A function $v : 2^N \to R_+$ such that $v(\emptyset) = 0$, where R_+ denotes non negative real numbers.
$v()$ is a characteristic function. There are games of the following nature

$v(N) = 0$
$v(S) = -v(N \setminus S)$ for all subsets S of N.
$v(\emptyset) = 0$.
$v(S \cup T) \geq v(S) + v(T)$ where S and T are disjoint subsets of N.

$v()$ is a characteristic function of zero -sum game.
Here in this case, the range of the characteristic function is all real number
$v : 2^N \to R$ such that $v(\emptyset) = 0$ and R denotes real numbers.

G^N denotes the set of all characteristic form games with transferable utility.
What do we mean by transferable utility.

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 0$
$\{2\}$	$v(\{2\}) = 0$
$\{3\}$	$v(\{3\}) = 0$
$\{1,2\}$	$v(\{1,2\}) = 1$
$\{1,3\}$	$v(\{1,3\}) = 1$
$\{2,3\}$	$v(\{2,3\}) = 1$
$\{1,2,3\}$	$v(\{1,2,3\}) = 3$

Here, the value of coalition can be shared in any way like $v(\{1,2\}) = 1$ can be shared as $(x_1, x_2) = (0.2, 0.8)$, $(x_1, x_2) = (0.5, 0.5)$. Each allocation gives a specific utility to each player. When we represent the game based only on characteristic function, we assume that payoffs or sharing of values can be done in any way and we dont need to specify the utility functions of each player.

v a particular type of characteristic function denotes a specific coalitional game, it is an element of G^N.

Given a game $v \in G^N$, for any player $A_i \in N$, for any coalition $S \subseteq N \setminus \{A_i\}$, the marginal contribution A_i makes to the expanded coalition $S \cup \{A_i\}$ by joining is $v(S \cup \{A_i\}) - v(S)$.
The marginal contribution of player A_i to the coalition S, if it joins.

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 0$
$\{2\}$	$v(\{2\}) = 0$
$\{3\}$	$v(\{3\}) = 0$
$\{1,2\}$	$v(\{1,2\}) = 1$
$\{1,3\}$	$v(\{1,3\}) = 1$
$\{2,3\}$	$v(\{2,3\}) = 1$
$\{1,2,3\}$	$v(\{1,2,3\}) = 3$

$v(\{1,2,3\}) - v(\{1,2\}) = 3 - 1 = 2$ is the contribution of player 3 in the coalition $\{1,2,3\}$.
$v(\{1,2,3\}) - v(\{1,3\}) = 3 - 1 = 2$ is the contribution of player 2 in the coalition $\{1,2,3\}$.

A game $v \in G^N$ is called super additive if $v(S \cup T) \geq v(S) + v(T)$ for all $S, T \subset N$ such that $S \cap T = \emptyset$. The worth of a coalition is higher than the worth of the individual players. The worth of a bigger coalition is higher than the worth of a smaller coalition.

A super additive game is cohesive if $v(N) \geq \sum_{j=1}^{k} v(S_j)$ for every partition $\{S_1, S_2, S_3 ... S_k\}$ of N.
A partition $\{S_1, S_2, S_3 ... S_k\}$ of set N means $\cup_{j=1}^{k} S_j = N$ and $S_i \cap S_j = \emptyset$ when $i \neq j$.

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 0$
$\{2\}$	$v(\{2\}) = 0$
$\{3\}$	$v(\{3\}) = 0$
$\{1,2\}$	$v(\{1,2\}) = 1$
$\{1,3\}$	$v(\{1,3\}) = 1$
$\{2,3\}$	$v(\{2,3\}) = 1$
$\{1,2,3\}$	$v(\{1,2,3\}) = 3$

It is super additive game.

A game $v \in G^N$ is called sub-additive if $v(s \cup T) \leq v(S) + v(T)$ for all $S, T \subset N$ such that $S \cap T = \emptyset$.
Cost sharing, as the size of the coalition increases, the aggregate cost decreases.

A game is additive if $v(s \cup T) = v(S) + v(T)$ for all $S, T \subset N$ such that $S \cap T = \emptyset$.

A game $vinG^N$ is called inessential if $v(N) = \sum_{A_i \in N} v(\{A_i\})$.

A game $vinG^N$ is called essential if $v(N) > \sum_{A_i \in N} v(\{A_i\})$.

coalitions	v()
$\emptyset$	0
$\{1\}$	$v(\{1\}) = 3$
$\{2\}$	$v(\{2\}) = 3$
$\{3\}$	$v(\{3\}) = 3$
$\{1, 2\}$	$v(\{1, 2\}) = 2$
$\{1, 3\}$	$v(\{1, 3\}) = 2$
$\{2, 3\}$	$v(\{2, 3\}) = 2$
$\{1, 2, 3\}$	$v(\{1, 2, 3\}) = 1$

Sub-additive game.

A game $v \in G^N$ is called symmetric if for any $s, T \subseteq N$ with $|S| = |T|$, we have $v(S) = v(T)$.

A game $v \in G^N$ is monotonic if $S \subseteq T \subseteq N$ implies that $v(S) \leq v(T)$.

A game $v \in G^N$ is constant sum if for all $S \subseteq N,\ v(S) + v(N \setminus S) = v(N)$.

Suppose there are seven players $\{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$, suppose there are two disjoint set L and R of these players. $R = \{A_1, A_4, A_7\}$ and $L = \{A_2, A_3, A_5, A_6\}$. Players of set R has right shoe and players in set L has left shoe. A pair of shoe contains two shoes - left and right. A pair of shoe worth 1 and if there is only left or only right, it has no value. Consider set $S = \{A_1, A_3, A_6\}$, so one right shoe. $S \cap R = \{A_1\}$ and $L \cap S = \{A_3, A_6\}$, two left shoes. If there is cooperation between the players in S, then it will have one pair of shoe. The minimum number of left or right shoes determine the number of pairs. So the worth of coalition is $\min\{|S \cap R|, |S \cap L|\}$, when $|S| \geq 2$. We get the following characteristic function.

$$v(s) = \begin{cases} 0, \ \ if \ \ |S| \in \{0,1\}, \\ \min\{|S \cap R|, |S \cap L|\}, \ \ if \ \ |S| \geq 2. \end{cases}$$

$$V(N) = \min\{|R|, |L|\}.$$