Compliant Controller Formulation

List of Variables

$\triangle t$: delta timestep

Sensor Readings at Timestep t -
$\mathbf{q_s(t)}$: Joint Positions
$\mathbf{\dot{q}_s(t)}$: Joint Velocities
$\boldsymbol{\tau}_\mathbf{s}\mathbf{(t)}$: Joint Torques

Command Sent to Joints at Timestep t (through current control loop) -
$\boldsymbol{\tau}_\mathbf{c}\mathbf{(t)}$: Joint Torque

Constants -
$\mathbf{K_p}$: Joint Compliance Proportional Gain matrix
$\mathbf{K_d}$: Joint Compliance Derivative Gain matrix
$\mathbf{K_s}$: Joint Stiffness matrix
$\mathbf{K_r}$: Rotor Inertia matrix
$\mathbf{K_l}$: Friction Observer Matrix 1
$\mathbf{K_{lp}}$: Friction Observer Matrix 2
α : Low Pass Filter Coefficient

Nominal Terms -
$\mathbf{q_n}$: Nominal Joint Positions
$\mathbf{\dot{q}_n}$: Nominal Joint Velocities
$\mathbf{\ddot{q}_n}$: Nominal Joint Accelerations

$\mathbf{q_d}$: Desired Joint Positions
$\mathbf{\dot{q}_d}$: Desired Joint Velocities
$\boldsymbol{\tau}^\mathbf{f}_\mathbf{s}\mathbf{(t)}$: Filtered Joint Torques for Timestep t using a Low Pass Filter
$\boldsymbol{\tau}_\mathbf{task}\mathbf{(t)}$: Joint Torques for Task for Timestep t
$\mathbf{f(t)}$: Friction for Timestep t
$\mathbf{g(q)}$: Gravity at Joint Position q

Joint Space Compliant Control -

The following computations are done in order:

Filtering Torque Sensor Readings:

$$\boldsymbol{\tau}^\mathbf{f}_\mathbf{s}\mathbf{(t)} = \alpha\boldsymbol{\tau}^\mathbf{f}_\mathbf{s}\mathbf{(t-1)} + \mathbf{(1-}\alpha\mathbf{)}\boldsymbol{\tau}_\mathbf{s}\mathbf{(t)} \tag{1}$$

Compute Joint Torque for Task:

$$\boldsymbol{\tau}_\mathbf{task}\mathbf{(t+1)} = \mathbf{-K_p(q_n(t) - q_d - {K_s}^{-1}g(q_s(t))) - K_d(\dot{q}_n(t) - \dot{q}_d) + g(q_s(t))} \tag{2}$$

Nominal Motor Plant:

$$\mathbf{\ddot{q}_n(t+1)} = \mathbf{K_r^{-1}}(\boldsymbol{\tau}_\mathbf{task}\mathbf{(t+1)} - \boldsymbol{\tau}^\mathbf{f}_\mathbf{s}\mathbf{(t)}) \tag{3}$$
$$\mathbf{\dot{q}_n(t+1)} = \mathbf{\dot{q}_n(t)} + \mathbf{\ddot{q}_n(t+1)}\triangle t \tag{4}$$
$$\mathbf{q_n(t+1)} = \mathbf{q_n(t)} + \mathbf{\dot{q}_n(t+1)}\triangle t \tag{5}$$

Nominal Friction:

$$\mathbf{f(t+1) = K_r K_l((\dot{q}_n(t+1) - \dot{q}_s(t)) + K_{lp}(q_n(t+1) - q_s(t)))} \tag{6}$$

Torque Command:

$$\boldsymbol{\tau}_\mathbf{c}\mathbf{(t+1)} = \boldsymbol{\tau}_\mathbf{task}\mathbf{(t+1)} + \mathbf{f(t+1)} \tag{7}$$

Additional List of Variables

Constants -
$\mathbf{K_{tp}}$: Task Compliance Proportional Gain matrix
$\mathbf{K_{td}}$: Task Compliance Derivative Gain matrix
Nominal Terms -
$\mathbf{x_n}$: Nominal End Effector Pose
$\mathbf{J_n}$: Jacobian at Nominal End Effector Pose

$\mathbf{x_d}$: Desired End Effector Pose

Task Space Compliant Control -

For the task space compliant controller, computations (1) and (3)-(7) are the same as the joint space compliant controller, while the $\boldsymbol{\tau}_{task}$ computation changes to the following:

Compute Nominal End Effector Position using Forward Kinematics:

$$\mathbf{x_n} = FK(\mathbf{q_n}) \tag{2'}$$

Let the task space error calculated b/w $\mathbf{x_n}$ and $\mathbf{x_d}$ be $\mathbf{x_e}$. Compute Joint Torque for Task:

$$\boldsymbol{\tau}_{\mathbf{task}}(\mathbf{t+1}) = -\mathbf{J_n}^T(\mathbf{K_{tp}}\mathbf{x_e} + \mathbf{K_{td}}\mathbf{J_n}\dot{\mathbf{q}}_\mathbf{n}(\mathbf{t})) + \mathbf{g}(\mathbf{q_s}(\mathbf{t})) \tag{2.1'}$$