Lab Number: **02**
Due Date: **Sept 16, 2020**
Student Details:

- Name: **AB Satyaprakash**
- Roll Number: **180123062**
- Department: **Mathematics and Computing**

Question 1:

Observations:

a. 17 values of U were generated with the help of linear congruence generator, with the following seed:
x0=23, **m**=4096, **a**=17, **b**=1

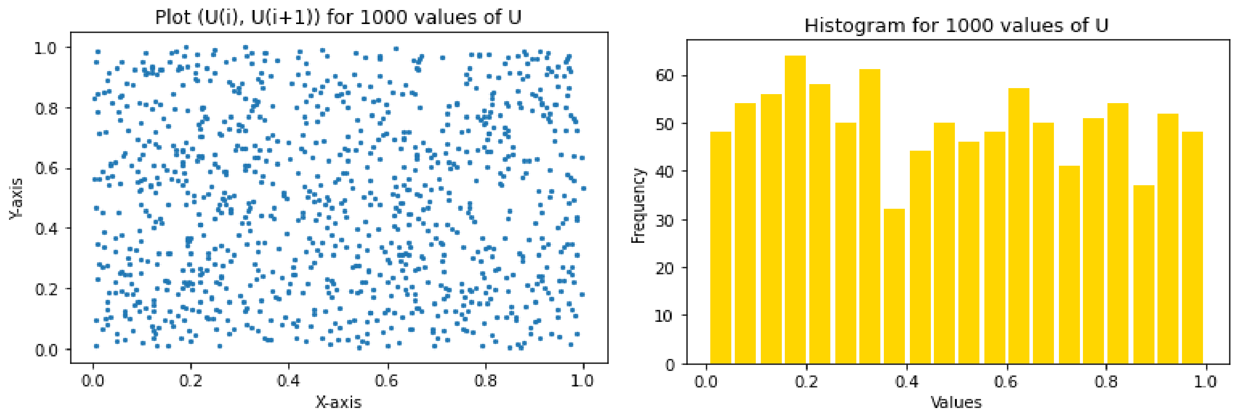
$$x_{i+1} = (ax_i + b) \bmod m$$
$$u_{i+1} = \frac{x_{i+1}}{m}$$

i.e. the formula for *Linear Congruence Generator*

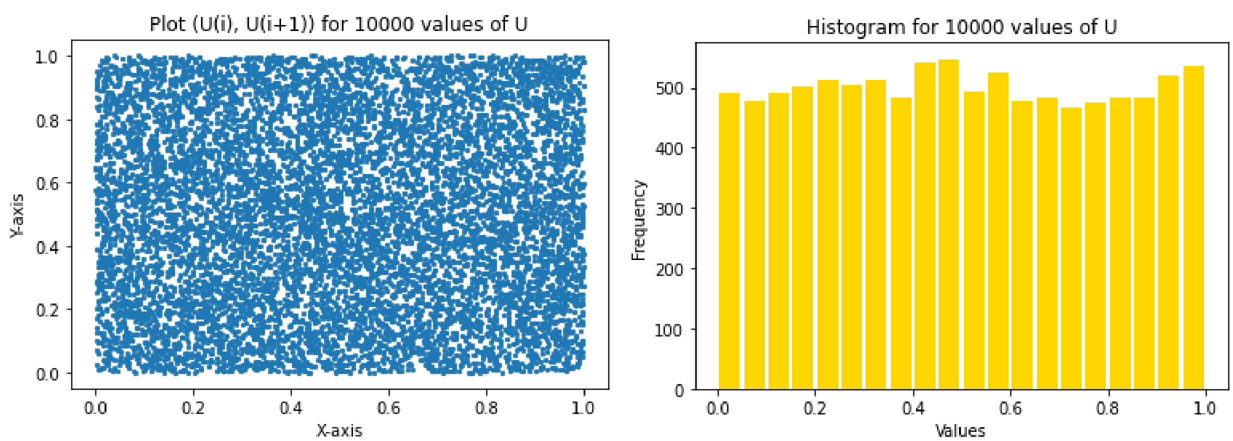
b. Using the formula for *lagged Fibonacci generator*, we generate 100000 values of U (and then use the first 1000, first 10000 and the 100000 values)

c. We can observe that the plot gets denser as we go from 1000 to 100000 values of U to plot (Ui, Ui+1). This means that the points are getting more randomized.
In other words, we are able to obtain random numbers with longer period length with the help of *Lagged Fibonacci* generators as compared to the *linear congruence generator.*
From the bar diagram we observe that the frequency values are not all the same. This means it’s not quite uniform in distribution between [0,1) like linear congruence generators. Also, uniformity increases with increase in the number of U values taken into consideration, i.e. uniformity increases from 1000 to 100000 values for U.

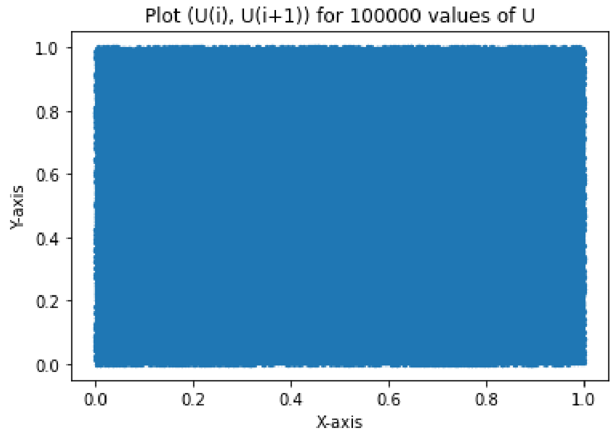
Output:

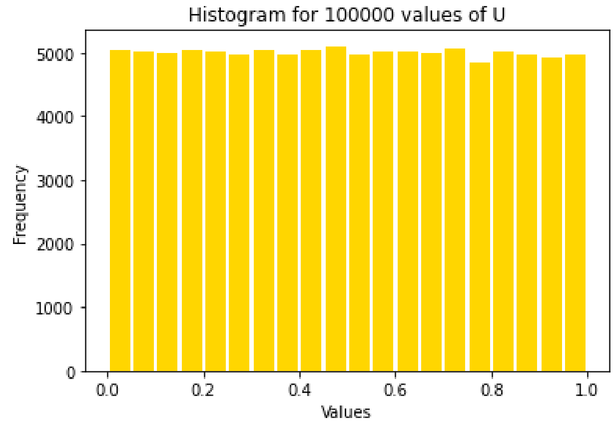


These are the plots for the first 1000 values of U generated by the *Lagged Fibonacci* generator.
Scatter Plot for (U(i), U(i+1)) is completely random, unlike Linear Congruence Generator.
Bar Diagram shows that the frequency values have large changes (from as high as 65 to as less as 30).



These are the plots for the first 10000 values of U generated by the *Lagged Fibonacci* generator.
Scatter Plot for (U(i), U(i+1)) has started to grow dense, indicating longer period length.
Bar Diagram shows that the frequency values have relatively smaller deviations.





These are the plots for the first 100000 values of U generated by the *Lagged Fibonacci* generator.
Scatter Plot for (U(i), U(i+1)) has grown very dense, indicating longest period length so far.
Bar Diagram shows that the frequency values are nearly equal showing a uniform random distribution.

Question 2:

Observations:

The mean of the function is taken as ϴ=0.5. The variance is thus 0.25 (since it's the CDF of an exponential function).

The U ~ $U[0,1]$ is generated via linear congruence generator for 100000 values with the following seed:

x0=23, **m**=4096, **a**=17, **b**=1

$$x_{i+1} = (ax_i + b) \bmod m$$
$$u_{i+1} = \frac{x_{i+1}}{m}$$

i.e. the formula for *Linear Congruence Generator*

Using the formula,

$$\boldsymbol{X = -\theta \log(1 - U)}$$

Inverse of exponential distribution, we get the values of X.

Using these values of X, we obtain F(x).

After that the 2 plots

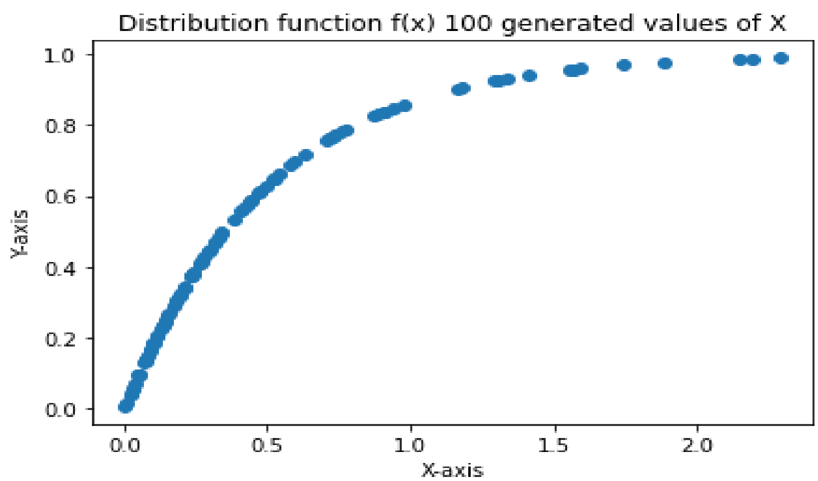
a. Cumulative Freq(x) vs interval [0,5] was plotted taking 0.05 interval lengths. The graph was plotted between cumulative-frequency(x) vs mid-points of intervals and joined by a smooth line.
b. The plot of F(x) vs x was plotted using the formula:
$$\boldsymbol{F(x) = 1 - e^{-x/\theta}}$$
were drawn for 1000, 10000, and 100000 values.

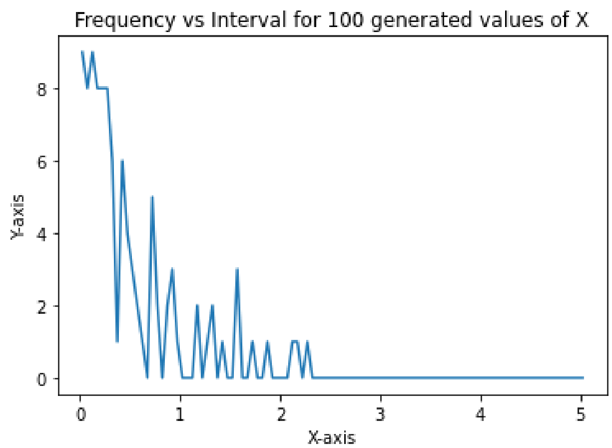
Both the plots were compared and were found to be nearly identical (they became more identical with increase in the number of values from 100 to 100000 values)

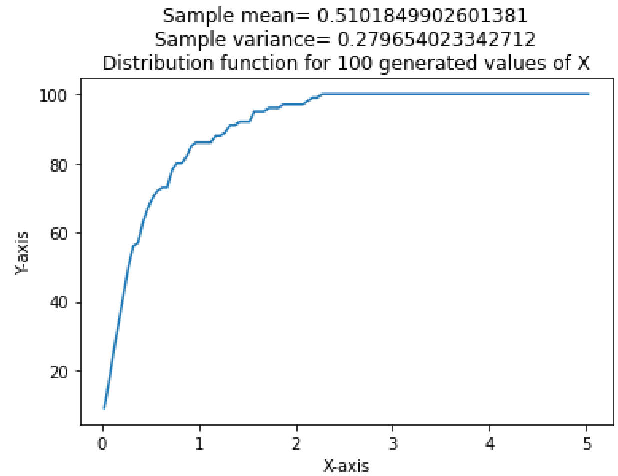
Also, the sample mean and sample variance were calculated in each case and were found to be close to the actual mean and variance.

Output:

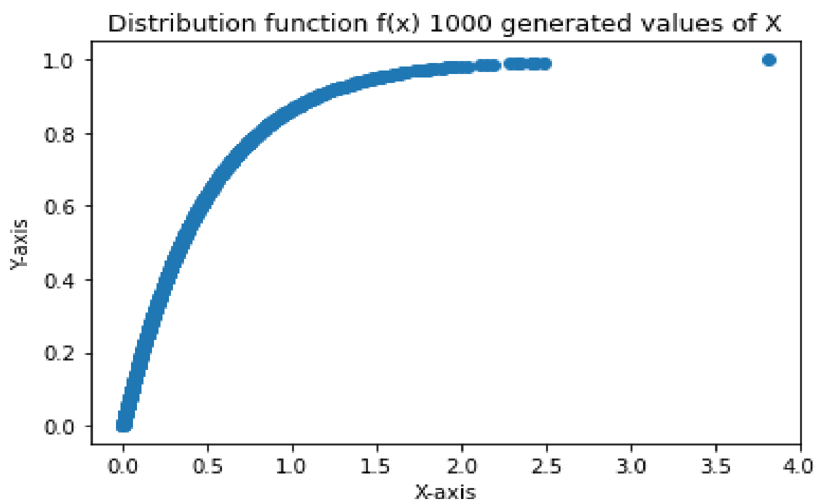
```
Actual Mean= 0.5
Actual Variance= 0.25
```

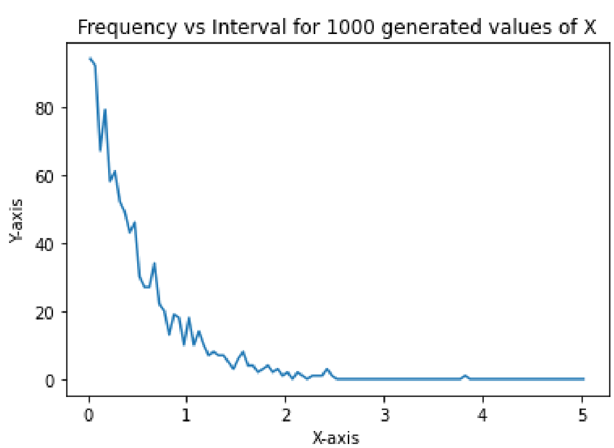


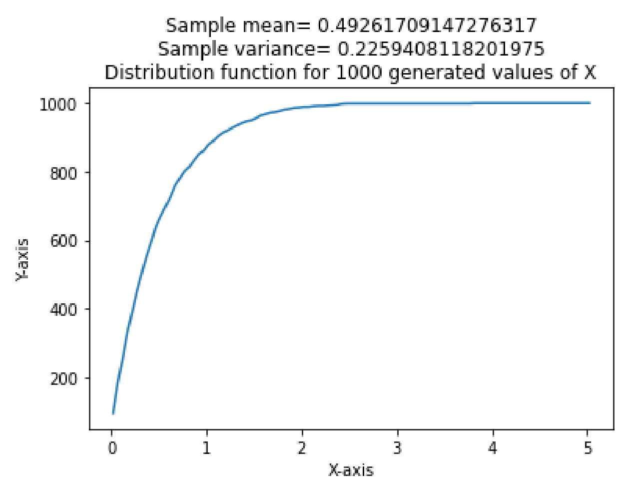




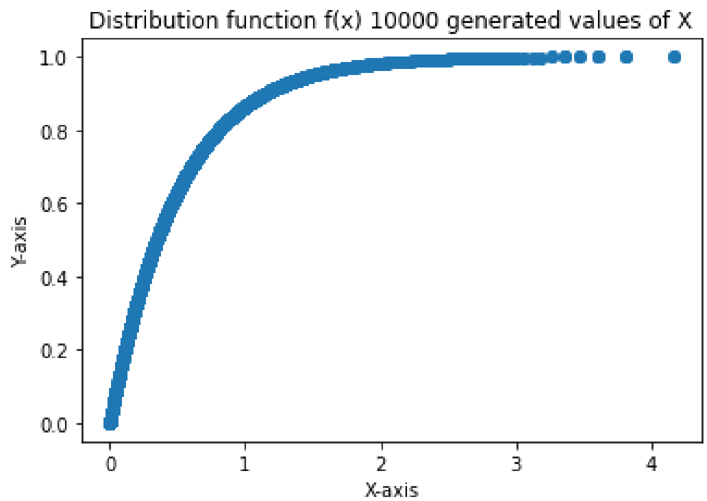
The first graph is f(x) vs x, for 100 generated values of x (scatter plot). The second graph is frequency(x) vs the interval [0,5], and the third is the cumulative frequency(x) vs interval [0,5]. It's clearly evident that the CDF (graph 3) resembles the actual CDF scatter plot of f(x).

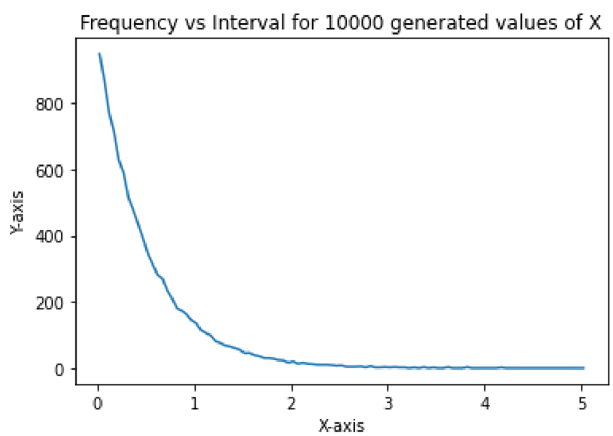


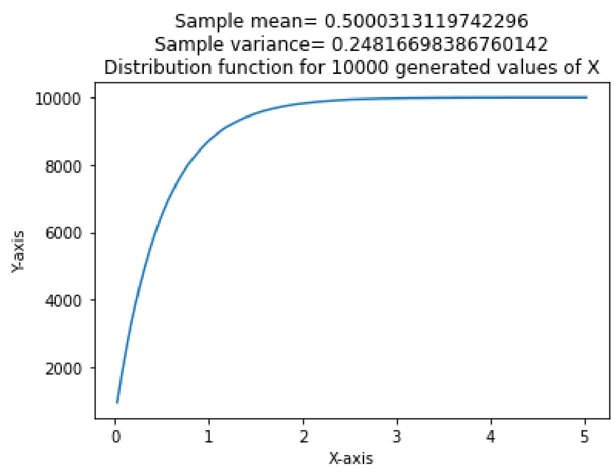




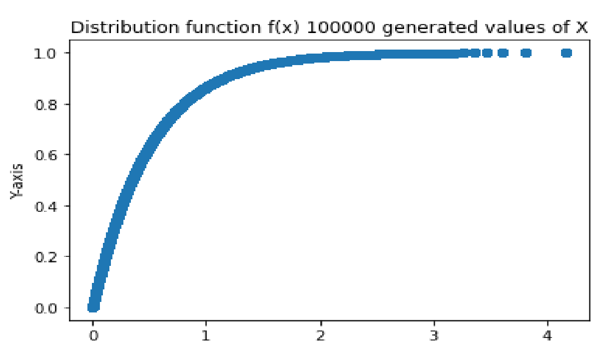
The first graph is f(x) vs x, for 1000 generated values of x (scatter plot). The second graph is frequency(x) vs the interval [0,5], and the third is the cumulative frequency(x) vs interval [0,5]. It's clearly evident that the CDF (graph 3) resembles the actual CDF scatter plot of f(x) even more in this case.

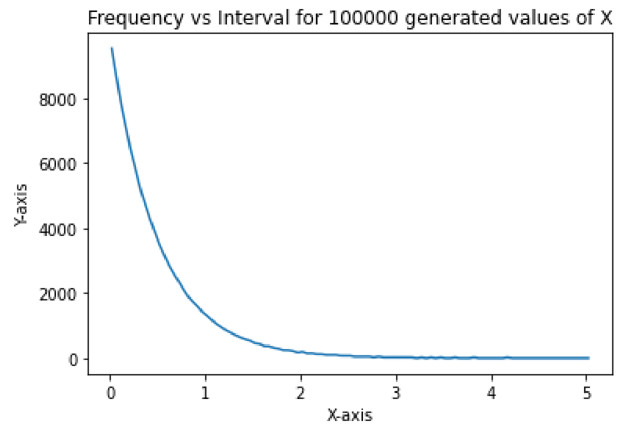


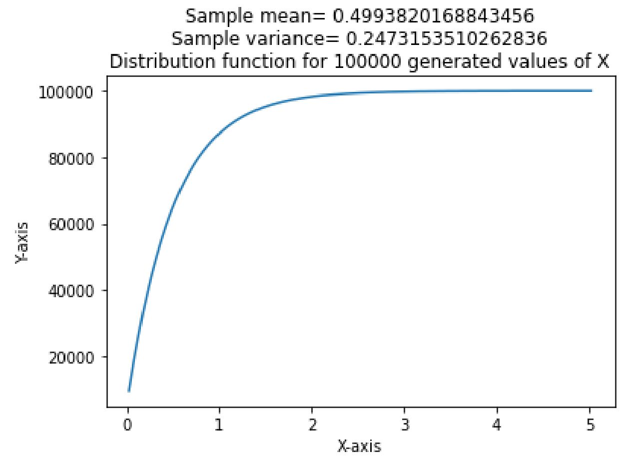




In this case we can see the deviation in the curves have reduced to a great extent. The sample mean and variance also looks very similar to the actual mean and variance.







This is the case for 100000 values of generated x. The sample mean = 0.4994 and variance=0.2473.

Question 3:

Observations:

The U ~ $U[0,1]$ is generated via linear congruence generator for 100000 values with the following seed:
x0=23, **m**=4096, **a**=17, **b**=1

$$x_{i+1} = (ax_i + b) \bmod m$$
$$u_{i+1} = \frac{x_{i+1}}{m}$$

Using the formula

$$\boldsymbol{X = \frac{1}{2} - \frac{1}{2}(\cos(u * \pi)}$$

i.e., *Inverse of arcsine distribution*, we get the values of X. And using these values of X we obtain F(x).

After that the 2 plots

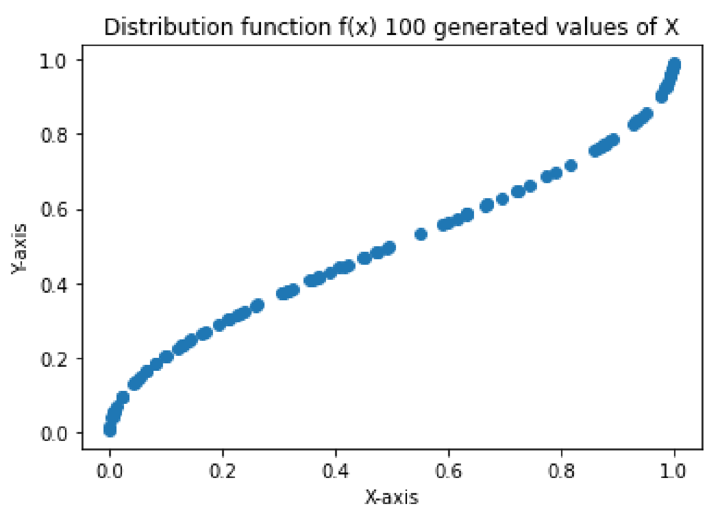
a. Cumulative Freq(x) vs interval [0,1] was plotted taking 0.05 interval lengths. The graph was plotted between cumulative-frequency(x) vs mid-points of intervals and joined by a smooth line
b. The plot of F(x) vs x was plotted using the formula:

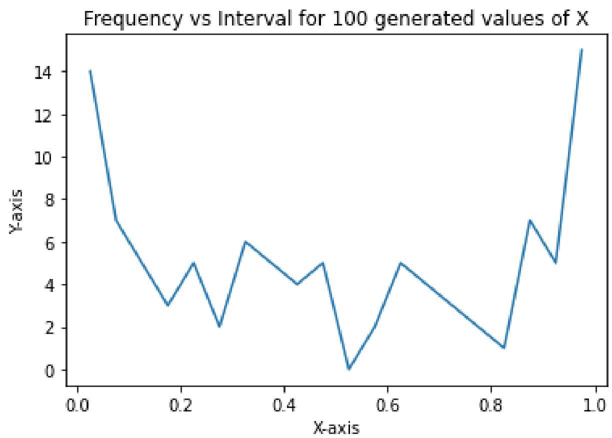
$$\boldsymbol{F(x) = \frac{2}{\pi}(\arcsin(\sqrt{x}))}$$

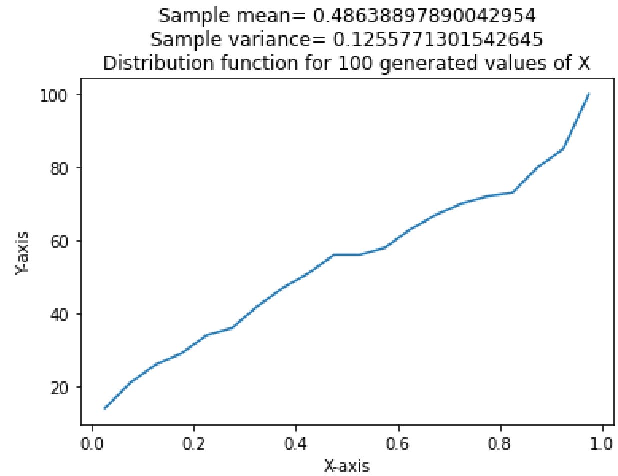
And was drawn for 1000, 10000, and 100000 values.

Both the plots were compared and were found to be nearly identical.
Also, the sample mean and sample variance were calculated in each case.

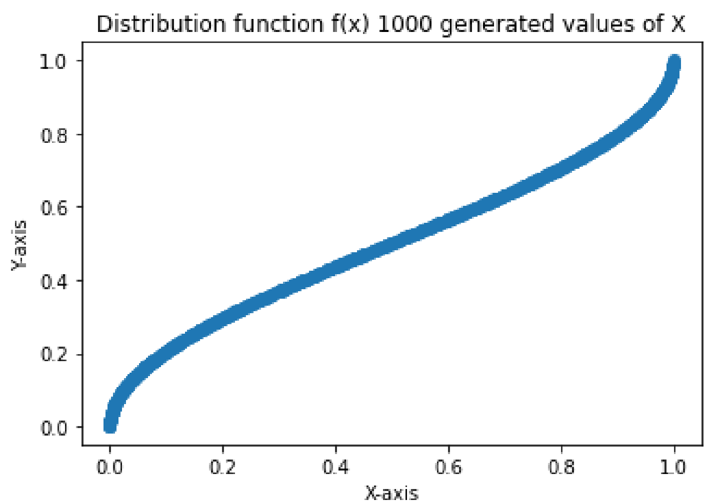
Output:

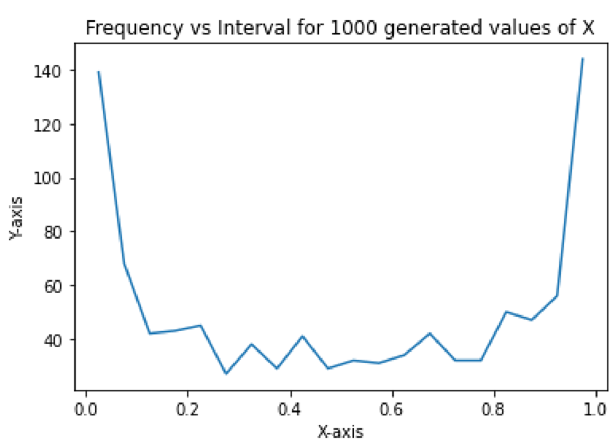


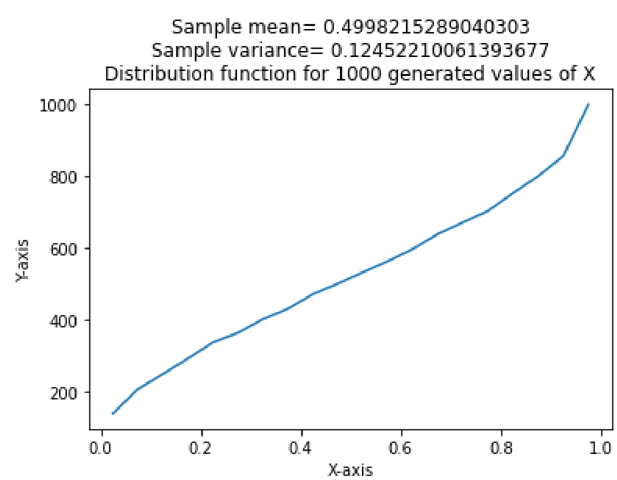




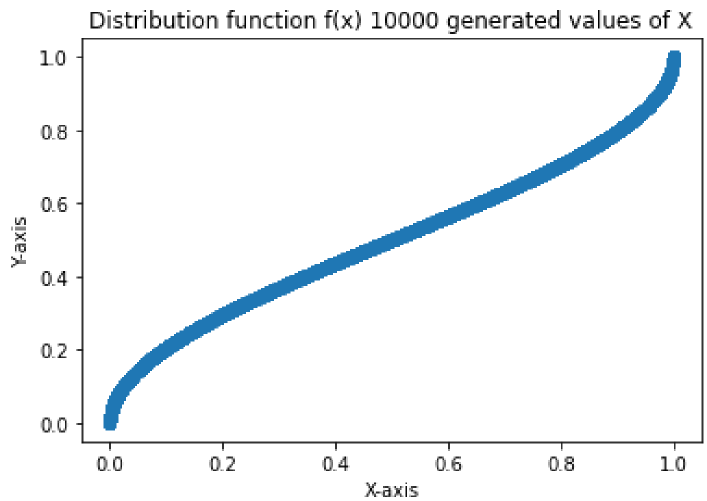
The first graph is f(x) vs x, for 100 generated values of x (scatter plot). The second graph is frequency(x) vs the interval [0,1], and the third is the cumulative frequency(x) vs interval [0,1]. It's clearly evident that the CDF (graph 3) resembles the actual CDF scatter plot of f(x).

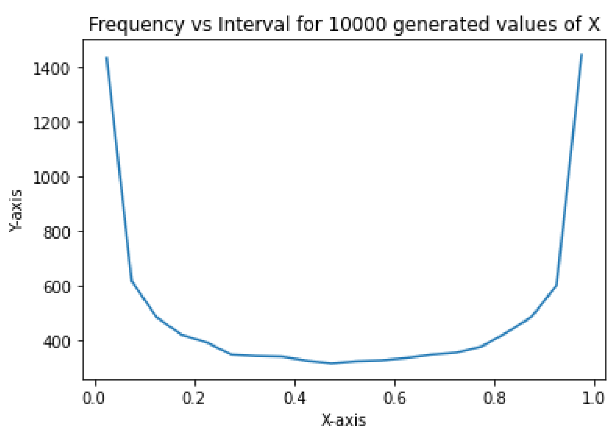


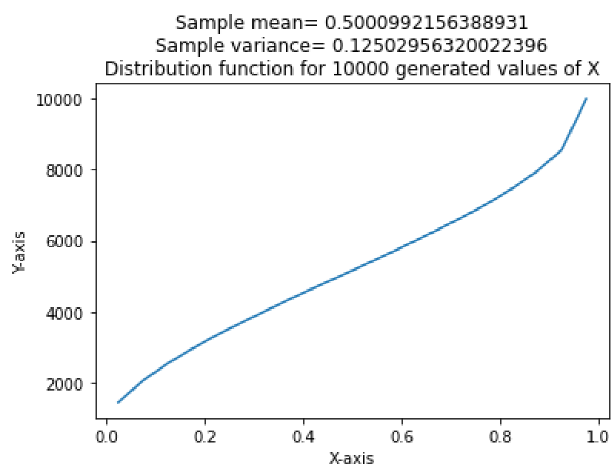




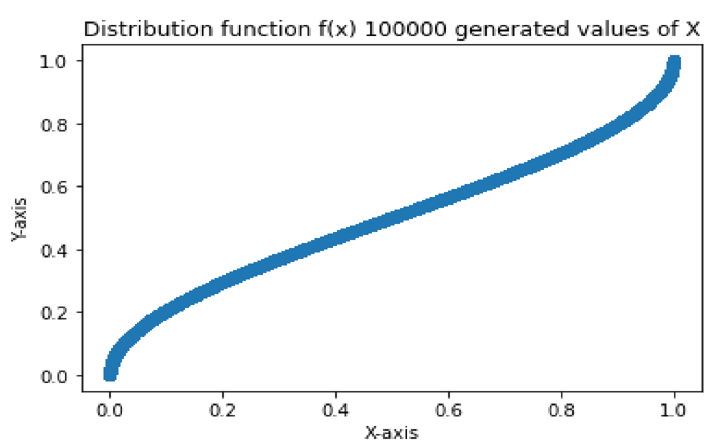
The first graph is f(x) vs x, for 1000 generated values of x (scatter plot). The second graph is frequency(x) vs the interval [0,1], and the third is the cumulative frequency(x) vs interval [0,1]. It's clearly evident that the CDF (graph 3) resembles the actual CDF scatter plot of f(x) even more in this case.

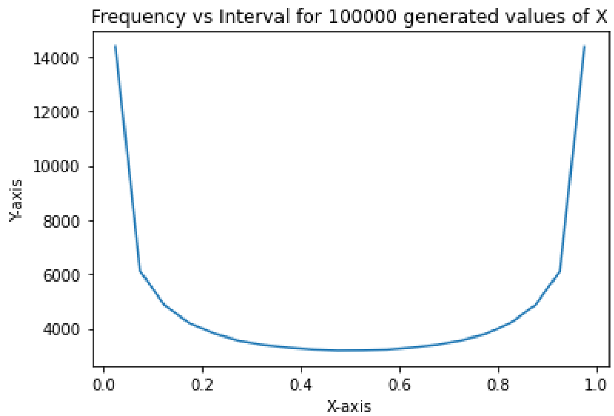


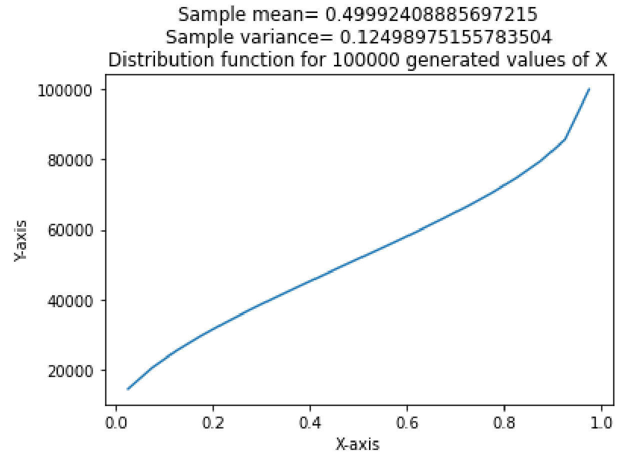




Graphs corresponding to 10000 values of generated x. The mean is ~0.5 and variance~0.125







Graph corresponding to 100000 generated values of x. We can see that the CDF we have generated is very similar to f(x) we obtain.