Differential Equations

Equation:

Such expression in which equality sign "=" is used is called an equation.

Differential Equation: (DE)

Such an equation in which derivative is involved is called differential equation.

i,e $\frac{dy}{dx} = \sin x.$

Ordinary Differential Equation (ODE)

If a DE contains only ordinary derivatives of one or more unknown functions with respect to a single independent variable, it is said to be an ODE

i,e $\frac{dy}{dx} = e^x - 5y.$

It can be written as

$$\frac{dy}{dx} + 5y = e^x$$

Partial Differential Equation (PDE)

An equation involving Partial derivatives of one or more Unknown functions of two or more independent variables is called a PDE.

i.e

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Order of DE

The highest derivative involve in the DE.

1- $\frac{dy}{dx} = \sin x \longrightarrow 1^{st}$ order

2- $\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx} = 3 \longrightarrow 1^{st}$ order

3- $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 4y = 4 \longrightarrow 2^{nd}$ order

Degree of DE

The Power of the highest derivative involve in the DE. i.e

$$\left(\frac{d^2y}{dx^2}\right)^1 + \left(\frac{dy}{dx}\right)^2 + y = 4$$

→ order is 2

→ degree is 1.

Linear Ordinary DE

There are three Properties:

1- Degree of dependent variable always equal to 1 or The Power of The derivative of the dependent valiable is 1.

2. If The dependent variable exists in DE whose degree must be 1. Dependent variable involve seperately, with not its derivative.

3. The Product of dependent variable and its derivative are not allowed.

i- $\frac{d^2y}{dx^2} + \frac{dy}{dx} + dy = 4 \rightarrow$ L.D.E

ii- $\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2y = 4 \rightarrow$ L.D.E

iii- $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 4 \rightarrow$ Not L.D.E

Difference in Equation and Identities

→ Equation is true for few values

→ Identities are equations, but it is true for all values.

$\cos^2 x + \sin^2 x = 1$.

Solutions

Equations d satisfies the equation

Q $\frac{dy}{dx} = x y^{1/2}$, $y = \frac{1}{16} x^4$, $(-\infty, \infty)$

Sol

$$L.H.S = \frac{dy}{dx}$$

$$= \frac{d(y)}{dx}$$

$$= \frac{d\left(\frac{1}{16}x^4\right)}{dx}$$

$$= \frac{1}{16}\frac{d}{dx} \cdot x^4$$

$$= \frac{1}{16} \cdot 4x^3$$

$$= \frac{1}{4}x^3$$

$$R.H.S = x y^{1/2}$$

$$= x\left(\frac{1}{16}x^4\right)^{1/2}$$

$$= x \cdot \frac{1}{4^{2\times\frac{1}{2}}} x^{4\times\frac{1}{2}} = x \cdot \frac{1}{4}x^2 = \frac{1}{4}x^3$$

$$= L.H.S$$