MA 322: Scientific Computing

Lecture - 1

Course Structure

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- Unit-I

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- Unit-I
  - Numerical methods for solving scalar nonlinear equations.

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More precisely, can we solve non-linear IVP exactly?

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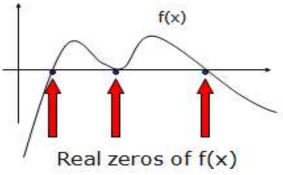

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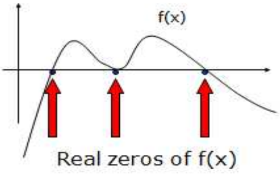

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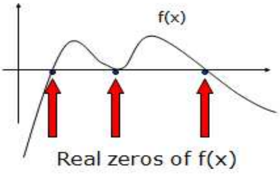

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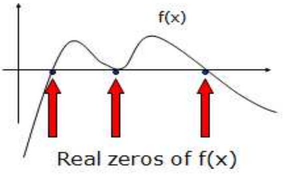

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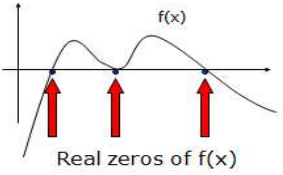

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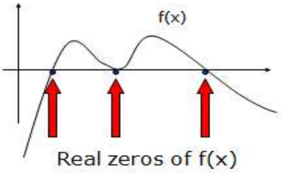

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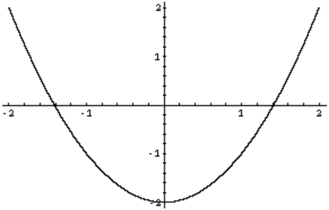

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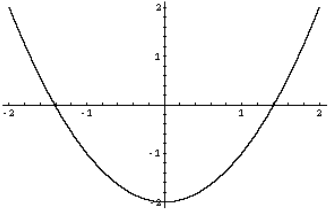

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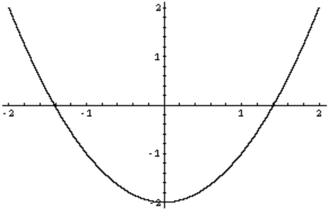

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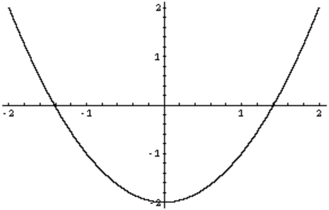

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$$f(c_n) \to f(c) = 0.$$

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- In general, for $n \geq 1$, we define (a_{n+1}, b_{n+1}) by

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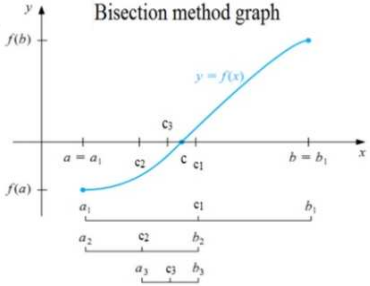

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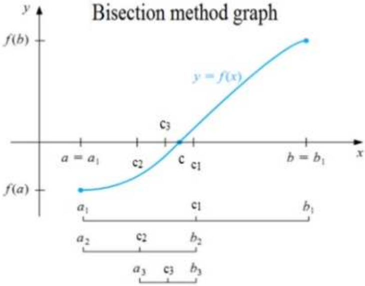

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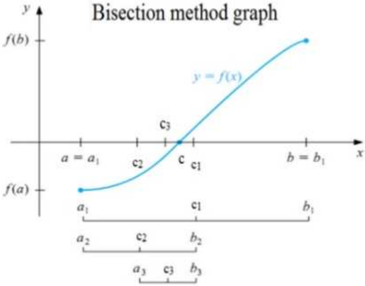

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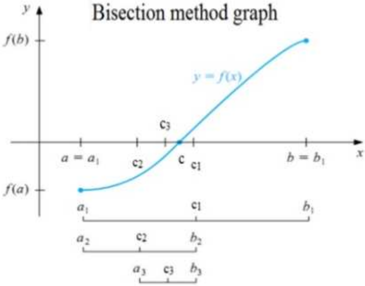

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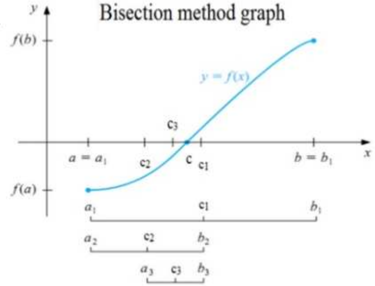

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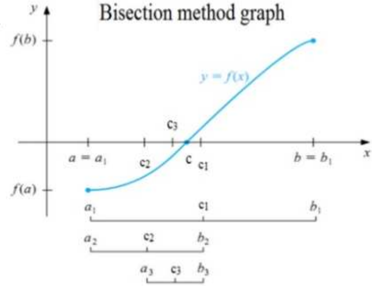

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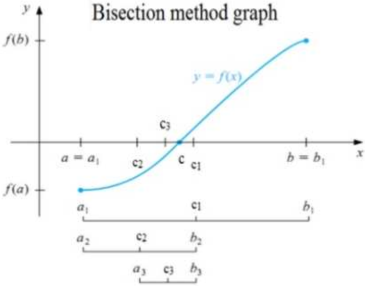

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Order of Convergence

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Definition: A sequence of iterates $\langle x_n \rangle$ is said to converge with order $p \geq 1$ to the actual root α if

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for some $C > 0$. If $p = 1$, the sequence is said to converge linearly to α.

Remark:

- For the linear convergence, the constant C must satisfy $C < 1$. (Homework)

Equivalent Definition: Let $\langle x_n \rangle$ be a sequence of values converging to α, such that

$$\lim_{n \to \infty} \frac{(\alpha - x_{n+1})}{(\alpha - x_n)^p} = C \tag{6}$$

for some some non-zero (finite) constant C, and some p. Then p is called the order of convergence for the sequence. Bisection method has linear convergence.

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Again, the quantity

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  - The method is guaranteed to converge (provided f is continuous on $[a, b]$).
  - At every step, we get upper and lower bounds on the root.