Lab Assessment Module 1 & 2

MA423 : Matrix Computations July-November, 2021 S. Bora

Marks: 15

Important instructions:

- **Put all your work in a single folder and submit a link to download it on MS Teams within the declared deadline.**
- **The most recent update time of the contents of the folder should be visible.**
- **Submitted folder should contain** *only* **the programs written for the purpose of the assessment and other supporting programs that are required for those programs to run. If there are any unnecessary programs in the folder, then upto 2 marks may be deducted from the score of every group member.**
- **The workspace file for question 2** *must* **be provided.**

1. Suppose A is a real positive definite matrix with a 2×2 partitioning and G be its Cholesky factor with a conformal partitioning so that

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{12}^T & A_{22} \end{array}\right] = \left[\begin{array}{c|c} G_{11}^T & \\ \hline G_{12}^T & G_{22}^T \end{array}\right] \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline & G_{22} \end{array}\right].$$

The relations $A_{11} = G_{11}^T G_{11}$, $A_{12} = G_{11}^T G_{12}$, $A_{22} = G_{12}^T G_{12} + G_{22}^T G_{22}$, may be used to design a blocked version of the outer product formulation for computing G.

Generalize this to a $\ell \times \ell$ partitioning of A of the form

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1\ell} \\ & A_{22} & \cdots & A_{2\ell} \\ & & \ddots & \vdots \\ & & & A_{\ell\ell} \end{bmatrix}.$$

where the diagonal blocks A_{ii} are of size $m_i \times m_i$, $i = 1, \ldots, \ell$, with $m_1 + \cdots + m_{\ell,\ell} = n$ and implement it via a Matlab function program $\texttt{G} = \texttt{cholB(A, m)}$ where $\texttt{m} = [\texttt{m}_1 \cdots \texttt{m}_\ell]$. The code should be efficient and make judicious use of other function programs written in the regular lab sessions. **(8)**

2. Running the Matlab script `matrix.m` provided to you will load a 5×5 matrix `A`. Use it to design a 5×5 system of equations $Ax = b$ whose exact solution `x` is known. Compute its solution via the `geppsolve.m` code and compare the exact relative error in the solution with respect to the norm $\|\cdot\|_\infty$ with the upper bounds obtained via $\kappa_\infty(A)$ and `skeel(A)` under the assumption that the backward errors are bounded above by unit roundoff `u`. Justify the choice of your upper bound via $\kappa_\infty(A)$ and choose the corresponding formulation for the upper bound via `skeel(A)`.

   Report all the values in `format long e` and give a full explanation of the reason for the tightness of one of the bounds over the other. **(3)**

3. Run the following code and give a full explanation of the values of x and k that you get. **(2)**

```
x = 1; k = 0;
while x + x > x
x = 2x; k = k+1;
end
```

4. Write a neat report of your experiments. **(2)**