भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 322: Scientific Computing Lab

Lab 03

AB Satyaprakash (180123062)

15th Feb 2021

Question 1.

Newton's forward difference formula for an interpolating polynomial of degree n is given as:

$$P_n(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \ldots + \frac{u(u-1)(u-2)\ldots(u-(n-1))}{n!}\Delta^n f(x_0)$$

where,

$\Delta^k f(x_0)\, are\, found\, using\, FD\, table$

$u = \frac{x-x_0}{h}\, and\, h = x_1 - x_0$

Using the above formulae:

a.

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Newton's interpolating polynomial of degree 1 using
nodes: 0.25, 0.5 is  4.278 x + 0.5792
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Approximated value of f(0.43) using the above is 2.4188032

Newton's interpolating polynomial of degree 2 using nodes: 0.25, 0.5, 0.75 is $5.551x^2 + 0.1151x + 1.273$

Approximated value of f(0.43) using the above is 2.34886312

Newton's interpolating polynomial of degree 3 using nodes: 0, 0.25, 0.5, 0.75 is $2.912x^3 + 1.183x^2 + 2.117x + 1$

Approximated value of f(0.43) using the above is 2.36060473408

b.

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Newton's interpolating polynomial of degree 1 using
nodes: 0.1, 0.2 is  -2.707 x - 0.0193
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Approximated value of f(0.18) using the above is -0.506647844

Newton's interpolating polynomial of degree 2 using nodes: 0.1, 0.2, 0.3 is $0.8763x^2 - 2.97x - 0.001777$

Approximated value of f(0.18) using the above is -0.508049852

Newton's interpolating polynomial of degree 3 using nodes: 0.1, 0.2, 0.3, 0.4 is
$-0.4855x^3 + 1.168x^2 - 3.024x + 0.001136$

Approximated value of f(0.18) using the above is -0.5081430744000002

Question 2.

Newton's backward difference formula for an interpolating polynomial of degree n is given as:

$$P_n(x) = f(x_n) + v\Delta f(x_{n-1}) + \frac{v(v+1)}{2!}\Delta^2 f(x_{n-2}) + \ldots + \frac{v(v+1)(v+2)\ldots(v+(n-1))}{n!}\Delta^n f(x_0)$$

where,

$\Delta^k f(x_0)\, are\, found\, using\, FD\, table$

$v = \frac{x-x_n}{h}\, and\, h = x_n - x_{n-1}$

Using the above formulae:

a.

Newton's interpolating polynomial of degree 1 using nodes: -0.5, -0.25 is 1.439 x + 0.6946

Approximated value of f(-0.3333333333333333) using the above is 0.21504166666666674

Newton's interpolating polynomial of degree 2 using nodes: -0.75, -0.5, -0.25 is $2.501x^2 + 3.314x + 1.007$

Approximated value of f(-0.3333333333333333) using the above is 0.18030555555555572

Newton's interpolating polynomial of degree 3 using nodes: -0.75, -0.5, -0.25, 0 is $x^3 + 4.001x^2 + 4.002x + 1.101$

Approximated value of f(-0.3333333333333333) using the above is 0.17451851851851852

b.

Newton's interpolating polynomial of degree 1 using nodes: 0.2, 0.3 is 2.906 x - 0.8652

Approximated value of f(0.25) using the above is -0.13869286499999999

Newton's interpolating polynomial of degree 2 using nodes: 0.1, 0.2, 0.3 is $-2.296x^2 + 4.054x - 1.003$

Approximated value of f(0.25) using the above is -0.13295220624999993

Newton's interpolating polynomial of degree 3 using nodes: 0.1, 0.2, 0.3, 0.4 is $-0.4732x^3 - 2.012x^2 + 4.002x - 1$

Approximated value of f(0.25) using the above is -0.13277477437499996

Question 3.

Given
$\Delta(P(0)) = 24, \Delta^3(P(0)) = 6, \Delta^2(P(0)) = 0 \, and \, \Delta P(x) = P(x+1) - P(x)$.

- Using the 3rd relation, i.e., Δ_2(P(0))=0,

  We can write this as P(x) = P(0) + ∆P(0)·x + x(x-1)(x-2) + x(x-1)(x-2)(x-3).

- To obtain Δ^2(P(10)) we will use Δ^2(P(x)) at x = 10, which will remove the constant and linear term.
- This makes Δ^2(P(x)) = Δ^2([$x^4 - 5x^3 + 8x^2 - 4x$]).
- Again using the fact that second differences will remove the const and linear term,

  We get Δ^2([$x^4 - 5x^3 + 8x^2$]) at x=10.

- Finally use: Δ^2([f(x)]) = f(x+2)- 2f(x+1)+ f(x).

  Thus, the value of Δ^2(P(10)) = 1140

Question 4.

Given $g(x) = \frac{\sin x}{x^2}$ and the following data:

x =	0.1	0.2	0.3	0.4	0.5
g(x) =	**9.9833**	**4.9667**	**3.2836**	**2.4339**	**1.9177**

We will use similar methods of forward-differences with a polynomial of degree 3 as in Question 1. to obtain the approximation of g(0.25) and 0.25(g(0.25)) in cases (i) and (ii) respectively.

We also obtain the error term using:

$$\frac{(x-0.1)(x-0.2)(x-0.3)(x-0.4)}{4!(0.1)^4}\Delta^4 f_0$$

a. Thus, the approximation of g(0.25) using direct interpolation = P(0.25) = **3.864718750000005**
   The error term is at x = 0.25 is 0.0468796874999999

b. Thus, the approximation of g(0.25) using interpolation on xg(x) table = P(0.25) = **3.9584725**
   The error term is at x = 0.25 is 5.62499999994317e-6

c. **Since the differences in (i) are oscillating and are not decreasing fast, the resulting error in interpolation would be large.**
   However, the differences in (ii) tend to become smaller in magnitude, we expect more accurate results in this case.

Question 5.

Given polynomials:

$P(x) = 3 - 2(x + 1) + 0(x + 1)(x) + (x + 1)(x)(x - 1)$
$Q(x) = -1 + 4(x + 2) - 3(x + 2)(x + 1) + (x + 2)(x + 1)(x)$

a. To show that both P and Q interpolate the data, we will find the values of P(x) and Q(x) at all the given nodes and show that the values are the same as the function data.

   The value of P(-2) = -1
   The value of P(-1) = 3
   The value of P(0) = 1
   The value of P(1) = -1
   The value of P(2) = 3

   The value of Q(-2) = -1
   The value of Q(-1) = 3
   The value of Q(0) = 1
   The value of Q(1) = -1
   The value of Q(2) = 3

   Thus, both cubic polynomials P(x) and Q(x) interpolate the given data

b. **Simplifying P(x) we get** $x^3 - 3x + 1$
   Simplifying Q(x) we get $x^3 - 3x + 1$
   Since we can clearly see that P(x) = Q(x), this ensures that the uniqueness property of interpolating polynomials is not violated

Question 6.

As we can simply observe if all 4th order forward differences are 1, this means that all 5th order forward differences will be 0, in other words, **the degree of the polynomial is 4**

Since we are given $P(0) = 4, P(1) = 9, P(2) = 15, P(3) = 18$, we can construct the FD Table:

The difference Table

x	$f(x_n)$	$\Delta f(x_n)$	$\Delta^2 f(x_n)$	$\Delta^3 f(x_n)$	$\Delta^4 f(x_n)$	$\Delta^5 f(x_n)$
0	4					
		5				
1	9		1			
		6		-4		
2	15		-3		1	
		3		??		0
3	18		??		1	
		??		??		0
4	??		??		1	
		??		??		
5	??		??			
		??				
6	??					

Also, we know $\Delta^4 f(x_0) = 1$, so we can construct the 4th-degree polynomial, without knowing about the **??** values.

The polynomial is thus given as

$$0.04167x^4 - 0.9167x^3 + 2.958x^2 + 2.917x + 4$$

Clearly, the coefficient of x^3 is -0.9166666666666666 or -11/12

Question 7.

Given $P(x) = 1 + 4x + 4x(x - 0.25) + \frac{16}{3}x(x - 0.25)(x - 0.5)$

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Using P(x) we find the following:
f[x0,x1,x2,x3] = 16/3
f[x0,x1,x2] = 4
f[x0,x1] = 4
f[x0] = 1

Using the above information and simple arithmetic, the
divided differences table is obtained as follows:
x         f
------------------------------------------------------------
0         1
0.25      2    4
0.5       3.5  6    4
0.75      6    10   8    5.333333333333333
```

Clearly, from the above table, we get f(0.75) = 6