Consider a weighted majority game;
$[10; 5, 8, 2, 3]$

$$v(S) = \begin{cases} 1 \text{ if } S \text{ is a winning coalition} \\ 0 \text{ if } S \text{ is not a winning coalition} \end{cases}$$

Take the order of players $1, 2, 2, 4$
The marginal contribution of player 1 is zero, marginal contribution of player 3 and 4 also zero.
The marginal contribution of player 2 is 1.
Here player 2 is a pivotal player.

Take another order of players $3, 4, 1, 2$
The marginal contribution of player 13 and 4 is zero, marginal contribution of player 1 also 1.
The marginal contribution of player 2 is 0.
Here player 1 is a pivotal player.

Result: In a weighted majority game, there is exactly one pivotal player in every order.

We add player one by one in a coalition, there is always a player i who turns a losing coalition into a winning coalition. Suppose, we need to add more then one player to turn a winning coalition into a winning coalition. This set of players can be partitioned where each partition contains only one player. Suppose there are k such partitions. Now, if we add one by one in the coalitions, after adding $k-1$ players, if the coalition is still not a winning coalition then we need to add the kth player. That makes kth player pivotal player. Thus, there is only one player who is pivotal. If $k-1$ players can make the coalition a winning coalition, in this case kth player is no more pivotal. If we drop the $k-1$ th player and the coalition is no more winning coalition then $k-1$ th player is pivotal. We can continu in this way, till it has only one player, and that one player is going to be pivotal. If that player is not pivotal, then the initial coalition is not a losing coalition. We have shown that only one player can be pivotal.

$[10; 5, 8, 2, 3]$, in this game, number of ways player 1, can be pivotal
If player 1 is in position 1, it is not possible.
If player 1 is in second position, $2134, 2143$; 2 ways
If player 1 is in third position, $3412, 4312$: 2 ways
If player 1 is in fourth position, the possible ways
$2341, 2431, 3421, 3241, 4231, 4321,$; not pivotal in any one of them.
In 4 ways player 1 can be pivotal out of 24 ways they can be positioned. So $\frac{4}{24}$ is the strength of player 1. Also Shapley value.

Player 2, number of ways it can be pivotal
If player 2 is position 1; zero number of ways
If player 2 is in second position:
$1234, 1243, 3214, 3241, 4213, 4231$; 6 ways
If player 2 is in third position : $3421, 4321, 1423, 4123, 1324, 3124$; 6 ways
If player 2 is in fourth position: not possible.
The strength/power of player 2 is $\frac{12}{24}$, its Shapley value.

Player 3, number of ways it can be pivotal
If player 3 is first position: not possible
If player 3 is in second position: $2314, 2341$: two ways
If player 3 is in third position: $1432, 4132$ two ways
If player 3 is in fourth position: not possible
Strength of player 3 is $\frac{4}{24}$, its shapley value.

Player 4, number of ways it can be pivotal

If player 4 is first position: not possible

If player 4 is in second position: $2413, 2431$: two ways

If player 4 is in third position: $1342, 3142$ two ways

If player 4 is in fourth position: not possible

Strength of player 4 is $\frac{4}{24}$, its Shapley value.

Result:

In a weighted majority game of N players, a players Shapley value is: $\frac{\text{number of times player is pivotal}}{\text{number of possible orders } (N!)}$

Shapley Shubik power index of players is $\frac{\text{number of times player is pivotal}}{\text{number of possible orders } (N!)}$

Cost Sharing

Cost sharing game
$(N; c)$ where c is the characteristic function denoting cost going to be shared between the members of the coalition.

The cost of setting up distribution network; cable TV, water supply, electricity etc

The cost allocation can be done using Shapley value

Order	1	2	3
123	24	12	24
132	24	12	24
213	18	18	24
231	0	18	42
312	0	12	48
321	0	12	48
Average	11	14	35

Shapley value is $(11, 14, 35)$

Another way to get this outcome
Every one uses the first segment (edge) , suppose segment 1. The segment (edge) connecting junction and player 2 is is only used by player 2, suppose segment 2. The segment (edge) connecting junction and 1 is shared between player 1 and 3, suppose 3. The segment connected player 1 and player 3 is only used by player 3, suppose segment 4.

Each segment is equally shared among the users.

segment	1	2	3
segment 1	2	2	2
Segment 2	0	12	0
Segment 3	9	0	9
Segment 4	0	0	24
aggregate	11	14	35

Result:

Equal sharing of cost of each segment among the players who use the segment gives Shapley value of the cost sharing game.