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Linear Models

Machine Learning Workshop Series

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Theory

Problem Definition

Provided a set $\mathcal{S}$ of (x_i, y_i) pairs:

$$\mathcal{S} = \{(x_1, y_1), (x_2, y_2), ..., (x_k, y_k)\}$$

Find a function f that maps $x_i \to y_i$:

$$f : \mathcal{X} \to \mathcal{Y}$$

f is a:

- **(Discriminate) Model**
- **Map Function**
- **Function Approximation**

for $\mathcal{S}$, such that:

$$f(x_i) = \hat{y}_i \approx y_i, \quad \forall i$$

Linear Model

f is a **Linear Model**, iff:

$$f(x_i) = w * x_i, \quad x_i \in \mathbb{R}^n; y_i \in \mathbb{R}^m; w \in \mathbb{R}^{m \times n} \tag{1}$$

For a finite $\mathcal{S}$ where $|\mathcal{S}| = k$, we express (1) in matrix format as:

$$f(\mathbf{X}) = \hat{\mathbf{y}} = \mathbf{X} * \mathbf{w}, \quad \mathbf{X} \in \mathbb{R}^{k \times n}; \mathbf{y} \in \mathbb{R}^{k \times m}; \mathbf{w} \in \mathbb{R}^{n \times m}$$

Matrices **X** and **y** are constructed by stacking the k pairs as row vectors.

Mini Demo

Metric

Use **Mean Squared Error (MSE)** as the metric function in order to evaluate how good/bad the model f is, such that:

$$MSE_f = \frac{1}{2k}\sum_{i=1}^{k}(y_i - \hat{y}_i)^2 = \frac{1}{2k}\sum_{i=1}^{k}(y_i - w_i * x_i)^2$$

In matrix form:

$$MSE_f = \frac{1}{2k}(\mathbf{y} - \mathbf{X} * \mathbf{w})^T(\mathbf{y} - \mathbf{X} * \mathbf{w}) \tag{2}$$

Target: minimize MSE_f in order to increase accuracy! Apply optimization algorithms to achieve it.

Solutions

Minimize the metric (a.k.a error) function.

Analytic Solution

Maximum Likelihood Estimator: differentiate 2 and find its minimum. There is a global minimiser, since MSE_f is convex (quadratic).

$$\mathbf{w_{MLE}} = (\mathbf{X}^T\mathbf{X})^{-1} * \mathbf{X}^T * \mathbf{y}$$

Computational Solution

Gradient Descent: make incremental updates, towards the correct solution, using the derivative of the error at each step.

$$w_{t+1} = w_t - \eta(y_i - \hat{y})x_i$$

Application

Context

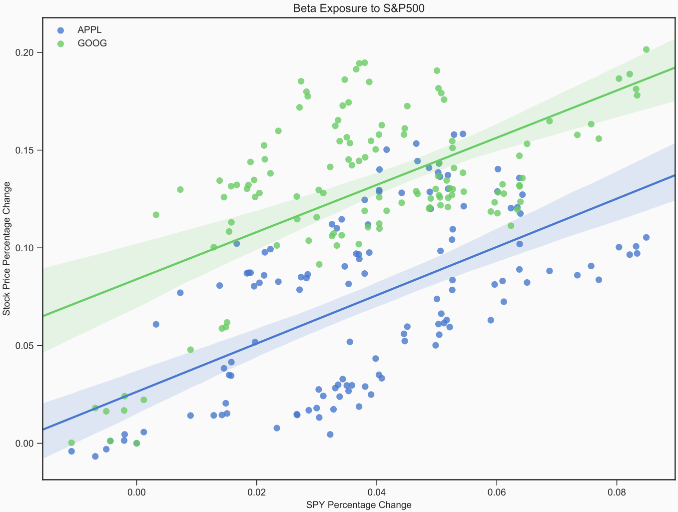
Factor models are a way of explaining the returns of one asset via a linear combination of the returns of other assets. The general form of a factor model is:

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n \tag{3}$$

An asset's β to another is just the β factor from (3).
Look for:

$$y_{stock} = \beta x_{benchmark} + \alpha = \begin{bmatrix} x_{benchmark} & 1 \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

Results


Beta Exposure to S&P500
APPL
GOOG
Stock Price Percentage Change
SPY Percentage Change
0.20
0.15
0.10
0.05
0.00
0.00
0.02
0.04
0.06
0.08

Questions?

Codelab

Setup

1. Create a `Github` account.
2. Sign-in `cocalc` using your `Github` credentials.
3. Create a new project in `cocalc`.
4. Clone (green button at top RHS) in `zip` format the **Linear Models** repository.
5. `Upload` the `zip` file to newly created `cocalc` project.
6. Click on the `zip` file and extract the compressed files.
7. Navigate to the extracted folder `~/Linear-Models-master/notebooks/Demo.ipynb`.
8. Select from the menu bar: `Kernel` → `Change Kernel` → `Python 3 (Anaconda)`.

Challenges

Beta Hedging

Come up with a Beta Hedging strategy using Linear Models

Generic Machine Learning API for Linear Models

Create a generic API, encapsulating all the different implementations and exposing simple methods:

- `fit`
- `predict`

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