MATH50003 Numerical Analysis

I.2 Divided Differences

Office Hour: Thursdays, 4pm, Huxley 6M40

Please come if you need help setting up Julia.
I can walk through the process.

Dr Sheehan Olver

Part I
Calculus on a Computer

1. Rectangular rules for integration

2. Divided differences for differentiation

3. Dual numbers for differentiation

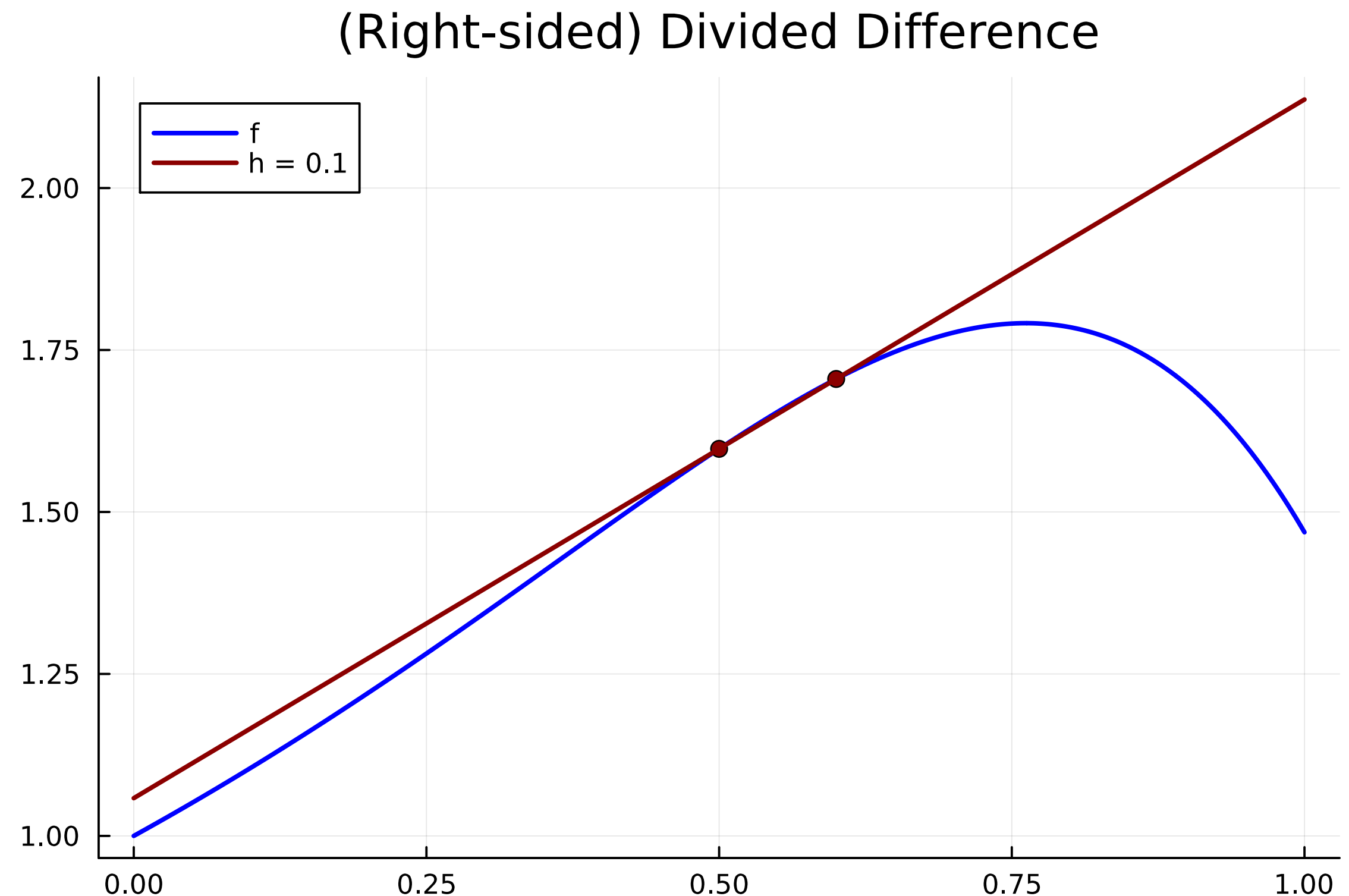# (Right-sided) divided differences
Approximating derivatives from function values

- Start with the definition of a derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Idea: make h small and use

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

(Right-sided) Divided Difference

f
h = 0.1

Divided differences is slope of line and approximates derivative

Proposition 1 (divided differences error). *Suppose that f is twice differrentable on the interval $(x, x + h)$. The error in approximating the derivative using divided differences is*

$$f'(x) = \frac{f(x+h) - f(x)}{h} + \delta$$

where $|\delta| \leq Mh/2$ for $M = \sup_{x \leq t \leq x+h} |f''(t)|$.

Other approximations to derivatives
Explored in Problem Sheets/Lab

Central differences

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Second-order divided differences

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Now for implementation
Can we get an idea of what goes wrong with h very small?

(Right-sided) Divided Difference