

Parsing Lab

Step by step solutions

Exercise 1

Given this grammar and lexicon

Lexicon
<i>Det</i> → <i>that</i> <i>this</i> <i>the</i> <i>a</i>
<i>Noun</i> → <i>book</i> <i>flight</i> <i>meal</i> <i>money</i>
<i>Verb</i> → <i>book</i> <i>include</i> <i>prefer</i>
<i>Pronoun</i> → <i>I</i> <i>she</i> <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i> <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i> <i>to</i> <i>on</i> <i>near</i> <i>through</i>

\mathcal{L}_1 in CNF
$S \rightarrow NP VP$
$S \rightarrow X1 VP$
$X1 \rightarrow Aux NP$
$S \rightarrow book \mid include \mid prefer$
$S \rightarrow Verb NP$
$S \rightarrow X2 PP$
$S \rightarrow Verb PP$
$S \rightarrow VP PP$
$NP \rightarrow I \mid she \mid me$
$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det Nominal$
$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal Noun$
$Nominal \rightarrow Nominal PP$
$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb NP$
$VP \rightarrow X2 PP$
$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$

Exercise 1

Parse the following sentence

Book	the	flight	through	Houston

Solution 1 – step 1

For cell (1, 1), look for rules of the form $* \rightarrow book$

<u>Book</u>	the	flight	through	Houston
Cell (1,1)				

Solution 1 – step 1.1

S -> book

<u>Book</u>	the	flight	through	Houston
S				

Solution 1 - step 1.2

Nominal -> book

<u>Book</u>	the	flight	through	Houston
S, Nominal				

Solution 1 - step 1.3

VP -> book

[illegible]

Solution 1 – step 1.4

Noun -> book

<u>Book</u>	the	flight	through	Houston
S, Nominal, VP, Noun				

Solution 1 – step 1.5

Verb -> book

<u>Book</u>	the	flight	through	Houston
S, Nominal, VP, Noun, Verb				

Solution 1 – step 2

For cell (2, 2), look for rules of the form * -> *the*

Book	<u>the</u>	flight	through	Houston
S, Nominal, VP, Noun, Verb				
	Cell (2,2)			

Solution 1 – step 2.1

Det -> the

Book	<u>the</u>	flight	through	Houston
S, Nominal, VP, Noun, Verb				
	Det			

Solution 1 – step 3

For cell (3, 3), look for rules of the form * -> flight

Book	the	<u>flight</u>	through	Houston
S, Nominal, VP, Noun, Verb				
	Det			
		Cell (3, 3)		

Solution 1 – step 3.1

Nominal -> flight

Book	the	<u>flight</u>	through	Houston
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal		

Solution 1 – step 3.2

Noun -> flight

Book	the	<u>flight</u>	through	Houston
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal, Noun		

Solution 1 – step 4

For cell (4, 4), Look for rules of the form * -> through

Book	the	flight	<u>through</u>	Houston
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal, Noun		
			Cell (4, 4)	

Solution 1 – step 4.1

Preposition -> through

Book	the	flight	<u>through</u>	Houston
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal, Noun		
			Preposition	

Solution 1 – step 5

For cell (5, 5), look for rules of the form * -> Houston

Book	the	flight	through	<u>Houston</u>
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal, Noun		
			Preposition	
				Cell (5, 5)

Solution 1 – step 5.1

NP -> Houston

Book	the	flight	through	<u>Houston</u>
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal, Noun		
			Preposition	
				NP

Solution 1 – step 5.2

Proper-Noun -> Houston

Book	the	flight	through	<u>Houston</u>
S, Nominal, VP, Noun, Verb				
	Det			
		Nominal, Noun		
			Preposition	
				NP, Proper-Noun

Solution 1 – step 6

For cell (1, 2), check cells (1,1) and (2,2)

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	Cell (1,2)			
	<u>Det</u>			
		Nominal, Noun		
			Preposition	
				NP, Proper-Noun

Solution 1 – step 6.1

No rule found! Eg, There is no rule of the form * -> S Det

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---			
	<u>Det</u>			
		Nominal, Noun		
			Preposition	
				NP, Proper-Noun

Solution 1 – step 7

For cell (2, 3), check cells (2,2) and (3,3)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---			
	<u>Det</u>	Cell (2, 3)		
		<u>Nominal, Noun</u>		
			Preposition	
				NP, Proper-Noun

Solution 1 – step 7.1

NP -> Det Nominal

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---			
	<u>Det</u>	NP		
		<u>Nominal, Noun</u>		
			Preposition	
				NP, Proper-Noun

Solution 1 – step 8

For cell (3, 4), check cells (3,3) and (4,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---			
	Det	NP		
		<u>Nominal, Noun</u>	Cell (3, 4)	
			<u>Preposition</u>	
				NP, Proper-Noun

Solution 1 – step 8.1

No rule found! Eg, no rule like * -> Nominal Preposition

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---			
	Det	NP		
		<u>Nominal, Noun</u>	---	
			<u>Preposition</u>	
				NP, Proper-Noun

Solution 1 – step 9

For cell (4, 5), check cells (4,4) and (5,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---			
	Det	NP		
		Nominal, Noun	---	
			<u>Preposition</u>	Cell (4,5)
				<u>NP, Proper-Noun</u>

Solution 1 – step 9.1

PP -> Preposition NP

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---			
	Det	NP		
		Nominal, Noun	---	
			<u>Preposition</u>	PP
				<u>NP, Proper-Noun</u>

Solution 1 – step 10

For cell (1, 3), first check (1,1) and (2,3)

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	Cell (1,3)		
	Det	<u>NP</u>		
		Nominal, Noun	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 10

then check (1,2) and (3,3)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	Cell (1,3)		
	Det	NP		
		<u>Nominal, Noun</u>	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 10.1

S -> Verb NP

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S		
	Det	<u>NP</u>		
		Nominal, Noun	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 10.2

VP -> Verb NP

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP		
	Det	<u>NP</u>		
		Nominal, Noun	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 10.3

X2 -> Verb NP

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2		
	Det	<u>NP</u>		
		Nominal, Noun	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 10.4

No rule found when we check (1,2) and (3,3)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	NP		
		<u>Nominal, Noun</u>	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 11

For cell (2, 4), first check (2, 2) and (3,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	<u>Det</u>	NP	Cell (2,4)	
		Nominal, Noun	---	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 11

then check (2, 3) and (4,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	<u>NP</u>	Cell (2,4)	
		Nominal, Noun	---	
			<u>Preposition</u>	PP
				NP, Proper-Noun

Solution 1 – step 11.1

No rule found when we check (2,2) and (3,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	<u>Det</u>	NP	---	
		Nominal, Noun	==	
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 11.2

No rule found when we check (2,3) and (4,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	<u>NP</u>	---	
		Nominal, Noun	---	
			<u>Preposition</u>	PP
				NP, Proper-Noun

Solution 1 – step 12

For cell (3,5), first check (3,3) and (4,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	NP	---	
		<u>Nominal, Noun</u>	---	Cell (3,5)
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 12

then check (3,4) and (5,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	NP	---	
		Nominal, Noun	==	Cell (3,5)
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 1 – step 12.1

Nominal -> Nominal PP

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	NP	---	
		<u>Nominal, Noun</u>	---	Nominal
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 12.2

No rule found when we check (3,4) and (5,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2		
	Det	NP	---	
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 1 – step 13

For cell (1,4), first check (1,1) and (2,4)

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2	Cell (1,4)	
	Det	NP	---	
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 13

then check (1,2) and (3,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	Cell (1,4)	
	Det	NP	---	
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 13

then check (1,3) and (4,4)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	<u>S, VP, X2</u>	Cell (1,4)	
	Det	NP	---	
		Nominal, Noun	---	Nominal
			<u>Preposition</u>	PP
				NP, Proper-Noun

Solution 1 – step 13.1

Check (1,1) and (2,4). No rule found!

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2	---	
	Det	NP	==	
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 13.2

Check (1,2) and (3,4). No rule found!

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	Det	NP	---	
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 13.3

Check (1,3) and (4,4). No rule found!

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	<u>S, VP, X2</u>	---	
	Det	NP	---	
		Nominal, Noun	---	Nominal
			<u>Preposition</u>	PP
				NP, Proper-Noun

Solution 1 – step 14

For cell (2,5), first check (2,2) and (3,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	<u>Det</u>	NP	---	Cell (2,5)
		Nominal, Noun	---	<u>Nominal</u>
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 14

then check (2,3) and (4,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	Det	<u>NP</u>	---	Cell (2,5)
		Nominal, Noun	---	Nominal
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 14

then check (2,4) and (5,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	Det	NP	==	Cell (2,5)
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 1 – step 14.1

Check (2,2) and (3,5). NP -> Det Nominal

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	<u>Det</u>	NP	---	NP
		Nominal, Noun	---	<u>Nominal</u>
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 14.2

Check (2,3) and (4,5). No rule found!

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	Det	<u>NP</u>	---	NP
		Nominal, Noun	---	Nominal
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 14.3

Check (2,4) and (5,5). No rule found!

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	
	Det	NP	==	NP
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 1 – step 15

For cell (1,5), first check (1,1) and (2,5)

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2	---	Cell (1,5)
	Det	NP	---	<u>NP</u>
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 15

then check (1,2) and (3,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	Cell (1,5)
	Det	NP	---	NP
		Nominal, Noun	---	<u>Nominal</u>
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 15

then check (1,3) and (4,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	<u>S, VP, X2</u>	---	Cell (1,5)
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 15

then check (1,4) and (5,5)

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	Cell (1,5)
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 1 – step 15.1

Check (1,1) and (2,5). S -> Verb NP

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2	---	S1
	Det	NP	---	<u>NP</u>
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 15.2

Check (1,1) and (2,5). VP -> Verb NP

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2	---	S1, VP
	Det	NP	---	<u>NP</u>
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 15.3

Check (1,1) and (2,5). X2 -> Verb NP

Book	the	flight	through	Houston
<u>S, Nominal, VP,</u> <u>Noun, Verb</u>	---	S, VP, X2	---	S1, VP, X2
	Det	NP	---	<u>NP</u>
		Nominal, Noun	---	Nominal
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 15.4

Check (1,2) and (3,5). No rule found!

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	S1, VP, X2
	Det	NP	---	NP
		Nominal, Noun	---	<u>Nominal</u>
			Preposition	PP
				NP, Proper-Noun

Solution 1 – step 15.5

Check (1,3) and (4,5). S → VP PP

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	<u>S, VP, X2</u>	---	S1, VP, X2, S2
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 15.6

Check (1,3) and (4,5). S -> X2 PP

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	<u>S, VP, X2</u>	---	S1, VP, X2, S2, S3
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	<u>PP</u>
				NP, Proper-Noun

Solution 1 – step 15.7

Check (1,4) and (5,5). No rule found!

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	S1, VP, X2, S2, S3
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 1 – final

The three S in (1,5) form three different parse trees.

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	S1 , VP, X2, S2 , S3
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Exercise 2

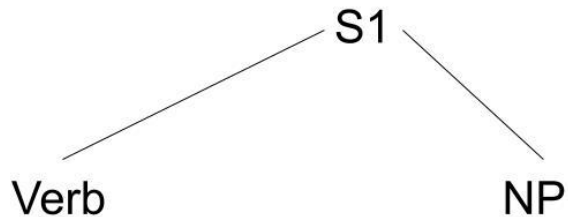
Retrieve the three parse trees from S1, S2 and S3

Book	the	flight	through	Houston
S, Nominal, VP, Noun, Verb	---	S, VP, X2	---	S1 , VP, X2, S2 , S3
	Det	NP	---	NP
		Nominal, Noun	---	Nominal
			Preposition	PP
				<u>NP, Proper-Noun</u>

Solution 2 – S1 – step 1

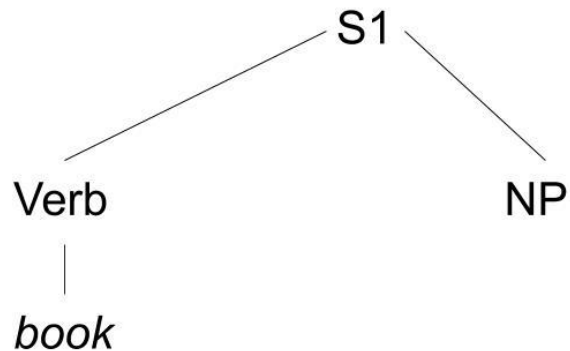
S1

Solution 2 – S1 – step 2



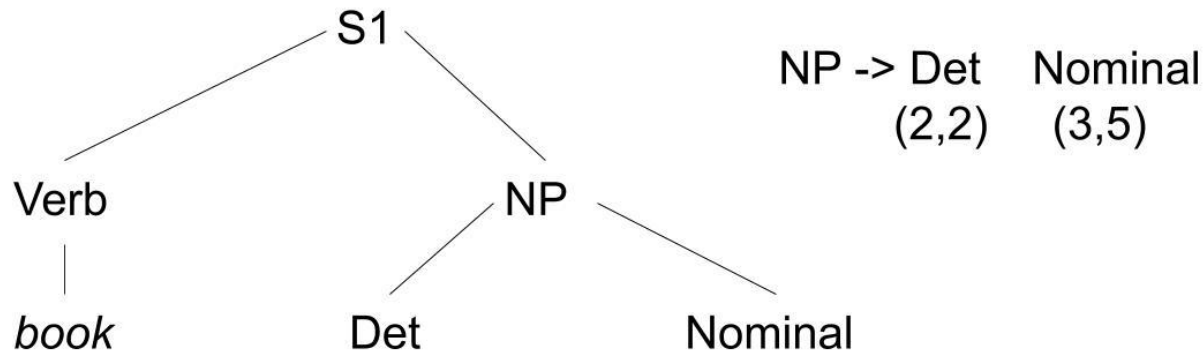
S -> Verb NP
 (1,1) (2,5)

Solution 2 – S1 – step 3

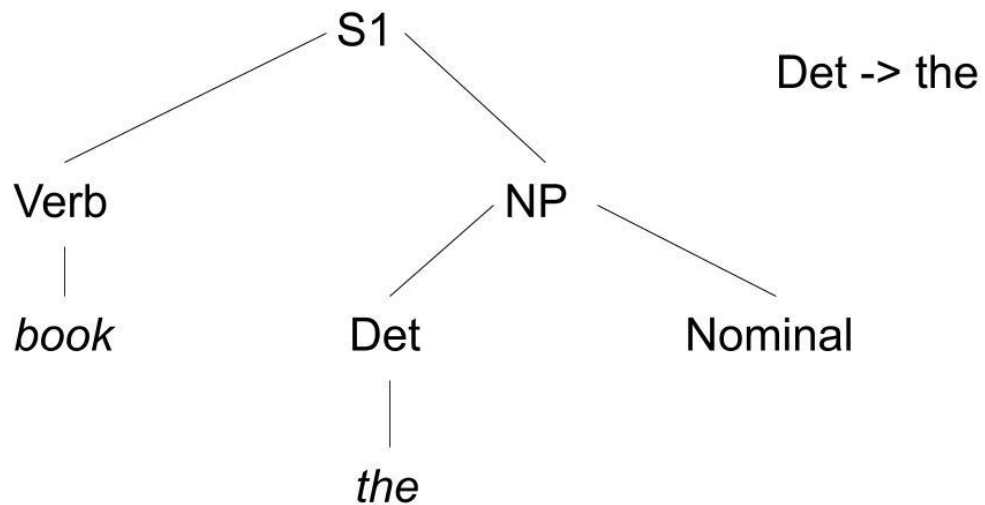


Verb -> book

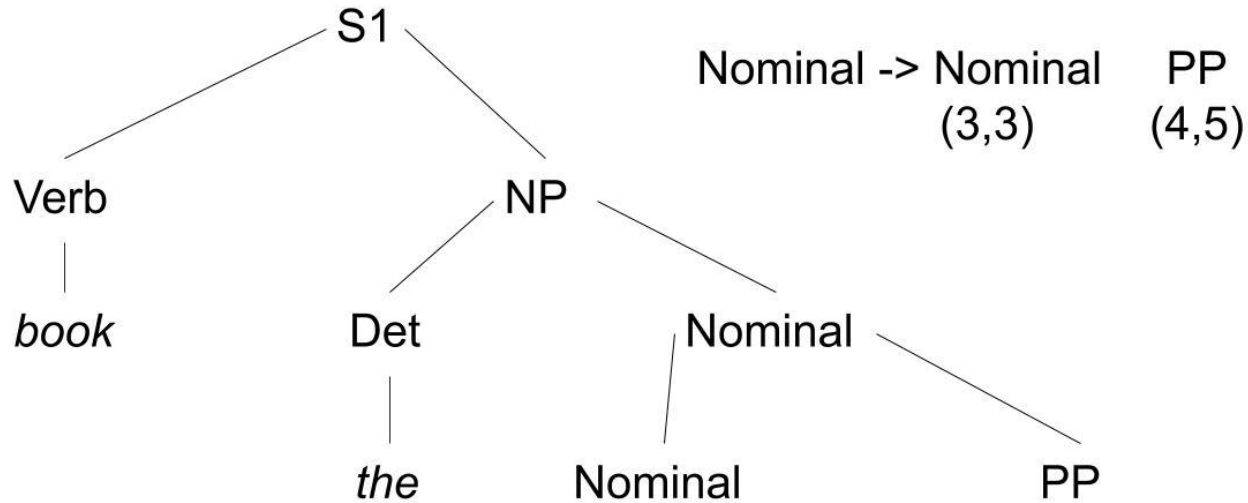
Solution 2 – S1 – step 4



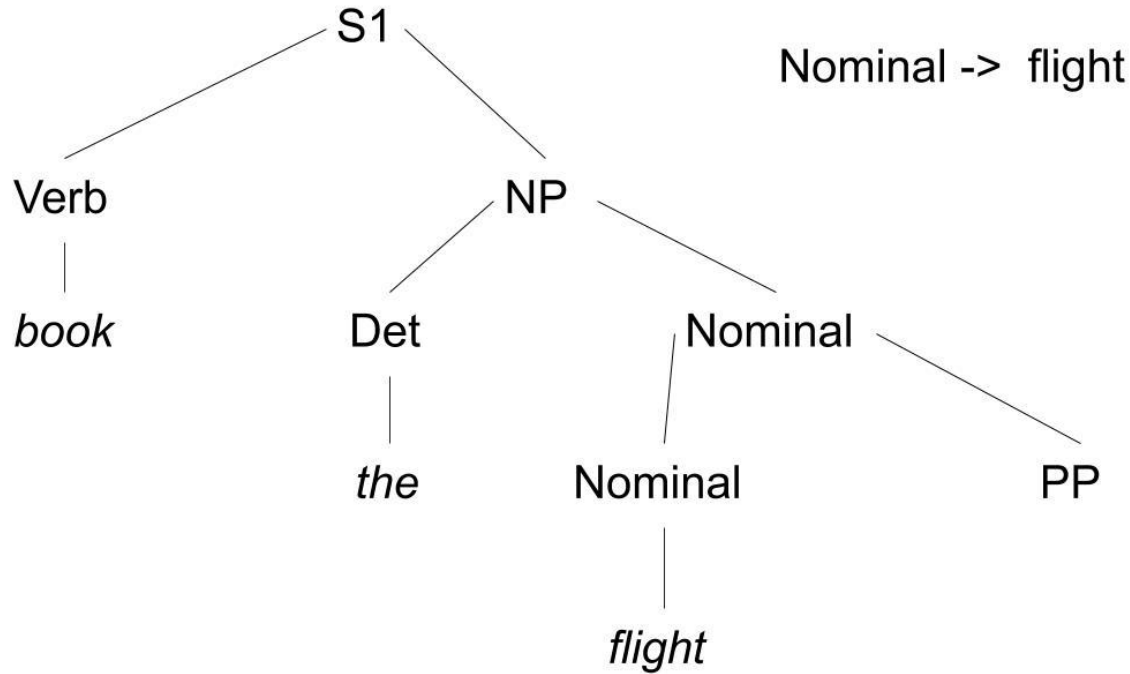
Solution 2 – S1 – step 5



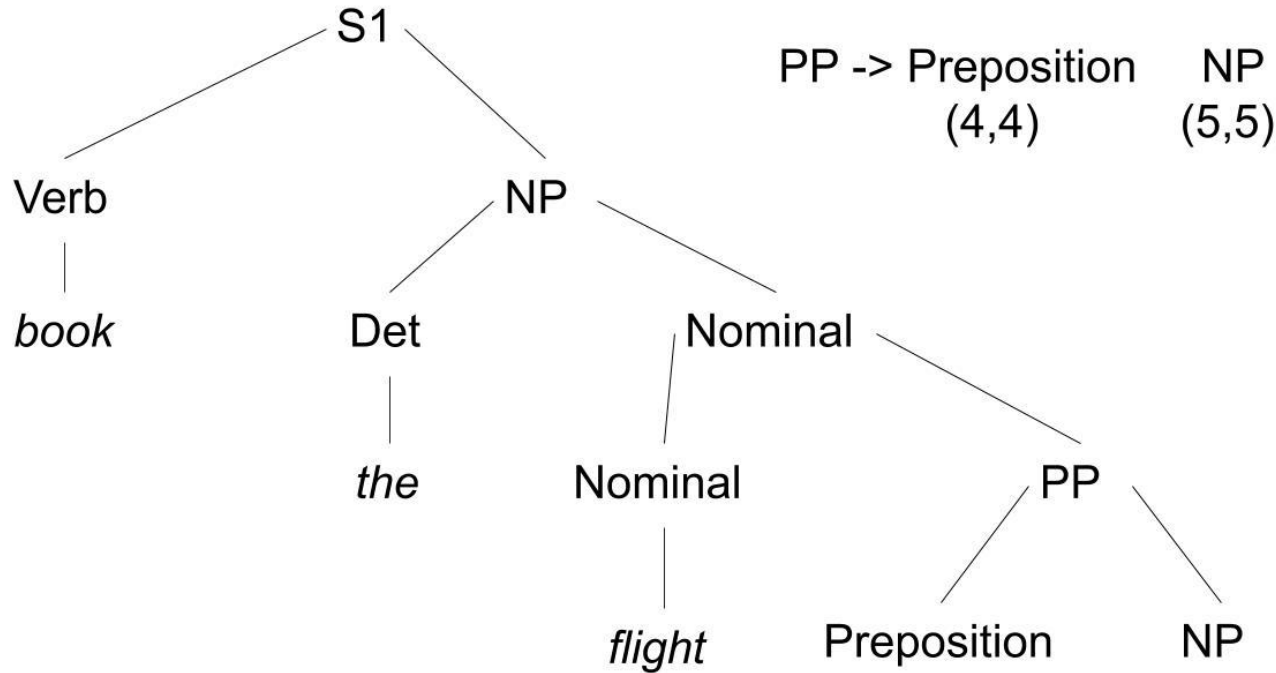
Solution 2 - S1 - step 6



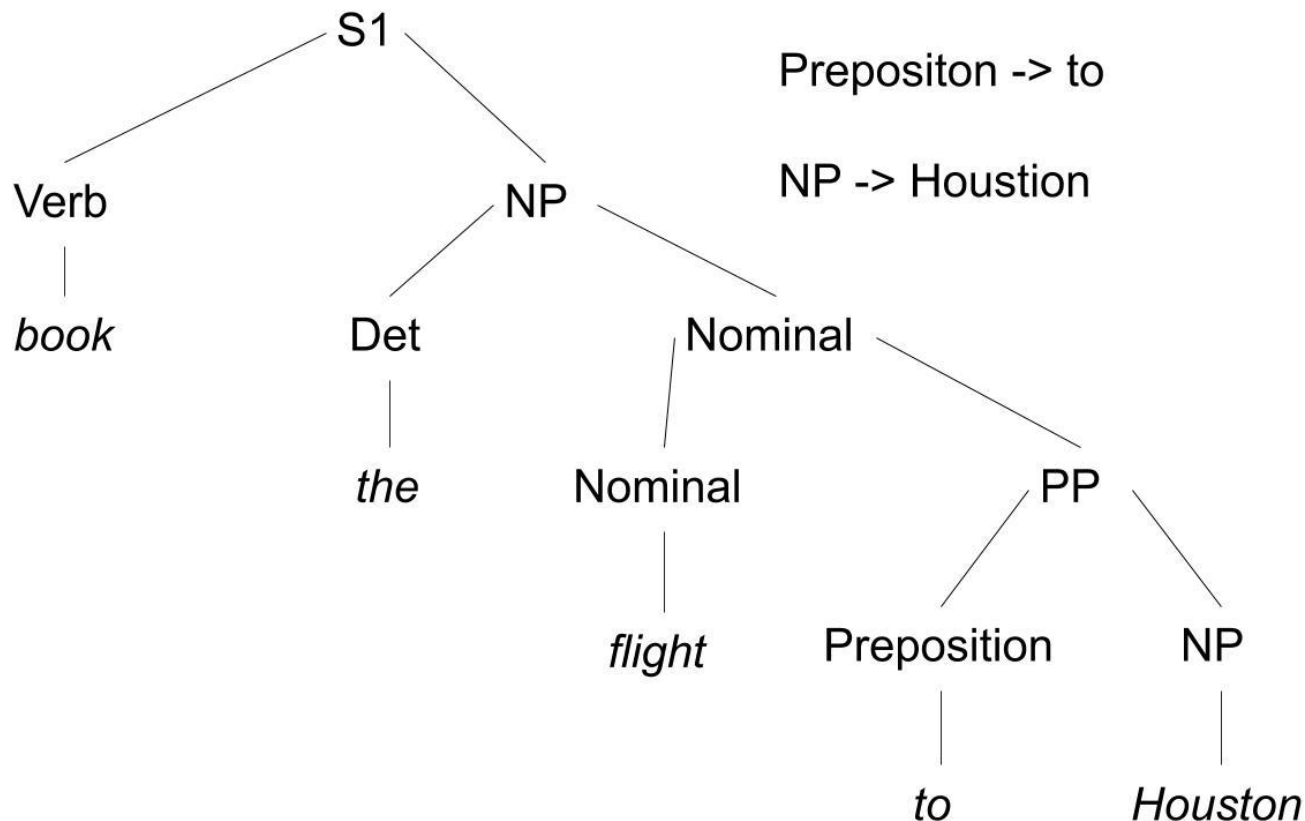
Solution 2 – S1 – step 7



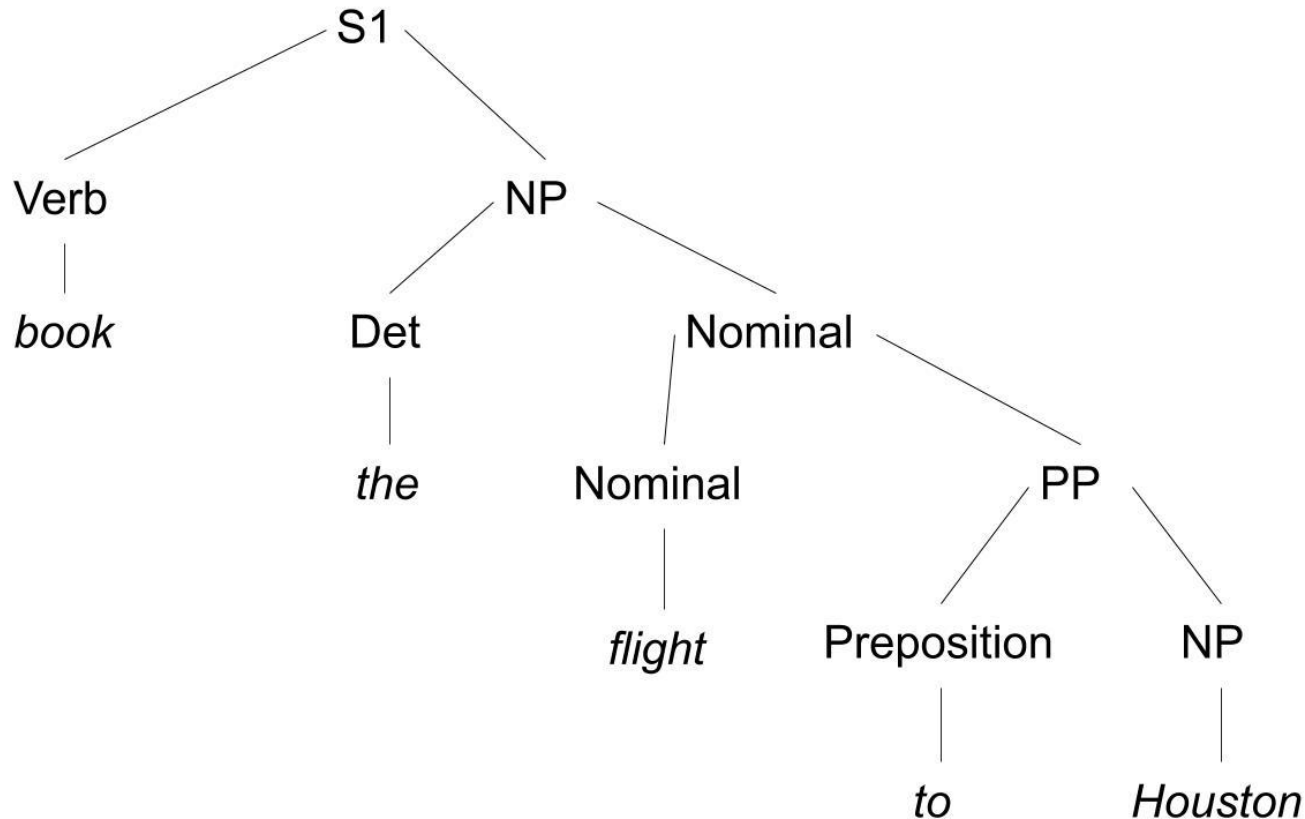
Solution 2 – S1 – step 8



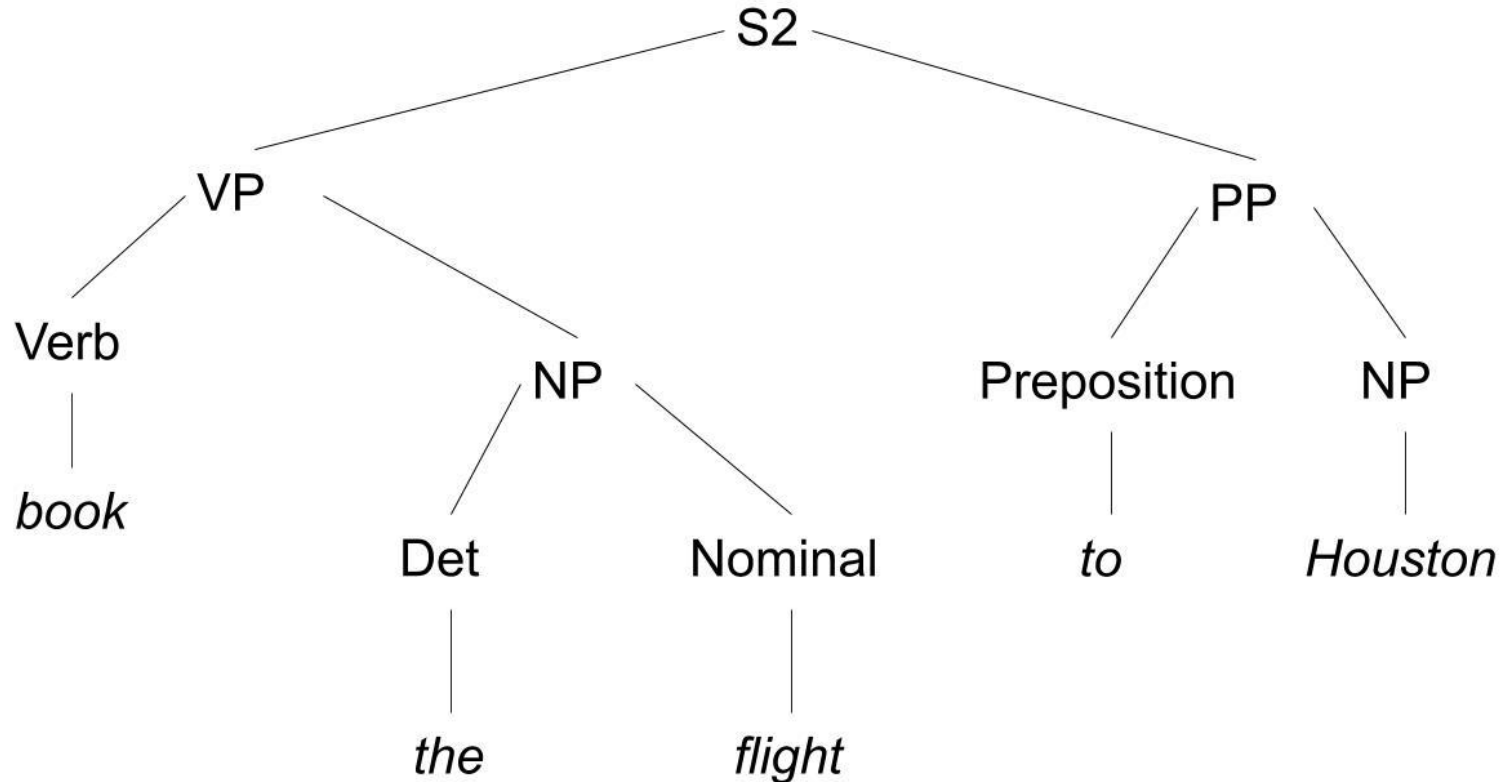
Solution 2 - S1 - step 9



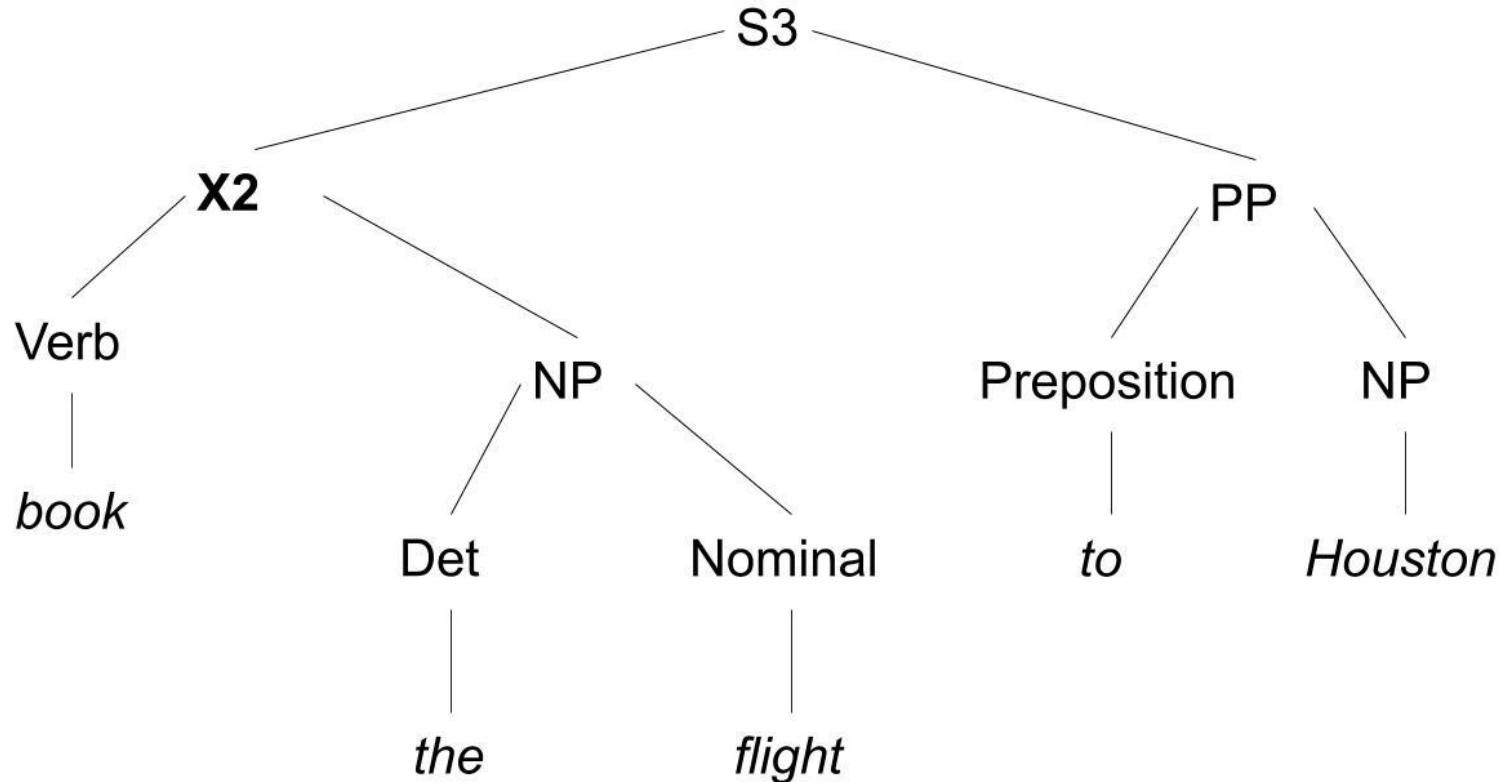
Solution 2 – S1 – final



Solution 2 - S2 - final



Solution 2 - S3 - final



Exercise 3

Given this corpus, show its complete PCFG:

```
( (S
  (NP-SBJ (DT The) (NN move))
  (VP (VBD followed)
    (NP
      (NP (DT a) (NN round))
      (PP (IN of)
        (NP
          (NP (JJ similar) (NNS increases))
          (PP (IN by)
            (NP (JJ other) (NNS lenders)))
          (PP (IN against)
            (NP (NNP Arizona) (JJ real) (NN estate) (NNS loans))))))
    (, ,)
  (S-ADV
    (NP-SBJ (-NONE- *))
    (VP (VBG reflecting)
      (NP
        (NP (DT a) (VBG continuing) (NN decline))
        (PP-LOC (IN in)
          (NP (DT that) (NN market))))))
  (. .)))
```

$T =$

$N =$

$S =$

$R =$

$q =$

Solution 3

$T = \{\text{the, move, followed, a, round, of, similar, increases, by, other, lenders, against, arizona, real, estate, loans, , , *, reflecting, continuing, decline, in, that, market, .}\}$

$N = \{\text{NP-SBJ, DT, NN, VP, VBD, NP, PP, IN, JJ, NNS, NNP, , , S-ADV, -NONE-, VBG, PP-LOC, .}\}$

$S = \{S\}$

Solution 3

$R = \{(\text{root} \rightarrow \text{S} .), (\text{S} \rightarrow \text{NP-SBJ VP}), (\text{NP-SBJ} \rightarrow \text{DT NN}), (\text{DT} \rightarrow \text{the}), (\text{NN} \rightarrow \text{move}),$
 $(\text{VP} \rightarrow \text{VBD NP} , \text{S-ADV}), (\text{VBD} \rightarrow \text{followed}), (\text{NP} \rightarrow \text{NP PP}), (\text{NP} \rightarrow \text{DT NN}), (\text{DT} \rightarrow \text{a}),$
 $(\text{NN} \rightarrow \text{round}), (\text{PP} \rightarrow \text{IN NP}), (\text{IN} \rightarrow \text{of}), (\text{NP} \rightarrow \text{JJ NNS}), (\text{JJ} \rightarrow \text{similar}), (\text{NNS} \rightarrow \text{increases}),$
 $(\text{IN} \rightarrow \text{by}), (\text{JJ} \rightarrow \text{other}), (\text{NNS} \rightarrow \text{lenders}), (\text{IN} \rightarrow \text{against}), (\text{NP} \rightarrow \text{NNP JJ NN NNS}),$
 $(\text{NNP} \rightarrow \text{Arizona}), (\text{JJ} \rightarrow \text{real}), (\text{NN} \rightarrow \text{estate}), (\text{NNS} \rightarrow \text{loans}), (, \rightarrow ,), (\text{S-ADV} \rightarrow \text{NP-SBJ VP}),$
 $(\text{NP-SBJ} \rightarrow \text{-NONE-}), (\text{VP} \rightarrow \text{VBG NP}), (\text{VBG} \rightarrow \text{reflecting}), (\text{NP} \rightarrow \text{NP PP-LOC}),$
 $(\text{NP} \rightarrow \text{DT VBG NN}), (\text{VBG} \rightarrow \text{continuing}), (\text{NN} \rightarrow \text{decline}), (\text{PP-LOC} \rightarrow \text{IN NP}), (\text{IN} \rightarrow \text{in}),$
 $(\text{DT} \rightarrow \text{that}), (\text{NN} \rightarrow \text{market}), (. \rightarrow .)\}$

Solution 3

For q,

$$P(\text{NP} \rightarrow \text{JJ NNS}) = \frac{\text{count}(\text{NP} \rightarrow \text{JJ NNS})}{\text{count}(\text{NP})} = 2/9$$

Solution 3

$$\begin{aligned} q = \{ & P(\text{root} \rightarrow S \text{ .}) = 1, P(S \rightarrow \text{NP-SBJ VP}) = 1, P(\text{NP-SBJ} \rightarrow \text{DT NN}) = \frac{1}{2}, P(\text{DT} \rightarrow \text{the}) = \frac{1}{4}, P(\text{NN} \rightarrow \text{move}) = \frac{1}{5}, \\ & P(\text{VP} \rightarrow \text{VBD NP, S-ADV}) = \frac{1}{2}, P(\text{VBD} \rightarrow \text{followed}) = 1, P(\text{NP} \rightarrow \text{NP PP}) = \frac{2}{9}, P(\text{NP} \rightarrow \text{DT NN}) = \frac{2}{9}, P(\text{DT} \rightarrow \text{a}) = \frac{2}{4}, \\ & P(\text{NN} \rightarrow \text{round}) = \frac{1}{5}, P(\text{PP} \rightarrow \text{IN NP}) = \frac{3}{3}, P(\text{IN} \rightarrow \text{of}) = \frac{1}{3}, P(\text{NP} \rightarrow \text{JJ NNS}) = \frac{2}{9}, P(\text{JJ} \rightarrow \text{similar}) = \frac{1}{3}, \\ & P(\text{NNS} \rightarrow \text{increases}) = \frac{1}{3}, P(\text{IN} \rightarrow \text{by}) = \frac{1}{3}, P(\text{JJ} \rightarrow \text{other}) = \frac{1}{3}, P(\text{NNS} \rightarrow \text{lenders}) = \frac{1}{3}, P(\text{IN} \rightarrow \text{against}) = \frac{1}{3}, \\ & P(\text{NP} \rightarrow \text{NNP JJ NN NNS}) = \frac{1}{9}, P(\text{NNP} \rightarrow \text{Arizona}) = 1, P(\text{JJ} \rightarrow \text{real}) = \frac{1}{3}, P(\text{NN} \rightarrow \text{estate}) = \frac{1}{5}, \\ & P(\text{NNS} \rightarrow \text{loans}) = \frac{1}{3}, P(, \rightarrow ,) = 1, P(\text{S-ADV} \rightarrow \text{NP-SBJ VP}) = 1, P(\text{NP-SBJ} \rightarrow \text{-NONE-}) = \frac{1}{2}, P(\text{VP} \rightarrow \text{VBG NP}) = \frac{1}{2}, \\ & P(\text{VBG} \rightarrow \text{reflecting}) = \frac{1}{2}, P(\text{NP} \rightarrow \text{NP PP-LOC}) = \frac{1}{9}, P(\text{NP} \rightarrow \text{DT VBG NN}) = \frac{1}{9}, P(\text{VBG} \rightarrow \text{continuing}) = \frac{1}{2}, \\ & P(\text{NN} \rightarrow \text{decline}) = \frac{1}{5}, P(\text{PP-LOC} \rightarrow \text{IN NP}) = 1, P(\text{IN} \rightarrow \text{in}) = \frac{1}{3}, P(\text{DT} \rightarrow \text{that}) = \frac{1}{4}, P(\text{NN} \rightarrow \text{market}) = \frac{1}{5}, P(. \rightarrow .) = 1 \} \end{aligned}$$

Exercise 4 – The CKY algorithm for PCFG

Example by Michael Collins

Given the grammar:

S	⇒	NP	VP	1.0
VP	⇒	Vi		0.4
VP	⇒	Vt	NP	0.4
VP	⇒	VP	PP	0.2
NP	⇒	DT	NN	0.3
NP	⇒	NP	PP	0.7
PP	⇒	IN	NP	1.0

Vi	⇒	sleeps	1.0
Vt	⇒	saw	1.0
NN	⇒	man	0.7
NN	⇒	woman	0.2
NN	⇒	telescope	0.1
DT	⇒	the	1.0
IN	⇒	with	0.5
IN	⇒	in	0.5

Generate the best parse tree for the sentence:

The woman saw the man with the telescope

Exercise 4 – The CKY algorithm for PCFG

Example by Michael Collins

Generate the best parse tree for the sentence:

The woman saw the man with the telescope

Solution 4

[illegible]

Solution 4 - step 1

[illegible]

DT -> the (1.0)

Solution 4 - step 2

[illegible]

NN -> woman (0.2)

Solution 4 - step 3

[illegible]

Solution 4 - step 4

[illegible]

Solution 4 - step 5

The	woman	saw	the	man	with	the	telescope
DT (1.0)							
	NN (0.2)						
		Vt (1.0)					
			DT (1.0)				
				NN (0.7)			

NN -> man (0.7)

Solution 4 - step 6

The	woman	saw	the	man	with	the	telescope
DT (1.0)							
	NN (0.2)						
		Vt (1.0)					
			DT (1.0)				
				NN (0.7)			
					IN (0.5)		

IN -> with (0.5)

Solution 4 - step 7

The	woman	saw	the	man	with	the	telescope
DT (1.0)							
	NN (0.2)						
		Vt (1.0)					
			DT (1.0)				
				NN (0.7)			
					IN (0.5)		
						DT (1.0)	

DT -> the (1.0)

Solution 4 - step 8

The	woman	saw	the	man	with	the	telescope
DT (1.0)							
	NN (0.2)						
		Vt (1.0)					
			DT (1.0)				
				NN (0.7)			
					IN (0.5)		
						DT (1.0)	
							NN (0.1)

NN -> telescope (0.1)

Solution 4 - step 9

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)						
	<u>NN (0.2)</u>						
		Vt (1.0)					
			DT (1.0)				
				NN (0.7)			
					IN (0.5)		
						DT (1.0)	
							NN (0.1)

NP -> DT NN (0.3)

$$\pi(1,2) = 0.3 \times 1.0 \times 0.2 = 0.06$$

Solution 4 – step 10

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)						
	<u>NN (0.2)</u>	---					
		<u>Vt (1.0)</u>					
			DT (1.0)				
				NN (0.7)			
					IN (0.5)		
						DT (1.0)	
							NN (0.1)

No rule found!

Solution 4 – step 11

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)						
	NN (0.2)	---					
		<u>Vt (1.0)</u>	---				
			<u>DT (1.0)</u>				
				NN (0.7)			
					IN (0.5)		
						DT (1.0)	
							NN (0.1)

No rule found!

Solution 4 - step 12

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)						
	NN (0.2)	---					
		Vt (1.0)	---				
			<u>DT (1.0)</u>	NP (0.21)			
				<u>NN (0.7)</u>			
					IN (0.5)		
						DT (1.0)	
							NN (0.1)

NP -> DT NN (0.3)

$$\pi(4,5) = 0.3 \times 1.0 \times 0.7 = 0.21$$

Solution 4 – step 13

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)						
	NN (0.2)	---					
		Vt (1.0)	---				
			DT (1.0)	NP (0.21)			
				<u>NN (0.7)</u>	---		
					<u>IN (0.5)</u>		
						DT (1.0)	
							NN (0.1)

No rule found!

Solution 4 – step 14

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)						
	NN (0.2)	---					
		Vt (1.0)	---				
			DT (1.0)	NP (0.21)			
				NN (0.7)	---		
					<u>IN (0.5)</u>	---	
						<u>DT (1.0)</u>	
							NN (0.1)

No rule found!

Solution 4 - step 15

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)						
	NN (0.2)	---					
		Vt (1.0)	---				
			DT (1.0)	NP (0.21)			
				NN (0.7)	---		
					IN (0.5)	---	
						<u>DT (1.0)</u>	NP (0.03)
							<u>NN (0.1)</u>

NP -> DT NN (0.3)

$$\pi(7,8) = 0.3 \times 1.0 \times 0.1 = 0.03$$

Solution 4 – step 16.1

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)	---					
	NN (0.2)	==					
		Vt (1.0)	---				
			DT (1.0)	NP (0.21)			
				NN (0.7)	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 16.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	<u>NP (0.06)</u>	---					
	NN (0.2)	---					
		<u>Vt (1.0)</u>	---				
			DT (1.0)	NP (0.21)			
				NN (0.7)	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 17.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	<u>NN (0.2)</u>	---	---				
		Vt (1.0)	==				
			DT (1.0)	NP (0.21)			
				NN (0.7)	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 17.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---				
			<u>DT (1.0)</u>	NP (0.21)			
				NN (0.7)	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 18.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		<u>Vt (1.0)</u>	---	VP (0.084)			
			DT (1.0)	<u>NP (0.21)</u>			
				NN (0.7)	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

VP -> Vt NP (0.4)

$$\pi(3,5) = 0.4 \times 1.0 \times 0.21 = 0.084$$

Solution 4 – step 18.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	==	VP (0.084)			
			DT (1.0)	NP (0.21)			
				<u>NN (0.7)</u>	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 19.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			<u>DT (1.0)</u>	NP (0.21)	---		
				NN (0.7)	---		
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 19.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	<u>NP (0.21)</u>	---		
				NN (0.7)	---		
					<u>IN (0.5)</u>	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 20.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				<u>NN (0.7)</u>	---	---	
					IN (0.5)	---	
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 20.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	
						<u>DT (1.0)</u>	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 21.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					<u>IN (0.5)</u>	---	PP (0.015)
						DT (1.0)	<u>NP (0.03)</u>
							NN (0.1)

PP -> IN NP (1.0)

$$\pi(6,8) = 1.0 \times 0.5 \times 0.03 = 0.015$$

Solution 4 – step 21.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---					
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							<u>NN (0.1)</u>

No rule found!

Solution 4 – step 22.1

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)	---	---				
	NN (0.2)	---	==				
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 22.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	<u>NP (0.06)</u>	---	---				
	NN (0.2)	---	---				
		Vt (1.0)	==	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 22.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---				
		Vt (1.0)	---	VP (0.084)			
			<u>DT (1.0)</u>	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 23.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	<u>NN (0.2)</u>	---	---	---			
		Vt (1.0)	---	<u>VP (0.084)</u>			
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 23.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	<u>NP (0.21)</u>	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 23.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)			
			DT (1.0)	NP (0.21)	---		
				<u>NN (0.7)</u>	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 24.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		<u>Vt (1.0)</u>	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 24.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	==	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	==	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 24.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	<u>VP (0.084)</u>	---		
			DT (1.0)	NP (0.21)	---		
				NN (0.7)	---	---	
					<u>IN (0.5)</u>	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 25.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			<u>DT (1.0)</u>	NP (0.21)	---	---	
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 25.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	<u>NP (0.21)</u>	---	---	
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 25.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	
					IN (0.5)	---	PP (0.015)
						<u>DT (1.0)</u>	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 26.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				<u>NN (0.7)</u>	---	---	---
					IN (0.5)	---	<u>PP (0.015)</u>
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 26.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	<u>NP (0.03)</u>
							NN (0.1)

No rule found!

Solution 4 – step 26.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---				
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							<u>NN (0.1)</u>

No rule found!

Solution 4 – step 27.1

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)	---	---	---			
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 27.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	<u>NP (0.06)</u>	---	---	S (.00504)			
	NN (0.2)	---	---	---			
		Vt (1.0)	---	<u>VP (0.084)</u>	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

S -> NP VP (1.0)

$$\pi(1,5) = 1.0 \times 0.06 \times 0.084 = 0.00504$$

Solution 4 – step 27.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	<u>NP (0.21)</u>	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 27.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	==	S (.00504)			
	NN (0.2)	---	---	---			
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				<u>NN (0.7)</u>	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 28.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	<u>NN (0.2)</u>	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 28.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 28.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	==	---	---		
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	==	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 28.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---		
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					<u>IN (0.5)</u>	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 29.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		<u>Vt (1.0)</u>	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 29.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	==	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	==	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 29.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	<u>VP (0.084)</u>	---	---	
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 29.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						<u>DT (1.0)</u>	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 30.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			<u>DT (1.0)</u>	NP (0.21)	---	---	---
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 30.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	<u>NP (0.21)</u>	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	<u>PP (0.015)</u>
						DT (1.0)	NP (0.03)
							NN (0.1)

NP -> NP PP (0.7)

$$\pi(4,8) = .7 \times .21 \times 0.015 = 0.002205$$

Solution 4 – step 30.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	<u>NP (0.03)</u>
							NN (0.1)

No rule found!

Solution 4 - step 30.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)			
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							<u>NN (0.1)</u>

No rule found!

Solution 4 – step 31.1

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	==		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 31.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	<u>NP (0.06)</u>	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 31.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	==	---	S (.00504)	---		
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	==	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 31.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	==	S (.00504)	---		
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	==	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 31.5

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	<u>S (.00504)</u>	---		
	NN (0.2)	---	---	---	---		
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					<u>IN (0.5)</u>	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 32.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	<u>NN (0.2)</u>	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 32.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 32.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	==	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	==	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 32.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	==	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	==	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 32.5

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						<u>DT (1.0)</u>	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 33.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		<u>Vt (1.0)</u>	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	<u>NP (.002205)</u>
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

VP -> Vt NP (0.4)

$$\pi(3,8) = .4 \times 1 \times .002205 = 0.000882$$

Solution 4 - step 33.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	==	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	==
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 33.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	<u>VP (0.084)</u>	---	---	VP (.000882) VP (.000252)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	<u>PP (0.015)</u>
						DT (1.0)	NP (0.03)
							NN (0.1)

VP -> VP PP (0.2)

$$\pi(3,8) = .2 \times .084 \times .015 = 0.000252$$

Solution 4 - step 33.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	<u>VP (0.084)</u>	---	---	VP (.000882) VP (.000252)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	<u>PP (0.015)</u>
						DT (1.0)	NP (0.03)
							NN (0.1)

$0.000252 < 0.000882$

So we keep the earlier VP (and its backpointers)

Solution 4 - step 33.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	<u>NP (0.03)</u>
							NN (0.1)

No rule found!

Solution 4 – step 33.5

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---		
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	<u>---</u>	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							<u>NN (0.1)</u>

No rule found!

Solution 4 – step 34.1

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	==	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 34.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	<u>NP (0.06)</u>	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 34.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	==	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	==	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 34.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	==	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	==	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 34.5

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	<u>S (.00504)</u>	---	---	
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 34.6

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	---	
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						<u>DT (1.0)</u>	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 35.1

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	<u>NN (0.2)</u>	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	<u>VP (.000882)</u>
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 35.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	<u>NP (.002205)</u>
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 35.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	==	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	==
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 35.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	<u>PP (0.015)</u>
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 35.5

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	==	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	<u>NP (0.03)</u>
							NN (0.1)

No rule found!

Solution 4 – step 35.6

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	
	NN (0.2)	---	---	---	---	<u>---</u>	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							<u>NN (0.1)</u>

No rule found!

Solution 4 – step 36.1

The	woman	saw	the	man	with	the	telescope
<u>DT (1.0)</u>	NP (0.06)	---	---	S (.00504)	---	---	---
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 36.2

The	woman	saw	the	man	with	the	telescope
DT (1.0)	<u>NP (0.06)</u>	---	---	S (.00504)	---	---	S (.00005292)
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	<u>VP (.000882)</u>
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

S -> NP VP (1.0)

$$\pi(1,8) = 1 \times .06 \times .000882 = .00005292$$

Solution 4 – step 36.3

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	==	---	S (.00504)	---	---	S (.00005292)
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	<u>NP (.002205)</u>
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 - step 36.4

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	==	S (.00504)	---	---	S (.00005292)
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	==
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 36.5

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	<u>S (.00504)</u>	---	---	S (.00005292)
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	<u>PP (0.015)</u>
						DT (1.0)	NP (0.03)
							NN (0.1)

No rule found!

Solution 4 – step 36.6

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	S (.00005292)
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	<u>NP (0.03)</u>
							NN (0.1)

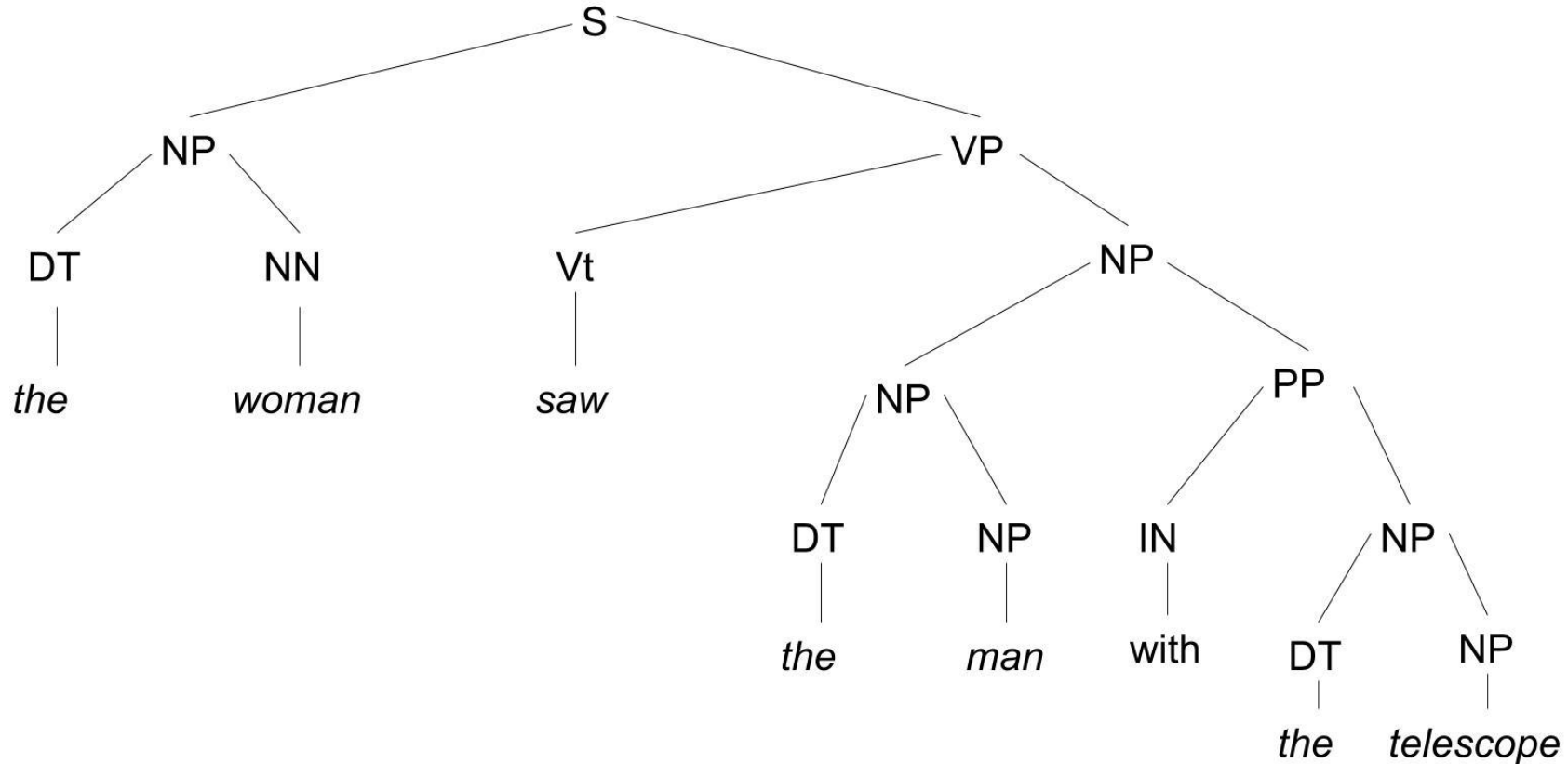
No rule found!

Solution 4 - step 36.7

The	woman	saw	the	man	with	the	telescope
DT (1.0)	NP (0.06)	---	---	S (.00504)	---	---	S (.00005292)
	NN (0.2)	---	---	---	---	---	---
		Vt (1.0)	---	VP (0.084)	---	---	VP (.000882)
			DT (1.0)	NP (0.21)	---	---	NP (.002205)
				NN (0.7)	---	---	---
					IN (0.5)	---	PP (0.015)
						DT (1.0)	NP (0.03)
							<u>NN (0.1)</u>

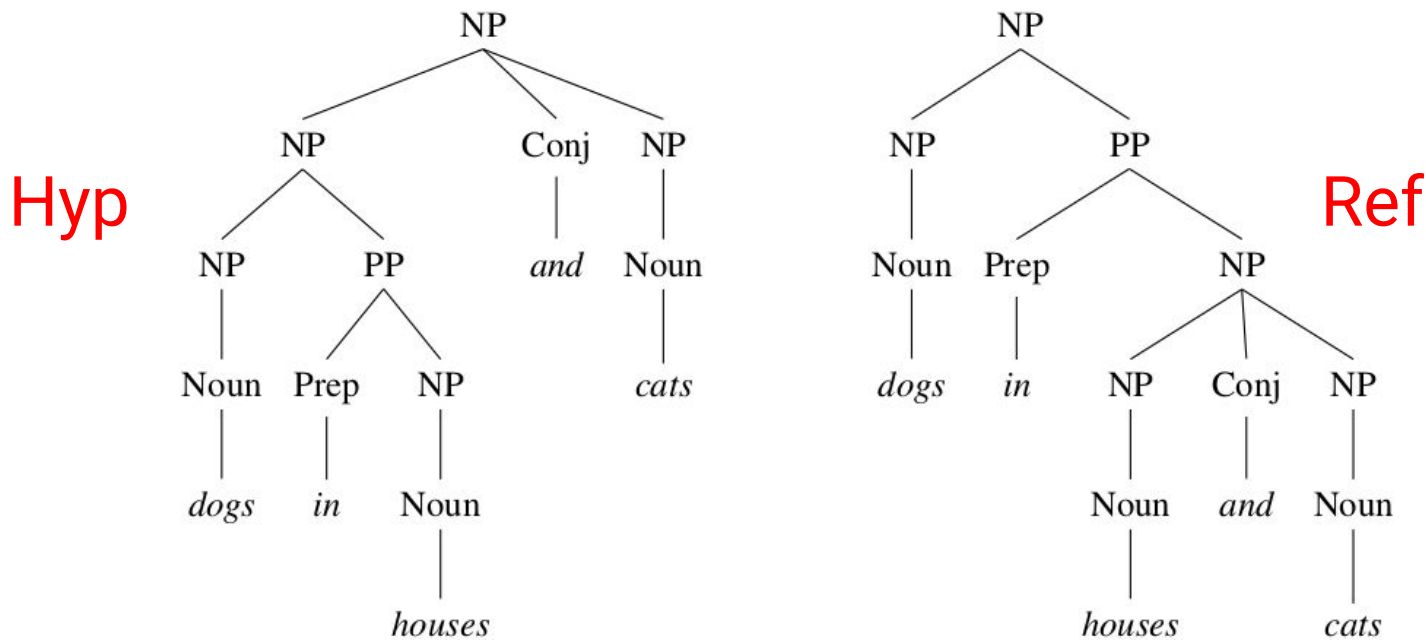
No rule found!

Solution 4 - final



Exercise 5

- Given the hyp and ref parse trees below, compute recall, precision and f-measure



Solution 5

Hypothesis

NP: dogs

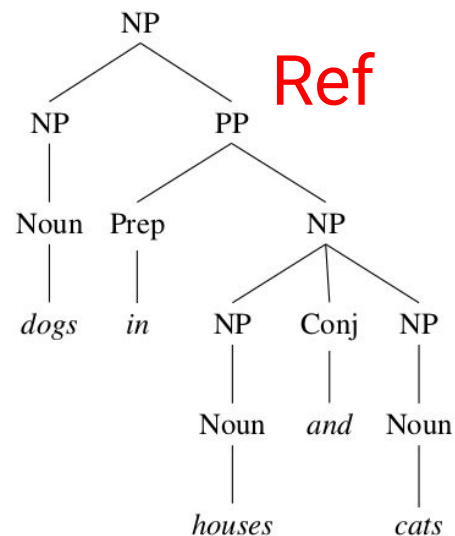
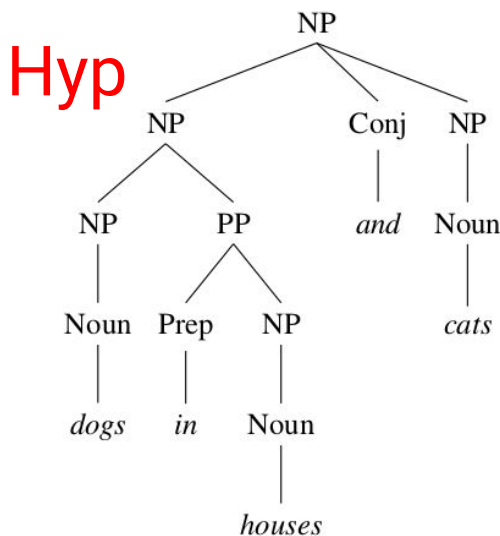
NP: houses

PP: in houses

NP: dogs in houses

NP: cats

NP: dogs in houses and cats



Reference

NP: dogs

NP: houses

PP: in houses and cats

NP: cats

NP: houses and cats

NP: dogs in houses and cats

Solution 5 – precision

Hypothesis

NP: **dogs**

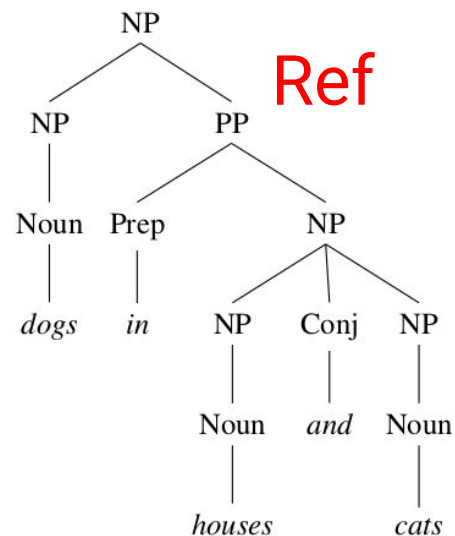
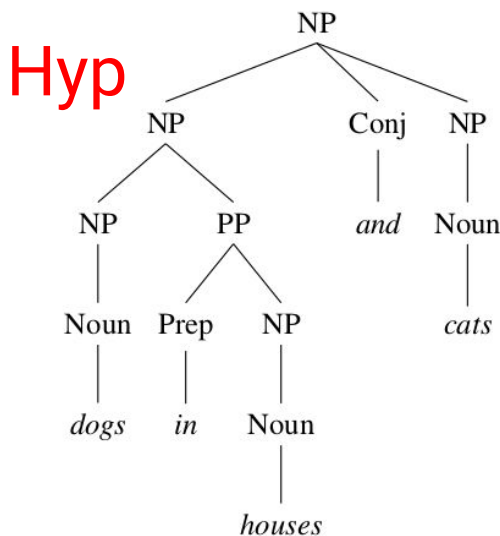
NP: **houses**

PP: in houses

NP: dogs in houses

NP: **cats**

NP: **dogs in houses and cats**



Reference

NP: dogs

NP: houses

PP: in houses and cats

NP: cats

NP: houses and cats

NP: dogs in houses and cats

Precision = **4** / 6

Solution 5 – recall

Hypothesis

NP: dogs

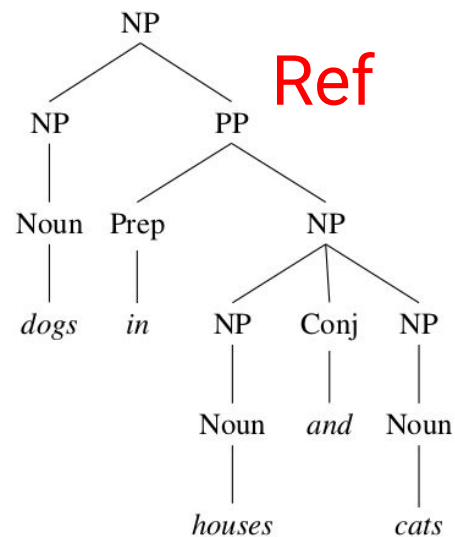
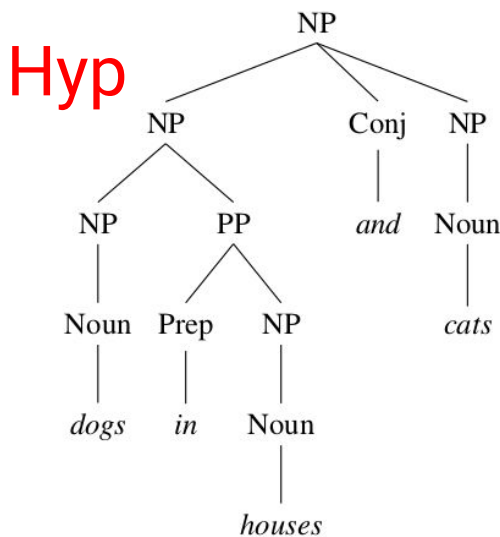
NP: houses

PP: in houses

NP: dogs in houses

NP: cats

NP: dogs in houses and cats



Reference

NP: **dogs**

NP: **houses**

PP: in houses and cats

NP: **cats**

NP: houses and cats

NP: **dogs in houses and cats**

$$\text{Recall} = 4 / 6$$

Solution 5 - F-measure

Hypothesis

NP: dogs

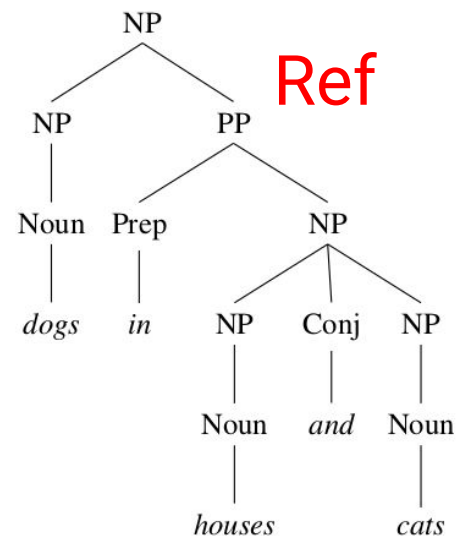
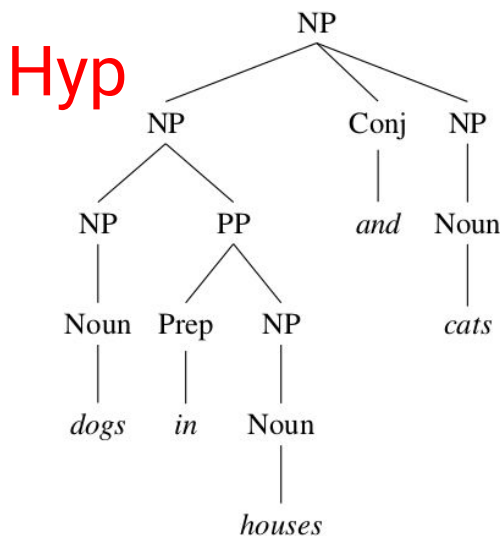
NP: houses

PP: in houses

NP: dogs in houses

NP: cats

NP: dogs in houses and cats



Reference

NP: dogs

NP: houses

PP: in houses and cats

NP: cats

NP: houses and cats

NP: dogs in houses and cats

$$\text{F-measure} = \frac{2(4/6)(4/6)}{4/6 + 4/6} = 4/6$$

Exercise 6 – Lexicalised CFG

Given the following CFG grammar, convert it into LCFG

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow VP PP$

$PP \rightarrow P NP$

$P \rightarrow \text{with}$

$V \rightarrow \text{saw}$

$NP \rightarrow NP PP$

$NP \rightarrow \text{astronomers}$

$NP \rightarrow \text{ears}$

$NP \rightarrow \text{saw}$

$NP \rightarrow \text{stars}$

$NP \rightarrow \text{telescope}$

Solution 6

For terminals,

NP(astronomers) \rightarrow astronomers

NP(ears) \rightarrow ears

NP(saw) \rightarrow saw

P(with) \rightarrow with

NP(stars) \rightarrow stars

V(saw) \rightarrow saw

NP(telescope) \rightarrow telescope

Solution 6

For non-terminals,

$S(v) \rightarrow_2 NP(n) VP(v)$ \forall n and v

$NP(n) \rightarrow_1 NP(n) PP(p)$ \forall n and p

$VP(v) \rightarrow_1 V(v) NP(n)$ \forall n and v

$VP(v) \rightarrow_1 VP(v) PP(p)$ \forall n and v

$PP(p) \rightarrow_1 P(p) NP(n)$ \forall n and p

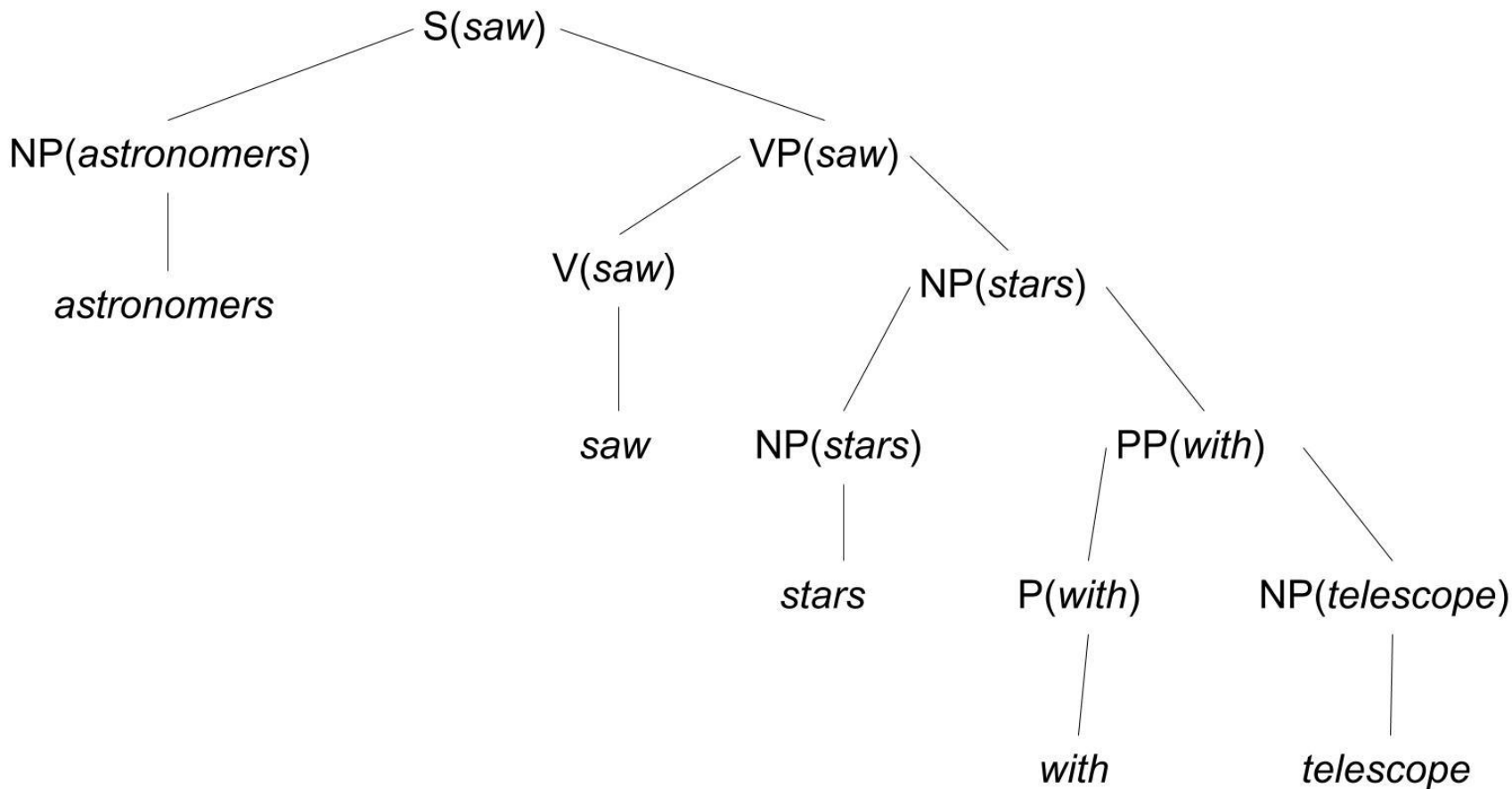
Solution 6 - example

astronomers saw stars with telescope

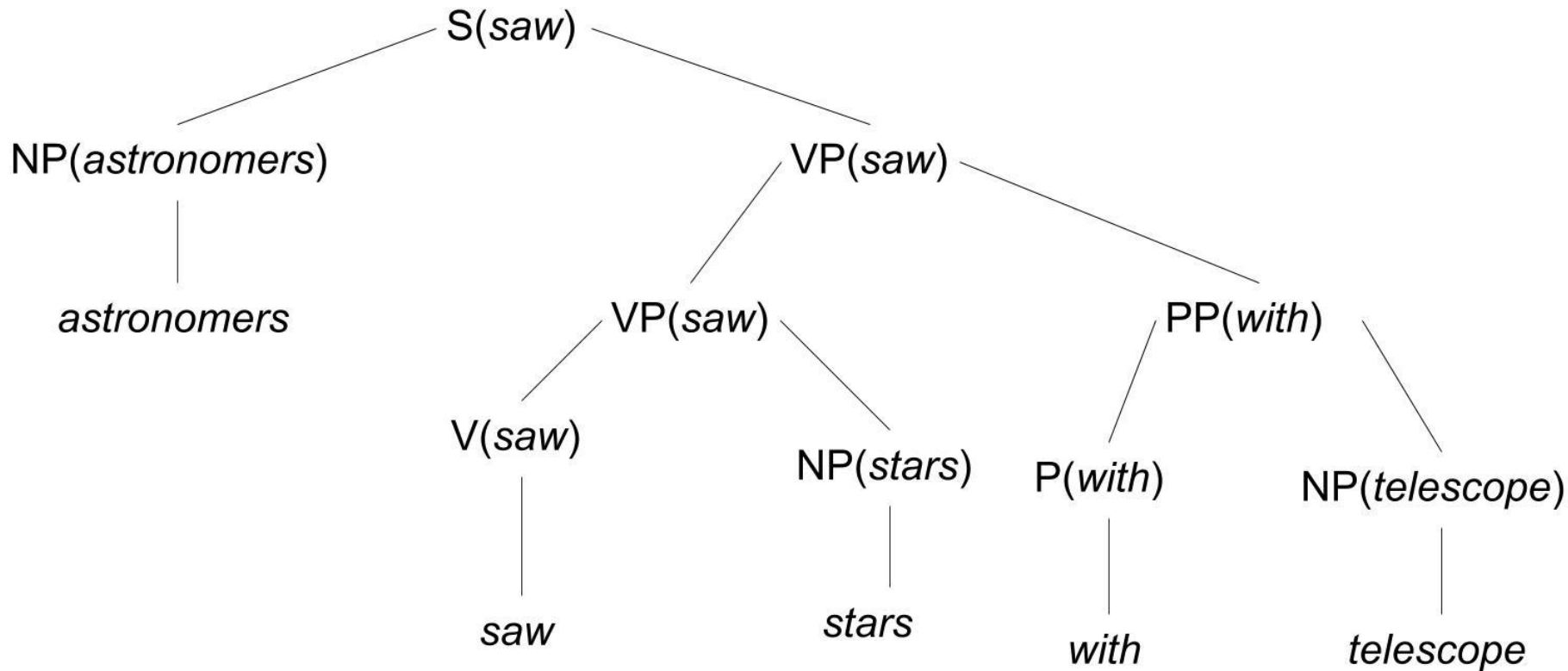
Solution 6 – example

We get two parse trees according to LCFG

Solution 6 - example - 1



Solution 6 - example - 2



Solution 6 – example

If we have a Probabilistic LCFG, then we can identify which of the two solutions is the most probable using the same dynamic programming algorithm used in Exercise 4.