भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 374: Financial Engineering Lab

Lab 11

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Note: ✍🏼

1. *Please wait for a little while for the simulations to run (~10 seconds)*
2. ***Please run python programs using python3***, *i.e.* `python3 <filename>.py`

Question 01.

The **Vasicek model**

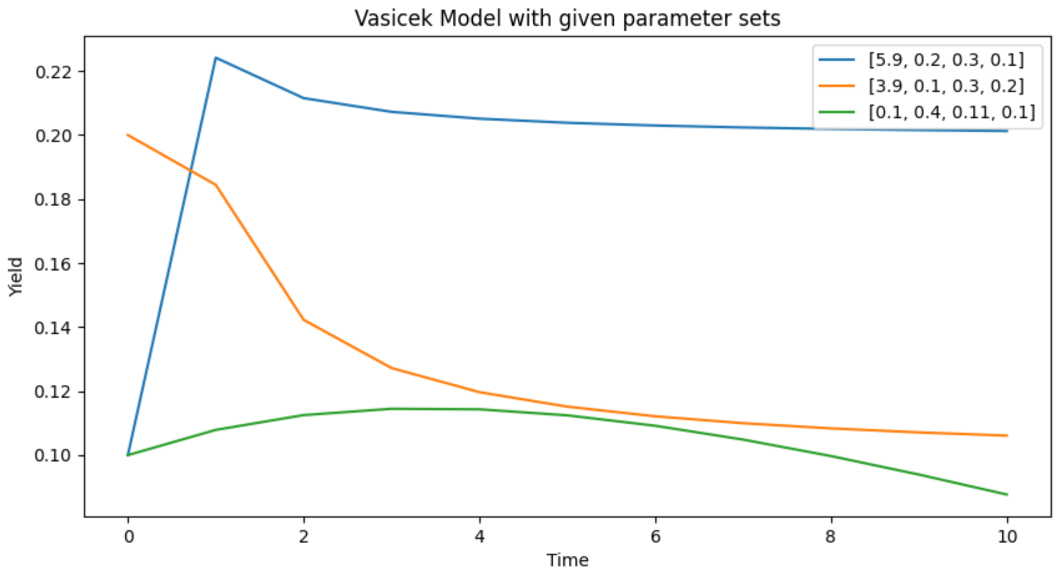
- The risk neutral process for r is given as:
  $dr = a(b - r)dt + \sigma dz$
- We now derive the yield using the zero-coupon bond price formula for Vasicek model -
  $P(t, T) = A(t, T)e^{-B(t,T)r(t)}$

  Here -
  - $B(t, T) = \frac{1 - e^{-a(T-t)}}{a}$
  - $A(t, T) = \exp(\left[\frac{(B(t,T) - T + t)(a^2 b - \sigma^2/2)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right])$

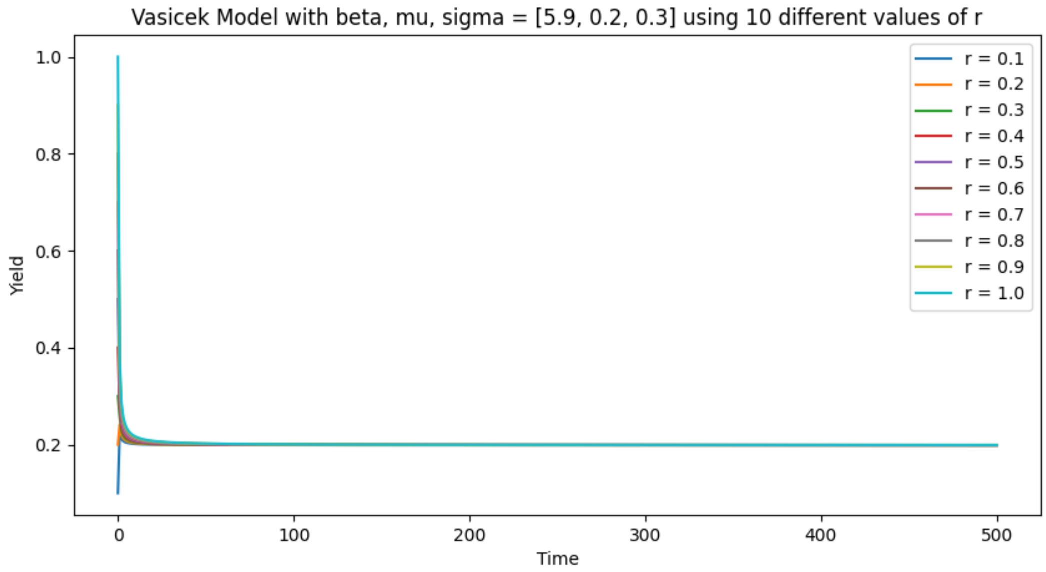
- Thus after we obtain P(t,T), we calculate the yield using:
  $y = -\frac{\log(P(t,T))}{T-t}$. In our setup, $a = \beta,\, b = \mu,\, and\, t = 0$

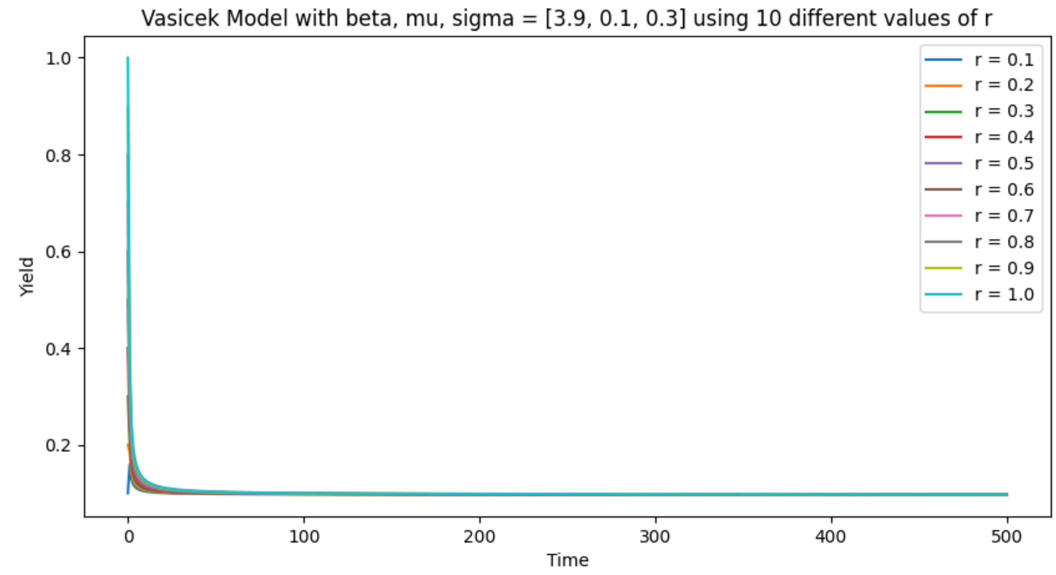
Plots and Observations

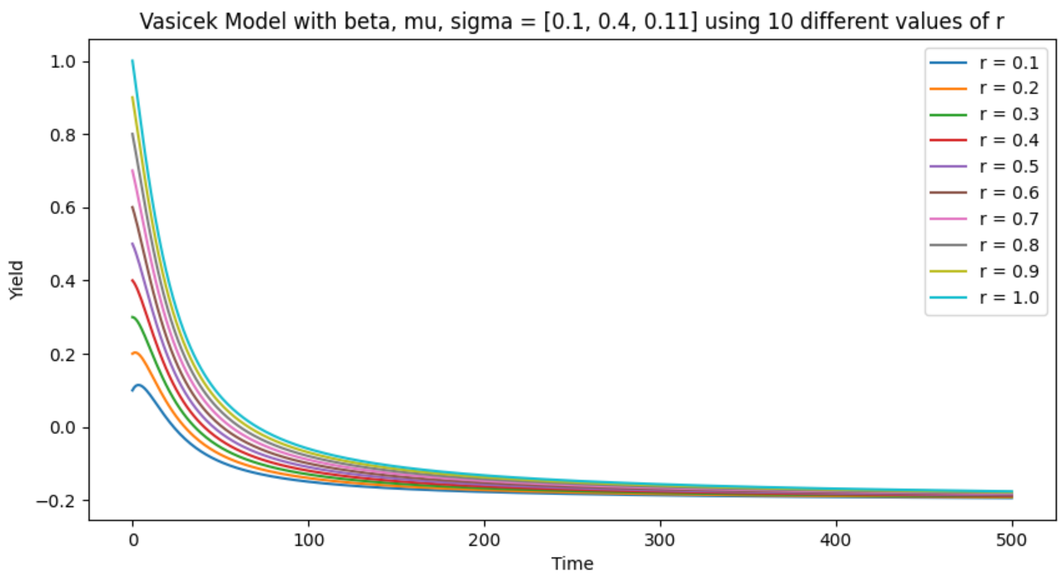
- The plot for term structure upto 10 time steps, for the three parameter sets [β, μ, σ, r(0)] given by [5.9, 0.2, 0.3, 0.1], [3.9, 0.1, 0.3, 0.2] and [0.1, 0.4, 0.11, 0.1] -
- Observations -
  - **Set 1**: Yield increases with time
  - **Set 2**: Yield decreases with time
  - **Set 3**: Yield increases with time



- Now for each of the three parameter sets, we plot yield curves versus maturity up to 500 time units for ten different values of r(0) -
- Observations -
  - For higher r(0), the yield is higher
  - The relation between yield and time to maturity is uncertain and depends on the value of other parameters like $\beta, \mu, and\, \sigma$.
  - For all r, the yields converge to a limit.




Vasicek Model with beta, mu, sigma = [3.9, 0.1, 0.3] using 10 different values of r
Yield
Time
r = 0.1
r = 0.2
r = 0.3
r = 0.4
r = 0.5
r = 0.6
r = 0.7
r = 0.8
r = 0.9
r = 1.0


Vasicek Model with beta, mu, sigma = [0.1, 0.4, 0.11] using 10 different values of r
Yield
Time
r = 0.1
r = 0.2
r = 0.3
r = 0.4
r = 0.5
r = 0.6
r = 0.7
r = 0.8
r = 0.9
r = 1.0

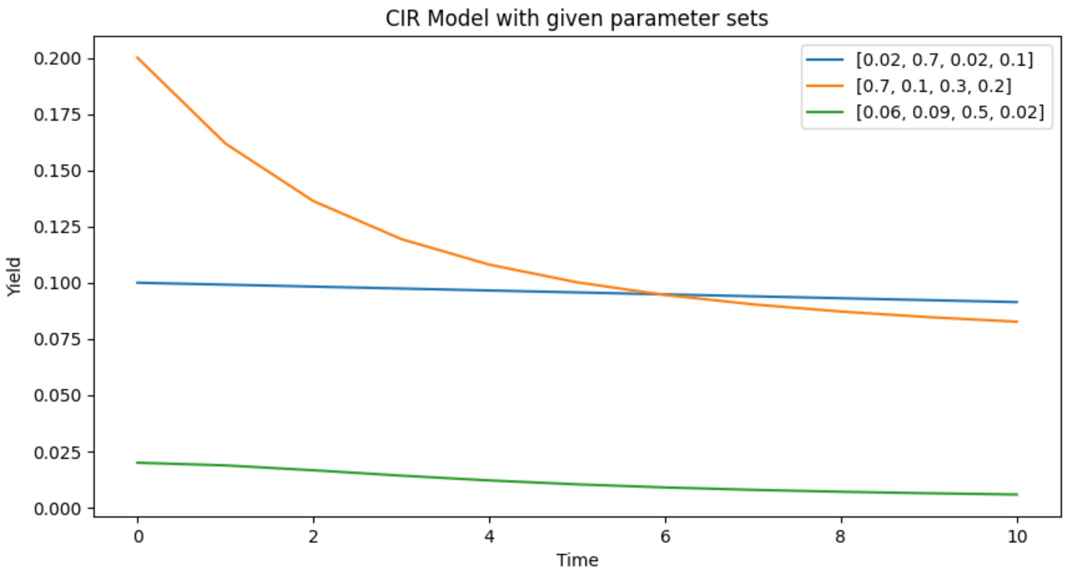
Question 02.

The **CIR model:**

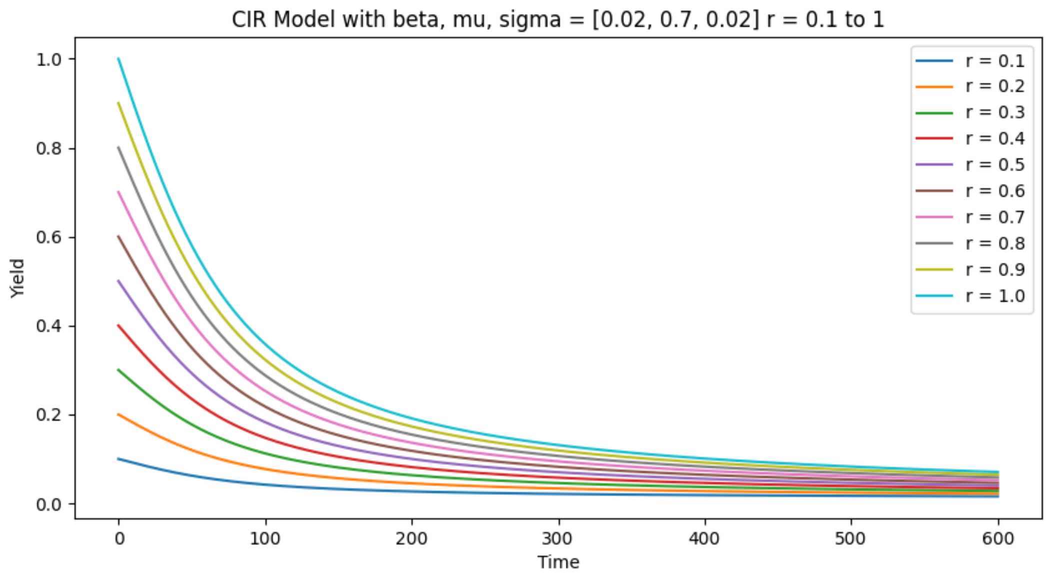
- The risk neutral process for r is given as:
  $dr = a(b-r)dt + \sigma\sqrt{r}dz$
- We now derive the yield using the zero-coupon bond price formula for CIR model -
  $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$
  Here -
  - $B(t,T) = \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+a)(e^{\gamma(T-t)}-1)+2\gamma}$
  - $A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma+a)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2ab/\sigma^2}$
  - $\gamma = \sqrt{a^2+2\sigma^2}$
- Thus after we obtain P(t,T), we calculate the yield using:
  $y = -\frac{\log(P(t,T))}{T-t}$. In our setup, $a = \beta,\, b = \mu,\, and\, t = 0$

Plots and Observations

- Plotting Yield vs Maturity Time for 3 parameter sets [0.02,0.7,0.02,0.1], [0.7,0.1,0.3,0.2], and [0.06,0.09,0.5,0.02] we have -
- Observations -
  - **Set 1**: Yield decreases (almost constant) with time
  - **Set 2**: Yield decreases with time
  - **Set 3**: Yield decreases with time



- For the parameter set [β, μ, σ] given by [0.02, 0.7, 0.02] and with r(0) = 0.1 : 0.1 : 1,yield curves versus maturity for 600 time units is plotted -



- The observations are exactly the same as in the Vasicek model.

Extras...

The following packages need to be installed for this lab. ***Kindly use pip3 since the code must be run in python-3 as mentioned previously.***

```
Numpy - pip3 install numpy
Matplotlib - pip3 install matplotlib
```