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MA 322: Scientific Computing Lab

Lab Quiz

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Question 1.

- Given $f(x) = \sqrt{x} - 1.1$
- The root of this equation in [0,2] will be 1.21

Part (a)

- For this part we use the bisection method, with a = 0, b = 2.
- Bisection root = **1.2100000083446503** & No of iterations = **26**
- Expected iteration (to nearest integer) count based on convergence analysis is **28**
- **Yes the iteration count matches the expectations, based on our convergence analysis**

Part (b)

- For this part we make use of **g(x) =** $(x + 1.21)/2$**.** Note that this also satisfies the **contraction-mapping theorem.**
- Fixed point iteration root = **1.2099999909847974** & No of iterations = **27**

Question 2.

- Given f(x) = **tan(πx)-6**
- The root of this equation is given as (1/π) arctan 6 ≈ **0.447431543**

Part (a)

- The root using 10 iterations of Bisection Method = **0.44765625**
- Error using bisection method = **0.00022470699999999066**

Part (b)

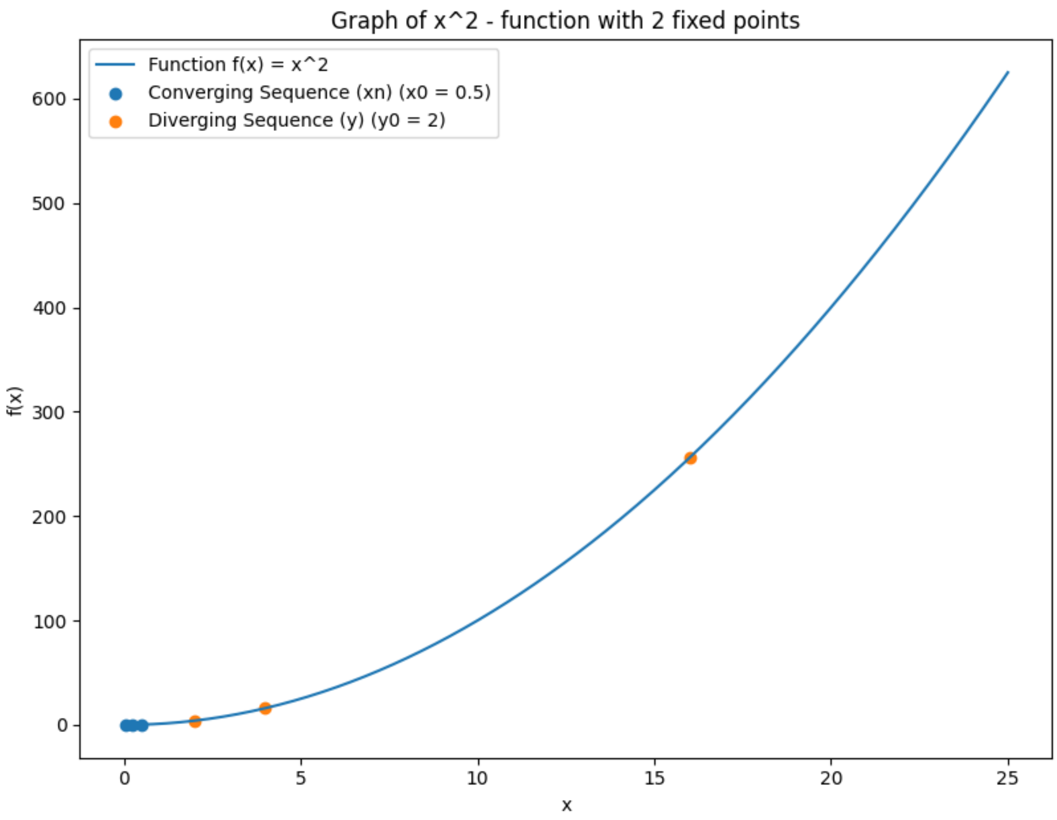
- The root using 10 iterations of Secant Method = **-2956.366770720262**
- Error using secant method = **2956.8142022632624**

<u>Reason why secant method is inaccurate -</u>

- **Clearly the Bisection Method is better in this case because of the smaller error value!**
- The reason why the secant method fails is because - function **tan(πx)-6 is too "wiggly" on the interval [x0,x1].**
- The reason for this is tan(πx)-6 tends to infinity as x goes to 0.5

Question 3.

- **For this question - we guess f(x) =** x^2**.**
- The guess is right because f(x) has 2 fixed points at 0 and 2.
- **Also if we choose x0 = 0.5 and y0 = 2, we will have the 2 sequences, {xn} and {yn} such that they have the properties that {xn} converges to one of the fixed point(=0) and the sequence {yn} goes away and diverges from 0.**
- The graph and the first three terms of both the sequences are below:



Question 4.

- We used Euler's and Runge-Kutta methods of order 2 and 4 for the IVP with y(0)=1,

$$\frac{dy}{dx} = 0.5(x - y)$$

- Taking h = [1, 1/2 , 1/4 , 1/8] we evaluated and compared the solutions with the exact solution

$$y(x) = 3exp(-x/2) + x - 2$$

- Clearly Rk4 with h = 1/8 gives the best approximation.
- We obtain the following tables :

	Euler	Runge-Kutta Order 2	Runge-Kutta Order 4	Solution
0	1.000	1.000000	1.000000	1.000000
1	0.500	0.875000	0.820312	0.819592
2	0.750	1.171875	1.104513	1.103638
3	1.375	1.732422	1.670186	1.669390

	Euler	Runge-Kutta Order 2	Runge-Kutta Order 4	Solution
0	1.000000	1.000000	1.000000	1.000000
1	0.750000	0.843750	0.836426	0.836402
2	0.687500	0.831055	0.819628	0.819592
3	0.765625	0.930511	0.917142	0.917100
4	0.949219	1.117587	1.103683	1.103638
5	1.211914	1.373115	1.359557	1.359514
6	1.533936	1.682121	1.669431	1.669390

	Euler	Runge-Kutta Order 2	Runge-Kutta Order 4	Solution
0	1.000000	1.000000	1.000000	1.000000
1	0.875000	0.898438	0.897491	0.897491
2	0.796875	0.838074	0.836404	0.836402
3	0.759766	0.814081	0.811870	0.811868
4	0.758545	0.822196	0.819594	0.819592
5	0.788727	0.858658	0.855787	0.855784
6	0.846386	0.920143	0.917102	0.917100
7	0.928088	1.003720	1.000589	1.000586
8	1.030827	1.106800	1.103641	1.103638
9	1.151973	1.227097	1.223960	1.223957
10	1.289227	1.362593	1.359517	1.359514
11	1.440573	1.511508	1.508521	1.508519
12	1.604252	1.672269	1.669393	1.669390

	Euler	Runge-Kutta Order 2	Runge-Kutta Order 4	Solution
0	1.000000	1.000000	1.000000	1.000000

1	0.937500	0.943359	0.943239	0.943239
2	0.886719	0.897717	0.897491	0.897491
3	0.846924	0.862406	0.862087	0.862087
4	0.817429	0.836801	0.836402	0.836402
5	0.797589	0.820315	0.819847	0.819847
6	0.786802	0.812395	0.811868	0.811868
7	0.784502	0.812524	0.811946	0.811946
8	0.790158	0.820213	0.819592	0.819592
9	0.803274	0.835005	0.834349	0.834348
10	0.823381	0.856469	0.855784	0.855784
11	0.850045	0.884203	0.883495	0.883495
12	0.882855	0.917825	0.917100	0.917100
13	0.921426	0.956980	0.956242	0.956242
14	0.965400	1.001333	1.000586	1.000586
15	1.014437	1.050569	1.049817	1.049817
16	1.068222	1.104392	1.103638	1.103638
17	1.126458	1.162524	1.161772	1.161772
18	1.188867	1.224705	1.223958	1.223957
19	1.255188	1.290690	1.289948	1.289948
20	1.325176	1.360248	1.359515	1.359514
21	1.398603	1.433162	1.432439	1.432439
22	1.475253	1.509231	1.508519	1.508519
23	1.554924	1.588262	1.587563	1.587562
24	1.637429	1.670076	1.669391	1.669390