Fair Allocation

Bankruptcy problem:

- Suppose a person borrows money from two creditors, c_1 and c_2.
- The borrower cannot repay the amount $c-1$ and c_2 as he has gone bankrupt.
- The worth of the asset created from these borrow in M, and $M \leq c_1 + c_2$.
- How do we allocate M between creditor 1 and 2?
- We have to do some form of rationing.

Surplus sharing problem:

- Suppose two person forms a joint venture.
- Person 1 invests c_1 amount and person 2 invest c_2 amount.
- Suppose the revenue generated from this joint venture is M and $M > c_1 + c_2$. It means the joint venture has made profit.
- How do we divide this M between the partners 1 and 2?
- If the $M < c_1 + c_2$, the joint venture has made loss. How to divide this M between the partners 1 and 2?

Rationing of medical supplies:

- Suppose there are two patients, each needs c_i amount of injections.
- The hospital authority has only M units and suppose $M < c_1 + c_2$.
- How do we divide this M between these two patients?

It is based on

- Allocation/division rules:
  - Proportional rule
  - Constrains equal award rule (equal sharing)
  - Constraint equal loss or constraint equal gain rule
- Allocation procedure:
  - Contested garment method
  - Rif method
  - O'Neill's division method.

Proportional method:

Given $((M, c)$ where c is claim vector and M is the endowment. $x = (x_1, x_2)$ is the final allocation to person 1 and 2 or claimant 1 and 2.

$$x_1 = \frac{c_1 M}{c_1 + c_2}, \quad x_2 = \frac{c_2 M}{c_1 + c_2}.$$

Each claimant gets proportional to its claim.

Constraint equal awards rule:

Given $((M, c)$ where c is claim vector and M is the endowment.
$x_1 = \min\{c_1, \lambda\}, \quad x_2 = \min\{c_2, \lambda\}$, where λ is such that
$\sum_{i=1}^{2} x_i = M$.

Each claims gets which ever is less its claim or an amount which is same for all the claimants.

First divide M equally $\frac{M}{N}$ among the players. If $\frac{M}{N} > c_i$ for some i players . Allocate c_i to those players and rest $M - \sum\limits_{i \in S} c_i$ where $S \subset N$, is allocated equally among the remaining players. In this remaining players, if there are some j players whose $\frac{M - \sum\limits_{i \in S} c_i}{|N - S|} < c_j$, allocate c_j to those players and the remaining $M - \sum\limits_{i \in S} c_i - \sum j \in S_1 c_j$ is equally divided among the rest of the claimants.

Constraint equal losses rule:

Given $((M, c)$ where c is claim vector and M is the endowment. $x_1 = \max\{c_1 - \theta, 0\}, \quad x_2 = \max\{c_2 - \theta, 0\}$ where θ is such that $x_1 + x_2 = M$.
$\theta = \dfrac{c_1 + c_2 - M}{2}$, if $\dfrac{c_1 + c_2 - M}{2} < c_i, i = 1, 2.$
$\theta = M - c_1 - c_2$ if any c_i is less than $\dfrac{c_1 + c_2 - M}{2}$. Both cannot be less because in that case $C_1 + c_2 > M$ is not satisfied.