ECEN 447, Fall 2021

Texas A&M University

Electrical and Computer Engineering Department

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Homework #4

Topic: Spatial domain enhancement, frequency domain enhancement

Due on October 28, 2021 6:29 pm

Instructions for the hand-written homework:

For the homework, you will be asked some basic concepts covered in your lectures, or to solve some problems based on a certain application in image processing. Most of the problems will be adapted from those solved during the lab. Please make sure to include necessary steps in your solutions. And we will evaluate your understanding of the problem in addition to the final results.

How to submit:

For hand-written homework problems, submit them on CANVAS. Please number the problems you are solving and clearly include steps necessary in your solutions.

PROBLEM (1)

Given a matrix $\mathrm{M} = \begin{bmatrix} 1 & 2 & 9 \\ 5 & 0 & 8 \\ 3 & 6 & 4 \end{bmatrix}$, please do the following calculations:

(1). Append 0s around the matrix, and write out the resulting 5×5 matrix

(2). Apply a 3×3 median filter to the matrix you get from (1), what is the result?

(3). Again append 0s around the result you get from (2), this time please try to apply a 3×3 mean filter, what can you get?

(4). Do you expect to get the same result if you reverse the order (i.e. first apply the mean filter and then apply the median filter; for this question you can just use Matlab to help you calculate if you are interested in the result)? Please include necessary explanations.

1) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 9 & 0 \\ 0 & 5 & 0 & 8 & 0 \\ 0 & 3 & 6 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 2 \\ 0 & 3 & 0 \end{bmatrix}$

3) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 6 & 8 & 7 \\ 9 & 11 & 10 \\ 8 & 10 & 9 \end{bmatrix}$

4) I would not expect the same result as the median filter is not a linear operator. In fact, with the 0 padding I know that each of the corners of my final answer will be 0.

PROBLEM (2)

Consider a 3×3 spatial mask that subtracts the sum of the four closest neighbors of a point (x,y) from four times the point itself.

Find the equivalent filter $H(u,v)$ in the frequency domain.

$$h(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$H(u,v) = \sum\sum h(x,y)\, e^{-j2\pi\frac{xu}{N}} e^{-j2\pi\frac{yv}{N}}$$

$$H(u,v) = \begin{bmatrix} -1,1 & 0 & 1 & 1 \\ -1,0 & 0,0 & 1\,0 & \\ -1\,-1 & 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & 6 \\ 3 & 0 & 3 \\ 6 & 3 & 6 \end{bmatrix}$$

$$\sum\sum h(x,y)\, e^{-j\frac{2\pi nx}{3}} e^{-j\frac{2\pi my}{3}}$$

$$\cos\frac{2\pi}{3} = \cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\sin\left(\frac{2\pi}{3}\right) = j\frac{\sqrt{3}}{2}$$

$$-1e^{-j\frac{2\pi}{3}} - 1e^{-j\frac{2\pi}{3}} + 4 - 1e^{+j\frac{2\pi}{3}} - 1e^{j\frac{2\pi}{3}}$$

$$= 4 - 2e^{-j\frac{2\pi}{3}} - 2e^{j\frac{2\pi}{3}} = 6$$

$$-1e^{-j\frac{2\pi}{3}} - 1 + 4 - 1 - 1e^{j\frac{2\pi}{3}}$$

$$= 3$$

$$-1e^{j\frac{2\pi}{3}} - 1e^{j\frac{2\pi}{3}} + 4 - 1e^{-j\frac{2\pi}{3}} - 1e^{j\frac{2\pi}{3}}$$

$$= 6$$

$$-1 - 1e^{-j\frac{2\pi}{3}} + 4 - 1e^{+j\frac{2\pi}{3}} - 1$$

$$= 3$$

$-1 \quad -1 \quad +4 \quad -1 \quad -1$

$= 0$

$-1 \quad -1e^{j\frac{2\pi}{3}} \quad +4 \quad -1e^{-j\frac{2\pi}{3}} \quad -1$

$= 3$

$-1e^{+j\frac{2\pi}{3}} \quad -1e^{-j\frac{2\pi}{3}} \quad +4 \quad -1e^{j\frac{2\pi}{3}} \quad -1e^{-j\frac{2\pi}{3}}$

$= 6$

$-1e^{\frac{2j\pi}{3}} \quad -1 \quad +4 \quad -1 \quad -1e^{-j\frac{2\pi}{3}}$

$= 3$

$-1e^{\frac{2j\pi}{3}} \quad -1e^{j\frac{2\pi}{3}} \quad +4 \quad -1e^{\frac{j2\pi}{3}} \quad -1e^{j\frac{2\pi}{3}}$

$= 6$

PROBLEM (3)

Given a matrix $M = \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix}$, please calculate its Discrete Fourier Transform and be sure to clearly include necessary steps in the calculation. You can use Matlab to check if you are correct when you finish it.

$M = \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix}$ $\quad f[M] = \begin{bmatrix} 13 & -7 \\ -7 & 5 \end{bmatrix}$

u, v

0, 0 $\quad 1+2+2+8$

1, 0 $\quad 1+(2+8)e^{-j\pi} + 2$

0, 1 $\quad 1+(2+8)e^{-j\pi} + 2$

1, 1 $\quad 1+(2+2)e^{-j\pi} + 8$