

Bijection from $[0,1]$ to \mathbb{R}

刘成锴

学号: 518030910425

Solution:

$$f(x) = \tan(x - \frac{1}{2})\pi$$

$f : [0, 1] \rightarrow \mathbb{R}$ is a simple injection.

$$g(x) = \begin{cases} 0 & x = -\infty \\ \frac{\arctan(x)}{\pi} + \frac{1}{2} & -\infty < x < \infty \\ 1 & x = \infty \end{cases}$$

$g : \mathbb{R} \rightarrow [0, 1]$ is a simple injection.

Hence, we find an explicit bijection from $[0, 1]$ to \mathbb{R}