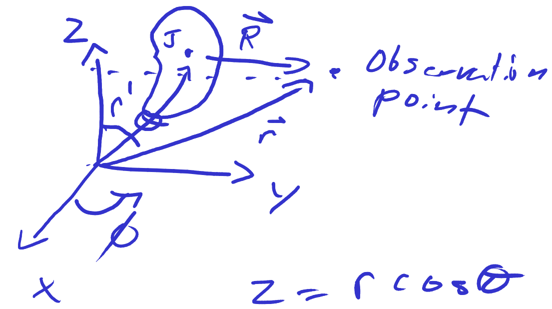
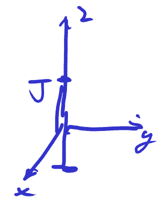
Homework #3 Ecen 636 Phased Arrays

1. Show that for a current distribution along the z axis, $|\vec{r}-\vec{r}'| \approx r - z'\cos\theta$ (use the binomial expansion, not the geometric argument.



$z = r\cos\theta$

$R = |\vec{r}-\vec{r}'| \approx r - z'\cos\theta$



$$|\vec{r} - z'a_z| = \left((r - r')(r - r')\right)^{1/2} = \left(r^2 - 2r\cdot r' + (r')^2\right)^{1/2} = \left(r^2 - 2rr'\cos\theta + (r')^2\right)^{1/2}$$

Binomial Expansion

$$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} \ldots \quad |x|<1$$

Let $x = \left(\frac{r'}{r}\right)^2 - \frac{2r'\cos\theta}{r}$

$$r(1+x)^{1/2} = r + \frac{r}{2}\left(\left(\frac{r'}{r}\right)^2 - \frac{2r'\cos\theta}{r}\right) - \frac{r}{8}\left(\left(\frac{r'}{r}\right)^4 - 4\frac{r'\cos\theta}{r}\left(\frac{r'}{r}\right)^2 + 4\left(\frac{r'}{r}\right)^2\cos^2\theta\right)$$

$$= r + \frac{(r')^2}{2r} - r'\cos\theta - \frac{r'^4}{8r^3} + \frac{1}{2}\frac{(r')^3\cos\theta}{r^2} - \frac{1}{2}\frac{(r')^2}{r}\cos^2\theta$$

$$+\frac{r}{16}\left(\left(\frac{r'}{r}\right)^6 - \frac{4r'\cos\theta}{r}\left(\frac{r'}{r}\right)^4 + 4\left(\frac{r'}{r}\right)^4\cos^2\theta - \frac{2r'\cos\theta}{r}\left(\frac{r'}{r}\right)^4 + 4\left(\frac{r'}{r}\right)^4\cos^2\theta - 8\left(\frac{r'}{r}\right)^3\cos^3\theta\right)$$

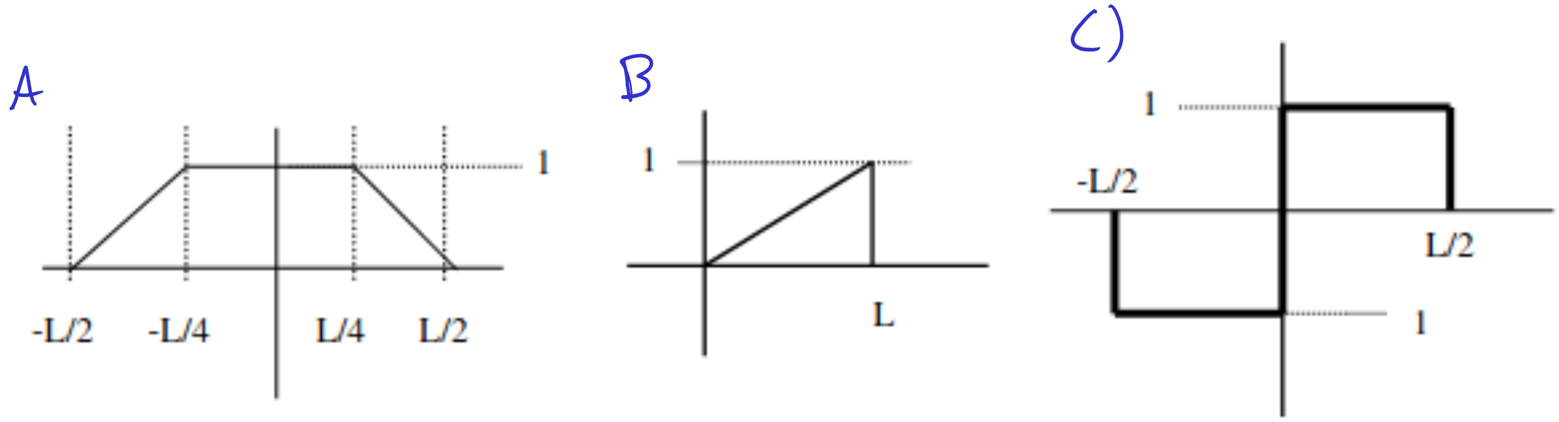
$$= r - r'\cos\theta + \frac{(r')^2}{2r}\left(1-\cos^2\theta\right) + \left(\frac{1}{2}\frac{(r')^3\cos\theta}{r^2} - \frac{1}{2}\frac{(r')^3\cos^3\theta}{r^2}\right) + \ldots$$

$$= r - r'\cos\theta + \frac{(r')^2}{2r}(\sin^2\theta) + \frac{(r')^3}{2r^2}\cos\theta\sin^2\theta + \ldots \qquad r' = z'$$

Problem 1 $\quad |r - r'| \approx r - z'\cos\theta$

Problem 2 $\quad |r - r'| \approx r - z'\cos\theta + \frac{(z')^2}{2r}\sin^2\theta + \frac{(r')^3}{2r^2}\cos\theta\sin^2\theta$

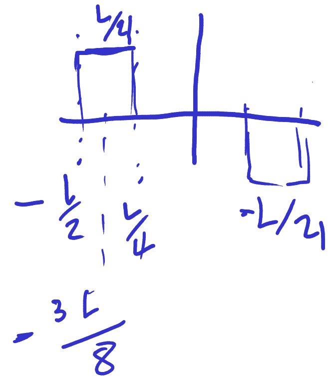
3. Find the far field patterns of the following current distributions by a method other than direct integration .



A $\frac{d}{dx} i(x) \Longleftrightarrow -jk_x i(k_x)$

$rect(L/8, 0) \Longleftrightarrow \frac{L}{4} sinc\left(\frac{k_x L}{8}\right)$

$\frac{d}{dx} A$



$\frac{d}{dx} A \Longleftrightarrow \frac{L}{4} sinc\left(\frac{k_x L}{8}\right) e^{-jk_x \frac{3L}{8}} - \frac{L}{4} sinc\left(\frac{k_x L}{8}\right) e^{jk_x \frac{3L}{8}}$

$= Z$

$Z = -jk_x i(k_x)$

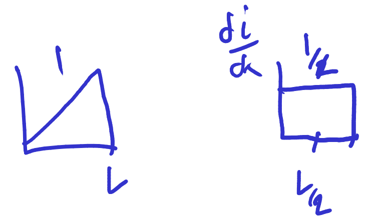
$i(k_x) = \frac{1}{-jk_x}\left(\frac{L}{4} sinc\left(\frac{k_x L}{8}\right)\left(e^{-jk_x \frac{3L}{8}} - e^{jk_x \frac{3L}{8}}\right)\right)$

$= \frac{L}{4} sinc\left(\frac{k_x L}{8}\right)\left(\frac{2\sin\left(k_x \frac{3L}{8}\right)}{k_x}\right) = \frac{L}{4} sinc\left(\frac{k_x L}{8}\right)\frac{3L}{4}\left(\frac{2\cdot 4\sin\left(k_x \frac{3L}{8}\right)}{3L k_x}\right)$

$= \frac{3L^2}{16} sinc\left(\frac{k_x L}{8}\right) sinc\left(\frac{k_x 3L}{8}\right)$

$\frac{\sin(k_x a)}{k_x a} = sinc(k_x a)$

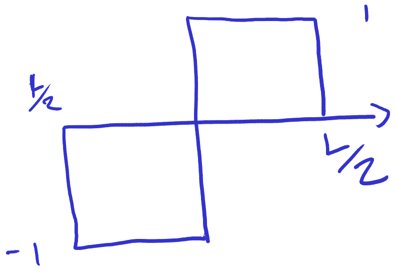
B)



$rect(1/L, L/2) \Longleftrightarrow \frac{1}{2L} sinc\left(\frac{k_x}{L}\right) e^{-jk_x \frac{L}{2}} = -jk_x i$

$i = \frac{j e^{-jk_x \frac{L}{2}}}{2L k_x} sinc\left(\frac{k_x}{L}\right)$

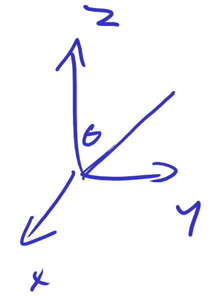
c)



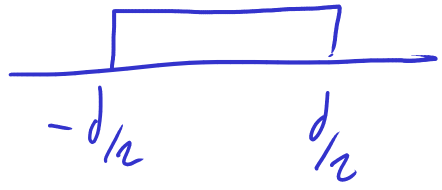
$$\mathcal{F}\{-\mathrm{rect}(1; \tfrac{L}{4}) + \mathrm{rect}(1, -\tfrac{L}{4})\}$$

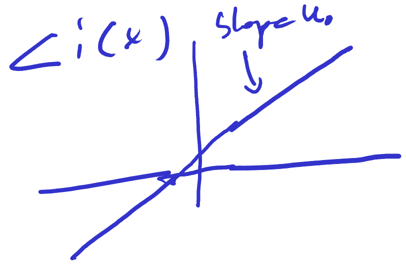
$$= -\mathrm{sinc}(k_x)\, e^{j k_x \frac{L}{4}} + \mathrm{sinc}(k_x)\, e^{-j k_x \frac{L}{4}}$$

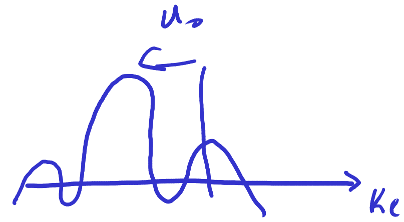
4.



$\theta = 60$







$(k_x + u_0) = 0$

$k_x = k \sin\theta \cos\phi$ $\qquad k = \frac{2\pi}{\lambda}$

$- \quad U_0 = \frac{2\pi}{\lambda} \sin\phi \cos\theta$

$U_0 = -\frac{2\pi}{\lambda} \cos 60° = \boxed{\frac{-\pi}{\lambda}}$

5. $U_0 = \frac{-\pi}{\lambda}$

$600\lambda \cdot \frac{-\pi}{\lambda} = -600\pi \% 2\pi = 0 = 2\pi$

phase shift of 2π