dual Cantor-Bernstein theorem

李孜睿 518030910424

March 8, 2020

(dual Cantor-Bernstein theorem). Let $f \in Y^X$ and $g \in X^Y$ be two surjectve maps. Assuming AC, show that there is a bijection $h \in Y^X$ such that $h \subseteq f \cup g^{-1}$.

Proof: Given any surjections $f \in Y^X$ and $g \in X^Y$, by AC (we can get a Right inverse of a surjective map) there are injections $u \subseteq f^{-1}$ and $v \subseteq g^{-1}$.

By Cantor-Bernstein Theorem, there exits a bijection $h \in Y^X$ such that $h \subseteq u^{-1} \cup v$.

As $u^{-1} \subseteq f$ and $v \subseteq g^{-1}$, $h \subseteq f \cup g^{-1}$. $\qquad\square$