Financial Engineering Lab (MA374)

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Assignment - 3
(24 January 2019)

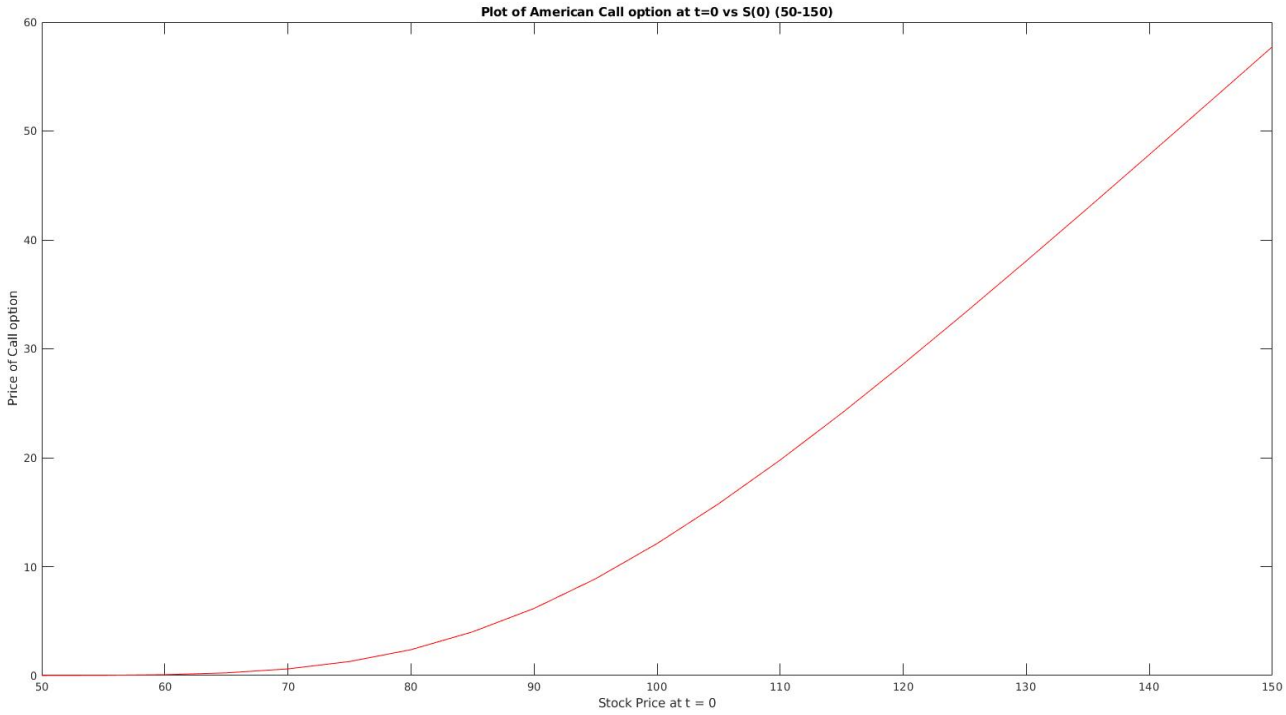
Question 1

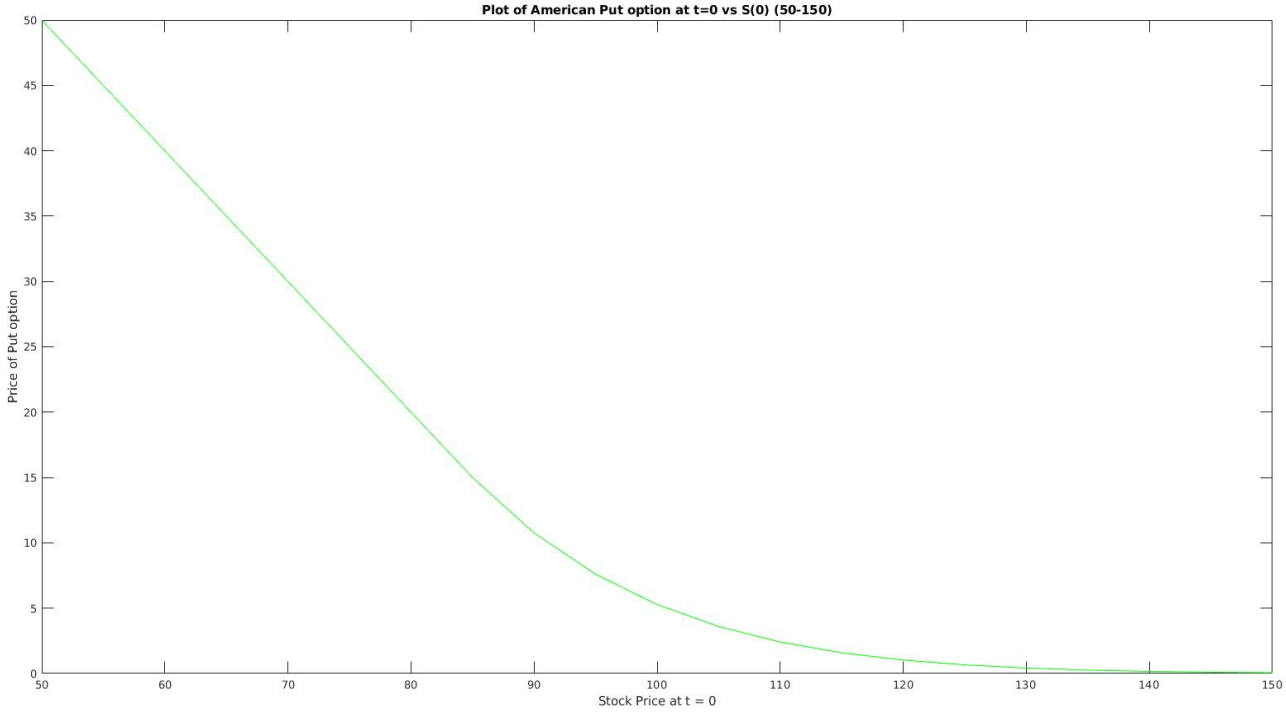
The Option Prices for the data -

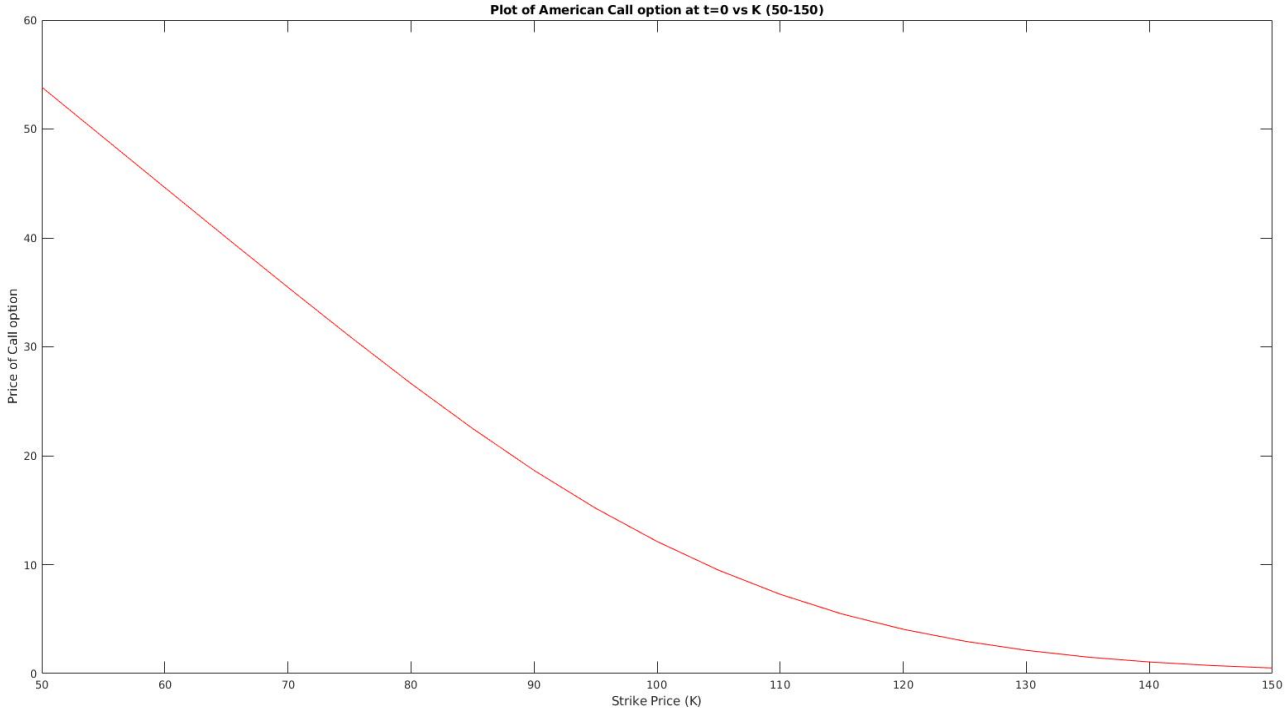
$$S(0) = 100;\ K = 100;\ T = 1;\ M = 100;\ r = 8\%;\ \sigma = 20\%$$

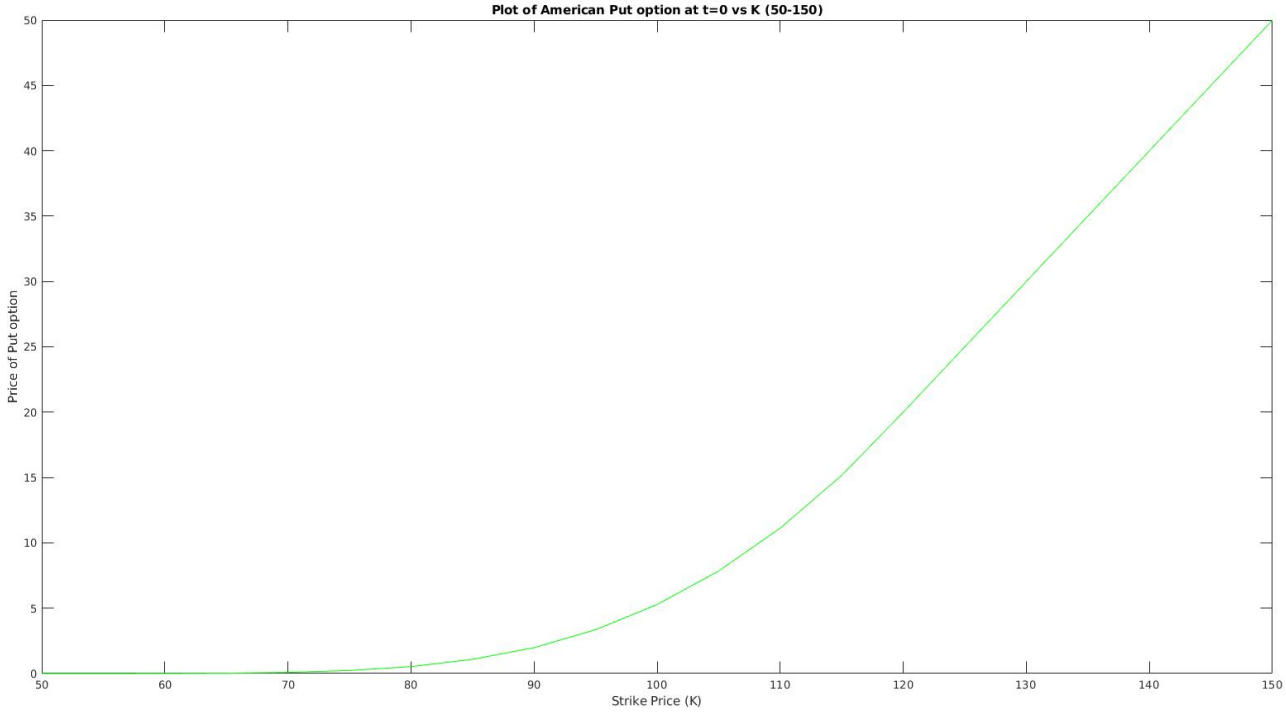
The American Call price is - 12.1230
The American Put price is - 5.2798

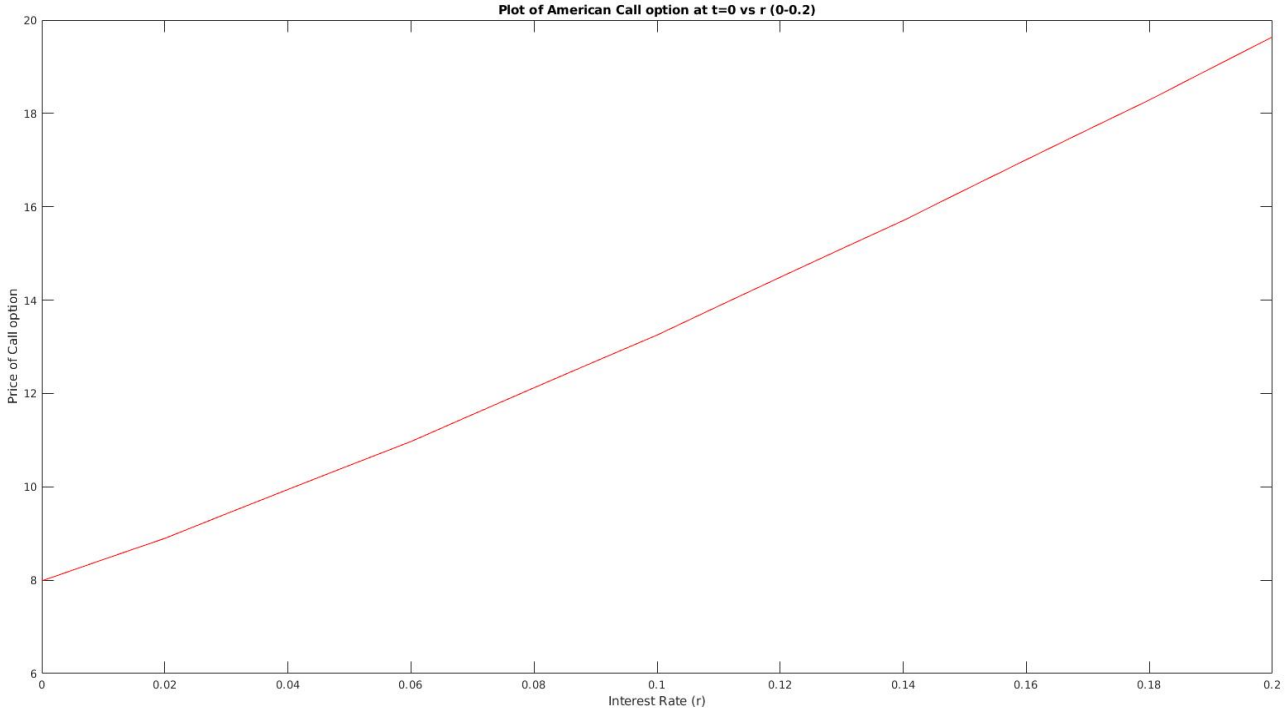
Sensitivity Analysis of option price variance with S(0), K, M, r, σ are done by plotting 2-D plots -

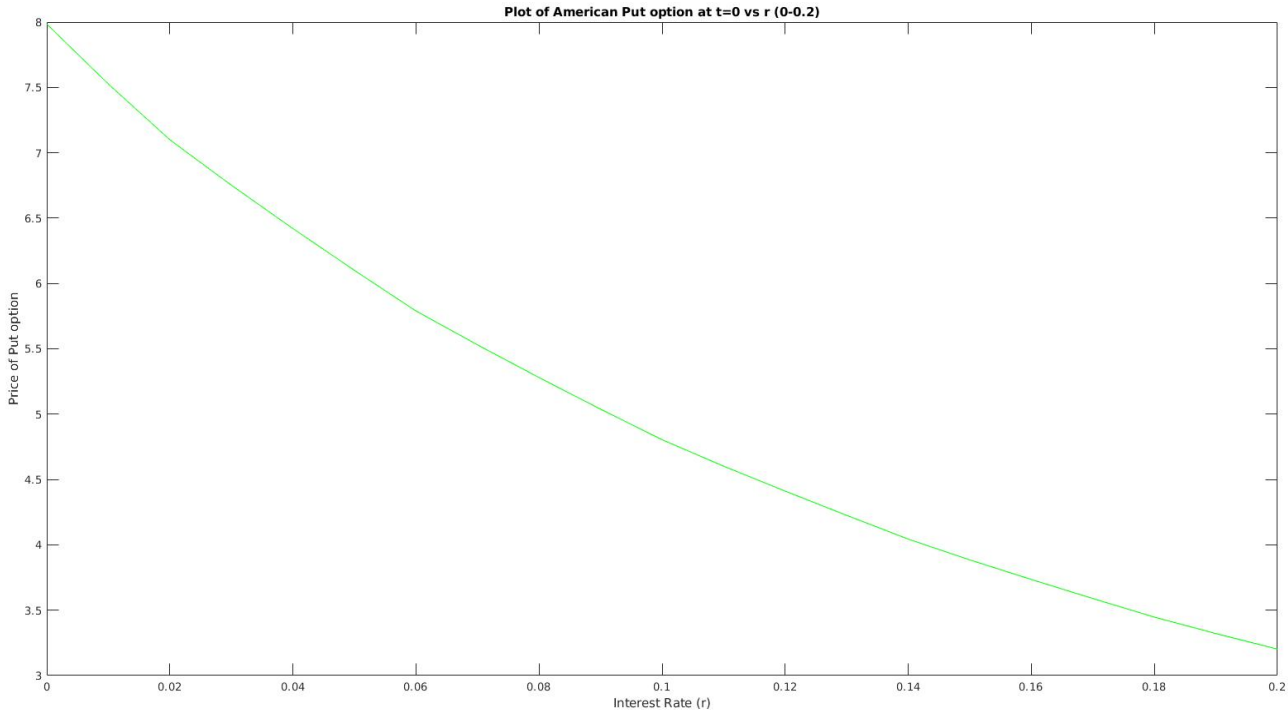


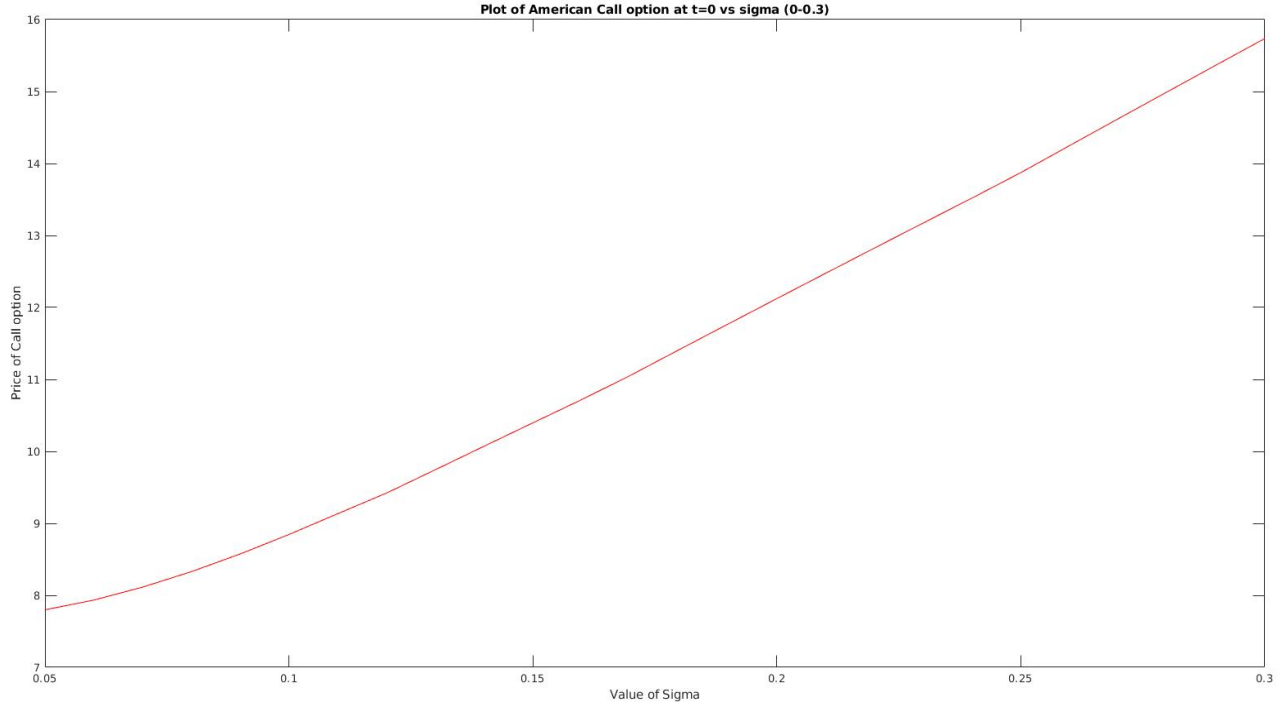

Plot of American Put option at t=0 vs S(0) (50-150)
Price of Put option
Stock Price at t = 0
0
5
10
15
20
25
30
35
40
45
50
50
60
70
80
90
100
110
120
130
140
150

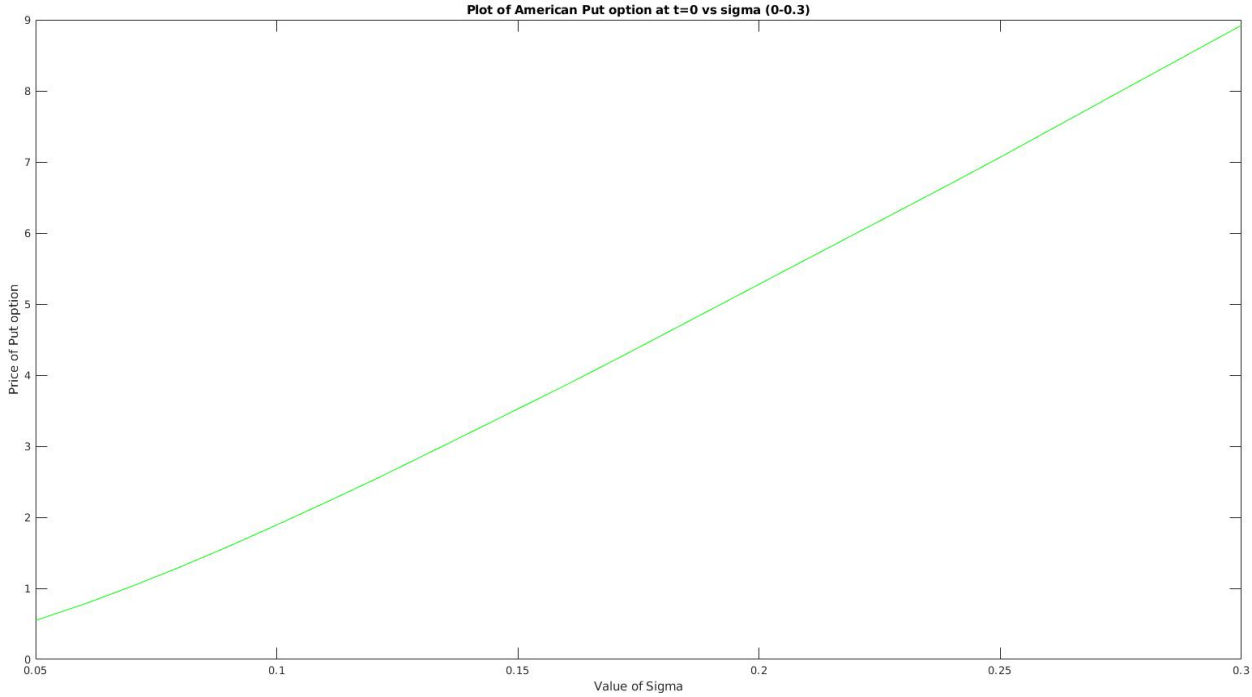

Plot of American Call option at t=0 vs K (50-150)
Price of Call option
Strike Price (K)
0
10
20
30
40
50
60
50
60
70
80
90
100
110
120
130
140
150

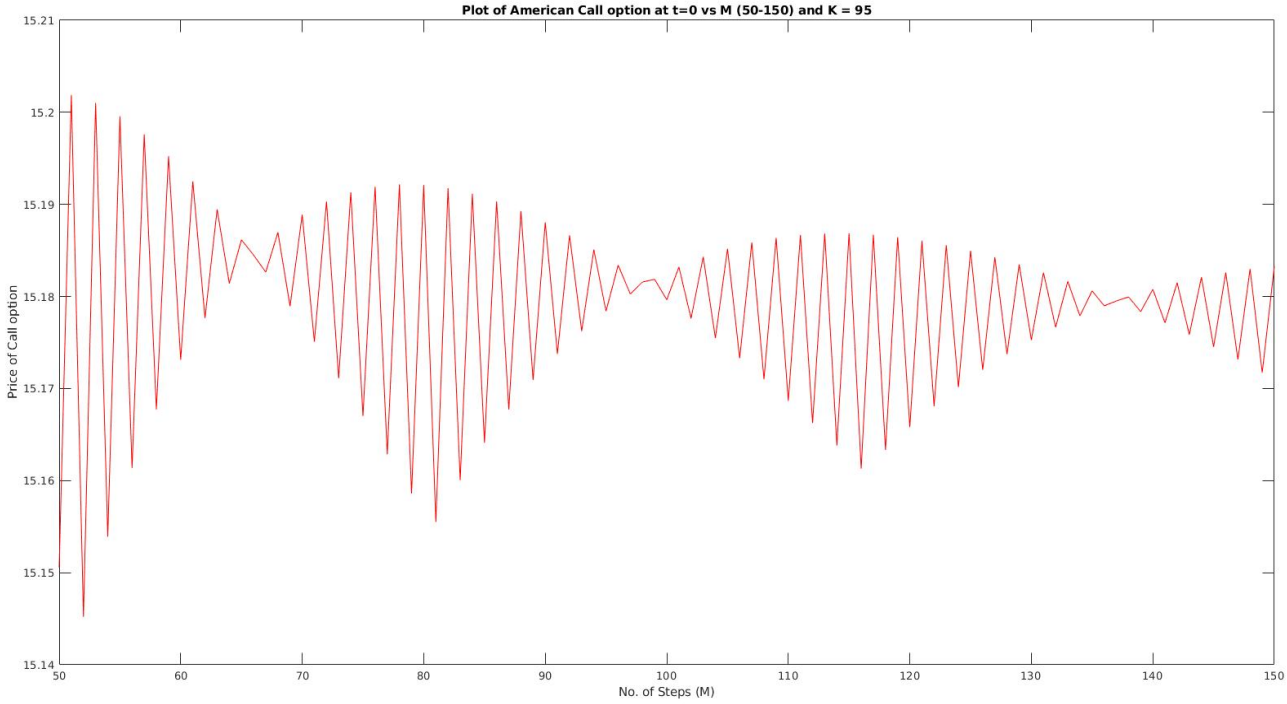

Plot of American Put option at t=0 vs K (50-150)
Price of Put option
Strike Price (K)
0
5
10
15
20
25
30
35
40
45
50
50
60
70
80
90
100
110
120
130
140
150

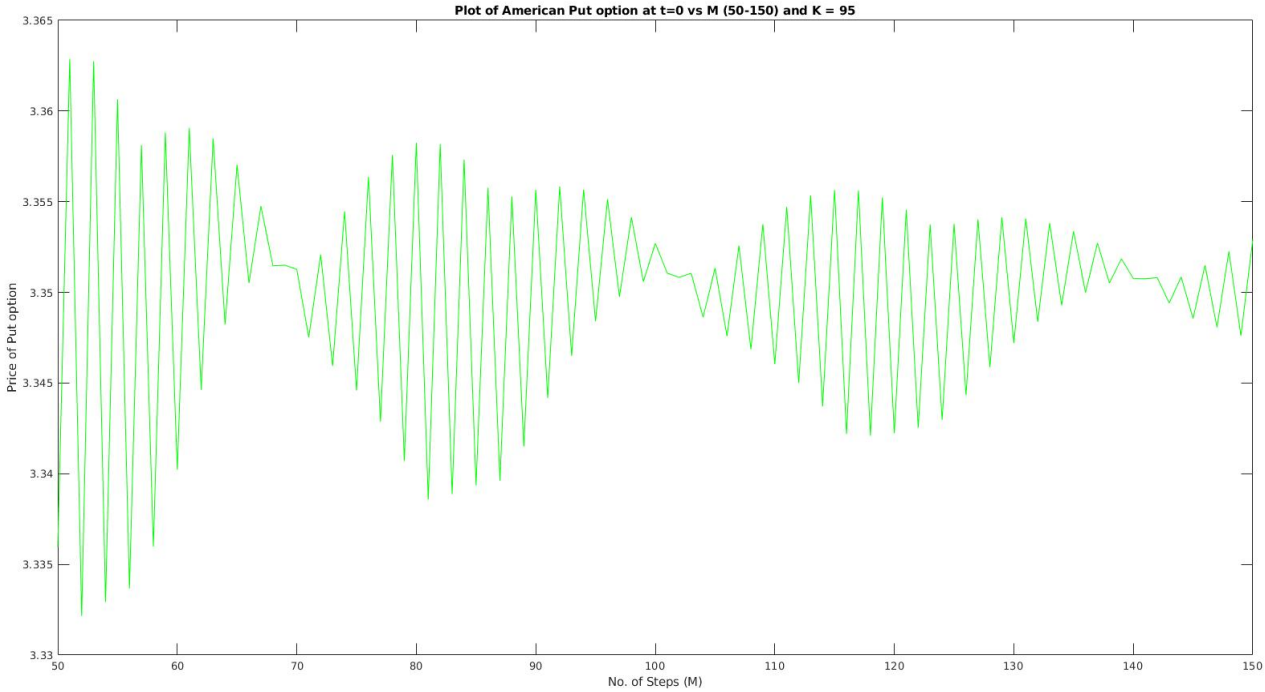

Plot of American Call option at t=0 vs r (0-0.2)
Price of Call option
Interest Rate (r)
6
8
10
12
14
16
18
20
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

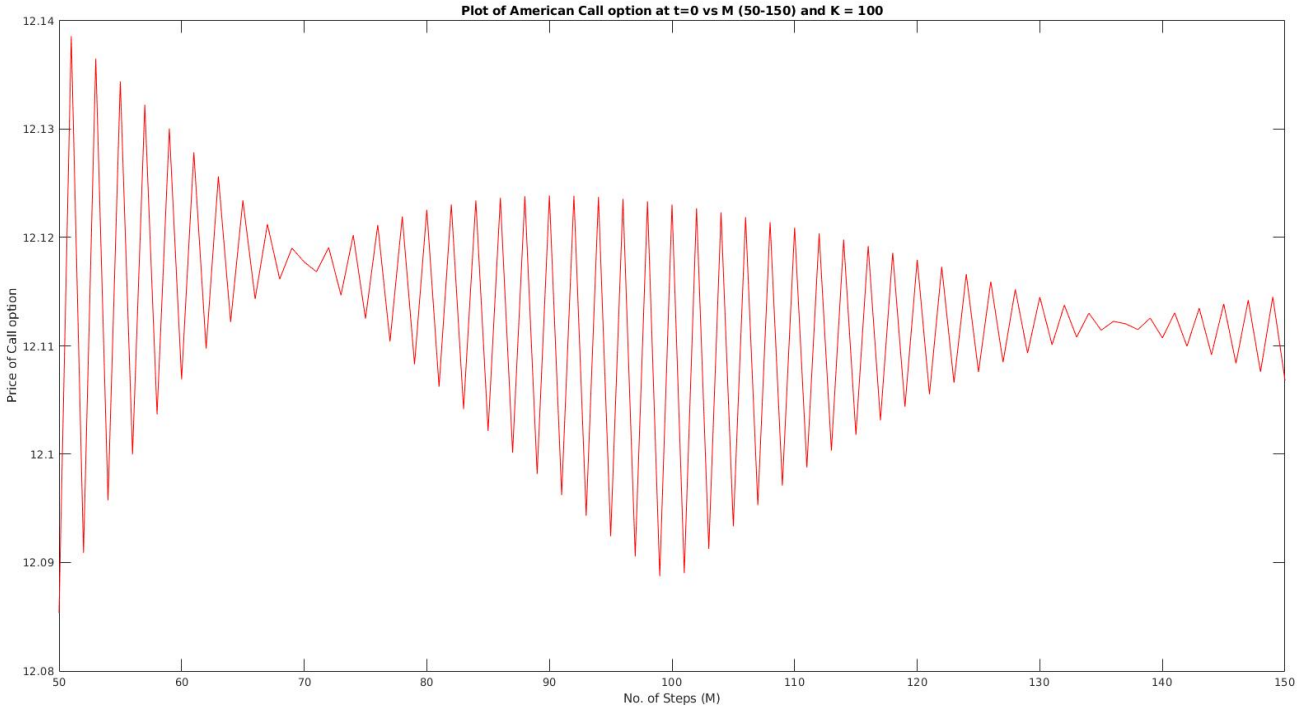

Plot of American Put option at t=0 vs r (0-0.2)
Price of Put option
Interest Rate (r)
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

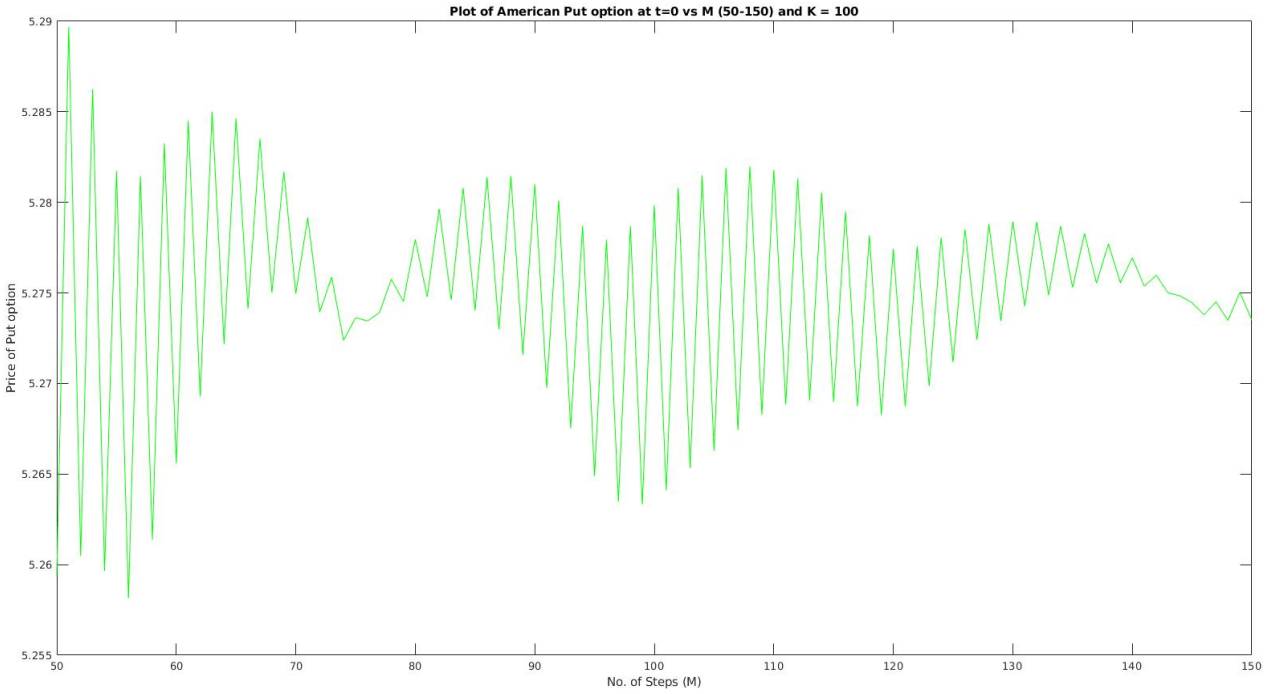

Plot of American Call option at t=0 vs sigma (0-0.3)
Price of Call option
Value of Sigma
16
15
14
13
12
11
10
9
8
7
0.05
0.1
0.15
0.2
0.25
0.3

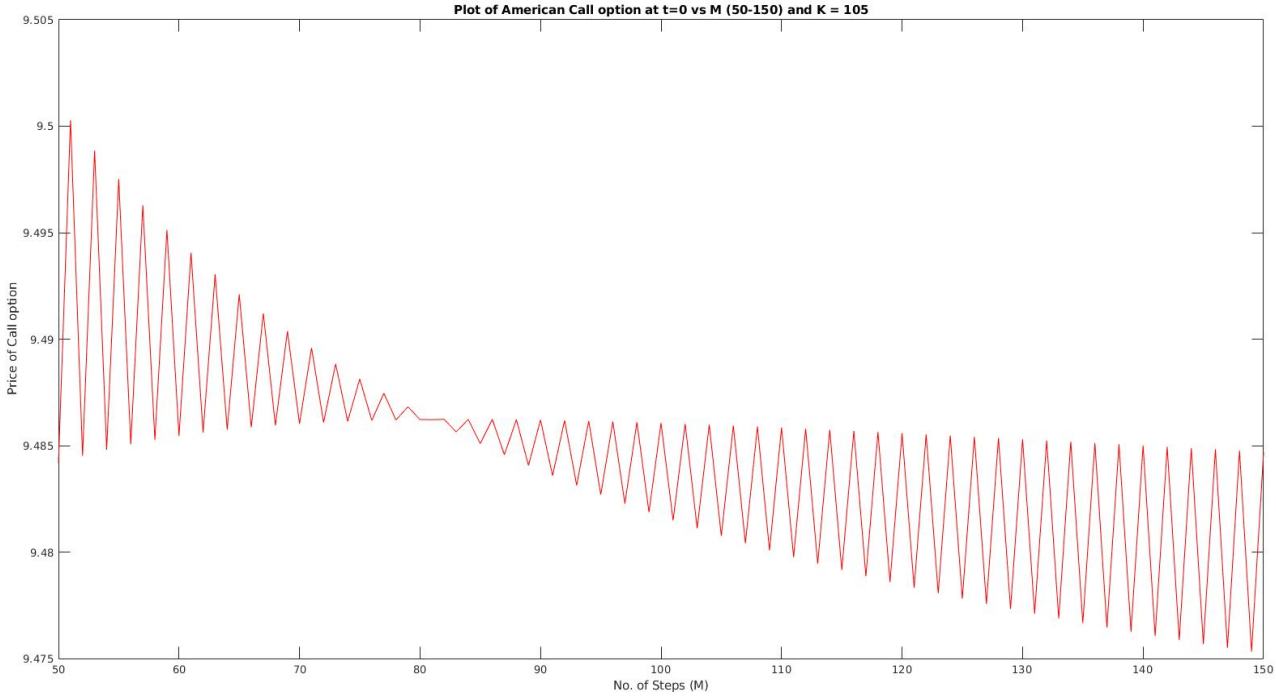

Plot of American Put option at t=0 vs sigma (0-0.3)
Price of Put option
Value of Sigma
0
1
2
3
4
5
6
7
8
9
0.05
0.1
0.15
0.2
0.25
0.3

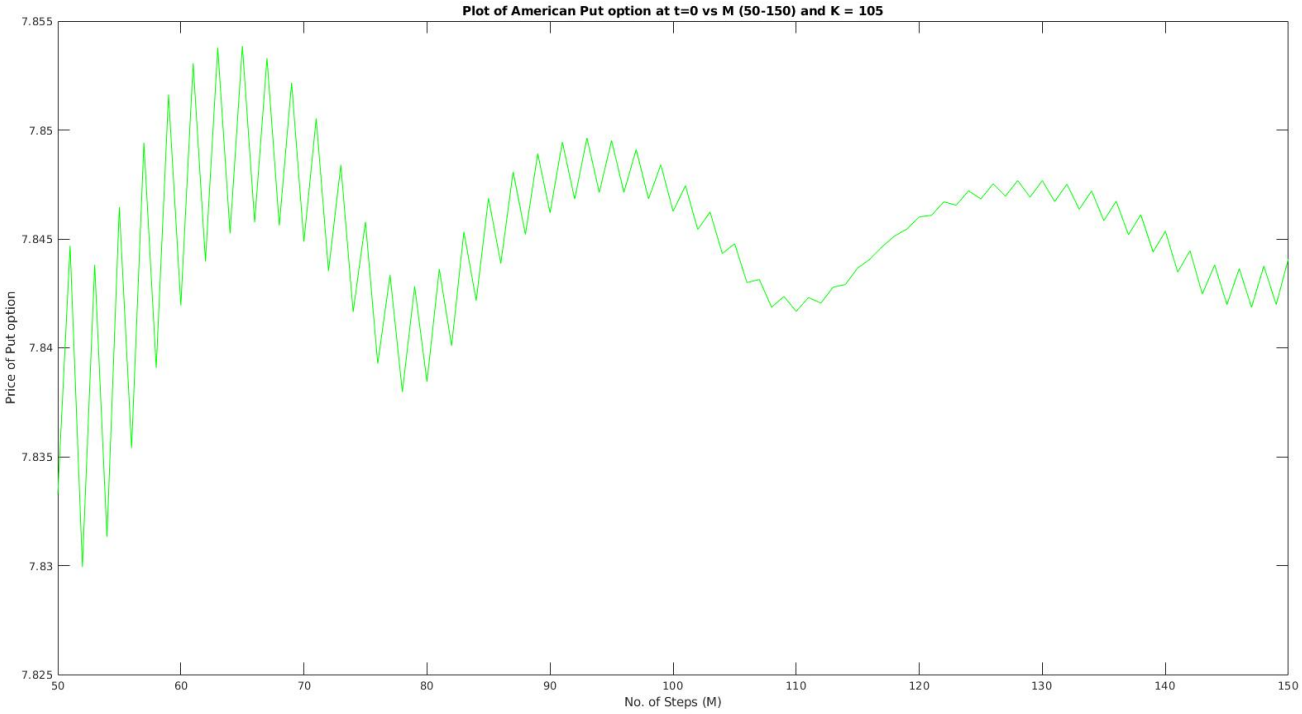

Plot of American Call option at t=0 vs M (50-150) and K = 95
Price of Call option
No. of Steps (M)
15.14
15.15
15.16
15.17
15.18
15.19
15.2
15.21
50
60
70
80
90
100
110
120
130
140
150


Plot of American Put option at t=0 vs M (50-150) and K = 95
Price of Put option
No. of Steps (M)
3.365
3.36
3.355
3.35
3.345
3.34
3.335
3.33
50
60
70
80
90
100
110
120
130
140
150


Plot of American Call option at t=0 vs M (50-150) and K = 100
Price of Call option
No. of Steps (M)
12.14
12.13
12.12
12.11
12.1
12.09
12.08
50
60
70
80
90
100
110
120
130
140
150


Plot of American Put option at t=0 vs M (50-150) and K = 100
Price of Put option
No. of Steps (M)
5.29
5.285
5.28
5.275
5.27
5.265
5.26
5.255
50
60
70
80
90
100
110
120
130
140
150


Plot of American Call option at t=0 vs M (50-150) and K = 105
Price of Call option
No. of Steps (M)
9.505
9.5
9.495
9.49
9.485
9.48
9.475
50
60
70
80
90
100
110
120
130
140
150



- # Question 2

The payoff of the lookback option is given by

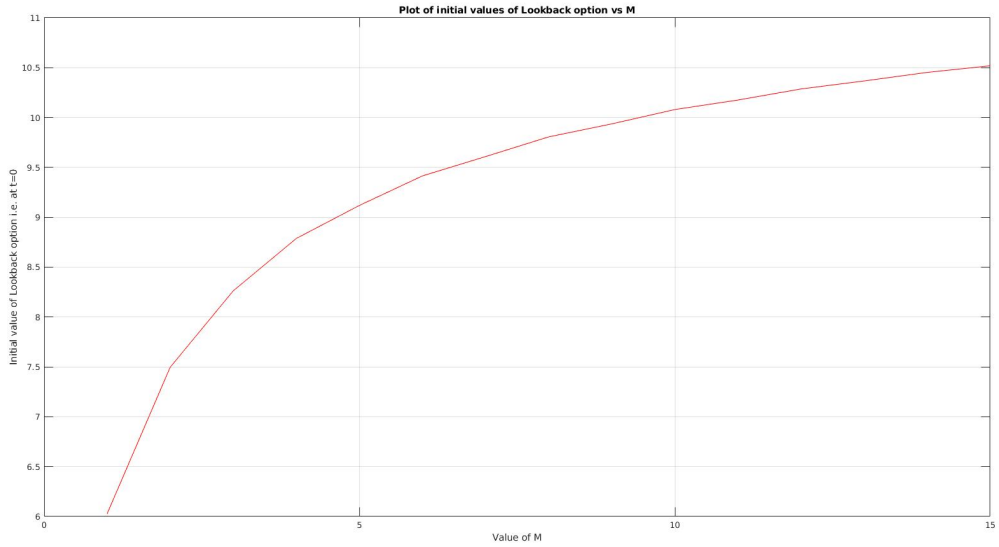
$$V = \max S(i) - S(M), \quad 0 <= i <= M$$

For **M = 5,** the initial price of Lookback Option = **9.1192989**
For **M = 10,** the initial price of Lookback Option = **10.080582**
For **M = 15,** the initial price of Lookback Option = **10.519165**

For **M = 25 and M = 50**, the amount of data required was too large for MATLAB to handle (Space Complexity exceeded). This will be addressed in question 3 using Markov based computationally efficient binomial algorithm.

b) From the graph it is observed that the initial values of Lookback Option tend to converge. Moreover for the initial values of M, *the increasing pattern of initial option value with M is observed.*

Plot Of Initial option price Vs value of M -



c) *The values of the options at all intermediate time points for M = 5 -*

T=0	T=0.2	T=0.4	T=0.6	T=0.8	T=1
9.119299	9.027951	8.548076	7.416771	5.501638	0
	9.504840	9.799119	9.955271	9.5713915	11.181413
		7.147915	6.201916	4.6004796	0
		12.168664	13.712862	15.631851	19.452691
			6.201916	4.6004796	0
			8.324614	8.0036137	9.349916
			7.148418	6.6808429	6.374517
			17.582062	21.188089	25.394563
				4.6004796	0
				8.0036137	9.349916
				3.8469288	0
				13.071381	16.266373
				3.8469288	0
				10.680904	13.578002
				10.680904	13.578002

				25.051229	29.482597
					0
					9.349916
					0
					16.266373
					0
					7.8184160
					5.3303822
					21.234976
					0
					7.8184160
					2.9013505
					18.805945
					2.9013504
					18.805945
					18.805945
					32.105394

• Question 3

The algorithm implemented in the the second question can handle value of M only upto 15. By implementing the efficient Markov based algorithm presented in Shreve Vol – 1, **the complexity has been brought down to $O(n^2)$ from $O(2^n)$**.

For **M = 20,** the initial price of Lookback Option = **10.7788**
For **M = 25,** the initial price of Lookback Option = **10.2972**
For **M = 50,** the initial price of Lookback Option = **10.5368**

Note : Due to limited profiency in MATLAB, I have implemented the Markov based efficient algorithm in C++11 language as it required usage of maps, vectors and pair data types. If required please run the code using the command - 'g++ -std=c++11 l3q3.cpp'.