The Baseline Constrained LAMBDA Method for Single Epoch, Single Frequency Attitude Determination applications

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BIOGRAPHY

Peter Buist graduated from Delft University of Technology in 1999. After a few years in the Japanese space/GPS industry, he returned at the end of 2006 to Delft University of Technology to work on his PhD in the field of attitude determination and formation flying.

ABSTRACT

This paper explains the principles of a baseline constrained version of the LAMBDA method. This version of the LAMBDA method is optimal in the way it makes use of the integerness of the initial integer ambiguities and the known baseline length.

This paper gives an overview of the theory behind the new method and compares the method with other existing ambiguity resolution methods applied for GNSS-based attitude determination. Emphasis is on the most challenging application: the single frequency, single epoch application.

Engineering aspects of the implementation of this method are discussed and the method is demonstrated using data collected in dynamic attitude environments: a slowly moving vessel and a more dynamic aircraft. The influence of the environment, especially dynamics and data quality (multipath) is known to have an impact on the performance of ambiguity resolution methods. In this paper we investigate this influence on the standard and baseline constrained version of the LAMBDA method.

Results of the standard LAMBDA method are compared with the baseline constrained version for the single frequency, single epoch, short constrained baseline application, and the latter version demonstrated a much better performance.

1 INTRODUCTION

GNSS-based attitude determination can be used for a number of different applications. Well-known is the 3 antennas/2 baseline (or more) configuration, which can perform full attitude determination onboard land- or water-based vehicles, aircraft or even spacecraft. With a 2 antennas/1 baseline configuration, it is possible to use GNSS as a pointing device similar to a magnetic compass [1]. With a 1 antenna/0 baseline configuration, GNSS could be used as a backup attitude sensor for certain applications [2].

Except for the 1 antenna method, GNSS-based attitude determination uses carrier phase observations. Carrier phase observations have as advantage over pseudorange observations that they are much more precise, but the drawback is that the user will have to solve the initial ambiguity problem. A large number of methods have been developed for this ambiguity resolution in GNSS-based attitude determination.

The LAMBDA method is currently the standard for a wide range of GNSS ambiguity resolution problems. The method is highly efficient and has a guaranteed optimal success rate performance. The LAMBDA method, in its original formulation, is applicable to all unconstrained and linearly constrained GNSS models. Its baseline constrained version is however also applicable to all quadratically constrained GNSS models.

In this paper the concepts of the baseline constrained LAMBDA method will be compared with other existing ambiguity resolution methods developed for GNSS-based attitude determination and a demonstration of the different steps using real data collected in dynamic environments will be given. The data was collected in two different dynamic environments; a slowly moving vessel and a more dynamic aircraft. Emphasis is laid on the practical implementation aspects for single frequency, single epoch applications.

2 OVERVIEW OF PREVIOUS WORK/ METHODS USED IN ATTITUDE DETERMINATION

In this section we will give a short overview of the different ambiguity resolution techniques, which were developed up until now. Table 1 summarizes the different ambiguity resolution techniques and divides them into different sub-groups.

A possible division for ambiguity resolution methods is motion-based and search-based. Examples of motion-based methods are [3] and [4]. This kind of methods are by definition not instantaneous as they make use of the abundance of information provided by the platform or GNSS satellite motion. This motion will result in relative carrier phase variations that can be exploited to identify the cycle ambiguities.

In the method developed by Cohen [3], the Cartesian position of the slave antenna relative to its unknown starting points is determined without knowledge of the integer ambiguities. If the platform moves by a large angle (or for a static platform if data is collected for a long time), a set of displacement vectors can be used to calculate an initial solution to initialize a nonlinear least-squares fit, which is a batch solution for a data set in which the integer terms remain constant over the collection period. By using the baseline constraints in the case of non-coplanar baseline configurations, motion about a single axis of rotation is adequate for ambiguity resolution. Hence drawbacks are that this method is not instantaneous, a significant amount of memory is required for storage of the batch data, a motion of the baseline is required (or data has to be collected for even a longer time), a priori attitude information is required and in case of a coplanar baseline configuration, motion about two axes is required.

In [4], some of the disadvantages of the motion-based method are improved by resolving the integer ambiguities without any a priori attitude knowledge, by requiring less computational effort and a noniterative implementation, but the method requires three noncoplanar baselines, and, since user motion is still required, can not be used instantaneous.

The search-based methods are motionless and therefore can be used instantaneous (i.e. using only observations from a single epoch). These methods find a solution that minimizes the error residuals at a specific time by searching through a list of all potential integers and rejecting all candidate solutions for which the weight function of the residual errors becomes too large. For the search-based approach, we would like to limit the number of potential solutions as much as possible and reject unlikely candidates as early as possible in the search process.

According to [5], the search-based GNSS ambiguity resolution methods itself can be divided into two different approaches: the Bayesian approach and the non-Bayesian approach. In the Bayesian approach not only the vector observable y is assumed to be random but also the two vectors of unknown parameters, for example the integer vector and baseline or attitude solution. In the non-Bayesian approach only the y vector is considered to be a random vector.

In [6] a review is given on Bayesian approach and its relation with the non-Bayesian approach. In [7] a Bayesian approach is developed especially for the GNSS-based attitude determination application. For this kind of approach the ambiguity resolution and attitude filtering is not being performed separately but a unified approach is proposed in which the ambiguity resolution and filtering processes are combined under a Gaussian Sum Filter (GSF). Although the Bayesian approach has not yet found a wide-spread use in any of the GNSS applications, the concept itself is of interest to justify further research in this field. An example of an experimental implementation of a Bayesian approach for formation flight is [8]. (For completeness we would like to mention that the LAMBDA method, which will be discussed next, and especially the decorrelation step of the LAMBDA method can be applied in the Bayesian approach [9].)

For the non-Bayesian approach the LAMBDA method is well known and will be explained in more detail later. The LAMBDA method is using an integer least squares (ILS) approach.

Not all search-based methods make use of an integer least squares. The Knight method [10] and ARCE [11] search for the relative positions of the GPS antennas, where as for [12] the attitude space is searched. These methods could work very well if some information about baseline length and/or initial attitude is known. In that case their search space could be much reduced and therefore the search time would be shortened while the success rate would increase.

The Knight method was originally developed for aircraft applications and it requires some initial knowledge of the attitude of the platform. In applications where the initially attitude is not known it is reported not to work optimal [13]. The novelty of this method is partially the way the large number of potential solutions for the integer ambiguity is reduced by eliminating all unlikely solutions in an early stage of the search process.

Ambiguity resolution for attitude determination is different from ambiguity search for kinematic positioning because for attitude determination the baseline is generally speaking relatively short and the baseline length is precisely known.

Table 1: Ambiguity Resolution Methods applied in GNSS Attitude/ Short Baseline Applications

Motion-based	Search-based	
	Bayesian	Non-Bayesian approach
Cohen [3] Crassidis [4]	GSF[7]	LAMBDA [19], Knight [10], ARCE [11], Sutton [14], Brown [15], LSAST [16], OMEGA [17], INU [12], SSTL [18]

For attitude determination systems the initial integer cycle ambiguity search is often performed in the solution space instead of the measurement space [14]. This paper from Sutton states that in other words the initial search is based on baseline length instead of the phase residual. The fundamental equation relating the GPS phase difference measurements from two antennas can be solved independently for each antenna separation vector. The two vectors can be used to compute the attitude associated with the plane containing the two vectors. A better approach [15] is to take advantage of the geometry constraint that the two vector are coplanar. Two angles are required to specify the direction of the second vector towards the first one. However if given that the 2 vectors must lie in the same plane only one angle is required to specify the direction of the 2 vectors relative to each other.

Another method that makes use of the same principle is ARCE[11]. ARCE stands for Ambiguity Resolution using Constraint Equation. It divides the n-dimensional integer ambiguity vector into independent and dependent parts. If the carrier phase observations are accurate enough it assumes that not n-dimensional but a 3 dimensional space can be searched to find the proper integer ambiguities.

If knowledge of the baseline vector is applied as a pseudo-measurement the search space is reduced to 2-dimensional. Because of the errors on the carrier-phase observations, it is not straightforward to determine a appropriate search space and there is no theoretical guide to divide the search space into independent and dependent parts. However the success-rate as reported in [11] is reasonable.

Algorithms as LSAST[16] and OMEGA[17] can also be classified as independent-ambiguity-search methods, but they also apply the variance-covariance matrix obtained from the float solution.

In [12] the ambiguity search is performed in combination with an initial attitude search, which makes it possible for the GPS receiver to start attitude determination without any initial information. In order to accomplish this, an instantaneous ambiguity search algorithm was developed. In order to limit the ambiguity search space, coarse attitude estimation is calculated using information of visible satellites. By simply applying information in which direction the GPS antennas are pointing, the computational time required for the ambiguity search was reduced.

Another example of an attitude-based search method is [18], which make use of the Gram-Schmidt orthogonalization procedure in order to find individual baseline solutions. The second step of this method narrows the search space by identifying the most likely combination of baselines by employing the known orientation of the baselines. The third step finds the initial attitude and employs an integrity check for the found solution.

3 LAMBDA METHOD FOR ATTITUDE DETERMINATION

The LAMBDA method, introduced in [19], is currently the standard for a wide range of GNSS ambiguity resolution problems. The method is highly efficient and has a guaranteed optimal success rate performance. A number of researchers have been working on different implementations for attitude determination applications of the LAMBDA method [20][21][22][23][24][25][26].

Psiaki [27] researched a number of implementation aspects for relative navigation and attitude determination applications using the LAMBDA method.

In [24] and [26], it is stated that if the baseline length is known it can be used to verify the found integer solution afterwards. Hence in these approaches the known baseline length is used for validation rather then estimation of the integer ambiguities. In [24], an interesting approach is also discussed in case the baseline length is not fixed or unknown.

The LAMBDA method, in its original formulation, is applicable to all unconstrained and linearly constrained GNSS models. Its baseline constrained version, which will be discussed in 3.2, is however also applicable to all quadratically constrained GNSS models. First results of the method applied to attitude determination were reported in [20] and [21], the theory of which is described in [22]. In [21] results were presented in combination with an ambiguity validation method for GNSS-based attitude determination.

In this section we will give, after a review of the LAMBDA method, an explanation of the baseline constrained LAMBDA method, in which the known baseline length is used to find the integer ambiguities. After the theoretical explanation, the different implementation aspects of the method will be demonstrated using data collected in dynamic attitude environments. In this paper we will concentrate on the single epoch, single frequency application.

For a multi-frequency and/or multi-epoch application using a short baseline, the standard LAMBDA method is expected to be sufficient, but as we will see next for our application the baseline constrained LAMBDA method will give superior performance.

3.1 THEORETICAL BACKGROUND OF THE STANDARD LAMBDA METHOD

The double difference carrier phases can be written as a system of linearized observation equations [28]:

$$y = Aa + Bb + e$$

Where y is the vector of observed minus estimated double difference carrier phases and/or code observations of the order m, a is the unknown vector of ambiguities expressed in cycles rather than range to maintain their integer character, b is the baseline vector, which for relative navigation applications is unknown but for which the length in attitude determination is known, B is the geometry matrix containing normalized line-of-sight vectors, e is the vector that contains the noise terms, A is a design matrix that links the data vector to the unknown vector a.

Note that for kinematic applications, both of the receivers will use their own clock, and therefore we prefer to use double difference carrier phase observations. However for the attitude determination application, as generally a single GNSS receiver is used in combination with a number of antennas, we could use the single difference observations and estimate the offset between the antennas ('linebias') as an extra state in b. In this paper we will follow the double difference approach.

The variance matrix of y is given by the positive definite matrix Q_y which is assumed to be known, hence we can write[22]:

$$E(y)=Aa+Bb,\ D(y)=Q_y$$

Where E is the mean or the expected value and D is the variance or dispersion of y.

As explained in [28], the least squares solution of the linear system of observation equations is obtained from:

$$\min_{a,b}\left\|y - Aa - Bb\right\|^2_{Q_y} \text{, where } \left\|..\right\|^2_{Q_y} = (..)^T Q_y^{-1}(..) .$$

It is important to note that the ambiguities are integers and therefore a and b are ($a \in Z^m$, $b \in R^3$) with Z^m being the m-dimensional space of integers. The solution of the linear system of observation equations is therefore a constrained least-squares problem. Of course it would be possible to express the integer ambiguities in the space of reals, after all the space of integers is a subset of the space of reals. In that case the solution could be found by an ordinary least squares solution. However this would imply that the ambiguities would not be integers anymore and we would not use all available information to solve the problem, information that could have, if used in an appropriate way, a very beneficial impact on our ability to estimate the unknown parameters.

The solution of the unconstrained least squares problem (the float solution) is hereafter referred to as $\hat{a}$ and $\hat{b}$, the solution of the constrained least squares problem (the fixed solution) is referred to as $\breve{a}$ and $\breve{b}$.

With the float solutions ($\hat{a} \in R^m$, $\hat{b} \in R^3$), the solution of the linear system of observations becomes the squared norm of the least squares residual vector: $\left\|y - A\hat{a} - B\hat{b}\right\|^2_{Q_y} = \left\|\hat{e}\right\|^2_{Q_y}$.

If we use the constraint that the solution of a has to be an integer ($\breve{a} \in Z^m$, $\breve{b} = \hat{b}(\breve{a}) \in R^3$), the minimum solution becomes:

$$\left\|y - A\breve{a} - B\breve{b}\right\|^2_{Q_y} = \left\|\breve{e}\right\|^2_{Q_y} = \left\|\hat{e}\right\|^2_{Q_y} + \left\|\hat{a} - \breve{a}\right\|^2_{Q_{\hat{a}}} .$$

The minimum of this equation is clearly equal to or larger than the float solution.

From the discussion above it is clear that we can use a multistep approach to solve the linear system of observation equations: in the first step the unconstrained least-squares problem is solved. In this step the float solutions for the ambiguity and baseline vectors are found in combination with the variance-covariance matrice.
The result of the first is the input for the second step. In the second step we solve the vector of integer least-squares estimates of the ambiguities $\breve{a}$:

$$\min\left\|\hat{a} - a\right\|^2_{Q_{\hat{a}}} \text{ with } a \in Z^m,$$

which can be written as $\breve{a} = \arg(\min\left\|\hat{a} - a\right\|^2_{Q_{\hat{a}}})$ with arg is vector of integers that minimize the term within the brackets.

A so called integer search is needed to find $\breve{a}$. The search space for this problem is defined as

$\left\{a \in \mathrm{Z}^{\mathrm{m}} \mid \left(\|\hat{a}-a\|_{Q_{\hat{a}}}^{2}\right) \leq \chi^{2}\right\}$, with χ^2 is a properly chosen constant.

The LAMBDA method, which stands for Least-squares AMBiguity Decorrelation Adjustment is an efficient way to do this. The method has been introduced by Teunissen [19][29] and is currently considered the world-standard for unconstrained and linearly constrained GNSS ambiguity resolution. The method's popularity stems from its high numerical efficiency and guaranteed optimal performance in terms of achieving the highest possible success rate [30].

For GNSS applications the search space, which boundary is determined by the variance matrix of the float solution of *a*, is very much influenced by the high correlation between the ambiguities. However a transformation can be applied that decorrelates the ambiguities and normally improves the computational efficiency of the search dramatically [22][28].

Once the solution $\breve{a}$ has been obtained, the residual $(\hat{a}-\breve{a})$ is used to adjust the float solution $\hat{b}$ of the first step, which is $\breve{b}=\hat{b}(\breve{a})$.

Therefore the final fixed baseline solution is obtained as:

$\breve{b}=\hat{b}(\breve{a})=\hat{b}-Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}(\hat{a}-\breve{a})$ with $Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}=-\left(B^{T}Q_{y}^{-1}B\right)^{-1}B^{T}Q_{y}^{-1}A$.

This equation shows the relation that exists between the float and fixed solution, $\hat{b}$ and $\breve{b}$. It shows that the difference between the two baseline estimates depends on the difference between float and the fixed least-squares ambiguity estimate $\hat{a}$ and $\breve{a}$.

Now we can calculate the variance-covariance matrix of $\breve{b}$ as [28]: $Q_{\breve{b}}=Q_{\hat{b}}-Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}Q_{\hat{a}\hat{b}}$, if we have confidence that the integer is estimated correctly [31]. From this equation it is clear that then $Q_{\breve{b}}<Q_{\hat{b}}$ and hence the fixed baseline estimation is more accurate than the float solution. However if the ambiguities are fixed incorrectly, the result might be that the fixed solution is worse than the float solution.

3.2 BASELINE CONSTRAINED LAMBDA METHOD

For GNSS-based attitude determination application, we can make use of the knowledge that the length of the baseline is known and constant. Hence the baseline constrained integer ambiguity resolution, can make use of the standard GNSS model by adding the length constraint (i.e. the quadratically or baseline constraint) of the baseline $\|b\|_{I_3}=l$, where l is known. The observation equations then become:

$E(y)=Aa+Bb$, $D(y)=Q_y$, $\|b\|_{I_3}=l$, $a\in Z^m$, $b\in R^3$

Using this transformation the least squares criterion reads

$$\min_{a\in Z^m,\, b\in R^3, \|b\|_{I_3}=l}\|y-Aa-Bb\|_{Q_y}^2$$

$$=\|\hat{e}\|_{Q_y}^2+\min_{a\in Z^m,\, b\in R^3, \|b\|_{I_3}=l}\left(\|\hat{a}-a\|_{Q_{\hat{a}}}^2+\|\hat{b}(a)-b\|_{Q_{\hat{b}(a)}}^2\right)$$

$$=\|\hat{e}\|_{Q_y}^2+\min_{a\in Z^m}\left(\|\hat{a}-a\|_{Q_{\hat{a}}}^2+\min_{b\in R^3, \|b\|_{I_3}=l}\left(\|\hat{b}(a)-b\|_{Q_{\hat{b}(a)}}^2\right)\right)$$

where $\hat{b}(a)$ is the least squares solution for b, assuming that a is known.

There are a number of methods available to solve this quadratically constrained least-squares problem. In this paper we will use a method based on an iteration of orthogonal projections onto an ellipsoid described in [22].

3.3 INTEGER SEARCH FOR BASELINE CONSTRAINED INTEGER AMBIGUITY RESOLUTION

Next we will have to search for the integer least-squares ambiguity vector $\breve{a}$ in the integer search space: $\psi\left(\chi^2\right)=\left\{a\in Z^m \mid \left(\|\hat{a}-a\|_{Q_{\hat{a}}}^2+\|\hat{b}(a)-\breve{b}(a)\|_{Q_{\hat{b}(a)}}^2\right)\leq\chi^2\right\}$

Note that this search space is not ellipsoidal anymore due to the presence of the residual baseline term.

For the baseline constrained approach we first use the standard LAMBDA method to collect all integer vectors inside the search space $\psi_0\left(\chi^2\right)=\left\{a\in Z^m \mid \left(\|\hat{a}-a\|_{Q_{\hat{a}}}^2\right)\leq\chi^2\right\}$, this search space contains all integer vectors of $\psi(\chi^2)$ and thus also the vector $\breve{a}$ where we are looking for. To limit the search space we only use those integers that satisfy the inequality $\|\hat{b}(a)-\breve{b}(a)\|_{Q_{\hat{b}(a)}}^2\leq\chi^2-\|\hat{a}-a\|_{Q_{\hat{a}}}^2$.

Of course one would like this search space to be as small as possible in order to finish the search in a reasonable time which is especially important for real-time applications. However in order to guarantee that the search space is not empty, the search space should neither be chosen to small.

Furthermore the computation of $\breve{b}(a)$ can be very time consuming if it has to be done for many integer candidates *a*. However if we change the search space we can avoid this by using the smallest and largest eigenvalues of $Q_{\hat{b}(a)}^{-1}$. It can namely be shown that the original search space has to be smaller than a search space expressed by the largest eigenvalue and larger than a search space expressed by the smallest eigenvalue [22].

In this work we will use the same approach as was used in Park and Teunissen[20], where the search space is defined as the smallest eigenvalue plus a value χ^2 which is increased until the search space is not empty.

3.4 THE SUCCESS RATE OF THE AMBIGUITY RESOLUTION METHOD

An ambiguity resolution method will always provide the integer values for the phase ambiguities, however there is no guarantee that these values are correct. The ambiguity success rate, i.e. probability of correct integer estimation, depends on three factors: the observation equations (functional model), the precision of the observations (the stochastic model), and the chosen method for ambiguity resolution [10]. The theoretical success rate can be calculated once the functional and stochastic models are available, even before the actual measurements are obtained. For a discussion on the theoretical ambiguity success rate see [31]. In this paper we will focus on the experimental or empirical success rate.

For analysis of the performance of the standard and the baseline constrained LAMBDA method, we compare the true integer ambiguity vector (the 'true solution' known from post-processing) and the estimated integer ambiguity vector at every epoch. The empirical success rate is defined as the number of epochs where the obtained integer ambiguity vector was equal to the true integer ambiguity vector divided by the total number of epochs.

4 DEMONSTRATION USING DATA COLLECTED IN DYNAMIC EXPERIMENTS

In the next section we will demonstrate the standard and baseline contrained LAMBDA method using data collected in an experiment onboard a vessel and an aircraft**.**

4.1 VESSEL EXPERIMENT

For the first demonstration of the baseline constrained LAMBDA method we will use data for which the orientation of the baseline vector is relatively slowly changing. This data was collected onboard a vessel. Details about this experiment can be found in [32]. For this experiment two receivers (1 Leica SR530, 1 Ashtech Z12) are used, both connected to their own GPS antenna. These two antennas form a baseline of about 2 meter. As the antennas are placed on their own mast, are using a choke-ring (Ashtech) or are survey grade (Leica AT502) and have a relatively free field-of-view, the impact of blocking and multipath on the observations is expected to be very small. Both receivers are of the surveying type providing very precise observations. As can be seen in Figure 1, there is one more antenna onboard the ship. As this data is only used to demonstrate the algorithm, the data from the third receiver is not analyzed in this paper.

Although double frequency observations are available for both receivers, we use only single frequency, single epoch observations as this is the most realistic scenario for attitude determination and the most challenging application.

The number of locked GPS satellites and PDOP are shown in Figure 2. As can be observed in this figure the number of locked channels is between 6 and 8. PDOP is between 2.2 and 4.2. Hence, the number and the geometry of observations for this experiment is reasonably good.

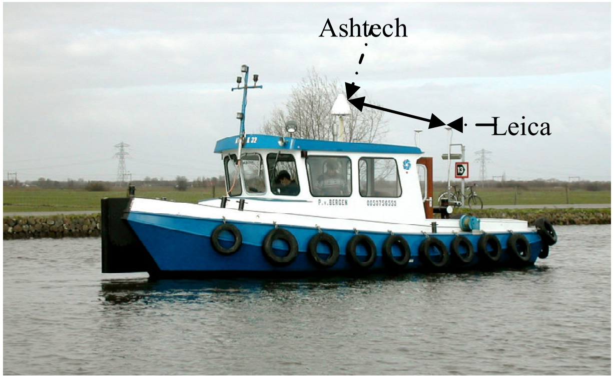

Figure 1 Experimental set-up for the vessel experiment

The horizontal relative vector between the two antennas is shown for x and y axis in Figure 3 and Figure 4. Figure 3 shows the float solutions as a function of time. It is very clear from this picture that the ship moves up and down on the same trajectory for 4 times and, as it is a vessel, all movements and rotations are done in one plane: the horizontal plane. Figure 4 shows the float and fixed solutions. It is clear from this figure that successful solutions are positioned on a circle with a diameter of 2 times the distance between the two antennas. So one can think of this figure as representing the master or reference antenna as being in the origin and the second antenna is placed on a circle around the origin at a distance equal to the distance between the two antennas, because both antennas are attached to the same moving body. Their relative position is only changing if the vessel makes a turn.

On epochs for which the algorithm failed to find the proper ambiguity solution, the fixed solution is, because of the baseline constraint, still positioned on the known distance from the reference antenna but the solution is situated out of the horizontal or x-y plane. This can be observed in Figure 5, which shows the estimated float and fixed solutions in 3D.

The overall empirical success rate and success rate subdivided for the number of locked GPS satellites at each epoch are summarized in Table 2. As can be seen in this table the success rate increases with the number of available observations but even with 6 locked GPS satellites for a single frequency the success rate is very high (>95 %). Also the success rate of the standard LAMBDA method using single frequency and single epoch observations is shown in Table 2. As is expected the success rate for the standard LAMBDA method also increases with the number of available measurements. Overall success rate of the standard LAMBDA method is 82.01%, for the constrained LAMBDA method this is improved to 99.5%. If the number of locked satellites is respectively 6, 7 and 8, this success rate improves from 28.33% to 96.51%, 81.15% to 99.77% and 96.67% to 99.97%. Hence especially when a limited number of observations is available the success rate increases spectacularly for the baseline constrained LAMBDA method.

As was explained in section 3.3 of this paper, at first the smallest value for the integer search space χ^2 is computed. If the search space is found to be empty, which means that the unconstrained least squares solution is and baseline constrained integer least solution is not located in the applied search space, the search process is redone after increasing the search space.

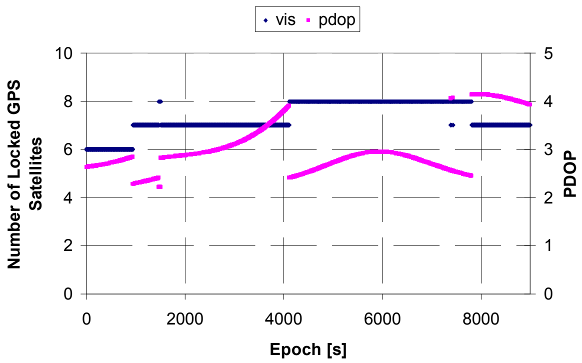

Figure 2 Number of locked satellites/PDOP as a function of experiment time

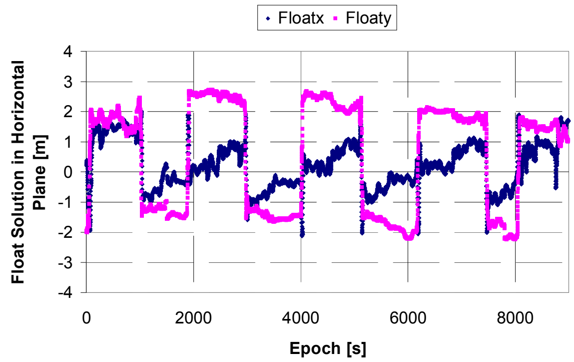

Figure 3 The float solution of x and y in the horizontal plane as a function of experiment time

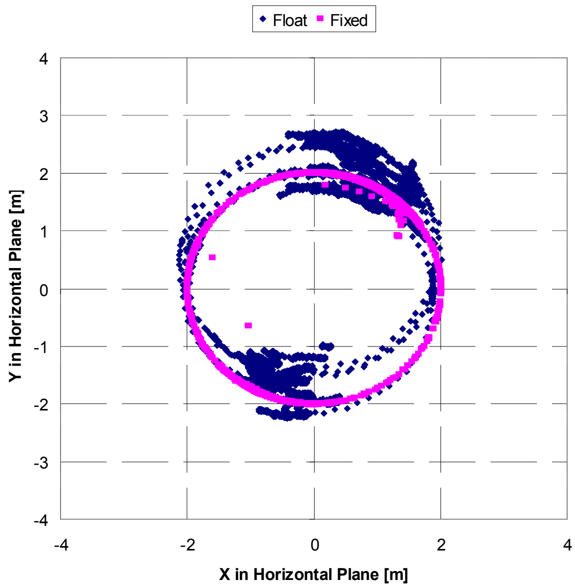

Figure 4 The float and the fixed solution of y as a function of the x solution in the horizontal plane

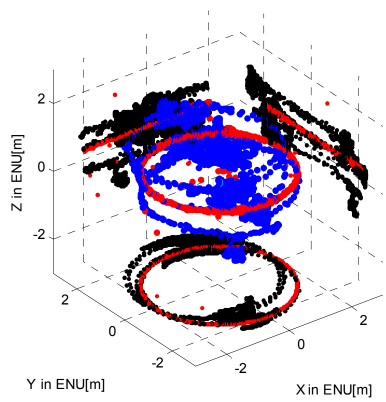

Figure 5 The float and fixed solution for x, y and z in ENU (East, North, Up)-Frame

Figure 6 shows the number of times that the initial value of χ^2 is doubled before the solution is accepted (i.e. the search space was not empty). Most of time the initial search space contained the constrained solution (>83%) and thus we can conclude that, as expected from the set-up of the experiment, the stochastic model is accurate. The figure also shows the empirical success rate, and it is clear from this figure that if the constrained solution was found using the initial value of the search space, it is most likely that the correct ambiguity solution was found. Of course this is the case as the observations at that epoch are reasonably accurate. The initial values and the applied values of χ^2 when a solution was accepted (i.e. both unconstrained and the constrained integer solution are located in the same search space) are shown in Figure 7.

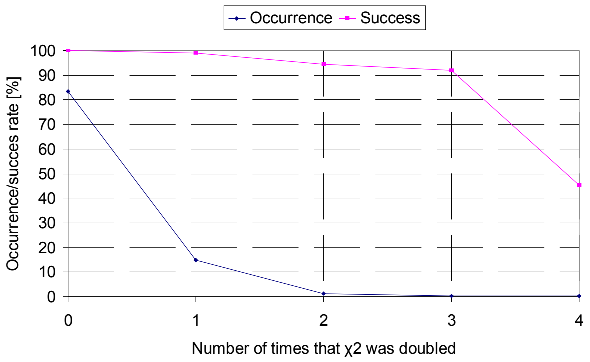

Figure 6 The occurrence and empirical success rate as a function of the times that the initial integer search space was doubled

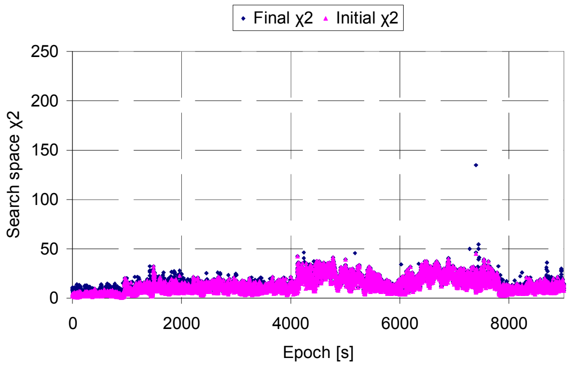

Figure 7 The initial and final integer search space

By comparing Figure 2 and Figure 7 it is clear that the size of the search space, as expected, is depending on the number of available observations. This relationship is shown in Figure 8. Important to note for single epoch (i.e. real time) applications is that the required search time will on average increase with the increase of search space (which is related to the number of observations) but that the number of potential solutions will increase as well. This is important for the computational load required to find the proper integer ambiguity. We will discuss this further in section 5.

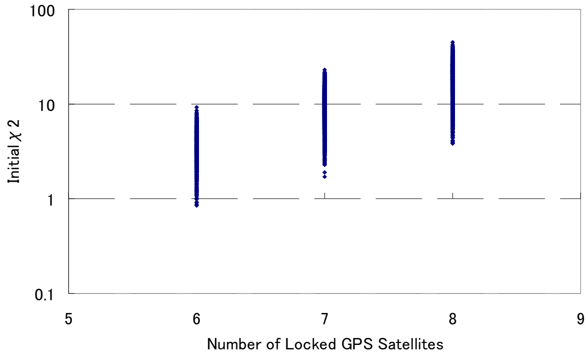

Figure 8 The initial integer search space vs. the number of locked satellites

4.2 AIRCRAFT EXPERIMENT

For this experiment, data was collected on 31 August 2006 onboard the Cessna Citation airplane of the faculty of Aerospace Engineering, Delft University of Technology. The first 500 seconds of data was collected before take-off, the rest of data was collected during flight. A three antenna configuration forming two baseline vectors was used (Figure 9), where each of the antennas was connected to the same receiver, manufactured by Septentrio. The first baseline has a length of about 5.5 meter and the second about 7.5 meter. In this paper we will use only data from the first baseline vector.

The data collected onboard the aircraft is more challenging than the vessel because it is collected in a more dynamic environment. This is partly because of the higher velocity and rate of the attitude maneuvers applied on the GPS receiver for this experiment but also due to flexibility in the baseline vector. This is especially true for the second baseline as one of the antennas that make up this baseline is mounted on a wing of the aircraft. The first baseline is located on the body of the aircraft, hence this baseline vector is expected to be much more constant. Furthermore, the GPS antennas are, in order to minimize their impact on the aerodynamical shape, mounted directly on the aircraft. For the vessel application, every antenna was mounted on its own mast and a choke-ring or survey grade antenna was applied (Figure 1), therefore the multipath environment of the aircraft will, compared with the vessel, be much more severe. Another point is that the reference baseline length is less well determined compared to the ship experiment.

Figure 10 shows the number of locked GPS satellites and the PDOP calculated with the line of sight vectors to these satellites. As can be observed in this figure the number of locked channels is between 6 and 8, and PDOP is between 1.4 and 3.5. Hence, the number and the geometry of observations for this experiment is also reasonably good. During the first 500 seconds before take-off the number of visible satellites change rapidly, due to blocking.

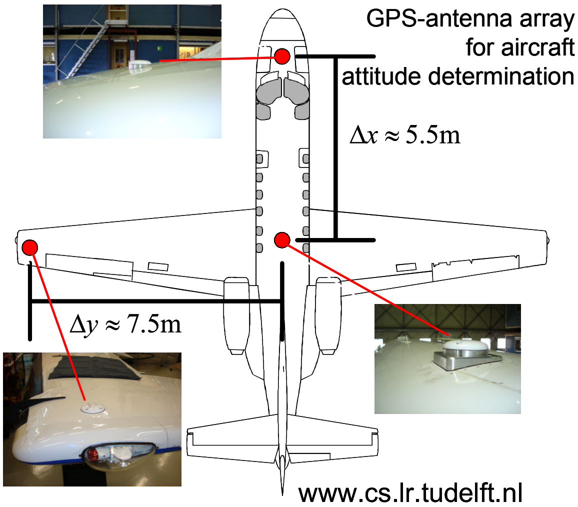

Figure 9 Experimental Setup for the aircraft experiment

A difference with the vessel application is that the attitude of an aircraft varies in three dimensions (two planes), and therefore all correct fixed solutions will not be placed on a single (horizontal) plane. For that reason no figure with the relative vector between the two antennas similar to Figure 4 and Figure 5 is included for the aircraft application.

The overall empirical success rate and success rate subdivided for the number of locked GPS satellites for each epoch are summarized in Table 2. As can be seen in this table the success rate increases with the number of available observations. As will be discussed later because of the difference in the environment, compared with the vessel experiment the success rate is lower. Also the success rate of the standard LAMBDA method is shown in Table 2. Overall success rate of the standard LAMBDA method using single frequency, single epoch observations is 14.30%, for the constrained LAMBDA method this is improved to 58.15%. If the number of locked satellites is respectively, 6, 7 and 8 this success rate improves from 12.70% to 31.75%, 8.29% to 45.70% and 17.22% to 63.26%. Hence the success rate increases spectacularly. From Figure 10 it is clear that especially for the case that 6 GPS satellites are locked (there are only a few epochs with 6 locked satellites), the number of epochs is too low to give a representative empirical success rate. This is expected to be the reason the success rate for the case of 6 locked channels is a little higher than for the case that 7 satellites are locked.

For this experiment the initial search space contained the constrained solution only around 15% of the time (Figure 11). The reason is that the stochastic model is not good enough to capture all the uncertainties – in contrast to the vessel experiment. Especially multipath errors are expected to be the cause of poor data quality, but they cannot be accounted for in the stochastic model. Hence there is a mismatch between the data and the model. The figure also shows the empirical success rate, again showing that when the initial search space does contain the constrained solution, that it is most likely that the ambiguities are fixed correctly. Obviously, in these epochs the quality of the observation data is good, so that the constrained and unconstrained ambiguity solutions do not lie far apart, or may even be identical.

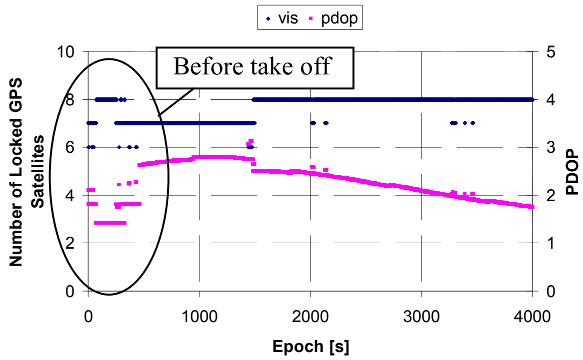

Figure 10 Number of locked satellites/PDOP as a function of experiment time

Figure 11 The occurrence and empirical success rate as a function of the times that the initial integer search space was doubled

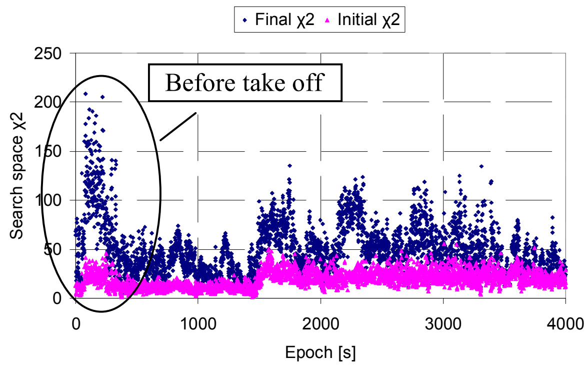

Figure 12 The initial and final integer search space

The initial values and the applied values of χ^2 when a solution was accepted are shown in Figure 12. By comparing Figure 7 and Figure 12 it is clear that the initial values for χ^2 are very similar, but that for the aircraft experiment, because of the less accurate observations, more often no solution was found for the initial search space and therefore the search space was increased until an acceptable solution was found or the maximum number of cycles (5) was reached. Compared with Figure 8, Figure 13 shows larger variation for the initial search space, which is also an indication for the data quality.

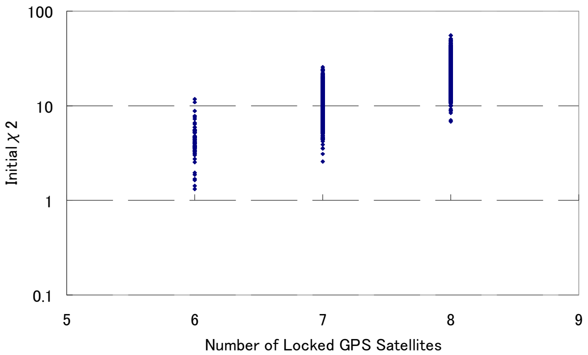

Figure 13 The initial integer search space vs. the number of locked satellites

The reason for the lower success rate is of course the poorer data quality not captured by the functional and stochastic model. This is a result of the fact the GPS antennas are mounted directly on the aircraft, and that the aircraft application is much more dynamic than the ship application. Especially in the first 500 seconds before take-off, multipath errors from the surroundings of the aircraft are dominant. Moreover for the ship experiment an elevation threshold of 10 degrees was used and therefore the receiver selects only GPS satellites on an elevation angle higher than this threshold. For the aircraft application, an elevation angle of 0 deg is used as a threshold and as is well known GPS observations from close to the local horizon are less accurate (i.e. more susceptible to multipath and thermal noise) than observations from a higher elevation angle.

Table 2 Single Frequency, Epoch by Epoch Ambiguity Resolution: Empirical success rates (%) with Unconstrained LAMBDA (UC) and Baseline Constrained LAMBDA (BC)

	Vessel Experiment		Aircraft Experiment	
	UC	BC	UC	BC
Overall	82.01	99.5	14.30	58.15
Number of locked sats				
6	28.33	96.51	12.70	31.75
7	81.15	99.77	8.29	45.70
8	96.67	99.97	17.22	63.26

However disregarding data from lower elevations did not increase the success rates for the aircraft experiment. For example a threshold of 1, 5 and 10 degrees was tried and the empirical success rate of 1 deg was 58.25%, 5 deg was 54% and for 10 degrees it became around 20%. This can be explained by the fact that a lower number of available observations will result in a lower success rate (Table 2).

5 DISCUSION/FURTHER WORK

This paper has given a short description of ambiguity resolution methods developed for GNSS-based attitude determination. For a more complete comparison between the existing methods and the baseline constrained LAMBDA method, we plan to implement some of the other methods in software as well and compare their performance (computational load, empirical success rate, etc) using the same experimental data.

Results of the standard LAMBDA method were compared with the baseline constrained version for the single frequency, single epoch, short constrained baseline application. The latter version demonstrated a much better performance, both for a low dynamic and a high dynamic environment.

It has been shown that the size of the initial search space and the success rate increases when more observations are available. This is as expected, but might be important to consider if we would like to implement the new ambiguity resolution in a real application. A high success rate is of course desirable, but a large search space might cause problems in terms of computation time. Problems could occur for example if the proposed method is implemented in a space-qualified CPU for a real-time application. From our experience we know that search times with the standard LAMBDA method are not an issue, even for real-time applications with multiple frequencies. But it should be investigated whether with the baseline constrained LAMBDA method, increasing the search space - especially if no solution is found in the initial search space - might cause problems. In that case one could opt for resolving only a subset of the ambiguities, but only if the success rate is still high enough.

In [22], a baseline constrained float solution was introduced. It is expected that implementation of this alternative initialization of the search process will improve the performance. We plan to investigate this, together with the influence of data quality algorithms to eliminate low quality observations.

Another interesting result from the aircraft experiment is that the probability of success decreases if no solution was found in the initial search space and thus the search space was increased. Therefore the number of times that the search space was increased could be of use for validation of the solution. This result is important because for real-time applications, it might be better to wait one epoch for new observations if there is some doubt about the solution. And this also demonstrates that a good validation method, as proposed in [21], is required for real time applications.

In this paper we restricted our selves to the single baseline application. An obvious next step is to combine data from multiple baselines. The data collected onboard the Cessna Citation, which was described in section 4.2 of this paper, could be used for that, but then we have to take into account that the second baseline is flexible as it is made up by an antenna on the body of the aircraft and the wing. We expect that combining data from multiple baselines can improve the success rate of the baseline constrained LAMBDA method even further.

The baseline constrained LAMBDA method is in this paper preliminary described as an approach to solve the initial ambiguity problem, but it could also be used as a method for combined integer ambiguity resolution and attitude determination[22]. Similar to vector methods as QUEST[33], this would have as advantage that no knowledge about the attitude dynamics is required. Further research in this field is required.

6 CONCLUSIONS

This paper provided a short overview of previous work in the field of ambiguity resolution methods for GNSS-based attitude determination.

A baseline constrained version of the well-known LAMBDA method was introduced and described mathematically. Engineering aspects of the implementation of this method were discussed and the method was demonstrated using data collected in dynamic attitude environments: a slowly moving vessel and a more dynamic aircraft. The influence of the environment, especially dynamics and data quality (multipath) is known to have an impact on the performance of ambiguity resolution methods. In this paper we investigated the influence of the environment on the standard and the baseline constrained version of the LAMBDA method.

The baseline constrained version of the LAMBDA method is optimal in the way that it makes use of the integerness of the initial integer ambiguities and the known baseline length.

The LAMBDA method is known as a very reliable ambiguity resolution method. Results of the standard LAMBDA method were compared with the baseline constrained version for single frequency, single epoch, short constrained baseline applications and the latter version demonstrated a much better performance.

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