Ecen 601 Homework Assignment 1

1. Which of the following expressions are statements?

   (a) Today is a nice day.
   (b) Go to sleep.
   (c) Is it going to snow tomorrow?
   (d) The U.S. has 49 states.
   (e) I like to eat fruit, and you often think about traveling to Spain.
   (f) If we go out tonight, the babysitter will be unhappy.
   (g) Call me on Thursday if you are home.

A statement or proposition is the content of an assertion that is either true or false but not both.

1. a) statement b) not a statement c) not a statement
   d) statement e) statement f) statement g) not a statement

2. Let X = "Fred has red hair," let Y = "Fred has a big nose" and R = "Fred likes to eat figs." Translate the following statements into symbols.

   (a) Fred does not like to eat figs.
   (b) Fred has red hair, and does not have a big nose.
   (c) Fred has red hair or he likes to eat figs.
   (d) Fred likes to eat figs, and he has red hair or he has a big nose.
   (e) Fred likes to eat figs and he has red hair, or he has a big nose.
   (f) It is not the case that Fred has a big nose or he has red hair.
   (g) It is not the case that Fred has a big nose, or he has red hair.
   (h) Fred has a big nose and red hair, or he has a big nose and likes to eat figs.

2a) $\neg R$ b) $X \wedge (\neg Y)$ c) $X \vee R$ d) $R \wedge (X \vee Y)$
e) $(X \wedge R) \vee Y$ f) $\neg(Y \vee X)$ g) $(\neg Y) \vee X$ h) $(Y \wedge X) \vee (Y \wedge R)$

3. Let E = "The house is blue," let F = "The house is 30 years old" and G = "The house is ugly." Translate the following statements into symbols.

(a) If the house is 30 years old, then it is ugly.

(b) If the house is blue, then it is ugly or it is 30 years old.

(c) If the house is blue, then it is ugly, or it is 30 years old.

(d) The house is not ugly if and only if it is 30 years old.

(e) The house is 30 years old if it is blue, and it is not ugly if it is 30 years old.

(f) For the house to be ugly, it is necessary and sufficient that it be ugly and 30 years old.

a) $F \to G$ b) $E \to (G \vee F)$

c) $(E \to G) \vee F$ d) $\neg G \leftrightarrow E$

e) $F \to E \wedge \neg G \to F$ f) $G \leftrightarrow (G \wedge F)$

The conditional connective

$P \to Q$ "if P then Q"

P is the antecedent

Q is the consequent

B: conditional

$P \leftrightarrow Q$ P if Q

4. Which of the following statements is a tautology, which is a contradiction and which is neither?

(a) $P \vee (\neg P \wedge Q)$.

(b) $(X \vee Y) \leftrightarrow (\neg X \to Y)$.

(c) $(A \wedge \neg B) \wedge (\neg A \vee B)$.

(d) $(Z \vee (\neg Z \vee W)) \wedge \neg(W \wedge U)$.

(e) $(L \to (M \to N)) \to (M \to (L \to N))$.

(f) $((X \leftrightarrow Z) \wedge (X \leftrightarrow Y)) \wedge X$.

(g) $((P \leftrightarrow \neg Q) \wedge P) \wedge Q$.

1 = T

0 = F

A tautology is a statement that is true for every valuation of its propositional variables

The negation of a tautology is a contradiction

4. a)

P	Q	P	V	¬P	∧	Q
0	0	0	0	1	0	0
0	1	0	1	1	1	1
1	0	1	1	0	0	0
1	1	1	1	0	0	1

Neither

b)

X	Y	(X	V	Y)	↔	(¬X	→	Y)
0	0	0	0	0	1	1	0	0
0	1	0	1	1	1	1	1	1
1	0	1	1	0	1	0	1	0
1	1	1	1	1	1	0	1	1

Tautology

c)

A	B	(A	∧	¬B)	∧	(¬A	V	B)
0	0	0	0	1	0	1	0	0
0	1	0	0	0	0	1	1	1
1	0	1	1	1	0	0	0	0
1	1	1	0	0	0	0	1	1

contradiction

4d) $(z \vee (\neg z \vee w)) \wedge \neg(w \wedge u)$

This is a tautology bc $z \vee \neg z \Leftrightarrow 1$

Tautology $\wedge$ X is simply X

$\neg(w \wedge u)$ is neither

4e) Tautology

L	M	N	$(L \to (M \to N)) \to (M \to (L \to N))$
0	0	0	1 1 1 1 1 1 1
0	0	1	1 1 1 1 1 1 1
0	1	0	1 1 0 1 1 1 1
0	1	1	1 1 1 1 1 1 1
1	0	0	1 1 1 1 0 1 1
1	0	1	1 1 1 1 1 1 1
1	1	0	1 0 1 0 1 1 0
1	1	1	1 1 1 1 1 1 1

4f) Neither

X	Y	Z	$(X \leftrightarrow Z) \wedge (X \leftrightarrow Y) \wedge X$
0	0	0	1 1 1 0 0
0	0	1	0 0 1 0 0
0	1	0	1 0 0 0 0
0	1	1	0 0 0 0 0
1	0	0	0 0 0 0 1
1	0	1	0 0 0 0 1
1	1	0	0 0 1 0 1
1	1	1	1 1 1 1 1

4g)

P	Q	$((P \leftrightarrow \neg Q) \wedge P) \wedge Q$
0	0	0 1 0 0
0	1	1 0 0 0
1	0	1 1 1 0
1	1	0 0 0 0

Contradiction

5. Suppose that the possible values of x and y are all cars. Let $L(x, y) =$ "x is as fast as y," let $M(x, y) =$ "x is as expensive as y" and $N(x, y) =$ "x is as old as y." Translate the following statements into words.

   (a) $(\exists x)(\forall y)L(x, y)$.

   (b) $(\forall x)(\exists y)M(x, y)$.

   (c) $(\exists y)(\forall x)[L(x, y) \vee N(x, y)]$.

   (d) $(\forall y)(\exists x)[\neg M(x, y) \rightarrow L(x, y)]$.

a) There exists an x for every y such that x is as fast as y

b) For every x there exists a y such that x is as expensive as y

c) There exists a y for every x such that x is as fast as y or x is as old as y

d) For every y there exists an x such that if there is not an x as expensive as y then x is as fast as y

6. Negate the following statement: For every real number $\epsilon > 0$ there exists a positive integer k such that for all positive integers n, it is the case that $|a_n - k^2| < \epsilon$.

Translating the statement to symbols

$$(\forall \epsilon > 0 \in \mathbb{R}\ \exists k \in \mathbb{Z}^+ \mid \forall n \in \mathbb{Z}^+\ |a_n - k^2| < \epsilon)$$

Negate the statement

$$\neg(\forall \epsilon > 0 \in \mathbb{R}\ \exists k \in \mathbb{Z}^+ \mid \forall n \in \mathbb{Z}^+\ |a_n - k^2| < \epsilon)$$

$$\exists \epsilon \leq 0 \in \mathbb{R}\ \forall k \in \mathbb{Z}^+ \mid \exists n \in \mathbb{Z}^+\ |a_n - k^2| \geq \epsilon$$

1. Let X = "I am happy," let Y = "I am watching a movie" and Z = "I am eating spaghetti." Translate the following statements into words.

(a) $Z \to X$.

(b) $X \leftrightarrow Y$.

(c) $(Y \vee Z) \to X$.

(d) $Y \vee (Z \to X)$.

(e) $(Y \to \neg X) \wedge (Z \to \neg X)$.

(f) $(X \wedge \neg Y) \leftrightarrow (Y \vee Z)$.

a) If I am eating spaghetti then I am happy

b) I am happy if and only if I am watching a movie

c) If I am watching a movie or I am eating spaghetti then I am happy

d) I am watching a movie or if I am spaghetti then I am happy

e) If I am watching a movie then I am not happy, and If I am eating spaghetti then I am not happy

f) I am happy and not watching a movie if and only if I am watching a movie or eating spaghetti.

2. Make a truth table for each of the following statements.

(a) $P \wedge \neg Q$.

(b) $(R \vee S) \wedge \neg R$.

(c) $X \vee (\neg Y \vee Z)$.

(d) $(A \vee B) \wedge (A \vee C)$.

(e) $(P \wedge R) \vee \neg(Q \wedge S)$.

1 = T

0 = F

a)

P	Q	P ∧ ¬Q
0	0	0
0	1	0
1	0	1
1	1	0

b)

R	S	(R ∨ S) ∧ ¬R
0	0	0
0	1	1
1	0	0
1	1	0

c)

X	Y	Z	X ∨ (¬Y ∨ Z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

d)

A	B	C	(A ∨ B) ∧ (A ∨ C)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

e)

P	Q	R	S	(P ∧ R) ∨ ¬(Q ∧ S)
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1