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INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 322: Scientific Computing Lab

Lab 07

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Question 1.

We apply the Runge-Kutta method of order four to determine how many units of potassium hydroxide will have been formed after 0.2s.

```
   Value of h       Units of KOH after 2 seconds
0    0.0001                 2079.566418
1    1e-05                  2079.408375
2    1e-06                  2079.408375
3    1e-07                  2079.408375
```

Question 2.

In the code we approximate the value of $y(0.2)$ for different values of h. We find that the Runge-Kutta Method of Order 2 and the Modified Euler Method give exactly the same approximations.

Runge Kutta method of order 2:

$$y_{n+1} = y_n + h \times f\left(t + \frac{h}{2}, y_n + \frac{h}{2} f(t, y_n)\right)$$

Modified Euler method:

$$y_{n+1} = y_n + \frac{h}{2}\left(f(t, y_n) + f(t + h, y_n + h f(t, y_n))\right)$$

$f(t, y) = -y + t + 1 \longrightarrow$ given. Thus, $f(t, y)$ is linear in both t and y.

Now,

$$y_n + \frac{h}{2}\left(f(t, y_n) + f(t + h, y_n + h f(t, y_n))\right)$$

$$= y_n + \frac{h}{2}\left(f(2t + h, 2y_n + h f(t, y_n))\right)$$

$$= y_n + h\left(f\left(t + \frac{h}{2}, y + \frac{h}{2} f(t, y_n)\right)\right)$$

Thus, modified Euler is equivalent to Runge Kutta of order 2.

```
Runge-Kutta Method of Order 2 with h = 0.2 gives y(0.2) =
1.02
Modified Euler Method h = 0.2 gives y(0.2) = 1.02
The difference between the two values = 0.0

Runge-Kutta Method of Order 2 with h = 0.1 gives y(0.2) =
1.0190249999999998
Modified Euler Method h = 0.1 gives y(0.2) =
1.0190249999999998
```

```
The difference between the two values = 0.0

Runge-Kutta Method of Order 2 with h = 0.05 gives y(0.2) =
1.0188015933618164
Modified Euler Method h = 0.05 gives y(0.2) =
1.0188015933618164
The difference between the two values = 0.0

Runge-Kutta Method of Order 2 with h = 0.01 gives y(0.2) =
1.018733502735248
Modified Euler Method h = 0.01 gives y(0.2) =
1.018733502735248
The difference between the two values = 0.0
```

Question 3.

In this question for both parts we made use of the modified Euler method to approximate the IVPs and compared the results to the actual values.

```
For part (a) y(1) is approximated as 2
Actual value of y(1) is given by 2.0

For part (a) y(1.5) is approximated as 2.3541666666666665
Actual value of y(1.5) is given by 2.354101966249684 7

For part (a) y(2.0) is approximated as 2.7417450827887775
Actual value of y(2.0) is given by 2.7416573867739413


------------------------------------------------------------
For part (b) y(1) is approximated as 2
Actual value of y(1) is given by 2.0
```

```
For part (b) y(1.25) is approximated as 1.4160750785402427
Actual value of y(1.25) is given by 1.4031989692799332

For part (b) y(1.5) is approximated as 1.0310110697781514
Actual value of y(1.5) is given by 1.0164101466785118

For part (b) y(1.75) is approximated as 0.7522666785837252
Actual value of y(1.75) is given by 0.7380097715499843

For part (b) y(2.0) is approximated as 0.5432450024334279
Actual value of y(2.0) is given by 0.5296870980395587
```

Question 4.

We used Euler's method with h= 0.025, the Runge-Kutta second-order method with h=0.05, and the Runge-Kutta fourth-order method with h= 0.1 and compared at the common mesh points of these methods 0.1,0.2,0.3,0.4, and 0.5.

```
         Euler     Runge-Kutta O-2    Runge-Kutta O-4
0.1      0.655498      0.657373           0.657414
0.2      0.825338      0.829213           0.829298
0.3      1.008933      1.014939           1.015070
0.4      1.205635      1.213908           1.214087
0.5      1.414726      1.425409           1.425638
```

Question 5.

We compare approximations from Adams-Bashforth 4 step method, Adams-Moulton 3 step method, with the actual values.

```
     Adams-Bashforth  Adams-Moulton      Actual Value
0.0      0.500000       0.500000          0.500000
0.2      0.829299       0.829299          0.829299
0.4      1.214088       1.214088          1.214088
0.6      1.648941       1.648941          1.648941
0.8      2.063312       2.058428          2.127230
1.0      2.467748       2.467638          2.640859
1.2      2.863037       2.862550          3.179942
1.4      3.223804       3.221079          3.732400
1.6      3.521194       3.517541          4.283484
1.8      3.725060       3.720532          4.815176
2.0      3.797737       3.791637          5.305472
```

Question 6.

We used starting values obtained from the Runge-Kutta method of order four, and Adams-Bashforth methods to approximate the solutions to the IVPs.

```
For part (a):
     Adams-Bashforth  Adams-Moulton  Actual Values
0.0      1.000000      1.000000      1.000000
0.1      1.163461      1.163461      1.188119
0.2      1.293578      1.293578      1.346154
0.3      1.387930      1.387930      1.467890
0.4      1.477652      1.476739      1.551724
0.5      1.530130      1.530356      1.600000
0.6      1.554577      1.553568      1.617647
0.7      1.553375      1.552207      1.610738
0.8      1.533699      1.532161      1.585366
0.9      1.500249      1.498741      1.546961
1.0      1.457819      1.456359      1.500000
```

```
For part (b):
     Adams-Bashforth  Adams-Moulton  Actual Values
1.0       -1.442695      -1.442695      -1.442695
1.1       -1.351959      -1.351959      -1.347823
1.2       -1.275317      -1.275317      -1.268299
1.3       -1.209660      -1.209660      -1.200611
1.4       -1.150446      -1.150389      -1.142245
1.5       -1.099135      -1.098789      -1.091357
1.6       -1.053603      -1.053392      -1.046560
1.7       -1.013395      -1.013112      -1.006794
1.8       -0.977370      -0.977108      -0.971233
1.9       -0.944992      -0.944714      -0.939222
2.0       -0.915664      -0.915396      -0.910239

For part (c):
     Adams-Bashforth  Adams-Moulton  Actual Values
1.0       -2.000000      -2.000000      -2.000000
1.2       -1.749986      -1.749986      -1.714286
1.4       -1.599988      -1.599988      -1.555556
1.6       -1.499990      -1.499990      -1.454545
1.8       -1.435055      -1.419216      -1.384615
2.0       -1.377152      -1.361768      -1.333333
2.2       -1.335557      -1.318329      -1.294118
2.4       -1.298086      -1.284178      -1.263158
2.6       -1.270288      -1.256675      -1.238095
2.8       -1.245699      -1.234034      -1.217391
3.0       -1.226093      -1.215068      -1.200000
```

Question 7.

Applying the Adams fourth-order predictor-corrector method with h=0.2 and starting values from the Runge-Kutta fourth order method to the initial-value problem, we get:

```
     Adam's Predictor-Corrector Method  Actual Value
0.0                            0.500000      0.500000
0.2                            0.811877      0.829299
0.4                            1.157676      1.214088
0.6                            1.527195      1.648941
0.8                            1.976773      2.127230
1.0                            2.458009      2.640859
1.2                            2.956443      3.179942
1.4                            3.459381      3.732400
1.6                            3.950021      4.283484
1.8                            4.407880      4.815176
2.0                            4.807995      5.305472
```