

In the Bankruptcy problem:

$(C; M)$ where C is vector of claims $C = (c_1, c_2, c_3 \dots c_N)$ and M is the endowment or the size of the cake that needs to be divided.

$$\sum_{i=1}^N c_i > M.$$

From the figure 1, it is clear that the claimants with higher claims prefer constraint equal loss rule and the claimants with lower claims prefer constraint equal awards rule.

The proportional rule is preferred to constraint equal award rule when the claims are higher.

The constraint equal loss rule is preferred to proportional rule for the claimants with higher claims.

The claimants with lower claims prefer constraint equal award rule.

In case of surplus sharing problem:

The proportional rule remains same.

The constraint equal awards rule is simply sharing equally the total value of the endowment (egalitarian rule).

The constraint equal loss rule is equal surplus sharing. The surplus amount $M - c_1 - c_2$ is equally shared.

Surplus sharing problem:

$(C; M)$ where C is vector of investments $C = (c_1, c_2, c_3 \dots c_N)$ and M is the total revenue or the size of the cake that needs to be divided.

$$\sum_{i=1}^N c_i < M.$$

From the figure 2, it is clear that the claimants with higher claims prefer proportional rule and the claimants with lower claims prefer constraint equal awards rule (egalitarian rule).

The proportional rule is preferred to equal surplus sharing rule when the claims are higher.

The equal surplus sharing rule is preferred to constraint equal awards rule (egalitarian rule) for the claimants with higher claims. The claimants with lower claims prefer constraint equal award rule (egalitarian rule).

Properties of rule

- Feasibility: Given (C, M) , $\sum_{i=1}^N = M$, sum of the allocations must be equal to the endowment or size of the cake.
- Bounds on each allocation, $0 \leq x_i \leq c_i$ for all $i \in N$.
- Equal treatment of equals: Given any claims problem (C, M) , and for any i, j claimants, if $c_i = c_j$, then $x_i(c, M) = x_j(C, M)$. If the claims of any two claimants are same, they must be allocated same amount.

- Claims monotonicity: Given a claims problem (C, M) , for any claimant i , if $c'_i > c_i$, then $x_i(C, M) \leq x_i(c'_i, c_{-i}, M)$.

If the claim of any claimant i increases from c_i to c'_i and the claim of the rest of the claimants remains same, the amount allocated to claimant i should be greater than equal to the amount allocated when his claim was c_i .

Here $C = (c_1, c_2, c_3, c_3 \dots c_i, c_{i+1} \dots c_N)$, the claim vector $c_{-i} = (c_1, c_2, c_3, c_3 \dots c_{i-1}, c_{i+1} \dots c_N)$, the claims of all the claimants except claimant i is an element of this vector.

In case of three claimants, $C = (c_1, c_2, c_3)$,

$c_{-2} = (c_1, c_3)$, $(c_2, c_{-2}) = (c_1, c_2, c_3)$.

- Order preservation in awards: Given any claims problem (C, M) , For any two claimants i, j , if $c_i \geq c_j$, then $x_i(C, M) \geq x_j(C, M)$.

If the claim of a claimant is greater than some other claimant, then the claimant with greater claims should be allocated greater amount.

Another version of this property is

Given any claims problem (C, M) , For any two claimants i, j , if $c_i \geq c_j$, then $x_i(C, M) \geq x_j(C, M)$ and $c_i - x_i(C, M) \geq c_j - x_j(C, M)$.

If the claim of a claimant is greater than some other claimant, then the claimant with greater claims should be allocated greater amount and also, the loss made by the claimants with higher claims is greater than the loss made by the claimant with lower claims.