A set of individuals V is almost decisive for x against y if xPy whenever xP_iy for every $i \in V$ and yP_ix for every $i \notin V$.
A set of individuals V is almost decision for a pair $x, y \in X$, $[xP_iy\ \forall i \in V\ \&\ yP_ix \forall i \notin V] \ \rightarrow\ xPy$.

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A set of individuals V is decisive for a pair $x, y \in X$ $[xP_iy \forall i \in V] \ \rightarrow\ xPy$.

Notice that it is possible that a set V can be almost decisive but not decisive.
Suppose $xP_iy \forall i \in V$ is true and there exists $j \notin V$ such that xR_jy and the social preference is yRx. It satisfies almost decisive but not decisive.
Therefore, it is not always true that $D(x, y) \to \bar{D}(x, y)$.

But $\bar{D}(x, y) \to D(x, y)$.
If $xP_iy \forall i \in V$ and $j \notin V$, xR_jy, and xPy then if part of the definition of almost decisive set is is false so the state is trivially true. It is decisive.

Lemma 1

If there is some individual J who is almost decisive for any ordered pair of alternative, then an Social welfare function satisfying conditions U, P, and I implies that J must be a dictator.

Proof: Suppose that person J is almost decisive for some x against some y, it means $\exists\ x,\ y \in X : D(x, y)$. Let z be another alternative and let i refers to all individuals other than J. Assume $xP_jy\&yP_Jz$, and that yP_ix & yP_iz. We have not specified the preferences of the persons other than J between x and z.
Now $[D(x, y)\&xP_jy\&yP_ix] \rightarrow xPy$. Further $[yP_jz\&yP_iz] \rightarrow yPz$ from the condition P Pareto principle. But $[xPy\&yPz] \rightarrow xPz$, by the transitivity of the strict social preference relation.

We have xPz without any assumption on the preference relation over x and z on part of the individuals other than J. We have only assumed yP_iz and yP_ix. Now, these ranking of x and y and the ranking of y and z has any effect on the ranking of x and z it violates condition I , independence of irrelevant alternatives..
Hence xPz must be independent of the assumptions on x and y and y and z. Also, xPz is the consequence of xP_jz alone without having any effect from the ordering of i s. This means that J is decisive for x against z.
We get $D(x, y) \rightarrow \bar{D}(x, z)$ (1).

Now suppose $zP_jx\&xP_jy$, and $zP_ix\&yP_ix$. We have zPx from Pareto condition. And $D(x,y)\&xP_Jy\&yP_ix$, we have xPy. Using transitivity of strict social preference relation we have,
$zPx\&xPy \rightarrow zPy$. Again, we have got the social relation over z and y without specifying the individual preference relation of i s. Hence J is decisive for z against y.
We get $D(x,y) \rightarrow \bar{D}(z,y)$ (2).
We can also show $D(x,z) \rightarrow \bar{D}(y,z)$ (3)
by interchanging z with y in (2).

Again putting x in place of z, z in place of y, and y in place of x, we obtain from (1)
$D(y, z) \to \bar{D}(y, x)$ (4).
We have $D(x, y) \to \bar{D}(x, z)$ from (1)
$\to D(x, z)$ using the definition of decisive and almost decisive
$\to \bar{D}(y, z)$ from (3)
$\to D(y, z)$ from definition
$\to \bar{D}(y, x)$ from (4).
We get that $D(x, y) \to \bar{D}(y, x)$ (5).
By interchanging x and y in (1), (2) and (5), we get
$D(x, y) \to [\bar{D}(y, z) \& \bar{D}(z, x) \& \bar{D}(x, y)]$ (6).

Now, $D(x, y) \rightarrow \bar{D}(y, x)$ from (5)
$\rightarrow D(y, x)$.
Hence from (6) we have
$D(x, y) \rightarrow [\bar{D}(y, z) \& \bar{D}(z, x) \& \bar{D}(x, y)]$ (7).
Combining (1), (2), (5) and (7), it is seen that $D(x, y)$ implies that individual J is decisive for every ordered pair of alternatives from the set of three alternative $\{x, y, z\}$ given the condition U, P and I. Thus, J is a dictator over any set of three alternatives containing x and y.

Now, consider a larger number of alternatives. Take any two alternatives u and v out of the entire set of alternatives. If u and v are so chosen that they are same as x and y, then $\bar{D}(u, v)$ holds, as can be shown by taking a triple consisting of u, v and any other alternative z. If one of u and v is same as one of x and y, say u and x are same but not v and y, then take the triple consisting of x(*or* u), y, and v. Since $D(x, y)$, holds it again follows that $\bar{D}(u, v)$ and also $\bar{D}(v, u)$.

Let both u and v be different from x and y. Now first take $\{x, y, u\}$ and we get $\bar{D}(x, u)$ which implies $D(x, u)$. Now take the triple $\{x, u, v\}$. Since $D(x, u)$, it follows from previous argument that $\bar{D}(u, v)$ and also $\bar{D}(v, u)$. Thus, $D(x, y)$ for some x and y, implies $\bar{D}(u, v)$ for all possible ordered pairs (u, v). Therefore, individual J is a dictator.