भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 374: Financial Engineering Lab

Lab 01

AB Satyaprakash (180123062)

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Checking the *no-arbitrage condition*

Before we make any computation regarding the prices, we first need to check for the no-arbitrage condition.

We know that,

$$E[S(t_1)|S(t_0)] = S(t_0)\exp(r\delta t)$$

$$\implies puS(t_0) + (1-p)dS(t_0) = S(t_0)\exp(r\delta t)$$

$$\implies p = \frac{\exp(r\delta t)-d}{u-d}$$

Hence, we will calculate this probability and see if $p \epsilon [0, 1]$, and if it does not, we know that the ***no-arbitrage principle is getting violated.***

Question 1.

The ***predicted payoff*** can be calculated using the Binomial Pricing Model. There will be a corresponding payoff for each possible path. Also, each path is associated with a probability. Thus we can evaluate the ***expected payoff***.

The initial option price can be obtained as:

$$V = P\exp(-rT)$$

where V = initial value, P = expected payoff and T = time to maturity.

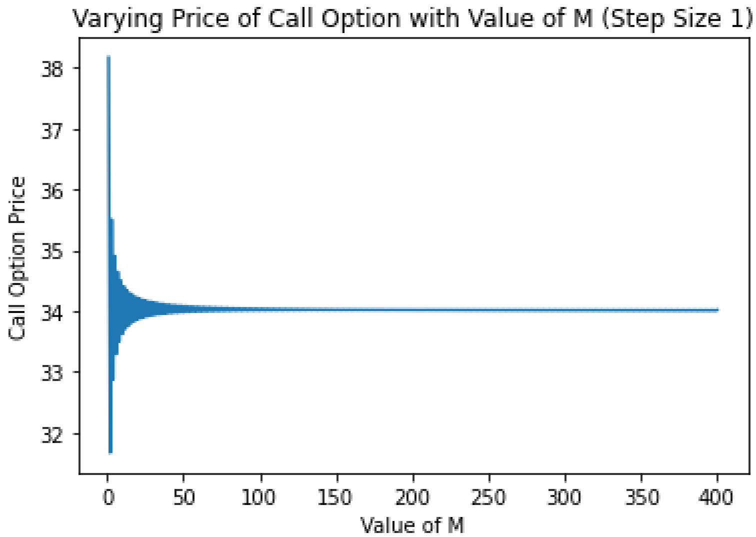
The following table is obtained after running the python code:

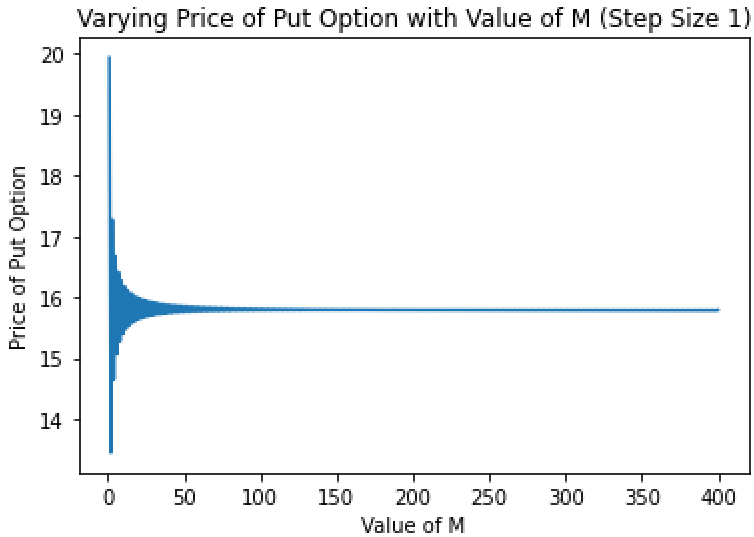
Step Size	Call Option Price	Put Option Price
1	38.167635	19.941717
5	34.906533	16.680615
10	33.625022	15.399104
20	33.859449	15.633532
50	33.981184	15.755267
100	34.011161	15.785243
200	34.019579	15.793661
400	34.019132	15.793214

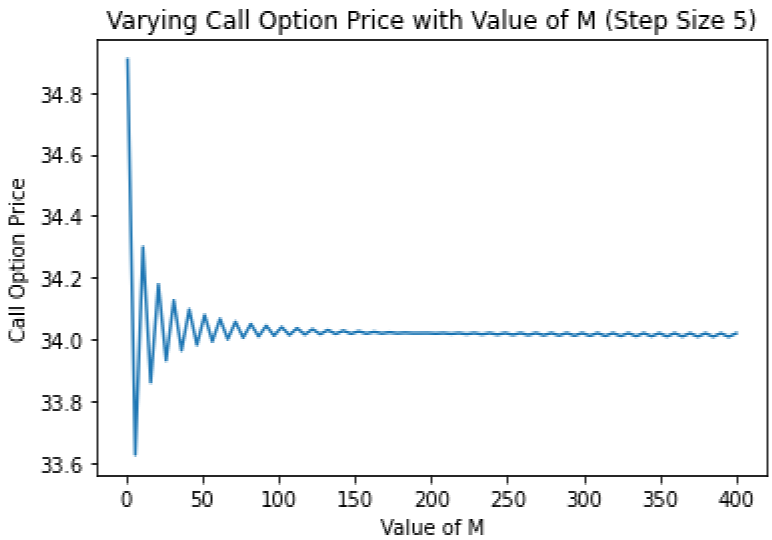
Question 2.

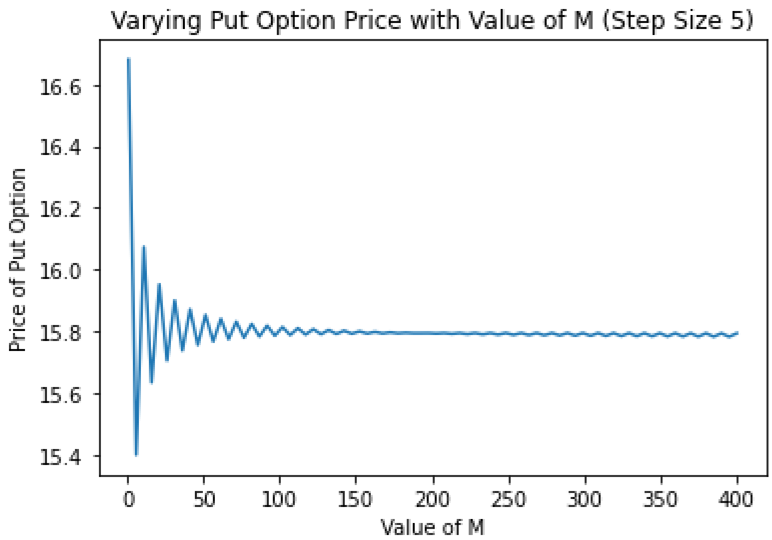
The option values at t=0 are obtained for various values of M by increasing M in steps of 1 and 5 respectively and plotting the corresponding graphs.

The following graphs were obtained on running the python code:









Observations:

1. We observe that the value of the **European call option** converges to **34.0** while the **European put option** converges to **15.7**.

2. This convergence is however not perfect. There is still some deviation.
3. On increasing the value of M, the prices of both the call and the put options slowly converge and the deviation between consecutive values reduces. However, on further increasing M, the error increases. This pattern repeats and the option value converges slowly.

Question 3.

The values of the options at **t = 0, 0.50, 1, 1.50, 3, 4.5** for the case **M = 20** are tabulated below:

Since for the ith step we have (i+1) possible values of the underlying asset, we can have (i+1) different values for the option.

Value of t	Price of Call Option	Price of Put Option
0	33.859449	15.633532
0.5	59.958769 31.893253 15.095873	8.479204 15.487143 24.672817
1	100.662666 57.699995 29.803955 13.469716 5.154831	3.504174 8.004223 15.269432 24.983287 35.965304
1.5	160.611388 98.438869 55.295356	0.942427 2.998250 7.436262

	27.573204 11.767497 4.121405 1.125003	14.963372 25.270960 36.970072 48.304951
3	519.099689 359.934184 242.030183 154.841699 91.193433 46.976188 19.725206 6.148520 1.235971 0.118330 0.0 0.0 0.0	0.0 0.0 0.008705 0.172103 1.235702 4.958186 13.221829 25.955024 40.533314 53.854842 64.433311 72.357695 78.228223
4.5	1419.424512 1024.993373 732.791598 516.323199 355.959465 237.159089 149.149606 83.950577 36.251494 8.149174 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.601546 8.281211 26.639984 46.277554 60.825424 71.602751 79.586791 85.501514 89.883248 93.129316 95.534063

Additional Lab requirements:

The following python packages have been used and need to be installed as shown.

1. **Pandas** : `pip install pandas`
2. **Matplotlib**: `pip install matplotlib`
3. **Numpy**: `pip install numpy`
4. **Ipython**: `pip install ipython`

Note: use pip or pip3 based on the python version.