Homework 8 Ecen 646: Cinlar 4.2(a,b), 4.4, 4.11

Problem 4.2 a)

$$P = \begin{array}{c|cccc} & r & w & b & y \\ r & 0 & 0 & 1 & 0 \\ w & 0 & 0.4 & 0.6 & 0 \\ b & 0.8 & 0 & .02 & 0 \\ y & 0.2 & 0.3 & 0 & 0.5 \end{array}$$

$P\{X_5=b, X_6=r, X_7=b, \cancel{X_8}=b \mid X_4=w\}$

$= 0.6 \cdot 0.8 \cdot 1 \cdot 0.2 = \boxed{0.096}$

b)

r	w	b	y
2	4	7	3

$E[f(X_5)f(X_6) \mid X_4=y] = \Big(2(0.2) \cdot (7) + 4(0.3) \cdot ((0.4)4 + (0.6)7) + 7(0) + 3(0.5) \cdot (2(0.2) + 4(0.3) + 3(0.5)) \Big)$

$.4 \quad 1.2 \quad +1.5$

$= 2.8 + 1.2(5.8) + 1.5 \cdot (3.1) = \boxed{14.41}$

$6.96 \qquad 4.65$

Problem 4.4:

a) States 1, 3 recurrent non-null aperiodic

State 2,4 transient

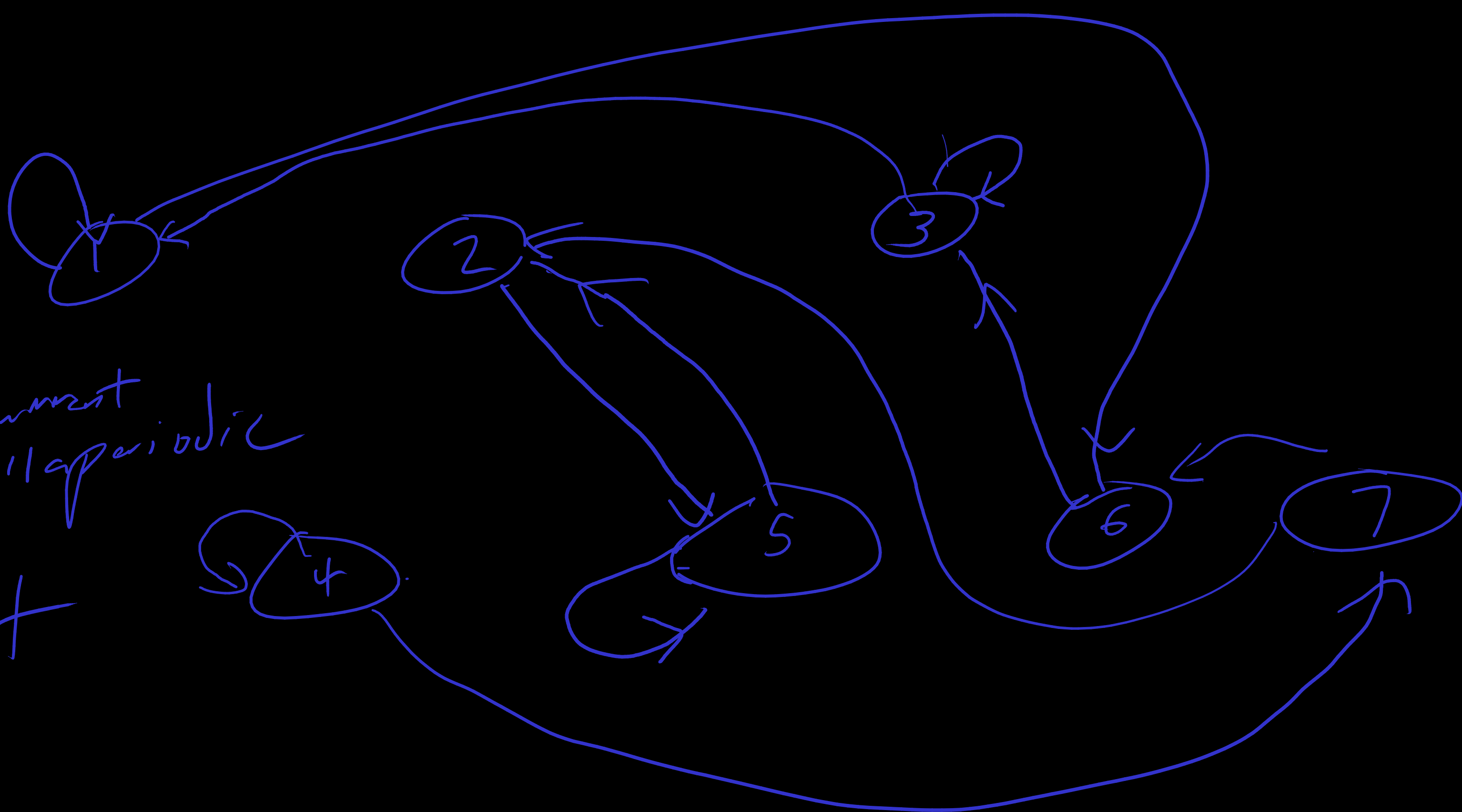
b) State 2, 5 transient

1, 3, 4 recurrent non null aperiodic

c) 2, 5 recurrent aperiodic non null

1, 3, 4, recurrent nonnull aperiodic

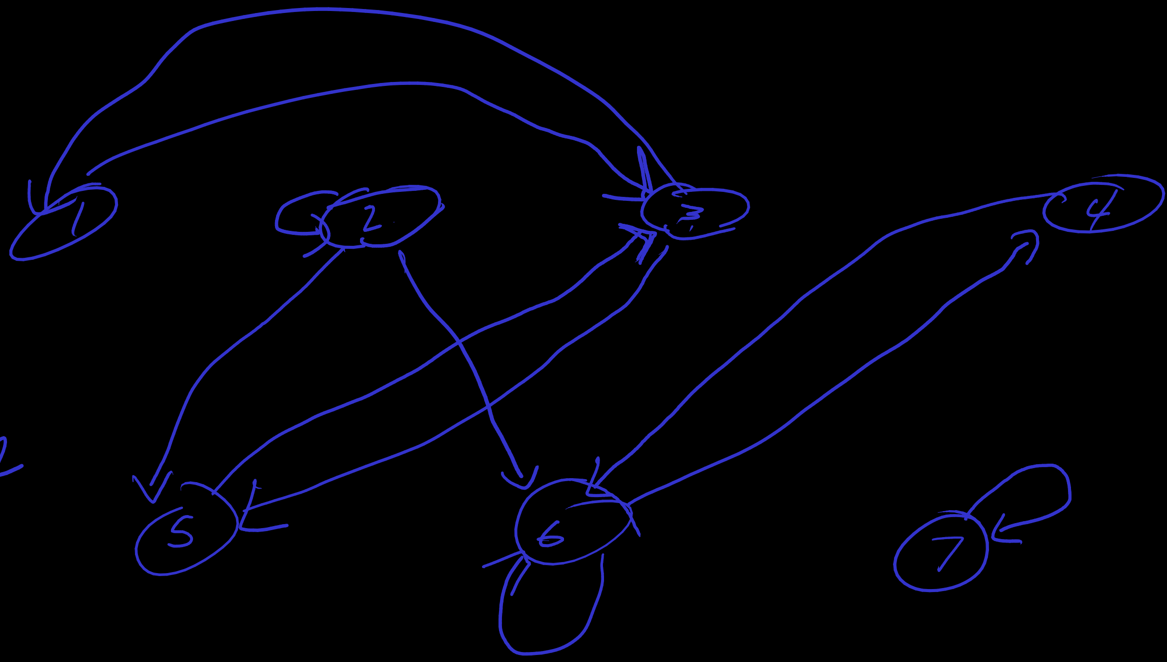
D



1, 2, 3, 5, 6 recurrent nonnull aperiodic

4, 7 Transient

E

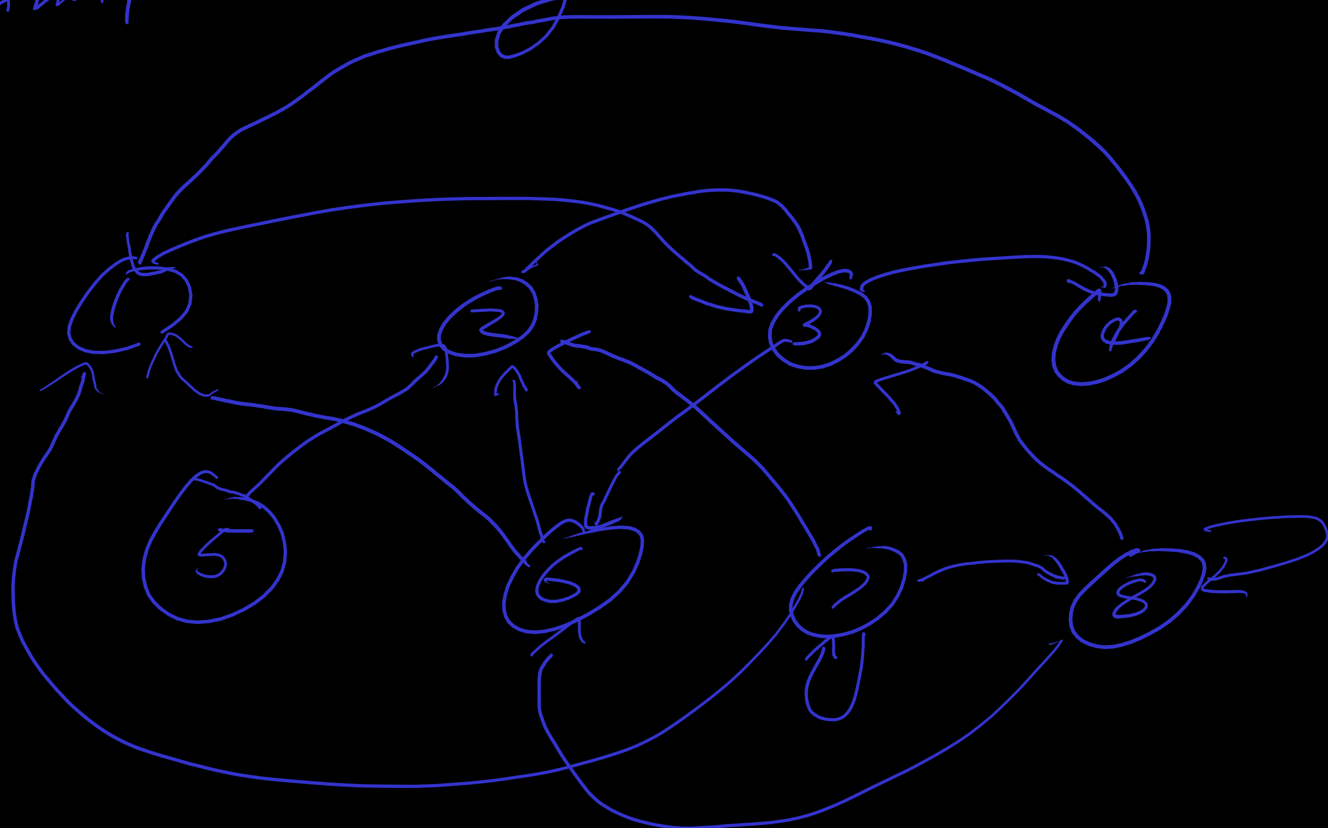


1,3,5
recurrent
periodic period 2

2 Transient

4,6 recurrent aperiodic non null | 7 absorbing

f)



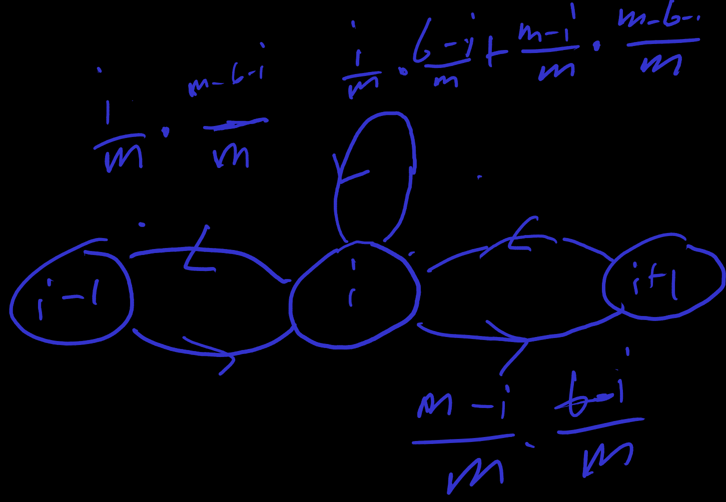
5,7,8
Transient

1,2,3,4,6 periodic period 3

(1,2)
(3)
(4,6)

4.11 Two urns containing m balls, b are black, 2m − b white

State i if first urn contains i black balls (m − i white)



$0 \le \forall i \le m$

$P[X_{n+1}=j+1 \mid X_0 \dots X_n]$

The transition probability for any state i depends only on i

$\therefore$ $P[X_{i+1} \mid X_0 \dots X_i] = P[X_{i+1} \mid X_i]$

Problem 4

a) $\forall i \in \{1,2,3,4,5,6\}$ $\pi(i) = \frac{1}{6}$

Markov Chain

$$P = \begin{bmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 2/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 1/2 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 2/3 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Yes

$0, 1, 2, 3, 4 \ldots$

$$P = \begin{bmatrix} 5/6 & 1/6 & 0 & \cdots \\ 0 & 5/6 & 1/6 & 0 & \cdots \\ \vdots & 0 & \ddots & \ddots \\ \vdots & & \ddots & \ddots \end{bmatrix}$$

c) time since the most recent six

Markov Chain

Yes

$$P = \begin{bmatrix} 1/6 & 5/6 & 0 & \cdots \\ 1/6 & 0 & 5/6 & \cdots \\ 1/6 & \cdots & & 5/6 \\ 1/6 & & & \ddots \\ 1/6 & & & \ddots \\ \vdots \end{bmatrix}$$

d) Yes it is a markov Chain

$$P = \begin{bmatrix} 1/6 & \frac{1}{6}\frac{5}{6} & \frac{1}{6}\left(\frac{5}{6}\right)^2 & \cdots \\ 1 & 0 & \cdots \\ 0 & 1 & \cdot \\ \cdot & \cdot & 1 & \cdot \\ & \cdot & & 1 & \cdot \\ \cdot & & & & 1 \end{bmatrix}$$

Problem 5

a) Yes as it is just a timeshift of a MC

$P[X_{m+r} \mid X_{m+r-1}]$

b) X_{2m} is a markov chain as

$P[X_{2m}] = P[X_{2m} \mid x_0 \dots X_{2m-2}]$ is independent of all but X_{2m-1} which is not present

c) $Y = (X_n, X_{n+1})$ $P[Y_n \mid Y_0 \dots Y_{n-1}]$

$= P[X_n, X_{n+1} \mid x_0 \dots X_n]$

$= P[X_n, X_{n+1} \mid X_{n-1}, X_n]$

$= \boxed{P[Y_n \mid Y_{n-1}]}$

Problem 6 $\{X_n : n \geq 1\}$ $S_n = \sum_{r=1}^{n} X_r$

a) S_n is a markov chain

$$P[S_n \mid S_0 \cdots S_{n-1}] = P\left[\sum_{r=0}^{n} X_n \,\middle|\, S_0 \cdots S_{n-1}\right]$$

$$= P[X_n + S_{n-1} \mid S_0 \cdots S_{n-1}]$$

$$= P[X_n + S_{n-1} \mid S_{n-1}]$$

Yes it is a markov chain

b) No as $P[Y_n \mid Y_0 \cdots Y_{n-1}] = P[Y_n - Y_{n-1} + Y_{n-1} - \cdots + Y_0 \mid Y_0 \cdots Y_{n-1}]$

$$= P[X_n + Y_{n-1} - Y_{n-2} + Y_{n-2} \cdots + Y_0 \mid Y_0 \cdots Y_{n-1}] \neq P[Y_n \mid Y_{n-1}]$$

c)

$$P[Z_n | Z_0 \cdots Z_{n-1}] = P[\sum_{r=0}^{n} S_n | Z_0 \cdots Z_{n-1}]$$

$$= P[S_n + \sum_{r=0}^{n-1} S_r | Z_0 \cdots Z_{n-1}] = P[S_n + Z_{n-1} | Z_0 \cdots Z_{n-1}]$$

$$= P[X_n + Z_{n-1} - Z_{n-2} + Z_{n-1} | Z_0 \cdots Z_{n-1}]$$

$$= P[X_n + 2Z_{n-1} - Z_{n-2} | Z_0 \cdots Z_{n-1}]$$

$$= P[X_n + 2Z_{n-1} - Z_{n-2} | Z_{n-2}\, Z_{n-1}]$$

Not a Markov Chain

D) Yes it is a markov chain

$$P[S_n, Z_n | (S_0, Z_0) \cdots (S_{n-1}, Z_{n-1})] = P[(X_n + S_{n-1}), (S_{n-1} + X_n + Z_{n-1}) | (S_0, Z_0) \cdots (S_{n-1}, Z_{n-1})]$$

$$= P[(X_n + S_{n-1}), (\underbrace{S_{n-1} + X_n}_{S_n} + Z_{n-1}) | (S_0, Z_0) \cdots (S_{n-1}, Z_{n-1})]$$