§2.1 导数概念

习题2.1解答

1. 解(1) $a=\lim\limits_{\Delta x\to 0}\frac{f(x_0-\Delta x)-f(x_0)}{\Delta x}=-\lim\limits_{\Delta x\to 0}\frac{f(x_0-\Delta x)-f(x_0)}{(x_0-\Delta x)-x_0}=-f'(x_0)$;

(2) $a=\lim\limits_{x\to 0}\frac{f(x)}{x}\underset{\because f(0)=0}{=}\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=f'(0)$ ($\because f'(0)\exists$);

(3) $a=\lim\limits_{h\to 0}\frac{f(x_0-h)-f(x_0+h)}{h}=\lim\limits_{h\to 0}\frac{f(x_0-h)-f(x_0)}{-((x_0-h)-x_0)}-\lim\limits_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$.

$=-f'(x_0)-f'(x_0)=-2f'(x_0)$.

2. 解:(1) $y'=2(x^{-\frac{1}{2}})'=2\times(-\frac{1}{2})\cdot x^{-\frac{3}{2}}=-x^{-\frac{3}{2}}$;

(2) $y'=(x^3\cdot x^{\frac{3}{5}}\cdot x^{-\frac{3}{2}})'=(x^{\frac{21}{10}})'=\frac{21}{10}x^{\frac{11}{10}}$;

3. 解: $\because y'\big|_{(\frac{\pi}{3},\frac{1}{2})}=-\sin x\big|_{x=\frac{\pi}{3}}=-\frac{\sqrt{3}}{2}$

$\therefore$ y在点$(\frac{\pi}{3},\frac{1}{2})$处切线方程 $y-\frac{1}{2}=-\frac{\sqrt{3}}{2}(x-\frac{\pi}{3})$, 即 $\frac{\sqrt{3}}{2}x+y-\frac{1}{2}(1+\frac{\pi}{\sqrt{3}})=0$

y在点$(\frac{\pi}{3},\frac{1}{2})$处法线方程 $y-\frac{1}{2}=\frac{2}{\sqrt{3}}(x-\frac{\pi}{3})$, 即 $\frac{2}{\sqrt{3}}x-y+\frac{1}{2}-\frac{2\pi}{3\sqrt{3}}=0$

4. 解(1) $\because \lim\limits_{x\to 0}x^3\sin\frac{1}{x}=0=y(0)$ $\therefore$ y在x为0处连续

又$\because$ $y'(0)=\lim\limits_{x\to 0}\frac{x^3\sin\frac{1}{x}-0}{x-0}=\lim\limits_{x\to 0}x^2\sin\frac{1}{x}=0$ $\therefore$ y在$x=0$处可导.

(2) $\because \lim\limits_{x\to 0}x\sin\frac{1}{x}=0=y(0)$ $\therefore$ y在$x=0$处连续

又$\because$ $y'(0)=\lim\limits_{x\to 0}\frac{x\sin\frac{1}{x}-0}{x-0}=\lim\limits_{x\to 0}\sin\frac{1}{x}$ 不存在, $\therefore$ y在$x=0$处不可导

5. $\because f(x)$在$x=1$处连续且可导 $\therefore \lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}x^3=1=\lim\limits_{x\to 1^+}(ax+b)=b+a$

$f'_+(1)=a=f'_-(1)=3$ $\therefore a=3$, $b=-2$

6. $f'_-(0)=-1$, $f'_+(0)=0$, $f'(0)$不存在

7. $f'(x)=\begin{cases}\cos x, & x<0\\ 2x, & x>0\end{cases}$, $\because f'_-(0)=1\neq 0=f'_+(0)$ $\therefore f'(0)$不存在.

§2.2 函数的求导法则

习题2.2解答

1、(1): $y'=x^2-2\ln^2 4^x+2e^{2x}$；

(2) $y'=\csc^2 x-\csc x\cot x$；

(3) $y'=\ln x\cdot\sin x+\sin x+x\ln x\cos x$；

(4) $y'=2\arcsin 2x\cdot\dfrac{1}{\sqrt{1-(2x)^2}}\cdot 2=\dfrac{4\arcsin 2x}{\sqrt{1-4x^2}}$

(5) $y'=\dfrac{1}{x+\sqrt{a^2+x^2}}\cdot\left(1+\dfrac{2x}{2\sqrt{a^2+x^2}}\right)=\dfrac{1}{\sqrt{a^2+x^2}}$

(6) $y'=\dfrac{1}{\cos x+\tan x}\cdot(-\sin x+\sec^2 x)=\dfrac{\sec^2 x-\sin x}{\tan x+\cos x}$

(7) $y'=\dfrac{\sec x\tan x+\csc^2 x}{\sec x-\cot x}$

(8) $y'=e^{2\arctan\sqrt{x}}\cdot 2\dfrac{1}{1+(\sqrt{x})^2}\cdot\dfrac{1}{2\sqrt{x}}=e^{2\arctan\sqrt{x}}\cdot\dfrac{1}{\sqrt{x}+x\sqrt{x}}=\dfrac{e^{2\arctan\sqrt{x}}}{\sqrt{x}(1+x)}$

(9) $y'=\left(-\dfrac{1}{\sqrt{1-x^2}}\cdot\arcsin x-\dfrac{1}{\sqrt{1-x^2}}\arccos x\right)/(\arcsin x)^2=\dfrac{\pi}{-2\sqrt{1-x^2}(\arcsin x)^2}$

2、解: $y'=f'(\sin^2 x)\cdot 2\sin x\cos x+f'(\cos^2 x)\cdot 2\cos x\cdot(-\sin x)$

$=\sin 2x\cdot(f'(\sin^2 x)-f'(\cos^2 x))$

3、解:(1) $y'=-e^{-x}(x^2-2x+3)+e^{-x}(2x-2)=e^{-x}(-x^2+4x-5)$

(2) $y'=\arcsin\dfrac{x}{2}+x\cdot\dfrac{1}{\sqrt{1-(\frac{x}{2})^2}}\cdot\dfrac{1}{2}+\dfrac{-2x}{2\sqrt{4-x^2}}=\arcsin\dfrac{x}{2}$

(3) $y'=\dfrac{1}{\cos\frac{1}{x}}\cdot\left(-\sin\dfrac{1}{x}\right)\cdot\left(-\dfrac{1}{x^2}\right)=\tan\dfrac{1}{x}/x^2$

(4) $y'=\dfrac{1}{\tan\frac{x}{2}}\sec^2\dfrac{x}{2}\cdot\dfrac{1}{2}-\cos x\ln\cot x-\sin x\cdot\dfrac{1}{\cot x}\cdot(-\csc^2 x)$

$=\csc x-\cos x\ln\cot x+\sec x=\sec x+\csc x-\cos x\ln\cot x$

§2.3 高阶导数

习题2.3解答

1.解：(1) $y'=2\tan x\cdot\sec^2 x$

$y''=2\sec^4 x+2\tan x\cdot 2\sec x\cdot\sec x\tan x=6\sec^4 x-4\sec^2 x$

(2) $y'=2x\arctan x+1$

$y''=2\arctan x+\dfrac{2x}{1+x^2}$

(3) $y'=\dfrac{1}{x+\sqrt{1+x^2}}\left(1+\dfrac{2x}{2\sqrt{1+x^2}}\right)=\dfrac{1}{\sqrt{1+x^2}}$

$y''=\left((1+x^2)^{-\frac{1}{2}}\right)'=-\frac{1}{2}\cdot(1+x^2)^{-\frac{3}{2}}\cdot 2x=-x(1+x^2)^{-\frac{3}{2}}=\dfrac{-x}{(1+x^2)^{\frac{3}{2}}}$

2.解：$y'=f'(x^3)\cdot 3x^2$，$y''=f''(x^3)\cdot 9x^4+6x\cdot f'(x^3)$

3.解：(1) $y^{(n)}=n!$

(2) $\because y=\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$，$\sin^{(n)} x=\sin(x+\frac{n\pi}{2})$，$y'=-\sin 2x$

$\therefore y^{(n)}=-2^{n-1}\sin(2x+\frac{n-1}{2}\pi)$

(3) $y'=1\cdot e^{2x}+2^1 xe^{2x}$

$y''=2e^{2x}+2e^{2x}+4xe^{2x}=2\times 2^1 e^{2x}+2^2 xe^{2x}$

$y'''=8e^{2x}+4e^{2x}+8xe^{2x}=12e^{2x}+8xe^{2x}=3\times 2^2 e^{2x}+2^3 xe^{2x}$

$\therefore y^{(n)}=n\cdot 2^{n-1}e^{2x}+2^n xe^{2x}$

4.解：$y^{(50)}=C_{50}^0(x^2)^{(0)}\cos^{(50)}5x+C_{50}^1(x^2)'\cos^{(49)}5x+C_{50}^2(x^2)''\cos^{(48)}5x$

$=x^2\cos(5x+25\pi)+50\times 2x\cos(5x+24\pi+\frac{\pi}{2})+\frac{50\times 49}{2}\times 2!\cos(5x+24\pi)$

$=-x^2\cos 5x-100x\sin 5x+2450\cos 5x$

§2·4 隐函数及参数方程的求导法

习题2.4解答

1、解：$y^2+x\cdot 2y\cdot y'-e^y\cdot y'=0$，$\therefore y'=\dfrac{y^2}{e^y-2xy}$

2、解：$y'=\sec^2(x+y)\cdot(1+y')$，$\therefore y'=-\sec^2(x+y)/\tan^2(x+y)=-1-\cot^2(x+y)$

$y''=2\cot(x+y)\cdot\csc^2(x+y)\cdot(1+y')$

$=-2\cot^3(x+y)\csc^2(x+y)$

3、解(1) $\ln y=x(\ln x-\ln(1+x))$ $\therefore \dfrac{y'}{y}=\ln x-\ln(1+x)+x\left(\dfrac{1}{x}-\dfrac{1}{x+1}\right)$

$\therefore y'=\left(\ln\dfrac{x}{1+x}+\dfrac{1}{1+x}\right)\cdot\left(\dfrac{x}{1+x}\right)^x$

(2) $\ln y=\dfrac{1}{2}\ln(x+3)+4\ln(4-x)-2\ln(x+2)$

$\dfrac{y'}{y}=\dfrac{1}{2(x+3)}-\dfrac{4}{4-x}-\dfrac{2}{x+2}$

$\therefore y'=\dfrac{\sqrt{x+3}\,(4-x)^4}{(x+2)^2}\left(\dfrac{1}{2(x+3)}-\dfrac{4}{4-x}-\dfrac{2}{x+2}\right)$

4、解：$y'(x)=\dfrac{y'(t)}{x'(t)}=\dfrac{2\cos 2t}{-\sin t}$，$y'|_{t=\frac{\pi}{4}}=0$，当$t=\frac{\pi}{4}$，$x=\frac{\sqrt{2}}{2}$，$y=1$

$\therefore$当$t=\frac{\pi}{4}$时，切线方程为：$y-1=0\left(x-\frac{\sqrt{2}}{2}\right)$，即$y=1$

法线方程为：$x=\frac{\sqrt{2}}{2}$

5、解：$y'(x)=\dfrac{y'(t)}{x'(t)}=\dfrac{1}{1+t^2}\Big/\dfrac{t}{1+t^2}=\dfrac{1}{t}$

$y''(x)=\dfrac{d(y')}{dt}\Big/x'(t)=\left(\dfrac{1}{t}\right)'\Big/\dfrac{t}{1+t^2}=-\dfrac{1}{t^2}\Big/\dfrac{t}{1+t^2}=-\dfrac{1+t^2}{t^3}$

6、解：$s(t)=\pi r^2(t)$，$v(t)=r'(t)=6\,m/s$，$r=6t$，$t=2s$

$s'(t)=2\pi r(t)\cdot r'(t)=2\pi\cdot 12\,(m)\cdot 6\,(m/s)=144\pi\,(m^2/s)$

答（略）

§2.5 函数的微分

习题2.5解答

1.解：(1) $dy = -2e^{-2x}\cdot\sin(5-2x)dx - 2e^{-2x}\cos(5-2x)dx$

$= -2e^{-2x}[\sin(5-2x)+\cos(5-2x)]dx$

(2) $dy = -\dfrac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\cdot\dfrac{-2xdx}{2\sqrt{1-x^2}} = \dfrac{x}{|x|}\cdot\dfrac{1}{\sqrt{1-x^2}}dx = \begin{cases}\dfrac{1}{\sqrt{1-x^2}}dx, & 0<x<1\\ -\dfrac{dx}{\sqrt{1-x^2}}, & -1<x<0\end{cases}$

(3) $dy = \dfrac{1}{1+\left(\frac{1-x^2}{1+x^2}\right)^2}\cdot\left(\dfrac{1-x^2}{1+x^2}\right)'dx = -\dfrac{2x}{1+x^4}dx$

2.解：(1) $d(\frac{4}{3}x^3) = 4x^2dx$；　(2) $d(-\cos x) = \sin x dx$；

(3) $d(\frac{1}{\omega}\sin\omega x) = \cos\omega x dx$；(4) $d(\ln(2+x)) = \dfrac{1}{2+x}dx$；

(5) $d(-\frac{1}{3}e^{-3x}) = e^{-3x}dx$；(6) $d(2\sqrt{1+x}) = \dfrac{1}{\sqrt{1+x}}dx$；

(7) $d(-\frac{1}{5}\cot 5x - x) = \cot 5x dx$.

3.解：设 $f(x) = \sqrt[6]{x}$，则 $f'(x) = \frac{1}{6}x^{-\frac{5}{6}}$，$f(x) \approx f(x_0) + f'(x_0)(x-x_0)$

由 $\sqrt[6]{65} = \sqrt[6]{2^6+1}$．取 $x_0 = 64 = 2^6$，$x = 65$，$\Delta x = x - x_0 = 1$

于是 $\sqrt[6]{65} \approx \sqrt[6]{64} + \frac{1}{6}\times(64)^{-\frac{5}{6}}\times 1 = 2 + \dfrac{1}{192} \approx 2.0052$