Homework # 1: Probability Theory

Rohan Kapur, ECPE 293A

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1. (a) We know that for each packet received, $P(a) = 2P(v)$ and $P(a) + P(v) = 1$. Therefore, $P(a) + P(v) = 2P(v) + P(v) = 3P(v) = 1 \rightarrow P(v) = \frac{1}{3} \rightarrow P(a) = \frac{2}{3}$. We need to find P(group discarded | 2nd packet is audio). The group is discarded if there are no video packets in the group. So by only considering the groups where the second packet is audio , we get:

$$P(\text{group discarded} \mid \text{2nd packet is audio}) = P(\{aaa\}) = \left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$$

   (b) Now by considering all $2^3 = 8$ groups of 3 packets in general, we get

$$P(\text{No video packets}) = P(\{aaa\}) = \left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$$

   the same answer as in part (a).

   (c) Again, by considering all possible groups of 3 packets, we get

$$P(\text{exactly 1 video packet}) = P(\{vaa, ava, aav\}) = \frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \boxed{\frac{12}{27}}$$

2. We know that in the general population, P(heart disease)=0.08, P(high cholesterol | heart disease)=0.7 and P(high cholesterol)=0.3. Given a patient with high cholesterol, we want to find their chance of having heart disease, i.e. **P(heart disease | high cholesterol)**. By applying conditional probability laws, we get:

$$P(\text{heart disease} \mid \text{high cholesterol}) = \frac{P(\text{high cholesterol} \mid \text{heart disease})P(\text{heart disease})}{P(\text{high cholesterol})}$$
$$= \frac{(0.7)(0.08)}{0.3} \approx 0.187 \rightarrow \boxed{18.7\%}$$

So the patient with high cholesterol has only an 18.7% chance of having heart disease, or a **81.3%** chance of not having it, making them **not at risk of heart disease**.