Question 02

We have 4 components mixture of bivariate Gaussian Distribution.

Let 4 components be f_1, f_2, f_3 and f_4. Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, be the corresponding parameters for respective bivariate Gaussian Distribution.

Then the mixture density will be:-

$$\boxed{f = \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3 + \lambda_4 f_4}$$, such that $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$

Suppose we have dataset as $\mathcal{D} = \{x_1, x_2, x_3 \ldots x_n\}$.
Now we introduce variables which are hidden, $z_1, z_2, z_3 \ldots z_n$; where-

$$z_i = \begin{cases} 0 & \text{if } x_i \in 1^{st} \text{ component} \\ 1 & \text{if } x_i \in 2^{nd} \text{ component} \\ 2 & \text{if } x_i \in 3^{rd} \text{ component} \\ 3 & \text{if } x_i \in 4^{th} \text{ component} \end{cases}$$

Let θ be our parameters.

Now we can write complete data log-likelihood as follows:-

$$\log p(x_i\, z_i \mid \theta) = \log \Big\{ (\lambda_1 f_1)^{\frac{(z_i-1)(z_i-2)(z_i-3)}{-6}} \; (\lambda_2 f_2)^{\frac{z_i(z_i-2)(z_i-3)}{2}} \; (\lambda_3 f_3)^{\frac{z_i(z_i-1)(z_i-3)}{-2}} \; (\lambda_4 f_4)^{\frac{z_i(z_i-1)(z_i-2)}{6}} \Big\}$$

where f_1, f_2, f_3 and f_4 are densities of 4 bivariate Gaussian Distribution present in mixture.