Proof of a Simple Lemma

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This is an easy exercise arranged in class. Nobody did it last week, so I give out a simple proof.

Lemma 1 Suppose that h is a Borel measurable function from R to R. Then

$$h(X) \in \mathcal{L}^1(\Omega, \mathcal{F}, P) \ if \ and \ only \ if \ h \in \mathcal{L}^1(R, \mathcal{B}, \Lambda_X)$$

and then

$$Eh(X) = \Lambda_X(h) = \int_R h(x)\Lambda_X(dx).$$

Proof. Proof is simple cause we can just feed everything into the standard machine. First if $h = I_B(B \in \mathcal{B})$, by the definition of Λ_X we know that

$$Eh(X) = \Lambda_X(h) = \int_R h(x)\Lambda_X(dx).$$

Then use linearity to tell that the conclusion is still true when h is a simple function on $(R, \mathcal{B})$. Next, when h is a non-negative function, by MON we can also get that

$$Eh(X) = \Lambda_X(h) = \int_R h(x)\Lambda_X(dx).$$

Use linearity again and we finish the proof. $\square$