

(5)

Truth table is:

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

The statement

$A \leftrightarrow B$  (A if and only if B)  
is true when  
both A and B <sup>are</sup> true,  
and when both  
A and B are false.

The statement is false when one of them  
A or B is false..

$$(A \rightarrow B) \leftrightarrow (\sim B \rightarrow \sim A)$$

A	B	$A \rightarrow B$	$\sim A$	$\sim B$	$\sim B \rightarrow \sim A$	$(A \rightarrow B) \leftrightarrow (\sim B \rightarrow \sim A)$
T	T	T	F	F	T	T
T	F	F	F	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

always true irrespective  
of truth value of A & B

is tautology.

5

$$(A \wedge B) \leftrightarrow (\neg A \vee \neg B) \leftrightarrow \neg(A \vee B)$$

A	B	$A \wedge B$	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg(\neg A \vee \neg B)$	$(A \rightarrow B) \rightarrow \neg(\neg A \vee \neg B)$
T	T	T	F	F	F	T	T
T	F	F	F	T	T	F	T
F	T	F	T	F	T	F	T
F	F	F	T	T	T	F	T

$$(A \wedge B) \leftrightarrow \neg(\neg A \vee \neg B) \text{ is a tautology.}$$

$$(A \rightarrow B) \leftrightarrow \neg(A \wedge \neg B)$$

A	B	$A \rightarrow B$	$\neg B$	$A \wedge \neg B$	$\neg(A \wedge \neg B)$	$(A \rightarrow B) \leftrightarrow \neg(A \wedge \neg B)$
T	T	T	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$$(A \rightarrow B) \leftrightarrow \neg(A \wedge \neg B) \text{ is a tautology.}$$

$$\neg(A \wedge B) \leftrightarrow \neg A \vee \neg B$$

A	B	$A \wedge B$	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg(A \wedge B) \leftrightarrow \neg A \vee \neg B$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

$$\sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B)$$

It is a tautology.

Try  $\sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B)$

Consider a set of statements like

~~A, B, C~~     A, B, C, D, E, F

- ①  $A \rightarrow B$
- ②  $\sim B \rightarrow C$
- ③  $D \rightarrow \bar{E}$
- ④  $E \wedge C \rightarrow F$
- ⑤  $B \rightarrow \sim \bar{E}$
- ⑥  $D$

$\sim A \wedge F$ .

The above set of statements are connected in the given way. Based on these statements we get the implication that  $\sim A \wedge F$ .

he will argue in the following way: (8)

Assume each of the ~~(6)~~ 6 statements  
~~are~~ is true.

(6) D is true and (3) is true  
so E is true.

~~For~~ For statement (5) to be true, B is  
true, ~~is~~ false.

so for statement (1) to be true  
A is false.

~~Since~~ Since B is false so  $\sim B$  is  
true. so statement (2) is true  
when C is true.

C is true and E is true so F is  
true, since statement (4) is true

---

~~We can conclude that~~  
Since A is false so  $\sim A$  is true.  
so  ~~$\sim A$  and~~  $\sim A \wedge F$  is true.

---