A <u>truth function</u> takes one or more truth values as arguments, and outputs a truth value. Truth functions can be defined by truth tables.

a	f(a)
T	F
F	T

$f(a)$ is $\neg a$

a	b	f(a, b)
T	T	F
T	F	T
F	T	T
F	F	F

$f(a)$ is a XOR b (exclusive or.)

We have defined symbols for NOT, AND, OR, IMPLIES, IFF.
$\neg$ $\wedge$ $\vee$ $\longrightarrow$ $\longleftrightarrow$

(1) How many truth functions are there on a fixed number n of input truth values?

Two truth functions are logically equivalent if they have the same truth tables.

A truth function is in disjunctive normal form (DNF) if it is expressed using $\neg, \vee, \wedge$ such that:

* no $\wedge$ occurs inside the scope of a $\neg$
* no $\vee$ occurs inside the scope of an $\wedge$ or a $\neg$.

Examples: a, $\neg a$, $a \wedge b$, $a \vee b$,
$(a \wedge \neg a) \vee b$, $(a \wedge \neg a) \vee (a \wedge b) \vee (b \wedge \neg b)$.

(2) Show that every truth function is logically equivalent to one in DNF.

Conjunctive normal form (CNF) is like DNF, except the priority of $\wedge, \vee$ are exchanged. (So no $\wedge$ occurs in the scope of an $\vee$.)

(3) Show that every truth function is logically equivalent to one in CNF.

A set of truth functions is <u>functionally complete</u> if they can be used to express every truth function.

(4) Show that the set $\{\neg, \rightarrow\}$ is functionally complete

A simple system for logical deduction is as follows. A <u>proof</u> is a list of expressions in $\{\neg, \rightarrow\}$. Each expression in the list is either an <u>axiom</u> (these are given to us initially) or a <u>deduction</u>. The only deduction rule is that if P and $P \rightarrow Q$ both appear in the list, then we may deduce Q. (Here P and Q are any logical expressions.)

(5) Show that if all of the axioms we are given are tautologies (always true), then every deduction is also a tautology.

A common axiom scheme is to take all expressions of the form $P \rightarrow (Q \rightarrow P)$, $(\neg Q \rightarrow \neg P) \rightarrow (P \rightarrow Q)$, $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$. (Here any logical expressions may be substituted for P, Q, R.)

(6) With the axioms given above, deduce

$$a \rightarrow a.$$

(Much harder!) Deduce $a \rightarrow \neg\neg a$.