Lab Number : **10**
Due Date : **Nov 18, 2020**
Student Details :

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Explanation:

For this lab, we first needed to obtain the Y¡s and $\widehat{Y}i$s , from the U¡s ~ U [0,1], using the formulae:

$Y i = exp(\sqrt{Ui})$

$\widehat{Y}i = 1/2(exp(\sqrt{Ui}) + exp(\sqrt{1 - Ui}))$

After getting the Yis and the $\widehat{Y}i$s, we now take their mean to get Im and $\widehat{Im}s$. We do all this for M values, where M is **100**, **1000**, **10000**, and **100000** in 4 different cases respectively.

Having got these values we just find the standard deviation, and then obtain the 95% confidence interval as we did in assignment 9:

$[Im - 1.96 * (\sigma/\sqrt{M}), Im + 1.96 * (\sigma/\sqrt{M})$ for a normal estimator and,

$[\widehat{Im} - 1.96 * (\widehat{\sigma}/\sqrt{M}), \widehat{Im} + 1.96 * (\widehat{\sigma}/\sqrt{M})$ for an antithetic estimator.

And we take the ratio of the length of normal to antithetic 95% conf intervals. The result is tabulated in the next section.

Results:

We don't need any specific libraries for this assignment. Only math, random, and statistics are used.
Also note that the tabulated values here might vary slightly, since they're randomly generated.

Value of M	Im	$\widehat{Im}$	95% confidence interval for Im	95% confidence interval for $\widehat{Im}$	Ratio
100	1.9780	2.0034	[1.8907, 2.0653]	[1.9967, 2.0101]	13.0661
1000	2.0019	2.0007	[1.9739, 2.0298]	[1.9986, 2.0027]	13.597
10000	2.0041	2.0003	[1.9955, 2.0127]	[1.9996, 2.0009]	13.3801
100000	1.9978	1.9999	[1.9951, 2.0005]	[1.9997, 2.0001]	13.4688

Inference:

- As can be seen, **the expected value for both, the normally obtained estimators and the antithetic variates estimators, are very close** and get closer with higher values of M. This is because as the sample size grows, the mean of the sample approaches the Expected Value of the distribution. Since Ui and $1 - Ui$ are both Uniformly distributed on the interval (0, 1), we get the above observation.
- Moreover, the ratio indicates that the length of the 95% confidence interval obtained by using antithetic variates is close to one-fourteenth of the length of the 95% confidence interval obtained using the normal method. Thus, the **method of antithetic variates reduces the variance successfully**.