The Non-Line of Sight Problem in Mobile Location Estimation

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Abstract— **We consider the problem of tracking mobile stations using the ranging measurements from multiple base stations (BSs), without apriori knowledge of which BSs (if any) have a direct line of sight (LOS) range measurement. The two key contributions of this paper are to show that (1) it is possible to discriminate between LOS versus non-LOS (NLOS) measurements at each BS by using the time history of its range measurements in a simple hypothesis testing problem and (2) it is possible to correct the NLOS ranging error by exploiting apriori knowledge of the statistical characteristics of the system's standard measurement noise. Simulation examples are presented and the rms error is compared to the Cramer Rao Lower Bound on location estimation.**

I. Problem Introduction

There are a number of possible applications for mobile location estimation [1], including Emergency-911, fleet management, and location-dependent information services. One method for locating a mobile terminal in two dimensions requires the measurement of the LOS distance between the mobile and at least three participating BSs. Each distance measurement generates a circle which is centered at the measuring base station and which has a radius equal to the mobile-to-base distance. In the absence of any measurement error, the intersection of the three circles unambiguously determines the location of the mobile.

When the range measurements are corrupted by noise, then the location of the mobile station can be determined by finding the solution that is most consistent with the measurements (e.g., in the least squares sense). However, in a dense urban environment, there may be no direct path from the mobile to the BS. Due to reflection and diffraction, the propagating wave may actually travel *excess* path lengths on the order of hundreds of meters and the direct path is blocked [2]. This phenomenon, which we refer to as the NLOS error, ultimately translates into a biased estimate of the mobile's location. This problem has been recognized by others, e.g., [2], [3], as a critical issue, possibly a "killer issue" for mobile location. In order to mitigate the effect of the measurement bias, it is necessary to develop location algorithms that are robust to the NLOS error.

Silventoinen and Rantalainen [4] conducted simulations and found that the mean location error increased linearly with the increase in the NLOS errors. One of the key contributions of their work was to suggest the possibility of detecting and correcting the NLOS measurements in order to decrease the bias in the location estimate.

This paper examines three aspects of location estimation in a multipath-dominated environment. In the first part of our work, we consider the problem of **NLOS identification**. Each BS measures the time of arrival of the mobile's transmission and converts the raw measurement into a range (mobile-to-BS distance). By using the time history of the range measurements in a simple hypothesis test, and exploiting our knowledge of the standard deviation of the standard measurement noise, we can determine if the measurements correspond to a LOS or NLOS environment. We also show that one can use the residuals from a simple least squares tracking estimator in order to further confirm the presence of NLOS measurements.

The second part of this paper concentrates on **LOS reconstruction**. If there is strong evidence that the measurements contain a NLOS error, then they must be corrected priori to location estimation. We show that NLOS error correction is possible when the standard measurement noise dominates the NLOS error, and there is some apriori knowledge of the support of the standard measurement noise over the real axis.

The last part of this paper presents the Cramer Rao Lower Bound on the rms estimation error, as a theoretical lower-bound on the estimator's rms performance. For a discussion of the use of the CRLB in related problems, see [5].

II. Problem Formulation

M BSs measure the time of arrival of a signal that has been sent to the mobile and then transponded back to the network. The arrival times are then converted to range measurements. The m^{th} range is modeled as

$$\begin{aligned} r_m(t_i) &= L_m(t_i) + n_m(t_i) + NLOS_m(t_i) \qquad (1) \\ m &= 1, \cdots, M \quad i = 0, \cdots, K-1, \qquad (2) \end{aligned}$$

where $L_m(t_i)$ is the LOS range

$$L_m(t_i) = |x(t_i) + j * y(t_i) - x_m - j * y_m|. \qquad (3)$$

$(x(t_i), y(t_i))$ and (x_m, y_m) are the coordinates of the mobile at time, t_i, and the m^{th} base (respectively).

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In a LOS environment, the BS measures range in the presence of the standard system measurement noise, $n_m(t_i)$. Under that condition, there is **no** NLOS error and we can set $NLOS_m(t_i) \equiv 0$. However, in a NLOS environment, the wave may be reflected or diffracted as it propagates and the direct path is blocked. The resulting range measurement is then corrupted by two independent sources of error: (1) the standard measurement noise, $n_m(t_i)$ and (2) the *positive* NLOS error, $NLOS_m(t_i)$.

One might expect the chief source of tracking error in a NLOS environment to be the positive bias of the range measurements when the NLOS error dominates the standard measurement noise. Measurements taken by Nokia in the GSM network [2] confirm that the NLOS problem tends to be the key source of error in the range measurements, and that the resulting range measurements are positively biased. Their measurements indicate that the mean and standard deviation of the range errors are on the order of 513m and 436m, respectively.

In our work, the objective is to first identify when the NLOS error is present in a time series of range measurements from each BS, then (if necessary) reconstruct the LOS from the corrupted measurements, and finally to track the coordinates of the mobile station as a function of time using the combined measurements taken from all of the bases. Although we focus here on time histories, a special case of the formulation is that of a single range measurement ($K = 1$).

III. NLOS Identification

It is not known apriori which range measurements (if any) contain NLOS errors. The NLOS measurements can be identified, however, if we consider the time history of the range measurement from each BS individually, combined with our apriori knowledge of the standard deviation of the standard measurement noise.

At each BS, the range measurements are first smoothed by modeling

$$r_m(t_i) = \sum_{n=0}^{N-1} a_m(n) t_i^n \tag{4}$$

and solving for the unknown coefficients, $\{a_m(n)\}_{n=0}^{N-1}$ by the least squares technique. The smoothed measurements are represented as

$$s_m(t_i) = \sum_{n-0}^{N-1} \hat{a}_m(n) t_i^n. \tag{5}$$

Essentially, the proposed NLOS identification technique requires comparison of the standard deviation of a sample statistic to the known standard deviation of that statistic under the null hypothesis that the measurements are LOS.

When the mobile has LOS with the BS, then the standard measurement noise affects the measured range in a predictable manner. On the average, the measured range deviates from the true LOS range by the standard deviation of $n_m(t)$. Let $\sigma_m^2 = E\left\{n_m^2(t)\right\}$, and suppose that we perform an N^{th} order polynomial fit to smooth the data. Then we would expect for

$$\hat{\sigma}_m = \sqrt{\frac{1}{K} \sum_{i=0}^{K-1} \left(s_m(t_i) - r_m(t_i)\right)^2} \sim O(\sigma_m). \tag{6}$$

If however, the NLOS error is present along with $n_m(t)$ and is *uncorrelated in time*, then we would expect for the measured range to have a significantly larger average deviation from the smoothed curve than σ_m. This assumption is strongly supported by the Nokia range error histogram [2], as we explain below.

The NLOS error is a nonnegative random variable which can be modeled as having approximately finite support over the positive real axis, $0 \le NLOS_m(t) \le \beta_m$. The standard measurement noise is modeled as a zero-mean random variable with (approximately) finite support over $-\alpha_m \le n_m(t) \le \alpha_m$. The composite noise term is a linear superposition of $NLOS_m(t)$ with $n_m(t)$, which has (approximately) finite support over $-\alpha_m \le n_m(t) + NLOS_m(t) \le \beta_m - \alpha_m$. The Nokia measurement results provide a valuable example that gives us some indication of the order of α_m and β_m for a particular system. For their measurements, $\alpha_m \sim 400$m, which implies that $\beta_m \sim 1300$m. One could loosely argue that the $n_m(t) \sim N(0, \sigma_m)$, where $\sigma_m \sim O(150m)$. This implies that the standard deviation of the NLOS error $\sim 409m$, and clearly indicates that the presence of the NLOS error **increases** the standard deviation of the measurements in a significant manner.

When the NLOS error is present, then the measured range will deviate from the smoothed curves on the average by

$$\hat{\sigma}_m \gg \sigma_m. \tag{7}$$

The presence of a large standard deviation will be used to discriminate between LOS versus NLOS measurements.

To summarize:

$$\begin{aligned} H_0 : \hat{\sigma}_m &= \sigma_m \\ H_1 : \hat{\sigma}_m &> \sigma_m. \end{aligned} \tag{8}$$

We reject the null hypothesis, H_0, (LOS case) for large values of $\hat{\sigma}_m$.

In the next section we show that under certain conditions, we can confirm our rejection of the null hypothesis by using a residual analysis rank test.

A. Residual Analysis Rank Test

In this section, we assume that the LOS hypothesis has been rejected at one or more BSs but that there is some uncertainty about the hypothesis testing results. Under that condition, we can use a second technique in order to confirm our belief that certain BSs are actually reporting NLOS measurements.

In order to apply the residual analysis rank test, we initially use all of the raw measurements in order to estimate the mobile's coordinates over time. At each instant, the least squares estimates, $(\hat{x}_{LS}(t_i), \hat{y}_{LS}(t_i))$, are chosen to minimize

$$F_i = \sum_{m=1}^{M} \left(r_m(t_i) - \hat{L}_m(t_i) \right)^2 \tag{9}$$

where $\hat{L}_m(t_i) = |\hat{x}(t_i) - x_m + j * \hat{y}(t_i) - j * y_m|$.

We define the residual as the difference between the measured range, $r_m(t_i)$, and the calculated range, as follows:

$$e_m(t_i) = r_m(t_i) - \hat{L}_m(t_i) \tag{10}$$

for $(i = 0, \cdots, K-1)$ and $(m = 1, \cdots, M)$. The NLOS error manifests itself as a highly inconsistent measurement, so it is reasonable to assume that if a large NLOS error exists at a particular BS, that on the average its residual will be larger in magnitude than the residuals at the other BSs. When there are several BSs which have NLOS measurements, then we can check to see if the magnitude of their residuals is, on the average, larger than the BSs that we believe to have LOS with the mobile.

To summarize the rank test:

1. Use all measurements at t_i to estimate $(x(t_i), y(t_i))$.
2. Calculate residual: $e_m(t_i) = r_m(t_i) - \hat{L}_m(t_i)$.
3. Count number of times $|e_m(t_i)|$ is largest error for each t_i.
4. Rank results from 3.

IV. LOS Reconstruction

When the null hypothesis is rejected at a BS, we have strong evidence that its range measurement is corrupted by the NLOS error. Under that condition, we employ a NLOS error correction technique.

The composite error is the superposition of the NLOS error and the standard measurement noise. We have shown that it can be modeled as a random variable that ranges over the values: $-\alpha_m \leq n_m(t) + NLOS_m(t) \leq \beta_m - \alpha_m$. This indicates that the maximum measurement error is roughly α_m (m) below the line of sight.

The LOS is reconstructed in two steps. First, we smooth the data using an N^{th} order polynomial fit, under the assumption that the *major* effect of the NLOS error is to just to bias the data. In the next step, we utilize our knowledge of α_m to correct the NLOS error. After the data is smoothed, we can calculate the deviation of each value of the measured range from the polynomial fit at each instant of time, $r_m(t_i) - s_m(t_i)$. Given a sufficient observation interval, and independent measurements, the maximum deviation of the measured range below the smoothed curve (occurring, say, at time t_n) will be a value that will be very nearly equal to $L_m(t_n) - \alpha_m$. If we vertically displace the smoothed curve down so that it passes through the point of maximum deviation, and then displace the curve upward by the value α_m, then the resulting corrected-smoothed curve is the estimate of the LOS range. Figure (1) demonstrates the central idea. NLOS measurements were produced by adding a NLOS error to the LOS distance at each instant of time. By using the histogram of ranging errors reported by Nokia in [2], we generated random variables that corresponded to the NLOS error. From their measurements, we determined that α_m = 400m, and used that value in order to separate out the measurement error from the NLOS error.

V. Cramer Rao Lower Bound

In its simplest form, the Cramer Rao Lower Bound (CRLB) is a lower bound on the rms error of any unbiased estimator. The vehicle's position in the x-y plane at any time is given by

$$\begin{aligned} x(t) &= x_0 + v_x t \\ y(t) &= y_0 + v_y t, \end{aligned} \tag{11}$$

where (x_0, y_0) is its initial coordinates and (v_x, v_y) represent its speed in the x and y directions, respectively. The vector of parameters to be estimated is

$$\vec{\theta} = \left[\begin{array}{cccc} x_0 & y_0 & v_x & v_y \end{array} \right]^T. \tag{12}$$

The CRLB is a covariance matrix, which lowerbounds the covariance of estimation errors as

$$\text{CRLB}_{\vec{\theta}} \leq E\left\{ \left[\hat{\vec{\theta}} - \vec{\theta}\right] \left[\hat{\vec{\theta}} - \vec{\theta}\right]^T \right\}. \tag{13}$$

By using the definition given in [5], we can show that

$$\text{CRLB}_{\vec{\theta}} = \left(\sum_{m=1}^{M} \frac{1}{\sigma_m^2} \mathbf{T}_m \mathbf{T}_m^T \right)^{-1} \tag{14}$$

where

$$\mathbf{T}_m = \begin{bmatrix} \cos(\theta_m(t_0)) & \cdots & \cos(\theta_m(t_{K-1})) \\ \sin(\theta_m(t_0)) & \cdots & \sin(\theta_m(t_{K-1})) \\ t_0 \cos(\theta_m(t_0)) & \cdots & t_{K-1} \cos(\theta_m(t_{K-1})) \\ t_0 \sin(\theta_m(t_0)) & \cdots & t_{K-1} \sin(\theta_m(t_{K-1})) \end{bmatrix} \tag{15}$$

and $\theta_m(t_i) = \angle \{x(t_i) - x_m + j * y(t_i) - j * y_m\}$.

The rms location error of the estimator is compared to the square root of the diagonal elements in $\text{CRLB}_{\vec{\theta}}$.

VI. Examples

In this section, we present a digest of simulation examples to demonstrate the performance of the NLOS identification and correction algorithm described in the preceding sections. In all of the examples, the vehicle's trajectory in the x-y plane is given by (11). The sampling period was $0.5s$, and 200 samples were taken. The velocity remained constant at v_x = 9.7 m/s and v_y = 16.8m/s. The BSs were randomly assigned to have either NLOS or LOS measurements, and the NLOS measurements were random variables that were generated using the Nokia histogram [2]. The standard measurement noise was i.i.d. $N(0, 150\text{m})$. From their results, we have taken α_m = 400m and β_m = 1300m. There were four BSs in each example, with three of them uniformly spaced around a circle of radius 5 km, with the fourth BS located at the center of the circle.

A. Example I

Two simulation examples were run. In the first simulation Ia, BSs 1 and 2 always had NLOS measurements, while in simulation Ib, all 4 BSs had NLOS measurements. The NLOS measurements were correctly identified by examining the standard deviation of the residuals, which are reported in Table 1 and 2. Figures 2 and 3 demonstrate the substantial improvement in the estimated vehicle trajectory *after* NLOS identification and error correction.

B. Example II

The purpose of this simulation was to demonstrate several situations in which it is possible to identify the NLOS measurement using residual analysis. Three simulations were conducted, with $x_0 = -118.3m$, $y_0 = -3.7m$, and the number of NLOS measurements ranged from 1 to 3. The number of times that each BS had the largest absolute residual was tallied, The results shown in Table 3 indicate how the BSs with NLOS errors were confirmed by using the residual analysis rank test.

C. Example III

The objectives of this simulation were to (1) compare the mean and standard deviation of the location error of the proposed algorithm to the output of the least squares estimator without any NLOS error correction and (2) to compare the performance of both schemes to the least squares estimator when all of the measurement were LOS. All results were compared to the square root CRLB. 500 independent trials were conducted, during which BSs 1 and 3 always had LOS with the mobile, while BSs 2 and 4 never had LOS. $x_0 = -118.3\text{m}$, $y_0 = -3.7\text{m}$. The results are reported in Table 4. The first column refers to the performance of the least squares estimator when there was no correction (NC) for the NLOS error. Column 2 indicates the performance of the NLOS identification and correction (C) algorithm, while column 3 presents simulation results when all of the measurements were LOS and corrupted only by i.i.d. $N(0, 150\text{m})$ noise. Finally, column 4 presents the square root CRLB. In general, the notation

$$\mu_a = \frac{1}{500}\sum_{n=0}^{499} \hat{a}(n) - a(n) \qquad (16)$$

$$\sigma_a = \sqrt{\frac{1}{500}\sum_{n=0}^{499} \left(\hat{a}(n) - a(n) - \mu_a\right)^2}. \qquad (17)$$

The location and speed errors in each coordinate were measured in meters and meters/second, respectively. The simulation results indicate that the NLOS error correction technique was able to significantly reduce the estimation bias (by orders of magnitude) as compared to the results without NLOS error correction.

D. Example IV

In this simulation, we estimated the probability of correctly detecting a NLOS measurement as a function of the number of samples (window length). The sampling period remained fixed at 0.5 s, while the number of samples per trial varied between 5 and 150. $x_0 = -200\text{m}$, $y_0 = 100\text{m}$. BSs 1 and 4 always had LOS, while BSs 2 and 3 did not. During each trial, we recorded the BS that had the largest residual for the largest fraction of time. This BS was declared NLOS. 500 independent trials were performed, and the fraction of time (out of 500) that each BS was declared NLOS was recorded. Figure 4 shows the very high probability of detecting the NLOS even for a small number of samples.

VII. Conclusion

We have presented a new tracking algorithm that is capable of discriminating between LOS versus NLOS range measurements and correcting the NLOS error by using apriori knowledge of the approximate support of the noise over the real axis. Simulation results indicate that the positive ranging bias, which is caused by the NLOS error, was reduced by several orders of magnitude after performing the LOS reconstruction technique, and that the standard deviation of error also decreased. The results encourage further investigation into variations of the algorithms and such issues as correlated measurements, other scenarios, sensitivities to parameters, etc.

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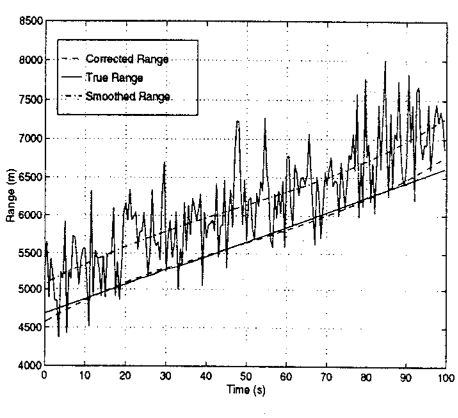

Fig. 1. Sample LOS reconstruction from NLOS measurements.

TABLE I

Sim. Ia: Standard Deviation of Measurements from Smoothed Curve for 2 NLOS Measurements

Base	NLOS	$\hat{\sigma}_m$ (m)
1	Yes	467.3
2	Yes	447.6
3	No	163.1
4	No	142.1

TABLE II

Sim. Ib: Standard Deviation of Measurements from Smoothed Curve for 4 NLOS Measurements

Base	NLOS	$\hat{\sigma}_m$ (m)
1	Yes	440.2
2	Yes	444.4
3	Yes	463.6
4	Yes	450.2

TABLE III

Sim. II: Percentage of Time BS had largest residual

	BS1	BS2	BS3	BS4
LOS	10	11.5	18.5	
NLOS				60
LOS	18.5	15		
NLOS			26.5	40
LOS	12.5			
NLOS		20	40.5	27

TABLE IV

Sim. III: Comparison of Estimator Performance

	NC	C	LOS	$\sqrt{\text{CRLB}}$
μ_{x_0}	297.8	-3.98	0.17	--
σ_{x_0}	32.9	28.30	16.42	15.88
μ_{y_0}	-306.1	-2.36	0.54	--
σ_{y_0}	55.5	45.13	14.15	14.18
μ_{v_x}	0.18	-0.09	-0.005	--
σ_{v_x}	0.55	0.49	0.27	0.27
μ_{v_y}	4.49	-.01	-0.005	--
σ_{v_y}	0.84	0.64	0.25	0.25

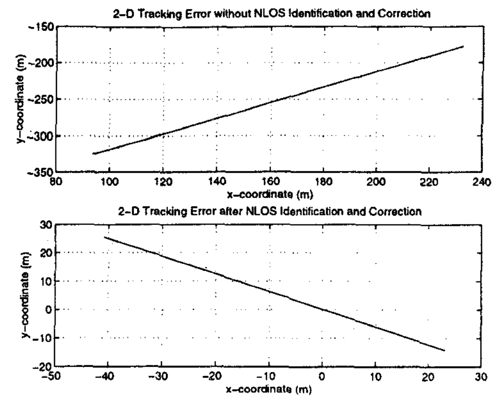

Fig. 2. Sim. Ia: 3 BSs located on circle of radius 5000 m, with serving BS at center. 2 NLOS measurements present. $x(t) = 126.9 + 9.7t$, $y(t) = 286.6 + 16.8t$.

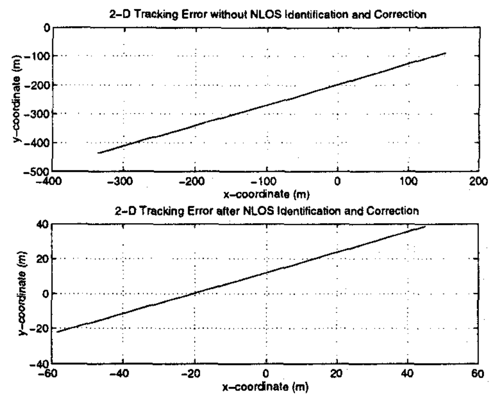

Fig. 3. Sim. Ib: 3 BSs located on circle of radius 5000 m, with serving BS at center. 4 NLOS measurements present. $x(t) = -190.7 + 9.7t$, $y(t) = 219.1 + 16.8t$.

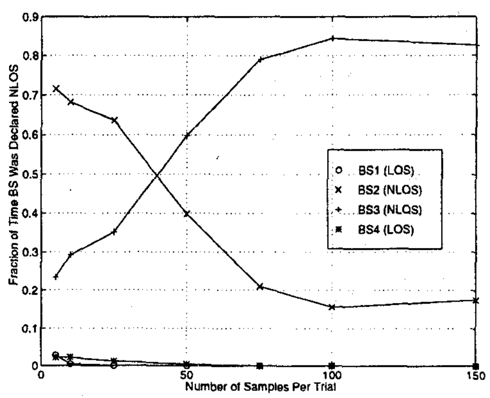

Fig. 4. Sim. IV: 3 BSs located on circle of radius 5000 m, with serving BS at center. 2 NLOS measurements present. $x(t) = -200 + 9.7t$, $y(t) = 100 + 16.8t$.