20200324 Exercise 4

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March 28, 2020

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Question 1. Let $(y_n)_{n\in\mathbb{N}}$ be a sequence of reals from [0,1] such that $\sum_{n\in\mathbb{N}} y_n = \infty$. Show that $\prod (1-y_n) = 0$.

solution Because $\sum_{n\in N} y_n = \infty$, so $\exists \epsilon > 0 \exists N \ \forall n > N, \ y_n > \epsilon$. Then we consider when n > N, $\prod_{N=1}^{M} (1-y_n) <= (1-\epsilon)^{M-N}$. So $\lim_{M\to\infty} \prod_{N=1}^{M} (1-y_n) <= (1-\epsilon)^{M-N} = 0$, which means $\prod_{N=1}^{M} (1-y_n) = 0$.