

# **MATH50003**

# **Numerical Analysis**

## **I.2 Divided Differences**

**Office Hour: Thursdays, 4pm, Huxley 6M40**

**Please come if you need help setting up Julia.  
I can walk through the process.**

# Part I

## Calculus on a Computer

1. Rectangular rules for integration
2. Divided differences for differentiation
3. Dual numbers for differentiation
4. Newton's method for root finding

# (Right-sided) divided differences

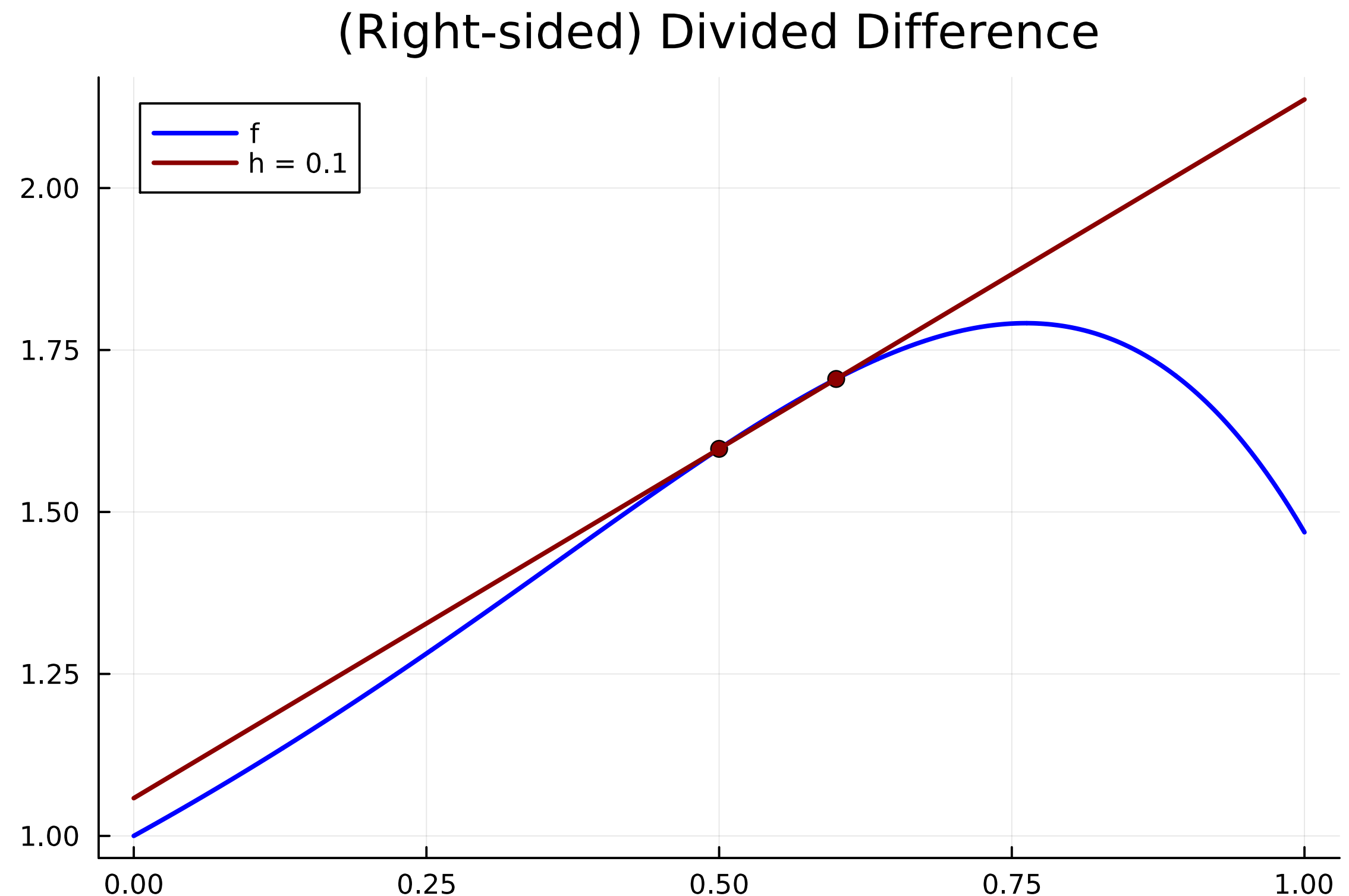
## Approximating derivatives from function values

- Start with the definition of a derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- Idea: make  $h$  small and use

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$



Divided differences is slope of line and approximates derivative

# (Right-sided) divided differences

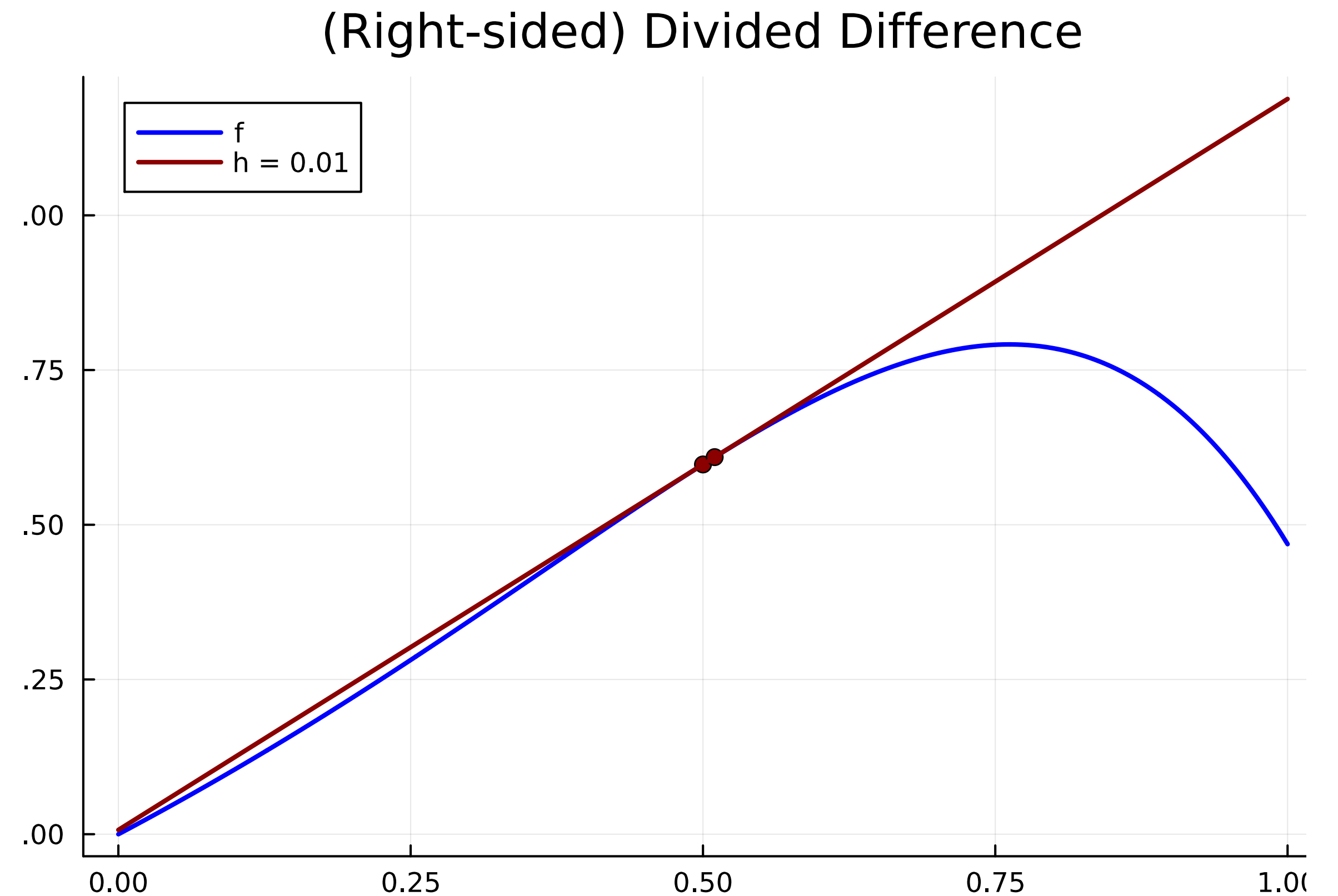
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Making  $h$  smaller gives more accurate results (or does it?)

# (Right-sided) divided differences

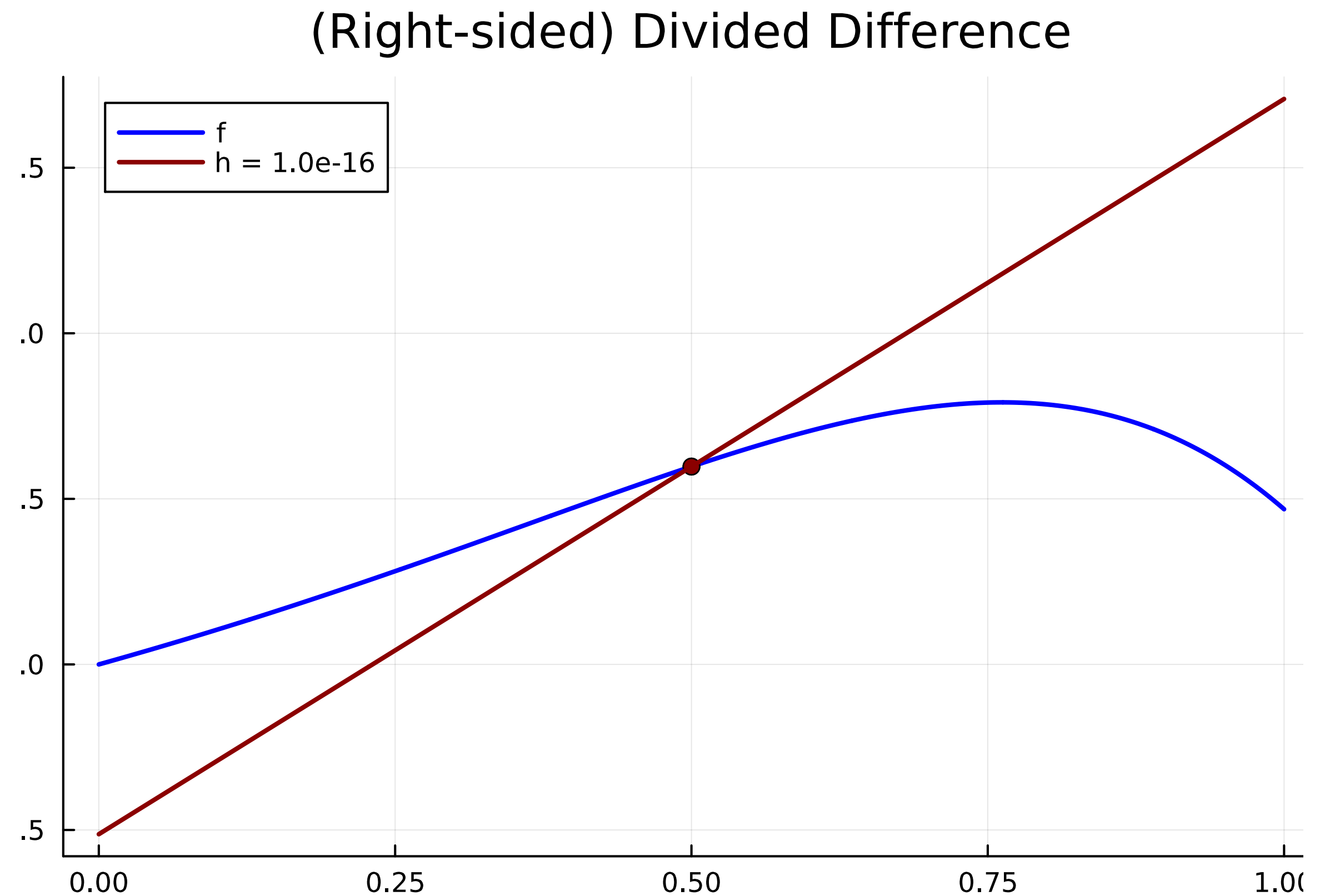
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Something has gone terribly wrong....

**Proposition 1** (divided differences error). *Suppose that  $f$  is twice differentiable on the interval  $(x, x + h)$ . The error in approximating the derivative using divided differences is*

$$f'(x) = \frac{f(x + h) - f(x)}{h} + \delta$$

where  $|\delta| \leq Mh/2$  for  $M = \sup_{x \leq t \leq x+h} |f''(t)|$ .



# Other approximations to derivatives

Explored in Problem Sheets/Lab

Central differences

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Second-order divided differences

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$



# Now for implementation

Can we get an idea of what goes wrong with  $h$  very small?

