Quiz 1 A.

1) $[(P \to Q) \wedge (Q \to S)] \to (P \to S)$

P	Q	S	$P \to Q$	$Q \to S$	$(P \to Q) \wedge (Q \to S)$	$P \to S$	$[(P \to Q) \wedge (Q \to S)] \to (P \to S)$
T	T	T	T	T	T	T	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	F	F	T	F	F	T
F	T	F	T	F	F	F	T
F	F	F	T	T	T	T	T

Tautology.

2) $\sim(P \to Q)$; $\sim(Q \to P)$

when P is true and Q is false

$\Rightarrow$ It implies $P \to Q$ is false

so $\sim(P \to Q)$ is true.

Since Q is false, so $Q \to P$ is true, so $\sim(Q \to P)$ is false.

- Then both the statements cannot be consistent. $=$

3). $\{x, y, z, u\}$.

R: $\{(x,x), (x,y), (y,z), (x,z)$ [illegible] (y,u) $(x,u)\}$.

It is transitive, but not reflexive and complete. $=$

4). $\{x, y, z, u\}$.

xPy, yPz, zIu, uIx, xPz
yIu.

It violates transitivity.
$x \in \emptyset$ belongs to choice set.
=.

5). Individual 1: $x\ y\ z\ t$

" 2: $t\ x\ y\ z$.

" 3: $y\ z\ t\ x$.

Set of Pareto optimal state

$\{x, t, y\}$.

6 $\{x, y, z, u, v, t\}$.

x y z u v t — 2. Individuals.

y z x v t x — 2 Individuals.

z u v t x y → 1 Individual

—.

$N(x P_i y) = 2 < N(y P_i x) = 3 \rightarrow \neg y P x$.

$N(y P_i z) = 4 > N(z P_i y) = 1 \rightarrow y P z$.

$N(z P_i u) = 4 > N(u P_i z) = 1 \rightarrow z P u$.

$N(u P_i v) = 5 > N(v P_i u) = 0 \rightarrow u P v$

$N(v P_i t) = 5 > N(t P_i v) = 0 \rightarrow v P t$.

$N(x P_i t) = 2 < N(t P_i x) =$.

6 $\{x, y, z, u, v, t\}$.

$x\ y\ z\ u\ v\ t$ – 2 Individuals.

$y\ z\ x\ u\ v\ t$ → 2 Individuals.

$z\ x\ y\ u\ v\ t$ → 1 Individual.

—

$N(x P_2 y) = 3 < N(y P_2 x) = 2 \rightarrow x P y.$

$N(y P_2 z) = 4 > N(z P_2 y) = 1 \Rightarrow y P z.$

$N(z P_2 x) = 3 > N(x P_2 z) = 2 \Rightarrow z P x$

A cycle over x, y, z.

rest, $x P u$, $x P v$, $x P t$.

$y P u$, $y P v$, $y P t$

$z P u$, $z P v$, $z P t$.

7). $\{x, y, z\}$

1: x y z.

2: y z x.

3: z y x.

4: x (zy).
2

Score: x: 2, y: 1, z: 1.

plurality vote: $\Rightarrow \begin{pmatrix} x \\ (yz) \end{pmatrix}$.

Score:

x: 2+2 = 4

y: 2+3 = 5

z: 2+2 = 4
2

Borda count: $\Rightarrow \begin{pmatrix} y \\ (xz) \end{pmatrix}$

8). Individual 1: x y z t

" 2: y x t z

" 3: x t y z.

- Individual 3 can be almost decisive over t against y.

Suppose 3 is almost decisive over t against y, then

t P y.

By Pareto, we have x P y,

x P t, y P z.

Thus, we have.

x t y z.

Individual 3 is decisive over all ordered pair. So it is dictator.

z

9. $\{x, y, u, v\}$.

$x P_1 y \rightarrow x P y$.

$u P_2 v \rightarrow u P v$.

Individual 1 — Individual 2.

$x\ y\ v\ u$ — $x\ y\ u\ v$.

Individual 3

$x\ v\ y\ u$.

$x P y$., $u P v$., $x P v$.

$x R u$.

$x \in$ Social choice set.
