Financial Engineering Lab (MA374)

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Assignment - 1
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- ## <u>Question **1**</u>

$S_o = 100$, K (strike price) = 105, T = 5, r = 0.05, σ = 0.3

The initial Option prices for M = 1,5,10,20,50,100,200,400 are as following :

M	Call Price	Put Price
1	38.168	19.942
5	34.907	16.681
10	33.625	15.399
20	33.859	15.634
50	33.981	15.755
100	34.011	15.785
200	34.02	15.794

The above prices are calculated using the backtracking approach where the option price at any step is Expected discounted value of payoff with respect to risk-neutral probability measure.

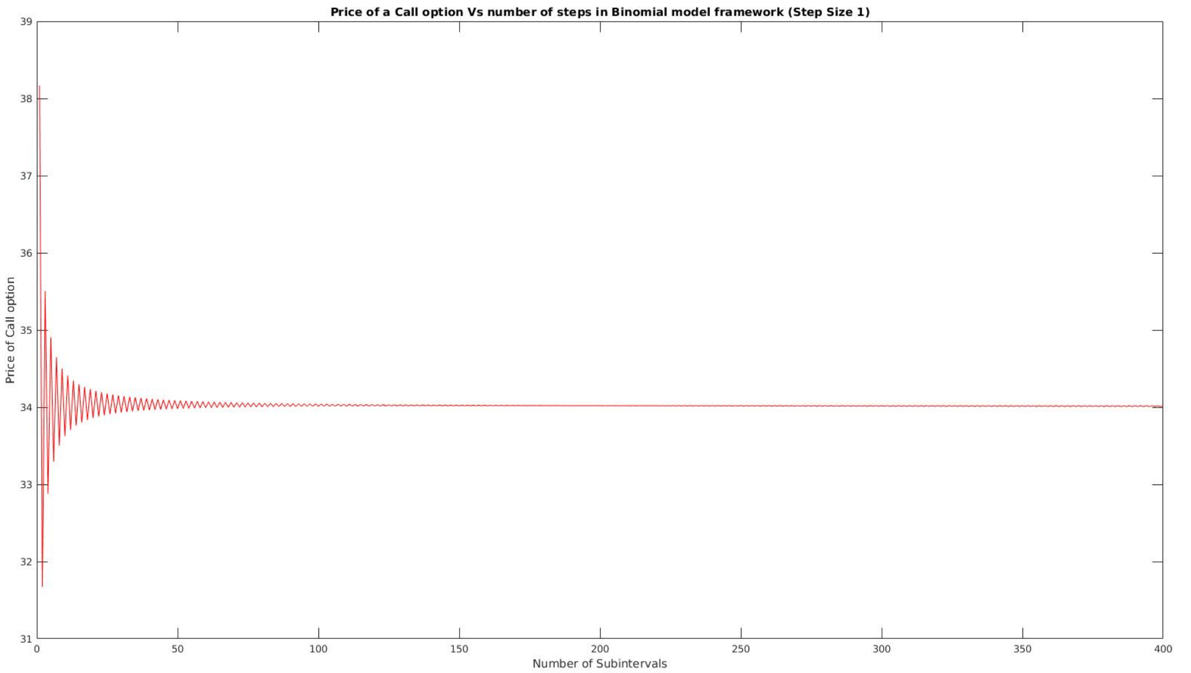
$$c = e(-rt) * (q * Pup + (1-q)Pdn)$$

q (risk-neutral probability) = $(e^{(rt)} - d)/(u - d)$
Pup = Payoff if stock goes up
Pdn = Payoff if stock goes down

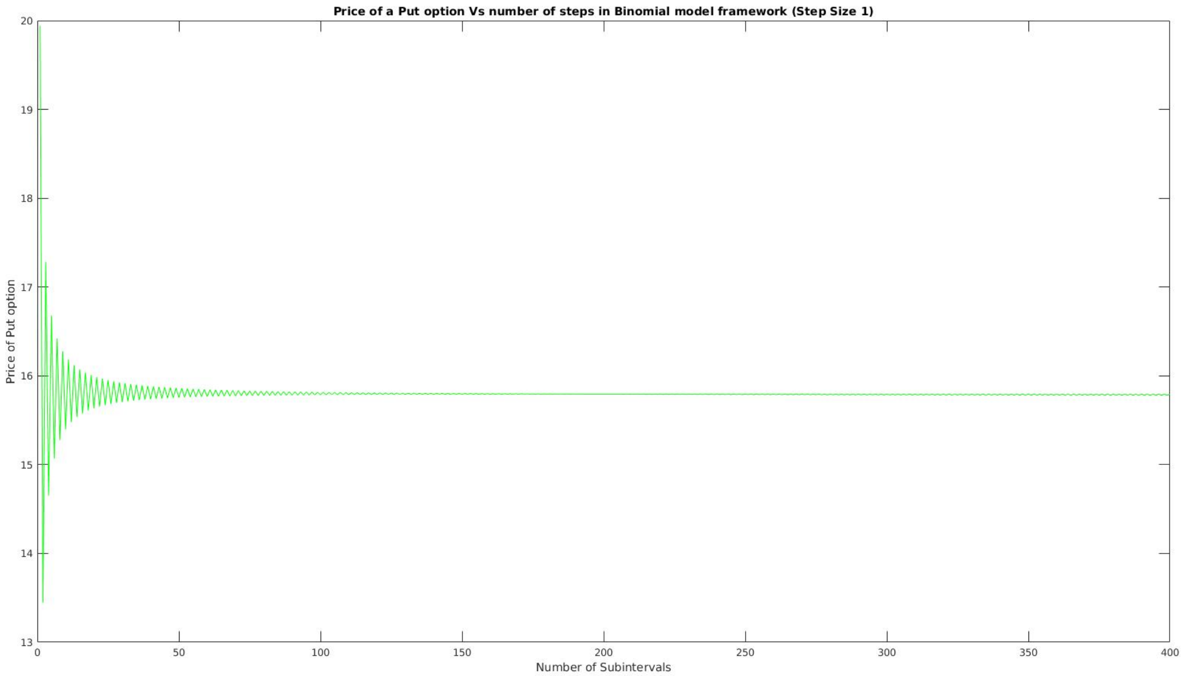
Call Option Payoff = $\max(S_n - K,0)$
Put Option Payoff = $\max(K-S_n,0)$

- # <u>Question **2**</u>

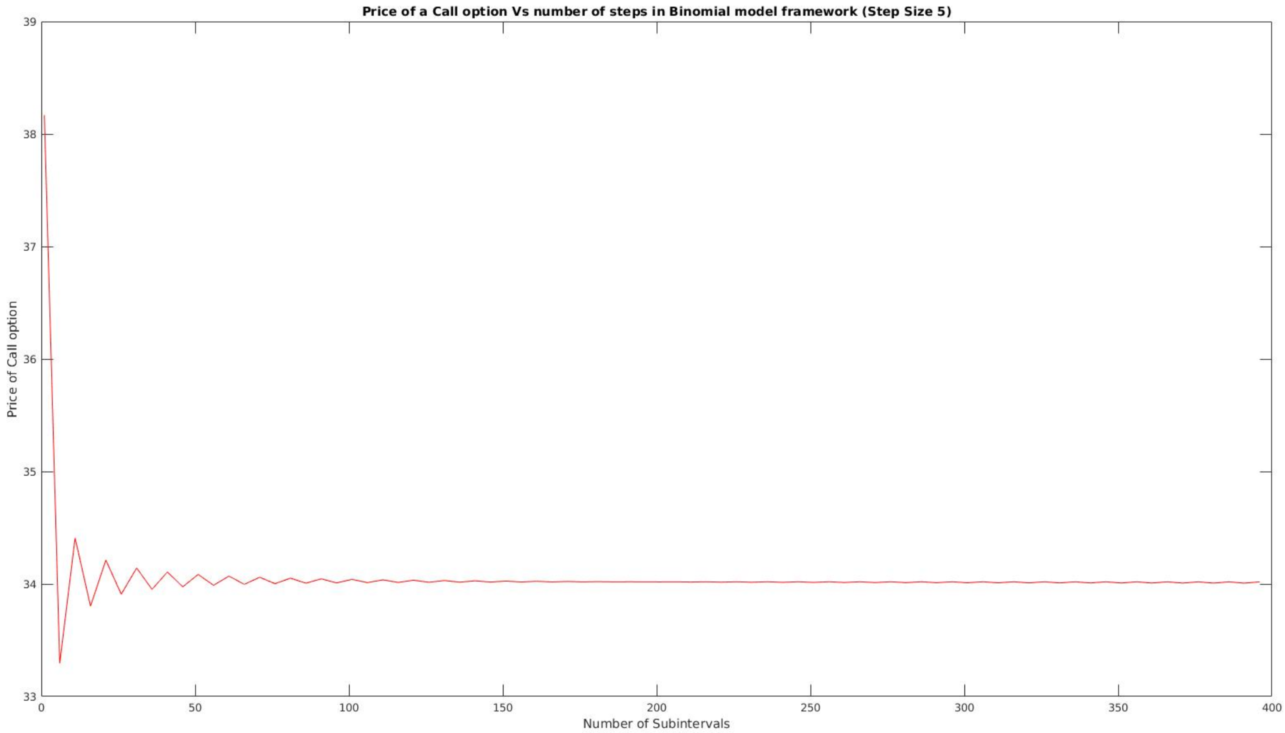
Following are the graphs of the initial option prices Vs number of subintervals :



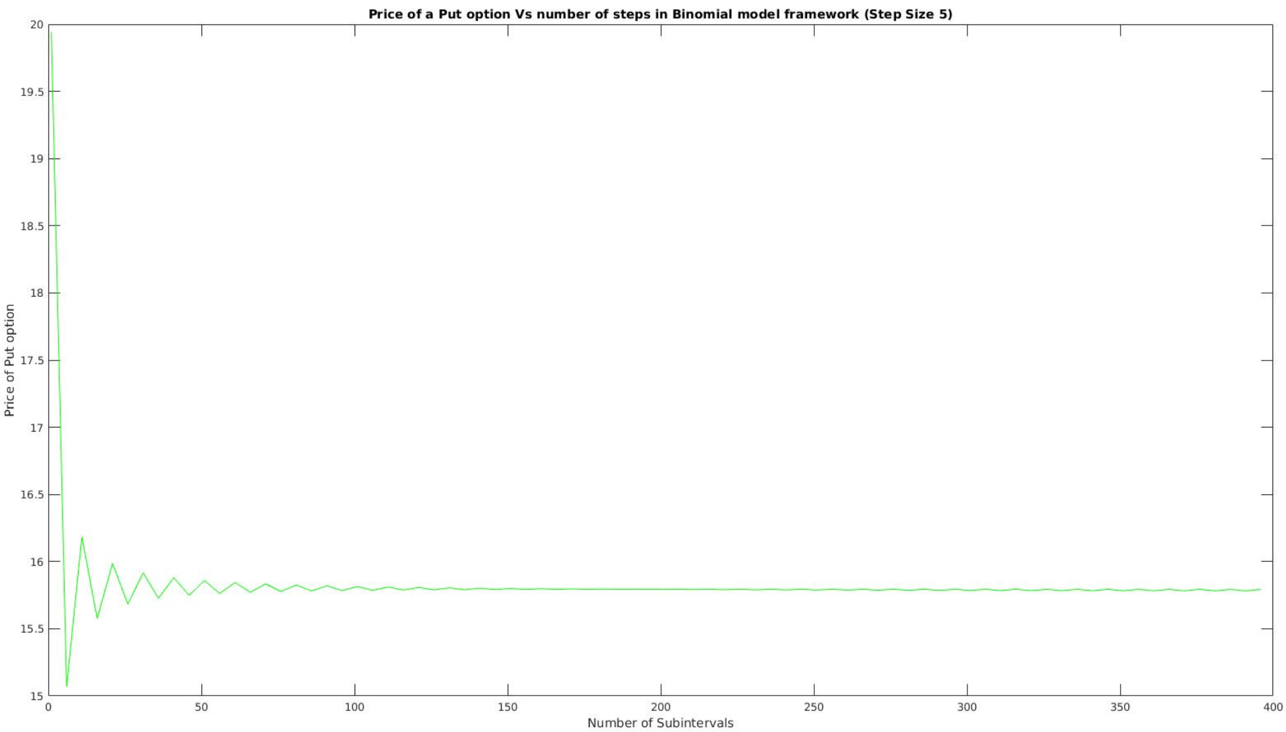
Price of Call Option Vs number of subintervals varied with step size 1



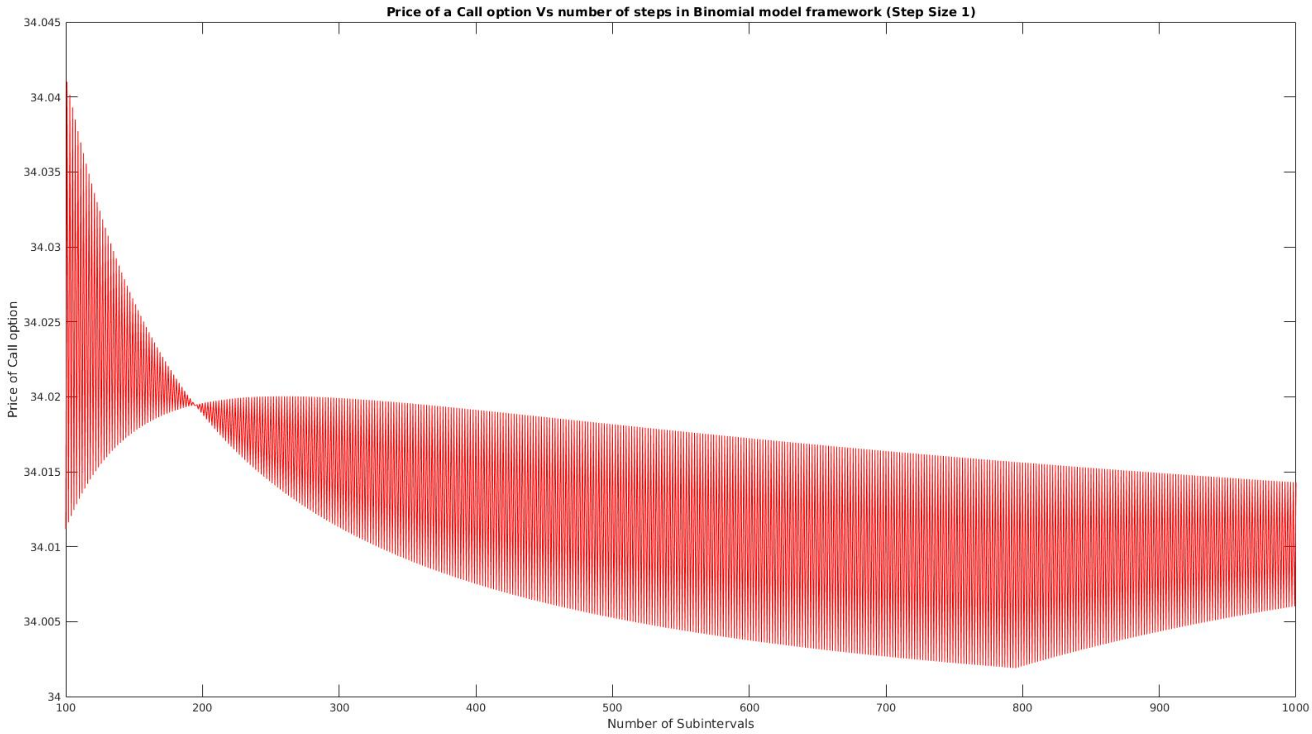
Price of Put Option Vs number of subintervals varied with step size 1



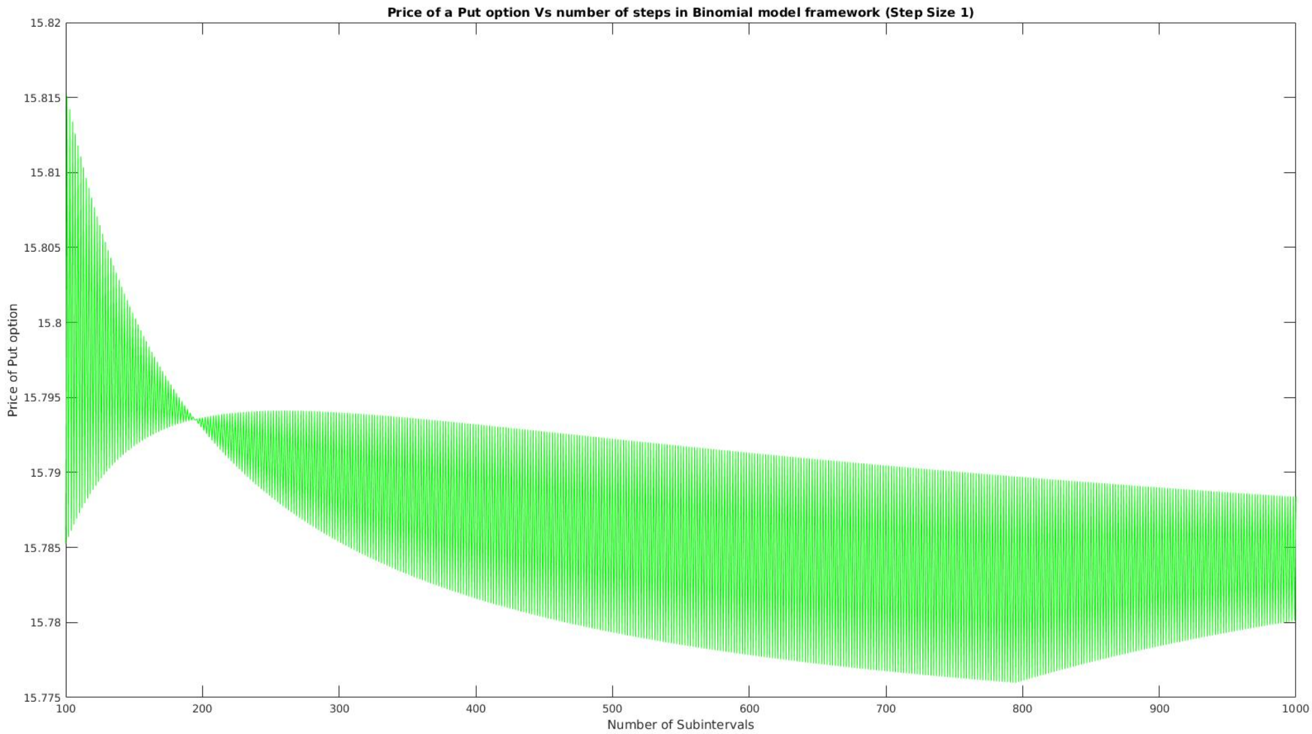
Price of Call Option Vs number of subintervals varied with step size 5



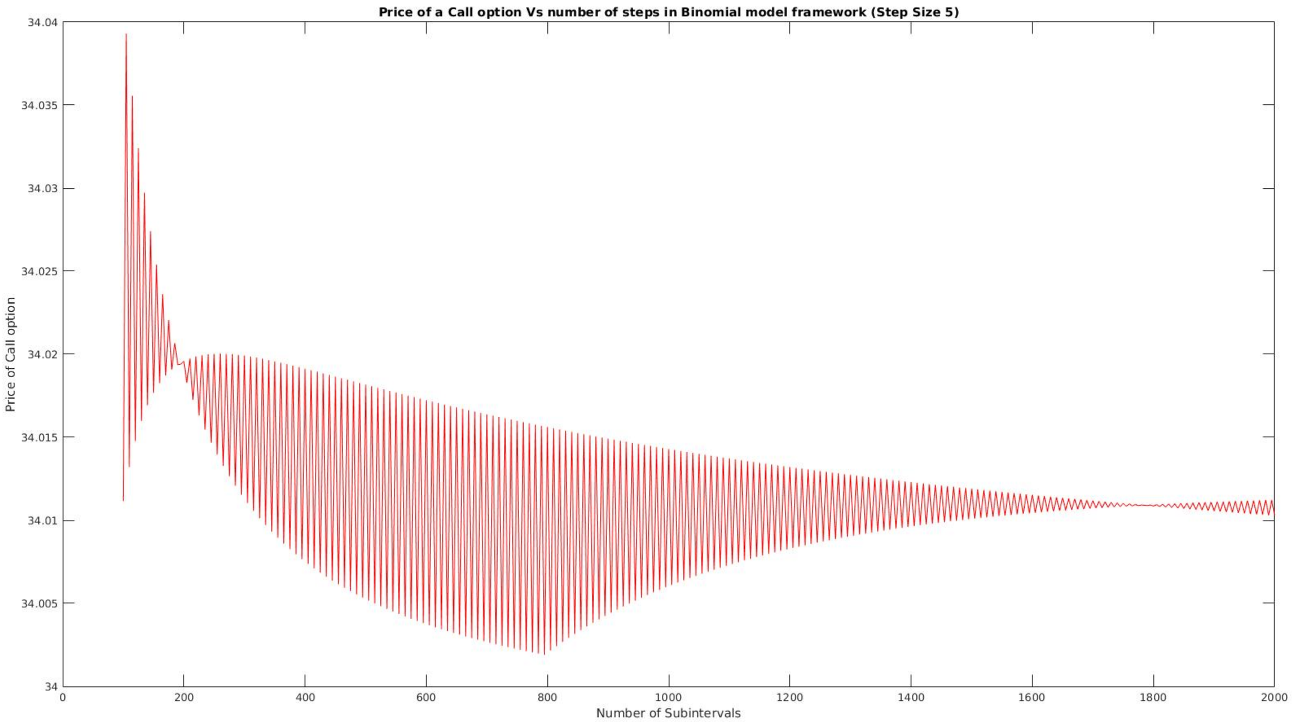
Price of Put Option Vs number of subintervals varied with step size 5



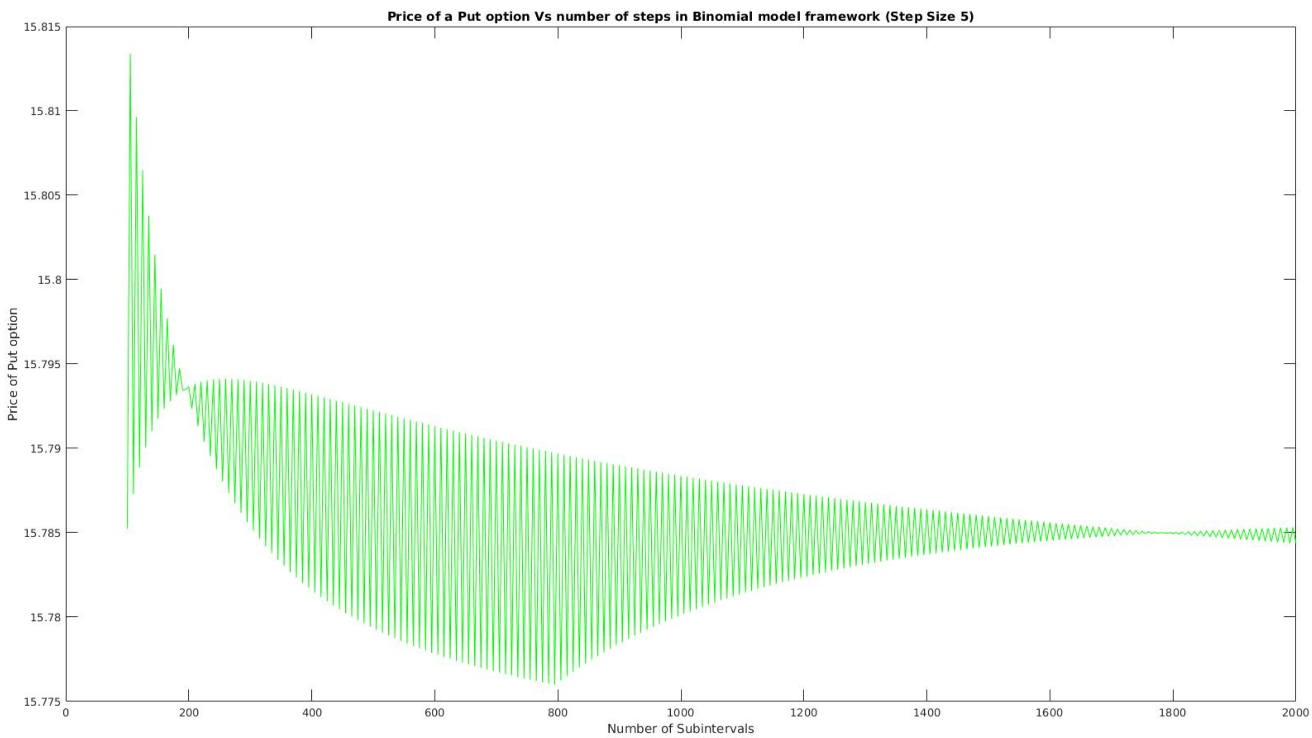
Price of Call Option Vs number of subintervals varied with step size 1



Price of Put Option Vs number of subintervals varied with step size 1



Price of Call Option Vs number of subintervals varied with step size 5



Price of Put Option Vs number of subintervals varied with step size 5

<u>Observations from the plots</u> -

1. We observe that value of European call option converges to 34.012 and European put option converges to 15.785.

2. We also observe that this convergesnce is not perfect. Some deviations occur from the converging values. For eg. , the call option price for M = 1 is 38.168 but it does not uniformly decreaces to 34.012 as M increaces. It fluctuates down to 34.003 for M = 800 and later increces to the converging value.

3. **However the deviations decreace as the value of M (number of subintervals) increces as can be seen in plots for higher values of M (~2000).**

◆ <u>Question **3**</u>

The price of the European call option at t = 0, 0.50, 1, 1.50, 3, 4.5 for M = 20 are as following :

Time (t)	0	0.5	1	1.5	3	4.5
	33.859	59.959	100.66	160.61	519.1	1419.4
		31.893	57.7	98.439	359.93	1025
		15.096	29.804	55.295	242.03	732.79
			13.47	27.573	154.84	516.32
			5.1548	11.767	91.193	355.96
				4.1214	46.976	237.16
				1.125	19.725	149.15
					6.1485	83.951
					1.236	36.251
					0.11833	8.1492
					0	0
					0	0
					0	0
						0
						0
						0
						0
						0
						0

The price of the European put option at t = 0, 0.50, 1, 1.50, 3, 4.5 for M = 20 are as following :

Time (t)	0	0.5	1	1.5	3	4.5
	15.634	8.4792	3.5042	0.94243	0	0
		15.487	8.0042	2.9982	0	0
		24.673	15.269	7.4363	0.0087053	0
			24.983	14.963	0.1721	0
			35.965	25.271	1.2357	0
				36.97	4.9582	0
				48.305	13.222	0
					25.955	0
					40.533	0.60155
					53.855	8.2812
					64.433	26.64
					72.358	46.278
					78.228	60.825
						71.603
						79.587
						85.502
						89.883
						93.129
						95.534

Note : The code checks for the **arbitrage** possibilities using the following conditions necessary for the market to be arbitrage free -

- $d < e^{(rt)}$
- $e^{(rt)} < u$