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INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 423: Matrix Computations Lab
Lab 07

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Question 1.

The following condition numbers and residual norms are obtained from the methods given in the assignment.

```
Method 1:
Condition num=5.634851e+13 and ||r|| = 8.184730e-13

Method 2:
Condition num=4.959021e+26 and ||r|| = 1.103722e-09

Method 3:
Condition num=1.961761e+14 and ||r|| = 8.183408e-13
```

Methods 1 and 3 give comparable errors so any one of them can be used as the best fit.

Question 2.

From the table below, we can conclude that with an increase in K, the compression ratio increases, and the error decreases.

K	Compression Ratio	Error
50	0.40625	0.019189
55	0.44688	0.017871
60	0.4875	0.016704
65	0.52812	0.015468
70	0.56875	0.014684
75	0.60938	0.014008
80	0.65	0.013315
85	0.69063	0.012416
90	0.73125	0.011749
95	0.77187	0.011097
100	0.8125	0.010434
105	0.85313	0.0099365
110	0.89375	0.0094439
115	0.93437	0.0088782
120	0.975	0.0083226
125	1.0156	0.0078731
130	1.0562	0.0073195
135	1.0969	0.0067562
140	1.1375	0.0063843
145	1.1781	0.0059139
150	1.2188	0.0054823
155	1.2594	0.0050779
160	1.3	0.0046985
165	1.3406	0.004308
170	1.3813	0.0038497
175	1.4219	0.0035138
180	1.4625	0.003091

Question 3.

From the below output, we conclude that SVD gives numerical rank = 89 for the original matrix as well as the perturbed matrix.
From the rank revealing QR decomposition's output we get E = I i.e. permutation matrix is equal to the identity matrix. This clearly means that no pivoting was done.

Also, R(90,90) = 1.903869e-03, i.e., numerical rank calculated from rank revealing QR decomposition is 90 which is not correct.
This experiment shows that the QR method failed to detect the rank of the Kahan matrix correctly.

```
Max Diagonal Entry = 1.000000e+00
Min Diagonal Entry = 1.903869e-03

Original Matrix
sigma(1,1) = 8.789335e+00
sigma(89,89) = 2.384233e-03
sigma(90,90) = 3.960644e-15
Rank of Original Matrix: 89


Perturbed Matrix
sigma(1,1) = 8.789335e+00
sigma(89,89) = 2.384233e-03
sigma(90,90) = 3.960653e-15
Rank of Perturbed Matrix: 89


Using QR decomposition
|| I - E||: 0.000000e+00
R(90,90) = 1.903869e-03
```