Hw 9 4.6 4.7 4.10 Finlay 8.7 8.10(a only)

4.6

If $P(j,j) > 0$ then it is

always possible to go back to the same state in 1 time. This means that the time to return can be 1 which means $\delta = 1$. Therefore j is recurrent aperiodic. For a finite irreducible MC, all recurrent states must be the same type. ∴ All states are aperiodic.

4.7 $t_0 = 0$ $t_1 = 1$ $t_2 = 2$ $\lambda = 4$

If State < 3 then = Demand gotten

New State = $8 - n$ p_n

or

New State = 0 if $n \geq 8$ with prob r_n

If State > 3

New state = $2 \min(\text{State} - 4, 0)$ p_n or r_{State}

Newstate > State prob 0

4.10

P_i = Prob that it lasts i more time

$$P\{Y_{n+1} = i-1 \mid Y_n = i\} = P\{\text{Lifetime} > n \mid n = i\}$$

$$= P\left\{\frac{\text{Lifetime} > n+1}{\text{Lifetime} > n}\right\} = \left\{\frac{\sum_{i=n+2}^{\infty} P_i}{\sum_{i=n+1}^{\infty} P_i}\right\}$$

$$= \left\{\frac{1 - \sum_{i=0}^{n+1} P_i}{1 - \sum_{i=0}^{n} P_i}\right\} = \begin{cases} 1 - P_1 & \text{if } i = 0 \\ \dfrac{1 - P_1 - \cdots - P_{i+1}}{1 - P_1 - \cdots - P_i} & i \geq 1 \end{cases}$$

b) Avg can't increase by more than 1

Prob you go to next state $= q_i$

Prob go to state $0 = 1 - q_i$

$$P = \begin{bmatrix} 1-q_0 & q_0 & 0 & \cdots \\ 1-q_1 & 0 & q_1 & 0 \\ \vdots & 0 & & q_2 \\ \vdots & 0 & & \end{bmatrix}$$

c) 0 is recurrent aperiodic non null

States at infinity become null.

8.10 $\lim_{n\to\infty} \frac{1}{n+1} \sum_{m=0}^{n} E_i[f(X_m)]$ Average X_i from 0 $\Rightarrow \infty$

$= \lim_{n\to\infty} E_i\left[\frac{1}{n+1}\sum_{m=0}^{n} f(X_m)\right] = E_i[f(\tilde{X}_m)]$

$= 0.2(3) - 1(0.1) + 2(0.3) - 5(0.2) + 3(0.2)$

$= .6 - .1 + .6 - 1 + .6 = \boxed{0.7}$

8.7

$$\sum_{x \in S} p^n(x,y) = 1 \quad \lim n \to \infty$$

$|S| = m$

$$= \sum_{x \in S} \pi(y) = 1 \qquad |S|\pi(y) = 1$$

$$\pi(y) = \frac{1}{|S|} = \frac{1}{m}.$$

Problem 6

row y = Col y

$\vec{V}_{y\cdot 0}$

$$\begin{bmatrix} 0 & P_{01} & \cdots & \cdots & \cdots \\ P_{01} & 0 & P_{13} & \cdots & \cdots \\ P_{02} & P_{12} & 0 & \cdots & \cdots \\ P_{63} & P_{13} & P_{23} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 1 & 1 & 1 & 1 & \cdots \end{bmatrix}$$

$\sum \text{row } y = 1 \Rightarrow \sum \text{Cols} = 1$ which means the matrix is doubly markov.

$\pi(y) = \frac{1}{n}$ for doubly markov M.