Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA 324)
Problem Set 03

1. Let $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(\theta, \theta)$, where $\theta > 0$ is unknown parameter. Is the minimal sufficient statistic complete?

2. Let $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} U(0, \theta)$, where $\theta > 0$. Show that $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$ is complete statistic.

3. Suppose that $X_1, X_2, \ldots, X_m \overset{i.i.d.}{\sim} Gamma(\alpha, \beta)$ and $Y_1, Y_2, \ldots, Y_n \overset{i.i.d.}{\sim} Gamma(\alpha, k\beta)$, where $\alpha > 0$ and $k > 0$ are known constants, and $\beta > 0$ is unknown parameter. Also assume that X_i's and Y_j's are independent. Is minimal sufficient statistic T complete?

4. Let $X_1, X_2, \ldots, X_n$ be *i.i.d.* RVs having Beta distribution with parameters $\alpha > 0$ and $\beta > 0$. Is the minimal sufficient statistic T complete? Is the conclusion remain same if $\alpha = \beta$?

5. Suppose that $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} U(-\theta, \theta)$ with unknown $\theta > 0$. Denote

$$T = \max\{|X_1|, |X_2|, \ldots, |X_n|\}, \qquad U_1 = \frac{|X_{(1)}|}{X_{(n)}}, \qquad \text{and} \qquad U_2 = \frac{(X_1 - X_2)^2}{X_{(1)}X_{(2)}}.$$

Is T distributed independently of $\boldsymbol{U} = (U_1, U_2)$?

6. Let $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(\mu_1, \sigma^2)$ and $Y_1, Y_2, \ldots, Y_m \overset{i.i.d.}{\sim} N(\mu_2, \sigma^2)$, where $\mu_1 \in \mathbb{R}$, $\mu_2 \in \mathbb{R}$, and $\sigma > 0$ are unknown parameters. Also assume that X_i's and Y_j's are independent. Denote

$$\begin{aligned}
T &= \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 + \sum_{i=1}^{m} \left(Y_i - \overline{Y}\right)^2, \\
V_1 &= \frac{\left(\overline{X} - Y_{(n)} - X_1 + Y_2\right)^2}{T}, \\
V_2 &= \frac{\left(\overline{X} - \overline{Y} - X_2 + Y_{(m)}\right)}{|X_{(n)} - X_{(1)}|}, \\
U_1 &= \left(\overline{X} - \overline{Y}\right)^3,
\end{aligned}$$

and

$$U_2 = \frac{\left(\overline{X} - \overline{Y}\right)^2}{T}.$$

Check whether the two dimensional statistics $\boldsymbol{U} = (U_1, U_2)$ and $\boldsymbol{V} = (V_1, V_2)$ are independent.

7. Let $X_1, X_2, \ldots, X_n$ be a RS from population with PDF

$$f(x, \theta, \sigma) = \begin{cases} \frac{1}{\sigma} e^{-\frac{x-\theta}{\sigma}} & \text{if } x > \theta \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Argue that $X_{(1)}$ and $\sum_{i=1}^{n} \left(X_i - X_{(1)}\right)$ are independently distributed.