Note: This document is a part of the lectures given during the Jan-May 2020 Semester.

Immunization:

We consider the problem of a portfolio manager whose portfolio consists exclusively of bonds and who is concerned about the interest-rate risk. The concept of bond we consider here is a very broad one: by bonds we mean the right to receive given payments at a future given dates.

Matching Durations:

A liability, that is, the obligation to pay a known amount at a future date, is equivalent to a short position in a pure discount bond maturing at that date. Consider a bank that collects money from savings accounts that pay interest. The deposited amount is equivalent to a short position in a pure discount bond with maturity zero: at any moment, the depositor can withdraw the balance in the account, so the account is equivalent to a bond that matures immediately. (Typically the depositor "renews" the investment by keeping the money in the account for sometime). Then, suppose that the bank invests the money by issuing a loan, say a 10 year mortgage at a fixed rate. If the interest on the savings account is 3% and the interest on the mortgage is 5%, the bank is poised to make a profit. Note that the bank has a portfolio of bonds: a short position in a bond with a very short maturity (the savings account) and a long position in a coupon bond with long maturity (the mortgage). Suppose now that the interest rates in the economy go up and that, in order to be able to keep the deposits of its customers, the bank has to raise the interest to 6%. At that rate the bank will loose money. The interest-rate rise has affected the present value of the mortgage. The value of that mortgage is now lower because the future payments will be discounted at higher interest rates. The simplest solution for hedging this type of risk is the construction of a dedicated portfolio. The idea is to match the liabilities (the short position in bonds) with the investments (the long position in bonds), so that the coupons in the long position are identical in size and timing to the payments required by the liabilities. However, a dedicated portfolio is not always feasible: the required pure discount bonds might not be available in the market, or, they might be available, but expensive, or too risky (and here we mean the risk of default). An alternative strategy for the bond portfolio manager is to construct a long portfolio of bonds whose duration matches the duration of the portfolio of liabilities.

Example: Hedging by Immunization

Suppose that the term structure is flat and the annual interest rate is 5%. We have a liability with a nominal value of 100, and the payment will take place in two years. In the market there are only two pure discount bonds, a one year pure discount bond with nominal value 100 and a four year pure discount bond with nominal value 100. We start with the present value of the liability, that is,

$$\frac{100}{(1.05)^2} = 90.70$$

in cash. In order to guarantee that we will be able to meet the payment in two years (approximately), we have to invest the money on a portfolio of bonds with the duration equal to the duration of the portfolio of short positions. In this case, there is only one liability with maturity in two years, equivalent to a short position in a pure discount bond with maturity in two years and, therefore, duration equal to two years. We want to invest in a portfolio with duration of two, as well. There are only two pure discount securities, with maturities (and, therefore, durations) equal to one and four. We now take advantage of the property: *If a bond portfolio comprises of n bonds with weights $w_1, w_2, \ldots, w_n$ and durations of $D_1, D_2, \ldots, D_n$, respectively, then the duration of the portfolio is given by,*

$$D = \sum_{i=1}^{n} w_i D_i.$$

We will invest a proportion α of the initial capital in the bond with duration 1 and the rest in the bond with duration 4. We solve,

$$\alpha D_1 + (1-\alpha)D_2 = D \Rightarrow \alpha + (1-\alpha)4 = 2 \Rightarrow \frac{2}{3}.$$

We therefore invest $90.70 \times \frac{2}{3} = 60.47$ in the one year bond and the rest $90.70 - 60.47 = 30.23$ in the four year bond. The price of the one year bond is

$$\frac{100}{1.05} = 95.24$$

and therefore, we buy,

$$\frac{60.47}{95.24} = 0.63$$

units of the one year bond.
Similarly, the price of the four year bond is

$$\frac{100}{(1.05)^4} = 82.27$$

and we buy,

$$\frac{30.23}{82.27} = 0.37$$

units of the four year bond.

1. Suppose that immediately after investing in such a portfolio, interest rates go up to 6% (and the term structure remains flat). The present values of both the liability and the portfolio of bonds drop. The new values of the liability and the portfolio of bonds are,

$$\frac{100}{(1.06)^2} = 89 \ , \ 0.63 \times \frac{100}{1.06} + 0.37 \times \frac{100}{(1.06)^4} = 88.74.$$

   The drop in the value of the portfolio roughly matches the drop in the value of the liability.

2. Suppose now that, immediately after forming the portfolio of bonds, the interest rate drops to 4% (and the term structure remains flat). In this case, the value of liability goes up. The new value is,

$$\frac{100}{(1.04)^2} = 92.46 \ , \ 0.63 \times \frac{100}{1.04} + 0.37 \times \frac{100}{(1.04)^4} = 92.20.$$

   Again, the increase in the value of the portfolio approximately matches the increase in the value of the liability.

There are several problems with the previous analysis. Firstly, this approach relies on the unrealistic assumptions that the term structure is flat and that all the interest rate movements will be in parallel shifts. Secondly, the duration changes as the interest rates move.

Example: Change in Duration

Consider the liability of the previous examples and the portfolio of bonds formed for immunization purposes, in case the rates go down to 4%. After the interest rate changes, the duration of the liability will still be equal to 2. However, the duration of the portfolio of bonds will not be equal to 2. Now, the new price of the portfolio for bonds is 92.20. The new weights of the one year and the four year bonds are,

$$\frac{0.63 \times \frac{100}{1.04}}{92.20} = 0.66, \ \frac{0.37 \times \frac{100}{(1.04)^4}}{92.20} = 0.34.$$

Duration of the portfolio is,

$$D = 0.66 \times 1 + 0.34 \times 4 = 2.02.$$

Therefore the bond is more sensitive to an increase in interest rates.