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INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 322: Scientific Computing Lab

Lab 01

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Question 1.

We will first take the range [1,2] and observe that x=g(x) obeys the fixed-point criteria.

a. To calculate n, we simply need to run a loop till,

   |x(n+1) - x(n)|<=10^-5. We will assume x0 = 1.

   The result of n comes out as 3.

b. Now that we have the value of √2 = 1.4143, we can use the formula given in lecture slide 1,

   $$\lim_{n\to\infty} \frac{(\alpha - x_{n+1})}{(\alpha - x_n)^p} = C$$

   We first see that since g(x) satisfies the fixed point criteria, theoretically the order should be 1, since g'(x)>0 in [1,2]. Now we will replace p=1, and take a small epsilon and observe that indeed, C converges to a constant.

   Thus the order of convergence =1.

Question 2.

For this question, we'll simply assume 7 different values for n, i.e

[1,10,100,1000,10000,100000,1000000], and in each case make a straight forward calculation as per the given algorithm to obtain the approximate roots.

We'll notice that the roots get accurate as the value of n increases.

Question 3.

For this question, we’ll first make use of an error factor of 0.005, and obtain an approximate root using the Bisection Method. We then make use of this **root (=1.8929322922508809),** as x0 for the equation for Newton’s method, and find Newton’s root correct to 7 decimal places = **1.895494267033981**.

Question 4.

For this question, we’ll proceed as previously to first find the approximate root by the bisection method. The next step is to change the equation to the form, x=g(x).

For fixed-point iteration we'll take x=g(x), with g(x)=sinx+x/2.
Note that this will ensure all criteria are satisfied since g(x) will in [**π**/2,**π**].
The order of convergence = 1 is then calculated using the formula:

$$q \approx \frac{\log_e\left(\left|\frac{x_{k+1}-x_k}{x_k-x_{k-1}}\right|\right)}{\log_e\left(\left|\frac{x_k-x_{k-1}}{x_{k-1}-x_{k-2}}\right|\right)}$$

Question 5.

For this question, we’ll straightaway use the secant method with the given x0 value and taking x1=0 since we observe that since f(0)*f(-1)

<0, and thus there is a root between 0 and -1. We then make use of stopping criteria, **to get the root = -0.5791589060508088**

Question 6,7,8.

For these questions, we first make use of the Bisection method, to get the value of an approximate root, taking epsilon as 0.005. We then simply make use of the given iteration in the question, and setting x0= the obtained root, and using the formula discussed in the solution to Question 4 we evaluate the order of convergence to be **1,1 and 2 respectively.**