Lab Number : 06
Due Date : Oct 16, 2020
Student Details :

- Name : AB Satyaprakash
- Roll Number : 180123062
- Department : Mathematics and Computing

Question 1 :

(1) For this question we are given, **μ** and **∑** (namely the mean vector and the covariance matrix) as follows :

$\boldsymbol{\mu = [\mu1\ \mu2] = [5\ 8]}$ and

$\boldsymbol{\sum = [\sigma1\ 2\ \sigma1\sigma2\ \rho\ \ \sigma1\sigma2\ \rho\ \ \sigma2^2] = [1\ 2a\ 2a\ 4]}$ (a 2D vector)

As we can quite clearly see, the value of $\sigma1$=1 and $\sigma2^2$=4, thus $\sigma1$=1 and $\sigma2$=2.Using this we obtain the value of ρ. Once we have ρ, we can generate the matrix A, given by

A = $\boldsymbol{[\sigma 1\ 0\ \ \rho\ \sigma 2\ p\ 1 - \rho\ \ 2\sigma 2]}$,

From using the entries of A, and generating (Z_1,Z_2) using the **Marsaglia and Bray** process for N(0,1), we have X = [X1 X2] as (from **Cholesky** Factorization):

$$X_1 = \mu1 + \sigma1 Z_1$$
$$X_2 = \mu2 + \rho\sigma2 Z_1 + \sqrt{1-\rho^2}\,\sigma2 Z_2$$

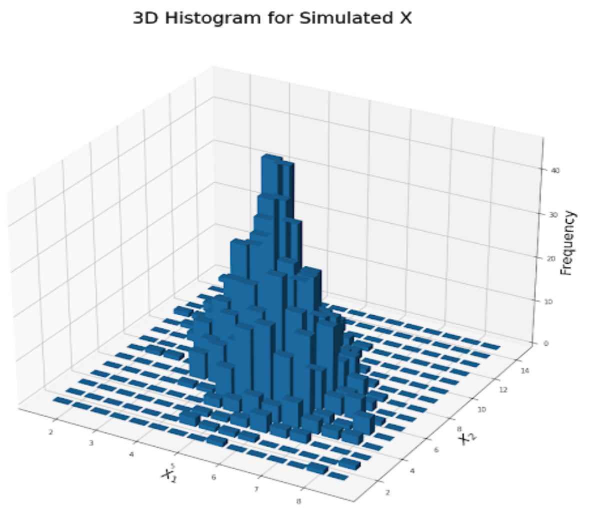
Now for 1000 values of (Z_1,Z_2) we obtain 1000 values of X for use in part 2 & 3.

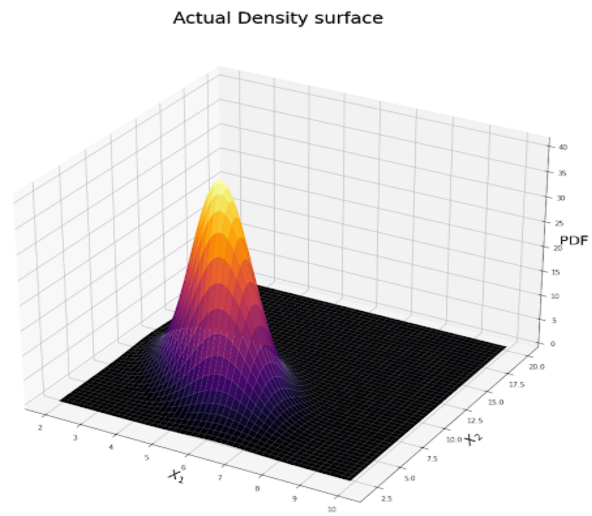
(2) For this question we need to plot the 3D bars for values of X as (X1,X2) on XY axis and frequencies on the Z axis.
This is achieved by various python libraries including numpy and matplotlib (see the code).

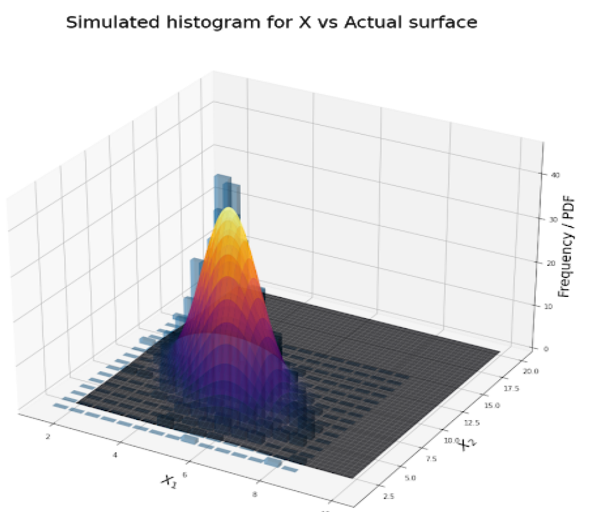
(3) In this part we have drawn the plots for both simulated and actual densities in the case of two dimensional and marginal one dimensional cases. We observe that in both cases, after appropriate scaling the curves are similar and thus prove correctness of the experiment.

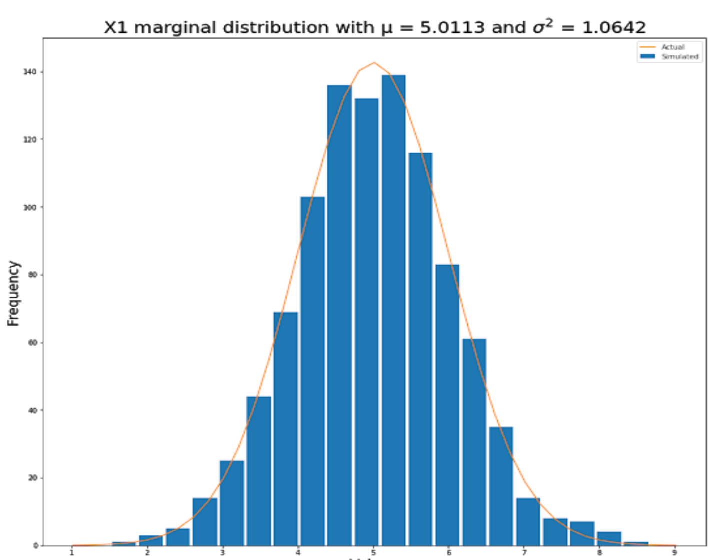
The output for various cases of a

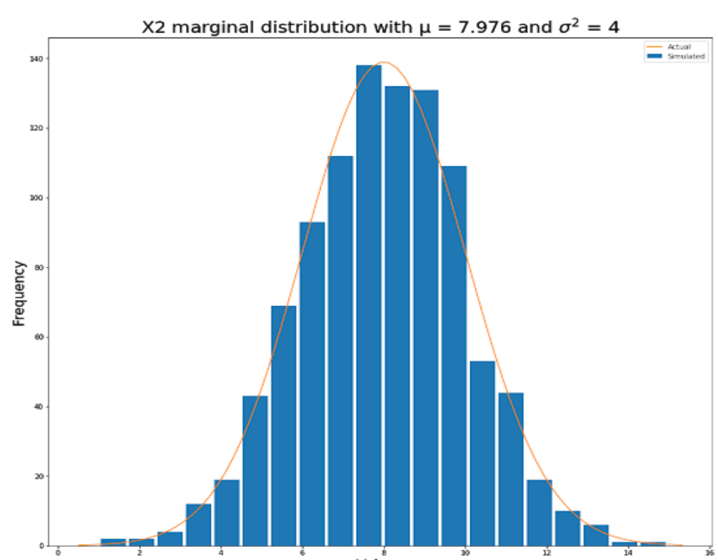
a. a=-0.5



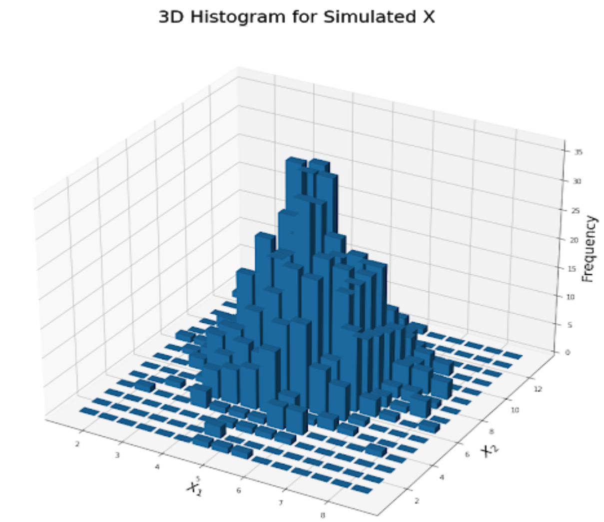


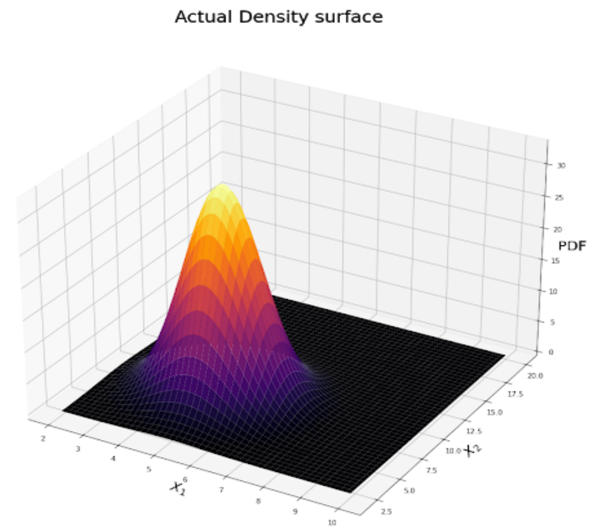


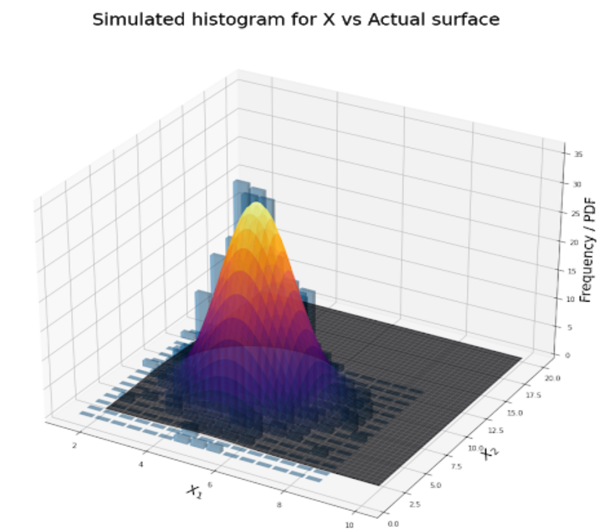


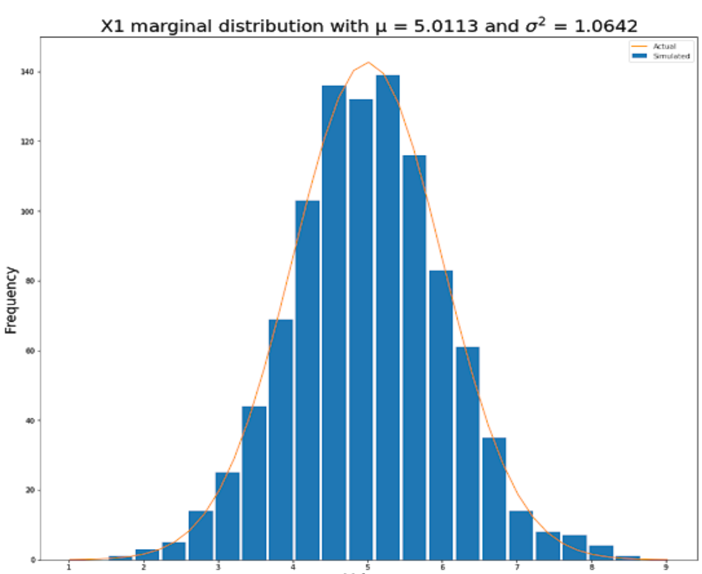


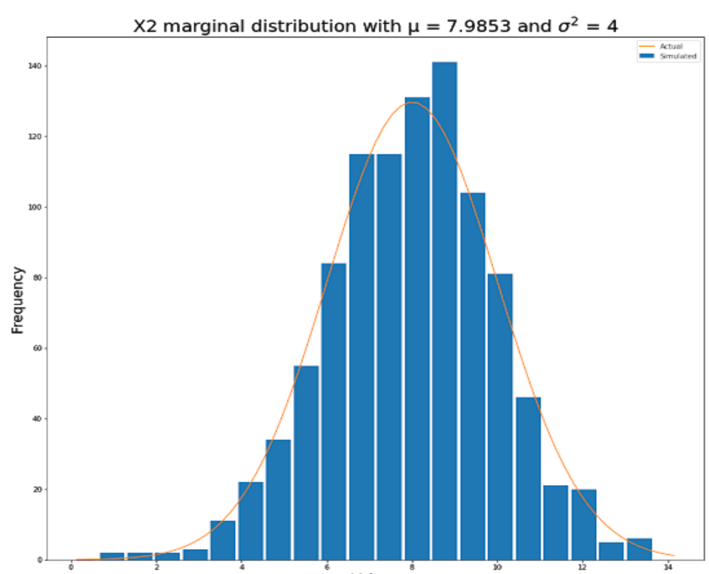
b. a=0.0


3D Histogram for Simulated X
Frequency
X_1
X_2

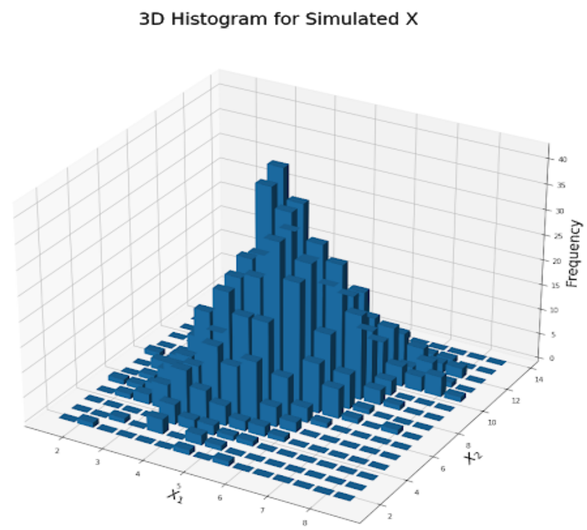

Actual Density surface
PDF
X_1
X_2

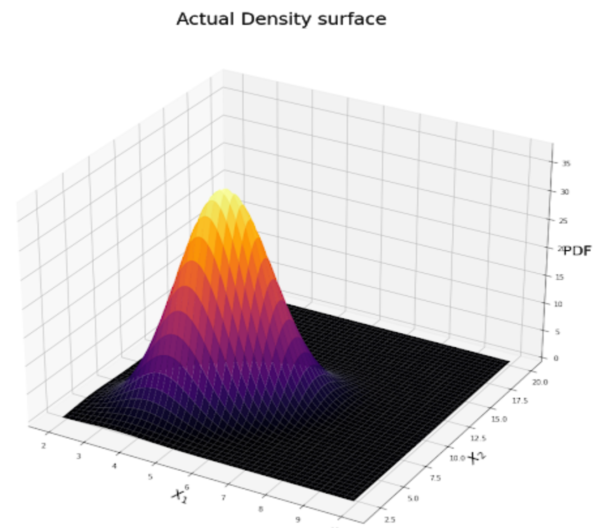

Simulated histogram for X vs Actual surface
Frequency / PDF
X_1
X_2

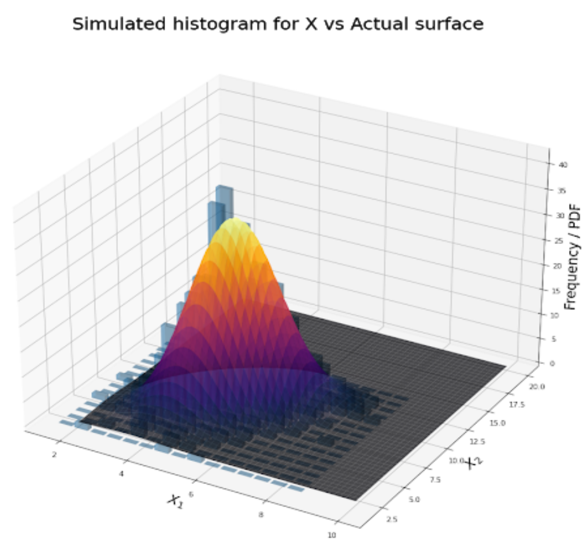

X1 marginal distribution with μ = 5.0113 and σ^2 = 1.0642
Actual
Simulated
Frequency
Values

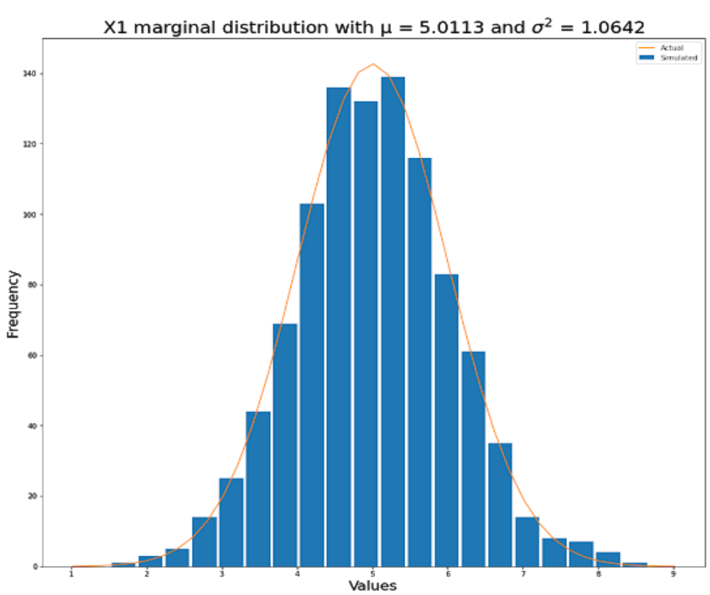

X2 marginal distribution with μ = 7.9853 and σ^2 = 4
Actual
Simulated
Frequency
Values

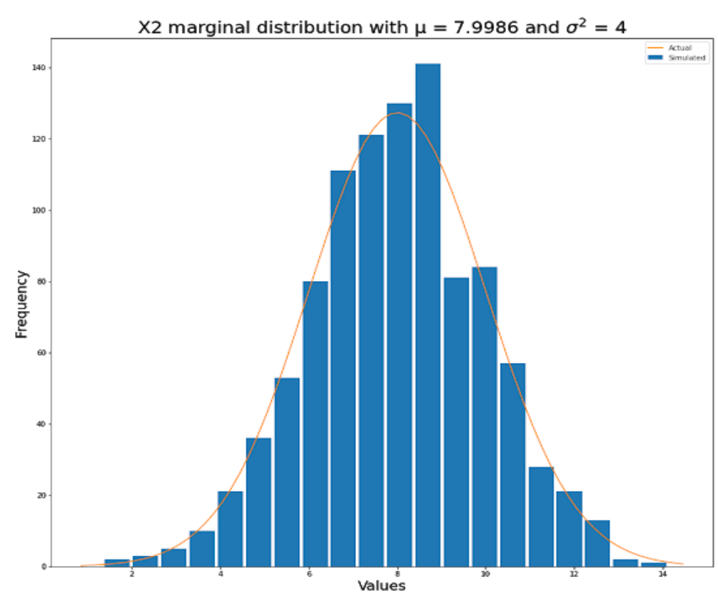
c. a=0.5


3D Histogram for Simulated X
Frequency
X_1
X_2

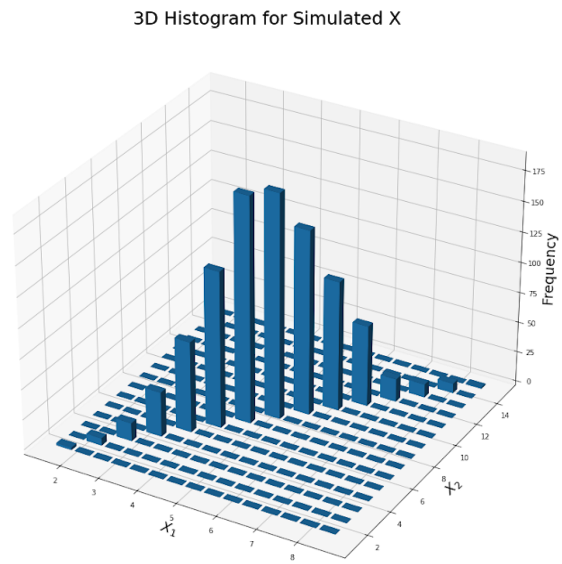

Actual Density surface
PDF
X_1
X_2

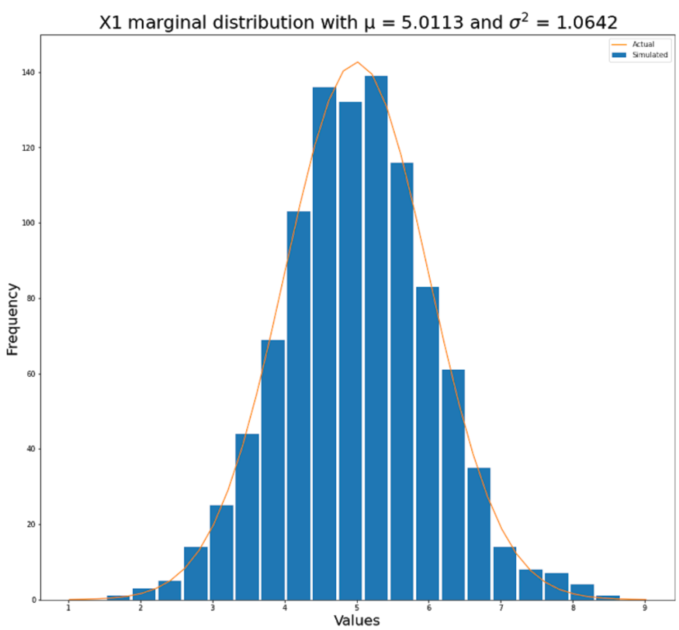

Simulated histogram for X vs Actual surface
Frequency / PDF
X_1
X_2

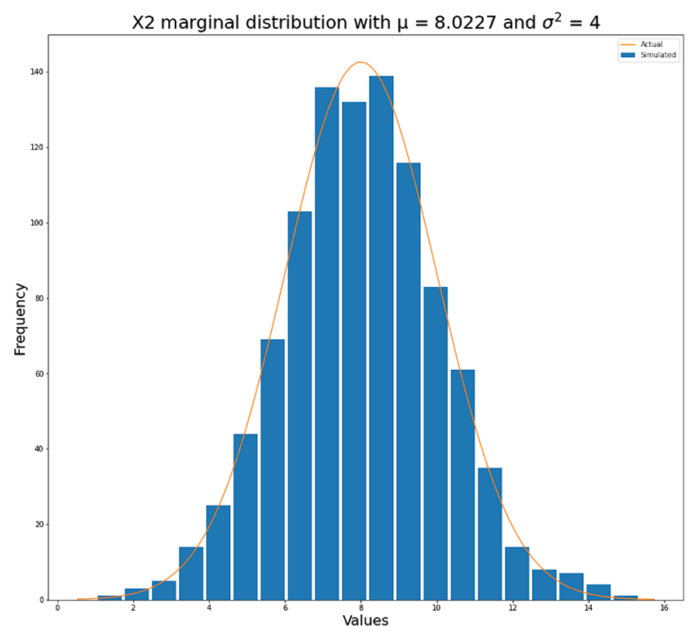

X1 marginal distribution with μ = 5.0113 and σ^2 = 1.0642
Actual
Simulated
Frequency
Values


X2 marginal distribution with μ = 7.9986 and σ^2 = 4
Actual
Simulated
Frequency
Values

d. a=1.0 (Special case)







In this case we print the following output in the terminal. This is because since the determinant for ∑ is 0, we don't have the actual PDF (it's indefinite by formula).

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The determinant for COV matrix is 0 and PDF does not exist in this case
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