

# LhARA linear optics documentation

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## 1 Introduction

The LhARA [1, 2] linear optics package was written to allow rapid calculations to initiate more detailed studies of the LhARA beam lines and to be used as a tool to check issues as they arise. The package has been written in Python so that it is accessible and can readily be updated, modified and maintained. At present the code treats proton beams only.

The documentation presented here is designed to present the approximations and notation in a consistent form and to summarise the module, class and data structures that have been adopted.

## 2 Coordinate systems

### 2.1 Laboratory coordinate system

The origin of the LhARA coordinate system, the “laboratory coordinate system” or “laboratory reference frame”, is at the position of the laser focus at the laser-target interaction point. The  $z$  axis is horizontal and parallel to the nominal capture axis, pointing in the downstream direction. The  $y$  axis points vertically upwards and the  $x$  axis completes a right-handed orthogonal coordinate system.

Unit vectors along the  $x$ ,  $y$  and  $z$  axes are  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  respectively. The position of the reference particle, its momentum and energy are described as functions of the distance it has travelled from the origin of coordinates to its current position. The distance the reference particle has travelled is defined to be  $s$ , making the position,  $\mathbf{r}_0$ , momentum,  $\mathbf{p}_0$ , and energy,  $E_0$ , of the reference particle at position  $s$ :

$$\begin{aligned} \mathbf{r}_0 &= \mathbf{r}_0(s); \\ \mathbf{p}_0 &= \mathbf{p}_0(s); \text{ and} \\ E_0 &= E_0(s). \end{aligned} \tag{1}$$

The magnitude of the reference particle velocity is  $v_0$  and the relativistic parameters that determine the reference particle energy and momentum are:

$$\begin{aligned} \beta_0 &= \frac{v_0}{c}; \text{ and} \\ \gamma_0 &= \frac{1}{\sqrt{1 - \beta_0^2}}; \end{aligned}$$

where  $c$  is the speed of light. The time,  $t$ , at which the reference particle is at  $s$  is also a function of  $s$ :

$$t = t(s) = \frac{s}{v_0} = \frac{s}{c} \frac{E_0}{cp_0}; \tag{2}$$

where  $p_0 = |\mathbf{p}_0|$ .



Figure 1: Reference particle local coordinate system. The trajectory of the reference particle is shown as the red line. The distance the reference particle has travelled, measured from the origin of coordinates in the laboratory frame, is labelled  $s$ . The origin of the “reference particle local coordinate (RPLC) system is coincident with the position of the reference particle. The directions of unit vectors along each of three righthanded, orthogonal coordinate axes are shown as black arrows labelled  $i_r$ ,  $j_r$ , and  $k_r$ .

## 2.2 Reference particle local coordinate system

A coordinate system defined relative to the position of the reference particle, the “reference particle local coordinate” (RPLC) system, may be defined using the direction in which the particle is travelling. The position of the particle defines the origin of the RPLC system, see figure 1. The tangent to the reference particle trajectory at  $s$  defines the  $z_r$  axis with unit vector  $k_r$ . In the laboratory frame, the presence of local electric or magnetic fields may cause the reference particle’s trajectory to change. In the neighbourhood of the particle, the curved trajectory may be described in terms of an arc of a circle. The  $x_r$  axis (with unit vector  $i_r$ ) is then taken to be in the direction pointing away from the centre of the circle. The third coordinate axis,  $y_r$ , is defined to complete the right-handed orthogonal coordinate system; the unit vector along the  $y_r$  axis being given by  $j_r = k_r \times i_r$ .

The trajectory of the reference particle is a straight line as it traverses a drift space and a variety of beam-line elements. Examples of such beam-line elements include solenoids, quadrupoles, etc. The reference trajectory is also undeviated by passage through an accelerating cavity placed such that the accelerating field is parallel to the reference-particle trajectory.

The RPLC coordinate system at  $s = 0$  is taken to coincide with the laboratory coordinate system. Beam-line elements are placed sequentially along the trajectory of the reference particle. The RPLC system at the entrance to a particular beam-line element is taken to coincide with the RPLC system defined at the exit of the beam-line

element after which it is placed.

## 40 2.3 Transforming to and from reference particle local coordinates to laboratory coordinates

In the RPLC system, the trajectory of the reference particle,  $\mathbf{R}_0$ , is:

$$\mathbf{R}_0(s) = \mathbf{0}. \quad (3)$$

The position of a test particle in the RPLC frame,  $\mathbf{R}$ , is described with reference to the position of the reference particle. In the laboratory frame, the position of the test particle is:

$$\mathbf{r}(s) = \mathbf{r}_0(s) + \delta\mathbf{r}(s); \quad (4)$$

where:

$$\delta\mathbf{r}(s) = \underline{\underline{R}}(s)\mathbf{R}(s); \text{ and} \quad (5)$$

$\underline{\underline{R}}(s)$  is a rotation matrix that takes the RPLCs at  $s$  to the laboratory frame coordinates.

In the laboratory frame, the unit vectors  $\mathbf{i}_r$ ,  $\mathbf{j}_r$  and  $\mathbf{k}_r$  are given by:

$$\begin{aligned} \mathbf{i}_r &= \begin{pmatrix} i_{rx} \\ i_{ry} \\ i_{rz} \end{pmatrix}; \\ \mathbf{j}_r &= \begin{pmatrix} j_{rx} \\ j_{ry} \\ j_{rz} \end{pmatrix}; \text{ and} \\ \mathbf{k}_r &= \begin{pmatrix} k_{rx} \\ k_{ry} \\ k_{rz} \end{pmatrix}. \end{aligned} \quad (6)$$

The rotation matrix,  $\underline{\underline{R}}$ , may now be written:

$$\underline{\underline{R}}(s) = \begin{bmatrix} i_{rx} & j_{rx} & k_{rx} \\ i_{ry} & j_{ry} & k_{ry} \\ i_{rz} & j_{rz} & k_{rz} \end{bmatrix}. \quad (7)$$

## 3 Phase space and trace space

45 The motion of particles passing through an accelerator is most often described using classical Hamiltonian mechanics; quantum mechanics being required only in particular cases such as the description of spin polarisation in a storage ring. In classical Hamiltonian mechanics the equations of motion are solved to give the evolution of the position, momentum, and energy as functions of a single independent parameter. The independent parameter is often taken to be time.

50 Relativistic mechanics exploits four-vector position,  $\underline{\mathcal{R}} = (r, ct)$ , and four-vector momentum  $\underline{\mathcal{P}} = (c\mathbf{p}, E)$ . In the Hamiltonian description of particle dynamics, these four vectors become functions of the independent variable, i.e.  $\underline{\mathcal{R}} = \underline{\mathcal{R}}(t)$  and  $\underline{\mathcal{P}} = \underline{\mathcal{P}}(t)$ . In the laboratory system, the position of the reference particle along its trajectory is directly related to the time coordinate by  $t = c\beta_0 s$ . This allows  $s$  to be taken as the independent variable and for the motion of particles in the beam to be derived as function of  $s$ .

55 The 6D phase-space coordinates of a particle as a function of  $s$  are given by the position and momentum three vectors. The particle energy may be determined from the invariant mass and the time coordinate from the invariant interval between the origin and the the position represented by  $s$ .

The “trace-space” coordinates of a particle are defined relative to the reference particle. Usually, a beam is understood to contain particles which follow trajectories that differ rather little from that of the reference particle. Trace space is defined such that the position, “momentum”, and “energy” coordinates are small for particles which follow trajectories close to that of the reference particle. The utility of this approach is that trace-space coordinates may be used to perform Taylor expansions of the Hamiltonian which may readily be solved to yield a description of particle transport using functions that are linear in the trace-space coordinates.

The notation used for the 6D phase and trace spaces are defined in this section.

### 65 3.1 Phase space

The 6D phase-space vector is defined in terms of the three-vector position and three vector momentum as:

$$\begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \end{bmatrix} \quad (8)$$

The trajectory of the particle may be evaluated as a function of time or  $s$ .

### 3.2 Trace space

Trace space is defined to simplify the calculation of the trajectory of particles through the accelerator lattice and is derived from the phase space expressed in the RPLC frame. Consider a particle with position  $(x_{\text{RPLC}}, y_{\text{RPLC}}, z_{\text{RPLC}})$  and momentum  $p_{\text{RPLC}}$  with components  $(p_{x\text{RPLC}}, p_{y\text{RPLC}}, p_{z\text{RPLC}})$ . Taking the momentum of the reference particle in the laboratory frame to be  $p_0$ , the trace-space coordinates are given by:

$$\phi = \begin{pmatrix} x_{\text{RPLC}} \\ x'_{\text{RPLC}} \\ y_{\text{RPLC}} \\ y'_{\text{RPLC}} \\ z_{\text{RPLC}} \\ \delta_{\text{RPLC}} \end{pmatrix}; \quad (9)$$

where:

$$x'_{\text{RPLC}} = \frac{\partial x}{\partial s} = \frac{cp_{x\text{RPLC}}}{cp_0}; \quad (10)$$

$$y'_{\text{RPLC}} = \frac{\partial y}{\partial s} = \frac{cp_{y\text{RPLC}}}{cp_0}; \quad (11)$$

$$z_{\text{RPLC}} = \frac{s}{\beta_0} - ct = \frac{\Delta s}{\beta_0}; \text{ and} \quad (12)$$

$$\delta_{\text{RPLC}} = \frac{E}{cp_0} - \frac{1}{\beta_0} = \frac{\Delta E}{cp_0}. \quad (13)$$

Here  $\Delta s = s - s_0$  and  $\Delta E = E - E_0$ , where  $s_0$  and  $E_0$  are the referene particle position and energy respectively;  
70  $E$  and  $s$  are the energy and position of a particular particle in the beam.

## 4 Transfer matrices

Description of beam transport often carried out by:

- Breaking lattice down into a series of “elements”, e.g. drift, quadrupole, dipole, solenoid, etc.;
- Transport of particle between the start and end of a particular element of the lattice linearised such that the trace space at the end of the element,  $\phi_f$ , is written in terms of the trace space at the start of the element,  $\phi_i$ :

$$\phi_f = \underline{\underline{T}} \phi_i ; \text{ and} \quad (14)$$

- The distance along the reference particle trajectory is increased by  $\delta s$ , where:

$$s_{\text{end}} = s_{\text{start}} + \delta s ; \quad (15)$$

and  $s_{\text{start}}$  and  $s_{\text{end}}$  are the length of the reference particle trajectory at the start and end of the beam-line element respectively.

75

Many excellent descriptions of derivation of transfer matrices,  $\underline{\underline{T}}$ , so only quote results here.

### 4.1 Drift

A “drift” space refers to a region in which the beam propagates in the absence of any electromagnetic fields. In a drift, particles propagate in straight lines, therefore:

$$\underline{\underline{T}}_{\text{drift}} = \begin{pmatrix} 1 & l & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & l & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{l}{\beta_0^2 \gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} ; \quad (16)$$

where  $l$  is the length of the drift. The increment in the reference particle trajectory is:

$$\delta s = l . \quad (17)$$

### 4.2 Quadrupole

80 The passage of a beam particle through a quadrupole magnet may be described by specifying the field gradient,  $g$ , within the magnet and the length,  $l_q$ , of the quadrupole measured along its axis. The impact of a quadrupole on the trajectory of a particle in the  $xy$  plane is independent of the impact of the magnet on the particle’s trajectory in the  $yz$  plane. In this sense quadrupole focusing in the  $xz$  and  $yz$  planes is said to be “uncoupled”.

If the field gradient along the  $x$  and  $y$  axes is identical, then:

$$g_x = \frac{\partial B_{qx}}{\partial x} = g_y = \frac{\partial B_{qy}}{\partial y} = g ; \quad (18)$$

where the field in the quadrupole,  $\mathbf{B}_q$ , has components  $(B_{qx}, B_{qy}, 0)$ .

In the “hard-edge” approximation, where the field falls to zero at the start and end of the quadrupole, the transfer matrix for a quadrupole focusing in the  $xz$  plane (a “focusing quadrupole”) may be written:

$$\underline{T}_{\text{Fquad}} = \begin{pmatrix} \cos(\sqrt{k_q}l_q) & \frac{\sin(\sqrt{k_q}l_q)}{\sqrt{k_q}} & 0 & 0 & 0 & 0 \\ -\sqrt{k_q}\sin(\sqrt{k_q}l_q) & \cos(\sqrt{k_q}l_q) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh(\sqrt{k_q}l_q) & \frac{\sinh(\sqrt{k_q}l_q)}{\sqrt{k_q}} & 0 & 0 \\ 0 & 0 & \sqrt{k_q}\sinh(\sqrt{k_q}l_q) & \cosh(\sqrt{k_q}l_q) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{l_q}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \quad (19)$$

where:

$$k_q = \frac{gc}{p} \times 10^{-3}, \quad (20)$$

and  $c$  is the speed of light in metres per second,  $p$  is the magnitude of the momentum of the particle in MeV/c, and the field gradient,  $g$ , is given in T/m. As before,  $\beta_0$  is the relativistic velocity of the reference particle and  $\gamma_0 = (1 - \beta_0^2)^{-\frac{1}{2}}$ . The increment in the reference particle trajectory is:

$$\delta s = l_q. \quad (21)$$

It is important to include a description of the effect of dispersion on focusing in the LhARA beam line since the laser-driven proton and ion source provides a broad energy spectrum. Reference [3] provides two methods for the description of dispersion in a linear approximation. The first is to use the reference momentum to calculate the quadrupole focusing strength ( $k_{0q} = \frac{gc}{p_0} \times 10^{-3}$ ) and to include terms in the expressions for  $x$ ,  $x'$ ,  $y$ , and  $y'$  dependent on  $\delta$ . The second is to use equation 20 to calculate the effective quadrupole focusing strength. The second approach has been adopted here.

In the same notation, the transfer matrix for a quadrupole focusing in the  $yz$  plane (a “defocusing quadrupole”) may be written:

$$\underline{T}_{\text{Dquad}} = \begin{pmatrix} \cosh(\sqrt{k_q}l_q) & \frac{\sinh(\sqrt{k_q}l_q)}{\sqrt{k_q}} & 0 & 0 & 0 & 0 \\ \sqrt{k_q}\sinh(\sqrt{k_q}l_q) & \cosh(\sqrt{k_q}l_q) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos(\sqrt{k_q}l_q) & \frac{\sin(\sqrt{k_q}l_q)}{\sqrt{k_q}} & 0 & 0 \\ 0 & 0 & -\sqrt{k_q}\sin(\sqrt{k_q}l_q) & \cos(\sqrt{k_q}l_q) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{l_q}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (22)$$

### 90 4.3 Solenoid

The passage of a beam particle through a solenoid is determined by the magnetic field strength,  $B_s$ , within the solenoid and the length of the solenoid,  $l$ , measured along its axis. As the particle enters the solenoid, the fringe fields impart momentum transverse to the axis of the magnet. This results in the particle executing a helical trajectory, the axis of the helix being parallel to the solenoid axis. The sense of the rotation depends on the particle charge and the polarity of the field. The helical motion means that the evolution of the particle motion in the  $xz$  plane is coupled with the evolution of the particle motion in the  $yz$  plane.

In the “hard-edge” approximation, the magnetic field inside the magnet is given by  $\mathbf{B}_s = (0, 0, B_{s0})$ , where the solenoid axis lies along the  $z_{\text{RPLC}}$  axis. The solenoid field strength parameter is then given by:

$$k_s = \left[ \frac{B_{s0}c}{2p} \times 10^{-3} \right]^2; \quad (23)$$

where  $B_{s0}$  is measured in T,  $p$  in MeV/c and  $c$  in m/s.

The transfer matrix for passage of a positive particle through a solenoid with field pointing in the positive  $z_{\text{RPLC}}$  direction may be written:

$$\underline{T}_{\text{Sol}} = \begin{pmatrix} \cos^2(\sqrt{k_s}l) & \frac{1}{2\sqrt{k_s}} \sin(\sqrt{k_s}l) & \frac{1}{2} \sin(2\sqrt{k_s}l) & \frac{1}{\sqrt{k_s}} \sin^2(\sqrt{k_s}l) & 0 & 0 \\ -\frac{\sqrt{k_s}}{2} \sin(2\sqrt{k_s}l) & \cos^2(\sqrt{k_s}l) & -\sqrt{k_s} \sin^2(\sqrt{k_s}l) & \frac{1}{2} \sin(2\sqrt{k_s}l) & 0 & 0 \\ -\frac{1}{2} \sin(2\sqrt{k_s}l) & -\frac{1}{\sqrt{k_s}} \sin^2(\sqrt{k_s}l) & \cos^2(\sqrt{k_s}l) & \frac{1}{2\sqrt{k_s}} \sin(2\sqrt{k_s}l) & 0 & 0 \\ \sqrt{k_s} \sin^2(\sqrt{k_s}l) & -\frac{1}{2} \sin(2\sqrt{k_s}l) & -\frac{\sqrt{k_s}}{2} \sin(2\sqrt{k_s}l) & \cos^2(\sqrt{k_s}l) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{l}{\beta_0^2 \gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

As in the case of the quadrupoles, dispersion is accounted for using equation 23.

#### 4.4 Non-neutral (electron) plasma (Gabor) lens

A dense gas of electrons confined in a Penning-Malmberg trap provides an electric field that can be used to focus a positive ion beam. The electron gas is confined axially in the lens by an electrostatic potential created using a central anode of length  $l$ . The gas is confined radially using the uniform field of a solenoid. Assuming a uniform electron density,  $n_e$ , the focusing parameter,  $k_G$ , may be written:

$$k_G = \frac{e}{2\epsilon_0} \frac{m_p \gamma}{p^2} n_e; \quad (25)$$

105 where  $e$  is the charge on the electron,  $\epsilon_0$  is the permittivity of free space, and  $m_p$  is the proton mass. As in the case of the quadrupoles and solenoid, dispersion is accounted for using equation 25. The force on a particle passing through the electron gas is towards the axis of the lens and is proportional to the radial distance of the particle from the axis. Focusing is therefore cylindrically symmetric and does not couple motion in the the  $xz$  and  $yz$  planes.

In the “hard-edge” approximation, the electric field inside the lens falls to zero at the end of the electron gas and the contribution of the magnetic field used to confine the electron gas in the transverse direction has a negligible effect on particles passing through the lens. The transfer matrix for passage of a positive particle through the lens may be written:

$$\underline{T}_G = \begin{pmatrix} \cos(\sqrt{k_G}l) & \frac{\sin(\sqrt{k_G}l)}{\sqrt{k_G}} & 0 & 0 & 0 & 0 \\ -\sqrt{k_G} \sin(\sqrt{k_G}l) & \cos(\sqrt{k_G}l) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos(\sqrt{k_G}l) & \frac{\sin(\sqrt{k_G}l)}{\sqrt{k_G}} & 0 & 0 \\ 0 & 0 & -\sqrt{k_G} \sin(\sqrt{k_G}l) & \cos(\sqrt{k_G}l) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{l}{\beta_0^2 \gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (26)$$

#### 105 4.5 Dipole

The reference particle trajectory in the beam elements described above passes along the axis of the element. In contrast, a dipole bends the reference trajectory so that it describes the arc of a circle (see figure 2). The codes provides for transport through a “sector dipole” in the hard-edge approximation. In this case, the field within the magnet is taken to be constant and parallel to  $j_{\text{RPLC}}$ , i.e.  $\mathbf{B}_D = (0, B_{D0}, 0)$ . No edge focusing is considered.

110 The passage of particles through a dipole may be described by defining the parameter,  $k_D$ :

$$k_D = \left[ \frac{B_{D0}c}{p} \times 10^{-3} \right]^2. \quad (27)$$



Figure 2: Schematic representation of the passage of the reference particle through a sector dipole. The outline of the sector dipole is shown by the solid black lines. The trajectory of the reference particle is shown as the dashed line. The length of the reference-particle trajectory inside the field of the sector dipole is  $l_D$ . The  $x_{\text{RPLC}}$  and  $z_{\text{RPLC}}$  coordinate axes at the entry and exit of the sector dipole are shown. The radius of curvature of the reference particle-trajectory trajectory inside the magnet is  $\rho$  and the angle through which the  $x_{\text{RPLC}}$  is rotated is  $\phi$ .

The momentum of the reference particle is related to the curvature.  $\rho$ , by:

$$p_0 = B_{D0}\rho; \quad (28)$$

so:

$$k_D = \frac{1}{\rho}; \quad (29)$$

and the angle  $\phi$  is given by:

$$\phi = \frac{l}{\rho}. \quad (30)$$

With these definitions the transfer matrix for passage through a dipole may be written:

$$T_{\equiv D} = \begin{pmatrix} \cos(\phi) & \rho \sin(\phi) & 0 & 0 & 0 & \frac{\rho}{\beta_0} (1 - \cos(\phi)) \\ -\frac{\sin(\phi)}{\rho} & \cos(\phi) & 0 & 0 & 0 & \frac{\sin(\phi)}{\beta_0} \\ 0 & 0 & 1 & l & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{\sin(\phi)}{\beta_0} & -\frac{\rho}{\beta_0} (1 - \cos(\phi)) & 0 & 0 & 1 & \frac{l}{\beta^2 \gamma^2} - \frac{l - \rho \sin(\phi)}{\beta_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (31)$$

## 5 Source

Para 1:

- A variety of options for generating the particle distribution at source are included in the package;
- The principle (default) option is the model of the TNSA mechanism presented in [4].



Para 2:

- The energy spectrum of the protons per unit energy and unit solid angle is given by:

$$\frac{dN}{dE} = \frac{n_{i0}c_s t}{(2EE_0)^{\frac{1}{2}}} \exp \left[ - \left( \frac{2E}{E_0} \right)^{\frac{1}{2}} \right]; \quad (32)$$

where  $n_{i0}$  is the ion number density at  $t = 0$ , the instant the laser pulse strikes the target;  $c_s$  is given by:

$$c_s = \left[ \frac{Zk_B T_e}{m_i} \right]^{\frac{1}{2}}; \quad (33)$$

where  $Z$  is the ion charge number,  $k_B$  is the Boltzmann constant;  $T_e$  is the electron temperature;  $m_i$  is the ion mass;  $t$  is the instant in time at which the spectrum is evaluated; and

$$E_0 = \left[ \frac{n_{e0}k_B T_e}{\epsilon_0} \right]^{\frac{1}{2}} \quad (34)$$

Para 3:

- To generate the proton spectrum at the source, a practical approach is taken. The leading behaviour of equation 32 is taken to be:

$$\frac{dN}{dE} = \Gamma \frac{\exp \left[ -E^{\frac{1}{2}} \right]}{E^{\frac{1}{2}}}; \quad (35)$$

- Normalisation of equation 35 between a minimum energy ( $E_{\min}$ ) and a maximum energy ( $E_{\max}$ ) is used to determine the constant  $\Gamma$ . Both  $E_{\min}$  and  $E_{\max}$  are user defined and set at the initialisation stage.

## References

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## A Set-up and run

### Introduction

This section summarises the steps needed to set-up and run the linear optics simulation of the LhARA beam line. A summary of the tasks that the software suite performs will be documented in due course. The code has been developed in python; python 3 is assumed.

### Getting the code

The linear optics package is maintained using the GitHub version-control system. The latest release can be downloaded from:

```
\centerline{
  \href{https://github.com/ImperialCollegeLondon/LhARALinearOptics.git}{https://github.com/ImperialCollegeLondon/LhARALinearOptics.git}
}
```

### Dependencies and required packages

The linear optics code requires the following packages:

- Python modules: `scipy` and `matplotlib`.

It may be convenient to run the package in a “virtual environment”. To set this up, after updating your python installation to python 3.9, execute the following commands:

1. `python3 -m venv --system-site-packages venv`
  - This creates the director `venv` that contains files related to the virtual environment.
2. `source venv/bin/activate`
3. `python -m pip install pandas scipy matplotlib`

To exit from the virtual environment, execute the command `deactivate`. The command `source venv/bin/activate` places you back into the virtual environment.

The Imperial HEP linux cluster provides python 3.9.18 by default.

### Unpacking the code, directories, and running the tests

After downloading the package from GitHub, or cloning the repository, you will find a “`README.md`” file which provides some orientation and instructions to run the code. In particular, a bash script “`startup.bash`” is provided which:

- Sets the “`LhARAOpticsPATH`” environment variable so that the files that hold constants etc. required by the code can be located; and
- Adds “`01-Code`” (see below) to the `PYTHONPATH`. The scripts in “`02-Tests`” (see below) may then be run with the command “`python 02-Tests/<filename>.py`”.

Below the top directory, the directory structure in which the code is presented is:

`01-Code`: contains the python implementation as a series of modules. Each module contains a single class or a related set of methods.

`02-Tests`: contains self-contained test scripts that run the various methods and simulation packages defined in the code directory.

`11-Parameters`: contains the parameter set used to specify the various beam lines presently implemented.

The instructions in the `README.md` file should be followed to set up and run the code.

## 170 Running the code

Execute `startup.bash` from the top directory (i.e. run the bash command `source startup.bash`). This will:

- Set up `LhARAOpticsPATH`; and
- Add `01-Code` to the `PYTHONPATH`. The scripts in `02-Tests` may then be run with the command  
175 `python 02-Tests/<filename>.py`;
- Example scripts are provided in `03-Scripts`, these can be used first to “Run” the simulation and then to “Read” the data file produced. Example scripts are provided for the DRACO, LION, and LhARA Stage 1 beam lines.