Basic Probability Theory

Week 1

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Exercises

Exercise 3

Solution:

$$f(x) = \tan(x - \frac{1}{2})\pi$$

$f : [0, 1] \to \mathbb{R}$ is a simple injection.

$$g(x) = \begin{cases} 0 & x = -\infty \\ \dfrac{\arctan(x)}{\pi} + \dfrac{1}{2} & -\infty < x < \infty \\ 1 & x = \infty \end{cases}$$

$g : \mathbb{R} \to [0, 1]$ is a simple injection.

Hence, we find an explicit bijection from [0, 1] to $\mathbb{R}$

Exercise 4

Solution: I would like to prove $|\mathbb{R}| = |\mathbb{R}^2|$.

$$|\mathbb{R}| \to |\mathbb{R}^2|$$

For any $a \in \mathbb{R}$, there is $(a, a) \in \mathbb{R}^2$. It's an injection.

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$$|\mathbb{R}^2| \to |\mathbb{R}|$$

利用 Exercise 3 中的 g，我们可以将一个属于 $\mathbb{R}$ 中的数映射到区间 [0, 1] 上。

Then, for any $(a, b) \in \mathbb{R}^2$, 我们可以通过映射，得到 $(g(a), g(b))$。

将 $g(a), g(b)$ 分别表示为：$0.a_1a_2a_3...$ 和 $0.b_1b_2b_3...$。

然后我们再找一个映射，得到一个实数 c, 且 $c \in [0, 1]$。$c = 0.a_1b_1a_2b_2a_3b_3...$。

综合以上映射，we get an injection from $\mathbb{R}^2$ to $\mathbb{R}$。

Hence, there is an bijection from $\mathbb{R}^2$ to $\mathbb{R}$.

That is, $|\mathbb{R}| = |\mathbb{R}^2|$.