भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 322: Scientific Computing Lab

Lab 08

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31 Mar 2021

Question 1.

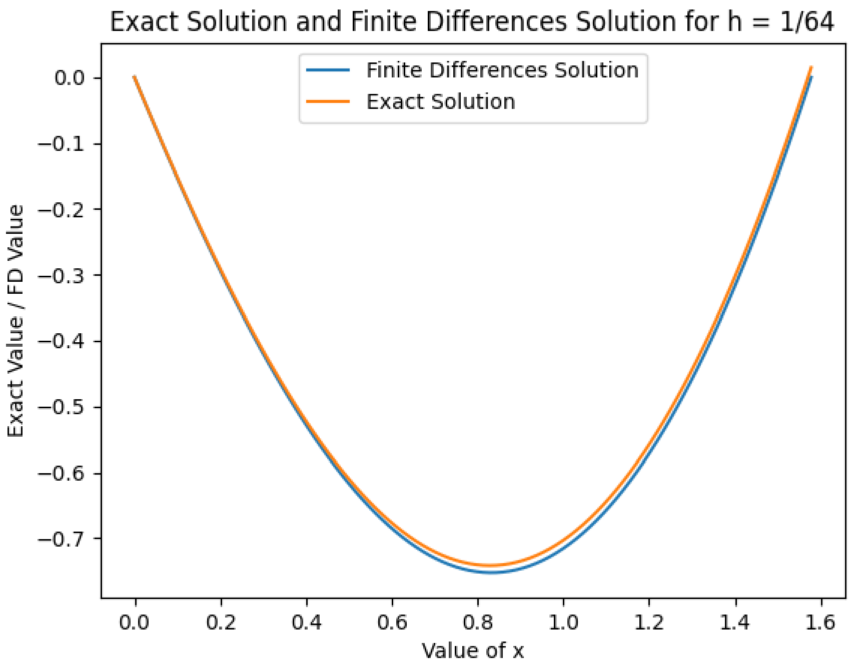
- The BVP is given as:

$$\frac{d^2y}{dx^2} + y(x) = 1 + x$$
$$y(0) = y\left(\frac{\pi}{2}\right) = 0$$

- **From this we can obtain the values of P = 1, Q = 1, R = 1 + x**
- Using second order scheme, we obtain the complete the table as follows:

	h	y(1/2)	f.d. solution at 1/2	error	ratio of error (absolute)
0	0.250000	-0.610088	-0.548903	0.061185	0.111468
1	0.125000	-0.610088	-0.665132	0.055044	0.082757
2	0.062500	-0.610088	-0.602438	0.007650	0.012698
3	0.031250	-0.610088	-0.602241	0.007847	0.013030
4	0.015625	-0.610088	-0.617165	0.007077	0.011467

- The plot of exact and FD solutions for h=1/64 is below:



Question 2.

- The BVP is given as:

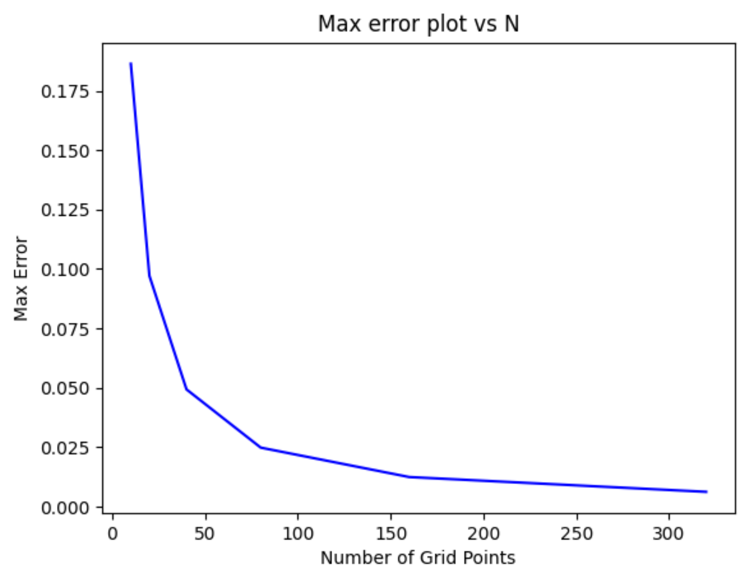
  $$-\frac{d^2u}{dx^2} = f(x)$$

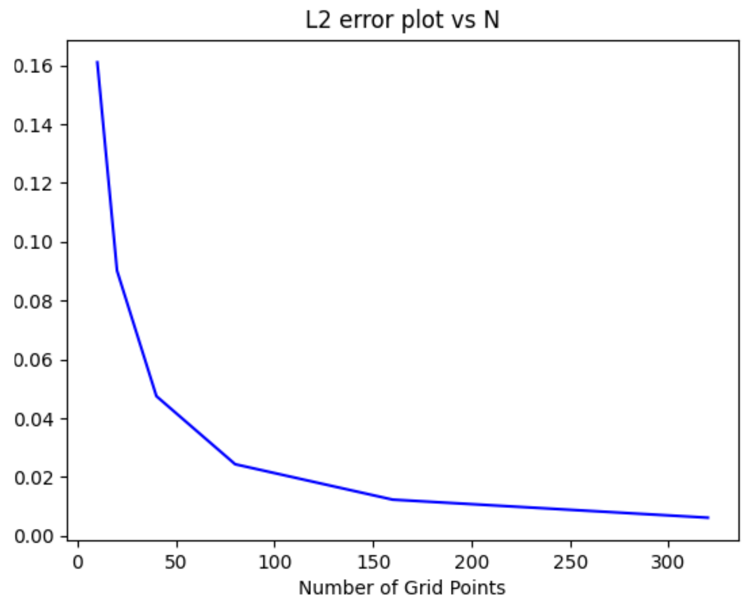
  $$u(0) = u(2\pi) = 0$$

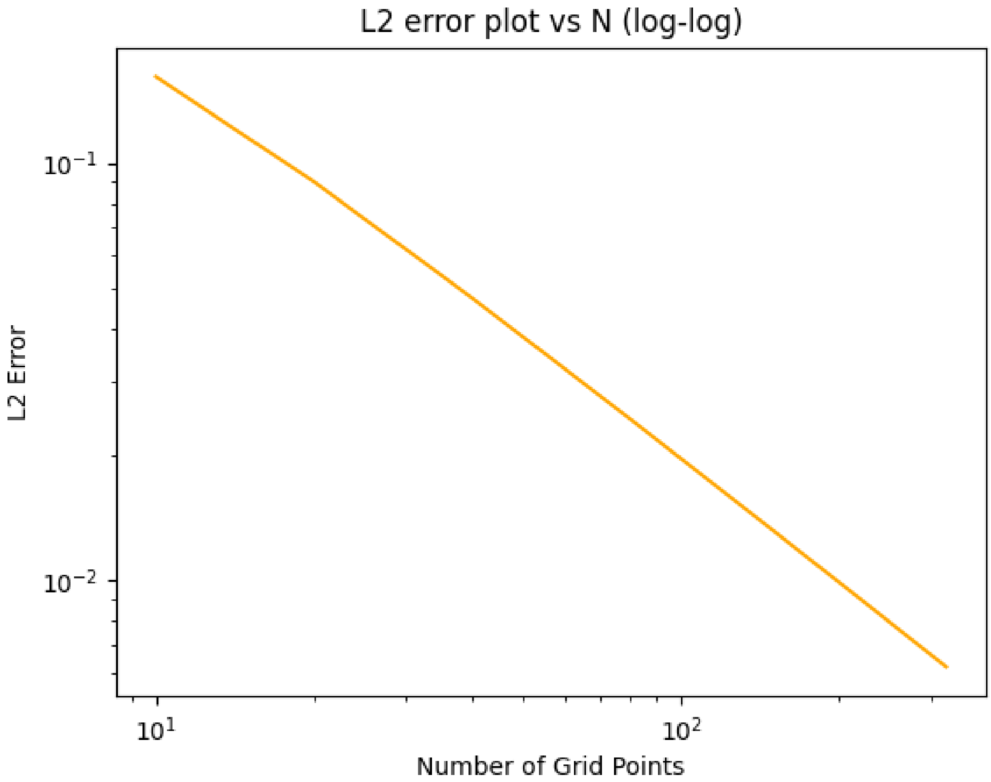
- The computer program is attached as a python file inside the Code folder, by the name q2.py
- The error in max and L^2 norm is tabulated as below:

	Grid Points	Max Error	L2 Norm Error
0	10	0.186308	0.161055
1	20	0.097122	0.090226
2	40	0.049332	0.047534
3	80	0.024839	0.024380
4	160	0.012460	0.012344
5	320	0.006240	0.006211

- The graphs for both of these norms are plotted in the normal and the log-log scale as below:







- The proof that truncation error is $O(h^4)$ is as follows:

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Given FD scheme

$$-D_x^+ D_x^- U_j = f_j + \frac{h^2}{12} D_x^+ D_x^- f_j \, , \quad j = 2, 3, \dots N-1.$$

$$D_x^+ D_x^- u(x_j) = \frac{u(x_{i-1}) - 2u(x_i) + u(x_{i+1})}{h^2}$$

By Taylor's Expansion,

$$u(x_{i+1}) = u(x_i) + hu'(x_i) + \frac{h^2}{2} u''(x_i) + \frac{h^3}{6} u'''(x_i) + \frac{h^4}{24} u''''(x_i) + \dots$$

$$u(x_{i-1}) = u(x_i) - hu'(x_i) + \frac{h^2}{2} u''(x_i) - \frac{h^3}{6} u'''(x_i) + \frac{h^4}{24} u''''(x_i) + \dots$$

Adding 2 eqns above,

$$u(x_i+1) + u(x_i-1) = 2u(x_i) + h^2 u''(x_i) + \frac{h^4}{12} u''''(x_i) + O(h^5)$$

So, $$\frac{u(x_i+1) - 2u(x_i) + u(x_i-1)}{h^2} = u''(x_i) + \frac{h^2}{12} u''''(x_i) + O(h^4)$$

But from question, LHS = $D_x^+ D_x^- u(x_i)$.

Thus, $$D_x^+ D_x^- f = f'' + \frac{h^2}{12} f'''' + O(h^4)$$

Since $f(x) = -u''(x)$

$$D_x^+ D_x^- f = -u'''' - \frac{h^2}{12} u^{(6)} + O(h^4)$$

$$\therefore -D_x^+ D_x^- u(x_i) = -u''(x_i) - \frac{h^2}{12} u''''(x_i) + O(h^4)$$

$$= f(x_i) + \frac{h^2}{12} D_x^+ D_x^- f(x_i) + O(h^4).$$

Thus, we get a truncation error of $O(h^4)$

Question 3.

- The BVP is given as -
  $$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 1,$$
  $$y(0) = 1, y(1) = 2(e-1)$$
- The absolute errors at the nodal points are tabulated below:

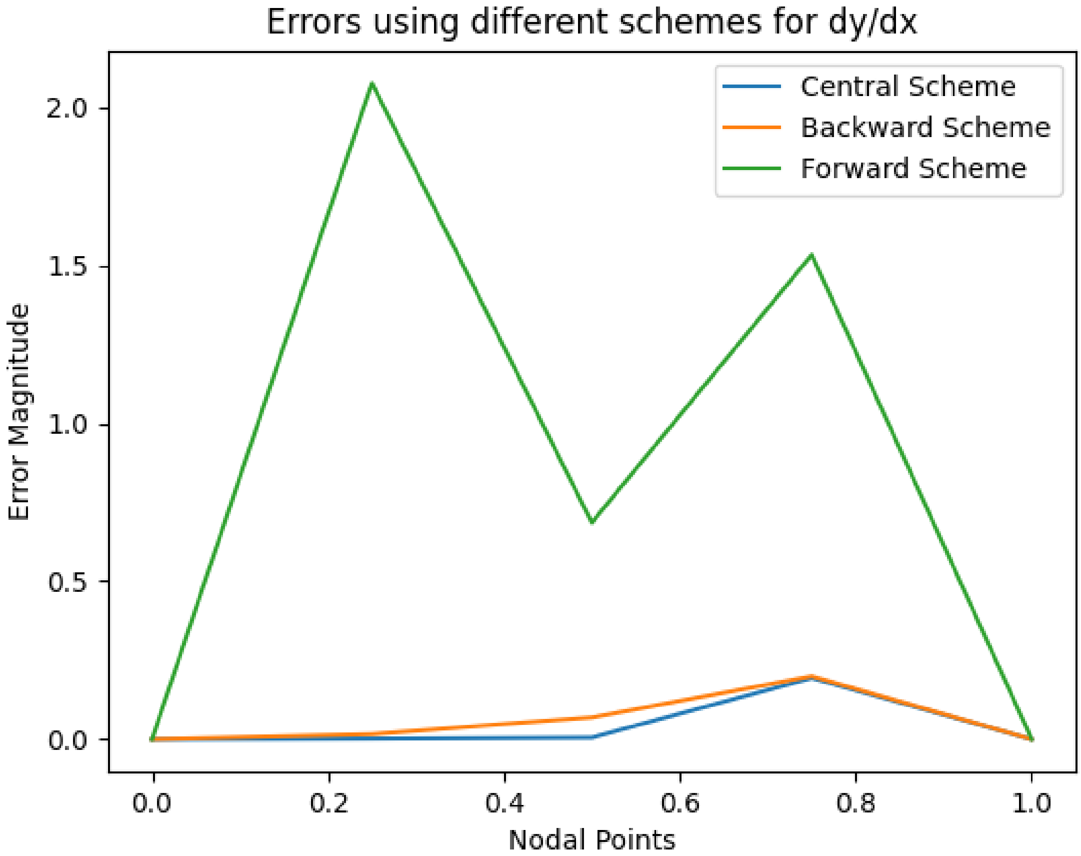
	Nodal points	Actual	Approx	Error
0	0	1.000000	1.000000	0.000000
1	1/3	1.457892	1.454869	0.003022
2	2/3	2.228801	2.225020	0.003782
3	1	3.436564	3.436564	0.000000

Question 4.

- The given BVP was solved using the 3 sets of schemes and the approximate values as well as error magnitudes are as below:

	Nodal points	Actual	Central Approximation	Central Error	Backward Approximation	Backward Error	Forward Approximation	Forward Error
0	0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1	1/4	0.000508	-0.001524	0.002032	0.016771	0.016264	2.076923	2.076415
2	2/4	0.006693	0.012195	0.005502	0.075472	0.068779	0.692308	0.685615
3	3/4	0.082043	-0.111280	0.193324	0.280922	0.198879	1.615385	1.533341
4	1	1.000000	1.000000	0.000000	1.000000	0.000000	1.000000	0.000000

- We now plot the magnitude of errors for each of the 3 schemes. From the plot we clearly observe that the central scheme has the least absolute error. This behavior is expected since the central scheme is second order convergent, while the forward and backward schemes are first order convergent.



Errors using different schemes for dy/dx
Central Scheme
Backward Scheme
Forward Scheme
Error Magnitude
Nodal Points
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0