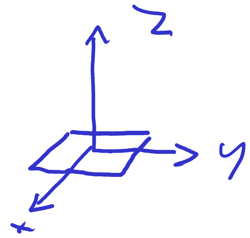
Ecen 636 Homework 2 part 2

2). Find the far fields of a rectangular aperture centered at the origin in the x-y plane. (Find the fields in the z>0 half-space.) The aperture field is:



$$\vec{E}_a(x,y) = \hat{a}_y\left(1-\frac{2|x|}{b}\right)\left(1-\frac{2|y|}{a}\right) \quad |x|\le\frac{b}{2}, |y|\le\frac{a}{2}$$

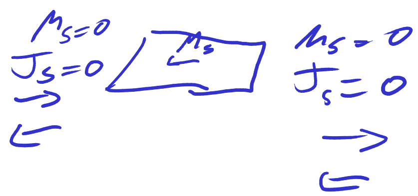
Use whatever shortcuts you prefer (be sure and explain them), and you can use the results in the notes.

$M_s = \hat{a}_n \times E_a$ Over the aperture

$J_s = ?$

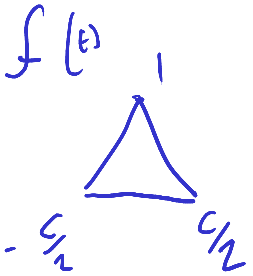
$M_s = ?$ $J_s = ?$ Everywhere else

PEC Replacement



For a 2D FT these two variables are seperable

$$M_s = -2\hat{a}_z \times E_a(x,y)$$
$$= -2\hat{a}_x\left(1-\frac{2|x|}{b}\right)\left(1-\frac{2|y|}{a}\right)$$



$$\int_{-\infty}^{\infty} f(t)e^{jk_t t}\,dt$$

$$F\{M_s\} = \tilde{M}_s = 2\hat{a}_x\left(\frac{b}{2}\right)\mathrm{Sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\mathrm{Sinc}^2\left(k_y\frac{a}{4}\right)$$

Triangle Function

$$\Lambda(b,x_0) = 1-\frac{|x-x_0|}{b}$$

$$\tilde{\Lambda} = b\,\mathrm{Sinc}\left(k_x\frac{b}{2}\right)e^{jk_x x_0}$$

$$\begin{bmatrix} r \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\tilde{M}_r = 2\left(\frac{b}{2}\right)\mathrm{Sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\mathrm{Sinc}^2\left(k_y\frac{a}{4}\right)\sin\theta\cos\phi$$

$$\tilde{M}_\theta = 2\left(\frac{b}{2}\right)\mathrm{Sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\mathrm{Sinc}^2\left(k_y\frac{a}{4}\right)\cos\theta\cos\phi$$

$$\tilde{M}_\phi = -2\left(\frac{b}{2}\right)\mathrm{Sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\mathrm{Sinc}^2\left(k_y\frac{a}{4}\right)\sin\phi$$

$\bar{A}(r) = \mu G(r) \tilde{\bar{J}}_s(k) = 0 \qquad \bar{F}(\bar{r}) = \varepsilon G(r) \tilde{\bar{M}}_s(k)$

In the far field, $H_r = E_r = 0$

$E_\theta = -jk G(r) \tilde{M}_\phi = jk G(r) \frac{ba}{2} \operatorname{sinc}^2\left(\frac{k_x b}{4}\right) \operatorname{sinc}^2\left(\frac{k_y a}{4}\right) \sin\phi$

$E_\phi = jk G(r) \tilde{M}_\theta = jk G(r) \frac{ba}{2} \operatorname{sinc}^2\left(\frac{k_x b}{4}\right) \operatorname{sinc}^2\left(\frac{k_y a}{4}\right) \cos\theta \cos\phi$

3) What are the vector effective lengths of:

a) the radiators in problems 1 and 2?

b) an ideal dipole (constant current) of length dl and current I_0

Vector Effective Length $\quad \bar{h}(\theta,\phi) = \int_V \bar{J}(\bar{r}') e^{j\Omega} dv'$

$\bar{A}(\bar{r}) = \mu G(r) \bar{h}(\theta,\phi)$

$\bar{E} = -j\omega \bar{A}_T$

$= -j\omega\mu G(r) \bar{h}(\theta,\phi)$

a) Problem 1

$$\tilde{\bar{M}} = \underbrace{2E_0 \operatorname{sinc}\left(\frac{k_x b}{2}\right) \operatorname{sinc}\left(\frac{k_y a}{2}\right) ab}_{\tilde{m}_z} \left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right)$$

$\bar{h}_r = \tilde{\bar{m}}_z \cos\theta \quad \bar{h}_\theta = \tilde{\bar{m}}_z \sin\theta$

$\bar{h}_\phi = 0$

Problem 2

$$\tilde{M}_r = \overbrace{2\left(\frac{b}{2}\right)\operatorname{sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\operatorname{sinc}^2\left(k_y \frac{a}{4}\right)}^{\tilde{\bar{m}}_x} \sin\theta\cos\phi$$

$$\tilde{M}_\theta = 2\left(\frac{b}{2}\right)\operatorname{sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\operatorname{sinc}^2\left(k_y \frac{a}{4}\right)\cos\theta\cos\phi$$

$$\tilde{M}_\phi = -2\left(\frac{b}{2}\right)\operatorname{sinc}^2\left(\frac{k_x b}{4}\right)\left(\frac{a}{2}\right)\operatorname{sinc}^2\left(k_y \frac{a}{4}\right)\sin\phi$$

$h_r = \tilde{\bar{M}}_x \sin\theta\cos\phi$

$h_\theta = \tilde{\bar{M}}_x \cos\theta\cos\phi$

$h_\phi = -\tilde{\bar{M}}_x \sin\phi$

b)

$$h = \int_V J(r) e^{j\Omega} dv' = \int_V \hat{a}_z I_0 dl\, \delta(x)\delta(y)\delta(z) e^{j\Omega} dv = \boxed{I_0 dl\, \hat{a}_z}$$