Any graph can be a minimum dependency graph

YANG Zonghan

April 7, 2020

Definition We call a dependency graph D *minimum*, if no dependency graph D' on the same set of events that have fewer edges in D.

Theorem Any graph can be a minimum dependency graph.

Proof. Let $G = (V, E)$. Assign every edge with a label of $[|E|]$, using bijection $\mathrm{num} : E \mapsto [|E|]$. Consider a set of mutually independent events $\{\mathcal{E}_1, \mathcal{E}_2, \cdots, \mathcal{E}_{|E|}\}$ on some probability space Ω. Construct event for every vertex $v \in V$ that

$$\mathcal{E}_v = \bigcap_{(u,v)\in E} \mathcal{E}_{\mathrm{num}\left((u,v)\right)}$$

The graph is obviously a dependency graph. Also, it's minimum: any graph G' with fewer edges doesn't have at least one edge in G, which is independent in the G' but not in G. □

Acknowledgement Thanks 毛昕渝 for checking the validity.