

PCET's & NMVPM's

Nutan College of Engineering & Research (NCER)

(Affiliated to Dr. Babasaheb Ambedkar Technological University, Lonere)

Assignment No: 01 of Subject

Finite Element Method (FEM)

By

Mr. Chaudhry Sufiyan Ahmad Imtiyaz Ahmad

(PRN: 50641920181162511002)

Guided By

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Department of Mechanical Engineering

NCER, Talegaon Dabhade

(2021-2022)

Name:- Sufyan Ahmad Chaudhary

Roll No:- 06

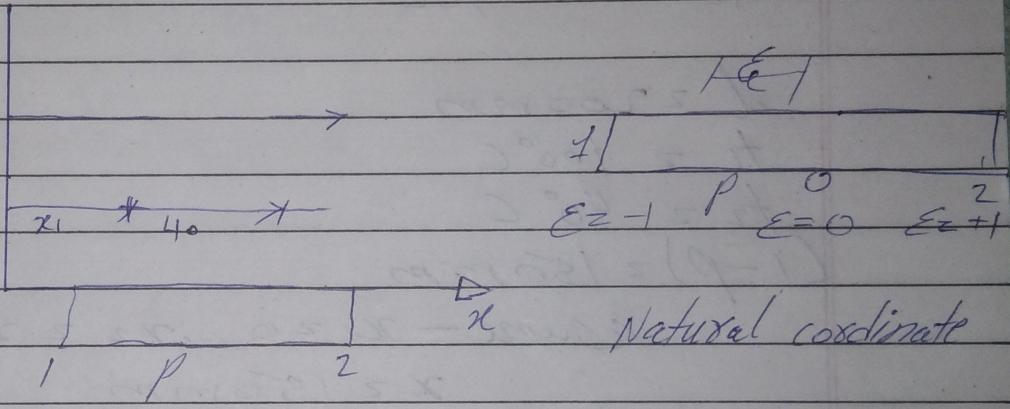
PRN :- 50641920181162511002

Semister :- 07

Assignment NO :- 01

Exp 1)

Ans →



local coordinate

$$x_1 = 200 \text{ mm}$$

$$y_1 = 303 \text{ mm}$$

$$x_2 = 360 \text{ mm}$$

$$y_2 = -0.05 \text{ mm}$$

$$l - p = 40 \text{ mm}$$

$$x = x_1 + l + p = 200 + 40 = 240 \text{ mm}$$

Natural co-ordinate

$$\epsilon = \frac{x - x_1}{x_2 - x_1} = \frac{2(240 - 200)}{(360 - 200)} - 1 = -0.5$$

Shape function

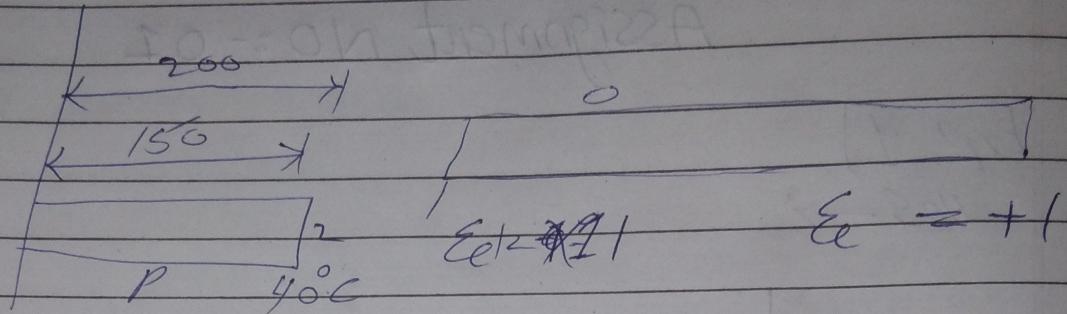
$$n_1 = \frac{1 - \epsilon}{2} = \frac{1 - (-0.5)}{2} = 0.75$$

$$n_2 = \frac{1 + \epsilon}{2} = \frac{1 + (0.5)}{2} = 0.25$$

Displacement

$$u_2 = N_1 u_1 + N_2 u_2 = 0.75(0.03) + 0.25(0.0)$$

$$u_2 = 0.01 \text{ mm}$$



$$l = 200 \text{ mm}$$

$$t_1 = 100^\circ\text{C}$$

$$t_2 = 40^\circ\text{C}$$

$$(1-p) = 150 \text{ mm}$$

$$\text{assume } x_1 = 0 \quad x_2 = 200$$

$$x = 150 \text{ mm}$$

natural co ordinate

$$\epsilon_e = \frac{2(x - x_1)}{x_2 - x_1} - 1 = \frac{2(150 - 0)}{200 - 0} - 1 = 0.5 - 1 = -0.5$$

Shape function

$$N_1 = \frac{1 - \epsilon_e}{2} = \frac{1 - 0.5}{2} = 0.25$$

$$N_2 = \frac{1 + \epsilon_e}{2} = \frac{1 + 0.5}{2} = 0.75$$

$$N_2 = \frac{1 + \epsilon_e}{2} = \frac{1 + 0.5}{2} = 0.75$$

temp at point p

$$= t_p N_1 t_1 + N_2 t_2$$

$$= 0.25(100) + 0.75(40)$$

$$= 25^\circ\text{C}$$

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Assignment No: 02 of Subject

Finite Element Method (FEM)

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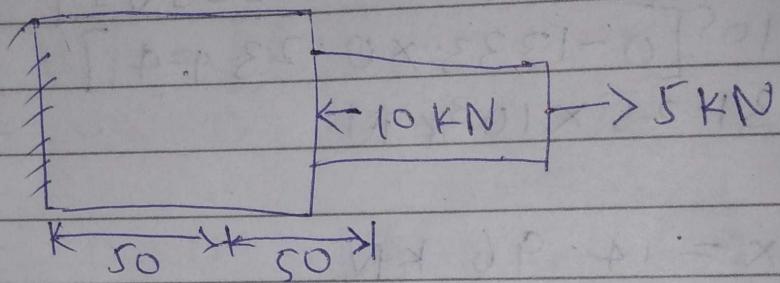
Assignment :- 2

Name = Sufiyan Ahmad chaudhary

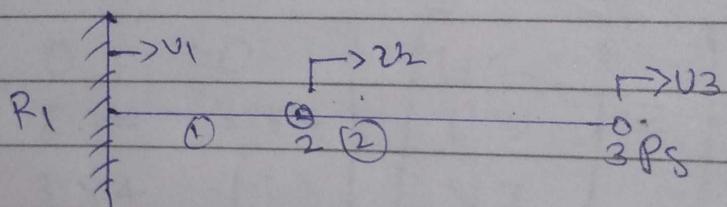
Roll No = 06

PRN = S0641920181162511002

Given:- $A_1 = 150 \text{ mm}^2$ $l_1 = 50 \text{ mm}$ $E_1 = 200 \text{ GPa}$
 $A_2 = 1000 \text{ mm}^2$ $l_2 = 50 \text{ mm}$ $E_2 = 70 \text{ GPa}$



Discretization



$$P_2 = 10 \text{ KN}$$

$$P_3 = 5 \text{ KN}$$

| Element No | Connectivity | Local node 1 | Local node 2 | Local node 3 |
|------------|---------------------|--------------|--------------|--------------|
| 1 | Global local node 1 | 1 | 2 | |
| 2 | Global local node 2 | 2 | | 3 |

element stiffness matrix

element 1

$$[K_1] = \frac{A_1 E_1}{L_1} \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\frac{150 \times 200 \times 10^3}{50} \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 60 & -60 \\ -60 & 60 \end{bmatrix}$$

element 2

$$[K_2] = \frac{A_2 E_2}{L_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{100 \times 70 \times 10^3}{40} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 14 & -14 \\ -14 & 14 \end{bmatrix}$$

Global vector

$$[F] = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} P \\ -10 \times 10^3 \\ 5 \times 10^3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Global stiffness matrix

$$[K] = [k_1] + [k_2]$$

$$= \begin{bmatrix} 60 & -60 & 0 \\ -60 & 60+14 & -14 \\ 0 & -14 & 14 \end{bmatrix}$$

load vector

$$[F] = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} R \\ -10x10^3 \\ 5x10^3 \end{Bmatrix} \quad \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

Global nodal displacement

$$[U]_n = \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} \quad \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$[K] [U] = F$$

$$10^4 \begin{bmatrix} 60 & -6 & 0 \\ -60 & 74 & -14 \\ 0 & -14 & 14 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = 0 \quad \begin{Bmatrix} R \\ -10 \times 10^3 \\ 5 \times 10^3 \end{Bmatrix}$$

Nodal displacement

$$10^4 \begin{bmatrix} 74 & -14 & 0 \\ -14 & 14 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} -10 \times 10^3 \\ 5 \times 10^3 \end{Bmatrix}$$

$$P_2 + \frac{14}{74} P_1$$

$$104 \begin{bmatrix} 74 & -14 \\ 0 & 14 \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} -10 \times 10^3 \\ 0.3 \times 10^3 \end{bmatrix}$$

$$104 \times 11.35 \times 10^3 = 0.311 \times 10^3$$

$$104 (74U_2 - 14U_3) = 10 \times 10^3$$

$$U_3 = 27.4 \times 10^3 \text{ mm}$$

$$U_2 = -8.33 \times 10^{-3} \text{ mm}$$

$$U_1 = 0 \text{ mm}$$

stress in element

$$1) \sigma_1 = \frac{E_1}{l_1} [-1 \ 1] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \frac{E_1}{l_1} (-U_1 + U_2)$$

$$= \frac{200 \times 10^3}{50} (10 - 8.33 \times 10^{-3})$$

$$\sigma_1 = -33.33 \text{ N/mm}^2$$

$$2) \frac{E_2}{l_2} (-U_2 + U_3)$$

$$= \frac{70 \times 10^3}{50} [-8.333 \times 10^{-3} + 27.4 \times 10^{-3}]$$

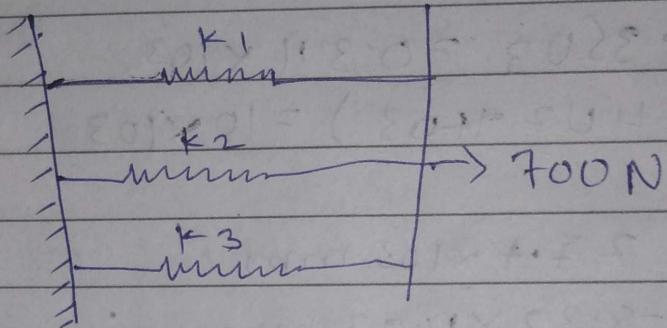
$$\sigma_2 = 50 \text{ N/mm}^2$$

Reaction at Support

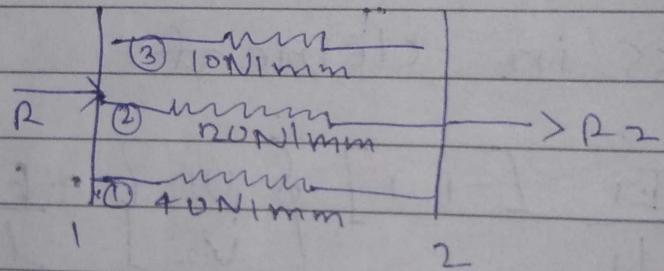
$$104(6000 - 60) = R$$

$$104((6000) - 60(-8.33 \times 10^3)) = R$$

$$R = -5 \times 10^3 \text{ N}$$



Discretization



Element Connectivity

| number | Global local Node(1) | number local node(2) |
|--------|-------------------------|-------------------------|
| 1 | 1 | 2 |
| 2 | 1 | 2 |
| 3 | 1 | 2 |

element 1 Connectivity

$$\text{Element } +1, [k] = k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$$

$$\text{Element } 2, [k_2] \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = k_2 \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}$$

$$\text{Element } 3, [k_3] k_3 \begin{bmatrix} k_3 & -k_3 \\ -k_3 & -k_3 \end{bmatrix} = \begin{bmatrix} 40 & 40 \\ -40 & -40 \end{bmatrix}$$

Global stiffness + $[k_3]$

$$\begin{bmatrix} 10 + 20 + 40 & -10 - 20 - 40 \\ -10 - 20 - 40 & 10 + 20 + 40 \end{bmatrix} = \begin{bmatrix} 70 & -70 \\ -70 & 70 \end{bmatrix}$$

Global load vector

$$\{F\} = \{P_1\} = \{P_2\} = \{700\}$$

Global nodal displacement

$$\{v\} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$[k] \{v\} = \{F\}$$

$$\begin{bmatrix} 70 & -70 \\ -70 & 70 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} P \\ 700 \end{bmatrix}$$

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Assignment No: 03 of Subject

Finite Element Method (FEM)

By

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(PRN: 50641920181162511002)

Guided By

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Department of Mechanical Engineering

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(2021-2022)

Assignment No: 3
 Name: Sufyan Ahmad Chaudhary
 Roll No: 06

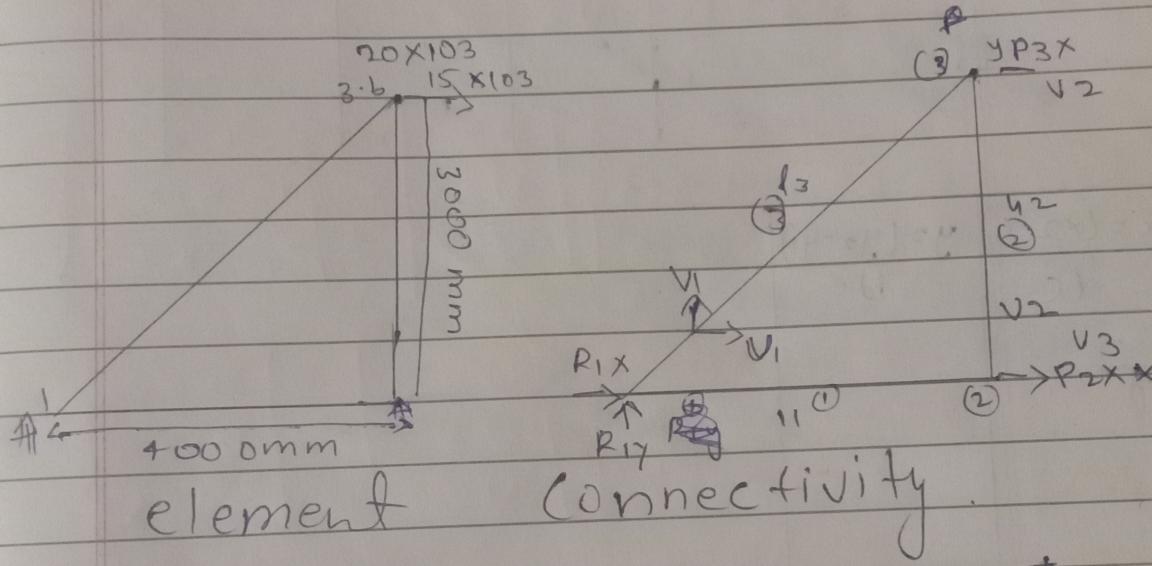
PRN = 50641920181162511002

Given: - $E = 200 \times 10^3 \text{ N/mm}^2$ $P_3x = 15 \times 10^3$

$P_3y = -20 \times 10^3 \text{ N}$ $J_1 = 4000 \text{ mm}$

$V_2 = 3000 \text{ mm}$ $A_1 = A_2 = A_3 = 2000 \text{ mm}^2$

$V_3 = \sqrt{J_1^2 + J_2^2} = \sqrt{(4000)^2 + (3000)^2} = 5000 \text{ mm}$



| Element No | Global node 1 | Node 2 | Local node |
|------------|---------------|--------|------------|
| 1 | 1 | 2 | |
| 2 | 2 | 3 | |
| 3 | 1 | 3 | |
| 4 | 1 | 1 | |

| Element number | length of element | Global Co-ordinate | | | | $x_1 - x_2$ | $y_1 - y_2$ |
|----------------|-------------------|--------------------|------------|------------|------------|-------------|-------------|
| | | local node | local mode | local mode | local mode | ue | ue |
| 1 | 4000 | 0 | 0 | 4000 | 0 | 1 | 0 |
| 2 | 3000 | 4000 | 0 | 4000 | 3000 | 0 | 1 |
| 3 | 5000 | 0 | 0 | 4000 | 3000 | 0.8 | 0.6 |

element stiffness matrix

$$[k_e] = A e E e \begin{bmatrix} cx_2 & cx & cy - (x^2 - cx \cdot cy) & 2j-1 \\ cx \cdot cy & cy^2 - cx \cdot cy & cy^2 & 2j \\ - (x^2 - cx \cdot cy) & cx^2 & cx \cdot cy & 2j-1 \\ - (cx \cdot cy - cy^2) & cx \cdot cy & cy^2 & 2j \end{bmatrix}$$

2j-1 2i 2j-1 2j

$$(1) i=1 \quad j=2 \quad : cx=1 \quad cy=0 \quad \begin{array}{l} 1 \ 2 \ 3 \ 4 \\ | \quad | \quad | \quad | \\ 1 \ 0 \ -1 \ 0 \quad 1 \\ 0 \ 0 \ 0 \ 0 \quad 2 \\ -1 \ 0 \ 1 \ 0 \quad 3 \\ 0 \ 0 \ 0 \ 0 \quad 4 \end{array}$$

$$[k]_{ij} = 2000 \times 200 \times 10^3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times 10^5 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(2) i=2 \quad j=3 \quad : cx=0 \quad cy=1 \quad \begin{array}{l} 3 \ 4 \ 5 \ 6 \\ | \quad | \quad | \quad | \\ 1 \ 0 \ -1 \ 0 \quad 3 \\ 0 \ 1.33 \ 0 \ 1.33 \quad 4 \\ -1 \ 0 \ 1 \ 0 \quad 5 \\ 0 \ -1.33 \ 0 \ 1.33 \quad 6 \end{array}$$

$$: 2 = 2000 \times 200 \times 10^3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1.33 & 0 & 1.33 \\ -1 & 0 & 1 & 0 \\ 0 & -1.33 & 0 & 1.33 \end{bmatrix} \times 10^5 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1.33 & 0 & 1.33 \\ -1 & 0 & 1 & 0 \\ 0 & -1.33 & 0 & 1.33 \end{bmatrix}$$

$$(3) i=1 \ j=3 \ c_x = 0.8 \ c_y = 0.6$$

$$k_3 = 2000 \times 200 \times 10^3$$

5000

| | | 1 2 5 6 |
|----------|---------------|---------|
| 10 - 10 | 054 04 - 016 | 1 |
| 00 00 | 48 03 - 24 03 | 2 |
| -10 - 10 | 4064 04343 | 5 |
| 00 00 | 0 + 8036078 | 6 |

$$[K] = [H_1] + [K_3] + [K_3]$$

| | | | | | | | |
|-------------------|--------|--------|----|-------|--------|--------|---|
| = 10 ⁵ | 1.512 | 0.384 | -1 | 0 | -0.312 | -0.384 | 1 |
| | 0.384 | 0.284 | 0 | 0 | -0.384 | -0.284 | 2 |
| | -1 | 0 | 1 | 0 | 0 | 0 | 3 |
| | 0 | 0 | 0 | 1.333 | 0 | -1.333 | 4 |
| | -0.512 | 0 | 0 | 1.333 | 0 | -0.512 | 5 |
| | 0 | -0.384 | 0 | 0 | 0.512 | -0.384 | 6 |
| | 0.384 | -2.88 | 0 | 1.333 | 0.384 | 1.621 | |
| | 1 | 2 | 3 | 4 | 5 | 6 | |

load vector

| | | | | |
|-----|---|-----------------|-----------------------|---|
| [F] | = | P _{1X} | R _{1X} | 1 |
| | | P _{1Y} | R _{1Y} | 2 |
| | = | P _{2X} | 0 | 3 |
| | | P _{2Y} | R _{2Y} | 4 |
| | | P _{3X} | 15 × 10 ³ | 5 |
| | | P _{3Y} | -20 × 10 ³ | 6 |

Nodal displacement

$$[U] = \begin{bmatrix} U_1 & 1 \\ V_1 & 2 \\ U_2 & 3 \\ V_2 & 4 \\ U_3 & 5 \\ V_3 & 6 \end{bmatrix}$$

$$[F] [U] = [f]$$

$$U_1 = 0 \quad V_2 = 0 \quad V_3 = 0$$

$$15 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.512 & 0.384 \\ 0 & 0.384 & 0.612 \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \times 10^3 \\ -20 \times 10^3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5.12 & 3.84 \\ 0 & 3.84 & 1.62 \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.5 \\ -31.25 \end{bmatrix}$$

$$U_2 = 0$$

$$5.12U_3 + 3.84V_3 = 1.5$$

$$13.33V_3 = -3.125$$

$$V_2 = 0$$

$$V_3 = -0.2354 \text{ mm}$$

$$5.12U_3 + 3.84(-0.2354) = 1.3$$

$$U_3 = 0.468 \text{ mm}$$

SUPPORT Reaction

$$\begin{aligned}
 R_{1x} &= 10^5 [1.52v_1 + 0.384v_1 - 0.2 - 0.512v_3 - 0.384v_2] \\
 &= 10^5 [0 + 0 + 0.384 + 0.288 \times 0.2344] \\
 &= -11.22 \times 10^3 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 R_{2y} &= 10^5 [1.333v_2 - 1.333v_3] \\
 &= 10^5 [0 - 1.333 \times 0.2344] \\
 &= 31.25 \times 10^3 \text{ kN}
 \end{aligned}$$

$$R_{1x} = 14.96 \text{ kN}$$

$$R_{1y} = -11.25 \text{ kN}$$

$$R_{2y} = 31.25 \text{ kN}$$