

# **MATH50003**

# **Numerical Analysis**

## **I.4 Newton's Method**

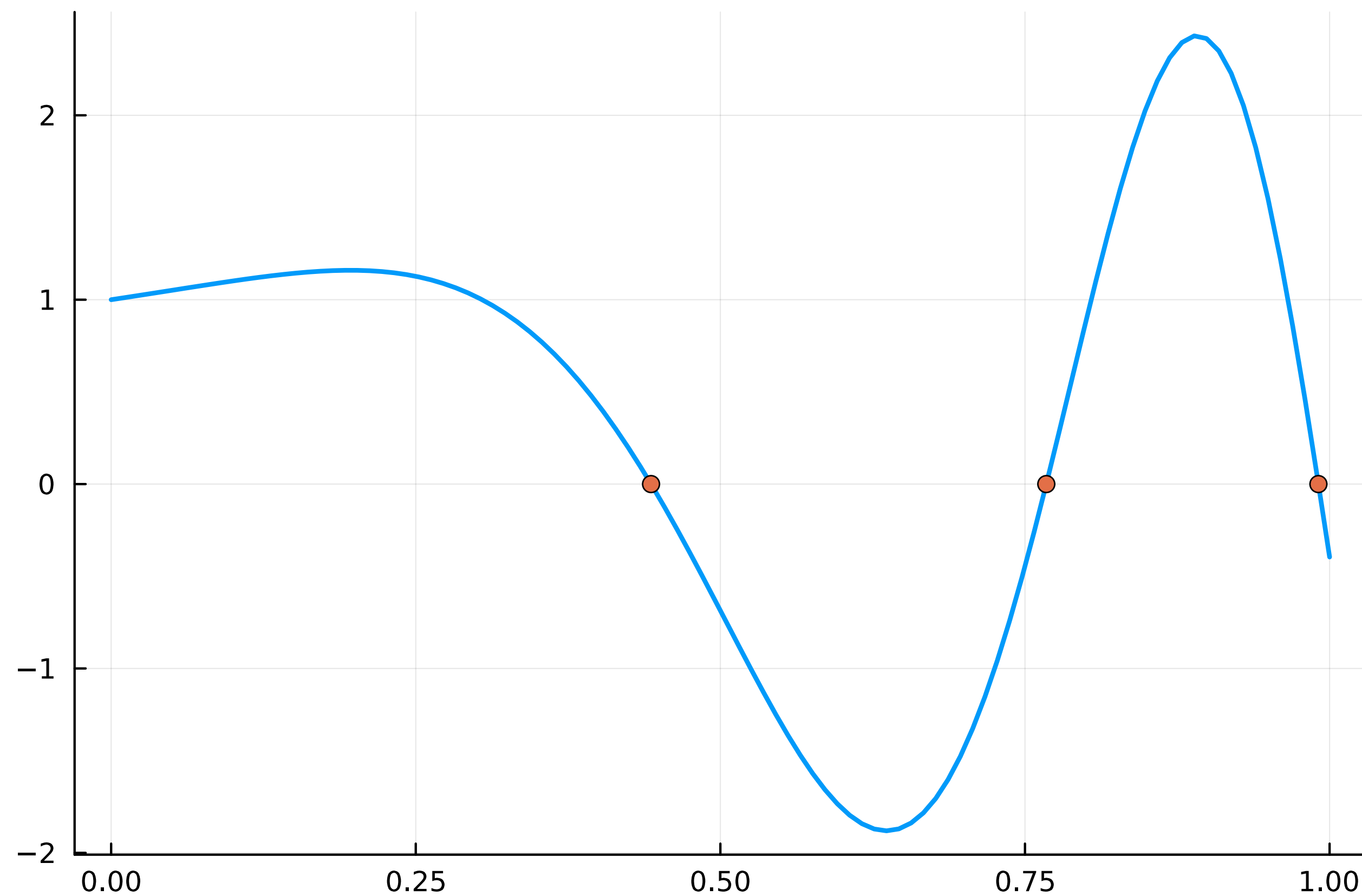
**Dr Sheehan Olver**

# Part I

## Calculus on a Computer

1. Rectangular rules for integration
2. Divided differences for differentiation
3. Dual numbers for differentiation
4. Newton's method for root finding

Given a function, how can we find  
*a single* root/zero?



# Newton's method

## Find roots of affine functions

Given initial guess  $x_0$ :

$$f(x) \approx f(x_0) + (x - x_0)f'(x)$$

Root of right-hand side:

$$f(x_0) + (x - x_0)f'(x_0) = 0 \quad \Leftrightarrow \quad x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

**Theorem 3** (Newton error). *Suppose  $f$  is twice-differentiable in a neighbourhood  $B$  of  $r$  such that  $f(r) = 0$ , and  $f'$  does not vanish in  $B$ . Denote the error of the  $k$ -th Newton iteration as  $\varepsilon_k := r - x_k$ . If  $x_k \in B$  then*

$$|\varepsilon_{k+1}| \leq M |\varepsilon_k|^2$$

where

$$M := \frac{1}{2} \sup_{x \in B} |f''(x)| \sup_{x \in B} \left| \frac{1}{f'(x)} \right|.$$





**Corollary 1** (Newton convergence). *If  $x_0 \in B$  is sufficiently close to  $r$  then  $x_k \rightarrow r$ .*





**Let's see how it works in practice**