Computational MRI (COMP0121) Coursework 3

Gary Hui Zhang, PhD

Due: January 18th, 2021 (16:00)

General Notes:

1. Please read each problem carefully; make sure that you answer each one as completely as possible.
2. The problems have been phrased in the MR terms that you are expected to be familiar with at this point of the module; if you find any of them to be unfamiliar, look them up in the book chapters, the lecture slides, and/or the worksheets.
3. The problems are deliberately non-prescriptive with some parameters. You are expected to know, or to be able to work out, the suitable choices of these, and can justify your choices.
4. For simulation/visualisation tasks, remember that the objective is to use these as a tool to better understand the underlying MR phenomena. So you are expected to describe what your simulation shows, explain if they make sense, and reflect on what you have learned from it.

Submission Guidance:

1. A written report: submit as a single pdf file; maximum three-page long; minimum font size 10; recommended to use Latex; include any figures at the end of the report, which will not be counted towards the page limit.
2. A code listing: submit as a single pdf file; no page limit
3. A collection of videos: submit as a single zip file; please label clearly which simulation visualisation each video corresponds to.

Problem 1: Digital Fourier Transform in 1-D

Objective: To develop an understanding of the essential characteristics of Fourier Transform

Task 1: Set up a 1-D k-space that conforms to the MRI convention, has 128 uniformly sampled points, with a Δk that corresponds to a FOV of 8 mm. In other words determine the 1-D array of values to sample in the k-space. Explain what is the MRI convention and how you determine the array. Similarly, set up the corresponding 1-D image space that conforms to the MRI convention and the requirement of Digital Fourier Transform.

Task 2: Define and visualise the impulse function $\delta(k)$ centred at $k = 0$ such that it takes a value of 1 at $k = 0$ and 0 elsewhere. Use Matlab's built-in Fast Fourier Transform (FFT) function `fft` to compute its inverse Fourier Transform. Visualise and describe the resulting function in the image space. Explain if the result makes sense.

Task 3: Define all the translated versions of $\delta(k)$, $\delta(k - k_0)$, with k_0 taking in turn each of the 128 samples in k-space defined in **Task 1**. Explain how the shift theorem can be used to determine their inverse Fourier Transforms. Implement this idea and compare against the brute-force implementation using FFT. Visualise a few select examples to explain how the function in the image space depends on k_0.

Task 4: Define a Gaussian function $G(k)$ centred at $k = 0$ and with $\sigma = 8\Delta k$. Show that the function $G(k)\delta(k - k_0) = G(k_0)\delta(k - k_0)$. Explain how the linearity of the Fourier Transform can be used to determine its inverse Fourier Transform. Implement this idea and compare against the brute-force implementation using FFT.

Task 5: Express the sampled $G(k)$ for the k-space points determined in **Task 1** as a summation of the scaled and translated impulse functions $G(k_0)\delta(k - k_0)$ investigated in **Tasks 3 and 4**. Explain how one can compute the inverse Fourier Transform of $G(k)$ with this summation, taking advantage of the linearity of the Fourier Transform. Implement this idea and compare against the brute-force implementation using FFT.

Task 6: Define a rect function rect(k) centred at $k = 0$ and with a width $W = 16\Delta k$. Apply this as a k-space filter on $G(k)$. Compute the point spread function corresponding to this filter, as well as the inverse Fourier Transform of the filtered $G(k)$ (in any way you like). Visualise your results and explain your findings.

Task 7: Define a Hanning function hann(k) centred at k = 0 and with a width W matching rect(k) defined above. Apply this as an additional filter on $G(k)$. Compute the point spread function corresponding to this filter, as well as the inverse Fourier Transform of the filtered $G(k)$. Visualise your results and explain your findings.

Problem 2: Digital Fourier Transform in 2-D

Objective: To generalise the basic understanding from *Problem 1* to 2-D.

You have been provided an axial slice through a human brain as a 256×256 array. The first dimension of the array corresponds to the x axis and the second dimension the y axis. Each pixel is 1mm-by-1mm in size.

Task 1: Set up the 2-D image space that conforms to the MRI convention, i.e. a 2-D array of the coordinates of each pixel in the provided image. Visualise the image with Matlab's built-in function `imagesc`. Take care to make sure that the image is displayed with the correct aspect ratio; remember that each pixel is square in shape. Also make sure that the first dimension is displayed along the horizontal axis and the second dimension the vertical axis. Finally, the larger y coordinates should be displayed above the lower ones; your choice is between "`axis ij`" and "`axis xy`". (Hint: your image should look like Figure 11.1c on pg 210 of the textbook.)

Task 2: Set up the corresponding 2-D k-space that conforms to the MRI convention. Explain how you deduce the number of samples and the sampling intervals. Compute the Fourier Transform of the image with this k-space with Matlab's built-in function `fft2`. Visualise the k-space image; consider carefully what you need to visualise. Take care again to display the image in line with the expectation given in **Task 1**. (Hint: your image(s) should look like Figure 11.1d of the textbook.)

Task 3: Define a 2-D rect function centred at $k_{x,y} = 0$ and with a width $W_{k_x,k_y} = 32\Delta k_{k_x,k_y}$. Apply this as a "low-pass" k-space filter on the image. Compute the point spread function corresponding to this filter, as well as the inverse Fourier Transform of the filtered image. Visualise your results and explain your findings.

Task 4: Define a "high-pass" filter function that is the complement of the rect function above and repeat **Task 3** for this filter.

Task 5: Define a “partial Fourier” filter that removes 20% of the k-space with the smallest k_x values while keeping everything else intact. Repeat **Task 3** for this filter.

Problem 3: **MR Imaging Parameters**

Objective: To understand the MR imaging parameters that influence image acquisition and image quality.

Task: Complete **Problem 12.7** from the textbook