Anchor-Free Localization: Estimation of Relative Locations of Sensors

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***Abstract*—Some applications of sensor networks do not require the knowledge of absolute locations of sensors; knowledge of relative locations of sensors is sufficient. We study the problem of estimating relative locations of sensors, which we refer to as *relative localization*, under the assumption that each sensor can measure the distances between neighbor sensors. We show that the relative localization can be performed by setting up a non-linear optimization problem and solving it by standard optimization techniques such as the steepest descent method. We theoretically and numerically investigate the characteristics of the proposed relative localization and obtain several findings. For example, the relative locations of sensors can be accurately estimated even if the distance measurements include large errors. Thus, primitive distance measurement techniques, including RSSI-based measurements, are applicable for the relative localization. We also find that the relative localization has a preferable scaling property; it performs better as the number of sensors increases.**

***Index Terms*—sensor, localization, relative location, anchor free, optimization**

I. Introduction

Recent progress in electronics and micromechanics allows us to have small, low-cost, and low-power sensors, which have built-in battery and wireless-communication capability to transmit sensed data to a data center via a single or multi-hop wireless link. Such sensors present us with great opportunities for a wide range of applications including tracking, surveillance, or environment monitoring. Even if each sensor provides little amount of data, we can obtain respectable information on a field of watch by aggregating large amount of data sensed by densely deployed sensors.

When aggregating sensed data, knowledge of the geometrical locations of sensors is very important in order to identify in which location each piece of data is collected. Knowing sensors' locations is also valuable for location-aware routing or topology control including clustering. A straightforward solution for knowing sensors' locations, usually referred to as *localization*, is to equip each sensor with a Global Positioning System (GPS) receiver, but it is prohibited because of the size, cost, and power consumption constraints of sensors.

One of common approaches for localization is to use a small number of GPS-enabled sensors, typically called *anchors*, in order to give information on locations to neighbor sensors. A considerable number of studies have been made on the localization using a small number of anchors; these are either range-based or range-free. In range-based localization, a sensor estimates its location based on anchors' locations and its ranges to anchors. In range-free localization, a sensor estimates its location based only on anchors' locations.

In some applications like shape recognition of a target object [1], [2], [3], [4], it is not necessary to know the absolute locations of sensors; knowledge of relative locations of sensors is sufficient. The knowledge of relative locations is helpful also for the absolute localization; if we know the relative locations of sensors, the absolute localization is possible simply by knowing the absolute locations of only three sensors. Motivated by the fact, in this work, we are concerned with the problem of estimating relative locations of sensors, which we refer to as *relative localization*. The relative localization can be formulated as a non-linear global optimization problem and a set of estimated relative locations of sensors is given as its solution. The optimization problem can easily be solved by the steepest descent method.

Because of the nature of the problem, our relative-localization algorithm is *anchor free*; it does not require any sensor with known position. On the other hand, each sensor needs to estimate its ranges to neighbor nodes. The estimated ranges are sent to the central server, which conducts all the computations for solving a non-linear optimization problem to obtain the relative locations of sensors. We have numerically found that our algorithm can accurately estimate the relative locations of sensors even if the distance measurements include large errors. Thus, we can use simple ranging methods like received-signal-strength-indicator-based (RSSI-based) ones, which do not require additional expensive devices. We also find that the relative localization has a preferable scaling property; it performs better as the number of sensors increases.

This article is organized as follows: in Sec. II, we mention some related work concerning the sensor localization. In Sec. III we explain several assumptions and notions used in this work. In Sec. IV we propose a method for relative localization by formulating it as a non-linear optimization problem. We also show several mathematical characteristics of the proposed method. In Sec. V we evaluate the effectiveness of our proposal using simulation experiments. Finally, we conclude the study in Sec.VI.

II. Related Work

Range-based localizations measure the distances to anchors based on signal strength or time of arrival in order to estimate the locations of sensors using trilateration or triangulation.

Several studies [5], [6], [7] use Received Signal Strength Indicator (RSSI) for distance measurement because most sensor nodes have radio transmitters and receivers. The RSSI-based distance measurement relies on theoretical or empirical model to translate signal strength into distance estimates. The RSSI-based measurement techniques, however, have a difficulty that multi-path effect, antenna orientation, or unknown path loss factor inevitably causes measurement errors. Time of Arrival (TOA) of ultrasound was used for distance estimation in [8], [9], where a small ultrasonic transducer attached to a target object emits a pulse of ultrasound and receivers in rooms record the time of arrivals of pulses from the transducer to determine its location using the multilateration. Time difference of arrival (TDOA) between simultaneously transmitted radio and ultrasound signals was used for distance estimation in Cricket [10] and AHLoS [11].

Sensors with unknown locations could help each other to estimate their locations by informing their neighbors of their location estimates. This observation yields a new paradigm of localization, called *cooperative localization* [12], [13], [14], which estimates the locations of sensors by solving some global optimization problem in a central or distributed fashion. For example, Shang *et al.* [15] estimate the locations of sensors using the classical multidimensional scaling (MDS) based on the measured distances (or connectivity) between all sensor pairs. If the distance between a pair of sensors is not known, it is assumed to be equal to the length of the shortest path interconnecting the pair of sensors. Their algorithm can be applied to the relative localization. Although their approach is similar with ours, we directly solve the optimization problem without using the MDS. If the solution by the MDS is used as an initial input, our algorithm would yield a result better than the MDS.

III. Preliminaries

A. Function of sensors

We consider a two-dimensional area where N sensors are deployed. We assume that each sensor can approximately estimate distances between neighbor sensors, and that each sensor can send estimation results to a central server via single long-range wireless link. The central server has the information on the distances between sensors, based on which it conducts all the computations for solving a non-linear optimization problem mentioned later to obtain the relative locations of sensors. Let $\boldsymbol{s}_k$ denote the location of sensor k $(k = 1, \ldots, N)$, and let d_{ij} and $d_{ij}^{(m)}$ respectively denote the real and estimated distance between sensors i and j. In general, $d_{ij}^{(m)} \neq d_{ji}^{(m)}$. We use the following variables:

$$m(i, j) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{sensor } i \text{ can estimate the distance to sensor } j (\neq i), \\ 0 & \text{otherwise,} \end{cases}$$

$$M(i) \stackrel{\text{def}}{=} \{ j \in (1, \ldots, N) | m(i, j) = 1 \}, \quad S \stackrel{\text{def}}{=} (\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N),$$

$$\boldsymbol{d} \stackrel{\text{def}}{=} \underbrace{(d_{12}, d_{13}, \ldots, d_{ij}, \ldots, d_{NN-1})}_{m(i,j)=1},$$

$$\boldsymbol{d}^{(m)} \stackrel{\text{def}}{=} \underbrace{(d_{12}^{(m)}, d_{13}^{(m)}, \ldots, d_{ij}^{(m)}, \ldots, d_{NN-1}^{(m)})}_{m(i,j)=1}.$$

Note that $\boldsymbol{d}$ ($\boldsymbol{d}^{(m)}$) is a vector sorting d_{ij} ($d_{ij}^{(m)}$) in a lexicographic order, while skipping d_{ij} ($d_{ij}^{(m)}$) if $m(i, j) = 0$[1].

B. Relative locations

The size and shape of an arbitrary object remains the same under the transformations of translation, rotation, and reflection. We use these basic transformations for the definition of the relative locations.

Definition 1. *We say that $P \stackrel{\text{def}}{=} (\boldsymbol{p}_1, \ldots, \boldsymbol{p}_N)$ is a set of relative locations of sensors with their co-ordinates S, if applying one of the three transformations (translation, rotation, and reflection) or its combination to S yields P.*

Let $P \stackrel{\mathcal{R}}{\sim} S$ denote the fact that P is a set of relative locations of S.

Note that relation $\mathcal{R}$ is an *equivalence relation* in $\mathbb{R}^{2N}$; $\mathcal{R}(S)$ defined below is its *equivalent class* with representative S:

$$\mathcal{R}(S) \stackrel{\text{def}}{=} \left\{ P \in \mathbb{R}^{2N} \middle| P \stackrel{\mathcal{R}}{\sim} S \right\}.$$

An arbitrary element of $\mathcal{R}(S)$ is a set of relative locations of S. That is, there are multiple (infinite) sets of relative locations of sensors even if absolute location of each sensor is uniquely given. We introduce another equivalent relation.

Definition 2. *We say that S is equivalent to P in the sense of inter distances if*

$$\forall i \in (1, \ldots, N), \ \forall j \in M(i) \quad |\boldsymbol{s}_i - \boldsymbol{s}_j| = |\boldsymbol{p}_i - \boldsymbol{p}_j|.$$

We denote it by $S \stackrel{\mathcal{R}_d}{\sim} P$.

Relation $\mathcal{R}_d$ is also an equivalent relation in $\mathbb{R}^{2N}$; $\mathcal{R}_d(S)$ defined below is its equivalent class with representative S:

$$\mathcal{R}_d(S) \stackrel{\text{def}}{=} \left\{ P \in \mathbb{R}^{2N} \middle| P \stackrel{\mathcal{R}_d}{\sim} S \right\}.$$

Definition 3. *If $\mathcal{R}_d(S) = \mathcal{R}(S)$, then we say that sensors are relatively localizable based on the inter distances.*

In this work, assuming the relative localizability of sensors, we consider the problem to determine a set of relative locations (an element of $\mathcal{R}(S)$) from $\boldsymbol{d}^{(m)}$.

IV. Relative Localization

A. Formulation as an Optimization Problem

We define the following mapping $\boldsymbol{d}^{(e)} : \mathbb{R}^{2N} \to \mathbb{R}^{\sum_i |M(i)|}$.

$$\begin{aligned} \boldsymbol{d}^{(e)}(X) &= \boldsymbol{d}^{(e)}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) \\ &= \underbrace{(|\boldsymbol{x}_1 - \boldsymbol{x}_2|, |\boldsymbol{x}_1 - \boldsymbol{x}_3|, \ldots, |\boldsymbol{x}_i - \boldsymbol{x}_j|, \ldots, |\boldsymbol{x}_{N-1} - \boldsymbol{x}_N|)}_{m(i,j)=1}. \end{aligned}$$

[1]Because of the limited effective range, the distance between two sensors separated far from each other cannot be measured.

Note that $\boldsymbol{d}^{(e)}(X)$, a vector sorting $|\boldsymbol{x}_i - \boldsymbol{x}_j|$ in a lexicographic order[2], is a set of distances between sensors when $\boldsymbol{x}_i$ is coordinate of sensor i. Using $\boldsymbol{d}^{(e)}(X)$, we define the following objective function;

$$\varepsilon\left(X; \boldsymbol{d}^{(m)}\right) \stackrel{\text{def}}{=} \|\boldsymbol{d}^{(e)}(X) - \boldsymbol{d}^{(m)}\|^2 \stackrel{\text{def}}{=} \sum_i \sum_{j \in M(i)} (|\boldsymbol{x}_i - \boldsymbol{x}_j| - d_{ij}^{(m)})^2. \tag{1}$$

If $\boldsymbol{x}_k$ is estimated relative location of sensor k, $(|\boldsymbol{x}_i - \boldsymbol{x}_j| - d_{ij}^{(m)})$ represents the difference between the estimated and measured distance between sensors i and j, and thus the objective function (1) represents the mean square error. The following result readily comes from the definition of the relative localizability.

Theorem 1. *If sensors are relatively localizable based on the inter distances, then*

$$P = \arg\min_X \varepsilon(X; \boldsymbol{d}) \iff P \in \mathcal{R}(S).$$

It follows from Theorem 1 that when $\boldsymbol{d} = \boldsymbol{d}^{(m)}$, the problem of finding the solution minimizing $\varepsilon\left(X; \boldsymbol{d}^{(m)}\right)$ is equivalent to finding the relative locations of sensors.

If $\boldsymbol{d} \neq \boldsymbol{d}^{(m)}$, that is, the distance measurements are not accurate, the solution minimizing $\varepsilon\left(X; \boldsymbol{d}^{(m)}\right)$ is not always a set of relative locations. We could, however, expect that the solution minimizing $\varepsilon\left(X; \boldsymbol{d}^{(m)}\right)$ should be a good approximation of a set of relative locations. So, in this work, we propose to use $\hat{P}$ defined below as an estimator of a set of relative locations of sensors.

$$\hat{P} = \arg\min_X \varepsilon\left(X; \boldsymbol{d}^{(m)}\right). \tag{2}$$

Since $\varepsilon\left(X; \boldsymbol{d}^{(m)}\right)$ remains the same under the three transformations (translation, rotation, and reflection), it follows from (2) that an arbitrary element of $\mathcal{R}(\hat{P})$ can also be used as an estimator of relative locations of sensors because they have the same mean square error (the same objective function value). That is, the optimization problem in the right-hand side of (2) has multiple (infinite number of) solutions. This comes from the nature of the problem of relative localization.

B. Characteristics of Estimator

Definition 4. *We say that $\boldsymbol{d} \in \mathbb{R}^{\sum_i |M(i)|}$ is feasible if there exists $X \in \mathbb{R}^{2N}$ such that $\boldsymbol{d} = \boldsymbol{d}^{(e)}(X)$.*

Let $\mathcal{D}_f \subset \mathbb{R}^{\sum_i |M(i)|}$ denote the set of feasible inter distances (feasible space) and for $\boldsymbol{d} \in \mathcal{D}_f$ we define

$$\mathcal{P}(\boldsymbol{d}) \stackrel{\text{def}}{=} \left\{X | \boldsymbol{d} = \boldsymbol{d}^{(e)}(X)\right\}.$$

Observe that

$$\min_X \varepsilon\left(X; \boldsymbol{d}^{(m)}\right) = \min_X \|\boldsymbol{d}^{(e)}(X) - \boldsymbol{d}^{(m)}\|^2 = \min_{\boldsymbol{d} \in \mathcal{D}_f} \|\boldsymbol{d} - \boldsymbol{d}^{(m)}\|^2.$$

[2]It skips $|\boldsymbol{x}_i - \boldsymbol{x}_j|$ if $m(i,j) = 0$.

Thus, solving the optimization problem in the right-hand side of (2) is equivalent to finding the projection from $\boldsymbol{d}^{(m)}$ to $\mathcal{D}_f$ defined below

$$\pi_f(\boldsymbol{d}^{(m)}) \stackrel{\text{def}}{=} \arg\min_{\boldsymbol{d} \in \mathcal{D}_f} \left\|\boldsymbol{d} - \boldsymbol{d}^{(m)}\right\|,$$

and obtaining an element of $\mathcal{P}(\pi_f(\boldsymbol{d}^{(m)}))$.

The estimation error of estimator $\hat{P}$ is upper bounded as shown in the next Lemma.

Lemma 1.

$$\left\|\boldsymbol{d}^{(e)}(\hat{P}) - \boldsymbol{d}\right\| \leq 2 \left\|\boldsymbol{d}^{(m)} - \boldsymbol{d}\right\|.$$

Proof: It follows from the definition of $\hat{P}$ that

$$\|\boldsymbol{d}^{(e)}(\hat{P}) - \boldsymbol{d}^{(m)}\| \leq \|\boldsymbol{d} - \boldsymbol{d}^{(m)}\|.$$

Then, the proof readily follows from the observation that

$$\begin{aligned}\|\boldsymbol{d}^{(e)}(\hat{P}) - \boldsymbol{d}\| &= \|\boldsymbol{d}^{(e)}(\hat{P}) - \boldsymbol{d}^{(m)} + \boldsymbol{d}^{(m)} - \boldsymbol{d}\| \\ &\leq \|\boldsymbol{d}^{(e)}(\hat{P}) - \boldsymbol{d}^{(m)}\| + \|\boldsymbol{d} - \boldsymbol{d}^{(m)}\| \\ &\leq 2\|\boldsymbol{d} - \boldsymbol{d}^{(m)}\|.\end{aligned}$$

■

We expect that $\hat{P}$ converges to a set of real relative locations as the sensor density tends to infinity if the distance measurement error is unbiased (its average is zero). To verify this conjecture, consider a case where many sensors are deployed and the following normalized objective function is used:

$$\varepsilon\left(X; \boldsymbol{d}^{(m)}\right) = \frac{1}{\sum_i |M(i)|} \sum_i \sum_{j \in M(i)} (|\boldsymbol{x}_i - \boldsymbol{x}_j| - d_{ij}^{(m)})^2. \tag{3}$$

Theorem 2. *If $|M(i)|$ tends to infinity as $N \to \infty$, $E[\boldsymbol{d}^{(m)}] = \boldsymbol{d}$, and $d_{ij}^{(m)}$ and $d_{kl}^{(m)}$ are independent for $(i, j) \neq (k, l)$, then*

$$\lim_{N \to \infty} \boldsymbol{d}^{(e)}(\hat{P}) = \boldsymbol{d}.$$

(The proof is omitted for lack of space.)

C. Solution of the Optimization Problem

Since $\varepsilon\left(X; \boldsymbol{d}^{(m)}\right)$ is a non-linear function of X, and thus the right-hand side of (2) cannot be obtained analytically. Fortunately, the right-hand side of (2) can be numerically obtained by the standard steepest descendent method. The gradient of $\varepsilon\left(X; \boldsymbol{d}^{(m)}\right)$ is given as below

$$\frac{\partial}{\partial \boldsymbol{x}_i} \varepsilon\left(X; \boldsymbol{d}^{(m)}\right) = 2 \sum_{j \in M(i)}^{N} \left(1 - \frac{d_{ij}^{(m)}}{|\boldsymbol{x}_i - \boldsymbol{x}_j|}\right)(\boldsymbol{x}_i - \boldsymbol{x}_j).$$

More sophisticated algorithm [16], [17] for solving the optimization problem can be obtained by observing

$$\varepsilon\left(\hat{P}; \boldsymbol{d}^{(m)}\right) = g(\hat{P}, \hat{P}) \leq g(\hat{P}, Q),$$

where

$$g(\hat{P}, Q) \stackrel{\text{def}}{=} \sum_{i=1}^{N} \sum_{j \in M(i)} \left\{|\hat{\boldsymbol{p}}_i - \hat{\boldsymbol{p}}_j|^2 + (d_{ij}^{(m)})^2 - 2(\hat{\boldsymbol{p}}_i - \hat{\boldsymbol{p}}_j)(\boldsymbol{q}_i - \boldsymbol{q}_j)\frac{d_{ij}^{(m)}}{|\boldsymbol{q}_i - \boldsymbol{q}_j|}\right\},$$

and $Q \stackrel{\text{def}}{=} (\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N)$. Assume that we have the nth location estimates $\hat{P}^{(n)}$. Minimizing $g(\hat{P}, \hat{P}^{(n)})$ is achieved by finding $\partial g(\hat{P}, \hat{P}^{(n)})/\partial \hat{\boldsymbol{p}}_i = 0$ where

$$\frac{\partial g(\hat{P}, \hat{P}^{(n)})}{\partial \hat{\boldsymbol{p}}_i} = 2 \sum_{j \in M(i)} \left\{ (\hat{\boldsymbol{p}}_i - \hat{\boldsymbol{p}}_j) - (\hat{\boldsymbol{p}}_i^{(n)} - \boldsymbol{p}_j^{(n)}) \frac{d_{ij}^{(m)}}{d_{ij}^{(e:n)}} \right\},$$

and

$$d_{ij}^{(e:n)} = |\hat{\boldsymbol{p}}_i^{(n)} - \hat{\boldsymbol{p}}_j^{(n)}|.$$

The condition $\partial g(\hat{P}, \hat{P}^{(n)})/\partial \hat{\boldsymbol{p}}_i = 0$ yields

$$\hat{\boldsymbol{p}}_i = \frac{\hat{\boldsymbol{p}}_i^{(n)}}{|M(i)|} \sum_{j \in M(i)}^{N} \frac{d_{ij}^{(m)}}{d_{ij}^{(e:n)}} + \frac{1}{|M(i)|} \sum_{j \in M(i)}^{N} \hat{\boldsymbol{p}}_j - \frac{1}{|M(i)|} \sum_{j \in M(i)}^{N} \hat{\boldsymbol{p}}_j^{(n)} \frac{d_{ij}^{(m)}}{d_{ij}^{(e:n)}},$$

from which we could obtain the following recursive formula:

$$\hat{\boldsymbol{p}}_i^{(n+1)} = \frac{\hat{\boldsymbol{p}}_i^{(n)}}{|M(i)|} \sum_{j \in M(i)}^{N} \frac{d_{ij}^{(m)}}{d_{ij}^{(e:n)}} + \frac{1}{|M(i)|} \sum_{j \in M(i)}^{N} \hat{\boldsymbol{p}}_j^{(n)} \left(1 - \frac{d_{ij}^{(m)}}{d_{ij}^{(e:n)}} \right). \quad (4)$$

Repeated substitution of the location estimates via (4) constantly decreases the objective function and should reach the point minimizing the objective function.

The solution obtained by the gradient descendent algorithm may have dependence on the initial condition (initial estimated locations) because of the existence of local minimums. We experimentally found that the gradient descendent algorithm or the repeated substitution through (4) reaches the optimal solution almost surely despite of the choice of initial conditions when all pair-wise distances of sensors are measurable (see Sec. V). More precisely, different initial conditions lead to different final results, but they have the same objective-function value. When the distances between all sensor pairs are measurable, the classical MDS algorithm [15] can be applied for the localization. In such a case, the objective function itself seems to have a good structure, where the standard gradient descent algorithm can find the optimal solution.

By contrast, when $m(i, j) = 0$ for some (i, j) pairs due to the constraint of radio range, different initial conditions generally lead to different final results whose objective-function values are different (see Sec. V-C). So, in order to obtain better results, we should carefully choose a good initial condition. We will state one promising choice of the initial condition in Sec. V-C.

V. Evaluation

A. Simulation Conditions

We evaluate the performance of the proposed relative localization through a set of simulations. In a square region with side of 100 m, sensors were randomly deployed. We used four different scales of sensor networks: the number of sensors (N) were respectively 25, 100, 250, and 1000. The distance measurement error was given by

$$d_{ij}^{(m)} - d_{ij} = e d_{ij} U, \quad (5)$$

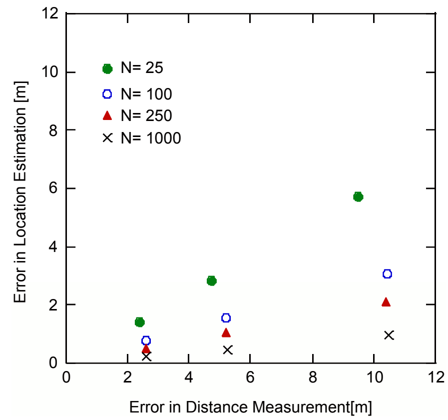

Fig. 1. Relationship between location estimation error and distance measurement error

where U is a random variable following the uniform distribution between -1 and 1, and e is a parameter for varying the error size. To solve the optimization problem in (2), we produced 100 initial conditions, each of which randomly chose the initial estimated locations of sensors within the region. Starting from 100 initial conditions, we respectively obtained the solutions by the steepest descendent method and, among those, we chose the best result that minimizes the objective function.

B. Simulation Results: Scenario 1

We first evaluated our proposal in a scenario (Scenario 1) where each sensor can measure the distances to all other sensors; that is, $m(i, j) = 1$ for all (i, j) pairs.

1) Accuracy of Localization: Figure 1 shows the relationship between the location estimation error and the distance measurement error. We carried out simulation experiments using ten different deployment patterns of sensors, and the figure shows the average for the ten deployment patterns. The error in location estimation means the average difference between estimated and real absolute locations of sensors. Note that our algorithm estimates the relative locations, not the absolute locations. Thus, we first applied the *co-ordinate transformation* to estimated *relative* locations in order to obtain the estimated *absolute* locations; more precisely, the combination of three basic transformations (translation, rotation, and reflection) was applied to relative locations so as to obtain a new set of locations *nearest* to the real absolute locations, where *nearest* means the smallest mean square error. The resultant locations were used as *estimated absolute* locations for qualifying the accuracy of the localization error.

In the figure, we find three features of the relative localization. First, the relationship between the location estimation error and the distance measurement error is almost linear. Second, the location estimation error is much smaller than the distance measurement error. For example, when $N = 1000$, the location estimation error was less than 1 m in average when the distance measurement error was larger than 10 m. Finally, the location estimation error decreases as the number of sensors

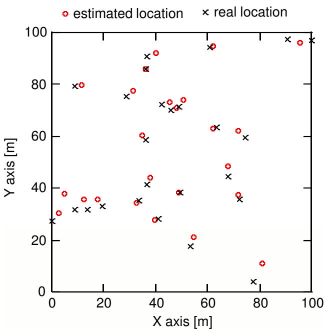

Fig. 2. Estimated and real sensor locations: $N = 25$

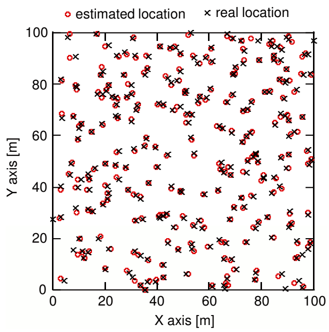

Fig. 3. Estimated and real sensor locations: $N = 250$

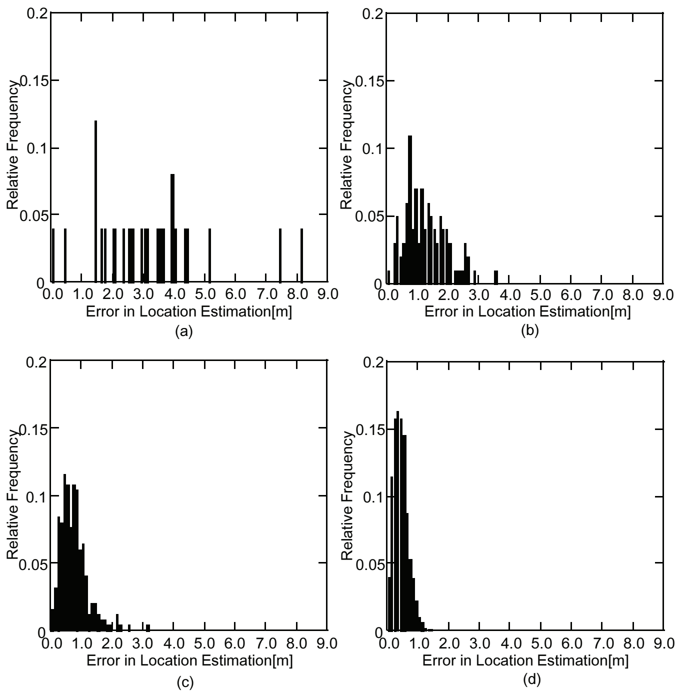

Fig. 4. Relative frequency of the location estimation error: (a) $N = 25$, (b) $N = 100$, (c) $N = 250$, (d) $N = 1000$.

increases. The final finding is consistent with Theorem 2; according to the theorem, the localization error converges to zero as the number of sensors tends to infinity.

Figures 2 and 3 respectively compare the estimated and real absolute locations of sensors when $N = 25$ and $N = 250$. The distance measurement errors were given by (5) with $e = 0.2$. For the comparison, the estimated absolute locations of sensors were also obtained by using the co-ordinate transformation. Both figures show that the discrepancy between estimated and real locations is insignificant. In particular, when $N = 250$, the estimated locations are very consistent with the actual locations.

Figures 4(a) – 4(d) show the relative frequency of the location estimation error. These figures show that the range of variation of location estimation errors becomes narrower as the number of sensors increases.

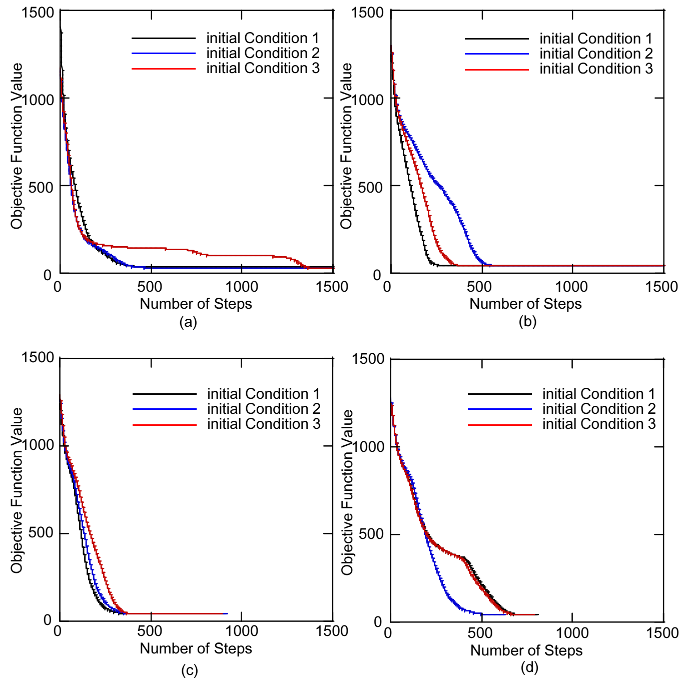

Fig. 5. Convergence of the steepest descent method: (a) $N = 25$, (b) $N = 100$, (c) $N = 250$, (d) $N = 1000$

2) Convergence of Steepest Descendent Method: Figures 5(a) – 5(d) show how the steepest descendent method converges to the final result. The vertical axis shows the value of the objective function. Each figure shows three lines, which correspond to the convergence behaviors respectively obtained with three different initial conditions (initial estimated locations). The figures show that the speed of the convergence does not depend on the network scale (number of sensors). The choice of the initial condition affects the convergence behavior, but it is not significant. We note that, in Scenario 1, the objective function converges to the same value in spite of the difference of initial solutions.

C. Simulation Results: Scenario 2

Next, we evaluated our proposal in another scenario (Scenario 2) where each sensor can measure distance between its neighbors located within a given radio range.

Figure 6 shows the relationship between the location estimation error and the radio range for distance measurement when the distance measurement error are given by (5) with $e = 0.4$. The radio range for distance measurement largely affects the accuracy of the location estimation; the accuracy of the location estimation is degraded more as the range, within which the distance can be measured, becomes shorter.

Figures 7(a) – 7(d) plot the objective function values of final solutions, which were respectively obtained with 100 different initial conditions when the radio range for distance measurement was 50 m and $e = 0.4$. The horizontal axis shows the IDs of initial conditions. In contrast to Scenario 1, we find the large dependence of solutions (estimated relative locations)

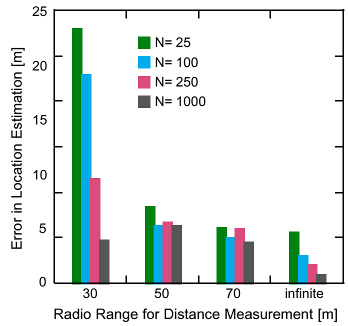

Fig. 6. Relationship between location estimation error and radio range for distance measurement

on the initial conditions. This result shows that the objective function should have many local minimums in Scenario 2 and thus the steepest descendent method may not obtain the global minimum.

If the distance between a pair of nodes cannot be measured, we can assume that it is equal to the length of the shortest path composed of links with known (measured) lengths, as used in [15]. The location estimation under this assumption can be used as an initial condition. In Figs. 7(a) – 7(d), we also show the objective function values of final solutions obtained by stating this initial condition, which is referred to as *initial condition with estimated distances* in the figures. Surprisingly, this initial condition always yields the smallest objective function value in each case. This indicates that, even in Scenario2, we could obtain good estimates of locations, avoiding the problem of local minimums.

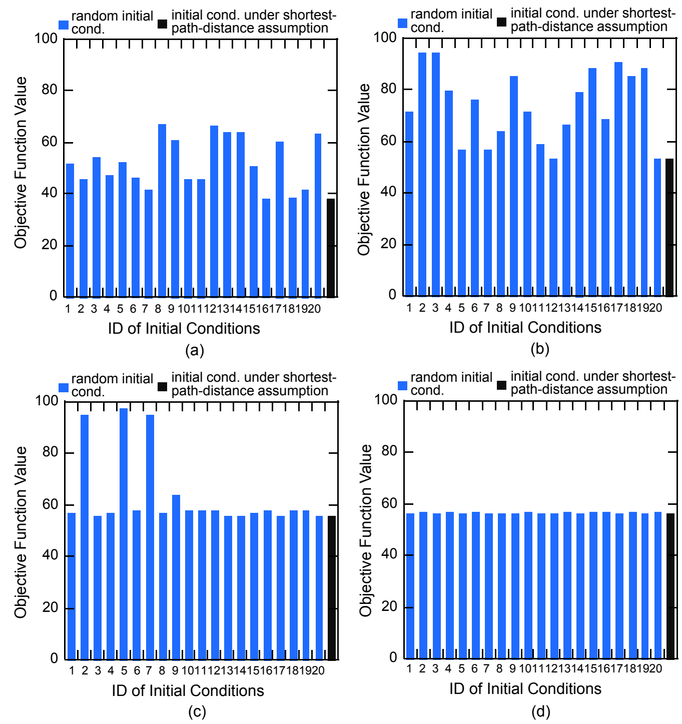

Fig. 7. Dependence of optimization results on initial conditions: (a) N = 25, (b) N = 100, (c) N = 250, (d) N = 1000

VI. Conclusion

We have studied the relative localization of sensors based on the measurement of distances between sensors. Our approach works very well when each sensor can measure the distances between all other sensors, but more sophisticated optimization technique than the steepest descendent method may be required when the radio range for the distance measurement is small. A remaining work is the evaluation of the feasibility using real sensor nodes in some testbed or real environments.

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