Note: This document is a part of the lectures given during the Jan-May 2020 Semester.

Price Yield Curve:

A price-yield curve is a curve in which a bond price is plotted as a function of the bond's yield. Prices are represented as percentages of the par value. As we have mentioned previously, the bond prices go down if the rates (and hence the yields), go up. Another obvious fact is that the bonds with a higher coupon rate cost more.

Term Structure of Interest Rates:

A thorough study of the bond market requires looking at the rates for bonds of all maturities as they move over time. The corresponding structure of various rates is called the term structure of interest rates.

Yield Curve:

The yield curve is the curve obtained by plotting the yields of bonds with different maturities against the maturity values. We focus on the yield curve of government bonds. The shape of the yield curve varies over time and from country to country. The difference between longer maturity yields and shorter maturity yields determines the slope of the term structure. Typically, yields of longer maturities are higher than the yields of shorter maturity, with the result that the yield curve is upward sloping. However, sometimes the short term bonds yield more than long term bonds and the curve is downward sloping, also called the inverted yield curve.

Spot Rate:

The annualized nominal interest of a pure discount bond is called the spot rate. For pure discount bonds, the spot rate is equal to the yield.

Example:

Consider a pure discount bond that matures in six months, has a nominal value of 100 and is trading at 98. The annualized compound rate r_{6m} is determined from,

$$98 = \frac{100}{(1 + r_{6m})^{1/2}},$$

with the solution $r_{6m} = 4.1233\%$. We say that the six month spot rate is 4.1233%.

This way of computing and quoting the six month spot rate uses the six month compounding convention. The simple rate convention can also be used. First, the implicit rate of return r is computed from,

$$98 = \frac{100}{(1 + r)} \Rightarrow r = 2.0408\%$$

Then the six-month annualized spot rate quoted is $2 \times 2.0408\% = 4.0816\%$.

Using the compound method or the simple method is a matter of convention.

Example:

We consider a two year pure discount bond with nominal value 100, which trades at 91. The two year spot rate r_{2y} is obtained from,

$$91 = \frac{100}{(1 + r_{2y})^2} \Rightarrow r_{2y} = 4.8285\%.$$

Alternatively we can compute the rate from,

$$91 = \frac{100}{(1+r)} \Rightarrow r_{2y} = 9.8901\%$$

and according to the simple interest rate this would give an annual rate of $0.5 \times 9.8901\% = 4.9451\%$.

The relationship between the yields of bonds with different maturities is of great interest to practitioners. The collection of the spot rates for all the maturities is called the term structure of interest rates. If we do not mention otherwise, we refer to the term structure of default free government bonds. In practice it is not possible to compute exactly the spot rates for all maturities simply because not all maturities are represented by the traded bonds. For example, there are no default-free bonds with maturity greater than 30 years.

Rates Arbitrage:

By arbitrage we mean making potential profits with zero investment, and no risk of losing money. For example, suppose that we can sell a zero-coupon bond that pays 100 at maturity T for a price of 95 today, and that we can also construct a portfolio of other securities that pays 100 at the same maturity for the initial investment of 94. Then we can make a risk-free arbitrage profit of 1 by selling the bond for 95 and investing 94 in the portfolio. At maturity, our positions cancel each other, while we have made 1 at the beginning. We shall assume for most of our subsequent discussion, that the market is free of arbitrage. Based on this assumption, often even when a default free zero-coupon bond is not traded, the spot rates can be computed from default-free bonds that pay coupons by no arbitrage arguments, as in the following example.

Example:

Consider a default-free coupon bond that will pay a coupon of 3.00 in six months and will make a final payment of 103.00 (the last coupon and the principal) in one year. This bond trades at 101.505. The coupon bond is equivalent to a portfolio of six month pure discount bond with a nominal value of 3.00 and another pure discount bond with a nominal value of 103.00. Assume also that a six month zero coupon bond is traded at 98.00, with a yield of 4.1233% (computed earlier). Using a combination of the six month pure discount bond and the coupon bond maturing in a year, we can construct a portfolio that behaves like a one-year pure discount bond. In this case we buy one unit of the coupon bond and sell 0.03 units of the six-month pure discount bond. The total cost of the basket of these bonds is $101.505 - 0.03 \times 98.00 = 98.565$. In six months the short position in the discount bond requires paying $0.03 \times 100 = 3.00$. Simultaneously the coupon for the same amount is paid resulting in zero profit/loss. In one year the principal and fixed coupon are paid generating a revenue of 103.00. No additional payments take place. The portfolio is thus equivalent to a pure discount bond with a cost of 98.565 with maturity of one year and a nominal value of 103.00. The corresponding one year spot rate r_{1y} is then derived from,

$$98.565 = \frac{103}{1+r_{1y}} \Rightarrow r_{1y} = 4.4996\%.$$

Alternatively we first compute the six-month spot rate $r_{6m} = 4.1233\%$ and then from the coupon bond using the six month spot rate, we compute the one year spot rate using,

$$101.505 = \frac{3}{(1+r_{6m})^{1/2}} + \frac{103}{1+r_{1y}} \Rightarrow r_{1y} = 4.4996\%.$$