KMB, 15/10/18

# M1F Foundations of Analysis, Problem Sheet 2.

**1.** What are the following sets? Justify your answers.

(a) $\bigcup_{n=0}^{\infty}[n, n+1)$.

(b) $\bigcup_{n=1}^{\infty}[1/n, 1]$.

(c) $\bigcup_{n=1}^{\infty}(-n, n)$.

(d) $\bigcap_{n=1}^{\infty}(-n, n)$.

**2.** Prove that the set $(0,1)$ (that is $\{x \in \mathbf{R} : 0 < x < 1\}$) has no largest element. (NB: by a "largest element" of a set $S$ I mean an element $x \in S$ such that $\forall y \in S, y \leq x$.)

**3.**
(a) Prove that if $n$ is an integer and 3 divides $n^2$ then 3 divides $n$.
(b) Deduce that $\sqrt{3}$ is irrational.

**4.** Are the following statements true or false? Proofs or counterexamples required.
(a) If $a$ is irrational and $b$ is irrational then $a+b$ must be irrational.
(b) If $a$ is irrational and $b$ is rational then $ab$ must be irrational.

**5.** Are the following statements true or false? Proof or counterexample required.
(a) $\forall x \in \mathbf{R}\, \exists y \in \mathbf{R}\, x + y = 2$.
(b) $\exists y \in \mathbf{R}\, \forall x \in \mathbf{R}\, x + y = 2$.

**6*.** Prove that $\sqrt{2} + \sqrt{6} < \sqrt{15}$ (NB you may assume the square roots exist).

**7.** Are the following numbers rational or irrational? Proofs required.
(a) $\sqrt{2} + \sqrt{3/2}$ (hint: if it were rational then its square would also be rational).
(b) $1 + \sqrt{2} + \sqrt{3/2}$.
(c) $2\sqrt{18} - 3\sqrt{8}$.