Discussion on Exercise 3

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The problem i encountered is that what the image for the endpoint 0,1 should be, since [0,1] is a closed interval while $\mathbb{R}$ is an open interval. But we know there is a simple bijection between (0, 1) and $\mathbb{R}$ with the help of *tan*. And from the reading of Was Cantor Surprised, we can take advantage of the injection Cantor constructed between [0,1] and [0, 1).

First the simple bijection $f = tan(\frac{\pi}{2}x - \frac{\pi}{2})(x \in (0,1))$ between (0, 1) and $\mathbb{R}$.
Next construct the bijection between $(0, 1)$ and [0, 1].

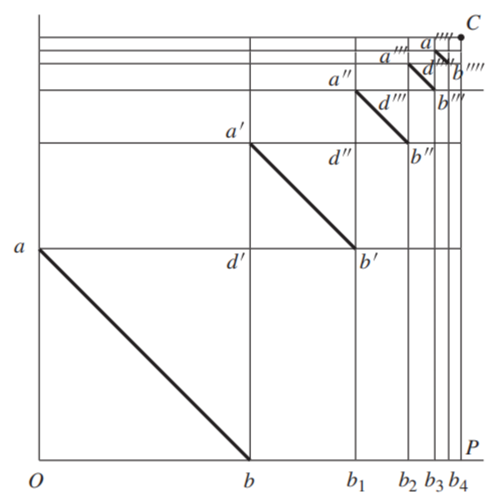

Figure 1. Cantor's function from [0, 1] to (0, 1].

图 1: Cantor's bijection

"The domain has been divided by a geometric progression, so $b = 1/2$,$b_1 = 3/4$, and so on; $a = (0, 1/2)$, $a' = (1/2, 3/4)$, etc. The point C is (1, 1). The points $d' = (1/2, 1/2)$,

$d'' = (3/4, 3/4)$, etc. give the corresponding subdivision of the main diagonal." (I can't explain the curve in english clearly, so i quote the discription in Was Cantor Surprised here). We can slightly change this bijection by ignoring the endpoint C, by doint so, we get the desired bijection g.

Also, in a $'modern'$ way(as Cantor would say), we can construct the bijection like this.

$$g(x) = \begin{cases} \dfrac{1}{2}, & x = 0 \\ \dfrac{1}{n+2}, & (x \in \{\dfrac{1}{n}, n \in \mathbb{N}\} \\ x, & (x \notin \{\dfrac{1}{n}, n \in \mathbb{N}\}) \end{cases} \tag{1}$$

By g, we map $0, 1, \frac{1}{2}$... to $\frac{1}{2}, \frac{1}{3}$..., since we have infinite $\frac{1}{n}$ in (0, 1), this can be done.

To sum up, the bijection between [0, 1] and $\mathbb{R}$ is : $f \circ g$