Problem:

Suppose a building worth 2000 per month to its owner. A cloth merchant is ready to pay a monthly rent of 2500, whereas a bank offers to pay 3000 per month. Find the core allocation of this game.

Solution:

First we have to formulate the characteristic function.
Suppose players are A, B and C, where A is owner, C is cloth merchant and B is the bank. $v_A = 2000$, $v_B = 0$, $v_C = 0$,
$v_{AC} = 2500$, $v_{AB} = 3000$, $v_{BC} = 0$ and $v_{ABC} = 3000$.
$x_A \geq 2000$, $x_B \geq 0$, $x_C \geq 0$.
$x_A + x_C \geq 2500$, $x_A + x_B \geq 3000$, $x_B + x_C \geq 0$

$x_A + x_C + x_B \geq 3000.$

Substituting $x_B + x_C \geq 0$ in $x_A + x_C + x_B = 3000$, since core allocation is an imputation we have, $x_A \leq 3000$ And we have $x_A \geq 2000$. So we get that
$2000 \leq x_A \leq 3000.$

Again we have $x_B \leq 500$, by substituting $x_A + x_C \geq 2500$ in $x_A + x_C + x_B = 3000$. This implies that $0 \leq x_B \leq 500$.

We have $x_C \leq 0$, by substituting $x_A + x_B \geq 3000$ in $x_A + x_C + x_B = 3000$. This implies that $0 = x_C$.
Thus, core allocations are
$2500 \leq x_A \leq 3000$, $0 \leq x_B \leq 500$, and $x_C = 0$.

Problem:
Suppose there are four players $\{A_1, A_2, A_3, A_4\}$, suppose there are two disjoint set L and R of these players. $R = \{A_1, A_2, \}$ and $L = \{A_3, A_4, \}$. Players of set R has right shoe and players in set L has left shoe. A pair of shoe contains two shoes - left and right. A pair of shoe worth 1 and if there is only left or only right, it has no value. Consider set $S = \{A_1, A_3, A_4\}$, so one right shoe. $S \cap R = \{A_1\}$ and $L \cap S = \{A_3, A_4\}$, two left shoes. If there is cooperation between the players in S, then it will have one pair of shoe. The minimum number of left or right shoes determine the number of pairs. So the worth of coalition is $\min\{|S \cap R|, |S \cap L|\}$, when $|S| \geq 2$.
We get the following characteristic function.

$$v(s) = \begin{cases} 0, \ \ if \ \ |S| \in \{0,1\}, \\ \min\{|S \cap R|, |S \cap L|\}, \ \ if \ \ |S| \geq 2. \end{cases}$$

$v(N) = \min\{|R|, |L|\}$.

We need to find core allocation.

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 0$
$\{2\}$	$v(\{2\}) = 0$
$\{3\}$	$v(\{3\}) = 0$
$\{4\}$	$v(\{4\}) = 0$
$\{1,2\}$	$v(\{1,2\}) = 0$
$\{1,3\}$	$v(\{1,3\}) = 1$
$\{1,4\}$	$v(\{2,3\}) = 1$
$\{2,3\}$	$v(\{2,3\}) = 1$
$\{2,4\}$	$v(\{2,4\}) = 1$
$\{4,3\}$	$v(\{4,3\}) = 0$
$\{1,2,3\}$	$v(\{1,2,3\}) = 1$
$\{1,2,4\}$	$v(\{1,2,4\}) = 1$

coalitions	v()
$\{1,3,4\}$	$v(\{1,4,3\}) = 1$
$\{2,3,4\}$	$v(\{4,2,3\}) = 1$
$\{1,2,3,4\}$	$v(\{1,4,2,3\}) = 2$

$x_i \geq 0,\ i = 1,2,3,4.$
$x_1 + x_3 \geq 1,\ x_1 + x_4 \geq 1,\ x_2 + x_3 \geq 1,\ x_2 + x_4 \geq 1,$
$x_1 + x_2 \geq 0,\ x_4 + x_3 \geq 0.$
$x_1 + x_2 + x_3 \geq 1,\ x_1 + x_2 + x_4 \geq 1,\ x_1 + x_4 + x_3 \geq 1,$
$x_4 + x_2 + x_3 \geq 1.$
$x_1 + x_2 + x_3 + x_4 \geq 2$

We have $x_1 + x_3 \leq 1$, by substituting $x_2 + x_4 \geq 1$ in $x_1 + x_2 + x_3 + x_4 = 2$. And we have $x_1 + x_3 \geq 1$, so $x_1 + x_3 = 1$ Similarly we have $x_2 + x_4 \leq 1$ and we have $x_2 + x_4 \geq 1$, so $x_2 + x_4 = 1$.

We also have $x_1 + x_4 = 1$ and $x_2 + x_3 = 1$.
We also get $x_1 \leq 1$ by substituting $x_4 + x_2 + x_3 \geq 1$ in $x_1 + x_2 + x_3 + x_4 = 2$. Similarly we have $x_2 \leq 1$, $x_3 \leq 1$, $x_4 \leq 1$.

From $x_1 + x_3 = 1$, $x_2 + x_4 = 1$, $x_1 + x_4 = 1$ $x_2 + x_3 = 1$ and $x_1 + x_2 + x_3 + x_4 = 2$. We have $x_1 = x_2$ and $x_3 = x_4$.

The core allocations are
$0 \leq x_i \leq 1, i = 1, 2, 3, 4$, $x_1 = x_2$, $x_3 = x_4$, and
$x_1 + x_2 + x_3 + x_4 = 2$.