\cdot e^{-2x}dx$$

故$\int sinx\cdot e^{-2x}dx=\frac{1}{5}e^{-2x}(-cosx-2sinx)+C$；

7、$\int x^2 \arctan x dx$

解：原式=$\frac{1}{3}\int arctanxd(x^3)=\frac{1}{3}x^3arctanx-\frac{1}{3}\int\frac{x^3}{1+x^2}dx$

$$=\frac{1}{3}x^3arctanx-\frac{1}{3}\int\frac{x(1+x^2)-x}{1+x^2}dx$$

$$=\frac{1}{3}x^3arctanx-\frac{1}{3}\int xdx+\frac{1}{6}\int\frac{d(1+x^2)}{1+x^2}$$

$$=\frac{1}{3}x^3arctanx-\frac{1}{6}x^2+\frac{1}{6}\ln(1+x^2)+C$$

8、$\int te^{-3t}dt$

解：原式=$-\frac{1}{3}\int td\left(e^{-3t}\right)=-\frac{1}{3}(te^{-3t}-\int e^{-3t}dt)$

$$=-\frac{1}{3}te^{-3t}-\frac{1}{9}e^{-3t}+C$$

9、$\int (x^2-1)\cos 2x dx$

解：原式= $\int x^2cos2xdx-\int cos2xdx$

$$=\frac{1}{2}x^2sin2x+\frac{1}{2}xcos2x-\frac{1}{4}sin2x-\int cos2xdx$$

$$=\frac{1}{2}x^2sin2x+\frac{1}{2}xcos2x-\frac{1}{4}sin2x-\frac{1}{2}sin2x+C$$

$$=\frac{1}{2}x^2sin2x+\frac{1}{2}xcos2x-\frac{3}{4}sin2x+C$$

10、$\int\frac{\ln^3 x}{x^2}\mathrm{d}x$

解：令$u=lnx\rightarrow du=\frac{1}{x}dx$

原式=$\int\frac{u^3}{e^u}du=-e^{-u}(u^3+3u^2+6u+6)+C$

$$=-\frac{1}{x}(ln^3x+3ln^2x+6lnx+6)+C$$

11、$\int\sin\ln x dx$

解：令$u=lnx\rightarrow x=e^u\rightarrow dx=e^udu$

原式=$\int sinu\cdot e^udu=\frac{1}{2}e^u(sinu-cosu)+C$

$$=\frac{1}{2}x(sinlnx-coslnx)+C$$

12、$\int(\arccos x)^2\mathrm{d}x$

解：令$u=arccosx\rightarrow x=cosu,u\in(0,\pi/2)$

原式=$\int u^2d(cosu)=u^2cosu-2\int cosu\cdot udu$

$$=u^2cosu-2\int ud(sinu)$$

$$=u^2cosu-2usinu+2\int sinudu$$

$$=u^2cosu-2usinu-2cosu+C$$

$$=x(arccosx)^2-2\sqrt{1-x^2}arccosx-2x+C$$

二．设 $f(x)$ 的一个原函数为 $\sin x$，求 $\int x^2 f''(x)\mathrm{d}x$

解：由题意得

$$\int f(x)dx = sinx + C \to f(x) = (sinx)' = cosx \to f'(x) = -sinx$$

故：$\int x^2 f''(x)dx = \int x^2 d\big(f'(x)\big) = x^2 f'(x) - 2\int f'(x)\cdot xdx$

$$= x^2 f'(x) - 2\int xdf(x)$$

$$= x^2 f'(x) - 2xf(x) + 2\int f(x)dx$$

$$= -x^2 sinx - 2xcosx + 2sinx + C$$

第四节　有理函数的积分

习题 4. 4

求下列积分：

1、$\int \frac{x^3}{x+2}\mathrm{d}x$

解：原式=$\int (x^2 - 2x + 4)dx - 8\int \frac{dx}{x+2}$

$$= \frac{1}{3}x^3 - x^2 + 4x - 8ln|x+2| + C$$

2、$\int \frac{2x+5}{x^2+x-12}\mathrm{d}x$

解：原式=$\int \frac{2x+5}{(x+4)(x-3)}dx = \frac{3}{7}\int \frac{dx}{x+4} + \frac{11}{7}\int \frac{dx}{x-3}$

$$= \frac{3}{7}ln|x+4| + \frac{11}{7}ln|x-3| + C$$

3、$\int \frac{x^2+1}{(x+1)^2(x-1)}\mathrm{d}x$

解：令$\frac{x^2+1}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$

$\rightarrow x^2+1 = (A+C)x^2 + (B+2C)x + (C-B-A)$

比较系数得

$$\begin{cases} A+C=1 \\ B+2C=0 \\ C-B-A=1 \end{cases} \rightarrow \begin{cases} A=1/2 \\ B=-1 \\ C=1/2 \end{cases}$$

原式$= \frac{1}{2}\int\frac{dx}{x+1} - \int\frac{dx}{(x+1)^2} + \frac{1}{2}\int\frac{dx}{x-1}$

$= \frac{1}{2}ln|x+1| + \frac{1}{x+1} + \frac{1}{2}ln|x-1| + C$

$= \frac{1}{2}ln|x^2-1| + \frac{1}{x+1} + C$

4、$\int \frac{x+5}{x^2-6x+13}\mathrm{d}x$

解：原式$=\int\frac{\frac{1}{2}(2x-6)+8}{x^2-6x+13}dx = \frac{1}{2}\int\frac{d(x^2-6x+13)}{x^2-6x+13} + 8\int\frac{dx}{(x-3)^2+4}$

$$= \frac{1}{2}\ln(x^2-6x+13) + 4\int\frac{d(\frac{x-3}{2})}{1+(\frac{x-3}{2})^2}$$

$$= \frac{1}{2}\ln(x^2-6x+13) + 4\arctan\left(\frac{x-3}{2}\right) + C$$

5、$\int \frac{\mathrm{d}x}{x^4+1}$

解：令$\frac{1}{x^4+1} = \frac{1}{(x^2+1)^2-(\sqrt{2}x)^2} = \frac{Ax+B}{x^2+\sqrt{2}x+1} + \frac{Cx+D}{x^2-\sqrt{2}x+1}$

比较系数得

$$\frac{1}{x^4+1} = \frac{1}{(x^2+1)^2-(\sqrt{2}x)^2} = \frac{\frac{\sqrt{2}}{4}x+\frac{1}{2}}{x^2+\sqrt{2}x+1} - \frac{\frac{\sqrt{2}}{4}x-\frac{1}{2}}{x^2-\sqrt{2}x+1}$$

原式=$\int \frac{\frac{\sqrt{2}}{4}x+\frac{1}{2}}{x^2+\sqrt{2}x+1}dx-\int \frac{\frac{\sqrt{2}}{4}x-\frac{1}{2}}{x^2-\sqrt{2}x+1}dx$

$$=\frac{\sqrt{2}}{8}ln\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}+\frac{\sqrt{2}}{4}\left[\arctan\left(\sqrt{2}x+1\right)+\arctan\left(\sqrt{2}x-1\right)\right]+C$$

6、$\int\frac{\mathrm{d}x}{2+\sin x}$

解：令$u=tan\frac{x}{2}\to sinx=\frac{2u}{1+u^2}$， $x=2arctanu, dx=\frac{2du}{1+u^2}$

原式 $=\int\frac{du}{1+u+u^2}=\frac{2}{\sqrt{3}}\arctan\left(\frac{2u+1}{\sqrt{3}}\right)+C$

$$=\frac{2}{\sqrt{3}}\arctan\left(\frac{2tan\frac{x}{2}+1}{\sqrt{3}}\right)+C$$

7、$\int\frac{\mathrm{d}x}{2\sin x-\cos x+5}$

解：设 $u=tan\frac{x}{2}\to sinx=\frac{2u}{1+u^2}$， $cosx=\frac{1-u^2}{1+u^2}, x=2arctanu$

$dx=\frac{2du}{1+u^2}$

原式=$\int\frac{du}{3u^2+2u+2}=\frac{1}{3}\int\frac{du}{(u+\frac{1}{3})^2+\frac{5}{9}}=\frac{1}{\sqrt{5}}arctan\frac{3u+1}{\sqrt{5}}+C$

$$=\frac{1}{\sqrt{5}}arctan\frac{3tan\frac{x}{2}+1}{\sqrt{5}}+C$$

8、$\int\frac{3\cos x-\sin x}{\sin x+\cos x}dx$

解：令$3cosx-sinx=A(sinx+cosx)+B(sinx+cosx)'$

即：$3cosx-sinx=A(sinx+cosx)+B(-sinx+cosx)$

$$\to\begin{cases}A-B=-1\\A+B=3\end{cases}\to\begin{cases}A=1\\B=2\end{cases}$$

原式=$\int \frac{sinx+cosx}{sinx+cosx}dx + 2\int \frac{d(sinx+cosx)}{sinx+cosx}$

$=x + 2ln|sinx + cosx| + C$

9、$\int \frac{\mathrm{d}x}{1+\sqrt[3]{x+1}}$

解：令$\sqrt[3]{x+1} = t \to x + 1 = t^3 \to dx = 3t^2dt$

原式= $3\int \frac{t^2dt}{1+t} = 3\int \frac{t^2-1+1}{1+t}dt = 3\int (t-1)dt + 3\int \frac{1}{1+t}dt$

$= 3\left(\frac{t^2}{2} - t + ln|1+t|\right) + C$

$= 3\left(\frac{(\sqrt[3]{x+1})^2}{2} - \sqrt[3]{x+1} + ln\left|1 + \sqrt[3]{x+1}\right|\right) + C$

10、$\int \frac{(\sqrt{x})^3+1}{\sqrt{x}+1}\mathrm{d}x$

解：令$\sqrt{x} = t \to x = t^2 \to dx = 2tdt$

原式= $\int \frac{t^3+1}{t+1}2tdt = 2\int \frac{(t+1)(t^2-t+1)}{t+1}tdt = 2\int (t^3 - t^2 + t)dt$

$= \frac{1}{2}t^4 - \frac{2}{3}t^3 + t^2 + C$

$= \frac{1}{2}x^2 - \frac{2}{3}(\sqrt{x})^3 + x + C$

11、$\int \frac{\mathrm{d}x}{\sqrt{x}+\sqrt[4]{x}}$

解：设$t = \sqrt[4]{x} \to x = t^4 \to dx = 4t^3dt$

原式= $4\int \frac{t^3}{t^2+t}dt = 4\int \left(t - 1 + \frac{1}{t+1}\right)dt = 2t^2 - 4t + 4ln|1+t| + C$

$= 2\sqrt{x} - 4\sqrt[4]{x} + 4ln\left|1 + \sqrt[4]{x}\right| + C$

12、$\int\sqrt{\frac{1-x}{1+x}}\cdot\frac{\mathrm{d}x}{x}$

解：令$\sqrt{\frac{1-x}{1+x}}=t\rightarrow x=\frac{1-t^2}{1+t^2}\rightarrow dx=\frac{-4t}{(1+t^2)^2}dt$

原式= $\int t\frac{-4t}{(1+t^2)^2}\cdot\frac{1+t^2}{1-t^2}dt=4\int\frac{t^2}{(t^2-1)(t^2+1)}dt=2\int\frac{1}{t^2-1}dt+2\int\frac{1}{t^2+1}dt$

$$=\int\left(\frac{1}{t-1}-\frac{1}{t+1}\right)dt+2\int\frac{dt}{t^2+1}$$

$$=ln\left|\frac{t-1}{t+1}\right|+2arctant+C$$

$$=ln\left|\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\right|+2arctan\sqrt{\frac{1-x}{1+x}}+C$$