

Optional Time Operation

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June 2, 2020

Problem 1 (Exercise 10.3 of Chapter E)

Suppose that S and T are stopping times (relative to $(\Omega, \mathcal{F}, \{\mathcal{F}_n\})$). Prove that $S \wedge T, S \vee T, S + T$ are stopping time.

Proof: Proof it by definition, $\{S \leq n\}, \{T \leq n\}, \{S = n\}, \{T = n\}$ all in \mathcal{F}_n ,

$$\begin{aligned}\{S \wedge T \leq n\} &= \{S \leq n\} \cup \{T \leq n\} \\ \{S \vee T \leq n\} &= \{S \leq n\} \cap \{T \leq n\} \\ \{S + T \leq n\} &= \bigcup_{k=0}^n \{S = k\} \cap \{T = n - k\}\end{aligned}$$

Hence the r.h.s of these equations are \mathcal{F}_n -measurable.

□