Quiz Balls

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Question 1. 罐子里有 *70* 个黑球和 *30* 个白球。每次从中取一个球直到罐子中只含单色球为止。最后罐子中剩的都是白球的概率为多少?

solution My intuition of the answer is $\frac{3}{10}$. We can discuss this question in a more general way. Suppose there are X black balls and Y white balls in the pot initially. First, We have $P = \binom{X+Y}{Y} = \frac{(X+Y)!}{X!Y!}$ different ways to take all the balls out of the pot. Next we consider how many different ways when last k balls are exactly white which means the last $(k+1)^{th}$ ball must be black, suppose the number is w_i, and the sum is W.

$$\begin{aligned} W = \sum_{i=1}^{Y} w_i &= \sum_{i=1}^{Y} \binom{X+Y-1-i}{X-1} \\ &= \sum_{i=1}^{Y} (\binom{X+Y-i}{X} - \binom{X+Y-i-1}{X}) \\ &= \binom{X+Y-1}{X} \\ &= \frac{(X+Y-1)!}{X!(Y-1)!}. \end{aligned}$$

Every way to take out the whole balls has equal probability. So the answer of the question is $\frac{W}{P} = \frac{Y}{X+Y}$.