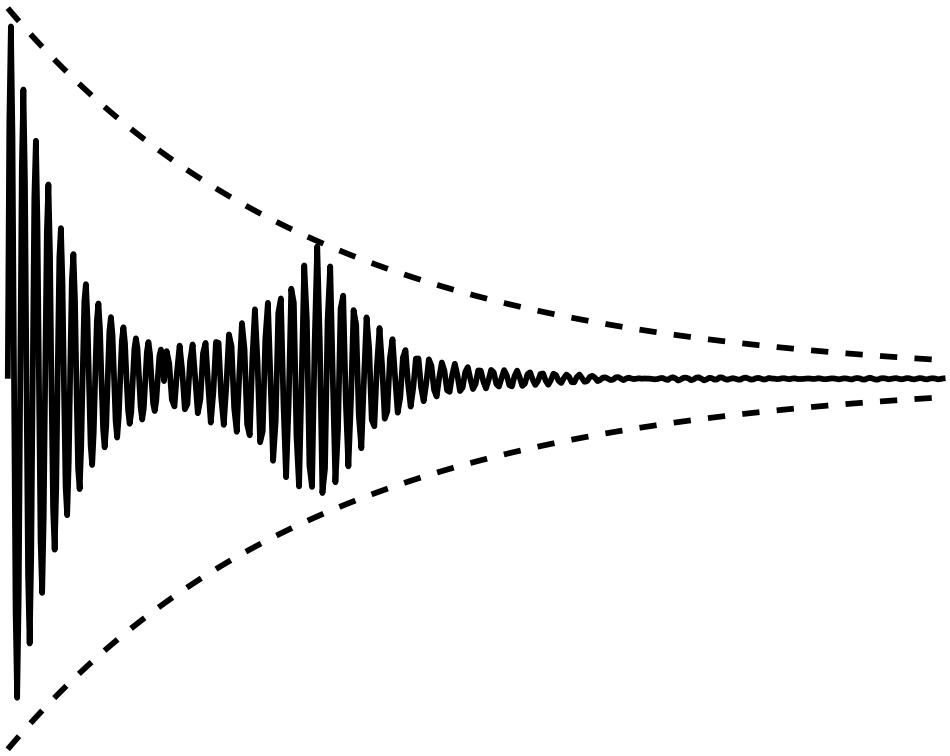
Coursework 1



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MRes - Coursework 1

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This report along with the zip file containing animations make up the coursework 1 submission.

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1 Spin Excess

Spin excess is given by the following equation:

$$\text{Spin excess} \approx N\frac{\hbar\omega_o}{2kT} \tag{1}$$

The factors involved in the calculation of the spin excess are: $\hbar$ is reduced Planck's constant which is Planck's constant (h) divided by 2π, it is a universal constant; ω_o is the Larmor frequency and is dependent on the gyromagnetic ratio γ and external static magnetic field B_o ($\omega_o = \gamma B_o$); gyromagnetic ratio is dependent on the proton of interest, this is typically a hydrogen proton; k is the Boltzmann's constant; T is the absolute temperature; N is the total number of spins in a sample.

Eqn. (1) can be thought of as a ratio between the quantum energy difference ($\hbar\omega_o$) to the thermal energy (kT), or spin excess is proportional to quantum energy difference and inversely proportional to thermal energy. Explicitly, the quantum energy difference is the energy difference between the parallel alignment to B_o versus anti-parallel alignment to B_o, where the parallel state is the lower energy state. The thermal energy is directly proportional to T. The quantum energy difference is directly proportional to B_o. Therefore, given a measurement of the same protons in the same sample (to ensure N and γ are constant), increasing B_o and/or decreasing T will increase the spin excess.

2 Forced precession with an on-resonance RF field

The following problem is answered by first defining the three dimensions $\mathbb{R}^3$ in the vector form $\mathbf{x} = [x,\ y,\ z]'$ with the associated unit vectors $\hat{x} = [1,\ 0,\ 0]'$, $\hat{y} = [0,\ 1,\ 0]'$ and $\hat{z} = [0,\ 0,\ 1]'$. The purpose of the rotation vectors are to rotate a vector define in $\mathbf{x}$ about each of the axes.

The rotation around any one of the axes corresponds to a positive clockwise rotation, by θ, on the orthogonal plane when viewing the plane from the negative direction of the axis. Intuitively, using your left-hand closing it into a fist with your thumb pointing out, the thumb represents the direction of the axis and the curl of your fingers represent a positive clockwise rotation on the orthogonal plane. Visualisations for the z, x and y axis rotations are given by the animations '2_1', '2_2' and '2_2', respectively.

2.1 Rotation around the z-axis. The rotation matrix around the z-axis is defined as:

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

To verify Eqn. (2) consider the unit vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$ defined previously. The unit vectors are rotated by θ clockwise around the z-axis to produce $\hat{x}_{rot}$, $\hat{y}_{rot}$ and $\hat{z}_{rot}$. The rotated unit vectors are calculated from $\hat{x}_{rot} = R_z(\theta)\cdot\hat{x}$, $\hat{y}_{rot} = R_z(\theta)\cdot\hat{y}$ and $\hat{z}_{rot} = R_z(\theta)\cdot\hat{z}$. Multiplying the rotation matrix by the unit vectors we find: $\hat{x}_{rot} = [\cos\theta,\ -\sin\theta,\ 0]'$, $\hat{y}_{rot} = [\sin\theta,\ \cos\theta,\ 0]'$ $\hat{z}_{rot} = [0,\ 0,\ 1]'$. These results show that rotating the vector preserves the magnitude whilst changing the x-y components, $\|\hat{x}_{rot}\| = 1$, $\|\hat{y}_{rot}\| = 1$ and $\|\hat{z}_{rot}\| = 1$. There is a reciprocal relationship between the x-y components of the vector when rotated. Furthermore, a positive θ results in a clockwise rotation on the x-y plane; $\hat{x}_{rot}|_{\theta=90^\circ} = [0,\ -1,\ 0]'$ and $\hat{y}_{rot}|_{\theta=90^\circ} = [1,\ 0,\ 0]'$.

2.2 & **2.3 Rotation around the x-axis and y-axis**. The following matrices correspond to rotations around the x-axis, Eqn. (3), and y-axis, Eqn. (4).

$$\mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \tag{3}$$

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{4}$$

Similar to problem 2.1 rotating around the x and y axes preserve the magnitude of the vector being rotated, whilst ensuring a positive θ corresponds to a clockwise rotation around the axis of rotation.

2.4 Forced procession.. The effects of T_1 and T_2 relaxation are ignored as the RF pulse is on a much smaller timescale than relaxation. Parameters given in the question are: $\Delta\theta = 90°$ flip angle, that the pulse is on-resonance and along the $\hat{x}'$ axis in the rotating frame of reference (FoR), and the duration of the pulse $\tau = 1$ ms. This information gives an insight into the dynamics. The on-resonance condition dictates that the applied RF frequency matches the Larmor frequency, meaning the resulting magnetic field B_1 is maximally synchronized to tip the spin around the x'-axis. The duration of pulse and flip angle dictate the B_1 strength as: $B_1 = \frac{\Delta\theta}{\gamma\tau} = 5.9\mu T$.

In the rotating FoR the flip of the spin is a 90° clockwise rotation around the x-axis. This means that the magnetisation vector is confined to the y-z plane. This is visualised by animation 2_4.

3 Free precession in the main static magnetic field.

In order visualise the precession of the spins as they relax it was necessary to set a fictitious Larmor frequency much lower than reality. In reality typical MRI scanners have a static magnetic field strength of the order $B_0 \approx \mathcal{O}(10^1)$ T, most clinical scanner are 1.5 or 3 T. The gyromagnetic ratio of protons are of order $\gamma \approx \mathcal{O}(10^8)$ rad/(sT), hydrogen protons are 2.68×10^8 rad/(sT). As $\omega_0 = \gamma B_0$, therefore $\omega_0 \approx \mathcal{O}(10^9)$. This has the consequence of many, many precessions per second meaning visualisation would be saturated by these spins. This is further compounded as the time scale for relaxation of the spin toward B_0 is over a lot longer time scales than the Larmor frequency. T_1 and T_2 relaxations are the longitudinal and transverse relaxations, respectively. Where the effect of relaxation is included, the values are $T_1 = 2$s and $T_2 = 1$s for all problems. These values were chosen as they were of similar order to reality (seconds to milliseconds depending on the material), additionally T_1 is typically longer than T_2. It was found that a Larmor frequency of $\omega_0 = 4\pi$ gave good visualisation, this corresponded to two rotations a second. The initial magnetization all of problem 3 was $M(0) = [0, 1, 0]'$. These dynamics are prescribed by the fact these problems pertain to only free precession (no RF pulse), and there is only a main static magnetic field (only B_0 no B_1).

The Bloch equations are the set of ordinary differential equations (ODE) that govern this system. This ODE has analytic solutions that will be used when solving the following problems. The solution consists of a rotation around the z-axis and exponential decays.

The transverse magnetisation is given as $M_\perp$ and consists of the x and y components of the magnetisation. The parallel magnetisation is given by $M_\parallel$ and is solely the z component of magnetisation.

3.1 Free precession with no relaxation.. As the initial magnetization vector is on the x-y plane and there is no relaxation, the free procession will include no z component. This is seen in animation '3_1' where there is a clockwise procession on the x-y plane, with two rotations per second, and no z component, as expected. This is because without relaxation the governing Bloch equations collapse to simply a reciprocal relationship between the x and y components that corresponds to a rotation of the magnetisation vector around the z-axis. The solution is $M(t) = \mathbf{R}_z(\omega_0 t) \cdot M(0)$. This is within the laboratory FoR.

3.2 Free precession with relaxation.. Including the relaxation provides more interesting, and more realistic dynamics. As the analytical solutions to the Bloch equations dictate that the transverse magnetisation exponentially decays to zero given $\exp(-\frac{t}{T_2})$. Furthermore, over time the parallel magnetisation decays toward one, toward the direction of the static magnetic field. The result is a precessing spin at the Larmor frequency, that moves toward $M(t = \infty) = [0, 0, 1]'$, as the transverse components decay and parallel component increase to align with the static magnetic field. This is visualised in animation '3_2'. Additionally, a stand-alone visualisation of the 3D magnetisation vector trajectory is given in '3_2_3D'.

3.3 Rotating FoR. In the rotating FoR the laboratory FoR axes rotate around the z-axis at the Larmor frequency. In the rotating FoR the circular precession of the spin at the Larmor frequency in the x-y plane is effectively halted, as these dynamics are captured by the fact that the FoR itself is rotating. Hence, problem 3.1 in the rotating frame corresponds to $M' = [0, 1, 0]'$, the magnetisation

vector is static in the rotating FoR. As this is a trivial solution it is not visualised. The solution to problem 3.2 in the rotating FoR is restricted to the y'-z' plane as, again, the rotation at the Larmor frequency is captured by the rotating FoR. This is visualised in animation '3_3'.

3.4 Isochromat precessing faster than Larmor frequency.. If the precession is faster than the Larmor frequency ($\omega_0 + \Delta\omega$) then in the rotating FoR the rotation is slower than the precession. This means that even though the rotating FoR is rotating at a similar frequency, the simulated spin will spin at the difference between the rotating FoR and the actual spin of the isochromat. Therefore the solution in the rotating FoR will be the same as visualised in '3_3' but with a x-y component due to a rotation of $\mathbf{R}_z(\Delta\omega t)$. This a clockwise rotation as $\Delta\omega$ is positive. this is seen in animation '3_4'. In the laboratory FoR the solution will be similar to '3_2' but with slightly more rotation around the z-axis at any given time..

3.5 Isochromat precessing slower than Larmor frequency. This is solution is very similar to the previous, the difference being that the precession is at a frequency of $\omega_0 - \Delta\omega$. Again $\Delta\omega$ is positive, so the the solution is identical in the rotating FoR although the rotation around the z-axis is anti-clockwise. In the laboratory FoR the solution is similar but with slightly less rotation around the z-axis at any given time. This is visualised in animation '3_5'.

A comparison of problem 3.4 and 3.5 in the laboratory frame of reference is given in animation '3_45_*comp*'.

4 Free induction decay and Inversion Recovery.

The sequences are best simulated and visualised using the magnetisation vector and transverse components. It is important to note that the transverse components are proportional to the signal, such that no transverse magnetisation will result in no signal. The reason for this is that the the magnetisation of each individual spin is extremely small. In the receive coils a changing magnetic field induces a current which can be measured, the Larmor frequency produces a very quickly changing magnetic field in the transverse plane, and the fact that it changes so quickly aids its detection. Additionally there are many spins in a given volume and this aids detection too.

For this problem the Larmor frequency for visualisation is a lot lower than reality, $\omega_0 = 50\pi$. This Larmor frequency will allow for the visualisation of the magnetisation vector and signal for the rest of the problems. For this problem we are only concerned with one isochromat, that the spins held within all precess at the same Larmor frequency.

4.1 Free induction decay (FID) sequence, The sequence consists of two parts:

1. Flip the spins from alignment with B_0 direction by $\pi/2$ radian (90°), using a RF pulse, to the transverse plane.

2. Free procession with relaxation for spin from the transverse plane to B_0 direction.

The time over which the flip occurs and the time over which relaxation occurs are orders of magnitudes different. This makes visualisation difficult. In order to capture the dynamics correctly the simulation was carried out in the rotating FoR. If simulation was done in the laboratory FoR either two Larmor frequencies (one for each part of the sequence) would have to be used or the time over which the flip takes place would have to be a lot longer. If not, then as the fictitious Larmor frequency used is so low it would look like there was little to no precession around the z-axis during a flip. In reality there would be many many rotations around the z-axis, therefore the dynamics are not captured correctly. Using a higher Larmor frequency for the flip part of the sequence is not realistic as there should only be one Larmor frequency, also a longer flip time would mean that relaxation effects could not be ignored. Therefore a rotating FoR is used, and it should be noted that in the laboratory FoR the spins are rotating around the z-axis at the Larmor frequency. The visualisation can be seen in '4_1', the FID gives a signal as transverse magnetisation occurs.

4.2 Inversion recovery sequence. The sequence consists of four parts:

1. Flip the spins from alignment with B_0 direction by π radians (180°), using a RF pulse, to the $-B_0$ direction.
2. Free procession with relaxation for spin from the $-B_0$ direction to B_0 direction.
3. Flip the spins by $\pi/2$ radians (90°), using a RF pulse. At the time of the flip the magnetisation vector have relaxed to either remain in the $-B_0$ direction, have zero magnetisation, or be in the B_0 direction. This will result in three possibilities when flipped by a $\pi/2$ RF pulse along the x'-axis. The time over which this relaxation occurs is T_I.
   (a) If in the $-B_0$ direction, the RF pulse will flip the spin onto the $-y'$-axis.
   (b) If the magnetisation is at zero, the RF pulse have no effect on the magnetisation, as the the is no parallel component to flip onto the transverse plane.
   (c) If in the B_0 direction, the RF pulse will flip the spin onto the y'-axis.
4. Where there is a transverse component, these spins will have a free procession with relaxation for spin from the transverse plane to B_0 direction. If no transverse component the spin will just continue to relax along the z-axis toward the B_0 direction,

Using the same reasoning as problem 4.1, the sequence was simulated in the rotating FoR. This sequence is commonly used to measure T_1. How this is done is understanding the dynamics. If T_I is such that when $M_z(t = T_I) = 0$ no signal will be obtained by the receiver coils. This T_I can be used to find T_1 by analysing the z-component of the Bloch equations: $M_z(t) = M_z(0)e^{-t/T_1} + M_0(1 - e^{-t/T_1})$, using the conditions that $t = T_I$, $M_z(t) = M_z(t = T_I) = 0$ (hence no signal when flipped $\pi/2$), $M_z(0) = M_z(t = 0) = -1$ (just been flipped π), and that $M_{z0} = 1$, it can be found that $T_I = T_1\ln(2)$. Three separate animations are done in order to visualise this effect of varying T_I: '4_2_*highTI*' has $T_I > T_1\ln(2)$; '4_2_*lowTI*' has $T_I < T_1\ln(2)$; and '4_2_*opt*' has $T_I = T_1\ln(2)$.

5 Spin echo

To simulate this sequence we deal with the rotating FoR, additionally a Larmor frequency of 50π was used. The time scales for the RF pulses are on the order of milliseconds and are on-resonance, and therefore can be thought of approximately instantaneous with no relaxation effects. This sequence consists of five parts:

1. Flip the collection of isochromats from alignment with B_0 direction by $\pi/2$ radians (90°), using a RF pulse along the x' axis, to the y' axis.
2. Free procession with relaxation of the isochromats from the transverse plane to B_0 direction. The transverse components dephase due to the collection of isochromats being of different Larmor frequencies.
3. Flip the spins by π radians (180°), using a RF pulse along the y' axis. This reverses the directions of the z and x components of magnetisation vectors. This happens at $t = \tau$ where t is defined from after the initial flip.
4. The transverse components rephase and a echo is acquired, $t = 2\tau = T_E$.
5. The transverse components begin to dephase again, $t = 2\tau = T_E$.

For both problems 5.1 and 5.2, the visualisation of the transverse components of the isochromats was prioritised. Additionally, relaxation effects were only taken into account for the normalised signal. Where, as previously discussed, the signal is proportional to the magnitude of the transverse components of magnetisation, $\mid M_\perp \mid$. Furthermore, as the collections of spins are not all at the same frequency they will dephase. Dephasing will cause the average transverse magnetisation across all isochromats to decay faster than purely thermodynamic effects from T_2. This additional dephasing decay can

be denoted T_2', and both effects are described by a decay time T_2^*. To account for the dephasing in the magnetisation, the average $\frac{1}{N}\sum M_\perp$ is used to as the transverse component of magnetisation, where N is the number of isochromats in the collection and the sum is over N. Therefore the signal is proportional to $|\ \frac{1}{N}\sum M_\perp\ |$. This is what will be plotted to give an indication of the signal from the collection of isochromats. The number of isochromats used was 1001, but only 11 were visualised. An echo is produced at $t = 2\tau = T_E = 1s$, where $t = \tau = 0.5s$ is the time between $\pi/2$ and π RF pulses. The signal at T_E is not effected by dephasing and follows the T_2 envelope.

5.1 Spin echo with uniform distribution. See animation '5_1' for visualisation of the Spin Echo sequence with uniform isochromat distribution. It can be seen that with a uniform distribution of isochromats gives rise to beats (approximately 0.25, 0.5 and 0.75 seconds), where there are momentary increases in signal due to certain isochromat transverse magnetisation components rephasing. As time goes on the effect of beats is diminished as less isochromats rephase, additionally the effect of T_2 decreases beats. From the animation it is easy to see $t = \tau$ when the π RF pulse occurs, and the subsequent echo at T_E.

5.2 Spin echo with Cauchy distribution. See animation '5_2' for visualisation of the Spin Echo sequence with Cauchy isochromat distribution. It can be seen that with a uniform distribution of isochromats does not give rise to regular beat pattern which is found with a uniform distribution. This due to the random isochromat values (which follow a Cauchy distribution) the components of the transverse magnetisation do not momentarily rephase in a uniform fashion. Instead there are random points of rephasing due to the random nature of the distribution. Again it is easy to see τ and T_E.

5.3 Right choice of isochromat make-up. To choose an appropriate make up of isochromats a brief sensitivity analysis was undertaken. Where a range of $\delta\omega$ for the uniform distribution, and a range of Δ and ω_0 for the Cauchy distributions, were used to investigate the right choice of for isochromat make-up. These can be respectively seen in visualisations '5_3_*dOmega*','5_3_*Delta*' and '5_3_*omega*'.

From 5_3_*dOmega* it can be seen that increasing $\delta\omega$ decreases the amplitude of the beats, increases the frequency of beats and increases the amount of dephasing, effectively T_2' increases with $\delta\omega$. A value of $\delta\omega = 16$ was deemed sufficient for visualisation of T_2^*.

From 5_3_*Delta* it can be seen that increasing Δ increase the amount of dephasing, effectively T_2' increases with δ. At very high values of Δ random oscillations can be seen. This is due to the nature of taking a continuous probability function and discretising it, random rephasing will occur. An infinite amount of isochromats should only decay. A value of $\Delta = 8$ was deemed sufficient for visualisation.

From 5_3_*omega* it can be seen that increasing ω_0 has no effect on the amount of dephasing. Changing values of ω_0 does not have an effect when viewing the magnitude of the average transverse components. Using the magnitude obscures the effect of ω_0. As ω_0 is equivalent to changing the mean Larmor frequency. In 5_3_*omega_comp* the $M_\perp = [M_{x'}, M_{y'}]'$ components are plotted and it can be seen that varying ω_0 causes the components to oscillate in the rotating FoR, see animation '5_3_*omega_comp*'. As the mean value of the distribution should be the rate at which the FoR rotates, it was chosen to keep $\omega_0 = 0$.

5.4 Hahn echo. The Hahn echo is visualised in 5_4. A Cauchy distribution of isochromats was used with $\omega_0 = 0$ and $\Delta = 8$. The first two steps are the same as the Spin echo sequence. In the third step the π RF pulse is replaced with a $\pi/2$ RF pulse along the x' axis. The second $pi/2$ pulse rotates the magnetisation vector onto the x'-z' plane. This eliminates the y' components of $M_\perp$, and causes $\frac{1}{N}\sum M_\perp \approx 0$. The spins then begin to rephase again due to there still being a x' component. At the echo time, as the y' components were eliminated, only half of the T_2 relaxed signal is recovered. See animation '5_4' for visualisation of the Hahn echo sequence with Cauchy isochromat distribution. The second pulse occurs at $t = \tau = 0.5s$, with a half intensity echo at $t = T_E = 1s$.

A Problem 1 code:

```
function SE = calcSE(T,B, Gamma)
%Function to calculate the approximate spin excess
%   Inputs
%   T = Temperature of the medium
%   B = Magnetic field strength
%   Gamma = Gyromagnetic ratio of medium
%
%   Output
%   SE = Spin excess

% Parameters

% Number of spins calculated from typical voxel volume 2*2*5mm = 0.02
% Avogadro's number = 6.02*10^23
% One voxel has 2 x 6.02 x10^23 x 0.02 / 18 protons
N = 1.338 * 10^21;

% Reduced planks constant
hbar = (6.62607015*10^(-34))/(2*pi);

% Boltzman constant
k = 1.38064852 * 10^(-23);

SE = N*(hbar*B*Gamma)/(2*k*T);
end
```

B Problem 2 code:

B.1 Rotation matrix z-axis:

```
function [rotated] = rotateZ(vector,theta)
%Rotate around z-axis
%   vector - the vector you want rotated, must have three dimensions
%   theta - angle in radians to rotate the vector around the z-axis
%   clockwise
%

rotated(1) = vector(1)*cos(theta) + vector(2)*sin(theta);
rotated(2) = vector(2)*cos(theta) - vector(1)*sin(theta);
rotated(3) = vector(3);
end
```

B.2 Rotation matrix x-axis:

```
function [rotated] = rotateX(vector,theta)
%Rotate around x-axis
%   vector - the vector you want rotated, must have three dimensions
%   theta - angle in radians to rotate the vector around the x-axis
%   clockwise
%

rotated(1) = vector(1);
rotated(2) = vector(2)*cos(theta) + vector(3)*sin(theta);
rotated(3) = vector(3)*cos(theta) - vector(2)*sin(theta);
end
```

B.3 Rotation matrix y-axis:

```
function [rotated] = rotateY(vector,theta)
%Rotate around y-axis
%   vector - the vector you want rotated, must have three dimensions
%   theta - angle in radians to rotate the vector around the y-axis
%   clockwise
%

rotated(1) = vector(1)*cos(theta) - vector(3)*sin(theta);
rotated(2) = vector(2);
rotated(3) = vector(3)*cos(theta) + vector(1)*sin(theta);
end
```

B.4 Animations of rotations:

```
% housekeeping
clc
clear

%% Rotation around the z-axis - visualisation
% Author - Imraj 11/11/2020

% initialise the video
video = VideoWriter(['2_1', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = 100;
video.set('FrameRate', frameRate);

% open video
video.open();

% initialise the figure
h = figure;

% define unit vectors
X = [1,0,0]';
Y = [0,1,0]';
Z = [0,0,1]';

% define angle of rotation 1 degree
theta = 2*pi/360;

% specify animation captures each degree of rotation
for i=0:360
    quiver3(0,0,0,X(1),X(2),X(3),'linewidth',2,'LineStyle','-','Color','k')
    hold on
    quiver3(0,0,0,Y(1),Y(2),Y(3),'linewidth',2,'LineStyle','--','Color','b')
    quiver3(0,0,0,Z(1),Z(2),Z(3),'linewidth',2,'LineStyle','-.','Color','r')

    % format title, legend, labels, limits, grid and box
    title(['z-axis rotation of unit vectors, $\theta =$', num2str(theta*i
        *360/(2*pi)),'$^\circ$'], interpreter, latex, fontsize, 15)
    legend('$\hat{x}$','$\hat{y}$','$\hat{z}$', interpreter, latex, fontsize, 15)
    xlabel(M_x, interpreter, latex, fontsize, 15)
    ylabel(M_y, interpreter, latex, fontsize, 15)
    zlabel(M_z, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
```

```
45      ylim([-1 1]);
46      zlim([-1 1]);
47      hold off
48
49      % assign frame and write it to the video
50      frame = getframe(h);
51      video.writeVideo(frame);
52
53      % rotate by theta
54      X = rotateZ(X,theta);
55      Y = rotateZ(Y,theta);
56      Z = rotateZ(Z,theta);
57  end
58
59  video.close();
60
61  %% Rotation around the x-axis - visualisation
62  % Author - Imraj
63
64  % initialise the video
65  video = VideoWriter(['2_2', '.mp4'], 'MPEG-4');
66
67  % set the frame rate
68  frameRate = 100;
69  video.set('FrameRate', frameRate);
70
71  % open video
72  video.open();
73
74  % initialise the figure
75  h = figure;
76
77  % define unit vectors
78  X = [1,0,0]';
79  Y = [0,1,0]';
80  Z = [0,0,1]';
81
82  % define angle of rotation 1 degree
83  theta = 2*pi/360;
84
85  % specify animation captures each degree of rotation
86  for i=0:360
87      quiver3(0,0,0,X(1),X(2),X(3),'linewidth',2,'LineStyle','-','Color','k')
88      hold on
89      quiver3(0,0,0,Y(1),Y(2),Y(3),'linewidth',2,'LineStyle','--','Color','b')
90      quiver3(0,0,0,Z(1),Z(2),Z(3),'linewidth',2,'LineStyle','-.','Color','r')
91
92      % format title, legend, labels, limits, grid and box
93      title(['x-axis rotation of unit vectors, $\theta =$', num2str(theta*i
            *360/(2*pi)),'$^\circ$'], interpreter, latex, fontsize, 15)
94      legend('$\hat{x}$','$\hat{y}$','$\hat{z}$', interpreter, latex, fontsize, 15)
95      xlabel(M_x, interpreter, latex, fontsize, 15)
96      ylabel(M_y, interpreter, latex, fontsize, 15)
97      zlabel(M_z, interpreter, latex, fontsize, 15)
98      grid on
99      box on
100     xlim([-1 1]);
101     ylim([-1 1]);
102     zlim([-1 1]);
103     hold off
104
```

```
105     % assign frame and write it to the video
106     frame = getframe(h);
107     video.writeVideo(frame);
108
109     % rotate by theta
110     X = rotateX(X,theta);
111     Y = rotateX(Y,theta);
112     Z = rotateX(Z,theta);
113 end
114
115 video.close();
116
117 %% Rotation around the y-axis - visualisation
118 % Author - Imraj
119
120 % initialise the video
121 video = VideoWriter(['2_3', '.mp4'], 'MPEG-4');
122
123 % set the frame rate
124 frameRate = 100;
125 video.set('FrameRate', frameRate);
126
127 % open video
128 video.open();
129
130 % initialise the figure
131 h = figure;
132
133 % define unit vectors
134 X = [1,0,0]';
135 Y = [0,1,0]';
136 Z = [0,0,1]';
137
138 % define angle of rotation 1 degree
139 theta = 2*pi/360;
140
141 % specify animation captures each degree of rotation
142 for i=0:360
143     quiver3(0,0,0,X(1),X(2),X(3),'linewidth',2,'LineStyle','-','Color','k')
144     hold on
145     quiver3(0,0,0,Y(1),Y(2),Y(3),'linewidth',2,'LineStyle','--','Color','b')
146     quiver3(0,0,0,Z(1),Z(2),Z(3),'linewidth',2,'LineStyle','-.','Color','r')
147
148     % format title, legend, labels, limits, grid and box
149     title(['y-axis rotation of unit vectors, $\theta =$', num2str(theta*i
            *360/(2*pi)),'$^\circ$'], interpreter, latex, fontsize, 15)
150     legend('$\hat{x}$','$\hat{y}$','$\hat{z}$', interpreter, latex, fontsize, 15)
151     xlabel(M_x, interpreter, latex, fontsize, 15)
152     ylabel(M_y, interpreter, latex, fontsize, 15)
153     zlabel(M_z, interpreter, latex, fontsize, 15)
154     grid on
155     box on
156     xlim([-1 1]);
157     ylim([-1 1]);
158     zlim([-1 1]);
159     hold off
160
161     % assign frame and write it to the video
162     frame = getframe(h);
163     video.writeVideo(frame);
164
```

```
    % rotate by theta
    X = rotateY(X,theta);
    Y = rotateY(Y,theta);
    Z = rotateY(Z,theta);
end

video.close();
```

B.5 Problem 2.4:

```
% housekeeping
clc
clear
%% Forced procession ignoring relaxation
% Author: Imraj Singh 03/11/2020

% Given parameters

% Magnitisation aligned along z-axis initially
M = [0, 0, 1]';

% Duration of the RF pulse
t = 1*10^(-3);

% Flip angle
dtheta = 90*pi/180;

% Gyromagnetic ratio for proton in hydrogen
gamma = 2.68*10^8;

% Calculate neccessary parameters

% The magnetic field strength caused by RF
B1 = dtheta/(gamma*t);

% Precession frequency
omega1 = gamma*B1;

% Define modelling parameters

% Duration and timestep of model
time = linspace(0,t,101);

Msoln = zeros(101,3);

Msoln(:,1) = M(1);
Msoln(:,2) = M(2)*cos(omega1.*time) + M(3)*sin(omega1.*time);
Msoln(:,3) = M(3)*cos(omega1.*time) - M(2)*sin(omega1.*time);

%% Animation

% initialise the video
video = VideoWriter(['2_4', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = 10;
video.set('FrameRate', frameRate);

video.open();

```

```
% initialise the figure
h = figure;

% specify animation captures each degree of rotation
for i=1:length(time)
    subplot(2,2,1)
    quiver3(0,0,0,0,0,1,'linewidth',2,'LineStyle','—','Color','k')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k—','linewidth',2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','–'
        ,'Color','r')

    % format title, legend, labels, limits, grid and box
    xlabel(M_{x'}, interpreter, latex, fontsize, 15)
    ylabel(M_{y'}, interpreter, latex, fontsize, 15)
    zlabel(M_{z'}, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(3)
    hold off


    subplot(2,2,2)
    quiver(0,0,0,1,'linewidth',2,'LineStyle','—','Color','k')
    hold on
    plot(Msoln(1:i,2),Msoln(1:i,3),'k—','linewidth',2)
    quiver(0,0,Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','–','Color','r')

    % format title, legend, labels, limits, grid and box
    xlabel(M_{y'}, interpreter, latex, fontsize, 15)
    ylabel(M_{z'}, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([0 1]);
    hold off

    subplot(2,2,3:4)
    plot(time(1:i)*1000,Msoln(1:i,2),'linewidth',2,'LineStyle','–','Color','r');
    hold on
    plot(time(1:i)*1000,Msoln(1:i,3),'linewidth',2,'LineStyle','–.','Color','b')
        ;

    % format title, legend, labels, limits, grid and box
    title(['Time: ', num2str(time(i)*1000),' ms'], interpreter, latex, fontsize, 15)
    xlabel(Time (ms), interpreter, latex, fontsize, 15)
    ylabel(Magnetisation, interpreter, latex, fontsize, 15)
    legend(M_{y'},M_{z'}, interpreter, latex, fontsize, 10)
    grid on
    box on
    xlim([0 max(time*1000)]);
    ylim([0 1]);
    hold off

    % assign frame and write it to the video
    frame = getframe(h);
    video.writeVideo(frame);
end
```

```
110
111 video.close();
```

C Problem 3 code:

C.1 Problem 3.1:

```
1  % housekeeping
2  clc
3  clear
4  %% Free procession ignoring relaxation
5  % Author: Imraj Singh 11/11/2020
6
7  % Given parameters
8  % Magnitisation aligned along y-axis initially
9  M = [0, 1, 0]';
10 M0 = [0, 0, 1]';
11
12 % Prescribe T1
13 T2 = 1;
14 T1 = T2 * 2;
15
16 % Calculate neccessary parameters
17
18 % Larmor frequency
19 omega0 = 2 * 2 * pi;
20
21 % End time
22 t = 4;
23 resolution = 100;
24
25 % Duration and timestep of model
26 time = linspace(0, t, t * resolution + 1);
27
28
29
30 Msoln(:,1) = M(1)*cos(omega0.*time) + M(2)*sin(omega0.*time);
31 Msoln(:,2) = M(2)*cos(omega0.*time) - M(1)*sin(omega0.*time);
32 Msoln(:,3) = M(3);
33
34
35 %% Animation module
36 % Author: Imraj Singh 03/11/2020
37
38 % initialise the video
39 video = VideoWriter(['3_1', '.mp4'], 'MPEG-4');
40
41 % set the frame rate quarter of real speed
42 frameRate = resolution/4;
43 video.set('FrameRate', frameRate);
44
45 video.open();
46
47 % initialise the figure
48 h = figure;
49
50 % specify animation captures each degree of rotation
51 for i=1:length(time)
52     subplot(2,2,1)
53     quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
```

```
54      hold on
55      plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
56      quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
            ,'Color','r')
57
58      % format legend, labels, limits, grid and box
59      xlabel(M_x, interpreter, latex, fontsize, 15)
60      ylabel(M_y, interpreter, latex, fontsize, 15)
61      zlabel(M_z, interpreter, latex, fontsize, 15)
62      grid on
63      box on
64      xlim([-1 1]);
65      ylim([-1 1]);
66      zlim([0 1]);
67      view(3)
68      hold off
69
70      subplot(2,2,2)
71      quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
            ,'Color','r')
72      hold on
73      plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
74      quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
75
76      % format legend, labels, limits, grid and box
77      xlabel(M_x, interpreter, latex, fontsize, 15)
78      ylabel(M_y, interpreter, latex, fontsize, 15)
79      zlabel(M_z, interpreter, latex, fontsize, 15)
80      grid on
81      box on
82      xlim([-1 1]);
83      ylim([-1 1]);
84      zlim([0 1]);
85      view(2)
86      hold off
87
88      subplot(2,2,3:4)
89      plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
90      hold on
91      plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
92
93      % format title, legend, labels, limits, grid and box
94      title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
95      xlabel(Time (s), interpreter, latex, fontsize, 15)
96      ylabel(Magnetisation, interpreter, latex, fontsize, 15)
97      legend(M_x,M_y, interpreter, latex, fontsize, 10)
98      grid on
99      box on
100     xlim([0 max(time)]);
101     ylim([-1 1]);
102     hold off
103
104
105     % assign frame and write it to the video
106     frame = getframe(h);
107     video.writeVideo(frame);
108 end
109
110 video.close();
```

C.2 Problem 3.2:

```
% housekeeping
clc
clear
%% Free procession with relaxation
% Author: Imraj Singh 03/11/2020

% Given parameters
% Magnitisation aligned along y-axis initially
M = [0, 1, 0]';
M0 = [0, 0, 1]';

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Larmor frequency
omega0 = 2 * 2 * pi;

% End time
t = 6;
resolution = 50;

% Duration and timestep of model
time = linspace(0, t, t * resolution + 1);

Msoln(:,1) = exp(-time./T2).*(M(1)*cos(omega0.*time) + M(2)*sin(omega0.*time));
Msoln(:,2) = exp(-time./T2).*(M(2)*cos(omega0.*time) - M(1)*sin(omega0.*time));
Msoln(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));


%% Animation module
% Author: Imraj Singh 03/11/2020

% initialise the video
video = VideoWriter(['3_2', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = resolution/4;
video.set('FrameRate', frameRate);

video.open();

% initialise the figure
h = figure;

% specify animation captures each degree of rotation
for i=1:length(time)
    subplot(2,2,1)
    quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
        ,'Color','r')

    % format legend, labels, limits, grid and box
    xlabel(M_x, interpreter, latex, fontsize, 15)
    ylabel(M_y, interpreter, latex, fontsize, 15)
```

```
    zlabel(M_z, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(3)
    hold off

    subplot(2,2,2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
        ,'Color','r')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k—','linewidth',2)
    quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','—','Color','k')

    % format legend, labels, limits, grid and box
    xlabel(M_x, interpreter, latex, fontsize, 15)
    ylabel(M_y, interpreter, latex, fontsize, 15)
    zlabel(M_z, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(2)
    hold off

    subplot(2,2,3:4)
    plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
    hold on
    plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
    plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','—','Color','k');

    % format title, legend, labels, limits, grid and box
    title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(Magnetisation, interpreter, latex, fontsize, 15)
    legend(M_x,M_y, M_z,interpreter, latex, fontsize, 10)
    grid on
    box on
    xlim([0 max(time)]);
    ylim([-1 1]);
    hold off


    % assign frame and write it to the video
    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();

%% Animation module - just the 3D
% Author: Imraj Singh 03/11/2020

% initialise the video
video = VideoWriter(['3_2_3D', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = resolution;
```

```
video.set('FrameRate', frameRate);

video.open();

% initialise the figure
h = figure;

% specify animation captures each degree of rotation
for i=1:length(time)
    quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','—','Color','k')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k—','linewidth',2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','–'
        ,'Color','r')

    % format title, legend, labels, limits, grid and box
    title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
    xlabel(M_x, interpreter, latex, fontsize, 15)
    ylabel(M_y, interpreter, latex, fontsize, 15)
    zlabel(M_z, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(3)
    hold off
    % assign frame and write it to the video
    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();
```

C.3 Problem 3.3:

```
% housekeeping
clc
clear
%% Free procession with relaxation rotating FoR
% Author: Imraj Singh 03/11/2020

% Given parameters
% Magnitisation aligned along y-axis initially
M = [0, 1, 0]';
M0 = [0, 0, 1]';

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Precession frequency
omega0 = 2 * 2 * pi;

% End time
t = 6;
resolution = 50;

```

```
% Duration and timestep of model
time = linspace(0, t, t * resolution + 1);


Msoln(:,1) = exp(-time./T2).*M(1);
Msoln(:,2) = exp(-time./T2).*M(2);
Msoln(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));


%% Animation

% initialise the video
video = VideoWriter(['3_3', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = resolution/4;
video.set('FrameRate', frameRate);

video.open();

% initialise the figure
h = figure;

% specify animation captures each degree of rotation
for i=1:length(time)
    subplot(2,2,1)
    quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
        ,'Color','r')

    % format legend, labels, limits, grid and box
    xlabel(M_{x'}, interpreter, latex, fontsize, 15)
    ylabel(M_{y'}, interpreter, latex, fontsize, 15)
    zlabel(M_{z'}, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(3)
    hold off

    subplot(2,2,2)
    quiver(0,0,Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-','Color','r')
    hold on
    plot(Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver(0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')

    % format legend, labels, limits, grid and box
    xlabel(M_{y'}, interpreter, latex, fontsize, 15)
    ylabel(M_{z'}, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([0 1]);
    hold off

    subplot(2,2,3:4)
    plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
```

```
    hold on
    plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
    plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','--','Color','k');

    % format title, legend, labels, limits, grid and box
    title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(Magnetisation, interpreter, latex, fontsize, 15)
    legend(M_{x'},M_{y'}, M_{z'},interpreter, latex, fontsize, 10)
    grid on
    box on
    xlim([0 max(time)]);
    ylim([0 1]);
    hold off


    % assign frame and write it to the video
    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();
```

C.4 Problem 3.4:

```
% housekeeping
clc
clear
%% Free procession with relaxation rotating FoR at Larmor
% Faster than Larmor by omega0/10
% Author: Imraj Singh 03/11/2020

% Given parameters
% Magnitisation aligned along y-axis initially
M = [0, 1, 0]';
M0 = [0, 0, 1]';

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Precession frequency
omega0 = 2 * 2 * pi;

% End time
t = 6;
resolution = 50;

% Duration and timestep of model
time = linspace(0, t, t * resolution + 1);



Msoln(:,1) = exp(-time./T2).*(M(1)*cos(omega0/10.*time) + M(2)*sin(omega0/10.*
    time));
Msoln(:,2) = exp(-time./T2).*(M(2)*cos(omega0/10.*time) - M(1)*sin(omega0/10.*
    time));
Msoln(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));
```

```
34
35
36 %% Animation module
37 % Author: Imraj Singh 03/11/2020
38
39 % initialise the video
40 video = VideoWriter(['3_4', '.mp4'], 'MPEG-4');
41
42 % set the frame rate
43 frameRate = resolution/4;
44 video.set('FrameRate', frameRate);
45
46 video.open();
47
48 % initialise the figure
49 h = figure;
50
51 % specify animation captures each degree of rotation
52 for i=1:length(time)
53     subplot(2,2,1)
54     quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
55     hold on
56     plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
57     quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
           ,'Color','r')
58
59     % format legend, labels, limits, grid and box
60     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
61     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
62     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
63     grid on
64     box on
65     xlim([-1 1]);
66     ylim([-1 1]);
67     zlim([0 1]);
68     view(3)
69     hold off
70
71     subplot(2,2,2)
72     quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
           ,'Color','r')
73     hold on
74     plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
75     quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
76
77     % format legend, labels, limits, grid and box
78     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
79     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
80     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
81     grid on
82     box on
83     xlim([-1 1]);
84     ylim([-1 1]);
85     zlim([0 1]);
86     view(2)
87     hold off
88
89     subplot(2,2,3:4)
90     plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
91     hold on
92     plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
```

```
    plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','--','Color','k');

    % format title, legend, labels, limits, grid and box
    title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(Magnetisation, interpreter, latex, fontsize, 15)
    legend(M_{x'},M_{y'}, M_{z'},interpreter, latex, fontsize, 10)
    grid on
    box on
    xlim([0 max(time)]);
    ylim([-1 1]);
    hold off


    % assign frame and write it to the video
    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();
```

C.5 Problem 3.4:

```
% housekeeping
clc
clear
%% Free procession with relaxation rotating FoR at Larmor
% Faster than Larmor by omega0/10
% Author: Imraj Singh 03/11/2020

% Given parameters
% Magnitisation aligned along y-axis initially
M = [0, 1, 0]';
M0 = [0, 0, 1]';

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Precession frequency
omega0 = 2 * 2 * pi;

% End time
t = 6;
resolution = 50;

% Duration and timestep of model
time = linspace(0, t, t * resolution + 1);



Msoln(:,1) = exp(-time./T2).*(M(1)*cos(omega0/10.*time) + M(2)*sin(omega0/10.*
    time));
Msoln(:,2) = exp(-time./T2).*(M(2)*cos(omega0/10.*time) - M(1)*sin(omega0/10.*
    time));
Msoln(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));


```

```
%% Animation module
% Author: Imraj Singh 03/11/2020

% initialise the video
video = VideoWriter(['3_4', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = resolution/4;
video.set('FrameRate', frameRate);

video.open();

% initialise the figure
h = figure;

% specify animation captures each degree of rotation
for i=1:length(time)
    subplot(2,2,1)
    quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
        ,'Color','r')

    % format legend, labels, limits, grid and box
    xlabel($M_{x'}$, interpreter, latex, fontsize, 15)
    ylabel($M_{y'}$, interpreter, latex, fontsize, 15)
    zlabel($M_{z'}$, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(3)
    hold off

    subplot(2,2,2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
        ,'Color','r')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')

    % format legend, labels, limits, grid and box
    xlabel($M_{x'}$, interpreter, latex, fontsize, 15)
    ylabel($M_{y'}$, interpreter, latex, fontsize, 15)
    zlabel($M_{z'}$, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([0 1]);
    view(2)
    hold off

    subplot(2,2,3:4)
    plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
    hold on
    plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
    plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','--','Color','k');

```

```
    % format title, legend, labels, limits, grid and box
    title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(Magnetisation, interpreter, latex, fontsize, 15)
    legend(M_x' ,M_y', M_z' ,interpreter, latex, fontsize, 10)
    grid on
    box on
    xlim([0 max(time)]);
    ylim([-1 1]);
    hold off


    % assign frame and write it to the video
    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();
```

C.6 Problem 3.5:

```
% housekeeping
clc
clear
%% Free procession with relaxation rotating FoR at Larmor
% Faster than Larmor by omega0/10
% Author: Imraj Singh 03/11/2020

% Given parameters
% Magnitisation aligned along y-axis initially
M = [0, 1, 0]';
M0 = [0, 0, 1]';

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Precession frequency
omega0 = 2 * 2 * pi;

% End time
t = 6;
resolution = 50;

% Duration and timestep of model
time = linspace(0, t, t * resolution + 1);



Msoln(:,1) = exp(-time./T2).*(M(1)*cos(-omega0/10.*time) + M(2)*sin(-omega0/10.*
    time));
Msoln(:,2) = exp(-time./T2).*(M(2)*cos(-omega0/10.*time) - M(1)*sin(-omega0/10.*
    time));
Msoln(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));


%% Animation module
% Author: Imraj Singh 03/11/2020
```

```
38
39 % initialise the video
40 video = VideoWriter(['3_4', '.mp4'], 'MPEG-4');
41
42 % set the frame rate
43 frameRate = resolution/4;
44 video.set('FrameRate', frameRate);
45
46 video.open();
47
48 % initialise the figure
49 h = figure;
50
51 % specify animation captures each degree of rotation
52 for i=1:length(time)
53     subplot(2,2,1)
54     quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
55     hold on
56     plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
57     quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
           ,'Color','r')
58
59     % format legend, labels, limits, grid and box
60     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
61     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
62     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
63     grid on
64     box on
65     xlim([-1 1]);
66     ylim([-1 1]);
67     zlim([0 1]);
68     view(3)
69     hold off
70
71     subplot(2,2,2)
72     quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
           ,'Color','r')
73     hold on
74     plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
75     quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
76
77     % format legend, labels, limits, grid and box
78     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
79     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
80     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
81     grid on
82     box on
83     xlim([-1 1]);
84     ylim([-1 1]);
85     zlim([0 1]);
86     view(2)
87     hold off
88
89     subplot(2,2,3:4)
90     plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
91     hold on
92     plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
93     plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','--','Color','k');
94
95     % format title, legend, labels, limits, grid and box
96     title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
```

```
97      xlabel(Time (s), interpreter, latex, fontsize, 15)
98      ylabel(Magnetisation, interpreter, latex, fontsize, 15)
99      legend(M_x' ,M_y', M_z' ,interpreter, latex, fontsize, 10)
100     grid on
101     box on
102     xlim([0 max(time)]);
103     ylim([-1 1]);
104     hold off
105
106
107    % assign frame and write it to the video
108     frame = getframe(h);
109     video.writeVideo(frame);
110 end
111
112 video.close();
```

C.7 Comparison between 3.4 and 3.5:

```
1  % housekeeping
2  clc
3  clear
4  %% Free procession with relaxation rotating FoR at Larmor
5  % Faster than Larmor by omega0/10
6  % Author: Imraj Singh 03/11/2020
7
8  % Given parameters
9  % Magnitisation aligned along y-axis initially
10 M = [0, 1, 0]';
11 M0 = [0, 0, 1]';
12
13 % Prescribe T1
14 T2 = 1;
15 T1 = T2 * 2;
16
17 % Calculate neccessary parameters
18
19 % Precession frequency
20 omega0 = 2 * 2 * pi;
21
22 % End time
23 t = 6;
24 resolution = 50;
25
26 % Duration and timestep of model
27 time = linspace(0, t, t * resolution + 1);
28
29 Msolnf(:,1) = exp(-time./T2).*(M(1)*cos(omega0*11/10.*time) + M(2)*sin(omega0
       *11/10.*time));
30 Msolnf(:,2) = exp(-time./T2).*(M(2)*cos(omega0*11/10.*time) - M(1)*sin(omega0
       *11/10.*time));
31 Msolnf(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));
32
33 Msolns(:,1) = exp(-time./T2).*(M(1)*cos(omega0*9/10.*time) + M(2)*sin(omega0
       *9/10.*time));
34 Msolns(:,2) = exp(-time./T2).*(M(2)*cos(omega0*9/10.*time) - M(1)*sin(omega0
       *9/10.*time));
35 Msolns(:,3) = M(3)*exp(-time./T1) + M0(3)*(1-exp(-time./T1));
36
37
```

```
38
39
40 %% Animation module - just the 3D
41 % Author: Imraj Singh 03/11/2020
42
43 % initialise the video
44 video = VideoWriter(['3_45_comp', '.mp4'], 'MPEG-4');
45
46 % set the frame rate
47 frameRate = resolution/4;
48 video.set('FrameRate', frameRate);
49
50 video.open();
51
52 % initialise the figure
53 h = figure;
54
55 % specify animation captures each degree of rotation
56 for i=1:length(time)
57     quiver3(0,0,0,Msolns(i,1),Msolns(i,2),Msolns(i,3),'linewidth',4,'LineStyle',
       '-','Color','r')
58     hold on
59     quiver3(0,0,0,Msolnf(i,1),Msolnf(i,2),Msolnf(i,3),'linewidth',4,'LineStyle',
       '-','Color','b')
60     quiver3(0,0,0,0,1,0,'linewidth',2,'LineStyle','--','Color','k')
61     plot3(Msolns(1:i,1),Msolns(1:i,2),Msolns(1:i,3),'r--','linewidth',2)
62     plot3(Msolnf(1:i,1),Msolnf(1:i,2),Msolnf(1:i,3),'b--','linewidth',2)
63
64     % format title, legend, labels, limits, grid and box
65     title(['Time: ', num2str(time(i)),' s'], interpreter, latex, fontsize, 15)
66     legend('Slower','Faster', interpreter, latex, fontsize, 10)
67     xlabel(M_x, interpreter, latex, fontsize, 15)
68     ylabel(M_y, interpreter, latex, fontsize, 15)
69     zlabel(M_z, interpreter, latex, fontsize, 15)
70     grid on
71     box on
72     xlim([-1 1]);
73     ylim([-1 1]);
74     zlim([0 1]);
75     view(3)
76     hold off
77     % assign frame and write it to the video
78     frame = getframe(h);
79     video.writeVideo(frame);
80 end
81
82 video.close();
```

D Problem 4 code:

D.1 Problem 4.1:

```
1 clc
2 clear
3
4 %% Problem 4.1 FID sequence magnetisation vector visualisation
5 % Author: Imraj Singh 11/11/2020
6
7 % Given parameters
8
```

```
9  % Magnitisation aligned along z-axis initially
10 M = [0, 0, 1]';
11
12 % Number of sample in each part of sequence
13 N = 101;
14
15 % Prescribe T1
16 T2 = 1;
17 T1 = T2 * 2;
18
19 % Calculate neccessary parameters
20
21 % Precession frequency
22 omega0 = 2 * 2 * pi;
23
24 % Flip time
25 INIT = 0;
26 tINIT = linspace(0, 0, N);
27
28 % Flip time
29 FT = .001;
30 tFT = linspace(0, FT, N);
31
32 % Relaxation time
33 RT = 8;
34 tRT = linspace(FT, RT + FT, N);
35
36 time = [tINIT, tFT, tRT];
37
38 % Indices
39 % Initial time
40 INITi = N;
41
42 % Flip time
43 FTi = INITi + N;
44
45 % Relaxation time
46 RTi = FTi + N;
47
48 Msoln = zeros(length(time),3);
49
50 % Initial time
51 Msoln(1:INITi,1) = M(1);
52 Msoln(1:INITi,2) = M(2);
53 Msoln(1:INITi,3) = M(3);
54
55 % Flip time
56 Vis90 = pi/2/FT;
57 M = Msoln(INITi,:);
58 Msoln(INITi:FTi,1) = M(1);
59 Msoln(INITi:FTi,2) = M(2)*cos(Vis90.*time(INITi:FTi)) + M(3)*sin(Vis90.*time(
       INITi:FTi));
60 Msoln(INITi:FTi,3) = M(3)*cos(Vis90.*time(INITi:FTi)) - M(2)*sin(Vis90.*time(
       INITi:FTi));
61
62 % Flip time 2
63 M = Msoln(FTi,:);
64 Msoln(FTi:RTi,1) = M(1).*exp(-time(FTi:RTi)./T2);
65 Msoln(FTi:RTi,2) = M(2).*exp(-time(FTi:RTi)./T2);
66 Msoln(FTi:RTi,3) = M(3)*exp(-time(FTi:RTi)./T1) + (1-exp(-time(FTi:RTi)./T1));
67
```

```
68
69 %% Animation module
70 % Author: Imraj Singh 03/11/2020
71
72 % initialise the video
73 video = VideoWriter(['4_1_rot', '.mp4'], 'MPEG-4');
74
75 % set the frame rate
76 frameRate = (N-1)/10;
77 video.set('FrameRate', frameRate);
78
79 video.open();
80
81 % initialise the figure
82 h = figure;
83
84 % specify animation captures each degree of rotation
85 for i=1:length(time)
86     subplot(2,2,1)
87     quiver3(0,0,0,0,0,1,'linewidth',2,'LineStyle','--','Color','k')
88     hold on
89     plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
90     quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-'
           ,'Color','r')
91
92     % format legend, labels, limits, grid and box
93     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
94     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
95     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
96     grid on
97     box on
98     xlim([-1 1]);
99     ylim([-1 1]);
100    zlim([0 1]);
101    view(3)
102    hold off
103
104    subplot(2,2,2)
105    quiver(0,0,Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-','Color','r')
106    hold on
107    plot(Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
108    quiver(0,0,0,1,'linewidth',2,'LineStyle','--','Color','k')
109
110    % format legend, labels, limits, grid and box
111    xlabel(M_{y'}, interpreter, latex, fontsize, 15)
112    ylabel(M_{z'}, interpreter, latex, fontsize, 15)
113    grid on
114    box on
115    xlim([-1 1]);
116    ylim([0 1]);
117    hold off
118
119    subplot(2,2,3:4)
120    plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
121    hold on
122    plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
123    plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','--','Color','k');
124
125    % format title, legend, labels, limits, grid and box
126
127    if i > 0 && i <= INITi
```

```
128             title('Magnetisation vector along $B_0$ initially', interpreter, latex,
                    fontsize, 15)
129         xlim([0 1]);
130         xlabel(Time (s), interpreter, latex, fontsize, 15)
131         elseif i > INITi && i <= FTi
132             title(['RF pulse $\pi / 2$, time: ',num2str(round(1000*time(i), 3)),' ms
                    '], interpreter, latex, fontsize, 15)
133         xlim([0 FT]);
134         xlabel(Time (s), interpreter, latex, fontsize, 15)
135         elseif i > FTi && i <= RTi
136             title(['Time from $\pi / 2$ flip: ', num2str(round(time(i) - INIT - FT,
                    3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)
137         xlim([0 RT]);
138         xlabel(Time (s), interpreter, latex, fontsize, 15)
139         end
140         ylabel(Magnetisation, interpreter, latex, fontsize, 15)
141         legend(M_{x'},M_{y'}, M_{z'},interpreter, latex, fontsize, 10)
142         grid on
143         box on
144         ylim([0 1]);
145         hold off
146
147
148         % assign frame and write it to the video
149         frame = getframe(h);
150         video.writeVideo(frame);
151     end
152
153     video.close();
```

D.2 Problem 4.2:

```
1   clc
2   clear
3
4   %% Problem 4.1 IR sequence magnetisation vector visualisation
5   % Author: Imraj Singh 11/11/2020
6
7   % Given parameters
8
9   % Magnitisation aligned along z-axis initially
10  M = [0, 0, 1]';
11
12  % Number of sample in each part of sequence
13  N = 101;
14
15  % Prescribe T1
16  T2 = 1;
17  T1 = T2 * 2;
18
19  % Calculate neccessary parameters
20
21  % Precession frequency
22  omega0 = 2 * 2 * pi;
23
24  % Flip time
25  INIT = 0;
26  tINIT = linspace(0, 0, N);
27
28  % Flip time 1
```

```
29 FT_1 = .002;
30 tFT_1 = linspace(0, FT_1, N);
31
32 % Relaxation time 1
33 RT_1 = .5;
34 tRT_1 = linspace(FT_1, RT_1 + FT_1, N);
35
36 % Flip time 2
37 FT_2 = .001;
38 tFT_2 = linspace(RT_1 + FT_1, RT_1 + FT_1 + FT_2, N);
39
40 % Relaxation time 2
41 RT_2 = 8;
42 tRT_2 = linspace(RT_1 + FT_1 + FT_2, + RT_1 + FT_1 + FT_2 + RT_2, N);
43
44 time = [tINIT, tFT_1, tRT_1, tFT_2, tRT_2];
45
46 % Indices
47 % Initial time
48 INITi_1 = N;
49
50 % Flip time 1
51 FTi_1 = INITi_1 + N;
52
53 % Relaxation time 1
54 RTi_1 = FTi_1 + N;
55
56 % Flip time 2
57 FTi_2 = RTi_1 + N;
58
59 % Relaxation time 2
60 RTi_2 = FTi_2 + N;
61
62 Msoln = zeros(length(time),3);
63
64 % Initial time
65 Msoln(1:INITi_1,1) = M(1);
66 Msoln(1:INITi_1,2) = M(2);
67 Msoln(1:INITi_1,3) = M(3);
68
69 % Flip time 1
70 tad = time;
71 Vis90 = pi/FT_1;
72 M = Msoln(INITi_1,:);
73 Msoln(INITi_1:FTi_1,1) = M(1);
74 Msoln(INITi_1:FTi_1,2) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*sin(Vis90.*
       tad(INITi_1:FTi_1));
75 Msoln(INITi_1:FTi_1,3) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*sin(Vis90.*
       tad(INITi_1:FTi_1));
76
77
78 % Relaxation time 1
79 tad = tad - FT_1;
80 M = Msoln(FTi_1,:);
81 Msoln(FTi_1:RTi_1,1) = M(1).*exp(-tad(FTi_1:RTi_1)./T2);
82 Msoln(FTi_1:RTi_1,2) = M(2).*exp(-tad(FTi_1:RTi_1)./T2);
83 Msoln(FTi_1:RTi_1,3) = M(3)*exp(-tad(FTi_1:RTi_1)./T1) + (1-exp(-tad(FTi_1:RTi_1
       )./T1));
84
85 % Flip time 2
86 tad = tad - RT_1;
```

```
Vis90 = pi/2/FT_2;
M = Msoln(RTi_1,:);
Msoln(RTi_1:FTi_2,1) = M(1);
Msoln(RTi_1:FTi_2,2) = M(2)*cos(Vis90.*tad(RTi_1:FTi_2)) + M(3)*sin(Vis90.*tad(RTi_1:FTi_2));
Msoln(RTi_1:FTi_2,3) = M(3)*cos(Vis90.*tad(RTi_1:FTi_2)) - M(2)*sin(Vis90.*tad(RTi_1:FTi_2));

% Relaxation time 2
tad = tad - FT_2;
M = Msoln(FTi_2,:);
Msoln(FTi_2:RTi_2,1) = M(1).*exp(-tad(FTi_2:RTi_2)./T2);
Msoln(FTi_2:RTi_2,2) = M(2).*exp(-tad(FTi_2:RTi_2)./T2);
Msoln(FTi_2:RTi_2,3) = M(3)*exp(-tad(FTi_2:RTi_2)./T1) + (1-exp(-tad(FTi_2:RTi_2)./T1));


%% Animation module
% Author: Imraj Singh 03/11/2020

% initialise the video
video = VideoWriter(['4_2_rot_opt', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = (N-1)/10;
video.set('FrameRate', frameRate);

video.open();

% initialise the figure
h = figure;

% specify animation captures each degree of rotation
for i=1:length(time)
    subplot(2,2,1)
    quiver3(0,0,0,0,0,1,'linewidth',2,'LineStyle','--','Color','k')
    hold on
    plot3(Msoln(1:i,1),Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver3(0,0,0,Msoln(i,1),Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-','Color','r')

    % format legend, labels, limits, grid and box
    xlabel(M_{x'}, interpreter, latex, fontsize, 15)
    ylabel(M_{y'}, interpreter, latex, fontsize, 15)
    zlabel(M_{z'}, interpreter, latex, fontsize, 15)
    grid on
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([-1 1]);
    view(3)
    hold off

    subplot(2,2,2)
    quiver(0,0,Msoln(i,2),Msoln(i,3),'linewidth',4,'LineStyle','-','Color','r')
    hold on
    plot(Msoln(1:i,2),Msoln(1:i,3),'k--','linewidth',2)
    quiver(0,0,0,1,'linewidth',2,'LineStyle','--','Color','k')

    % format legend, labels, limits, grid and box
    xlabel(M_{y'}, interpreter, latex, fontsize, 15)
```

```
144     ylabel(M_z', interpreter, latex, fontsize, 15)
145     grid on
146     box on
147     xlim([-1 1]);
148     ylim([-1 1]);
149     hold off
150
151     subplot(2,2,3:4)
152     plot(time(1:i),Msoln(1:i,1),'linewidth',2,'LineStyle','-','Color','r');
153     hold on
154     plot(time(1:i),Msoln(1:i,2),'linewidth',2,'LineStyle','-.','Color','b');
155     plot(time(1:i),Msoln(1:i,3),'linewidth',2,'LineStyle','--','Color','k');
156
157     % format title, legend, labels, limits, grid and box
158
159     if i > 0 && i <= INITi_1
160         title('Magnetisation vector along $B_0$ initially', interpreter, latex,
                fontsize, 15)
161         xlim([0 1]);
162         xlabel(Time (s), interpreter, latex, fontsize, 15)
163     elseif i > INITi_1 && i <= FTi_1
164         xlim([0 FT_1]);
165         title(['RF pulse $\pi $, time: ',num2str(round(1000*time(i), 3)),' ms'],
              interpreter, latex, fontsize, 15)
166         xlabel(Time (s), interpreter, latex, fontsize, 15)
167     elseif i > FTi_1 && i <= RTi_1
168         title(['Time from $\pi$ flip: ', num2str(round(time(i) - INIT - FT_1, 3)
              ),' s (Relaxation)'], interpreter, latex, fontsize, 15)
169         xlim([0 FT_1+RT_1+FT_2+RT_2]);
170         xlabel(Time (s), interpreter, latex, fontsize, 15)
171     elseif i > RTi_1 && i <= FTi_2
172         title(['RF pulse $\pi / 2$, time: ',num2str(round(1000*(time(i) - FT_1 -
              RT_1), 3)),' ms'], interpreter, latex, fontsize, 15)
173         xlim([0 FT_1+RT_1+FT_2+RT_2]);
174         xlabel(Time (s), interpreter, latex, fontsize, 15)
175     elseif i > FTi_2 && i <= RTi_2
176         title(['Time from $\pi / 2$ flip: ', num2str(round(time(i) - INIT - FT_1
              - RT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)
177         xlim([0 FT_1+RT_1+FT_2+RT_2]);
178         xlabel(Time (s), interpreter, latex, fontsize, 15)
179     end
180
181     ylabel(Magnetisation, interpreter, latex, fontsize, 15)
182     legend(M_x', M_y', M_z', interpreter, latex, fontsize, 10)
183     grid on
184     box on
185     ylim([-1 1]);
186     hold off
187
188
189     % assign frame and write it to the video
190     frame = getframe(h);
191     video.writeVideo(frame);
192 end
193
194 video.close();
```

E Problem 5 code:

E.1 Problem 5.1:

```
clc
clear

%% Free procession with Dephasing
% Author: Imraj Singh 03/11/2020

% Given parameters

% Number of sample in each part of sequence
N = 101;

% Number of isochromats
Num = 1001;

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Precession frequency
omega0 = 50 * pi;
deltaomega = 16;
omegaovector = linspace(-deltaomega,deltaomega,Num);

% Inital time 1
INIT_1 = 0;
tINIT_1 = linspace(0, INIT_1, N);

% Flip time 1
FT_1 = .001;
tFT_1 = linspace(0, FT_1, N);

% Dephasing time 1
DP_1 = .5;
tDP_1 = linspace(FT_1, DP_1 + FT_1, N);

% Flip time 2
FT_2 = .001;
tFT_2 = linspace(DP_1 + FT_1, DP_1 + FT_1 + FT_2, N);

% Rephasing time
RP = DP_1;
tRP = linspace(DP_1 + FT_1 + FT_2, DP_1 + FT_1 + FT_2 + RP, N);

% Inital time 2
INIT_2 = 0;
tINIT_2 = linspace(DP_1 + FT_1 + FT_2 + RP,  DP_1 + FT_1 + FT_2 + RP, N);

% Dephasing time 2
DP_2 = DP_1;
tDP_2 = linspace(DP_1 + FT_1 + FT_2 + RP, + DP_1 + FT_1 + FT_2 + RP + DP_2, N);

time = [tINIT_1, tFT_1, tDP_1, tFT_2, tRP, tINIT_2, tDP_2];

% Indices
% Initial time
INITi_1 = N;
```

```
59
60 % Flip time 1
61 FTi_1 = INITi_1 + N;
62
63 % Dephasing time 1
64 DPi_1 = FTi_1 + N;
65
66 % Flip time 2
67 FTi_2 = DPi_1 + N;
68
69 % Rephasing time
70 RPi = FTi_2 + N;
71
72 % Initial time
73 INITi_2 = RPi + N;
74
75 % Dephasing time 2
76 DPi_2 = INITi_2 + N;
77
78
79 Msoln = zeros(length(time),3, length(omegaovector));
80
81 for i = 1:length(omegaovector)
82     % Initial time
83     Msoln(1:INITi_1,1,i) = 0;
84     Msoln(1:INITi_1,2,i) = 0;
85     Msoln(1:INITi_1,3,i) = 1;
86
87     % Flip time 1
88     tad = time;
89     Vis90 = pi/2/FT_1;
90     M = Msoln(INITi_1,:,i);
91     Msoln(INITi_1:FTi_1,1,i) = M(1);
92     Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*sin(
           Vis90.*tad(INITi_1:FTi_1));
93     Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*sin(
           Vis90.*tad(INITi_1:FTi_1));
94
95     % Dephase time 1
96     tad = tad - FT_1;
97     M = Msoln(FTi_1,:,i);
98     Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) + M(2)
           *sin(omegaovector(i).*tad(FTi_1:DPi_1)));
99     Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) - M(1)
           *sin(omegaovector(i).*tad(FTi_1:DPi_1)));
100    Msoln(FTi_1:DPi_1,3,i) = M(3);
101
102    % Flip time 2
103    tad = tad - DP_1;
104    Vis90 = pi/FT_2;
105    M = Msoln(DPi_1,:,i);
106    Msoln(DPi_1:FTi_2,1,i) = M(1)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(Vis90
           .*tad(DPi_1:FTi_2));
107    Msoln(DPi_1:FTi_2,2,i) = M(2);
108    Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(Vis90
           .*tad(DPi_1:FTi_2));
109
110    % Rephase time
111    tad = tad - FT_2;
112    M = Msoln(FTi_2,:,i);
```

```
113     Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*sin(
            omegaovector(i).*tad(FTi_2:RPi));
114     Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*sin(
            omegaovector(i).*tad(FTi_2:RPi));
115     Msoln(FTi_2:RPi,3,i) = M(3);
116
117     % Echo time
118     tad = tad - RP;
119     M = Msoln(RPi,:,i);
120     Msoln(RPi:INITi_2,1,i) = M(1);
121     Msoln(RPi:INITi_2,2,i) = M(2);
122     Msoln(RPi:INITi_2,3,i) = M(3);
123
124
125     % Dephase time 2
126     tad = tad - INIT_2;
127     M = Msoln(INITi_2,:,i);
128     Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2)) + M
            (2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
129     Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2)) - M
            (1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
130     Msoln(INITi_2:DPi_2,3,i) = M(3);
131 end
132 Mperp = zeros(length(time),1);
133 for i=1:length(time)
134     Mperp(i) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i,2,:))^2)/
            Num;
135 end
136
137 %% Animation module
138
139 video = VideoWriter(['5_1', '.mp4'], 'MPEG-4');
140
141 % set the frame rate
142 frameRate = N/8;
143 video.set('FrameRate', frameRate);
144
145 video.open();
146
147 h = figure;
148
149
150 for i=INITi_1:length(time)
151     subplot(2,2,1)
152
153     quiver3(0,0,0,Msoln(i,1,1),Msoln(i,2,1),Msoln(i,3,1),'linewidth',5,'
            LineStyle','-','Color','r')
154     hold on
155     for j=1:100:length(omegaovector)
156         quiver3(0,0,0,Msoln(i,1,j),Msoln(i,2,j),Msoln(i,3,j),'linewidth',5,'
                LineStyle','-','Color','r')
157     end
158     grid on
159     box on
160     hold off
161     xlim([-1 1]);
162     ylim([-1 1]);
163     zlim([-1 1]);
164     xlabel(M_x', interpreter, latex, fontsize, 15)
165     ylabel(M_y', interpreter, latex, fontsize, 15)
166     zlabel(M_z', interpreter, latex, fontsize, 15)
```

```
167
168     subplot(2,2,2)
169     quiver3(0,0,0,Msoln(i,1,1),Msoln(i,2,1),Msoln(i,3,1),'linewidth',5,'
            LineStyle','-','Color','r')
170     hold on
171     for j=1:100:length(omegaovector)
172         quiver3(0,0,0,Msoln(i,1,j),Msoln(i,2,j),Msoln(i,3,j),'linewidth',5,'
                LineStyle','-','Color','r')
173     end
174     grid on
175     box on
176     xlim([-1 1]);
177     ylim([-1 1]);
178     zlim([-1 1]);
179     hold off
180     view(2)
181     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
182     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
183     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
184
185
186     subplot(2,2,3:4)
187
188     plot(time(1:i),Mperp(1:i),'linewidth',2,'LineStyle','-','Color','k')
189     hold on
190     plot(time(:),exp(-time(:)/T2),'linewidth',2,'LineStyle','--','Color','k')
191     hold off
192     grid on
193     box on
194     ylim([0 1]);
195     xlim([0 max(time)]);
196     xlabel(Time (s), interpreter, latex, fontsize, 15)
197     ylabel(Normalised signal, interpreter, latex, fontsize, 15)
198     legend(|\frac{1}{N}\sum M_{\perp}|,T_2 envelope, interpreter, latex, fontsize, 10)
199
200     % Initial
201     if i > 0 && i <= INITi_1
202         title('Magnetisation vector along $B_0$ initially', interpreter, latex,
                fontsize, 15)
203
204         % First flip
205     elseif i > INITi_1 && i <= FTi_1
206         xlim([0 FT_1]);
207         title('RF pulse $\pi /2$ ', interpreter, latex, fontsize, 15)
208
209         % First Dephase
210     elseif i > FTi_1 && i <= DPi_1
211         title(['Isocromats dephasing, t = ', num2str(round(time(i) - INIT_1 -
                FT_1, 3)),' s'], interpreter, latex, fontsize, 15)
212
213         % Second flip
214     elseif i > DPi_1 && i <= FTi_2
215         title(['RF pulse $\pi$, $t = \tau$ = ', num2str(round(DP_1, 3)),' s'],
                interpreter, latex, fontsize, 15)
216
217         % Rephasing
218     elseif i > FTi_2 && i <= RPi
219         title(['Isocromats rephasing, $t =$ ', num2str(round(time(i) - INIT_1 -
                FT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)
220
221         % Echo
```

```
    elseif i > RPi && i <= INITi_2
        title(['Echo time, $t = T_E = 2 \tau =$ ',num2str(round(DP_1 * 2, 3)),' s'], interpreter, latex, fontsize, 15)

        % Dephasing
    elseif i > INITi_2 && i <= DPi_2
        title(['Isocromats dephasing, $t =$ ', num2str(round(time(i) - INIT_1 - FT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)

    end

    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();
```

E.2 Problem 5.2:

```
clc
clear

%% Free procession with Dephasing
% Author: Imraj Singh 03/11/2020

% Given parameters

% Number of sample in each part of sequence
N = 11;

% Number of isochromats
Num = 1001;

% Prescribe T1
T2 = 1;
T1 = T2 * 2;

% Calculate neccessary parameters

% Precession frequency
omega0 = 50 * pi;
deltaomega = 0;
delta = 8;
omegaovector = deltaomega+delta*tan(pi*(rand(Num,1)-1/2));

% Inital time 1
INIT_1 = 0;
tINIT_1 = linspace(0, INIT_1, N);

% Flip time 1
FT_1 = .001;
tFT_1 = linspace(0, FT_1, N);

% Dephasing time 1
DP_1 = .5;
tDP_1 = linspace(FT_1, DP_1 + FT_1, N);

% Flip time 2
FT_2 = .001;
tFT_2 = linspace(DP_1 + FT_1, DP_1 + FT_1 + FT_2, N);
```

```
42
43 % Rephasing time
44 RP = DP_1;
45 tRP = linspace(DP_1 + FT_1 + FT_2, DP_1 + FT_1 + FT_2 + RP, N);
46
47 % Inital time 2
48 INIT_2 = 0;
49 tINIT_2 = linspace(DP_1 + FT_1 + FT_2 + RP,  DP_1 + FT_1 + FT_2 + RP, N);
50
51 % Dephasing time 2
52 DP_2 = DP_1;
53 tDP_2 = linspace(DP_1 + FT_1 + FT_2 + RP, + DP_1 + FT_1 + FT_2 + RP + DP_2, N);
54
55 time = [tINIT_1, tFT_1, tDP_1, tFT_2, tRP, tINIT_2, tDP_2];
56
57 % Indices
58 % Initial time
59 INITi_1 = N;
60
61 % Flip time 1
62 FTi_1 = INITi_1 + N;
63
64 % Dephasing time 1
65 DPi_1 = FTi_1 + N;
66
67 % Flip time 2
68 FTi_2 = DPi_1 + N;
69
70 % Rephasing time
71 RPi = FTi_2 + N;
72
73 % Initial time
74 INITi_2 = RPi + N;
75
76 % Dephasing time 2
77 DPi_2 = INITi_2 + N;
78
79
80 Msoln = zeros(length(time),3, length(omegaovector));
81
82 for i = 1:length(omegaovector)
83     % Initial time
84     Msoln(1:INITi_1,1,i) = 0;
85     Msoln(1:INITi_1,2,i) = 0;
86     Msoln(1:INITi_1,3,i) = 1;
87
88     % Flip time 1
89     tad = time;
90     Vis90 = pi/2/FT_1;
91     M = Msoln(INITi_1,:,i);
92     Msoln(INITi_1:FTi_1,1,i) = M(1);
93     Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*sin(
           Vis90.*tad(INITi_1:FTi_1));
94     Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*sin(
           Vis90.*tad(INITi_1:FTi_1));
95
96     % Dephase time 1
97     tad = tad - FT_1;
98     M = Msoln(FTi_1,:,i);
99     Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) + M(2)
           *sin(omegaovector(i).*tad(FTi_1:DPi_1)));
```

```
100     Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) - M(1)
            *sin(omegaovector(i).*tad(FTi_1:DPi_1)));
101     Msoln(FTi_1:DPi_1,3,i) = M(3);
102
103     % Flip time 2
104     tad = tad - DP_1;
105     Vis90 = pi/FT_2;
106     M = Msoln(DPi_1,:,i);
107     Msoln(DPi_1:FTi_2,1,i) = M(1)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(Vis90
            .*tad(DPi_1:FTi_2));
108     Msoln(DPi_1:FTi_2,2,i) = M(2);
109     Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(Vis90
            .*tad(DPi_1:FTi_2));
110
111     % Rephase time
112     tad = tad - FT_2;
113     M = Msoln(FTi_2,:,i);
114     Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*sin(
            omegaovector(i).*tad(FTi_2:RPi));
115     Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*sin(
            omegaovector(i).*tad(FTi_2:RPi));
116     Msoln(FTi_2:RPi,3,i) = M(3);
117
118     % Echo time
119     tad = tad - RP;
120     M = Msoln(RPi,:,i);
121     Msoln(RPi:INITi_2,1,i) = M(1);
122     Msoln(RPi:INITi_2,2,i) = M(2);
123     Msoln(RPi:INITi_2,3,i) = M(3);
124
125
126     % Dephase time 2
127     tad = tad - INIT_2;
128     M = Msoln(INITi_2,:,i);
129     Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2)) + M
            (2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
130     Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2)) - M
            (1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
131     Msoln(INITi_2:DPi_2,3,i) = M(3);
132 end
133 Mperp = zeros(length(time),1);
134 for i=1:length(time)
135     Mperp(i) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i,2,:))^2)/
            Num;
136 end
137
138 %% Animation module
139
140 video = VideoWriter(['5_2', '.mp4'], 'MPEG-4');
141
142 % set the frame rate
143 frameRate = N/8;
144 video.set('FrameRate', frameRate);
145
146 video.open();
147
148 h = figure;
149
150
151 for i=INITi_1:length(time)
152     subplot(2,2,1)
```

```
153
154     quiver3(0,0,0,Msoln(i,1,1),Msoln(i,2,1),Msoln(i,3,1),'linewidth',5,'
            LineStyle','-','Color','r')
155     hold on
156     for j=1:100:length(omegaovector)
157         quiver3(0,0,0,Msoln(i,1,j),Msoln(i,2,j),Msoln(i,3,j),'linewidth',5,'
                LineStyle','-','Color','r')
158     end
159     grid on
160     box on
161     hold off
162     xlim([-1 1]);
163     ylim([-1 1]);
164     zlim([-1 1]);
165     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
166     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
167     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
168
169     subplot(2,2,2)
170     quiver3(0,0,0,Msoln(i,1,1),Msoln(i,2,1),Msoln(i,3,1),'linewidth',5,'
            LineStyle','-','Color','r')
171     hold on
172     for j=1:100:length(omegaovector)
173         quiver3(0,0,0,Msoln(i,1,j),Msoln(i,2,j),Msoln(i,3,j),'linewidth',5,'
                LineStyle','-','Color','r')
174     end
175     grid on
176     box on
177     xlim([-1 1]);
178     ylim([-1 1]);
179     zlim([-1 1]);
180     hold off
181     view(2)
182     xlabel(M_{x'}, interpreter, latex, fontsize, 15)
183     ylabel(M_{y'}, interpreter, latex, fontsize, 15)
184     zlabel(M_{z'}, interpreter, latex, fontsize, 15)
185
186
187     subplot(2,2,3:4)
188
189     plot(time(1:i),Mperp(1:i),'linewidth',2,'LineStyle','-','Color','k')
190     hold on
191     plot(time(:),exp(-time(:)/T2),'linewidth',2,'LineStyle','--','Color','k')
192     hold off
193     grid on
194     box on
195     ylim([0 1]);
196     xlim([0 max(time)]);
197     xlabel(Time (s), interpreter, latex, fontsize, 15)
198     ylabel(Normalised signal, interpreter, latex, fontsize, 15)
199     legend(|\frac{1}{N}\sum M_{\perp}|, T_2 envelope, interpreter, latex, fontsize, 10)
200
201    % Initial
202     if i > 0 && i <= INITi_1
203         title('Magnetisation vector along $B_0$ initially', interpreter, latex,
                fontsize, 15)
204
205         % First flip
206     elseif i > INITi_1 && i <= FTi_1
207         xlim([0 FT_1]);
208         title('RF pulse $\pi/2$ ', interpreter, latex, fontsize, 15)
```

```
209
210         % First Dephase
211     elseif i > FTi_1 && i <= DPi_1
212         title(['Isocromats dephasing, t = ', num2str(round(time(i) - INIT_1 -
                FT_1, 3)),' s'], interpreter, latex, fontsize, 15)
213
214         % Second flip
215     elseif i > DPi_1 && i <= FTi_2
216         title(['RF pulse $\pi$, $t = \tau$ = ', num2str(round(DP_1, 3)),' s'],
                interpreter, latex, fontsize, 15)
217
218         % Rephasing
219     elseif i > FTi_2 && i <= RPi
220         title(['Isocromats rephasing, $t =$ ', num2str(round(time(i) - INIT_1 -
                FT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)
221
222         % Echo
223     elseif i > RPi && i <= INITi_2
224         title(['Echo time, $t = T_E = 2 \tau =$ ',num2str(round(DP_1 * 2, 3)),'
                s'], interpreter, latex, fontsize, 15)
225
226         % Dephasing
227     elseif i > INITi_2 && i <= DPi_2
228         title(['Isocromats dephasing, $t =$ ', num2str(round(time(i) - INIT_1 -
                FT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)
229
230     end
231
232     frame = getframe(h);
233     video.writeVideo(frame);
234 end
235
236 video.close();
```

E.3 Problem 5.3:

```
1  % house keeping
2  clc
3  clear
4  %% Sensitivity of Uniform Distribution to delta omega
5  % Author: Imraj Singh
6
7
8  % Given parameters
9
10 % Number of sample in each part of sequence
11 N = 101;
12
13 % Number of isochromats
14 Num = 10001;
15
16 % Prescribe T1
17 T2 = 1;
18 T1 = T2 * 2;
19
20 % Calculate neccessary parameters
21
22 % Inital time 1
23 INIT_1 = 0;
24 tINIT_1 = linspace(0, INIT_1, N);
```

```
25
26 % Flip time 1
27 FT_1 = .001;
28 tFT_1 = linspace(0, FT_1, N);
29
30 % Dephasing time 1
31 DP_1 = .5;
32 tDP_1 = linspace(FT_1, DP_1 + FT_1, N);
33
34 % Flip time 2
35 FT_2 = .001;
36 tFT_2 = linspace(DP_1 + FT_1, DP_1 + FT_1 + FT_2, N);
37
38 % Rephasing time
39 RP = DP_1;
40 tRP = linspace(DP_1 + FT_1 + FT_2, DP_1 + FT_1 + FT_2 + RP, N);
41
42 % Inital time 2
43 INIT_2 = 0;
44 tINIT_2 = linspace(DP_1 + FT_1 + FT_2 + RP,  DP_1 + FT_1 + FT_2 + RP, N);
45
46 % Dephasing time 2
47 DP_2 = DP_1;
48 tDP_2 = linspace(DP_1 + FT_1 + FT_2 + RP, + DP_1 + FT_1 + FT_2 + RP + DP_2, N);
49
50 time = [tINIT_1, tFT_1, tDP_1, tFT_2, tRP, tINIT_2, tDP_2];
51
52 % Indices
53 % Initial time
54 INITi_1 = N;
55
56 % Flip time 1
57 FTi_1 = INITi_1 + N;
58
59 % Dephasing time 1
60 DPi_1 = FTi_1 + N;
61
62 % Flip time 2
63 FTi_2 = DPi_1 + N;
64
65 % Rephasing time
66 RPi = FTi_2 + N;
67
68 % Initial time
69 INITi_2 = RPi + N;
70
71 % Dephasing time 2
72 DPi_2 = INITi_2 + N;
73
74 %% Sensitivity of Uniform Distribution to delta omega
75 % Author: Imraj Singh
76
77 % Precession frequency
78 omega0 = 50 * pi;
79 delta = [1 2 4 8 16 32 64 128];
80
81 Mperp = zeros(length(time),length(delta));
82 for Di = 1:length(delta)
83     omegaovector = linspace(-delta(Di),delta(Di),Num);
84     Msoln = zeros(length(time),3, length(omegaovector));
85
```

```
86      for i = 1:length(omegaovector)
87          % Initial time
88          Msoln(1:INITi_1,1,i) = 0;
89          Msoln(1:INITi_1,2,i) = 0;
90          Msoln(1:INITi_1,3,i) = 1;
91
92          % Flip time 1
93          tad = time;
94          Vis90 = pi/2/FT_1;
95          M = Msoln(INITi_1,:,i);
96          Msoln(INITi_1:FTi_1,1,i) = M(1);
97          Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*
                sin(Vis90.*tad(INITi_1:FTi_1));
98          Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*
                sin(Vis90.*tad(INITi_1:FTi_1));
99
100         % Dephase time 1
101         tad = tad - FT_1;
102         M = Msoln(FTi_1,:,i);
103         Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) +
                M(2)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
104         Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) -
                M(1)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
105         Msoln(FTi_1:DPi_1,3,i) = M(3);
106
107         % Flip time 2
108         tad = tad - DP_1;
109         Vis90 = pi/FT_2;
110         M = Msoln(DPi_1,:,i);
111         Msoln(DPi_1:FTi_2,1,i) = M(1);
112         Msoln(DPi_1:FTi_2,2,i) = M(2)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(
                Vis90.*tad(DPi_1:FTi_2));
113         Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(
                Vis90.*tad(DPi_1:FTi_2));
114
115         % Rephase time
116         tad = tad - FT_2;
117         M = Msoln(FTi_2,:,i);
118         Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*
                sin(omegaovector(i).*tad(FTi_2:RPi));
119         Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*
                sin(omegaovector(i).*tad(FTi_2:RPi));
120         Msoln(FTi_2:RPi,3,i) = M(3);
121
122         % Echo time
123         tad = tad - RP;
124         M = Msoln(RPi,:,i);
125         Msoln(RPi:INITi_2,1,i) = M(1);
126         Msoln(RPi:INITi_2,2,i) = M(2);
127         Msoln(RPi:INITi_2,3,i) = M(3);
128
129
130         % Dephase time 2
131         tad = tad - INIT_2;
132         M = Msoln(INITi_2,:,i);
133         Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
                + M(2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
134         Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
                - M(1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
135         Msoln(INITi_2:DPi_2,3,i) = M(3);
136     end
```

```
    for i=1:length(time)
        Mperp(i,Di) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i
            ,2,:))^2)/Num;
    end
end

video = VideoWriter(['5_3_dOmega', '.mp4'], 'MPEG-4');

% set the frame rate
frameRate = .25;
video.set('FrameRate', frameRate);

video.open();

h = figure;
for i=1:length(delta)
    plot(time(:),Mperp(:,i),'linewidth',2,'LineStyle','-','Color','k')

    grid on
    box on
    xlim([0 max(time)]);
    ylim([0 1]);
    view(2)
    title(['$\delta \omega $ = ', num2str(delta(i))], interpreter, latex, fontsize, 15)
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(|\frac{1}{N}\sum M_{\perp}|, interpreter, latex, fontsize, 15)

    frame = getframe(h);
    video.writeVideo(frame);
end

video.close();

%% Sensitivity of Cauchy to delta
% Author: Imraj Singh

% Precession frequency
omega0 = 50 * pi;
delta = [1 2 4 8 16 32 64 128];
deltaomega = 0;
Mperp = zeros(length(time),length(delta));
for Di = 1:length(delta)
    omegaovector = deltaomega+delta(Di)*tan(pi*(rand(Num,1)-1/2));
    Msoln = zeros(length(time),3, length(omegaovector));

    for i = 1:length(omegaovector)
        % Initial time
        Msoln(1:INITi_1,1,i) = 0;
        Msoln(1:INITi_1,2,i) = 0;
        Msoln(1:INITi_1,3,i) = 1;

        % Flip time 1
        tad = time;
        Vis90 = pi/2/FT_1;
        M = Msoln(INITi_1,:,i);
        Msoln(INITi_1:FTi_1,1,i) = M(1);
        Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*
            sin(Vis90.*tad(INITi_1:FTi_1));
        Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*
            sin(Vis90.*tad(INITi_1:FTi_1));

```

```
195         % Dephase time 1
196         tad = tad - FT_1;
197         M = Msoln(FTi_1,:,i);
198         Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) +
                M(2)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
199         Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) -
                M(1)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
200         Msoln(FTi_1:DPi_1,3,i) = M(3);
201
202         % Flip time 2
203         tad = tad - DP_1;
204         Vis90 = pi/FT_2;
205         M = Msoln(DPi_1,:,i);
206         Msoln(DPi_1:FTi_2,1,i) = M(1);
207         Msoln(DPi_1:FTi_2,2,i) = M(2)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(
                Vis90.*tad(DPi_1:FTi_2));
208         Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(
                Vis90.*tad(DPi_1:FTi_2));
209
210         % Rephase time
211         tad = tad - FT_2;
212         M = Msoln(FTi_2,:,i);
213         Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*
                sin(omegaovector(i).*tad(FTi_2:RPi));
214         Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*
                sin(omegaovector(i).*tad(FTi_2:RPi));
215         Msoln(FTi_2:RPi,3,i) = M(3);
216
217         % Echo time
218         tad = tad - RP;
219         M = Msoln(RPi,:,i);
220         Msoln(RPi:INITi_2,1,i) = M(1);
221         Msoln(RPi:INITi_2,2,i) = M(2);
222         Msoln(RPi:INITi_2,3,i) = M(3);
223
224
225         % Dephase time 2
226         tad = tad - INIT_2;
227         M = Msoln(INITi_2,:,i);
228         Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
                + M(2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
229         Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
                - M(1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
230         Msoln(INITi_2:DPi_2,3,i) = M(3);
231     end
232     for i=1:length(time)
233         Mperp(i,Di) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i
                ,2,:))^2)/Num;
234     end
235 end
236
237 h = figure;
238 for i=1:length(delta)
239     plot(time(:),Mperp(:,i),'linewidth',2,'LineStyle','-','Color','k')
240
241     grid on
242     box on
243     xlim([0 max(time)]);
244     ylim([0 1]);
245     view(2)
246     title(['$\Delta $ = ', num2str(delta(i))], interpreter, latex, fontsize, 15)
```

```
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(|$\frac{1}{N}\sum M_{\perp}$|, interpreter, latex, fontsize, 15)

    frame(i) = getframe(h);
end
video = VideoWriter(['5_3_Delta', '.mp4'], 'MPEG-4');
frameRate = .25;
video.set('FrameRate', frameRate);
video.open();
video.writeVideo(frame);
video.close();

%% Sensitivity of Cauchy to omega
% Author: Imraj Singh

% Precession frequency
omega0 = 50 * pi;
delta1 = 4;
deltaomega = [-100 -50 -25 -10 -5 0 5 10 25 50 100];
Mperp = zeros(length(time),length(delta));
for Di = 1:length(delta)
    omegaovector = delta(Di)+delta1*tan(pi*(rand(Num,1)-1/2));
    Msoln = zeros(length(time),3, length(omegaovector));

    for i = 1:length(omegaovector)
        % Initial time
        Msoln(1:INITi_1,1,i) = 0;
        Msoln(1:INITi_1,2,i) = 0;
        Msoln(1:INITi_1,3,i) = 1;

        % Flip time 1
        tad = time;
        Vis90 = pi/2/FT_1;
        M = Msoln(INITi_1,:,i);
        Msoln(INITi_1:FTi_1,1,i) = M(1);
        Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*
            sin(Vis90.*tad(INITi_1:FTi_1));
        Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*
            sin(Vis90.*tad(INITi_1:FTi_1));

        % Dephase time 1
        tad = tad - FT_1;
        M = Msoln(FTi_1,:,i);
        Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) +
            M(2)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
        Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) -
            M(1)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
        Msoln(FTi_1:DPi_1,3,i) = M(3);

        % Flip time 2
        tad = tad - DP_1;
        Vis90 = pi/FT_2;
        M = Msoln(DPi_1,:,i);
        Msoln(DPi_1:FTi_2,1,i) = M(1);
        Msoln(DPi_1:FTi_2,2,i) = M(2)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(
            Vis90.*tad(DPi_1:FTi_2));
        Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(
            Vis90.*tad(DPi_1:FTi_2));

        % Rephase time
        tad = tad - FT_2;
```

```
302             M = Msoln(FTi_2,:,i);
303             Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*
                    sin(omegaovector(i).*tad(FTi_2:RPi));
304             Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*
                    sin(omegaovector(i).*tad(FTi_2:RPi));
305             Msoln(FTi_2:RPi,3,i) = M(3);
306
307             % Echo time
308             tad = tad - RP;
309             M = Msoln(RPi,:,i);
310             Msoln(RPi:INITi_2,1,i) = M(1);
311             Msoln(RPi:INITi_2,2,i) = M(2);
312             Msoln(RPi:INITi_2,3,i) = M(3);
313
314
315             % Dephase time 2
316             tad = tad - INIT_2;
317             M = Msoln(INITi_2,:,i);
318             Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
                        + M(2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
319             Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
                        - M(1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
320             Msoln(INITi_2:DPi_2,3,i) = M(3);
321         end
322         for i=1:length(time)
323             Mperp(i,Di) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i
                    ,2,:))^2)/Num;
324         end
325     end
326
327
328     % set the frame rate
329
330     h = figure;
331     for i=1:length(delta)
332         plot(time(:),Mperp(:,i),'linewidth',2,'LineStyle','-','Color','k')
333
334         grid on
335         box on
336         xlim([0 max(time)]);
337         ylim([0 1]);
338         view(2)
339         title(['$\omega_0 $ = ', num2str(delta(i))], interpreter, latex, fontsize, 15)
340         xlabel(Time (s), interpreter, latex, fontsize, 15)
341         ylabel(|$\frac{1}{N}\sum M_{\perp}$|, interpreter, latex, fontsize, 15)
342         frame(i) = getframe(gcf);
343     end
344     video = VideoWriter(['5_3_omega', '.mp4'], 'MPEG-4');
345     frameRate = .25;
346     video.set('FrameRate', frameRate);
347     video.open();
348     video.writeVideo(frame);
349     video.close();
```

E.4 Problem 5.3 components plot:

```
1   clc
2   clear
3
4   %% Sensitivity of Cauchy to omega_0
```

```
5  % Author: Imraj Singh
6
7  % Given parameters
8
9  % Number of sample in each part of sequence
10 N = 101;
11
12 % Number of isochromats
13 Num = 10001;
14
15 % Prescribe T1
16 T2 = 1;
17 T1 = T2 * 2;
18
19 % Calculate neccessary parameters
20
21 % Inital time 1
22 INIT_1 = 0;
23 tINIT_1 = linspace(0, INIT_1, N);
24
25 % Flip time 1
26 FT_1 = .001;
27 tFT_1 = linspace(0, FT_1, N);
28
29 % Dephasing time 1
30 DP_1 = .5;
31 tDP_1 = linspace(FT_1, DP_1 + FT_1, N);
32
33 % Flip time 2
34 FT_2 = .001;
35 tFT_2 = linspace(DP_1 + FT_1, DP_1 + FT_1 + FT_2, N);
36
37 % Rephasing time
38 RP = DP_1;
39 tRP = linspace(DP_1 + FT_1 + FT_2, DP_1 + FT_1 + FT_2 + RP, N);
40
41 % Inital time 2
42 INIT_2 = 0;
43 tINIT_2 = linspace(DP_1 + FT_1 + FT_2 + RP,  DP_1 + FT_1 + FT_2 + RP, N);
44
45 % Dephasing time 2
46 DP_2 = DP_1;
47 tDP_2 = linspace(DP_1 + FT_1 + FT_2 + RP, + DP_1 + FT_1 + FT_2 + RP + DP_2, N);
48
49 time = [tINIT_1, tFT_1, tDP_1, tFT_2, tRP, tINIT_2, tDP_2];
50
51 % Indices
52 % Initial time
53 INITi_1 = N;
54
55 % Flip time 1
56 FTi_1 = INITi_1 + N;
57
58 % Dephasing time 1
59 DPi_1 = FTi_1 + N;
60
61 % Flip time 2
62 FTi_2 = DPi_1 + N;
63
64 % Rephasing time
65 RPi = FTi_2 + N;
```

```
66
67 % Initial time
68 INITi_2 = RPi + N;
69
70 % Dephasing time 2
71 DPi_2 = INITi_2 + N;
72
73
74 % Precession frequency
75 omega0 = 50 * pi;
76 delta1 = 4;
77 delta = [0 5 10 25 50 100 200];
78 Mperp = zeros(length(time),length(delta));
79 for Di = 1:length(delta)
80     omegaovector = delta(Di)+delta1*tan(pi*(rand(Num,1)-1/2));
81     Msoln = zeros(length(time),3, length(omegaovector));
82
83     for i = 1:length(omegaovector)
84         % Initial time
85         Msoln(1:INITi_1,1,i) = 0;
86         Msoln(1:INITi_1,2,i) = 0;
87         Msoln(1:INITi_1,3,i) = 1;
88
89         % Flip time 1
90         tad = time;
91         Vis90 = pi/2/FT_1;
92         M = Msoln(INITi_1,:,i);
93         Msoln(INITi_1:FTi_1,1,i) = M(1);
94         Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*
               sin(Vis90.*tad(INITi_1:FTi_1));
95         Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*
               sin(Vis90.*tad(INITi_1:FTi_1));
96
97         % Dephase time 1
98         tad = tad - FT_1;
99         M = Msoln(FTi_1,:,i);
100        Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) +
               M(2)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
101        Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) -
               M(1)*sin(omegaovector(i).*tad(FTi_1:DPi_1)));
102        Msoln(FTi_1:DPi_1,3,i) = M(3);
103
104        % Flip time 2
105        tad = tad - DP_1;
106        Vis90 = pi/FT_2;
107        M = Msoln(DPi_1,:,i);
108        Msoln(DPi_1:FTi_2,1,i) = M(1);
109        Msoln(DPi_1:FTi_2,2,i) = M(2)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(
               Vis90.*tad(DPi_1:FTi_2));
110        Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(
               Vis90.*tad(DPi_1:FTi_2));
111
112        % Rephase time
113        tad = tad - FT_2;
114        M = Msoln(FTi_2,:,i);
115        Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*
               sin(omegaovector(i).*tad(FTi_2:RPi));
116        Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*
               sin(omegaovector(i).*tad(FTi_2:RPi));
117        Msoln(FTi_2:RPi,3,i) = M(3);
118
```

```
        % Echo time
        tad = tad - RP;
        M = Msoln(RPi,:,i);
        Msoln(RPi:INITi_2,1,i) = M(1);
        Msoln(RPi:INITi_2,2,i) = M(2);
        Msoln(RPi:INITi_2,3,i) = M(3);


        % Dephase time 2
        tad = tad - INIT_2;
        M = Msoln(INITi_2,:,i);
        Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
            + M(2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
        Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2))
            - M(1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
        Msoln(INITi_2:DPi_2,3,i) = M(3);
    end
    for i=1:length(time)
        Mperp(i,1,Di) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i
            ,2,:))^2)/Num;
        Mperp(i,2,Di) = exp(-time(i)/T2)*sum(Msoln(i,1,:))/Num;
        Mperp(i,3,Di) = exp(-time(i)/T2)*sum(Msoln(i,2,:))/Num;
    end
end

h = figure;
for i=1:length(delta)
    plot(time(:),Mperp(:,2,i),'linewidth',2,'LineStyle','-','Color','r')
    hold on
    plot(time(:),Mperp(:,3,i),'linewidth',2,'LineStyle','-','Color','b')
    plot(time(:),exp(-time(:)/T2),'linewidth',2,'LineStyle','--','Color','k')
    plot(time(:),-exp(-time(:)/T2),'linewidth',2,'LineStyle','--','Color','k')
    hold off
    grid on
    box on
    xlim([0 max(time)]);
    ylim([-1 1]);
    title(['$\Delta $ = ', num2str(delta(i))], interpreter, latex, fontsize, 15)
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(Normalised average M_⊥, interpreter, latex, fontsize, 15)
    legend(1/N ∑ M_x' ,1/N ∑ M_y' ,T_2 envelope, interpreter, latex, fontsize, 10)

    frame(i) = getframe(h);
end
video = VideoWriter(['5_3_omega_comp', '.mp4'], 'MPEG-4');
frameRate = .25;
video.set('FrameRate', frameRate);
video.open();
video.writeVideo(frame);
video.close();
```

E.5 Problem 5.4:

```
clc
clear

%% Hahn echo
% Author: Imraj Singh 03/11/2020

% Given parameters
```

```
8
9  % Number of sample in each part of sequence
10 N = 11;
11
12 % Number of isochromats
13 Num = 1001;
14
15 % Prescribe T1
16 T2 = 1;
17 T1 = T2 * 2;
18
19 % Calculate neccessary parameters
20
21 % Precession frequency
22 omega0 = 50 * pi;
23 deltaomega = 0;
24 delta = 2;
25 omegaovector = deltaomega+delta*tan(pi*(rand(Num,1)-1/2));
26
27 % Inital time 1
28 INIT_1 = 0;
29 tINIT_1 = linspace(0, INIT_1, N);
30
31 % Flip time 1
32 FT_1 = .001;
33 tFT_1 = linspace(0, FT_1, N);
34
35 % Dephasing time 1
36 DP_1 = .5;
37 tDP_1 = linspace(FT_1, DP_1 + FT_1, N);
38
39 % Flip time 2
40 FT_2 = .002;
41 tFT_2 = linspace(DP_1 + FT_1, DP_1 + FT_1 + FT_2, N);
42
43 % Rephasing time
44 RP = DP_1;
45 tRP = linspace(DP_1 + FT_1 + FT_2, DP_1 + FT_1 + FT_2 + RP, N);
46
47 % Inital time 2
48 INIT_2 = 0;
49 tINIT_2 = linspace(DP_1 + FT_1 + FT_2 + RP,  DP_1 + FT_1 + FT_2 + RP, N);
50
51 % Dephasing time 2
52 DP_2 = DP_1;
53 tDP_2 = linspace(DP_1 + FT_1 + FT_2 + RP, + DP_1 + FT_1 + FT_2 + RP + DP_2, N);
54
55 time = [tINIT_1, tFT_1, tDP_1, tFT_2, tRP, tINIT_2, tDP_2];
56
57 % Indices
58 % Initial time
59 INITi_1 = N;
60
61 % Flip time 1
62 FTi_1 = INITi_1 + N;
63
64 % Dephasing time 1
65 DPi_1 = FTi_1 + N;
66
67 % Flip time 2
68 FTi_2 = DPi_1 + N;
```

```
69
70 % Rephasing time
71 RPi = FTi_2 + N;
72
73 % Initial time
74 INITi_2 = RPi + N;
75
76 % Dephasing time 2
77 DPi_2 = INITi_2 + N;
78
79
80 Msoln = zeros(length(time),3, length(omegaovector));
81
82 for i = 1:length(omegaovector)
83     % Initial time
84     Msoln(1:INITi_1,1,i) = 0;
85     Msoln(1:INITi_1,2,i) = 0;
86     Msoln(1:INITi_1,3,i) = 1;
87
88     % Flip time 1
89     tad = time;
90     Vis90 = pi/2/FT_1;
91     M = Msoln(INITi_1,:,i);
92     Msoln(INITi_1:FTi_1,1,i) = M(1);
93     Msoln(INITi_1:FTi_1,2,i) = M(2)*cos(Vis90.*tad(INITi_1:FTi_1)) + M(3)*sin(
           Vis90.*tad(INITi_1:FTi_1));
94     Msoln(INITi_1:FTi_1,3,i) = M(3)*cos(Vis90.*tad(INITi_1:FTi_1)) - M(2)*sin(
           Vis90.*tad(INITi_1:FTi_1));
95
96     % Dephase time 1
97     tad = tad - FT_1;
98     M = Msoln(FTi_1,:,i);
99     Msoln(FTi_1:DPi_1,1,i) = (M(1)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) + M(2)
           *sin(omegaovector(i).*tad(FTi_1:DPi_1)));
100    Msoln(FTi_1:DPi_1,2,i) = (M(2)*cos(omegaovector(i).*tad(FTi_1:DPi_1)) - M(1)
           *sin(omegaovector(i).*tad(FTi_1:DPi_1)));
101    Msoln(FTi_1:DPi_1,3,i) = M(3);
102
103    % Flip time 2
104    tad = tad - DP_1;
105    Vis90 = pi/2/FT_2;
106    M = Msoln(DPi_1,:,i);
107    Msoln(DPi_1:FTi_2,1,i) = M(1);
108    Msoln(DPi_1:FTi_2,2,i) = M(2)*cos(Vis90.*tad(DPi_1:FTi_2)) + M(3)*sin(Vis90
           .*tad(DPi_1:FTi_2));
109    Msoln(DPi_1:FTi_2,3,i) = M(3)*cos(Vis90.*tad(DPi_1:FTi_2)) - M(2)*sin(Vis90
           .*tad(DPi_1:FTi_2));
110
111    % Rephase time
112    tad = tad - FT_2;
113    M = Msoln(FTi_2,:,i);
114    Msoln(FTi_2:RPi,1,i) = M(1)*cos(omegaovector(i).*tad(FTi_2:RPi)) + M(2)*sin(
           omegaovector(i).*tad(FTi_2:RPi));
115    Msoln(FTi_2:RPi,2,i) = M(2)*cos(omegaovector(i).*tad(FTi_2:RPi)) - M(1)*sin(
           omegaovector(i).*tad(FTi_2:RPi));
116    Msoln(FTi_2:RPi,3,i) = M(3);
117
118    % Echo time
119    tad = tad - RP;
120    M = Msoln(RPi,:,i);
121    Msoln(RPi:INITi_2,1,i) = M(1);
```

```
122     Msoln(RPi:INITi_2,2,i) = M(2);
123     Msoln(RPi:INITi_2,3,i) = M(3);
124
125
126     % Dephase time 2
127     tad = tad - INIT_2;
128     M = Msoln(INITi_2,:,i);
129     Msoln(INITi_2:DPi_2,1,i) = M(1)*cos(omegaovector(i).*tad(INITi_2:DPi_2)) + M
            (2)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
130     Msoln(INITi_2:DPi_2,2,i) = M(2)*cos(omegaovector(i).*tad(INITi_2:DPi_2)) - M
            (1)*sin(omegaovector(i).*tad(INITi_2:DPi_2));
131     Msoln(INITi_2:DPi_2,3,i) = M(3);
132 end
133 Mperp = zeros(length(time),1);
134 for i=1:length(time)
135     Mperp(i) = exp(-time(i)/T2)*sqrt(sum(Msoln(i,1,:))^2 + sum(Msoln(i,2,:))^2)/
            Num;
136 end
137
138 %% Animation module
139
140 video = VideoWriter(['5_4', '.mp4'], 'MPEG-4');
141
142 % set the frame rate
143 frameRate = N/8;
144 video.set('FrameRate', frameRate);
145
146 video.open();
147
148 h = figure;
149
150
151 for i=INITi_1:length(time)
152     subplot(2,2,1)
153
154     quiver3(0,0,0,Msoln(i,1,1),Msoln(i,2,1),Msoln(i,3,1),'linewidth',5,'
            LineStyle','-','Color','r')
155     hold on
156     for j=1:100:length(omegaovector)
157         quiver3(0,0,0,Msoln(i,1,j),Msoln(i,2,j),Msoln(i,3,j),'linewidth',5,'
                LineStyle','-','Color','r')
158     end
159     grid on
160     box on
161     hold off
162     xlim([-1 1]);
163     ylim([-1 1]);
164     zlim([-1 1]);
165     xlabel(M_x', interpreter, latex, fontsize, 15)
166     ylabel(M_y', interpreter, latex, fontsize, 15)
167     zlabel(M_z', interpreter, latex, fontsize, 15)
168
169     subplot(2,2,2)
170     quiver3(0,0,0,Msoln(i,1,1),Msoln(i,2,1),Msoln(i,3,1),'linewidth',5,'
            LineStyle','-','Color','r')
171     hold on
172     for j=1:100:length(omegaovector)
173         quiver3(0,0,0,Msoln(i,1,j),Msoln(i,2,j),Msoln(i,3,j),'linewidth',5,'
                LineStyle','-','Color','r')
174     end
175     grid on
```

```
    box on
    xlim([-1 1]);
    ylim([-1 1]);
    zlim([-1 1]);
    hold off
    view(2)
    xlabel(M_x', interpreter, latex, fontsize, 15)
    ylabel(M_y', interpreter, latex, fontsize, 15)
    zlabel(M_z', interpreter, latex, fontsize, 15)


    subplot(2,2,3:4)

    plot(time(1:i),Mperp(1:i),'linewidth',2,'LineStyle','-','Color','k')
    hold on
    plot(time(:),exp(-time(:)/T2),'linewidth',2,'LineStyle','--','Color','k')
    hold off
    grid on
    box on
    ylim([0 1]);
    xlim([0 max(time)]);
    xlabel(Time (s), interpreter, latex, fontsize, 15)
    ylabel(Normalised signal, interpreter, latex, fontsize, 15)
    legend(| 1/N ∑ M_⊥ |,T_2 envelope, interpreter, latex, fontsize, 10)

    % Initial
    if i > 0 && i <= INITi_1
        title('Magnetisation vector along $B_0$ initially', interpreter, latex,
            fontsize, 15)

        % First flip
    elseif i > INITi_1 && i <= FTi_1
        xlim([0 FT_1]);
        title('RF pulse $\pi /2$ ', interpreter, latex, fontsize, 15)

        % First Dephase
    elseif i > FTi_1 && i <= DPi_1
        title(['Isocromats dephasing, t = ', num2str(round(time(i) - INIT_1 -
            FT_1, 3)),' s'], interpreter, latex, fontsize, 15)

        % Second flip
    elseif i > DPi_1 && i <= FTi_2
        title(['RF pulse $\pi$, $t = \tau$ = ', num2str(round(DP_1, 3)),' s'],
            interpreter, latex, fontsize, 15)

        % Rephasing
    elseif i > FTi_2 && i <= RPi
        title(['Isocromats rephasing, $t =$ ', num2str(round(time(i) - INIT_1 -
            FT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)

        % Echo
    elseif i > RPi && i <= INITi_2
        title(['Echo time, $t = T_E = 2 \tau =$ ',num2str(round(DP_1 * 2, 3)),'
            s'], interpreter, latex, fontsize, 15)

        % Dephasing
    elseif i > INITi_2 && i <= DPi_2
        title(['Isocromats dephasing, $t =$ ', num2str(round(time(i) - INIT_1 -
            FT_1 - FT_2, 3)),' s (Relaxation)'], interpreter, latex, fontsize, 15)

    end
```

```
231
232         frame = getframe(h);
233         video.writeVideo(frame);
234     end
235
236     video.close();
```

Code for all the plots