Optimization Problem (OP) (or Mathematical Programming Problem)

OP: Minimize $f(x)$
Subject to $x \in F$.

$f \rightarrow$ objective function (or cost function)

$x = (x_1, x_2, \dots x_n)^T \in \mathbb{R}^n \rightarrow$ decision variables

$F \subseteq \mathbb{R}^n \rightarrow$ constraint set or feasible set or feasible region or feasible space

Minimize $f(x)$

subject to $g_i(x) \geq 0$, $i = 1, 2, \dots, m$

$h_j(x) = 0$, $j = 1, 2, \dots, p$

Here, $F = \{ x \in \mathbb{R}^n \mid \underline{g}(x) \geq 0, \underline{h}(x) = 0 \}$

$x \in F \rightarrow$ feasible solution or feasible point

Defn: A feasible point $x^* \in F$ is an optimal, (for minimization problem) or a global optimal solution if

$$f(x) \geq f(x^*) \quad \forall x \in F$$

Defn: A point $x^0 \in F$ is called a local or relative minimum of f over F if $\exists\ \delta > 0$ such that

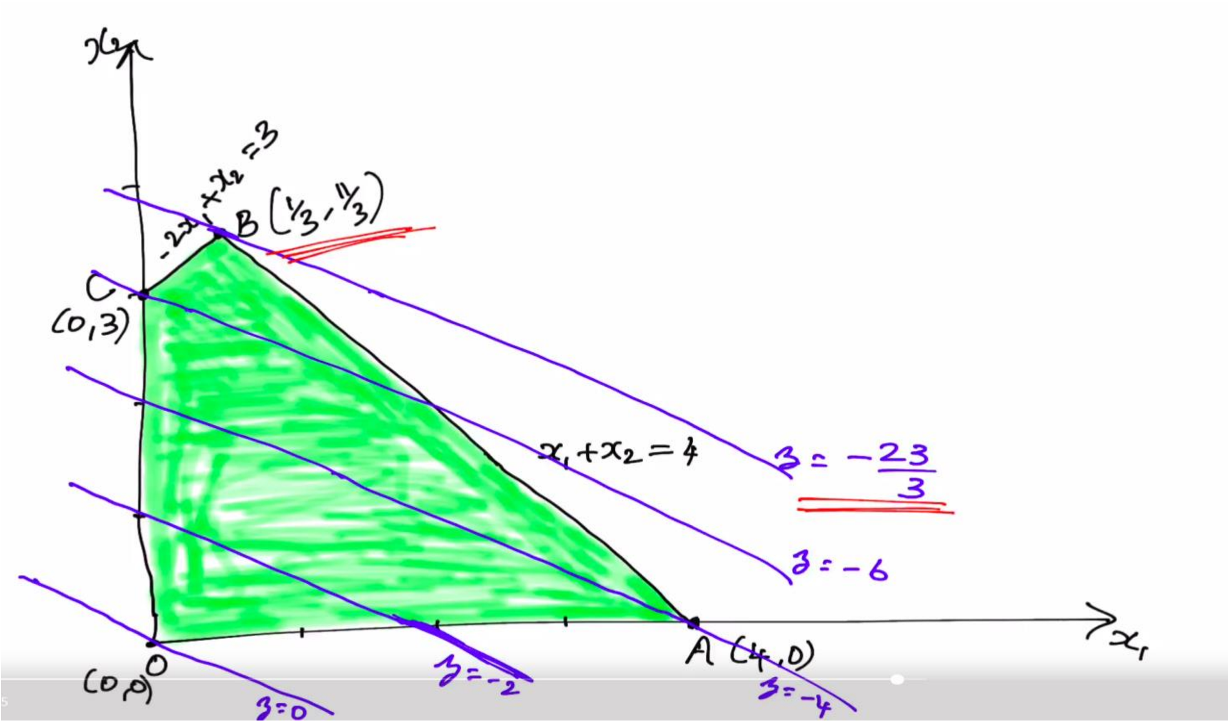
$$f(x) \geq f(x^0) \quad \forall\, x \in F \cap N_\delta(x^0).$$

where $N_\delta(x^0)$ is the neighbourhood of x^0 and has a radius δ.

OP-1: Minimize $z = -x_1 - 2x_2$

subject to
$$x_1 + x_2 \leq 4$$
$$-2x_1 + x_2 \leq 3$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

Linear Programming Problem (LPP)



OP-2

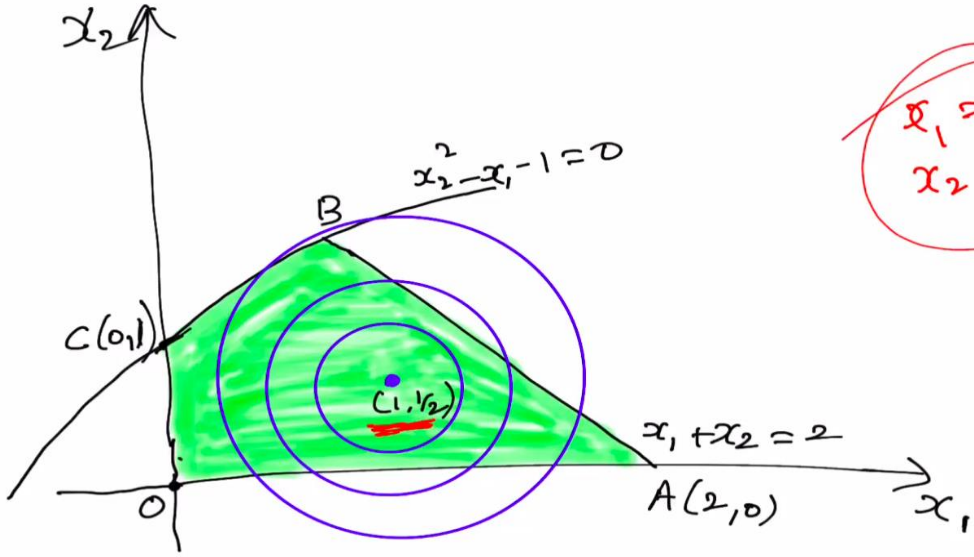
Minimize $z=(x_1-1)^2+\left(x_2-\frac{1}{2}\right)^2$

subject to

$$x_2^2-x_1-1 \le 0$$

$$x_1+x_2 \le 2$$

$$x_1 \geq 0,\ x_2 \geq 0.$$



$x_1 = 1$
$x_2 = 1/2$

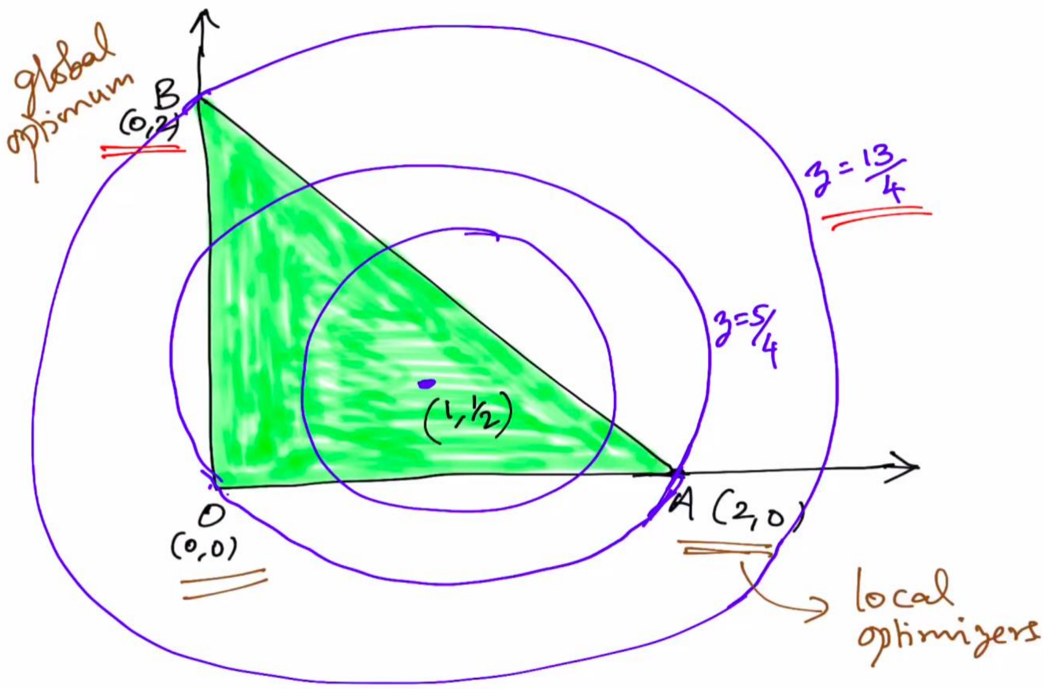
OP-3

Maximize $z = (x_1 - 1)^2 + (x_2 - \frac{1}{2})^2$

Subject to

$$x_1 + x_2 \leq 2$$

$$x_1 \geq 0, \quad x_2 \geq 0.$$



global optimum
B
(0,2)
$z = \frac{13}{4}$
$z = \frac{5}{4}$
(1, 1/2)
O
(0,0)
A (2,0)
local optimizers