MATH50003
Numerical Analysis

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https://github.com/Imperial-MATH50003/MATH50003NumericalAnalysis

Office Hour: Monday 11am, Huxley 6M40

What is Numerical Analysis?

Algorithms for continuous problems

Implementation in software

Analysis of convergence and stability

Assessment

Computer-based

- Labs
- Practice Computer-based Exam
- Computer-based Exam

Pen-and-Paper

- Problem sheets
- Final Exam

Julia is a programming language designed by MIT for Scientific Computing and Machine Learning

Compiled: generates efficient high performance code and allows us to see what the computer is actually doing

Easy to add custom types to understand mathematical concepts

COURSE CONTENT

I. Calculus on a Computer
   - Integration, differentiation, root finding
II. Representing Numbers
   - Modular arithmetic, floating point numbers, bounding errors
III. Numerical Linear Algebra
   - Data regression, differential equations, least squares
IV. Approximation Theory
   - Fourier series, orthogonal polynomials, Gaussian quadrature

Part I
Calculus on a Computer

1. Rectangular rules for integration

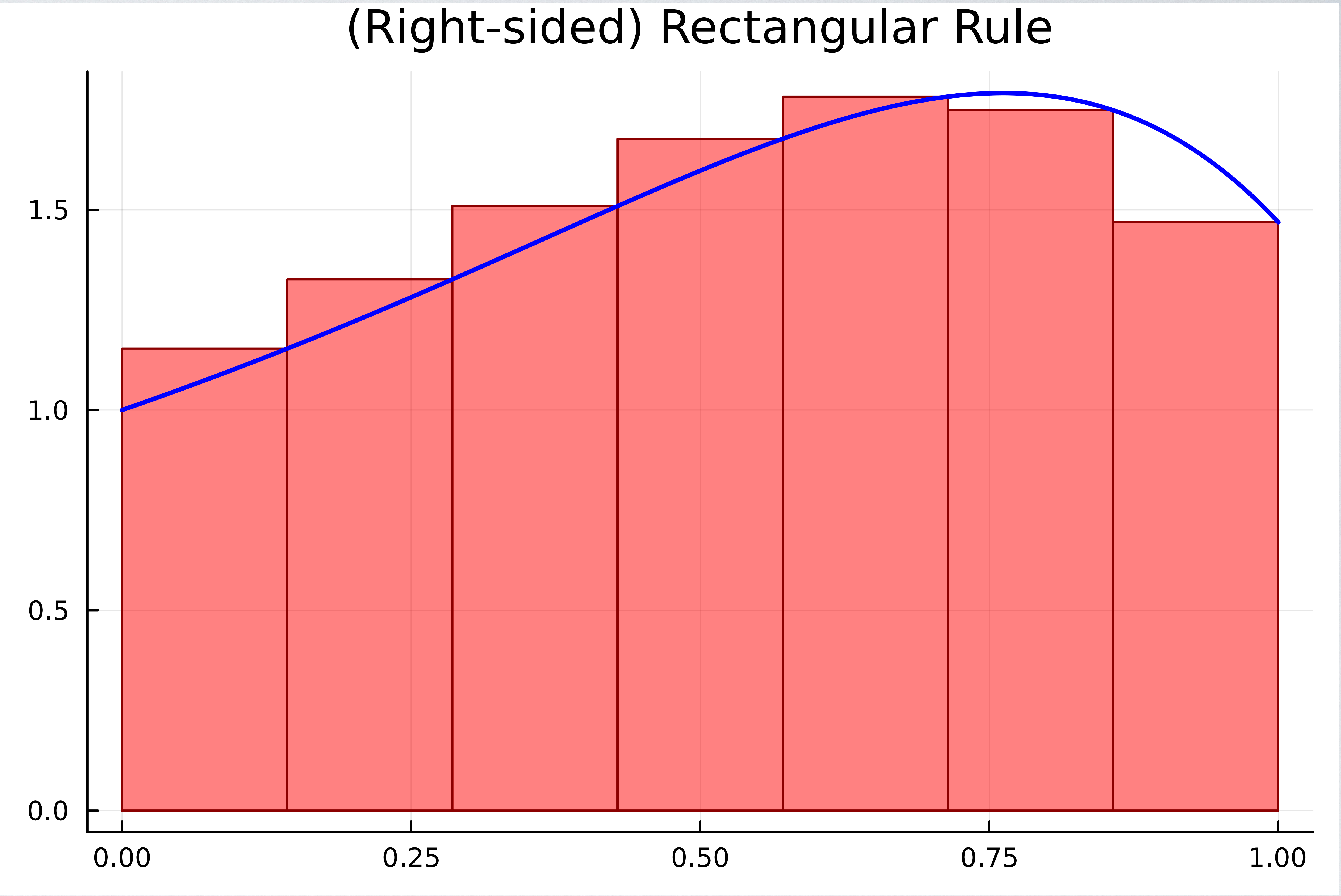
2. Divided differences for differentiation

3. Dual numbers for differentiation

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty} h \sum_{j=1}^{n} f(x_j)$$

where

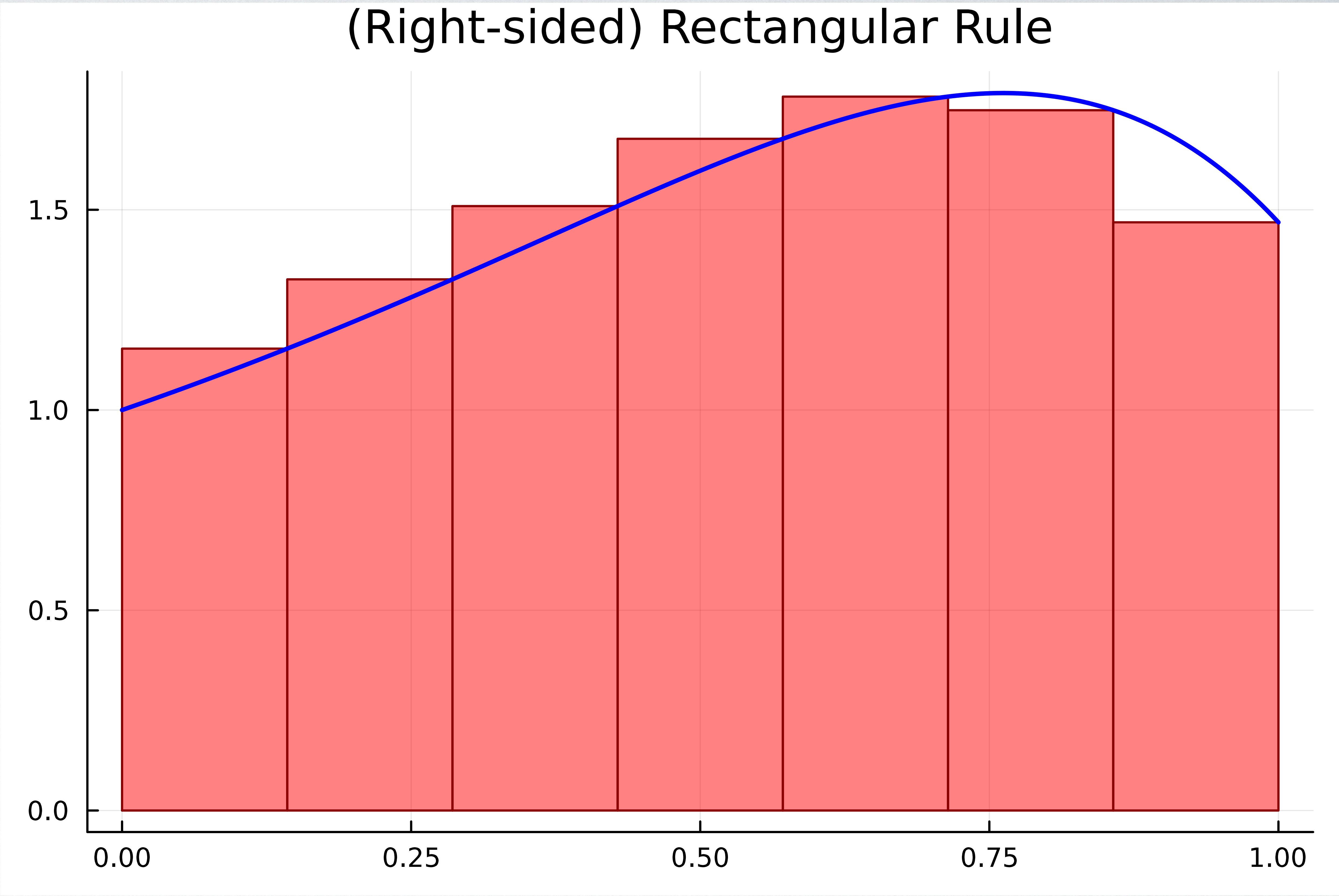
$$h = \frac{b-a}{n}$$



$$\int_a^b f(x)\mathrm{d}x \approx h \sum_{j=1}^{n} f(x_j)$$

where

$$h = \frac{b-a}{n}$$



Lemma 1 ((Right-sided) Rectangular Rule error on one panel). *Assuming f is differentiable we have*

$$\int_a^b f(x)\mathrm{d}x = (b-a)f(b) + \delta$$

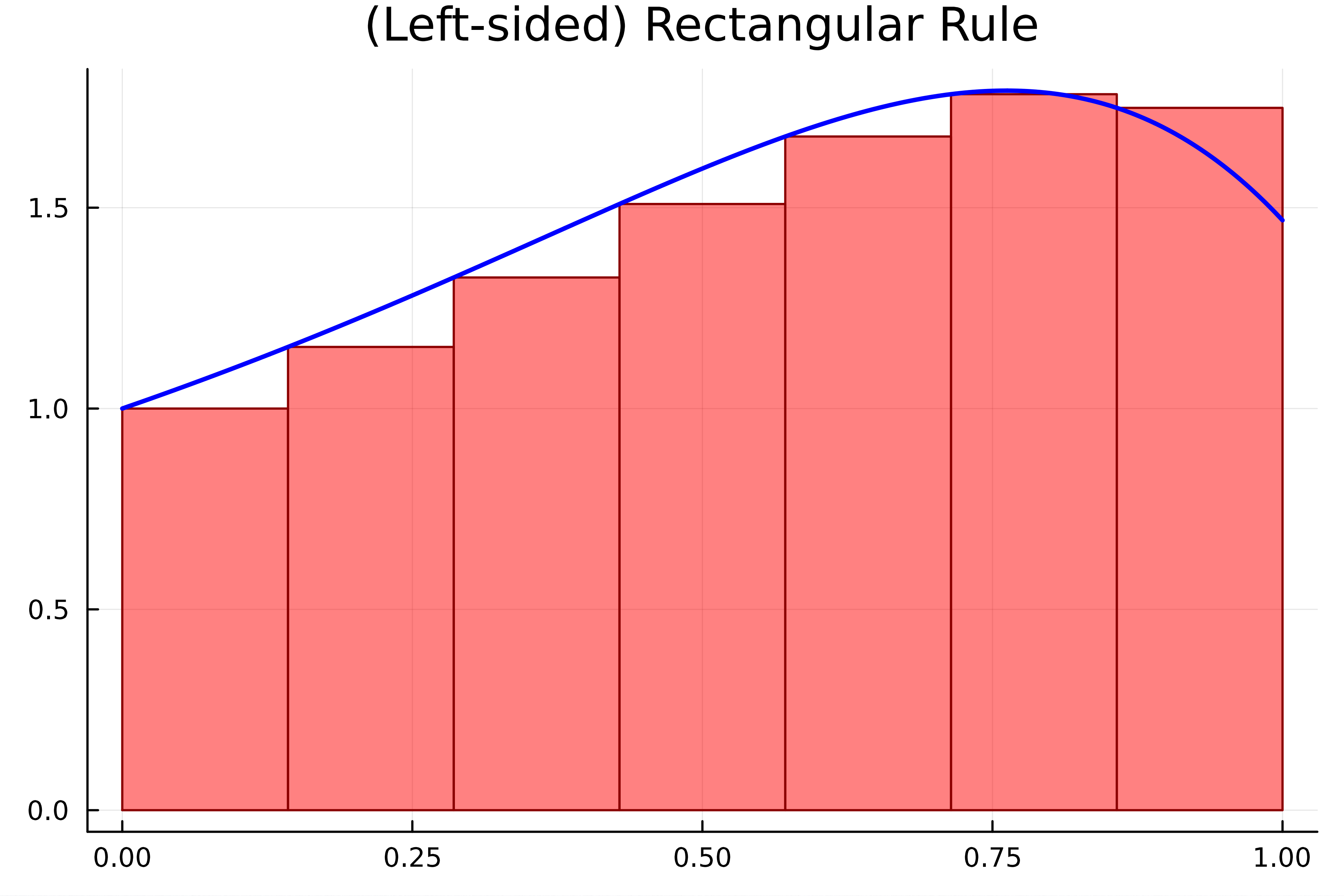
where $|\delta| \leq M(b-a)^2$ *for* $M = \sup_{a\leq x\leq b}|f'(x)|$.

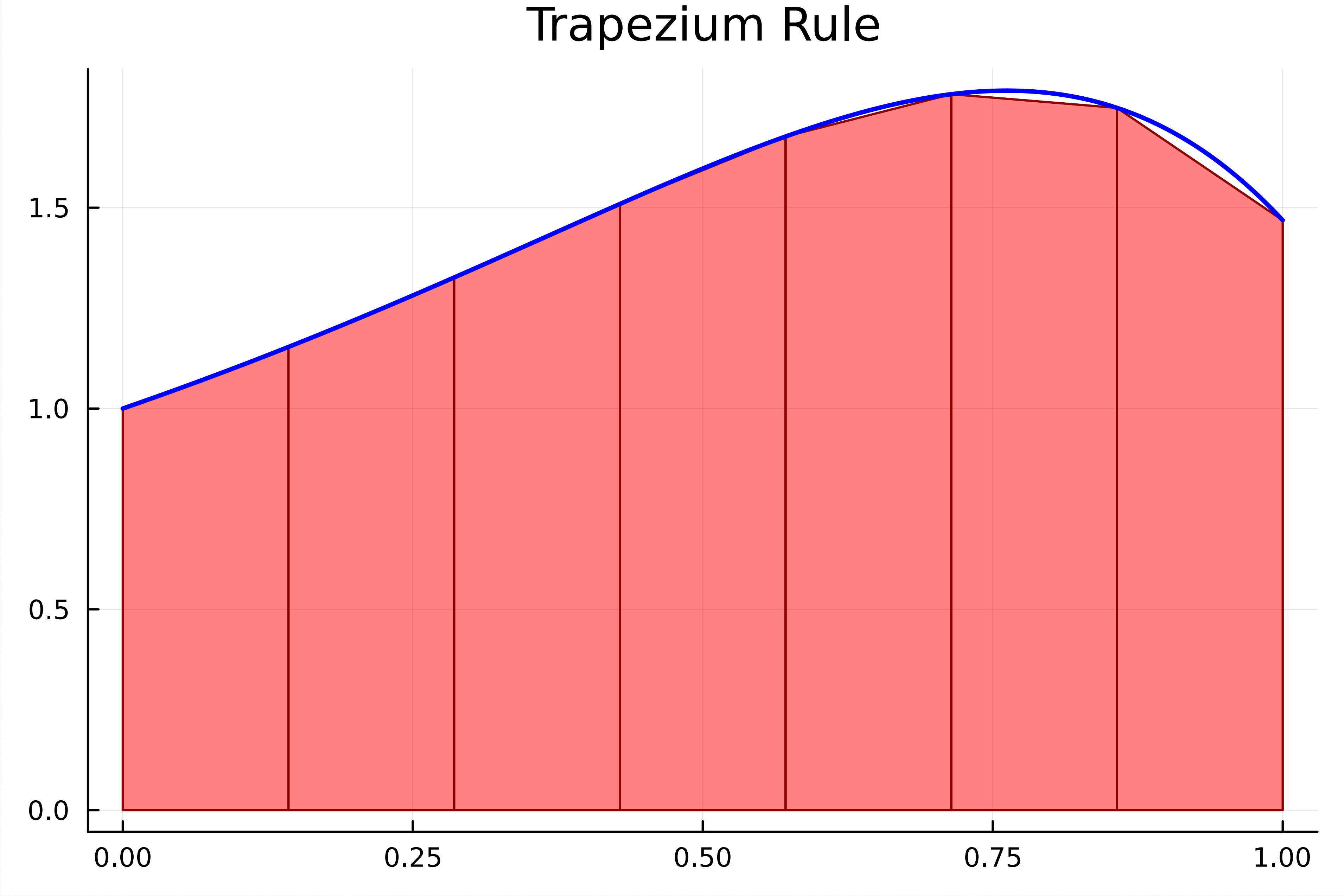
Theorem 1 (Rectangular Rule error). *Assuming f is differentiable we have*

$$\int_a^b f(x)\mathrm{d}x = h\sum_{k=1}^{n} f(x_k) + \delta$$

where $|\delta| \leq M(b-a)h$ *for* $M = \sup_{a\leq x\leq b}|f'(x)|$, $h = (b-a)/n$ *and* $x_k = a + kh$.

Other Approximations





$$h \sum_{j=0}^{n-1} f(x_j)$$

$$h \left[\frac{f(x_0)}{2} + \sum_{j=1}^{n-1} f(x_j) + \frac{f(x_n)}{2} \right]$$

How to do it in practice?