

Distribution function may not be left-continuous

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Exercise 8. *Construct an example to show that the distribution function of a random variable may not be left-continuous.*

Solution. Let X be a random variable on $([0, 1], \mathcal{B}[0, 1], \text{Leb})$.

$$X(a) := \begin{cases} 0 & \text{if } a > \frac{1}{2} \\ 1 & \text{if } a \leq \frac{1}{2} \end{cases} \quad (1)$$

Then $F_X(1) = P(X \leq 1) = P([0, 1]) = 1$, while $\lim_{a \rightarrow 1^-} F_X(a) = P(X < 1) = P([0, \frac{1}{2})) = \frac{1}{2} \neq F_X(1)$. So F_X is not left-continuous. \square