Chapter 1

The Basics of Credit Risk Management

Why is credit risk management an important issue in banking? To answer this question let us construct an example which is, although simplified, nevertheless not too unrealistic: Assume a major building company is asking its house bank for a loan in the size of ten billion Euro. Somewhere in the bank's credit department a senior analyst has the difficult job to decide if the loan will be given to the customer or if the credit request will be rejected. Let us further assume that the analyst knows that the bank's chief credit officer has known the chief executive officer of the building company for many years, and to make things even worse, the credit analyst knows from recent default studies that the building industry is under hard pressure and that the *bank-internal rating*[1] of this particular building company is just on the way down to a low *subinvestment grade*.

What should the analyst do? Well, the most natural answer would be that the analyst should reject the deal based on the information she or he has about the company and the current market situation. An alternative would be to grant the loan to the customer but to *insure* the loss potentially arising from the engagement by means of some credit risk management instrument (e.g., a so-called *credit derivative*).

Admittedly, we intentionally exaggerated in our description, but situations like the one just constructed happen from time to time and it is never easy for a credit officer to make a decision under such difficult circumstances. A brief look at any typical banking portfolio will be sufficient to convince people that defaulting obligors belong to the daily business of banking the same way as credit applications or ATM machines. Banks therefore started to think about ways of *loan insurance* many years ago, and the insurance paradigm will now directly lead us to the first central building block credit risk management.

[1] A rating is an indication of creditworthiness; see Section 1.1.1.1.

1.1 Expected Loss

Situations as the one described in the introduction suggest the need of a *loss protection* in terms of an *insurance*, as one knows it from car or health insurances. Moreover, history shows that even good customers have a potential to default on their financial obligations, such that an insurance for not only the critical but all loans in the bank's credit portfolio makes much sense.

The basic idea behind insurance is always the same. For example, in health insurance the costs of a few sick customers are covered by the total sum of revenues from the fees paid to the insurance company by all customers. Therefore, the fee that a man at the age of thirty has to pay for health insurance protection somehow reflects the insurance company's experience regarding *expected costs* arising from this particular group of clients.

For bank loans one can argue exactly the same way: Charging an appropriate *risk premium* for every loan and collecting these risk premiums in an internal bank account called *expected loss reserve* will create a capital cushion for covering losses arising from defaulted loans.

In probability theory the attribute *expected* always refers to an *expectation* or *mean value*, and this is also the case in risk management. The basic idea is as follows: The bank assigns to every customer a *default probability* (DP), a loss fraction called the *loss given default* (LGD), describing the fraction of the loan's exposure expected to be lost in case of default, and the *exposure at default* (EAD) subject to be lost in the considered time period. The loss of any obligor is then defined by a *loss variable*

$$\tilde{L} \;=\; \text{EAD} \times \text{LGD} \times L \qquad \text{with} \qquad L = \mathbf{1}_D, \quad \mathbb{P}(D) = \text{DP}, \qquad (1.\;1)$$

where D denotes the *event* that the obligor defaults in a certain period of time (most often one year), and $\mathbb{P}(D)$ denotes the probability of D. Although we will not go too much into technical details, we should mention here that underlying our model is some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, consisting of a *sample space* Ω, a *σ-Algebra* $\mathcal{F}$, and a probability measure $\mathbb{P}$. The elements of $\mathcal{F}$ are the *measurable events* of the model, and intuitively it makes sense to claim that the event of default should be measurable. Moreover, it is common to identify $\mathcal{F}$ with the *information* available, and the information if an obligor defaults or survives should be included in the set of measurable events.

Now, in this setting it is very natural to define the *expected loss* (EL) of any customer as the expectation of its corresponding loss variable $\tilde{L}$, namely

$$\mathrm{EL} \;=\; \mathbb{E}[\tilde{L}] \;=\; \mathrm{EAD} \times \mathrm{LGD} \times \mathbb{P}(D) \;=\; \mathrm{EAD} \times \mathrm{LGD} \times \mathrm{DP}, \tag{1. 2}$$

because the expectation of any *Bernoulli* random variable, like $\mathbf{1}_D$, is its event probability. For obtaining representation (1. 2) of the EL, we need some additional assumption on the constituents of Formula (1. 1), for example, the assumption that EAD and LGD are constant values. This is not necessarily the case under all circumstances. There are various situations in which, for example, the EAD has to be modeled as a random variable due to uncertainties in amortization, usage, and other drivers of EAD up to the chosen planning horizon. In such cases the EL is still given by Equation (1. 2) if one can assume that the exposure, the loss given default, and the default event D are independent and EAD and LGD are the expectations of some underlying random variables. But even the independence assumption is questionable and in general very much simplifying. Altogether one can say that (1. 2) is the most simple representation formula for the expected loss, and that the more simplifying assumptions are dropped, the more one moves away from closed and easy formulas like (1. 2).

However, for now we should not be bothered about the independence assumption on which (1. 2) is based: The basic concept of expected loss is the same, no matter if the constituents of formula (1. 1) are independent or not. Equation (1. 2) is just a convenient way to write the EL in the first case. Although our focus in the book is on *portfolio risk* rather than on *single obligor risk* we briefly describe the three constituents of Formula (1. 2) in the following paragraphs. Our convention from now on is that the EAD always is a deterministic (i.e., nonrandom) quantity, whereas the *severity* (SEV) of loss in case of default will be considered as a random variable with expectation given by the LGD of the respective facility. For reasons of simplicity we assume in this chapter that the severity is independent of the variable L in (1. 1).

1.1.1 The Default Probability

The task of assigning a default probability to every customer in the bank's credit portfolio is far from being easy. There are essentially two approaches to default probabilities:

- *Calibration of default probabilities from market data.*

  The most famous representative of this type of default probabilities is the concept of *Expected Default Frequencies* (EDF) from KMV[2] Corporation. We will describe the KMV-Model in Section 1.2.3 and in Chapter 3.

  Another method for calibrating default probabilities from market data is based on credit spreads of traded products bearing credit risk, e.g., corporate bonds and credit derivatives (for example, *credit default swaps*; see the chapter on *credit derivatives*).

- *Calibration of default probabilites from ratings.*

  In this approach, default probabilities are associated with *ratings*, and ratings are assigned to customers either by external rating agencies like *Moody's Investors Services*, *Standard & Poor's* (S&P), or *Fitch*, or by bank-internal rating methodologies. Because ratings are not subject to be discussed in this book, we will only briefly explain some basics about ratings. An excellent treatment of this topic can be found in a survey paper by Crouhy et al. [22].

The remaining part of this section is intended to give some basic indication about the calibration of default probabilities to ratings.

1.1.1.1 Ratings

Basically ratings describe the *creditworthiness* of customers. Hereby quantitative as well as qualitative information is used to evaluate a client. In practice, the rating procedure is often more based on the judgement and experience of the rating analyst than on pure mathematical procedures with strictly defined outcomes. It turns out that in the US and Canada, most issuers of public debt are rated at least by two of the three main rating agencies Moody's, S&P, and Fitch.

[2]KMV Corp., founded 13 years ago, headquartered in San Francisco, develops and distributes credit risk management products; see www.kmv.com.

Their reports on *corporate bond defaults* are publicly available, either by asking at their local offices for the respective reports or conveniently per web access; see www.moodys.com, www.standardandpoors.com, www.fitchratings.com.

In Germany and also in Europe there are not as many companies issuing traded debt instruments (e.g., bonds) as in the US. Therefore, many companies in European banking books do not have an external rating. As a consequence, banks need to invest[3] more effort in their own bank-internal rating system. The natural candidates for assigning a rating to a customer are the credit analysts of the bank. Hereby they have to consider many different *drivers* of the considered firm's economic future:

- Future *earnings* and *cashflows*,
- *debt*, *short- and long-term liabilities*, and *financial obligations*,
- *capital structure* (e.g., *leverage*),
- *liquidity* of the firm's assets,
- situation (e.g., political, social, etc.) of the firm's home *country*,
- situation of the *market* (e.g., *industry*), in which the company has its main activities,
- *management quality*, company *structure*, etc.

From this by no means exhaustive list it should be obvious that a rating is an *attribute of creditworthiness* which can not be captured by a pure mathematical formalism. It is a best practice in banking that ratings as an outcome of a statistical tool are always re-evaluated by the rating specialist in charge of the rating process. It is frequently the case that this re-evaluation moves the rating of a firm by one or more notches away from the "mathematically" generated rating. In other words, *statistical tools provide a first indication* regarding the rating of a customer, but due to the various *soft factors* underlying a rating, the

[3]Without going into details we would like to add that banks always should base the decision about creditworthiness on their bank-internal rating systems. As a main reason one could argue that banks know their customers best. Moreover, it is well known that external ratings do not react quick enough to changes in the economic health of a company. Banks should be able to do it better, at least in the case of their long-term relationship customers.

responsibility to assign a final rating remains the duty of the rating analyst.

Now, it is important to know that the rating agencies have established an ordered *scale* of ratings in terms of a letter system describing the creditworthiness of rated companies. The rating categories of Moody's and S&P are slightly different, but it is not difficult to find a mapping between the two. To give an example, Table 1.1 shows the rating categories of S&P as published[4] in [118].

As already mentioned, Moody's system is slightly different in meaning as well as in rating letters. Their rating categories are Aaa, Aa, A, Baa, Ba, B, Caa, Ca, C, where the creditworthiness is highest for Aaa and poorest for C. Moreover, both rating agencies additionally provide ratings on a *finer scale*, allowing for a more accurate distinction between different credit qualities.

1.1.1.2 Calibration of Default Probabilities to Ratings

The process of assigning a default probability to a rating is called a *calibration*. In this paragraph we will demonstrate how such a calibration works. The end product of a calibration of default probabilities to ratings is a mapping

$$\text{Rating} \mapsto \text{DP}, \qquad e.g., \qquad \{AAA, AA, ..., C\} \to [0,1], \quad R \mapsto \text{DP}(R),$$

such that to every rating R a certain default probability $\text{DP}(R)$ is assigned.

In the sequel we explain by means of Moody's data how a calibration of default probabilities to external ratings can be done. From Moody's website or from other resources it is easy to get access to their recent study [95] of *historic corporate bond defaults*. There one can find a table like the one shown in Table 1.2 (see [95] Exhibit 40) showing historic default frequencies for the years 1983 up to 2000.

Note that in our illustrative example we chose the *fine ratings scale* of Moody's, making finer differences regarding the creditworthiness of obligors.

Now, an important observation is that for best ratings no defaults at all have been observed. This is not as surprising as it looks at first sight: For example rating class *Aaa* is often calibrated with a default probability of 2 bps ("bp" stands for *'basispoint'* and means 0.01%),

[4] Note that we use shorter formulations instead of the exact wording of S&P.

TABLE 1.1: S&P Rating Categories [118].

AAA	*best credit quality* *extremely reliable with regard to financial obligations*
AA	*very good credit quality* *very reliable*
A	*more susceptible to economic conditions* *still good credit quality*
BBB	*lowest rating in investment grade*
BB	*caution is necessary* *best sub-investment credit quality*
B	*vulnerable to changes in economic conditions* *currently showing the ability to meet its financial obligations*
CCC	*currently vulnerable to nonpayment* *dependent on favourable economic conditions*
CC	*highly vulnerable to a payment default*
C	*close to or already bankrupt* *payments on the obligation currently continued*
D	*payment default on some financial obligation has actually occurred*

TABLE 1.2: Moody's Historic Corporate Bond Default Frequencies.

Rating	1983	1984	1985	1986	1987	1988
Aaa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Baa1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Baa2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Baa3	0.00%	1.06%	0.00%	4.82%	0.00%	0.00%
Ba1	0.00%	1.16%	0.00%	0.88%	3.73%	0.00%
Ba2	0.00%	1.61%	1.63%	1.20%	0.95%	0.00%
Ba3	2.61%	0.00%	3.77%	3.44%	2.95%	2.59%
B1	0.00%	5.84%	4.38%	7.61%	4.93%	4.34%
B2	10.00%	18.75%	7.41%	16.67%	4.30%	6.90%
B3	17.91%	2.90%	13.86%	16.07%	10.37%	9.72%

Rating	1989	1990	1991	1992	1993	1994
Aaa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa3	1.40%	0.00%	0.00%	0.00%	0.00%	0.00%
A1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Baa1	0.00%	0.00%	0.76%	0.00%	0.00%	0.00%
Baa2	0.80%	0.00%	0.00%	0.00%	0.00%	0.00%
Baa3	1.07%	0.00%	0.00%	0.00%	0.00%	0.00%
Ba1	0.79%	2.67%	1.06%	0.00%	0.81%	0.00%
Ba2	1.82%	2.82%	0.00%	0.00%	0.00%	0.00%
Ba3	4.71%	3.92%	9.89%	0.74%	0.75%	0.59%
B1	6.24%	8.59%	6.04%	1.03%	3.32%	1.90%
B2	8.28%	22.09%	12.74%	1.54%	4.96%	3.66%
B3	19.55%	28.93%	28.42%	24.54%	11.48%	8.05%

Rating	1995	1996	1997	1998	1999	2000
Aaa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Baa1	0.00%	0.00%	0.00%	0.00%	0.00%	0.29%
Baa2	0.00%	0.00%	0.00%	0.32%	0.00%	0.00%
Baa3	0.00%	0.00%	0.00%	0.00%	0.34%	0.98%
Ba1	0.00%	0.00%	0.00%	0.00%	0.47%	0.91%
Ba2	0.00%	0.00%	0.00%	0.61%	0.00%	0.66%
Ba3	1.72%	0.00%	0.47%	1.09%	2.27%	1.51%
B1	4.35%	1.17%	0.00%	2.13%	3.08%	3.25%
B2	6.36%	0.00%	1.50%	7.57%	6.68%	3.89%
B3	4.10%	3.36%	7.41%	5.61%	9.90%	9.92%

essentially meaning that one expects a Aaa-default in average twice in $10,000$ years. This is a long time to go; so, one should not be surprised that quite often best ratings are lack of any default history. Nevertheless we believe that it would not be correct to take the historical zero-balance as an indication that these rating classes are risk-free opportunities for credit investment. Therefore, we have to find a way to assign small but positive default probabilities to those ratings.

Figure 1.1 shows our "quick-and-dirty working solution" of the problem, where we use the attribute "quick-and-dirty" because in practice one would try to do the calibration a little more sophisticatedly[5].

However, for illustrative purposes our solution is sufficient, because it shows the main idea. We do the calibration in three steps:

1. Denote by $h_i(R)$ the historic default frequency of rating class R for year i, where i ranges from 1983 to 2000. For example, $h_{1993}(Ba1) = 0.81\%$. Then compute the mean value and the standard deviation of these frequencies over the years, where the rating is fixed, namely

$$m(R) = \frac{1}{18} \sum_{i=1983}^{2000} h_i(R) \qquad \text{and}$$

$$s(R) = \frac{1}{17} \sum_{i=1983}^{2000} \big(h_i(R) - m(R)\big)^2 .$$

   The mean value $m(R)$ for rating R is our first guess of the potential default probability assigned to rating R. The standard deviation $s(R)$ gives us some insight about the volatility and therefore about the error we eventually make when believing that $m(R)$ is a good estimate of the default probability of R-rated obligors. Figure 1.1 shows the values $m(R)$ and $s(R)$ for the considered rating classes. Because even best rated obligors are not free of default risk, we write "not observed" in the cells corresponding to $m(R)$ and $s(R)$ for ratings R=Aaa,Aa1,Aa2,A1,A2,A3 (ratings where no defaults have been observed) in Figure 1.1.

2. Next, we plot the mean values $m(R)$ into a coordinate system, where the x-axis refers to the rating classes (here numbered from

[5] For example, one could look at investment and sub-investment grades separately.

1 (Aaa) to 16 ($B3$)). One can see in the chart in Figure 1.1 that on a *logarithmic scale* the mean default frequencies $m(R)$ can be fitted by a regression line. Here we should add a comment that there is strong evidence from various empirical default studies that default frequencies grow exponentially with decreasing creditworthiness. For this reason we have chosen an exponential fit (linear on logarithmic scale). Using standard regression theory, see,e.g.,[106]Chapter 4, or by simply using any software providing basic statistical functions, one can easily obtain the following exponential function fitting our data:

$$\mathrm{DP}(x) \;=\; 3 \times 10^{-5}\, e^{0.5075\,x} \qquad (x = 1, ..., 16).$$

3. As a last step, we use our regression equation for the estimation of default probabilities $\mathrm{DP}(x)$ assigned to rating classes x ranging from 1 to 16. Figure 1.1 shows our result, which we now call *a calibration of default probabilities to Moody's ratings*. Note that based on our regression even the best rating Aaa has a small but positive default probability. Moreover, our hope is that our regression analysis has smoothed out sampling errors from the historically observed data.

Although there is much more to say about default probabilities, we stop the discussion here. However, later on we will come back to default probabilities in various contexts.

1.1.2 The Exposure at Default

The EAD is the quantity in Equation (1. 2) specifying the exposure the bank does have to its borrower. In general, the exposure consists of two major parts, the *outstandings* and the *commitments*. The outstandings refer to the portion of the exposure already drawn by the obligor. In case of the borrower's default, the bank is exposed to the total amount of the outstandings. The commitments can be divided in two portions, *undrawn* and *drawn*, in the time before default. The total amount of commitments is the exposure the bank has promised to lend to the obligor at her or his request. Historical default experience shows that obligors tend to draw on committed lines of credit in times of financial distress. Therefore, the commitment is also subject to loss in case of the obligor's default, but only the drawn (prior default) amount

Rating	Mean	Standard-Deviation	Default Probability
Aaa	not observed	not observed	0.005%
Aa1	not observed	not observed	0.008%
Aa2	not observed	not observed	0.014%
Aa3	0.08%	0.33%	0.023%
A1	not observed	not observed	0.038%
A2	not observed	not observed	0.063%
A3	not observed	not observed	0.105%
Baa1	0.06%	0.19%	0.174%
Baa2	0.06%	0.20%	0.289%
Baa3	0.46%	1.16%	0.480%
Ba1	0.69%	1.03%	0.797%
Ba2	0.63%	0.86%	1.324%
Ba3	2.39%	2.35%	2.200%
B1	3.79%	2.49%	3.654%
B2	7.96%	6.08%	6.070%
B3	12.89%	8.14%	10.083%

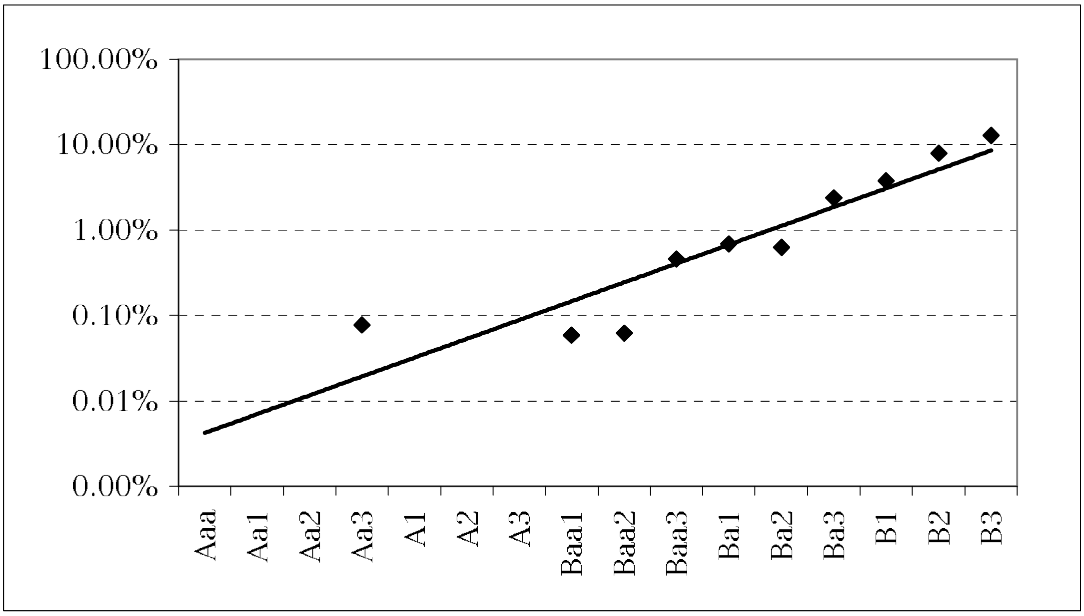

FIGURE 1.1
Calibration of Moody's Ratings to Default Probabilities

of the commitments will actually contribute to the loss on loan. The fraction describing the decomposition of commitments in drawn and undrawn portions is a random variable due to the optional character commitments have (the obligor has the right but not the obligation to draw on committed lines of credit). Therefore it is natural to define the EAD by

$$\text{EAD} = \text{OUTST} + \gamma \times \text{COMM}, \tag{1. 3}$$

where OUTST denotes the outstandings and COMM the commitments of the loan, and γ is the expected portion of the commitments likely to be drawn prior to default. More precisely, γ is the expectation of the random variable capturing the uncertain part of the EAD, namely the utilization of the undrawn part of the commitments. Obviously, γ takes place in the unit interval. Recall that we assume the EAD to be a deterministic (i.e., nonrandom) quantity. This is the reason why we directly deal with the expectation γ, hereby ignoring the underlying random variable.

In practice, banks will calibrate γ w.r.t. the creditworthiness of the borrower and the type of the facility involved.

Note that in many cases, commitments include various so-called *covenants*, which are embedded options either the bank has written to the obligor or reserved to itself. Such covenants may, for example, force an obligor in times of financial distress to provide more collateral[6] or to renegotiate the terms of the loan. However, often the obligor has some informational advantage in that the bank recognizes financial distress of its borrowers with some delay. In case of covenants allowing the bank to close committed lines triggered by some early default indication, it really is a question of time if the bank picks up such indications early enough to react before the customer has drawn on her or his committed lines. The problem of appropriate and quick action of the lending institute is especially critical for obligors with former good credit quality, because banks tend to focus more on critical than on good customers regarding credit lines (bad customers get much more attention, because the bank is already "alarmed" and will be more sensitive in case of early warnings of financial instability). Any stochastic modeling of EAD should take these aspects into account.

[6]Collateral means assets securing a loan, e.g., mortgages, bonds, guarantees, etc. In case a loan defaults, the value of the collateral reduces the loss on the defaulted loan.

The *Basel Committee on Banking Supervision*[7] in its recent consultative document [103] defines the EAD for on-balance sheet transactions to be identical to the *nominal* amount of the exposure.

For off-balance sheet transactions there are two approaches: For the *foundation approach* the committee proposes to define the EAD on commitments and revolving credits as 75% of the off-balance sheet amount of the exposure. For example, for a committed line of one billion Euro with current outstandings of 600 million, the EAD would be equal to $600 + 75\% \times 400 = 900$ million Euro.

For the *advanced approach*, the committee proposes that banks eligible for this approach will be permitted to use their own internal estimates of EAD for transactions with uncertain exposure. From this perspective it makes much sense for major banks to carefully think about some rigorous methodology for calibrating EAD to borrower- and facility-specific characteristics. For example, banks that are able to calibrate the parameter γ in (1. 3) on a finer scale will have more accurate estimates of the EAD, better reflecting the underlying credit risk. The more the determination of regulatory capital tends towards risk sensitivity, the more will banks with advanced methodology benefit from a more sophisticated calibration of EAD.

1.1.3 The Loss Given Default

The LGD of a transaction is more or less determined by "1 minus recovery rate", i.e., the LGD quantifies the portion of loss the bank will really suffer in case of default. The estimation of such loss quotes is far from being straightforward, because recovery rates depend on many driving factors, for example on the *quality of collateral* (securities, mortgages, guarantees, etc.) and on the *seniority* of the bank's claim on the borrower's assets. This is the reason behind our convention to consider the loss given default as a random variable describing the *severity* of the loss of a facility type in case of default. The notion LGD then refers to the expectation of the severity.

A bank-external source for recovery data comes from the rating agencies. For example Moody's [95] provides recovery values of defaulted bonds, hereby distinguishing between different seniorities.

[7] The Basle Commitee coordinates the rules and guidelines for banking supervision. Its members are central banks and other national offices or government agencies responsible for banking supervision.

Unfortunately many banks do not have good internal data for estimating recovery rates. In fact, although LGD is a key driver of EL, there is in comparison with other risk drivers like the DP little progress made in moving towards a sophisticated calibration. There are initiatives (for example by the ISDA[8] and other similar organisations) to bring together many banks for sharing knowledge about their practical LGD experience as well as current techniques for estimating it from historical data.

However, one can expect that in a few years LGD databases will have significantly improved, such that more accurate estimates of the LGD for certain banking products can be made.

1.2 Unexpected Loss

At the beginning of this chapter we introduced the EL of a transaction as an insurance or loss reserve in order to cover losses the bank expects from historical default experience. But holding capital as a cushion against *expected* losses is not enough. In fact, the bank should in addition to the expected loss reserve also save money for covering *unexpected* losses exceeding the average experienced losses from past history. As a measure of the magnitude of the deviation of losses from the EL, the standard deviation of the loss variable $\tilde{L}$ as defined in (1. 1) is a natural choice. For obvious reasons, this quantity is called the *Unexpected Loss* (UL), defined by

$$\mathrm{UL} \;=\; \sqrt{\mathbb{V}[\tilde{L}]} \;=\; \sqrt{\mathbb{V}\big[\mathrm{EAD}\times\mathrm{SEV}\times L\big]}\,.$$

1.2.1 Proposition *Under the assumption that the severity and the default event D are uncorrelated, the unexpected loss of a loan is given by*

$$UL \;=\; EAD\times\sqrt{\mathbb{V}[SEV]\times DP + LGD^2\times DP(1-DP)}\,.$$

Proof. Taking $\mathbb{V}[X]=\mathbb{E}[X^2]-\mathbb{E}[X]^2$ and $\mathbb{V}[1_D]=\mathrm{DP}(1-\mathrm{DP})$ into account, the assertion follows from a straighforward calculation. □

[8]International Swap Dealers Association.

1.2.2 Remark Note that the assumption of zero correlation between severity and default event in Proposition 1.2.1 is not always realistic and often just made to obtain a first approximation to the "real" unexpected loss. In fact, it is not unlikely that on average the recovery rate of loans will drop if bad economic conditions induce an increase of default frequencies in the credit markets. Moreover, some types of collateral bear a significant portion of market risk, such that unfavourable market conditions (which might also be the reason for an increased number of default events) imply a decrease of the collateral's market value. In Section 2.5 we discuss a case where the severity of losses and the default events are random variables driven by a common underlying factor.

Now, so far we have always looked at the credit risk of a single facility, although banks have to manage large portfolios consisting of many different products with different risk characteristics. We therefore will now indicate how one can model the total loss of a credit portfolio.

For this purpose we consider a portfolio consisting of m loans

$$\tilde{L}_i = \mathrm{EAD}_i \times \mathrm{SEV}_i \times L_i \ , \qquad \text{with} \qquad (1.\ 4)$$

$$L_i = \mathbf{1}_{D_i} \ , \qquad \mathbb{P}(D_i) = \mathrm{DP}_i \ .$$

The *portfolio loss* is then defined as the random variable

$$\tilde{L}_{PF} = \sum_{i=1}^{m} \tilde{L}_i = \sum_{i=1}^{m} \mathrm{EAD}_i \times \mathrm{SEV}_i \times L_i \ . \qquad (1.\ 5)$$

Analogously to the "standalone" quantities EL and UL we now obtain portfolio quantities EL_{PF} and UL_{PF}, defined by the expectation respectively standard deviation of the portfolio loss. In case of EL we can use the additivity of expectations to obtain

$$\mathrm{EL}_{PF} = \sum_{i=1}^{m} \mathrm{EL}_i = \sum_{i=1}^{m} \mathrm{EAD}_i \times \mathrm{LGD}_i \times \mathrm{DP}_i \ . \qquad (1.\ 6)$$

In case of the UL, additivity holds if the loss variables $\tilde{L}_i$ are pairwise uncorrelated (see Bienaymés Theorem in [7] Chapter 8). If the loss variables are correlated, additivity is lost. Unfortunately this is the standard case, because correlations are "part of the game" and a main driver of credit risk. In fact, large parts of this book will essentially be dealing with correlation modeling. The UL of a portfolio is the first risk quantity we meet where correlations respectively covariances play a fundamental role:

$$\mathrm{UL}_{PF} = \sqrt{\mathbb{V}[\tilde{L}_{PF}]} \tag{1. 7}$$

$$= \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m} \mathrm{EAD}_i \times \mathrm{EAD}_j \times \mathrm{Cov}[\mathrm{SEV}_i \times L_i, \mathrm{SEV}_j \times L_j]}\ .$$

Looking at the special case where severities are constant, we can express the portfolio's UL by means of *default correlations*, namely

1.2.3 Proposition *For a portfolio with constant severities we have*

$$UL_{PF}^2 = \sum_{i,j=1}^{m} EAD_i \times EAD_j \times LGD_i \times LGD_j \times$$

$$\times \sqrt{DP_i(1-DP_i)DP_j(1-DP_j)}\,\rho_{ij}$$

where $\rho_{ij} = Corr[L_i, L_j] = Corr[\mathbf{1}_{D_i}, \mathbf{1}_{D_j}]$ *denotes the* default correlation *between counterparties* i *and* j*.*

Proof. The proposition is obvious. □

Before continuing we want for a moment to think about the meaning and interpretation of correlation. For simplicity let us consider a portfolio consisting of two loans with LGD= 100% and EAD= 1. We then only deal with L_i for $i = 1, 2$, and we set $\rho = \mathrm{Corr}[L_1, L_2]$ and $p_i = \mathrm{DP}_i$. Then, the squared UL of our portfolio is obviously given by

$$\mathrm{UL}_{PF}^2 = p_1(1-p_1) + p_2(1-p_2) + 2\rho\sqrt{p_1(1-p_1)}\sqrt{p_2(1-p_2)}\ . \tag{1. 8}$$

We consider three possible cases regarding the default correlation ρ:

- $\rho = 0$. In this case, the third term in (1. 8) vanishes, such that UL_{PF} attains its minimum. This is called the case of *perfect diversification*. The concept of diversification is easily explained. Investing in many different assets generally reduces the overall portfolio risk, because usually it is very unlikely to see a large number of loans defaulting all at once. The less the loans in the portfolio have in common, the higher the chance that default of one obligor does not mean a lot to the economic future of other loans in the portfolio. The case $\rho = 0$ is the case, where the loans in the portfolio are completely unrelated. Interpreting the UL as a substitute[9] for portfolio risk, we see that this case minimizes the overall portfolio risk.

- $\rho > 0$. In this case our two counterparties are interrelated in that default of one counterparty increases the likelihood that the other counterparty will also default. We can make this precise by looking at the conditional default probability of counterparty 2 under the condition that obligor 1 already defaulted:

$$\mathbb{P}[L_2 = 1 \mid L_1 = 1] = \frac{\mathbb{P}[L_1 = 1, L_2 = 1]}{\mathbb{P}[L_1 = 1]} = \frac{\mathbb{E}[L_1 L_2]}{p_1} \qquad (1.\ 9)$$

$$= \frac{p_1 p_2 + \mathrm{Cov}[L_1, L_2]}{p_1} = p_2 + \frac{\mathrm{Cov}[L_1 L_2]}{p_1}\ .$$

  So we see that positive correlation respectively covariance leads to a conditional default probability higher (because of $\mathrm{Cov}[L_1, L_2] > 0$) than the unconditional default probability p_2 of obligor 2. In other words, in case of positive correlation any default in the portfolio has an important implication on other facilities in the portfolio, namely that there might be more losses to be encountered. The extreme case in this scenario is the case of *perfect correlation* ($\rho = 1$). In the case of $p = p_1 = p_2$, Equation (1. 8) shows that in the case of perfect correlation we have $\mathrm{UL}_{PF} = 2\sqrt{p(1-p)}$, essentially meaning that our portfolio contains the risk of only one obligor but with double intensity (*concentration risk*). In this situation it follows immediately from (1. 9) that default of one obligor makes the other obligor defaulting almost surely.

- $\rho < 0$. This is the mirrored situation of the case $\rho > 0$. We therefore only discuss the extreme case of perfect anti-correlation ($\rho = -1$). One then can view an investment in asset 2 as an almost *perfect hedge* against an investment in asset 1, if (additionally to $\rho = -1$) the characteristics (exposure, rating, etc.) of the two loans match. Admittedly, this terminology makes much

[9] Note that in contrast to the EL, the UL is the "true" uncertainty the bank faces when investing in a portfolio because it captures the deviation from the expectation.

more sense when following a *marked-to-market*[10] approach to loan valuation, where an increase in market value of one of the loans immediately (under the assumption $\rho = -1$) would imply a decrease in market value of the other loan. However, from (1. 8) it follows that in the case of a perfect hedge the portfolio's UL completely vanishes ($\mathrm{UL}_{PF} = 0$). This means that our perfect hedge (investing in asset 2 with correlation -1 w.r.t. a comparable and already owned asset 1) completely eliminates (neutralizes) the risk of asset 1.

We now turn to the important notion of economic capital.

1.2.1 Economic Capital

We have learned so far that banks should hold some capital cushion against unexpected losses. However, defining the UL of a portfolio as the *risk capital* saved for cases of financial distress is not the best choice, because there might be a significant likelihood that losses will exceed the portfolio's EL by more than one standard deviation of the portfolio loss. Therefore one seeks other ways to quantify risk capital, hereby taking a *target level* of *statistical confidence* into account.

The most common way to quantify risk capital is the concept of *economic capital*[11] (EC). For a prescribed level of confidence α it is defined as the α-quantile of the portfolio loss $\tilde{L}_{PF}$ minus the EL of the portfolio,

$$\mathrm{EC}_\alpha = q_\alpha - \mathrm{EL}_{PF} , \qquad (1.\ 10)$$

where q_α is the α-quantile of $\tilde{L}_{PF}$, determined by

$$q_\alpha = \inf\{q > 0 \mid \mathbb{P}[\tilde{L}_{PF} \le q] \ge \alpha\} . \qquad (1.\ 11)$$

For example, if the level of confidence is set to $\alpha = 99.98\%$, then the risk capital EC_α will (on average) be sufficient to cover unexpected losses

[10]In a marked-to-market framework loans do not live in a two-state world (default or survival) but rather are evaluated w.r.t. their *market value*. Because until today loans are only traded "over the counter" in *secondary markets*, a marked-to-market approach is more difficult to calibrate. For example, in Europe the secondary loan market is not as well developed as in the United States. However, due to the strongly increasing market of credit derivatives and securitised credit products, one can expect that there will be a transparent and well-developed market for all types of loans in a few years.

[11]Synonymously called *Capital at Risk* (CaR) or credit *Value-at-Risk* (VaR) in the literature.

in 9,998 out of 10,000 years, hereby assuming a planning horizon of one year. Unfortunately, under such a calibration one can on the other side expect that in 2 out of 10,000 years the economic capital $EC_{99.98\%}$ will not be sufficient to protect the bank from insolvency. This is the downside when calibrating risk capital by means of quantiles. However, today most major banks use an EC framework for their internal credit risk model.

The reason for reducing the quantile q_α by the EL is due to the "best practice" of decomposing the total risk capital (i.e., the quantile) into a first part covering expected losses and a second part meant as a cushion against unexpected losses. Altogether the pricing of a loan typically takes several cost components into account. First of all, the price of the loan should include the costs of administrating the loan and maybe some kind of upfront fees. Second, expected losses are charged to the customer, hereby taking the creditworthiness captured by the customer's rating into account. More risky customers have to pay a higher risk premium than customers showing high credit quality. Third, the bank will also ask for some compensation for taking the risk of unexpected losses coming with the new loan into the bank's credit portfolio. The charge for unexpected losses is often calculated as the *contributory* EC of the loan in reference to the lending bank's portfolio; see Chapter 5. Note that there is an important difference between the EL and the EC charges: The EL charge is independent from the composition of the reference portfolio, whereas the EC charge strongly depends on the current composition of the portfolio in which the new loan will be included. For example, if the portfolio is already well diversified, then the EC charge as a cushion against unexpected losses does not have to be as high as it would be in the case for a portfolio in which, for example, the new loan would induce some concentration risk. Summarizing one can say the EL charges are *portfolio independent*, but EC charges are *portfolio dependent*. This makes the calculation of the contributory EC in pricing tools more complicated, because one always has to take the complete reference portfolio into account. *Risk contributions* will be discussed in Chapter 5.

An alternative to EC is a risk capital based on *Expected Shortfall* (ESF). A capital definition according to ESF very much reflects an insurance point of view of the credit risk business. We will come back to ESF and its properties in Chapter 5.

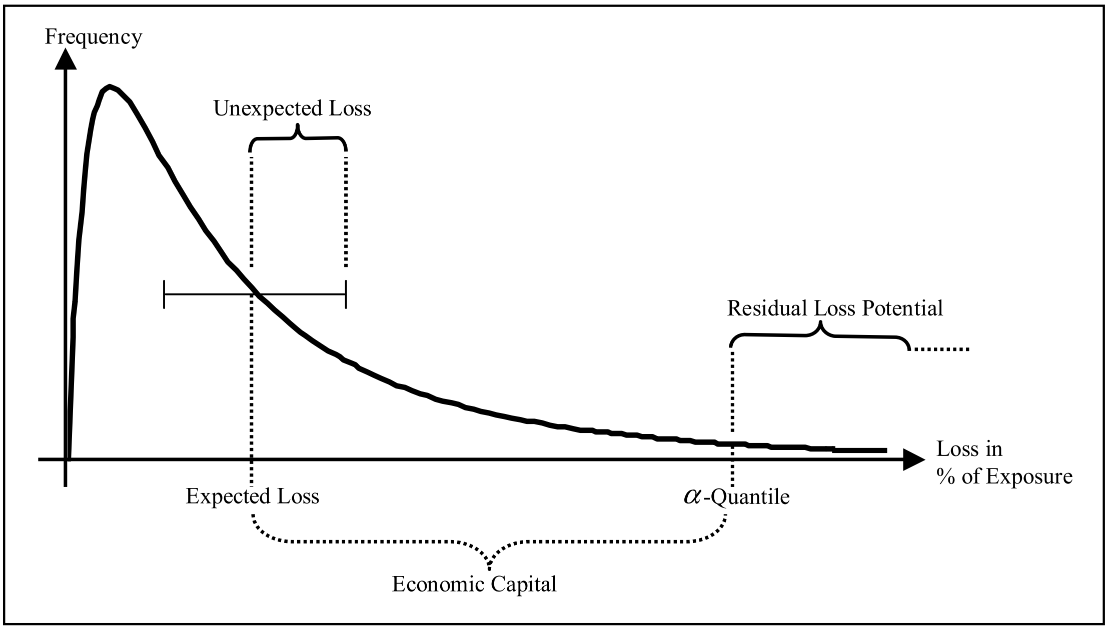

FIGURE 1.2
The portfolio loss distribution

1.2.2 The Loss Distribution

All risk quantities on a portfolio level are based on the portfolio loss variable $\tilde{L}_{PF}$. Therefore it does not come much as a surprise that the distribution of $\tilde{L}_{PF}$, the so-called *loss distribution* of the portfolio, plays a central role in credit risk management. In Figure 1.2 it is illustrated that all risk quantities of the credit portfolio can be identified by means of the loss distribution of the portfolio. This is an important observation, because it shows that in cases where the distribution of the portfolio loss can only be determined in an empirical way one can use empirical statistical quantities as a proxy for the respective "true" risk quantities.

In practice there are essentially two ways to generate a loss distribution. The first method is based on *Monte Carlo simulation*; the second is based on a so-called *analytical approximation*.

1.2.2.1 Monte Carlo Simulation of Losses

In a Monte Carlo simulation, losses are simulated and tabulated in form of a *histogram* in order to obtain an *empirical loss distribution*