Slides:

goo.gl/jvxqZB

Repo:

goo.gl/AfkCn6

Convolutional Neural Networks

Machine Learning Workshop Series

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Theory

Motivating example - image classification

Problem: Given a set $\mathcal{S}$ of images with associated class labels:

$$\mathcal{S} = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$$

where

$$x_i \in \mathbb{R}^{W \times H \times C}$$

(W, H, C - width, height and channels), and

$$y_i = 0, 1, \ldots, K$$

find a function f that maps $x_i \to y_i$:

$$f : \mathcal{X} \to \mathcal{Y}$$

A typical classification setup.

Example - Fashion-MNIST

Label	Description	Examples
0	T-Shirt/Top	
1	Trouser	
2	Pullover	
3	Dress	
4	Coat	
5	Sandals	
6	Shirt	
7	Sneaker	
8	Bag	
9	Ankle boots	

First attempt - standard neural network

Flatten each image to 1D vector.

Compute the network predictions:

$$f(\mathbf{X}) = \hat{\mathbf{y}} = \sigma(\mathbf{X}\mathbf{W_1} + \mathbf{b_1})\mathbf{W_2} + \mathbf{b_2}$$

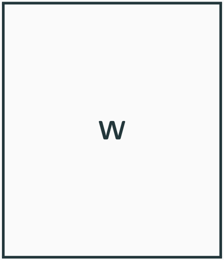
Compute the loss function:

$$\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})$$

Find the gradients of the loss function wrt. weights through backpropagation and update the parameters.

Problem - large number of parameters

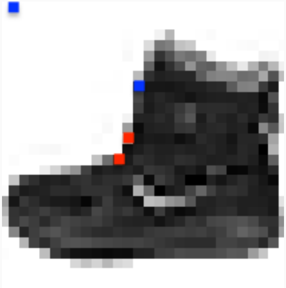
For a 256×256 RGB image (not large for today's standards!), a single neuron would have $256 \times 256 \times 3 = 196,608$ parameters. For a hidden layer of size 64, that's 12,582,912 parameters just for the first layer.



(input size $\times$ number of neurons)

Problem - spatial relationships

Every neuron in layer l_{i-1} is connected to every neuron in layer l_i $\rightarrow$ spatial relationships are ignored.



Problem - feature location

In classification, we don't need to know the exact location of a feature (e.g. a face) - its presence is enough.

Image source

A llama is a llama regardless of its location in the image

Convolutional neural networks

Convolutional neural network is a composition of differentiable functions (layers), just like a standard neural network. Each layer takes an input **volume** (3D array) and transforms it into another volume:

$$f : \mathbb{R}^{W_{in} \times H_{in} \times C_{in}} \to \mathbb{R}^{W_{out} \times H_{out} \times C_{out}}$$

Convolutional neural networks - building blocks

The basic convnet layers are:

- **convolutional layer**
- **pooling layer**
- **nonlinearity** - identical to standard neural nets, e.g. ReLU ($\max(0, x)$)
- **fully-connected layer** - equivalent to a standard neural net layer

Convolutional layer

Instead of multiplying by the input by the weight matrix:

$$f(\mathbf{X}) = \mathbf{XW} + \mathbf{b}$$

convolve the input with a *kernel*.

Kernels

Kernel:

$$\mathbf{W} \in \mathbb{R}^{M \times N \times C_{in}}$$

where M, N - hyperparameters

C_{in} - number of input channels

Forward pass:

$$\begin{aligned} f(\mathbf{X}_{ij}) &= (\mathbf{W} * \mathbf{X}_{ij}) + \mathbf{b} \\ &= \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{c=0}^{C_i} \mathbf{X}_{i-m,j-n,c} \mathbf{W}_{m,n,c} + \mathbf{b} \end{aligned}$$

Produces a 2D *activation map*.

Repeat for every kernel in the layer - each activation map is a new 'channel'.

Convolutional layer - hyperparameters

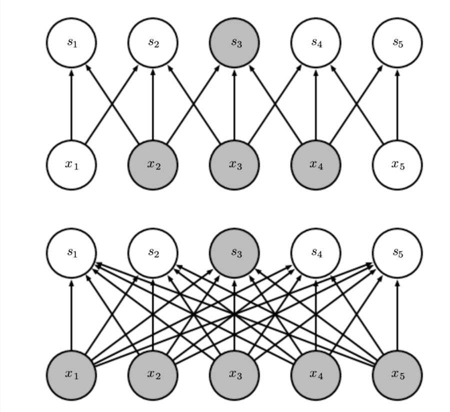
- **the width and height of each kernel** (M, N) - often $M = N$
- **the number of kernels to use** $\rightarrow$ the number of output channels
- **stride and padding** - will be discussed in the Codelab

Why use convolutions?

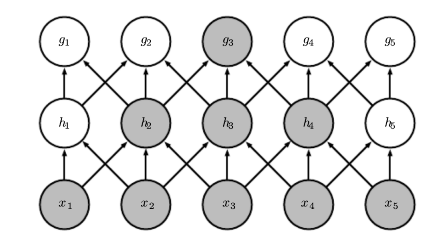
Convolutions adapt the neural network framework specifically for images through:

- **local connectivity**
- **shared parameters (tied weights)**
- **equivariance to translation**

Local connectivity



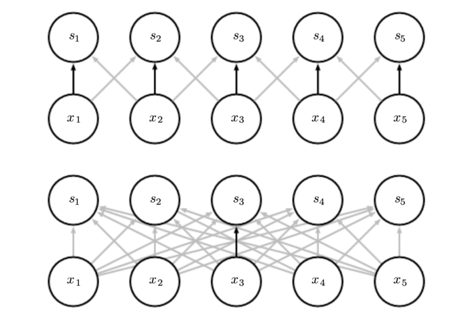
Local connectivity



Shared parameters

The same kernel is used for the whole input volume $\rightarrow$ less parameters.

Assuming $\mathbf{W} \in \mathbb{R}^{3\times3\times3}$ that's $3^3 = 27$ parameters (+ 1 for bias) per neuron.



Translational equivariance

Let $g(\mathbf{X}) = \mathbf{X}'$ be a transformation such that $\mathbf{X}'_{ij} = \mathbf{X}_{(i-1)j}$, that is, shifting (translating) all pixels one unit to the right. Then,

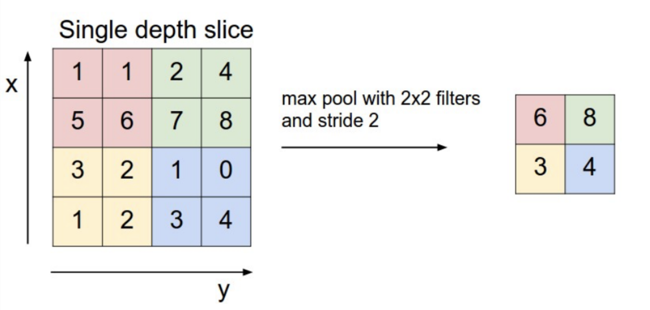
$$
\begin{aligned}
(\mathbf{W} * g(\mathbf{X}))_{ij} = (\mathbf{W} * \mathbf{X}')_{ij} &= \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{c=0}^{C_{in}} \mathbf{X}'_{(i-m)(j-n)c} \mathbf{W}_{mnc} \\
&= \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{c=0}^{C_{in}} \mathbf{X}_{(i-1-m)(j-n)c} \mathbf{W}_{mnc} \\
&= (\mathbf{W} * \mathbf{X})_{(i-1)j} \\
&= g((\mathbf{W} * \mathbf{X})_{ij})
\end{aligned}
$$

The neuron will fire at location corresponding to the translated feature.

Pooling

Keeps only the maximum value from a region.

- reduces spatial resolution
- reduces the number of parameters required
- increases invariance to small changes in input



Fully-connected layers

For image classification - stack a shallow standard (dense) neural network to output the class probabilities. It has the usual affine layers of the form:

$$f(\mathbf{X}) = \mathbf{XW} + \mathbf{b}$$

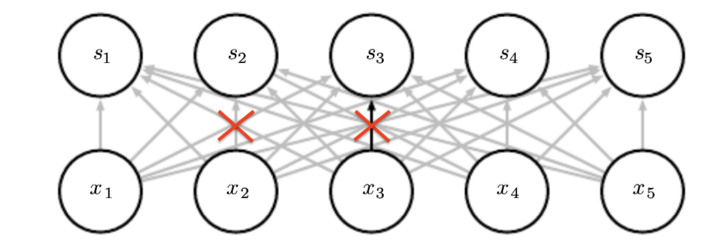
followed by the *softmax activation*:

$$\sigma(\mathbf{X}) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

- it simply normalizes the scores so that they can be interpreted as probabilities.

Dropout

Set every activation to 0 with probability p.



This acts as regularization $\rightarrow$ reduces overfitting, also increases training speed (usually only done for fully-connected layers).

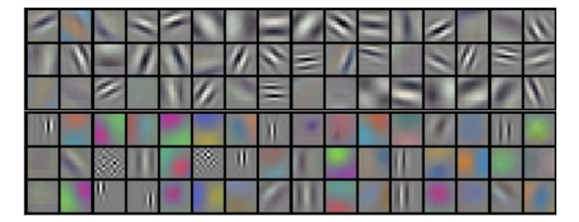
Convolutional neural network - full architecture

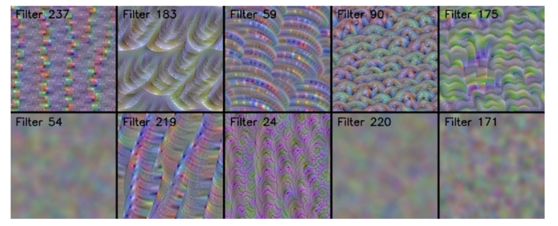
input $\rightarrow$ [**conv** $\rightarrow$ **ReLU** $\rightarrow$ **max pool**] $\times n$ $\rightarrow$ [**FC** $\rightarrow$ **dropout**] $\times m$

Architecture design is tricky - use one of the published architectures, e.g.

- VGG
- Inception v3
- ResNet
- Xception

What does a convnet learn?





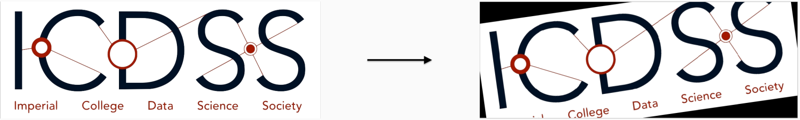
Transfer learning

Features learned by lower layers are very general $\rightarrow$ useful in different problems. Take the learned parameters (conv kernels) from a pre-trained network, 'freeze' them and only train the fully-connected layers.

Data augmentation

A simple way to increase the amount of training data - apply geometrical transforms (rotations, shears, etc.).

Can improve generalization - convnets are invariant to small shifts in the input, but not necessarily other transforms.



Questions?

Codelab

Setup

1. Create a `Github` account.
2. Sign-in `cocalc` using your `Github` credentials.
3. Create a new project in `cocalc`.
4. Clone (green button at top RHS) in `zip` format the **Neural Networks** repository.
5. `Upload` the `zip` file to newly created `cocalc` project.
6. Click on the `zip` file and extract the compressed files.
7. Navigate to the extracted folder `Neural-Networks-master/notebooks/Demo.ipynb`
8. Change the `kernel` by: `Kernel` → `Change Kernel` → `Python 3 (Anaconda)`

References i

cocalc.
Collaborative Calculation in the Cloud, 2018.
Online; accessed 28 Feb 2018; available at
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Github.
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