Core:

Given a game $v \in G^N$, the core of v is the set of all imputations x in $I(c)$ such that $x(S) \geq v(s)$ for all non-empty coalitions $S \subset N$. The core of a game $v \in G^N$ is denoted by $C(v)$.

Example 1

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 0$
$\{2\}$	$v(\{2\}) = 0$
$\{3\}$	$v(\{3\}) = 0$
$\{1,2\}$	$v(\{1,2\}) = 1$
$\{1,3\}$	$v(\{1,3\}) = 1$
$\{2,3\}$	$v(\{2,3\}) = 1$
$\{1,2,3\}$	$v(\{1,2,3\}) = 3$

We want to find the core allocation of the above coalition game.
We have $v(\{1\}) = 0 \leq x_{A_1},\ v(\{2\}) = 0 \leq x_{A_2},\ v(\{3\}) = 0 \leq x_{A_3}$.
$v(\{1,2\}) = 1 \leq x_{A_1} + x_{A_2},\ v(\{1,3\}) = 1 \leq x_{A_1} + x_{A_3},\ v(\{2,3\}) = 1 \leq x_{A_2} + x_{A_3}$.
$v(\{1,2,3\}) = 3 \leq x_{A_1} + x_{A_2} + x_{A_3}$.

Substituting $v(\{2,3\}) = 1 \leq x_{A_2} + x_{A_3}$ in $v(\{1,2,3\}) = 3 = x_{A_1} + x_{A_2} + x_{A_3}$. We have $2 \geq x_{A_1}$, similarly we get $2 \geq x_{A_2}$ and $2 \geq x_{A_3}$.
Therefore, core allocations are $2 \geq x_{A_1} \geq 0$, $2 \geq x_{A_2} \geq 0$, $2 \geq x_{A_3} \geq 0$ and $3 = x_{A_1} + x_{A_2} + x_{A_3}$.
It is shown in figure 1 and 2.

Example 2

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 1$
$\{2\}$	$v(\{2\}) = 1$
$\{3\}$	$v(\{3\}) = 1$
$\{1,2\}$	$v(\{1,2\}) = 2$
$\{1,3\}$	$v(\{1,3\}) = 2$
$\{2,3\}$	$v(\{2,3\}) = 2$
$\{1,2,3\}$	$v(\{1,2,3\}) = 3$

We want to find the core allocation of the above coalition game.
We have $v(\{1\}) = 1 \leq x_{A_1},\ v(\{2\}) = 1 \leq x_{A_2},\ v(\{3\}) = 1 \leq x_{A_3}$.
$v(\{1,2\}) = 2 \leq x_{A_1} + x_{A_2},\ v(\{1,3\}) = 2 \leq x_{A_1} + x_{A_3},\ v(\{2,3\}) = 2 \leq x_{A_2} + x_{A_3}$.
$v(\{1,2,3\}) = 3 \leq x_{A_1} + x_{A_2} + x_{A_3}$.

Substituting $v(\{2,3\}) = 2 \leq x_{A_2} + x_{A_3}$ in $v(\{1,2,3\}) = 3 = x_{A_1} + x_{A_2} + x_{A_3}$. We have $1 \geq x_{A_1}$, similarly we get $1 \geq x_{A_2}$ and $1 \geq x_{A_3}$.
And we have
$v(\{1\}) = 1 \leq x_{A_1},\ v(\{2\}) = 1 \leq x_{A_2},\ v(\{3\}) = 1 \leq x_{A_3}$.

Therefore, core allocation is $(x_{A_1}, x_{A_2}, x_{A_3}) = (1,1,1)$.
See figure 3 and 4.

Example 3

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 1$
$\{2\}$	$v(\{2\}) = 1$
$\{3\}$	$v(\{3\}) = 1$
$\{1, 2\}$	$v(\{1, 2\}) = 2$
$\{1, 3\}$	$v(\{1, 3\}) = 2$
$\{2, 3\}$	$v(\{2, 3\}) = 2$
$\{1, 2, 3\}$	$v(\{1, 2, 3\}) = 4$

We want to find the core allocation of the above coalition game.
We have $v(\{1\}) = 1 \leq x_{A_1},\ v(\{2\}) = 1 \leq x_{A_2},\ v(\{3\}) = 1 \leq x_{A_3}$.
$v(\{1,2\}) = 2 \leq x_{A_1} + x_{A_2},\ v(\{1,3\}) = 2 \leq x_{A_1} + x_{A_3},\ v(\{2,3\}) = 2 \leq x_{A_2} + x_{A_3}$.
$v(\{1,2,3\}) = 4 \leq x_{A_1} + x_{A_2} + x_{A_3}$.

Substituting $v(\{2,3\}) = 2 \leq x_{A_2} + x_{A_3}$ in $v(\{1,2,3\}) = 4 = x_{A_1} + x_{A_2} + x_{A_3}$. We have $2 \geq x_{A_1}$, similarly we get $2 \geq x_{A_2}$ and $2 \geq x_{A_3}$.
And we have
$v(\{1\}) = 1 \leq x_{A_1},\ v(\{2\}) = 1 \leq x_{A_2},\ v(\{3\}) = 1 \leq x_{A_3}$.
Therefore, core allocation are
$2 \geq x_{A_1} \geq 1$, $2 \geq x_{A_2} \geq 1$, $2 \geq x_{A_3} \geq 1$ and $4 = x_{A_1} + x_{A_2} + x_{A_3}$
It is shown in figure 5 and 6.

Example 4

Suppose one person A owns an old car which values nothing to him. There are two potential buyers, Buyer B values it at 1000 and buyer C values it at 1050. The trade between these people can be analysed based on coalition formation.

Coalitions	Value or worth of coalitions
$\{A\}$	0
$\{B\}$	0
$\{C\}$	0
$\{A, B\}$	1000
$\{A, C\}$	1050
$\{B, C\}$	0
$\{A, B, C\}$	1050
$\{\emptyset\}$	0

What is the core allocation?

We have $x_A \geq 0$, $x_B \geq 0$, $x_C \geq 0$,
$x_A + x_B \geq 1000, \quad x_A + x_C \geq 1050, \quad x_B + x_C \geq 0$
$x_A + x_B + x_C \geq 1050$.

Substituting $x_A + x_C \geq 1050$ in $x_A + x_B + x_C = 1050$, we get $x_B \leq 0$. We have $x_B \geq 0$, thus $x_B = 0$.
Substituting $x_A + x_B \geq 1000$ in $x_A + x_B + x_C = 1050$, we get $x_C \leq 50$. We have $x_C \geq 0$, thus $50 \geq x_C \geq 0$.
From this we get the core allocation as
$C(v) = \{(x_A, x_B, x_c) = (1050 - d, 0, d) | 0 \leq d \leq 50\}$.