

A game is played between players, where each player has to choose some strategies or strategy and each player gets payoff.

Example chess, tic-tac-toe,

Actions of players are : moving their pieces at their turn. The turns come sequentially. Payoff: the satisfaction of winning a game of chess.

Actions of players are: marking different positions at their turn. The turn comes sequentially. Payoff: satisfaction of winning a game of tic-tac-toe.

Three friends can jointly invest and put effort in a project and get a profit from this project. How to share the profit among the three friends. It may happen only two form a coalition out of the three.

Action: to form coalition. Payoff: share of the profit.

There are two sellers selling an object and one buyer who wants to buy only one unit of the object. Both the seller wants to sell to the buyer. The buyer can buy from seller 1 or seller 2.

Action: seller 1 and seller 2 to sell and buy to buy. It can be understood as coalition $\{seller1, buyer\}$, $\{seller2, buyer\}$, $\{seller1, seller2, buyer\}$.

In the first two coalition, if they form then transaction happens. In the third coalition, their may be sharing between seller 1 and seller 2 in the transaction with the buyer. The payoffs are gain from the trade or transaction to the buyer and seller.

We study strategic situations using game. While choosing action or strategy a player always consider how others are going to react or behave. We use Nash equilibrium to study such situation.

We are going to study only coalition formation game. We expand the profit sharing game.

Suppose there are three students $\{A, B, C\}$. This is the set of players. Jointly they can produce the following values or profits.

Coalitions	Value or worth of coalitions
$\{A\}$	10
$\{B\}$	10
$\{C\}$	10
$\{A, B\}$	20
$\{A, C\}$	20
$\{B, C\}$	30
$\{A, B, C\}$	50

Here the question are

How to divide the worth or value of the coalition and which one of the coalitions are going to be formed?

Suppose one person A owns an old car which values nothing to him. There are two potential buyers, Buyer B values it at 1000 and buyer C values it at 1050. The trade between these people can be analysed based on coalition formation.

Coalitions	Value or worth of coalitions
$\{A\}$	0
$\{B\}$	0
$\{C\}$	0
$\{A, B\}$	1000
$\{A, C\}$	1050
$\{B, C\}$	0
$\{A, B, C\}$	1050

Suppose seller sells at p_B to B , then B will buy if $p_B \leq 1000$.
Thus, the value of A is p_B and value of B is $1000 - p_B$. So
$$v(\{A, B\}) = p_B + 1000 - p_B = 1000.$$

Suppose seller sells at p_C to C , then C will buy if $p_C \leq 1050$.
Thus, the value of A is p_C and value of C is $1050 - p_C$. So
$$v(\{A, C\}) = p_C + 1050 - p_C = 1050.$$

Suppose seller A sells at p_C to C and C offers p_c to seller and $1000 - p_c$ to buyer B . So the values are p_c to A , $1000 - p_c$ to buyer B and $1050 - p_c - (1000 - p_c)$ to buyer C . The worth of the coalition is

$$v(\{A, B, c\}) = p_c + (1000 - p_c) + (1050 - p_c - (1000 - p_c)) = 1050.$$

Suppose seller A sells at p_C to C and C offers p_c to seller and buyer B gets zero. So the values are P_C to A, 0 to buyer B and $1050 - p_c$ to buyer C. The worth of the coalition is

$$v(\{A, B, c\}) = p_C + (1000 - p_c) + 0 = 1050.$$

What is going to be the outcome?

There are going to be n players, $N = \{A_1, A_2, A_3 \dots A_N\}$ a finite set of players and $n \geq 2$.

A subset S of N is called a coalition of N .

The entire set is called the grand coalition.

If there are N players then we have 2^N possible coalitions.

If we have 2 players, 8 coalitions are possible;

$\{\emptyset\}, \{A_1\}, \{A_2\}, \{A_3\}, \{A_1, A_2\}, \{A_1, A_3\}, \{A_2, A_3\}, \{A_1, A_2, A_3\}$

Suppose $S \subset N$, it means S is coalition then complement of S is

$N \setminus S$ is the set of all players not in S but in N .

$|S|$ denotes the number of players in the coalition S .

The worth of a coalition is defined based on a function called characteristic function.

A function $v : 2^N \rightarrow R_+$ such that $v(\emptyset) = 0$, where R_+ denotes non negative real numbers.

An example

Coalitions	Value or worth of coalitions
$\{A\}$	0
$\{B\}$	0
$\{C\}$	0
$\{A, B\}$	1000
$\{A, C\}$	1050
$\{B, C\}$	0
$\{A, B, C\}$	1050
$\{\emptyset\}$	0

