भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 473: Computational Finance Lab

Lab 01

AB Satyaprakash (180123062)

15 Aug 2021

When there are N time points t and M space points x:

- *The time complexity of FTCS is O(NM).*
- *The BTCS method takes O(NM3) - assuming that the backslash solver of Matlab takes O(M3) time for a matrix of size MxM.*
- *In the same way, the Crank-Nicolson method takes O(NM3) time.*
- *For the error plots visit the pages after error computation.*

Question 1.

Error table for FTCS:

M	Error	Order_of_Convergence
2	1.1481e-17	-50.404
4	0.01711	0.91744
8	0.0090588	0.98488
16	0.0045771	0.99694
32	0.0022934	0.99926
64	0.0011473	NaN

Error table for BTCS:

M	Error	Order_of_Convergence
2	3.9533e-18	-50.284
4	0.0054191	0.82156
8	0.0030663	0.95737
16	0.0015791	0.98966
32	0.00079524	0.99741
64	0.00039833	NaN

Error table for CN:

M	Error	Order_of_Convergence

2	7.9939e-18	-50.139
4	0.0099132	0.89645
8	0.0053254	0.98089
16	0.0026982	0.99584
32	0.001353	0.99898
64	0.00067698	NaN

Question 2.

For the case f(x) = sin(πx)

Error table for FTCS:

M	Error	Order_of_Convergence
2	0.25	1.6431
4	0.080044	1.1149
8	0.036957	1.0251
16	0.01816	1.0061
32	0.0090418	1.0015
64	0.0045161	NaN

Error table for BTCS:

M	Error	Order_of_Convergence
2	0.14211	0.27347
4	0.11757	0.78043
8	0.068449	0.94199
16	0.035629	0.98531
32	0.017997	0.99631
64	0.0090214	NaN

Error table for CN:

M	Error	Order_of_Convergence

2	0.026072	-0.11969
4	0.028328	0.73521
8	0.017017	0.93577
16	0.008896	0.98393
32	0.0044978	0.99598
64	0.0022552	NaN

For the case f(x) = x(1-x)

Error table for FTCS:

M	Error	Order_of_Convergence
2	0.25	1.6431
4	0.080044	1.1149
8	0.036957	1.0251
16	0.01816	1.0061
32	0.0090418	1.0015
64	0.0045161	NaN

Error table for BTCS:

M	Error	Order_of_Convergence
2	0.14211	0.27347
4	0.11757	0.78043
8	0.068449	0.94199
16	0.035629	0.98531
32	0.017997	0.99631
64	0.0090214	NaN

Error table for CN:

M	Error	Order_of_Convergence
2	0.026072	-0.11969
4	0.028328	0.73521
8	0.017017	0.93577
16	0.008896	0.98393
32	0.0044978	0.99598
64	0.0022552	NaN

Question 3.

Error table for FTCS:

M	Error	Order_of_Convergence
2	0.61872	-0.15086
4	0.68692	-0.0015529
8	0.68766	0.00025313
16	0.68754	6.8848e-05
32	0.68751	1.1074e-05
64	0.6875	NaN

Error table for BTCS:

M	Error	Order_of_Convergence
2	0.035806	0.64079
4	0.022964	0.58206
8	0.01534	0.86348
16	0.0084315	0.94618
32	0.004376	0.97643
64	0.002224	NaN

Error table for CN:

M	Error	Order_of_Convergence
2	0.17958	-0.32044
4	0.22424	-0.022064
8	0.2277	-0.0016953
16	0.22797	-0.00010132
32	0.22798	-3.9197e-07
64	0.22798	NaN

Question 4.

Error table for FTCS:

M	Error	Order_of_Convergence
2	0.21973	1.1125
4	0.10162	1.0573
8	0.048832	1.0292
16	0.023926	1.0148
32	0.011841	1.0075
64	0.0058899	NaN

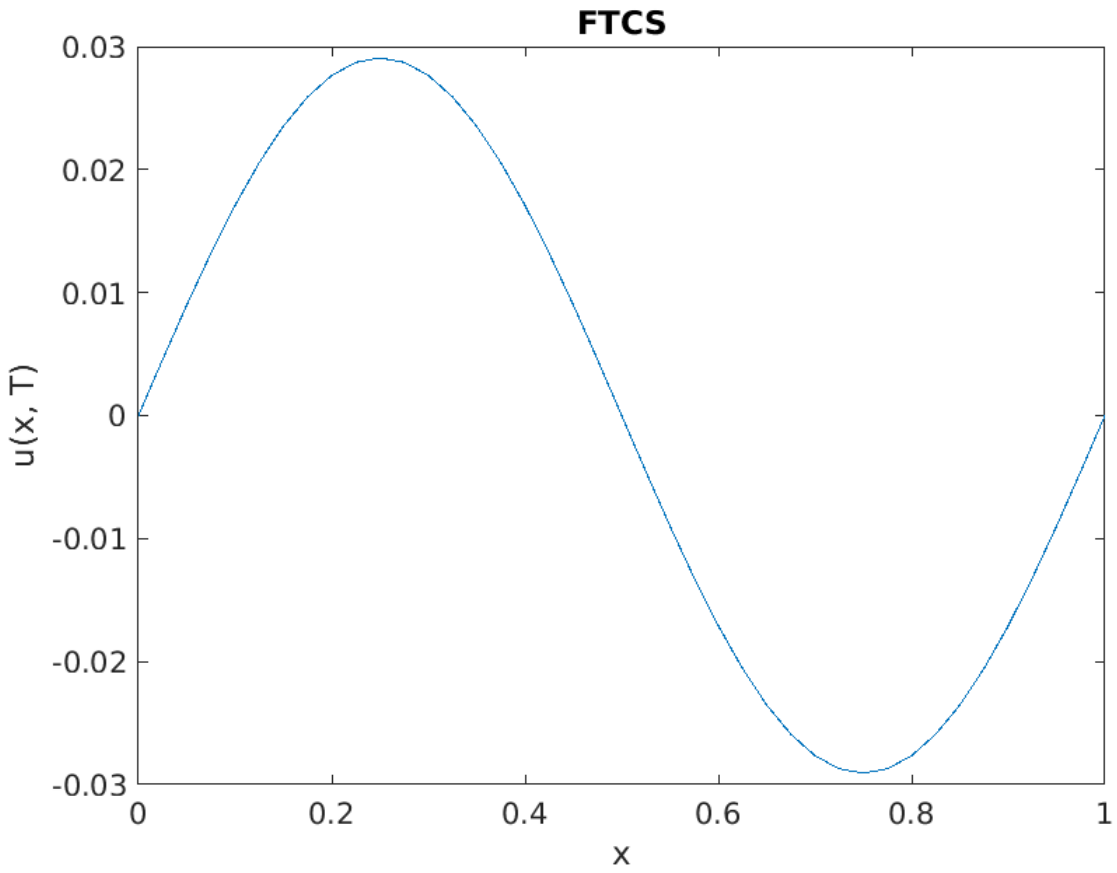
Error table for BTCS:

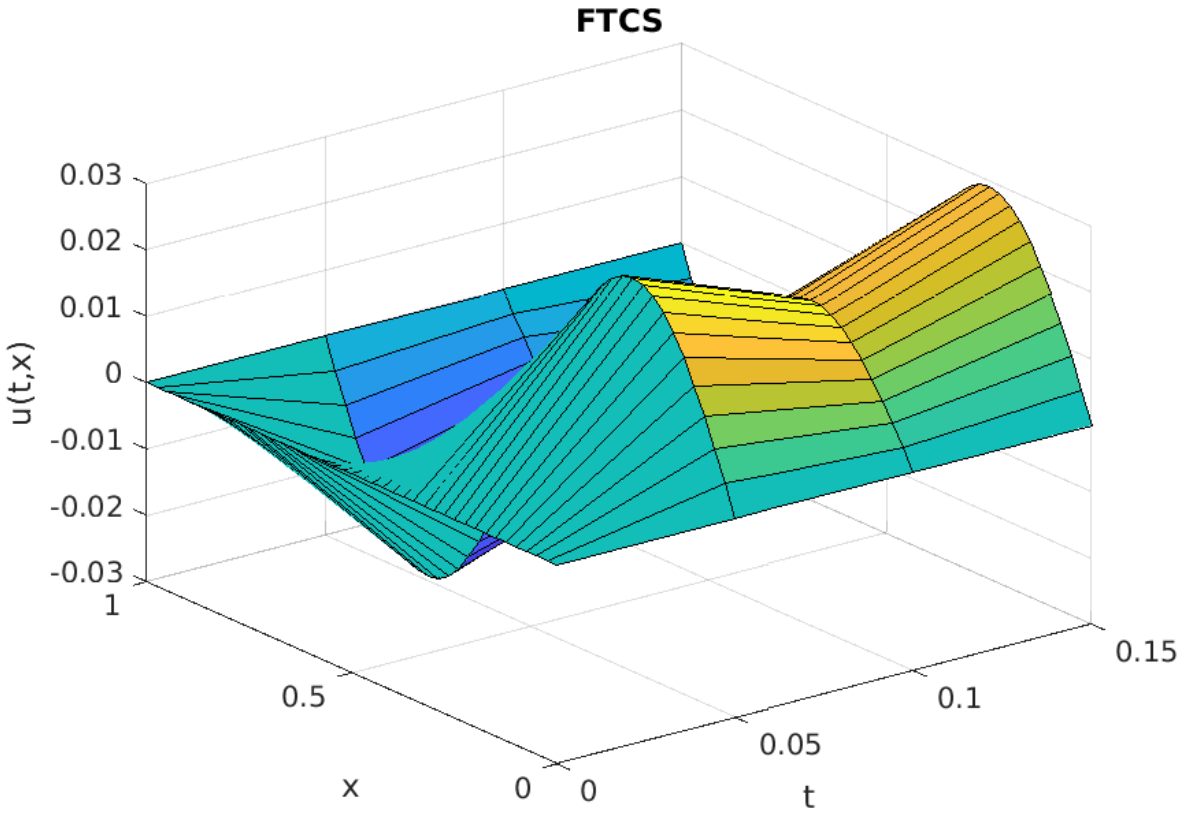
M	Error	Order_of_Convergence
2	0.10965	0.53509
4	0.075673	0.85857
8	0.041734	0.91466
16	0.022138	0.96068
32	0.011375	0.9816

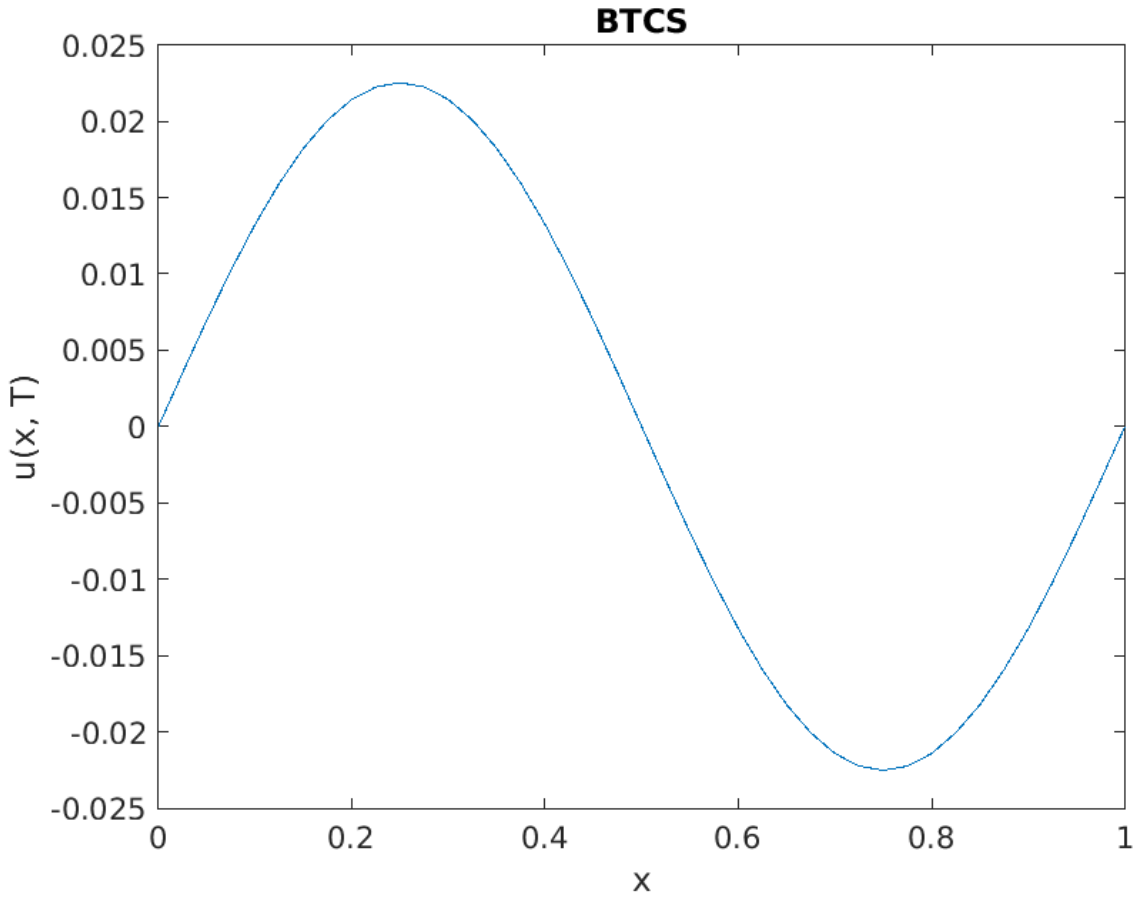
```
     64    0.0057605           NaN

Error table for CN:
     M     Error     Order_of_Convergence
     __    _________________________

     2     0.0051555      -0.52105
     4     0.0073981           0.70477
     8     0.0045391           0.92922
     16    0.0023837           0.98241
     32    0.0012064           0.99561
     64    0.00060506               NaN
```

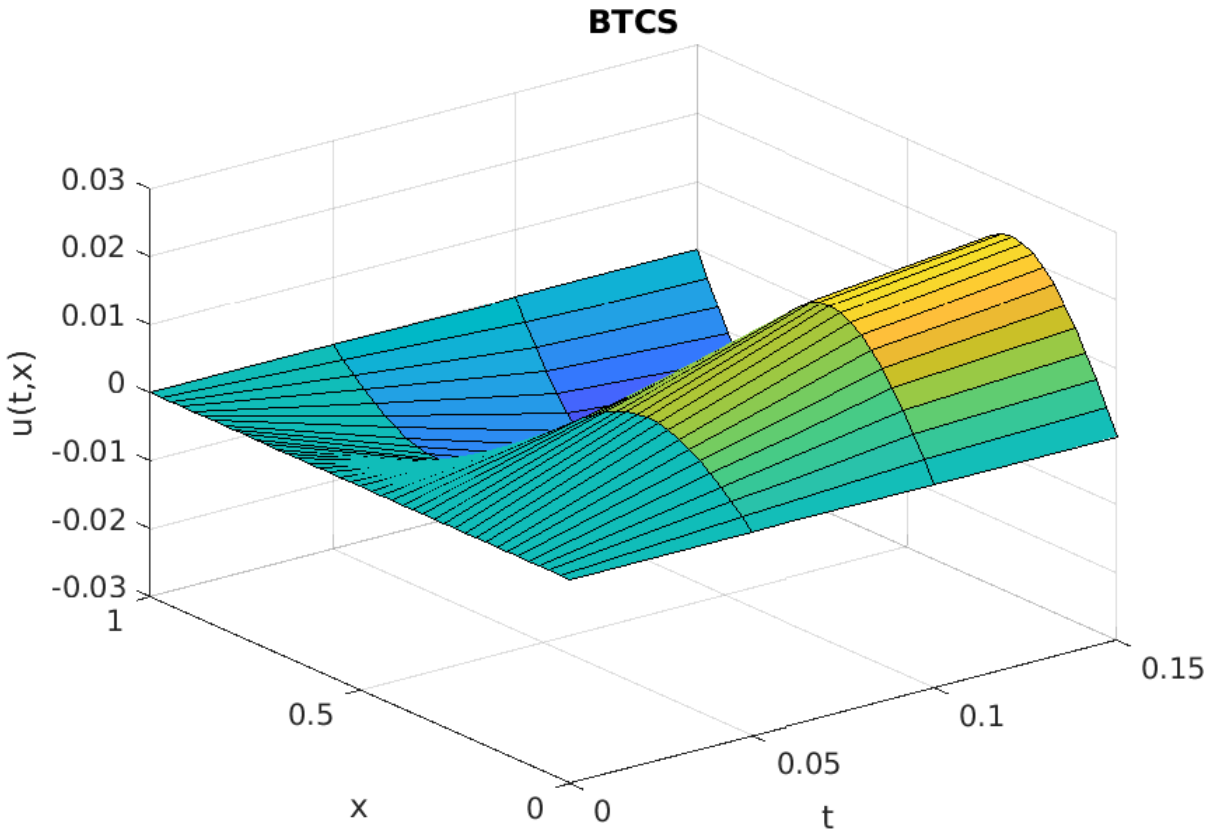
Question 1.

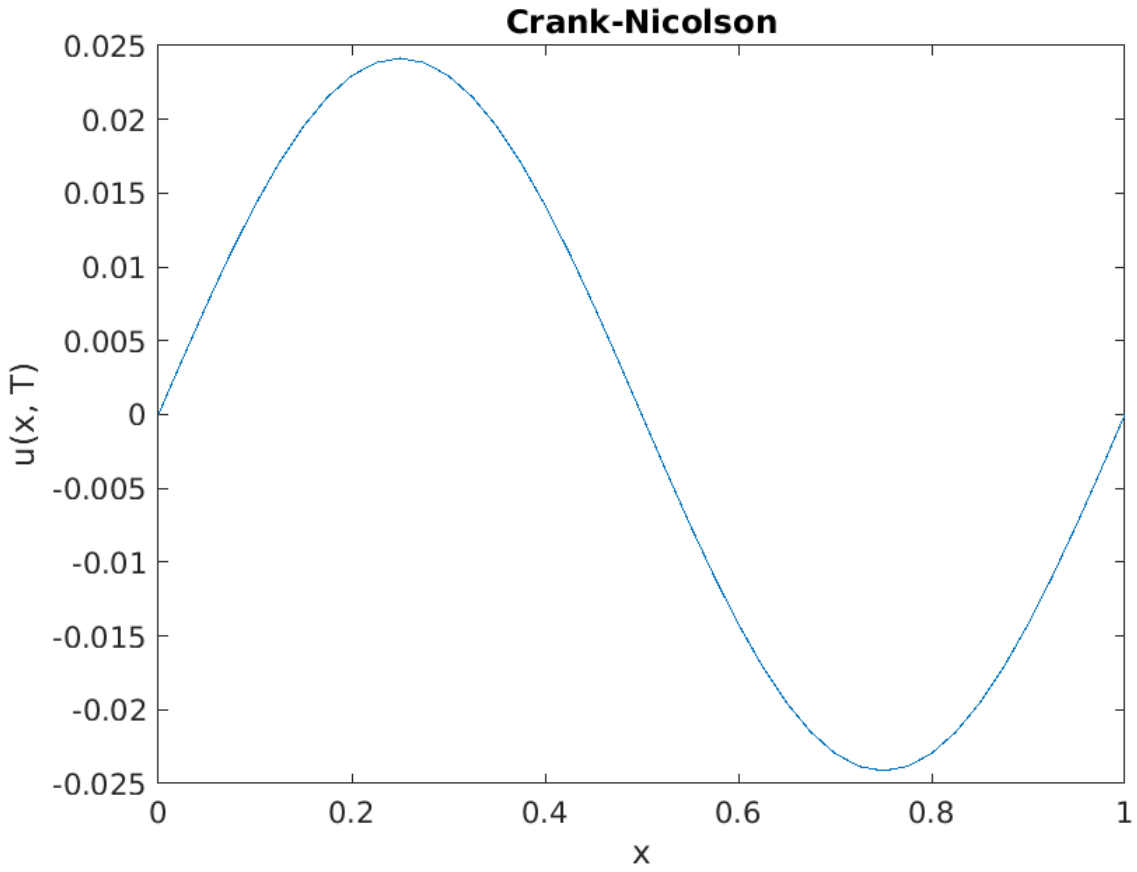
```
Enter spatial step-size: 0.025
Enter time step-size: 0.05
Enter number of time-levels: 3
```

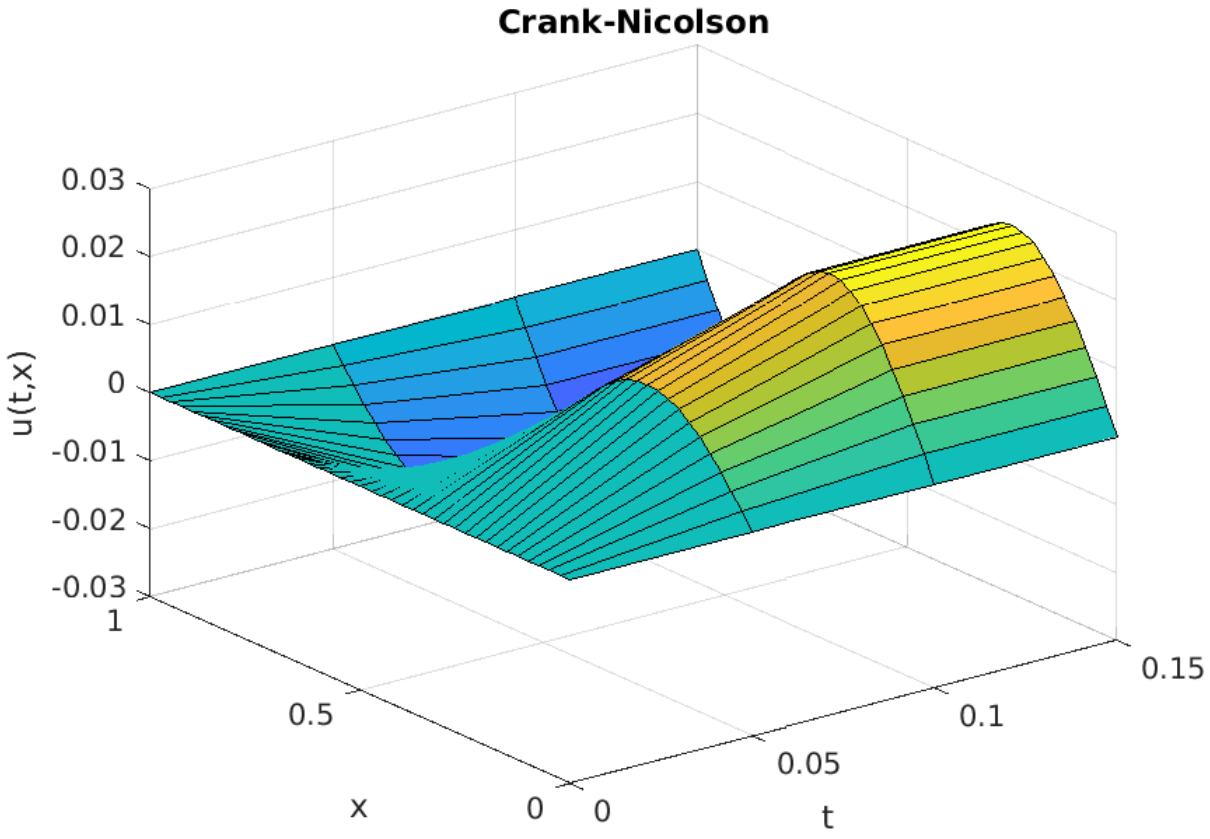



FTCS
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
u(t,x)
1
0.5
0
x
0
0.05
0.1
0.15
t


BTCS
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
u(x, T)
0
0.2
0.4
0.6
0.8
1
x


BTCS
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
u(t,x)
1
0.5
0
x
0
0.05
0.1
0.15
t

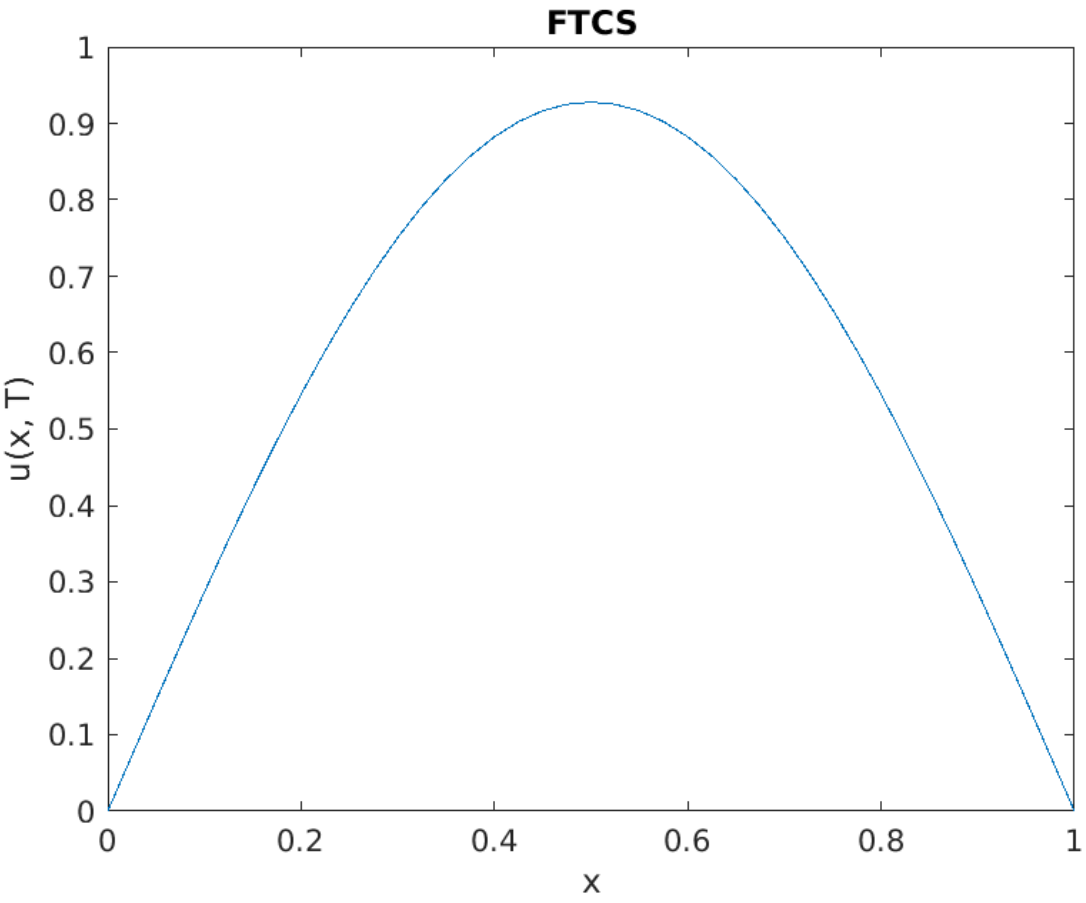

Crank-Nicolson
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
u(x, T)
0
0.2
0.4
0.6
0.8
1
x

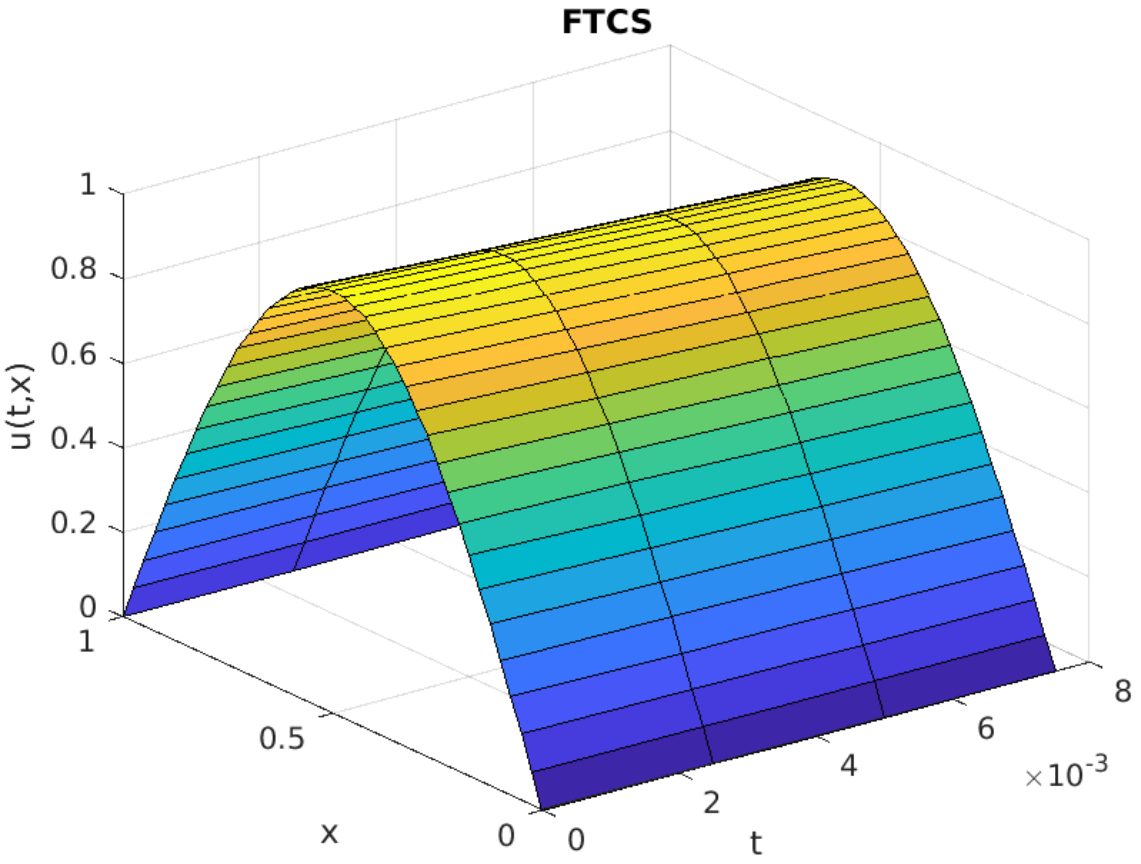

Crank-Nicolson
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
u(t,x)
1
0.5
0
x
0
0.05
0.1
0.15
t

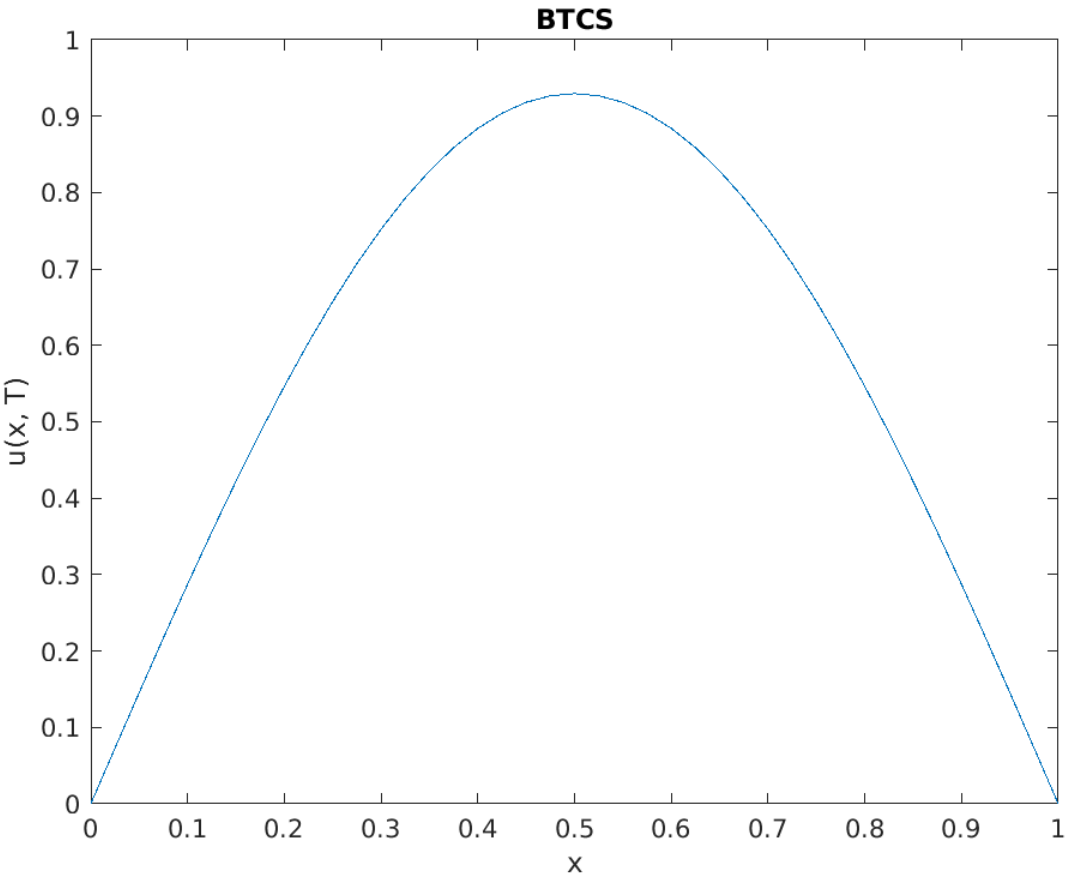
Question 2.

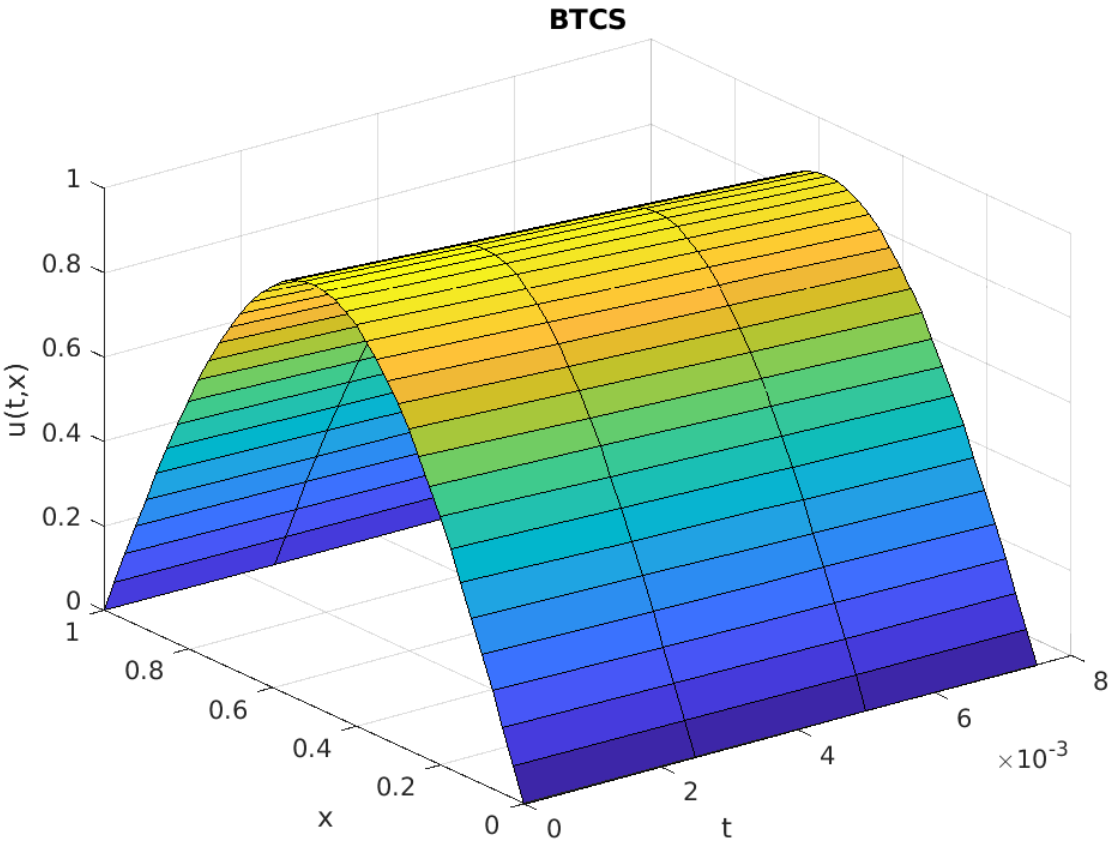
For the case f(x) = sin(πx)

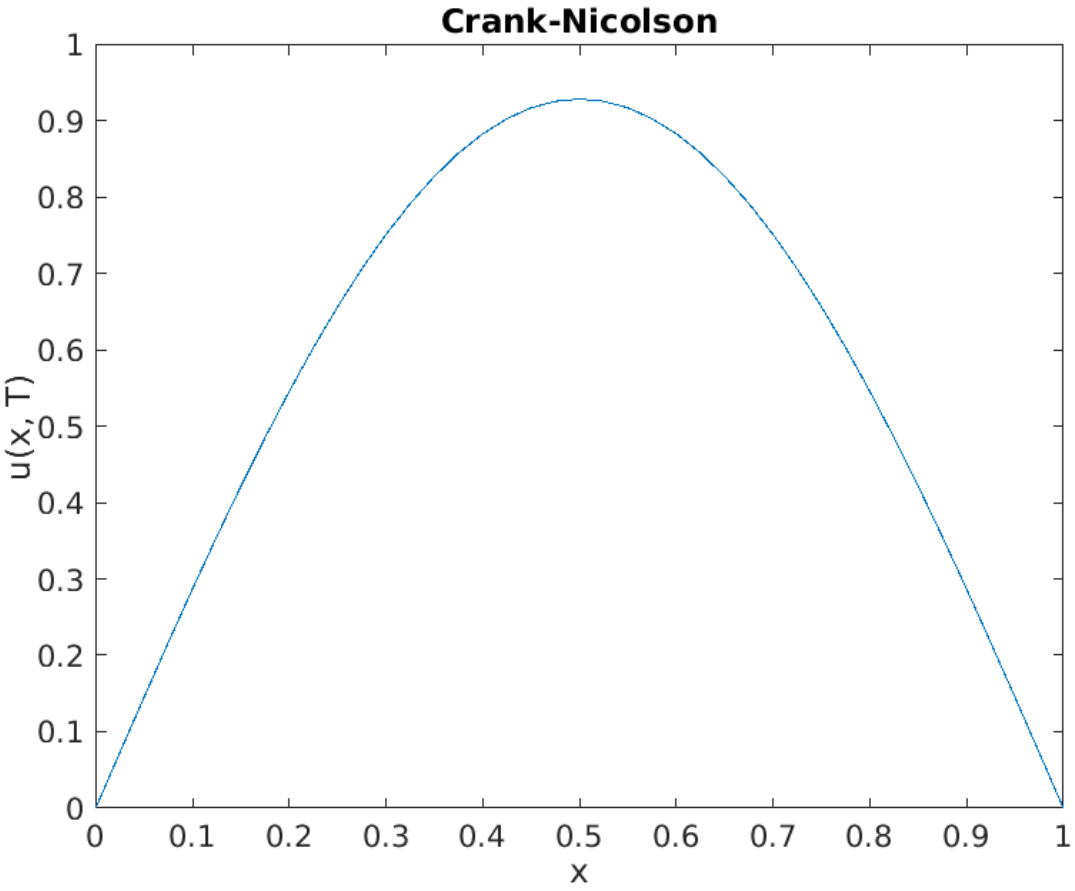
```
Enter spatial step-size: 0.025

Enter time step-size: 0.0025

Enter number of time-levels: 3
```

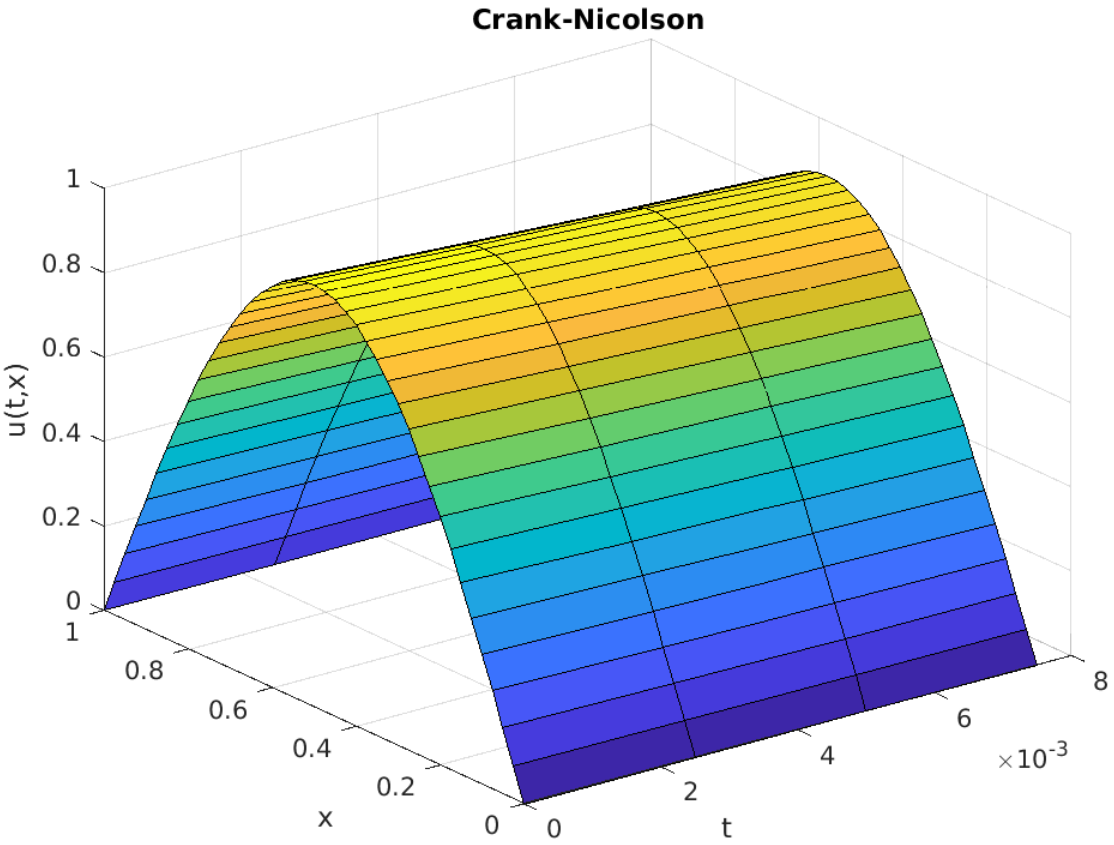



FTCS
u(t,x)
1
0.8
0.6
0.4
0.2
0
1
0.5
0
x
0
2
4
6
8
×10-3
t


BTCS
u(x, T)
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

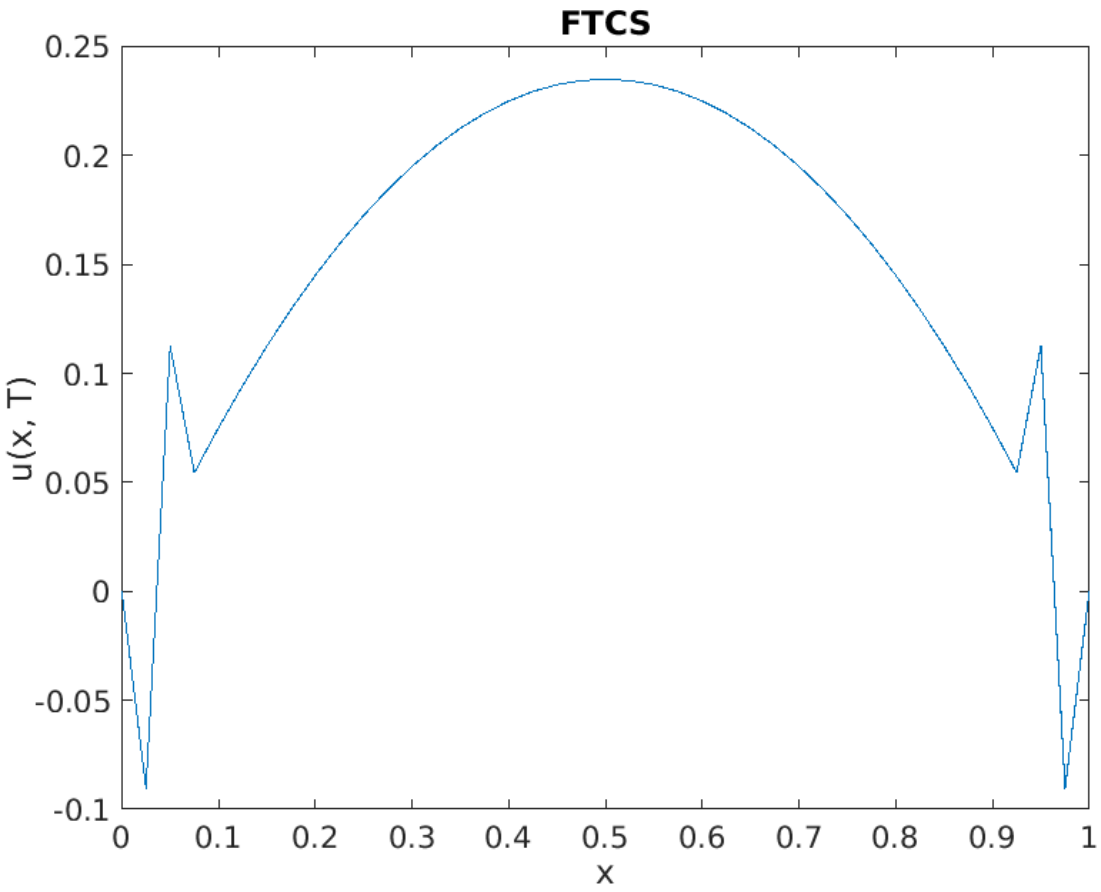

BTCS
u(t,x)
1
0.8
0.6
0.4
0.2
0
x
t
×10^-3
2
4
6
8

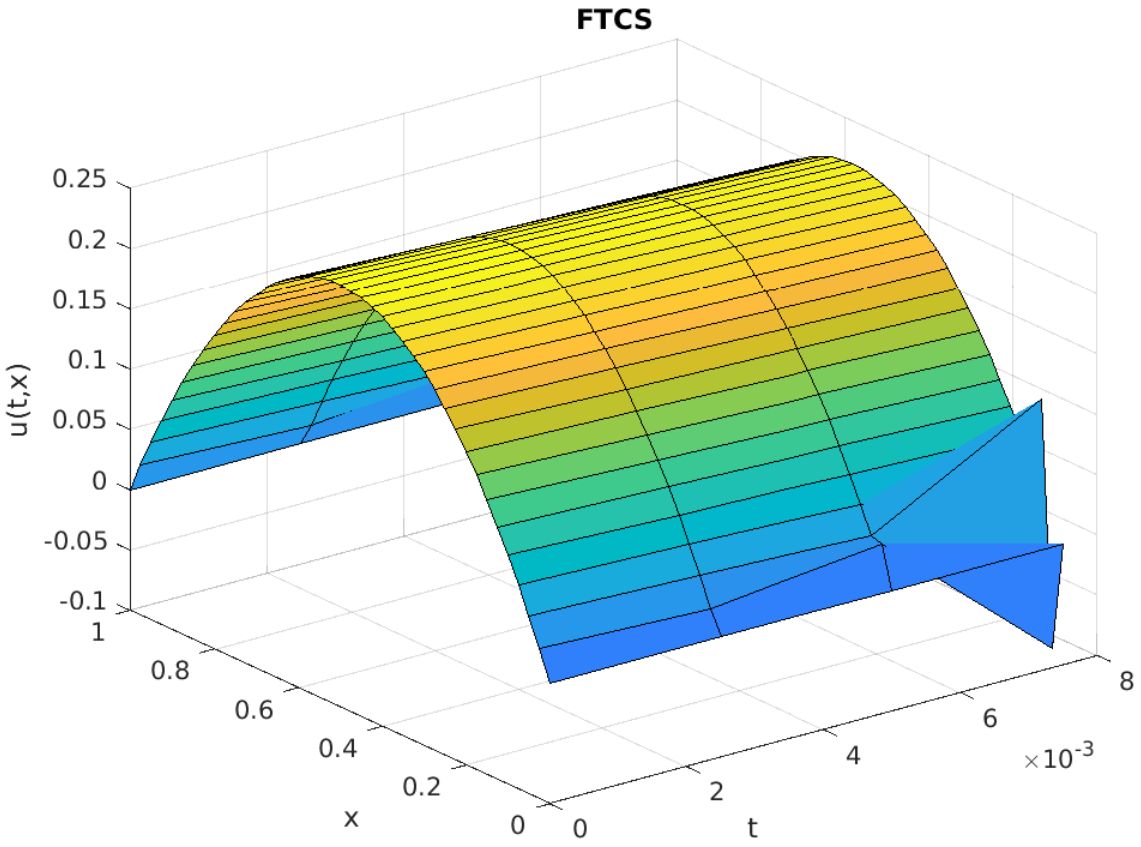

Crank-Nicolson
u(x, T)
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

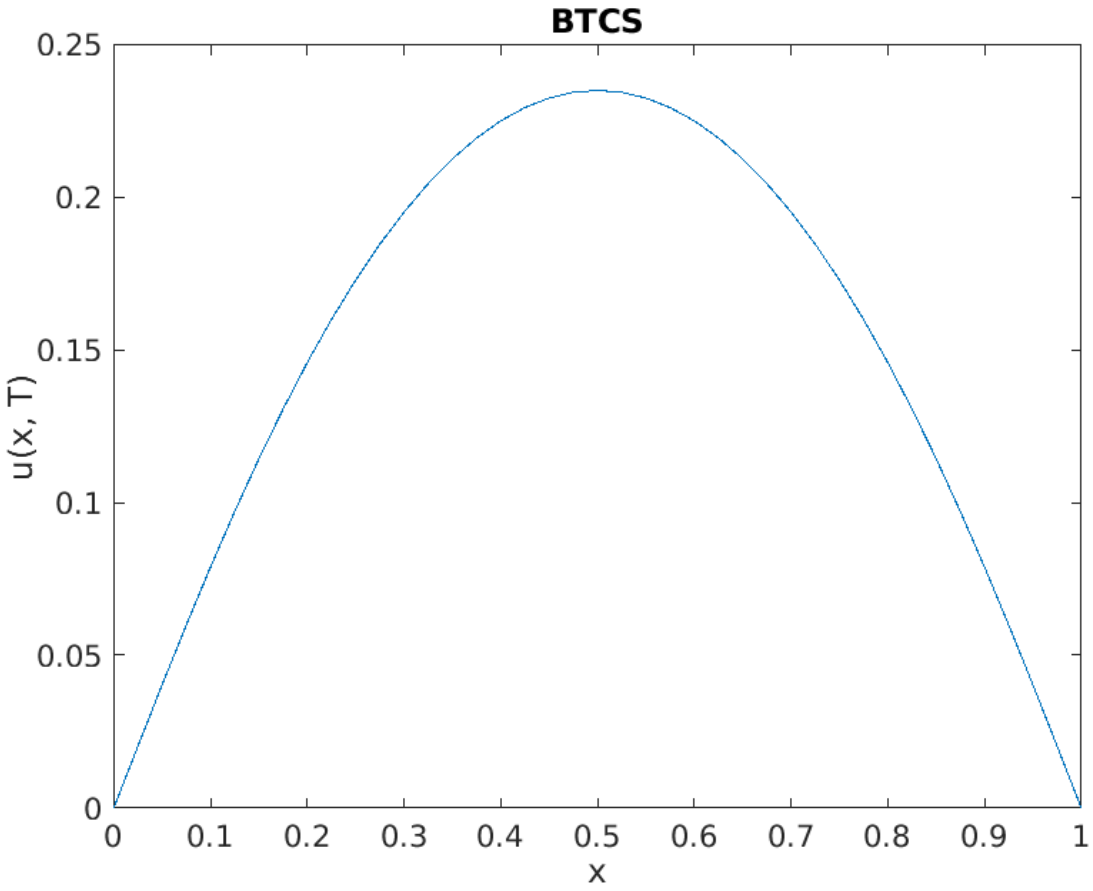

Crank-Nicolson
u(t,x)
1
0.8
0.6
0.4
0.2
0
x
t
×10^-3

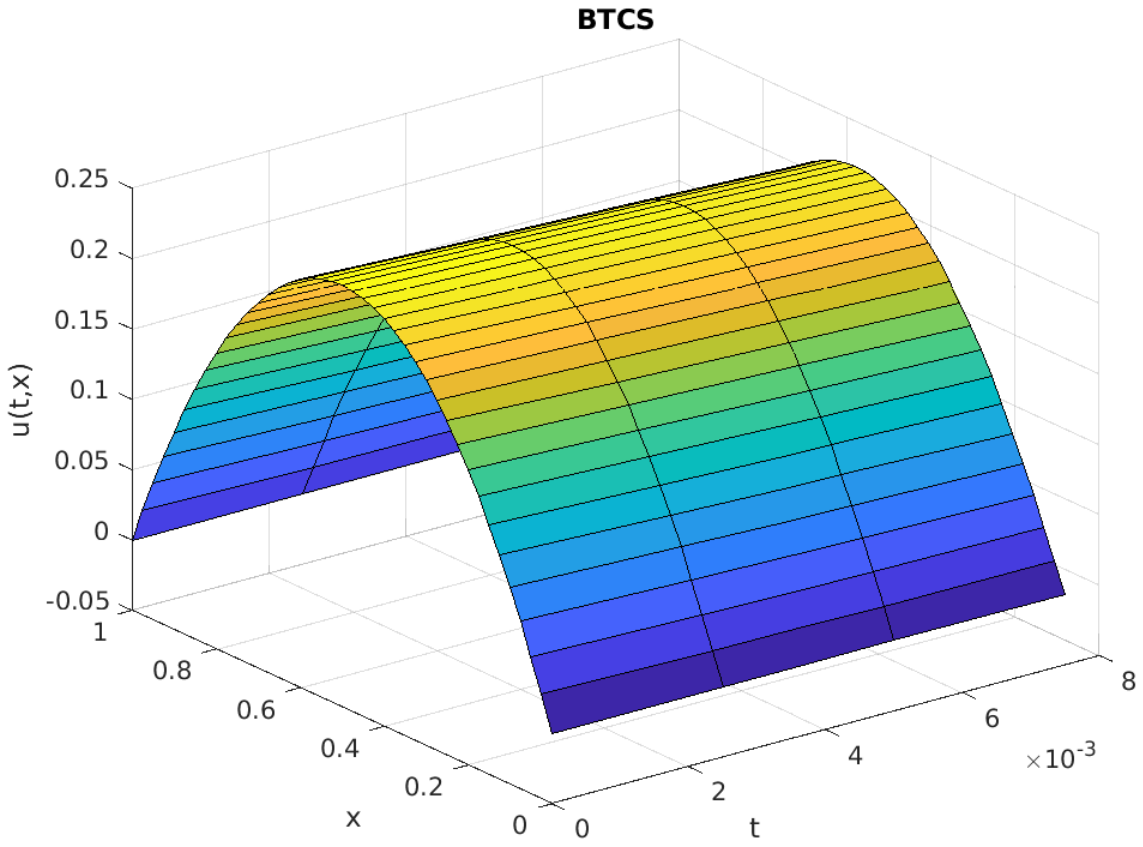
For the case f(x) =x(1-x)

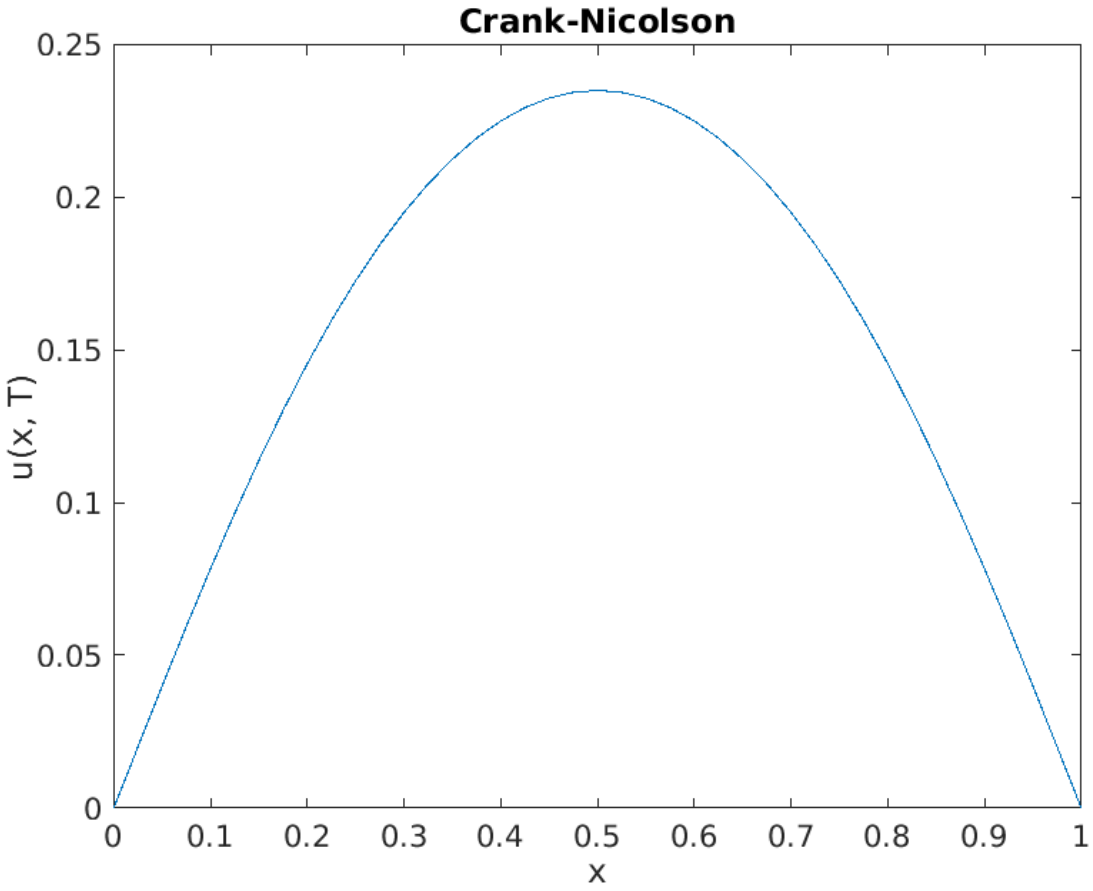
```
Enter spatial step-size: 0.025

Enter time step-size: 0.0025
Enter number of time-levels: 3
```

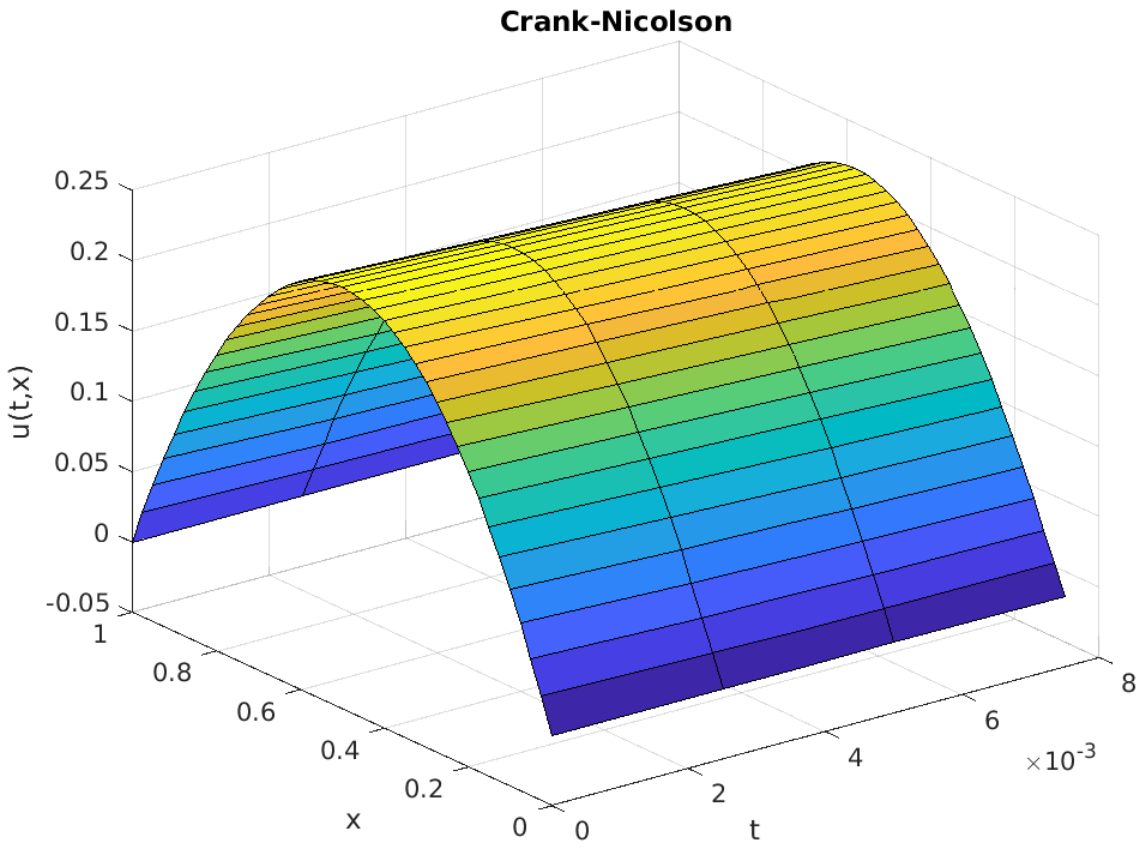



FTCS
u(t,x)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
1
0.8
0.6
0.4
0.2
0
x
0
2
4
6
8
×10-3
t


BTCS
0.25
0.2
0.15
0.1
0.05
0
u(x, T)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

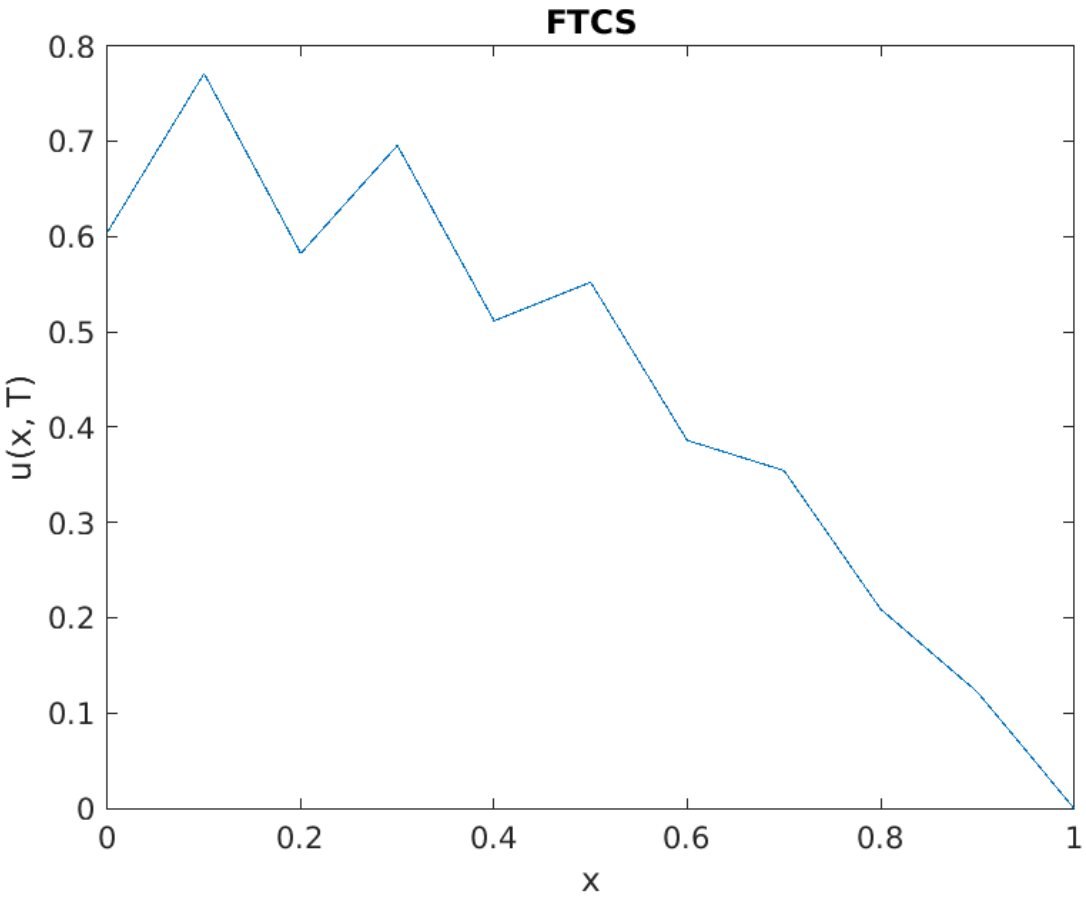

BTCS
u(t,x)
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
0.8
0.6
0.4
0.2
0
x
0
2
4
6
8
×10-3
t

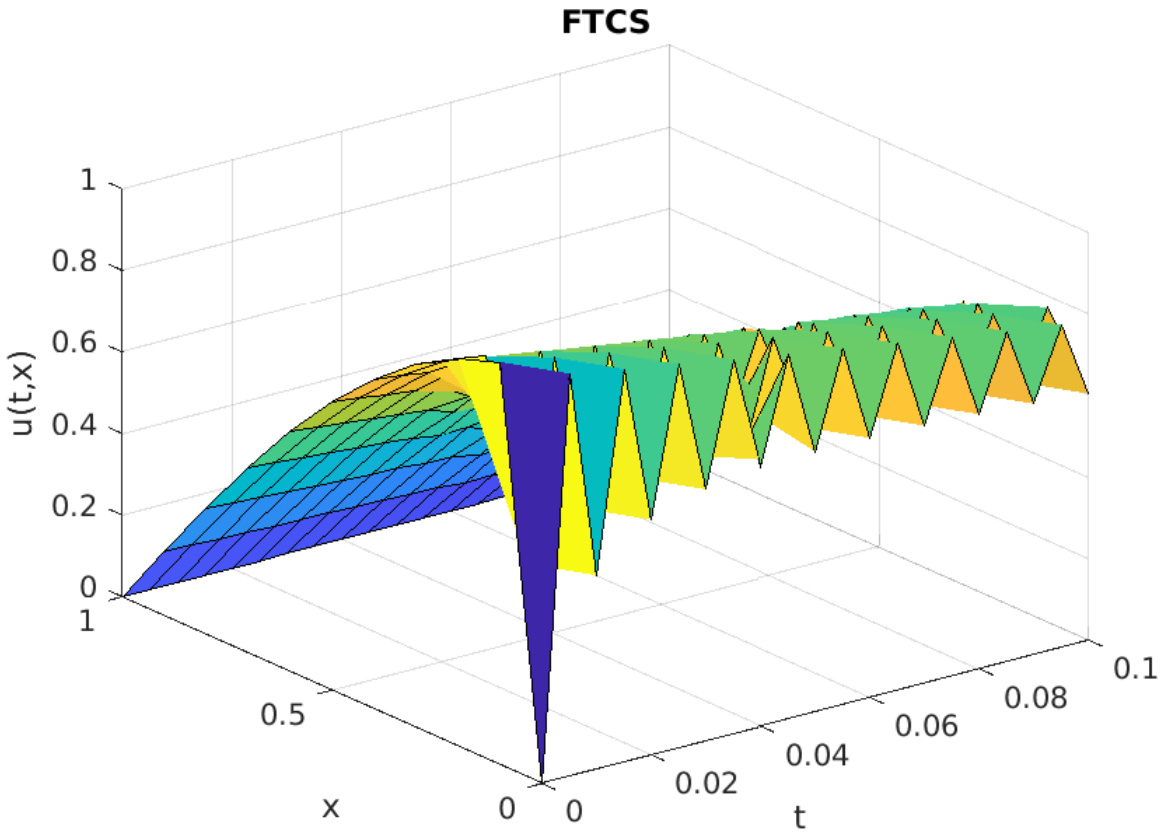

Crank-Nicolson
0.25
0.2
0.15
0.1
0.05
0
u(x, T)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

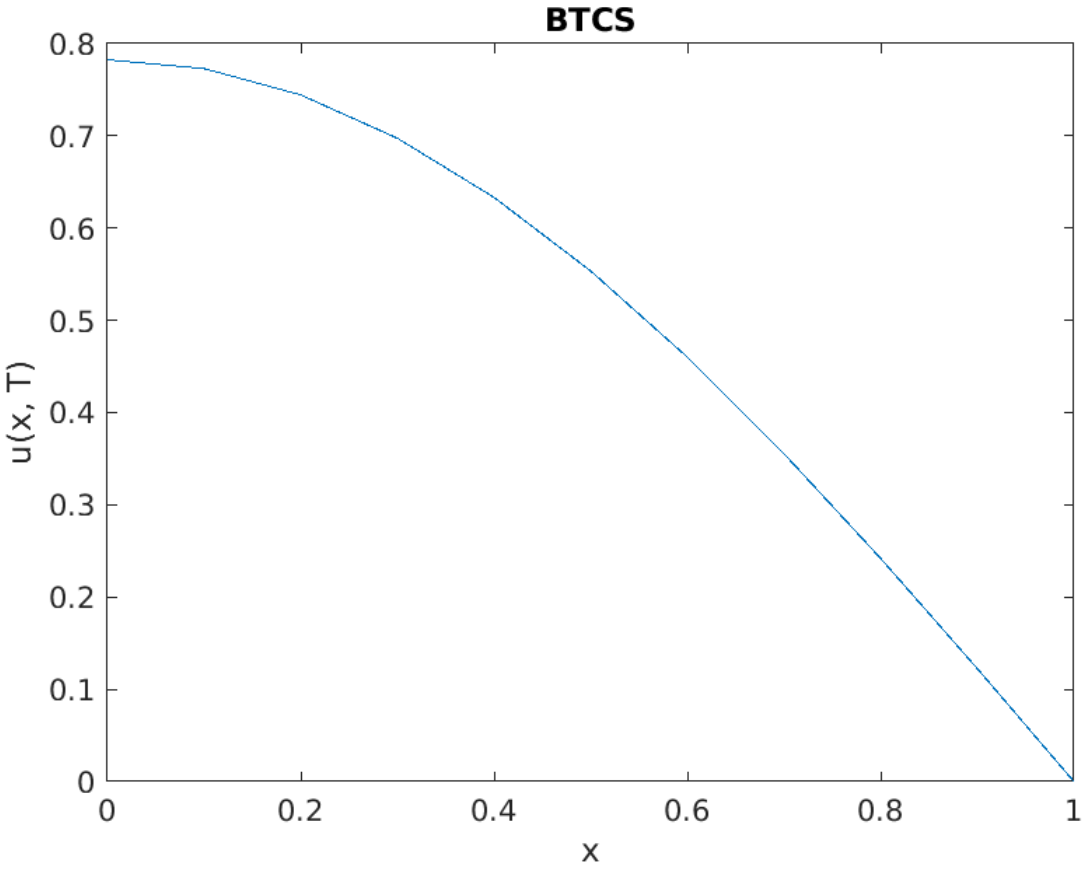

Crank-Nicolson
u(t,x)
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
0.8
0.6
0.4
0.2
0
x
0
2
4
6
8
×10-3
t

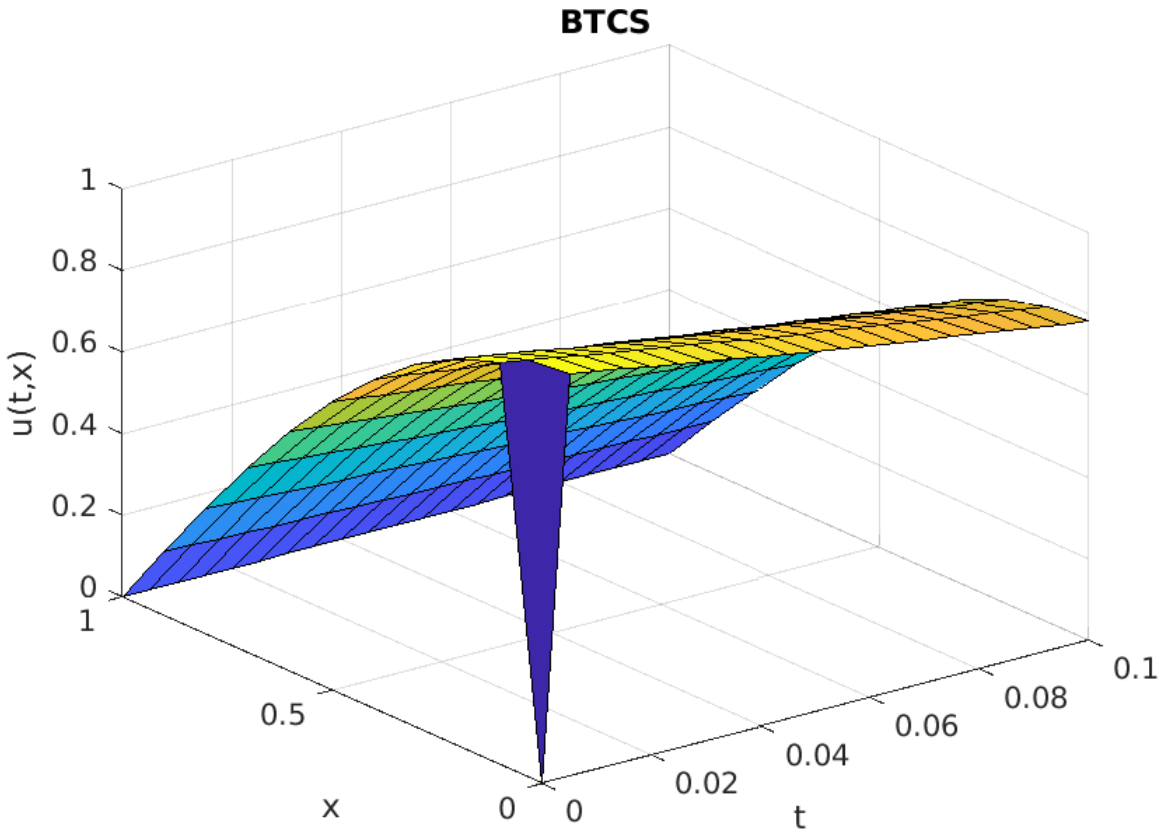
Question 3.

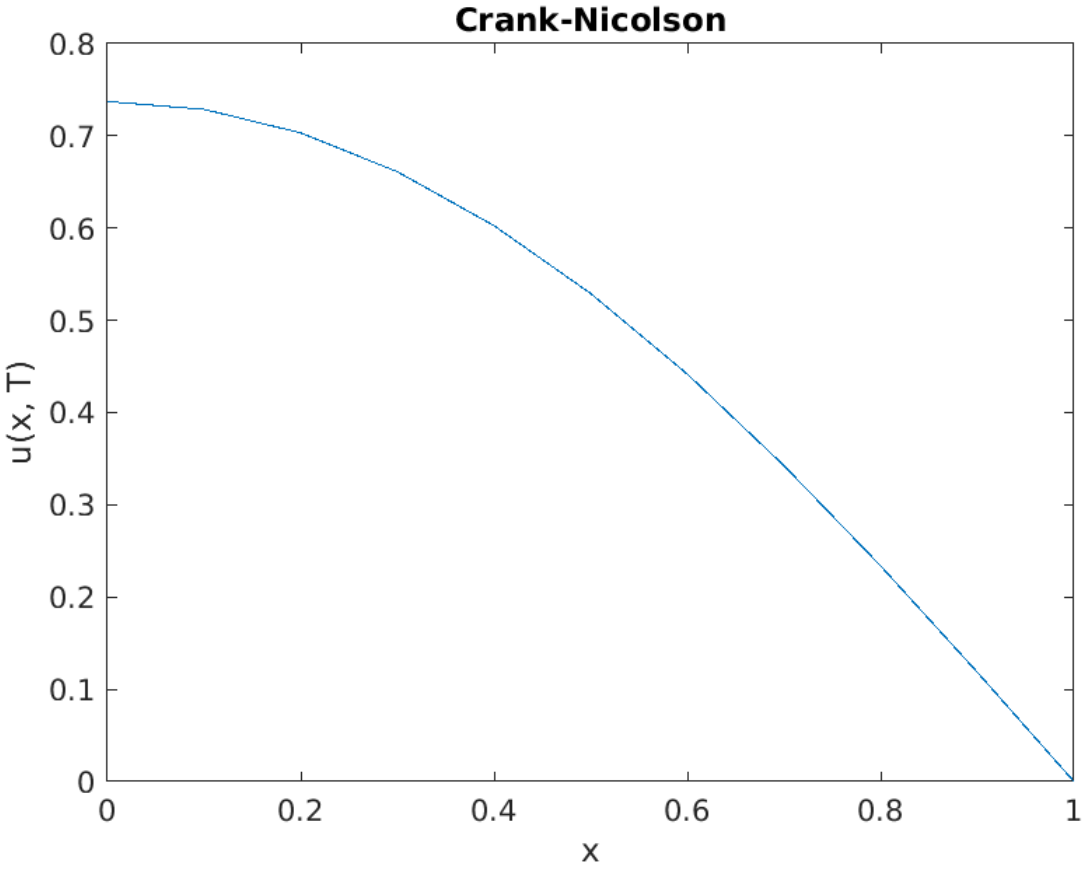
```
Enter spatial step-size: 0.1
Enter time step-size: 0.005
Enter number of time-levels: 20
```

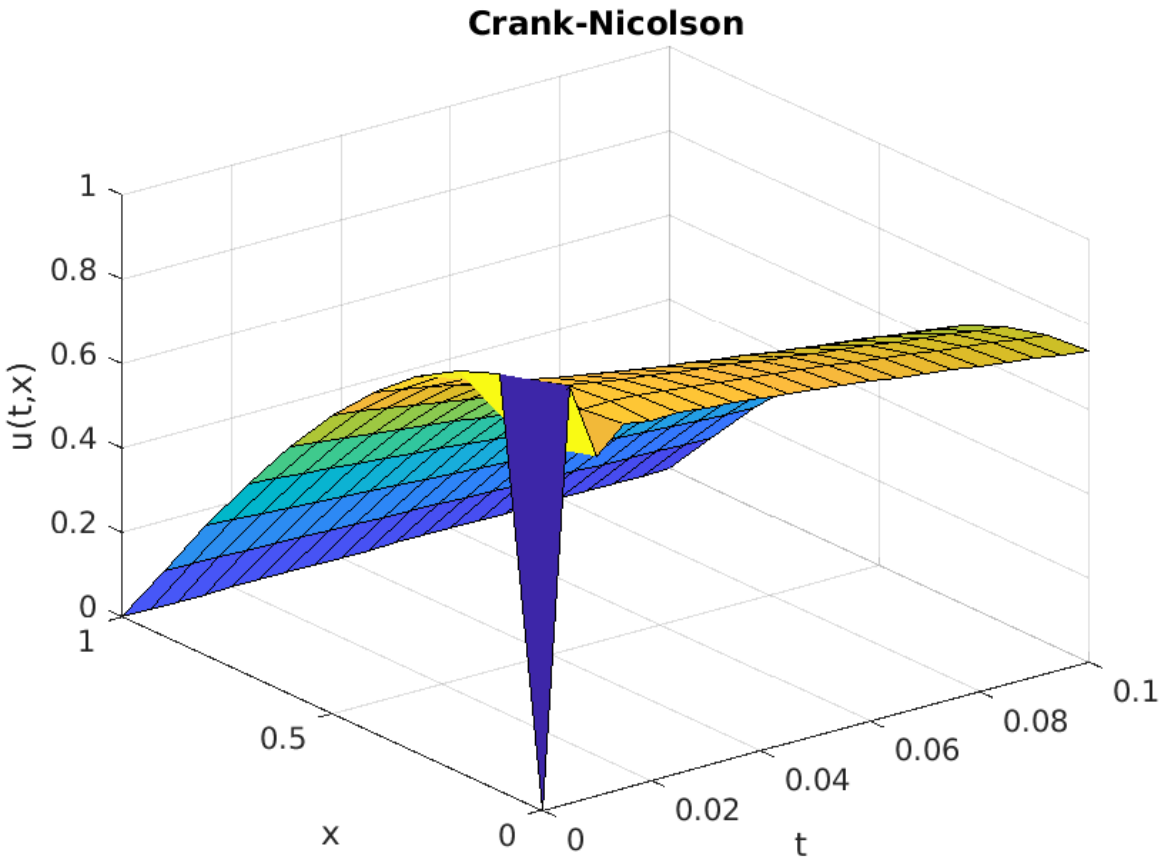



FTCS
u(t,x)
1
0.8
0.6
0.4
0.2
0
1
0.5
0
x
0
0.02
0.04
0.06
0.08
0.1
t


BTCS
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
u(x, T)
0
0.2
0.4
0.6
0.8
1
x

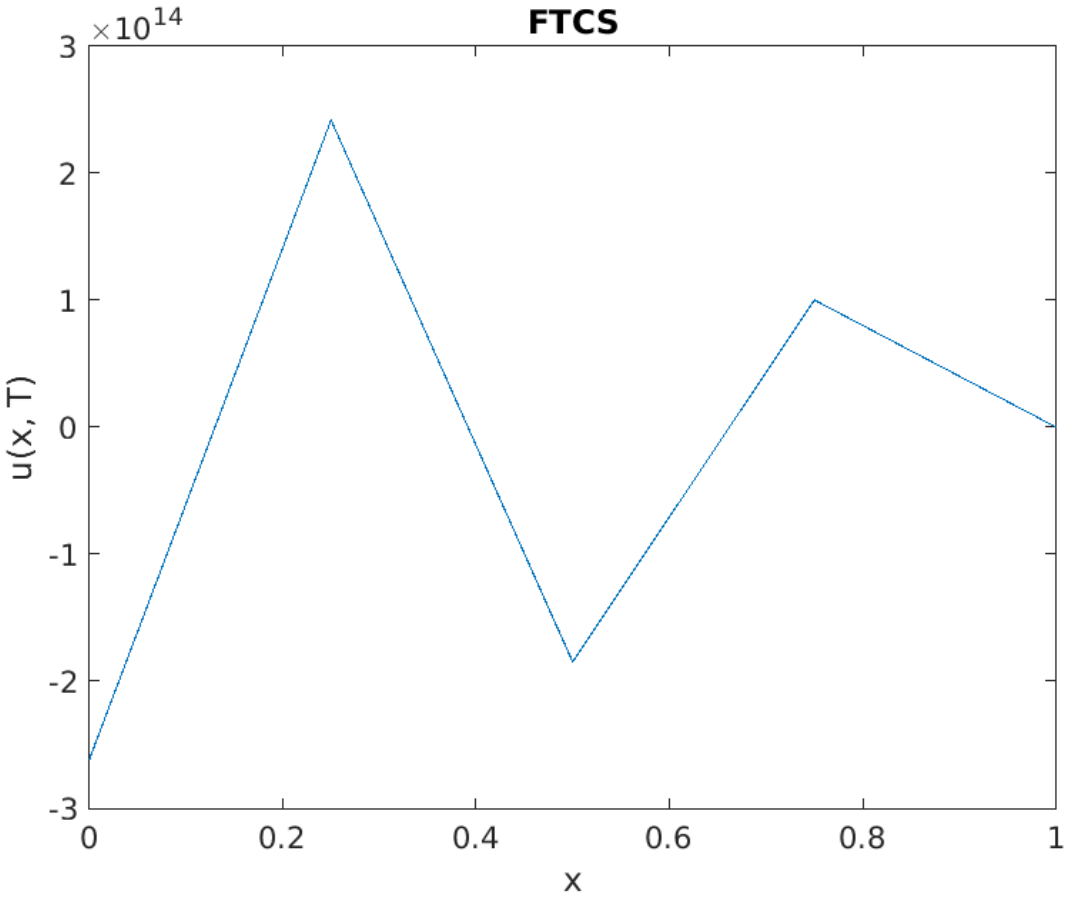

BTCS
u(t,x)
1
0.8
0.6
0.4
0.2
0
1
0.5
0
x
0
0.02
0.04
0.06
0.08
0.1
t

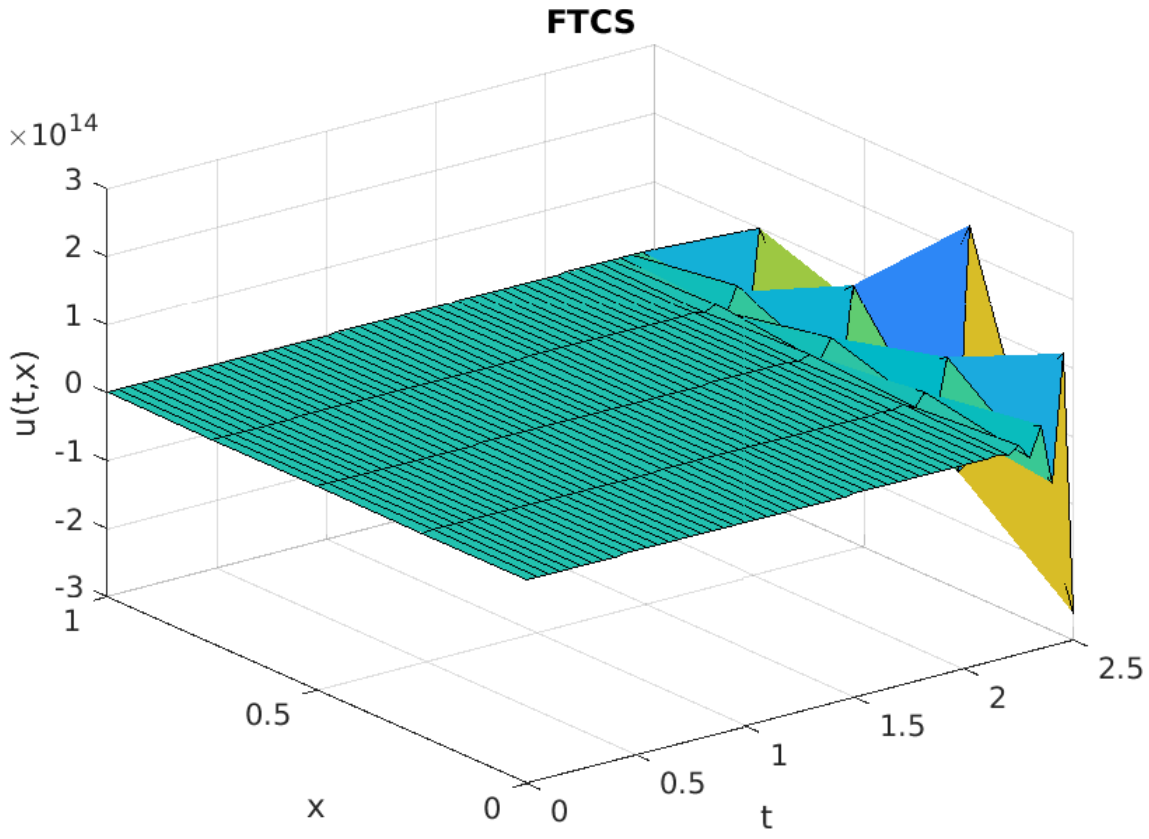

Crank-Nicolson
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
u(x, T)
0
0.2
0.4
0.6
0.8
1
x

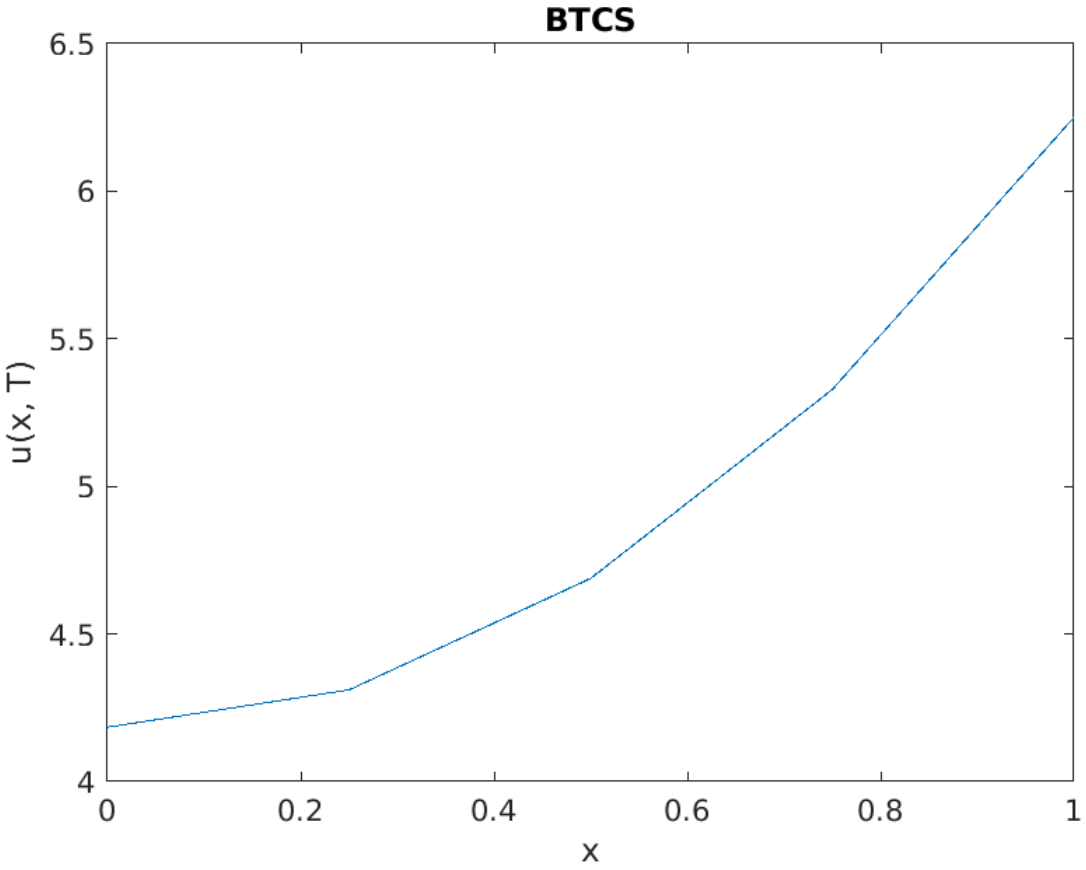

Crank-Nicolson
u(t,x)
1
0.8
0.6
0.4
0.2
0
1
0.5
0
x
0
0.02
0.04
0.06
0.08
0.1
t

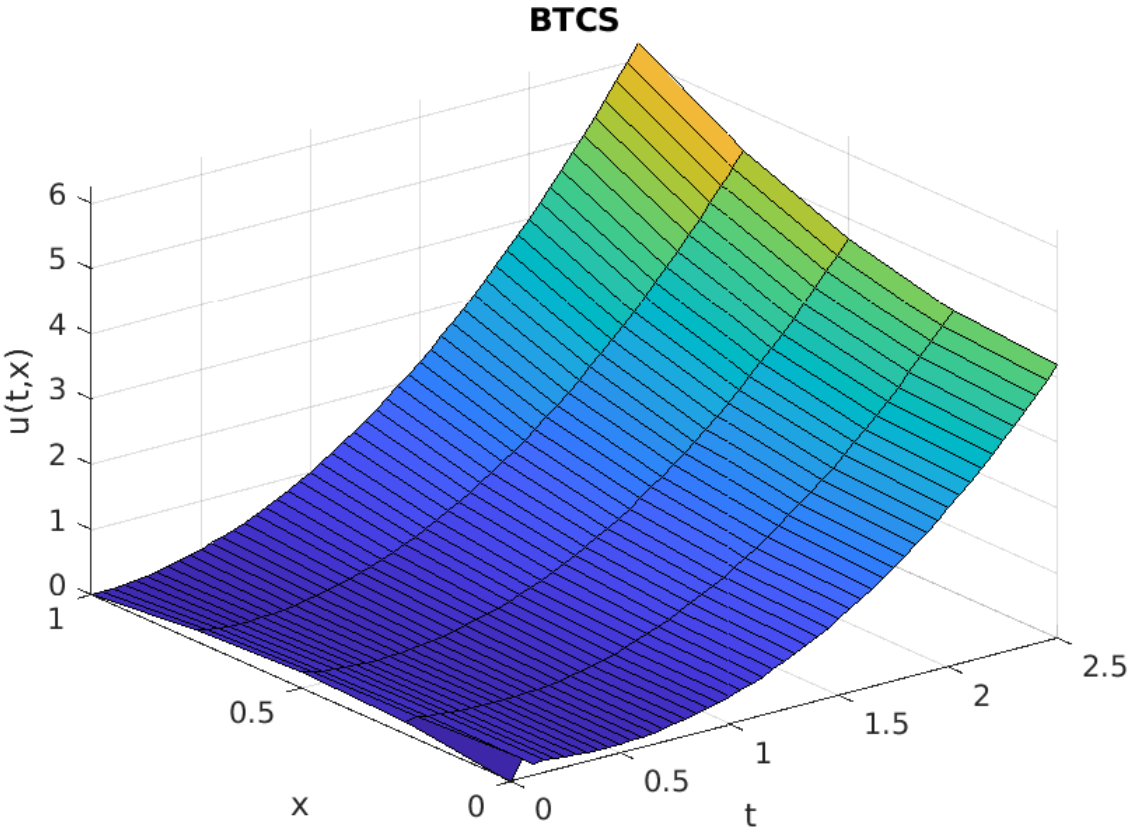
Question 4.

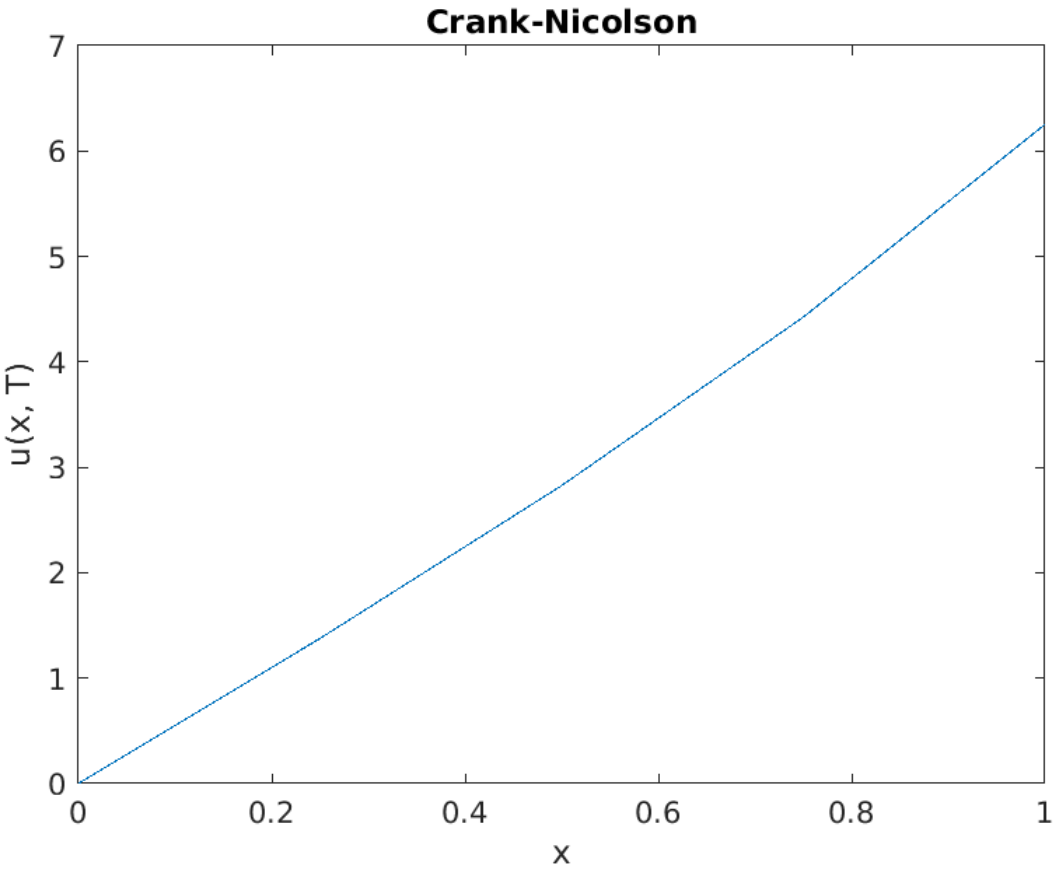
```
Enter spatial step-size: 0.25
Enter time step-size: 0.05
Enter number of time-levels: 50
```

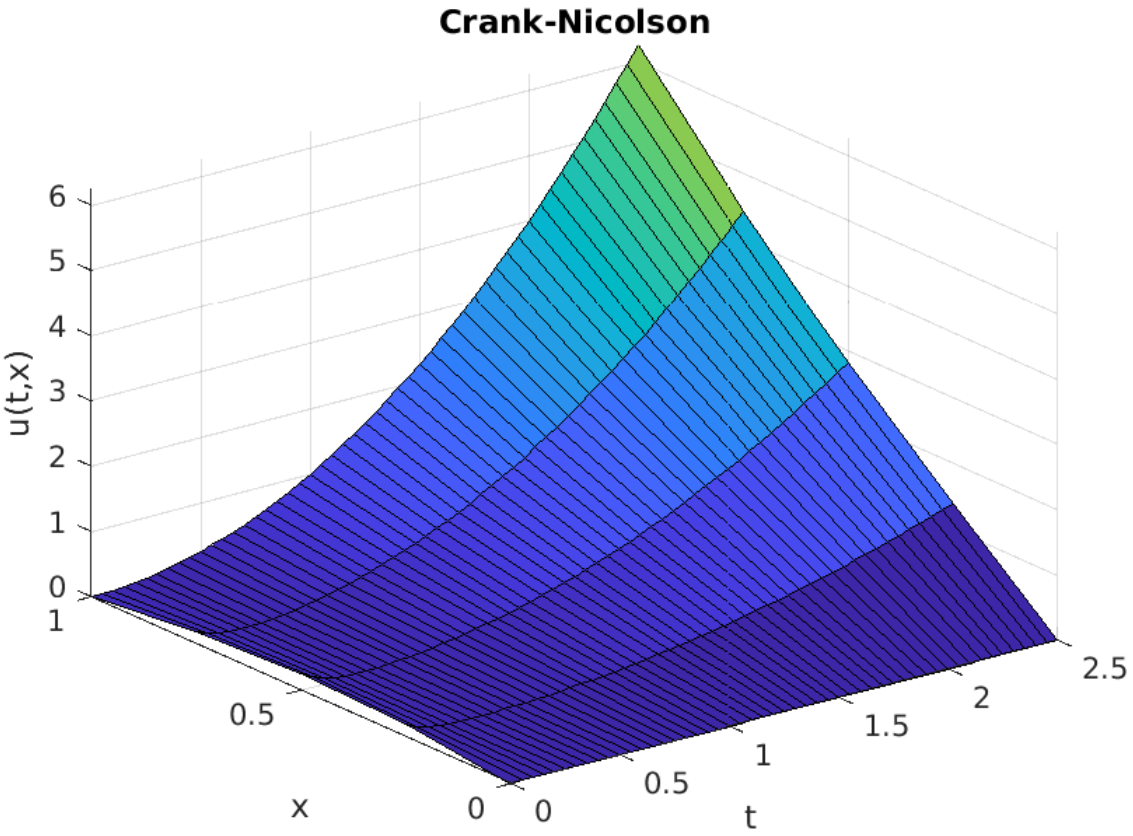



FTCS
×10^14
3
2
1
0
-1
-2
-3
u(t,x)
1
0.5
0
x
0
0.5
1
1.5
2
2.5
t


BTCS
6.5
6
5.5
5
4.5
4
u(x, T)
0
0.2
0.4
0.6
0.8
1
x


BTCS
u(t,x)
6
5
4
3
2
1
0
1
0.5
x
0
0
0.5
1
1.5
2
2.5
t


Crank-Nicolson
u(x, T)
x
0
1
2
3
4
5
6
7
0
0.2
0.4
0.6
0.8
1


Crank-Nicolson
u(t,x)
6
5
4
3
2
1
0
1
0.5
x
0
0
0.5
1
1.5
2
2.5
t