Advanced Control for a Robotic Quadrotor-Top Inverted Pendulum (Q-TIP)

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Introduction

A 6 DOF (degrees of freedom) robotic inverted pendulum, which can stabilize while moving, was analyzed and improved. The project aims to rebuild the dynamic modelling of the previously constructed robot in different degree of freedom. Different controllers were designed based on the mathematic models, only the pendulum system was focused in this paper. All the forces from the cart were treated as disturbances.

Design

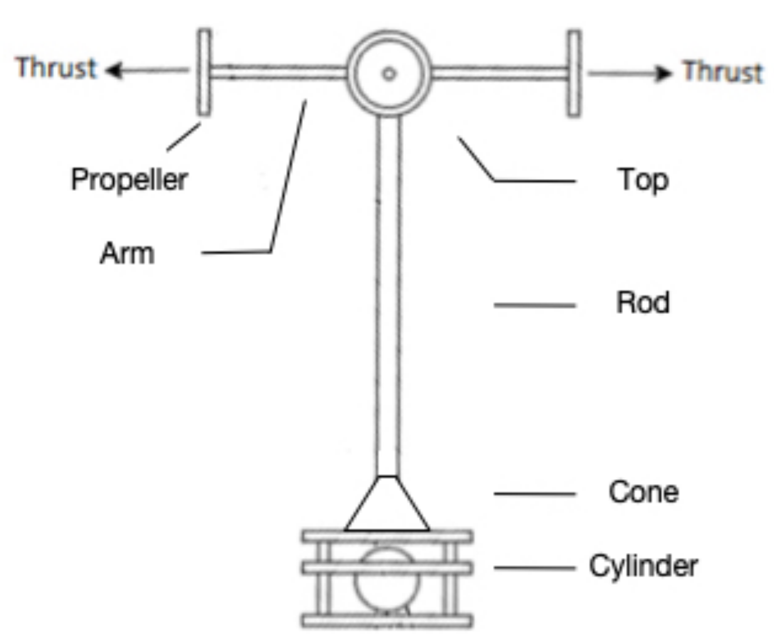

Figure 1: QTIP concept

QTIP was designed by previous students based on the basic mechanical structure of an inverted pendulum on a cart. The pendulum system utilized quadrotor propellers on top to balance while the bottom cart moves around. Two orthogonal pairs of propellers were set on the top, one pair for each degree of freedom. Those propellers can produce a lateral thrust to stabilize the pendulum arm in the upright position. Such quadrotor with a cart can be used to the real-life application, such as farm monitoring.

Construction

As shown in Figure.2, the inverted pendulum connects to the cart with a fixed ball joint, which allows the system to move freely with respect to the cart. Power supply and controller were set in the cart frame; thus, the cart can both translate and rotate on a flat surface by three omnidirectional wheels.

The cart is driven by differential drive motors, which can communicate the information from a Bluetooth module to an Android device. The propellers were controlled by ESCs and can be mounted at different angles to increase the controllability of the Euler's angles. An MPU6050 accelerometer was fixed at the cross centre of the pendulum arm.

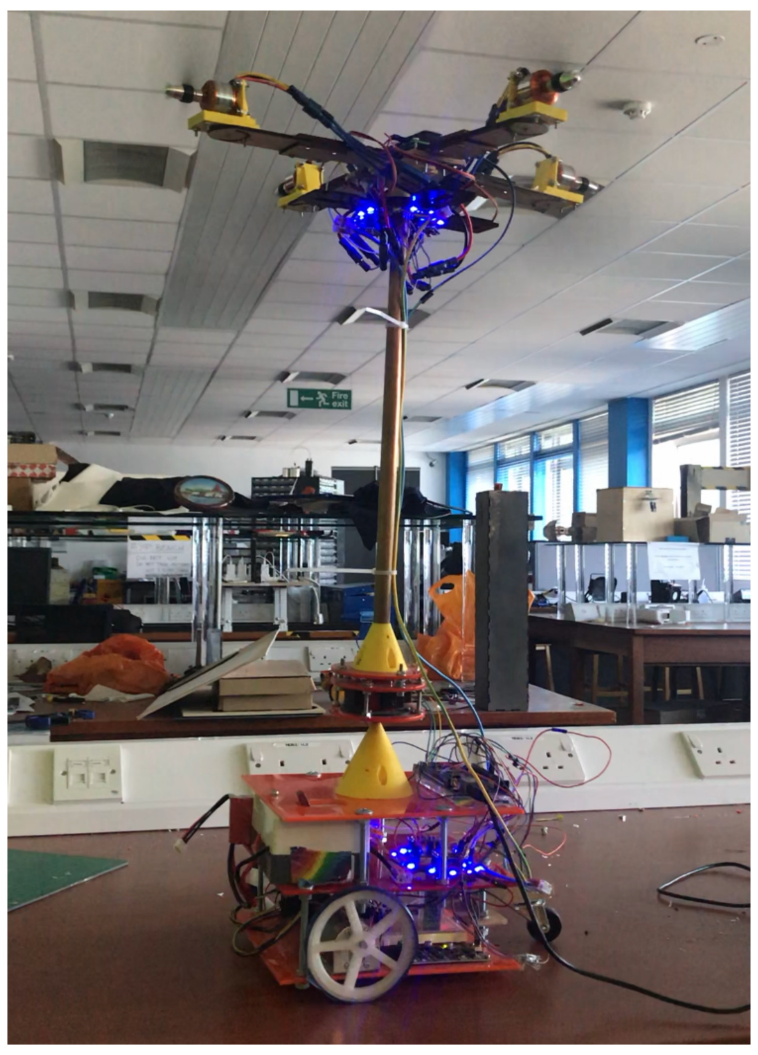

Figure 2: QTIP Construction

1 DOF Modelling

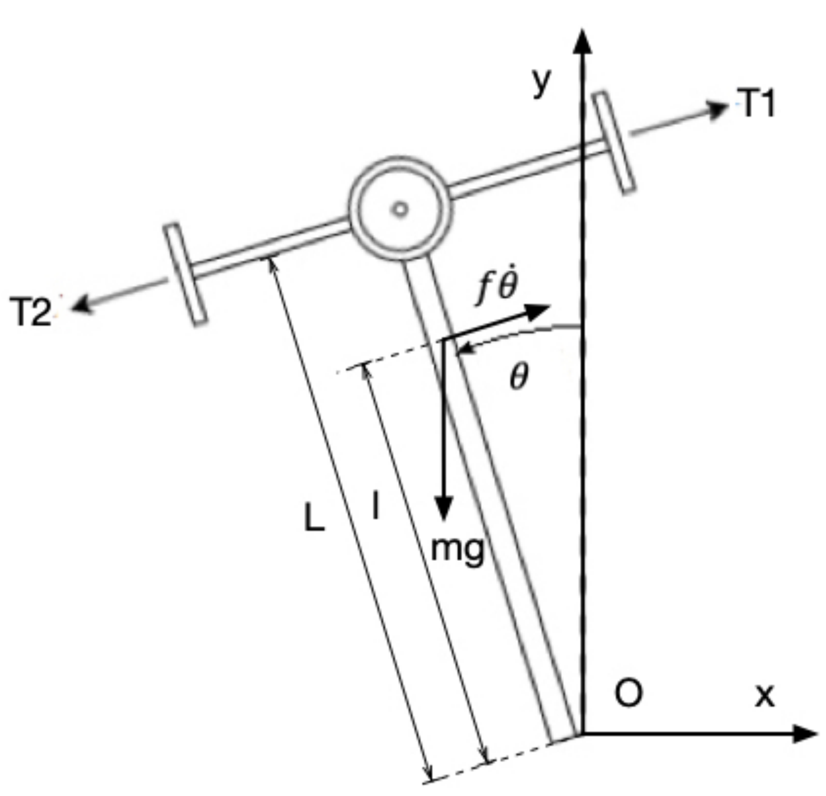

Figure 4: 1 DOF Diagram

The 1 DOF modeling was based on Newton's second law for rotational motion, which states the torque can be expressed as:

$$\tau = dF = J\ddot{\theta}$$

, where F is the force, and d is the perpendicular distance from the force to the pivot.

Pendulum stays upward was set as the initial position. The angle θ represents the angle (rad) between the pendulum and y-axis upward. θ can move from −90 to 90 degrees due to its physical constraints. The input of the system was set as the difference between the thrust forces produced by motor 1&2. The state of the system is $[\theta \quad \dot{\theta}]^T$.

2 DOF Modelling

From Figure.4, l_{xz} , and l_{yz} are the projection lengths of l on the x-z and y-z plane, respectively. Those lengths are in the triangle geometry relation. The angle θ_T represents the angle (rad) between the pendulum and z-axis upward. θ_x and θ_y are the angles between the z-axis and the projection of pendulum on the x-z plane and y-z plane, respectively.

2 DOF modelling can be treated as a coupled or a decoupled system. For the coupled system, the projection lengths are related to each other but not the same. Thus, the linear system can be created by finding equilibrium points. For the decoupled system, Assuming angles are very small compared to 1. The two subsystems on x-z and y-z planes can be treated as decoupled in the perpendicular direction of the local coordination. The three projection lengths can be treated as the same.

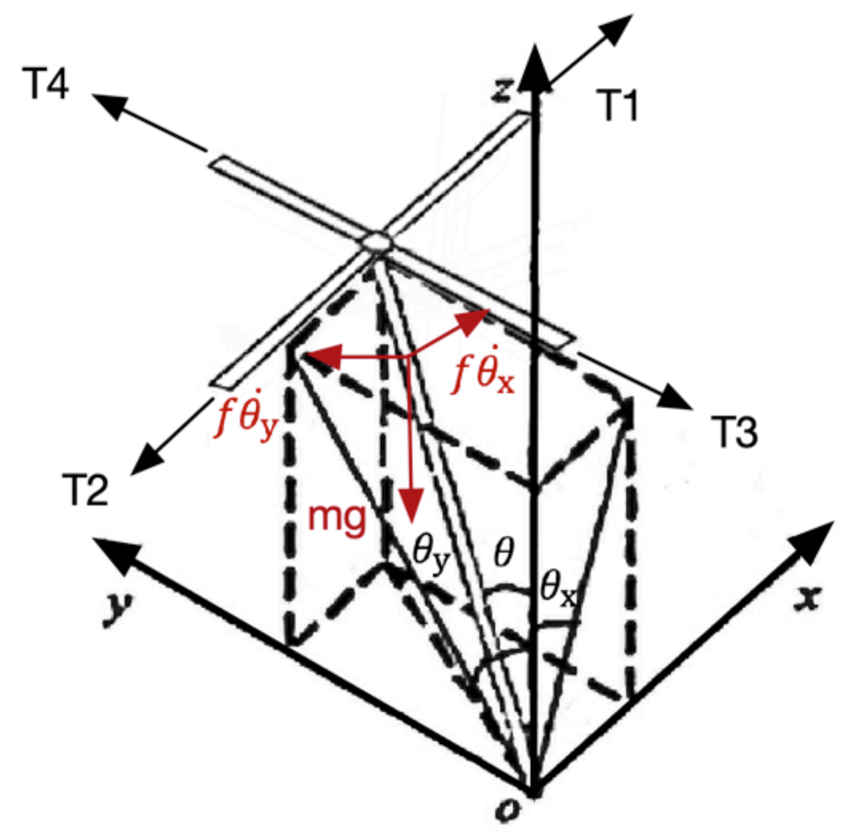

Figure 4: 2 DOF Diagram

The thrust force T2 is generated by the adjustment of the Motor 2's direction. Motor 2 was set to rotate an α degree that provides the component forces in x and y direction on the x-y plane. The y-component force will cause a rotational along the z-axis. The coordinate system of pendulum P was set around the origin O, which is the pivot connecting the pendulum and cart. The coordinates of pendulum body-follower frame is represented as P = [x y z]. The model was built by using the Lagrange's theorem, energy equilibrium, and rotational moment equilibrium.

3 DOF Modelling

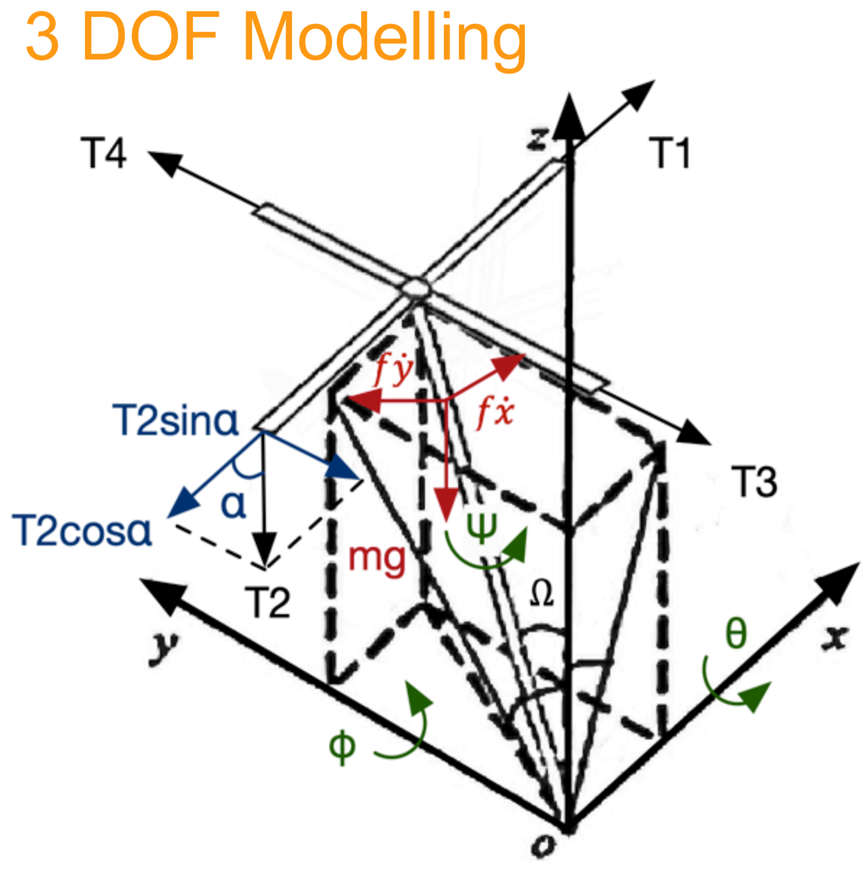

Figure 5: 3 DOF Diagram

This pendulum's attitude can be described by Euler angles. The roll, pitch and yaw are indicated as Figure.6. R matrices were created as the transformation matrices that can convert the pendulum's attitude from the body system to the world system.

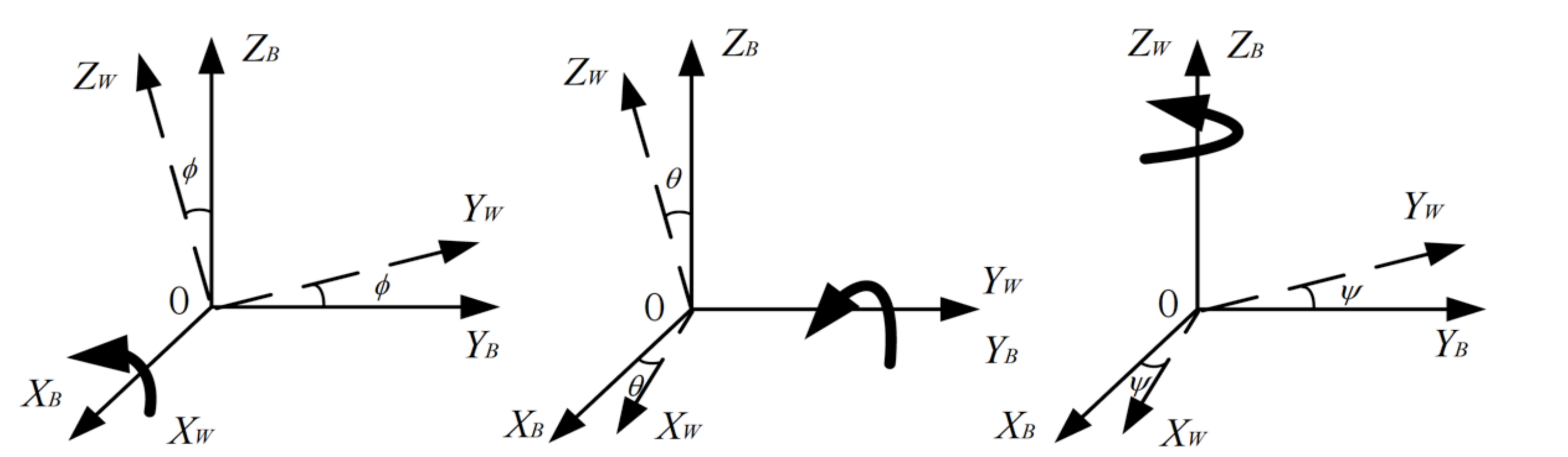

Figure 6: Euler Angle Definition (Yan Wei, 2017)

Control

Nonlinear	1DOF: PID, LQR, pole place controllers were built. PID was directly built depends on the nonlinear equation by Simulink. LQR and pole place were built depends on the linearized state-space system by Simulink and ODE45. The controllers from the linear system were used to control the nonlinear system.
	2DOF: For the decoupled system, the controllers were designed by simply double the 1DOF's control system. For the coupled system, the similar way to apply the LQR and pole place but difficult to the PID control.
Linear	1DOF: LQR and pole place controllers were built based on the state space system, which is created by linearizing the nonlinear equations.
	2DOF: LQR and pole place controllers for the coupled system, and PID for the decoupled system. The linear matrices were derived by finding the equilibrium points and then applying the Jacobian linearization.

References

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Acknowledgements

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