IMPERIAL

Early Career Researcher Institute

Regression Modelling in R

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1. **Theory (~30 min)**
   - Background
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   - Logistic regression

   Break (5 min)

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   Break (5 min)

3. **R practical in RStudio (part II): Logistic Regression (~30 min)**

   Break (5 min)

4. **Interpreting a Study (~15 min)**

Main idea of regression modelling

The problem: We have loads of data and we want to **describe the relationship**.

A solution: We build a **regression model**. There are many regression models. Today we're focussing on:

A. Linear regression
B. Logistic regression

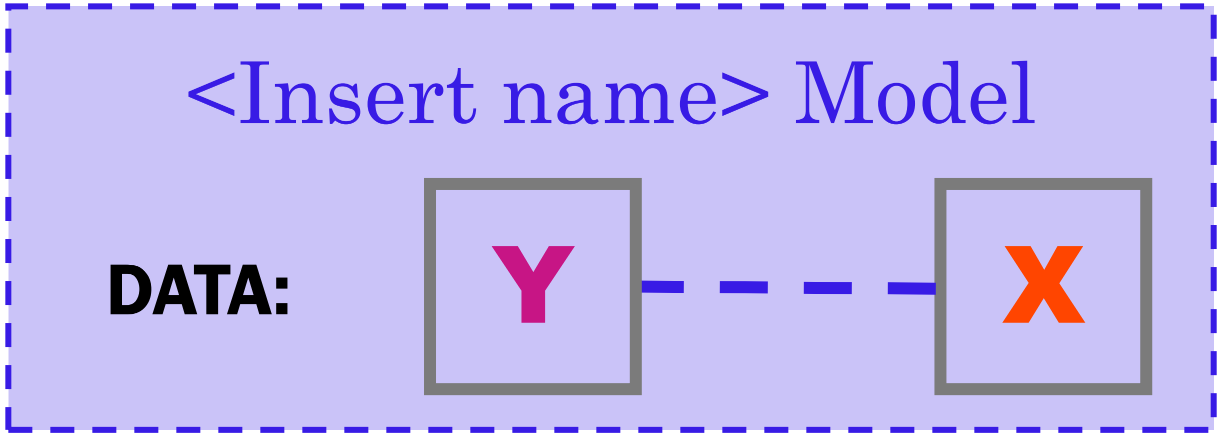
Height	Weight
1.1	0.4
1.9	1.2
1.7	1.9
2.8	2.0
2.3	2.8

What is regression modelling?

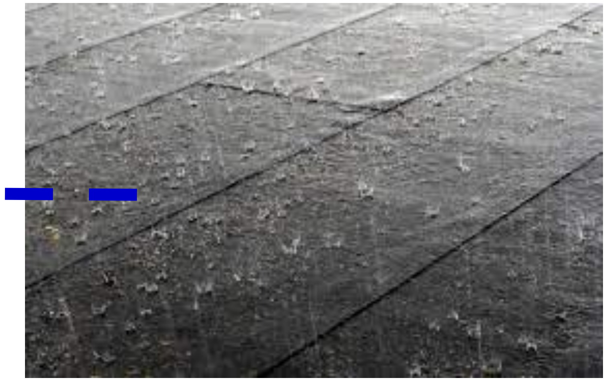
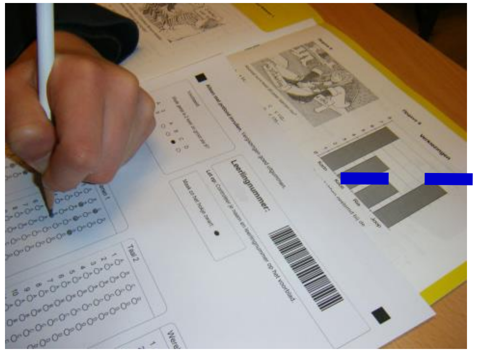
In statistics, regression modelling is a process for **estimating** a **line** or **curve** that **best represents the general trend** between one **outcome variable (Y)** and one or more **predictor variables (X).**

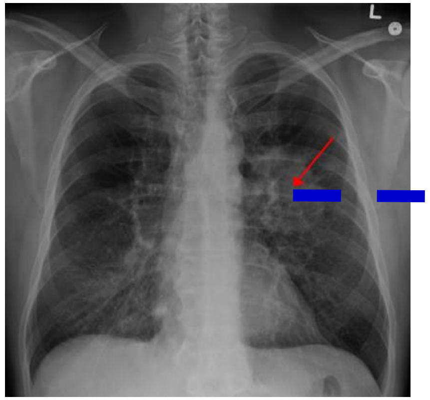
dependent variable, response, label, prediction

independent variables, exposures, explanatory variables, features



When do we need regression modelling?




Linear regression

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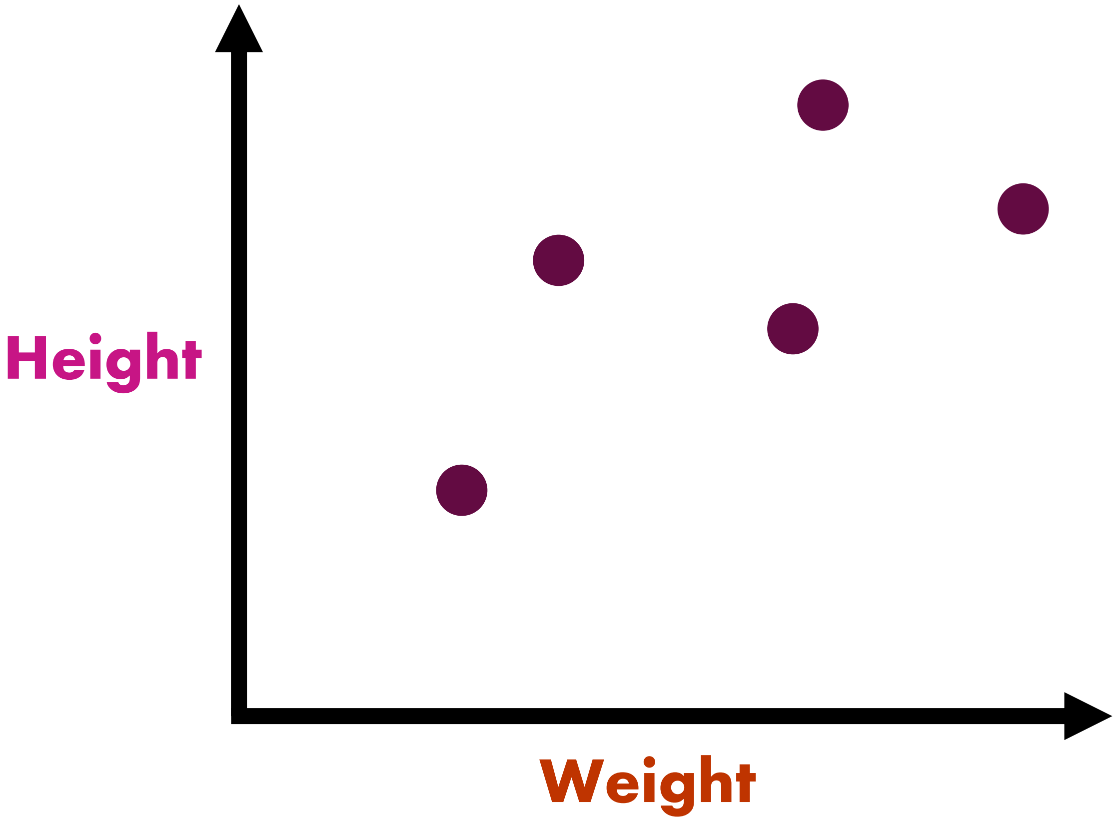
2.3	2.8



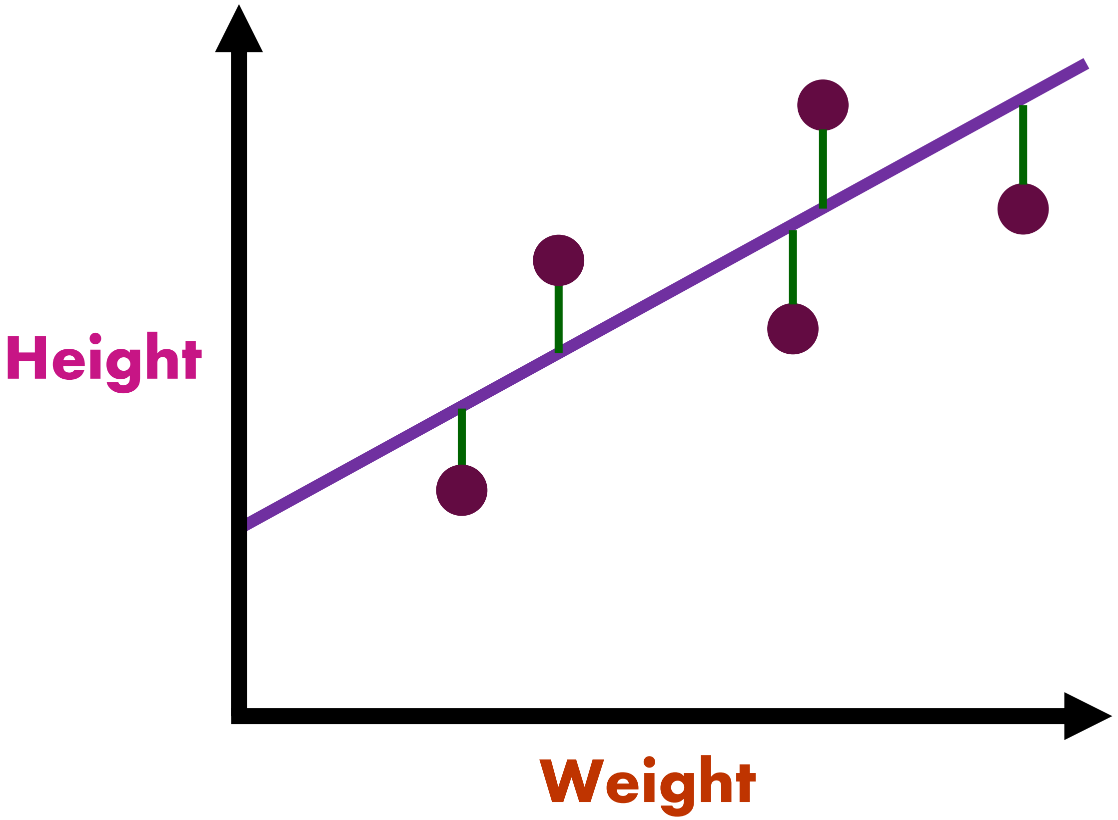
Linear regression

Use **least-squares** to fit a line to the data.

Least-squares minimises the
Sum of the Squared Residuals (SSR)

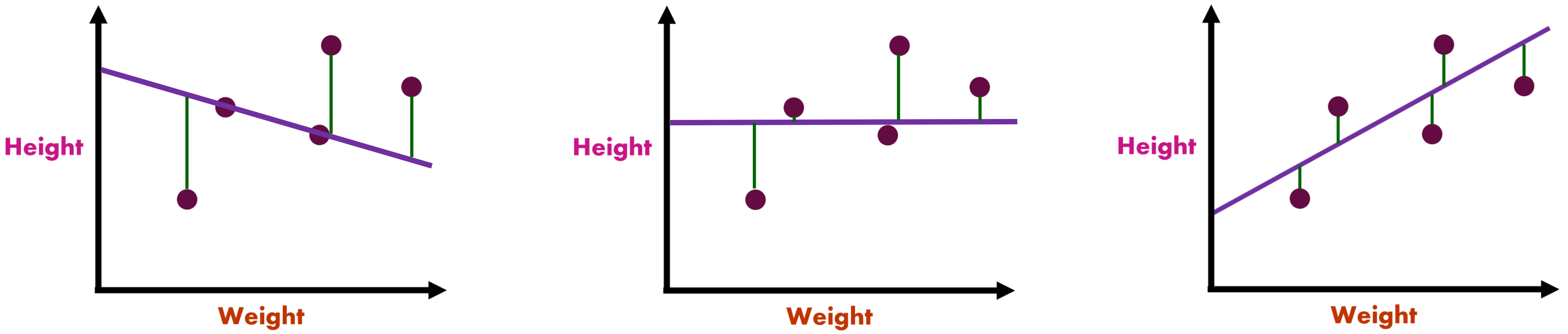
Residual = Observed - Fitted

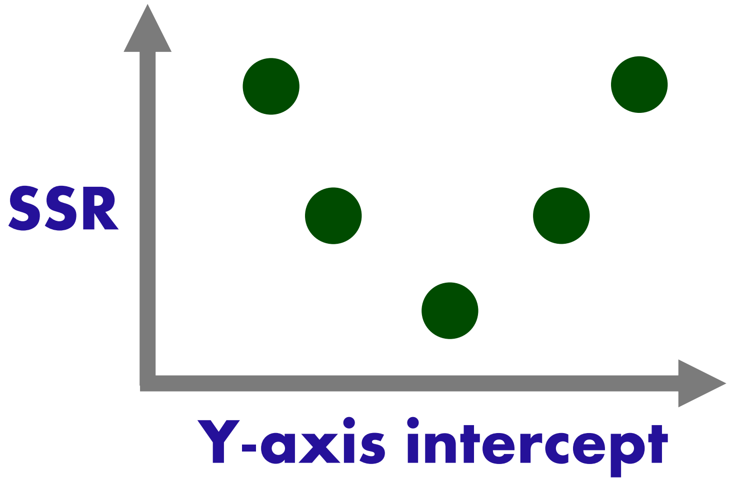
$$SSR = \sum_{i=1}^{n} (Observed_i - Fitted_i)^2$$



Linear regression

Use **least-squares** to fit a line to the data.





y = **mx** + **c**, where
y = how far up
x = how far along
m = slope (also β)
c = the y-intercept

Height = slope x Weight + intercept
Height = 0.5 x Weight + 1.1

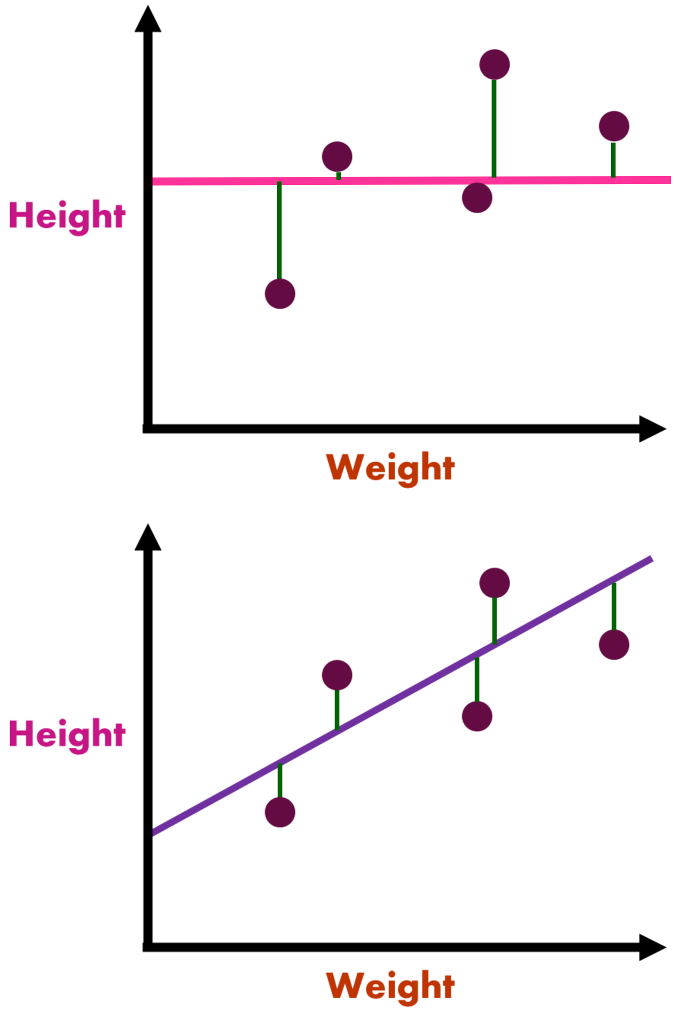
Linear regression

Calculate the R^2

R^2 is the proportion of the variation in the outcome that is explained by the predictor.

$$R^2 = \frac{SSR(mean) - SSR(fitted\ line)}{SSR(mean)}$$

$$R^2 = \frac{1.6 - 0.5}{1.6} = 0.7$$

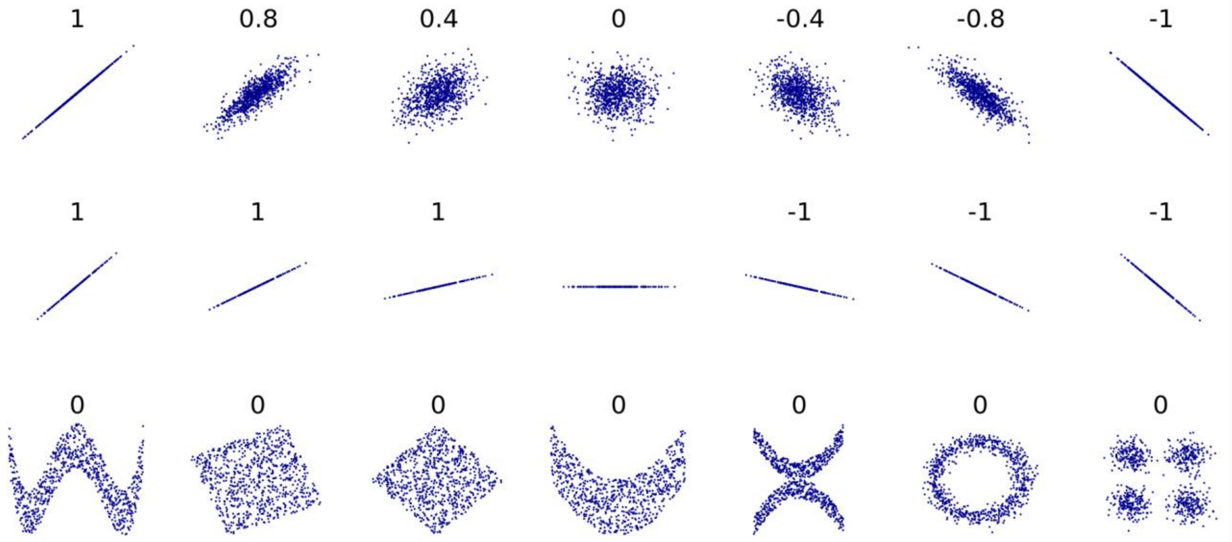


Linear regression

Pearson correlation coefficient (ρ)

The Pearson correlation coefficient (ρ, or rho) is the measure of **linear correlation** between two variables.

The word Correlation is made of **Co**- (meaning "together"), and **Relation**.



In the case of simple linear regression,

$\rho^2 = r^2 = R^2$

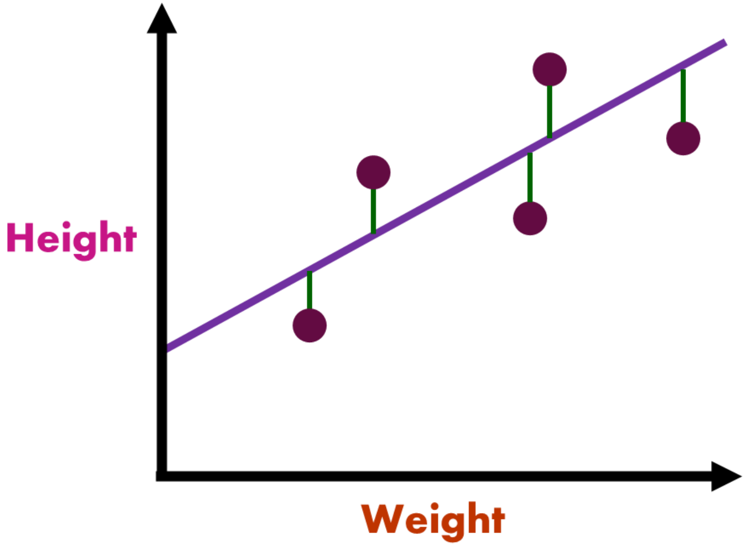
- Correlation is **Positive** when the values **increase** together, and
- Correlation is **Negative** when one value **decreases** as the other increases
- The value shows how good the correlation is (not how steep the line is), and if it is positive or negative.

Linear regression

Calculate *p*-value for R^2

The *p*-value for our R^2 tells us the probability that random data could result in a similar or better R^2.

In general, *p*-values below 0.05 give us a large confidence in the results of our analysis.



Height = 0.5 x Weight + 1.1

$$R^2 = 0.7$$

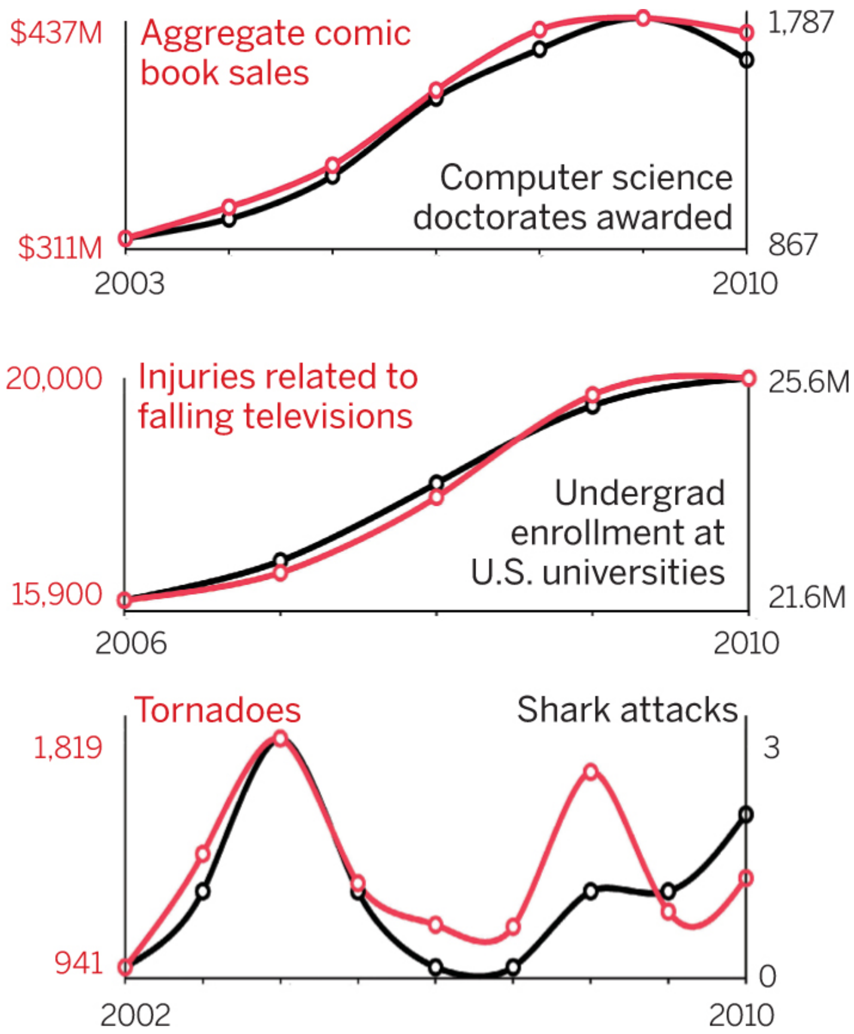
***p*-value = 0.1**

Correlation is not always causation

Height ~ Weight

Height <- Weight

Height -> Weight



Source: Tyler Vigen for Science Magazine

One outcome and multiple predictors

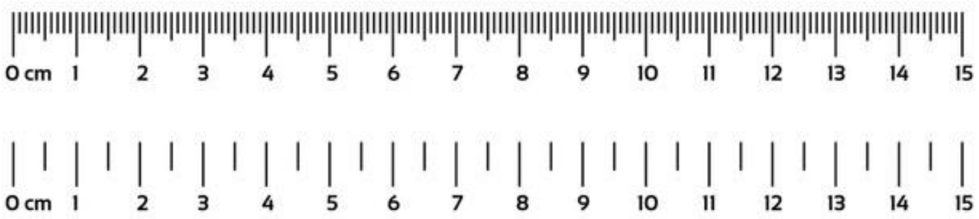
Height	Weight	Shoe size	Favourite colour
1.1	0.4	36	Green
1.9	1.2	41	Blue
1.7	1.9	39	Blue
2.8	2.0	43	Orange
2.3	2.8	44	Yellow

Continuous and discrete data

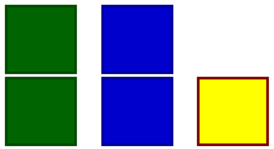
Height	Weight	Shoe size	Favourite colour
1.1	0.4	36	Green
1.9	1.2	41	Blue
1.7	1.9	39	Blue
2.8	2.0	43	Orange
2.3	2.8	44	Yellow

Continuous data (numeric) is measurable and can take any numeric value within a range.

The precision of the measurements is only limited by the tools we use, e.g. height in cm or mm:



Discrete data (factor) is countable and only takes specific values. We count the number of people who sit in the categories.

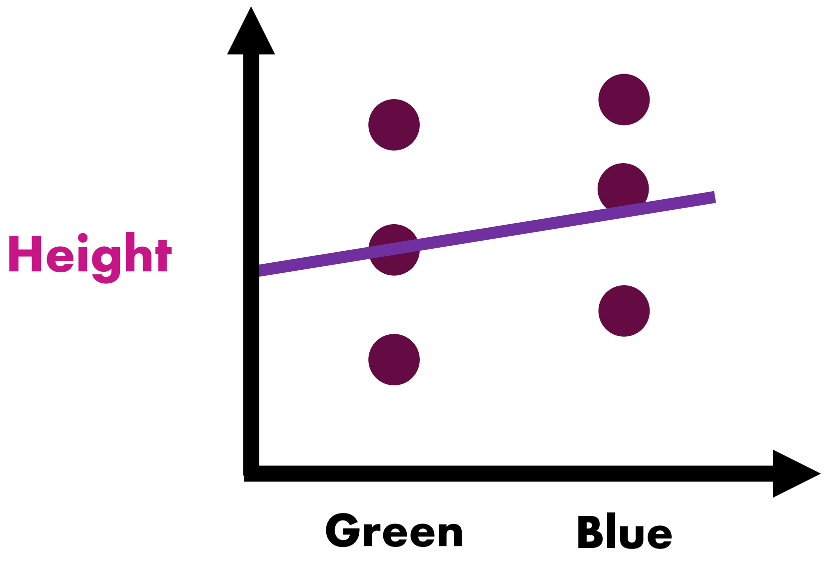


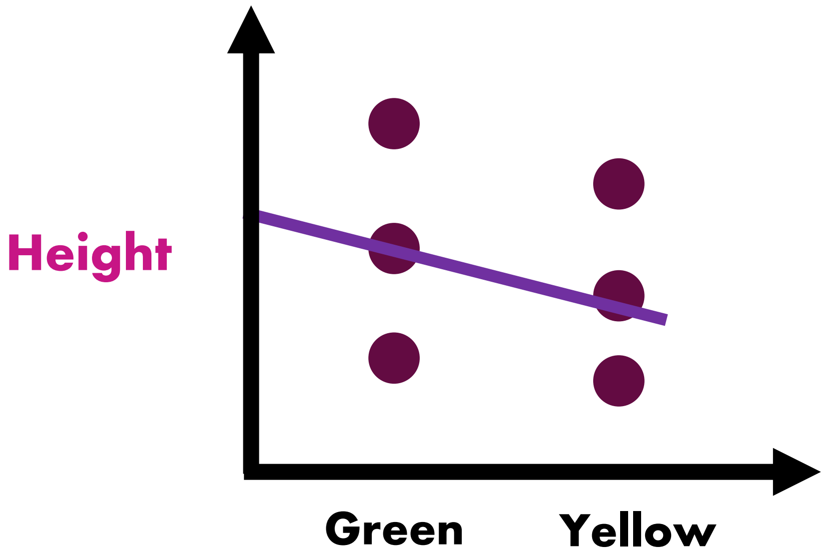
Two people love the colour green, two blue, and one yellow.

Note: some variables, e.g. “Shoe size”, can be coded as numeric or factor

Linear regression with discrete measurements (factors)

Height	Favourite colour	Blue	Yellow
1.1	Green	0	0
1.9	Blue	1	0
1.7	Blue	1	0
2.8	Green	0	0
2.3	Yellow	0	1





Simple linear regression: **Height** = m x **Weight** + c

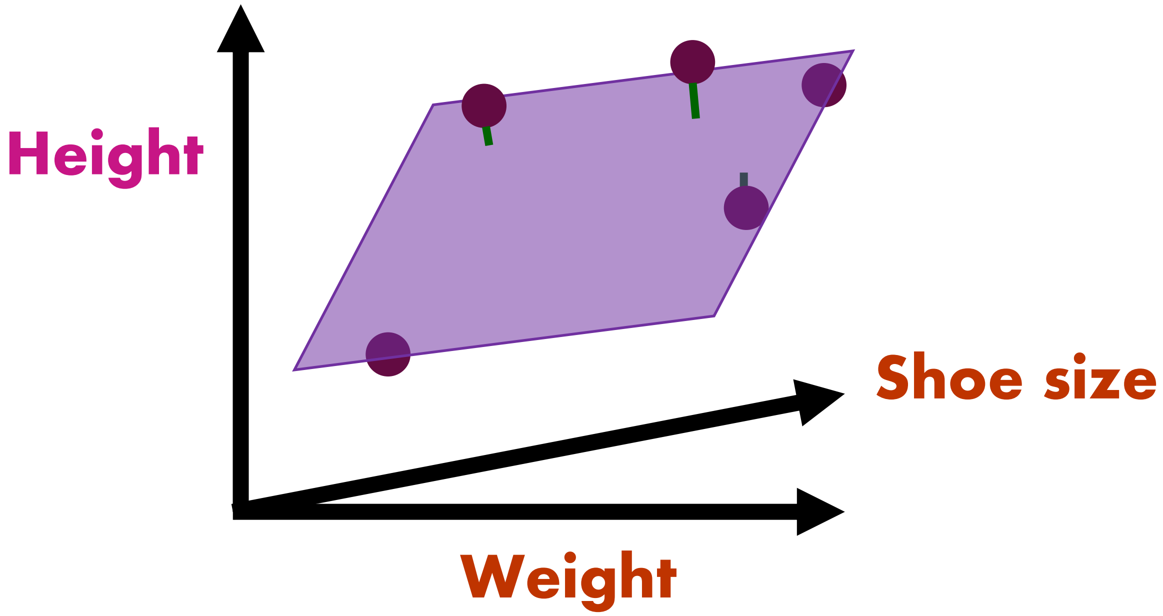
Simple linear regression (with factors): **Height** = m_1 x Blue + m_2 x Yellow + c

Multiple linear regression

Simple linear regression: **Height** = m x **Weight** + c

Multiple linear regression: **Height** = m_1 x **Weight** **+** m_2 x **Shoe size** + c

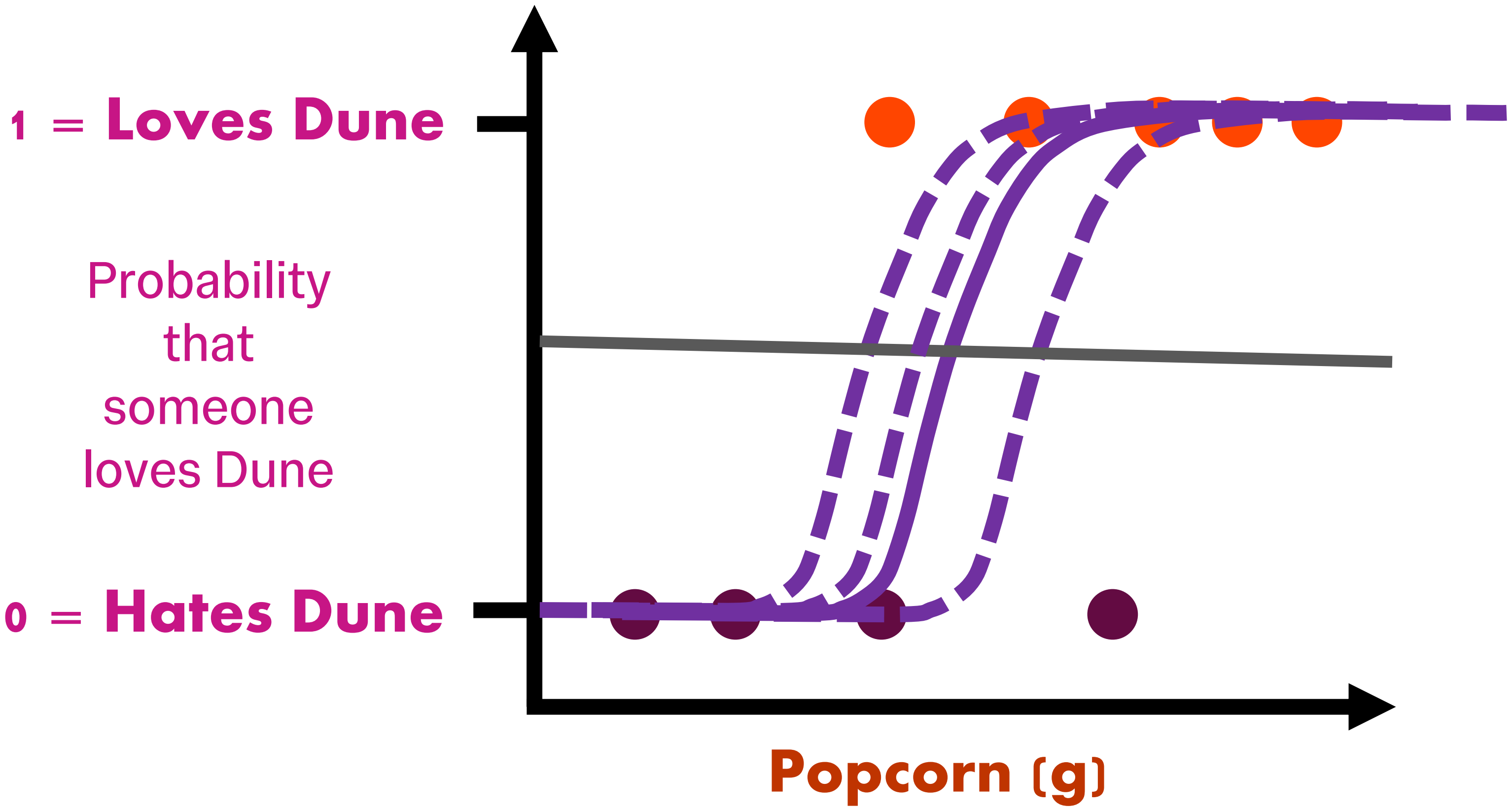
Height	Weight	Shoe size
1.1	0.4	36
1.9	1.2	41
1.7	1.9	39
2.8	2.0	43
2.3	2.8	44



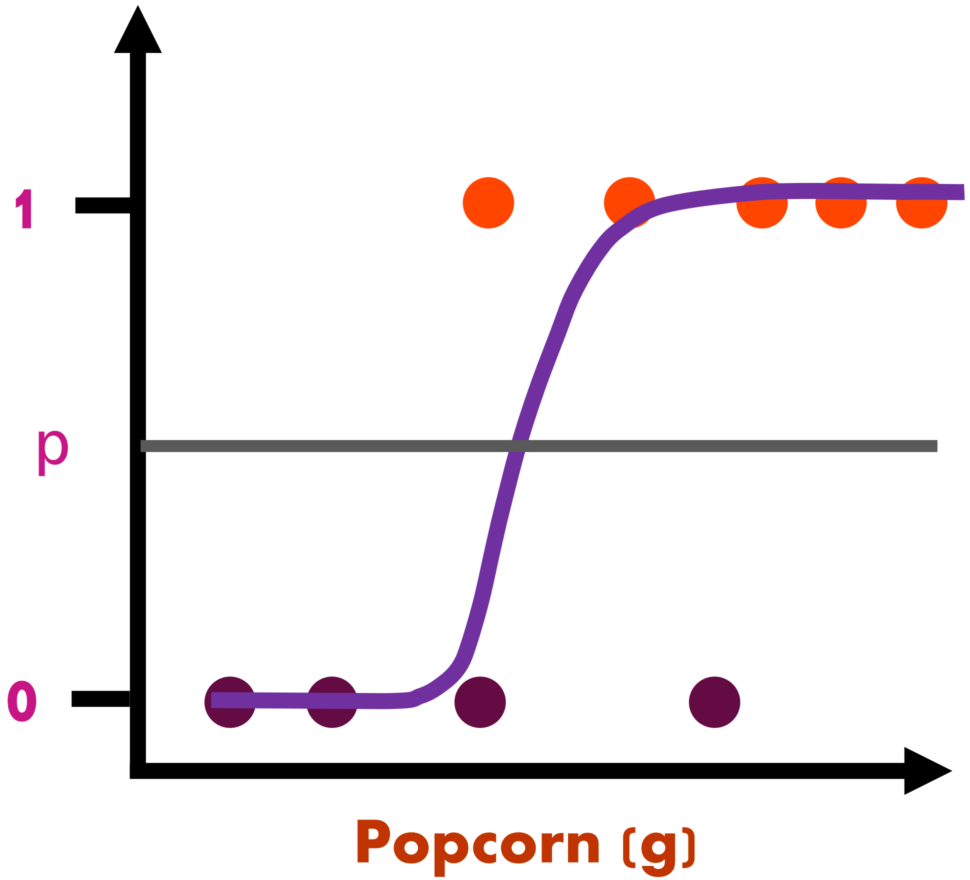
Logistic regression

Use **maximum likelihood** to fit an S-shaped logistic function to the data.

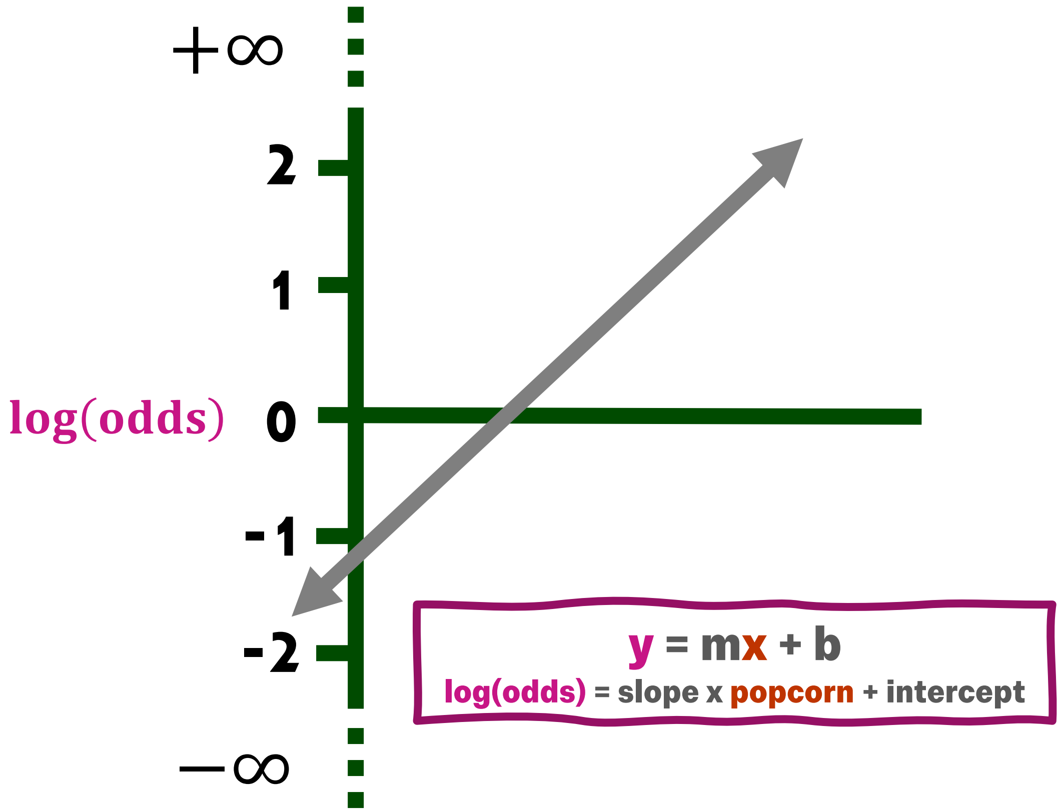
Loves Dune	Popcorn (g)
1	95
0	50
1	100
1	85
0	60



Logistic regression



Use logit function:
$\log(\frac{p}{1-p})$



Multiple logistic regression

As with linear regression, we can use multiple discrete and continuous predictors.

Loves Dune	Popcorn (g)	Loves Hacksaw Ridge	Astrological sign
1	95	0	Aquarius
0	50	1	Virgo
1	100	0	Taurus
1	85	1	Gemini
0	60	1	Leo

Practical session – but why use R?

Practical session – but why use R?

Mental health and caregiving experiences of family carers supporting people with psychosis (Sin *et al*, 2021)

tinyurl.com/2as79xtv

Workshop Questions

Spend 5 minutes to skim through the Abstract and Table 1-3.

1. **What was the aim of the study?**
2. **What were the outcome and predictor variables?**
3. **Interpret the regression coefficients in Table 3.**

1. What was the aim of the study?

To explore the associations between demographic, carer characteristics, and mental health outcomes of family carers supporting an individual with psychosis.

2. What were the dependent and independent variables?

Dependent variable: Warwick-Edinburgh Mental Wellbeing Scale (WEMWBS); range 14-70, higher score better wellbeing

Independent variable: (9) age, gender, ethnicity, employment status, highest education level achieved, marital status, relationship with CfP, living arrangement, duration of care.

Workshop answers

3. Interpret the regression coefficients in Table 3.

e.g. Age of CfP
For every unit increase in age of CfP (1 year):

- **(Coefficient + CI)** WEMWBS on average slightly increases by 0.29 with a 95% CI 0.1 to 0.5, after adjusting for other variables in the model
- **(*p*-value)** there is a strong evidence (p<0.01) that this association is not caused by random chance

Next Steps

Resources:

YouTube: StatQuest
Book: R for Data Science (2nd Edition)
Courses: Imperial Graduate School, Coursera, DataCamp

Statistics fundamentals: histograms, probability distributions, hypothesis testing

Machine learning: regression, classification, clustering, dimensionality reduction

Learning Outcomes

1. **Define and explain** fundamental concepts of regression modelling.
   - Regression models contain one outcome and one or multiple predictors.
   - Regression modelling consists of fitting a line or curve to the data and calculating the R^2 and p-value.

2. **Formulate, apply, and compare** regression models based on a research question.
   - Formulate and apply bespoke lm(y ~ x) and glm(y ~ x, family = binomial) models.
   - Identify potential covariates or confounding variables that should be considered in a regression model.

3. **Estimate** regression coefficients using R and **interpret** them in the context of the question.
   - Assess the fit of a regression model using measures such as R-squared and adjusted R-squared.

4. **Interpret** regression model results from scientific papers.