



**ASIA PACIFIC UNIVERSITY OF
TECHNOLOGY & INNOVATION (APU)**

DIGITAL ELECTRONICS

GROUP ASSIGNMENT

TITLE OF PROJECT	Passcode Authentication System
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DATE SUBMITTED	6th December 2023

Work Breakdown Structure

S/No.	Tasks Assigned	Name
1	Introduction	Shah Saud
2	Proposed Application	Shah Saud
3	Circuit Design	Kevin Sebastian Muhammad Hanzalah Hussain
4	Simulation and Physical Results	Darren Liem
5	Discussion	Imtiaz Ahmed Muhammad Hanzalah Hussain
6	Conclusion	Shah Saud
7	References	Combined
8	Appendix	Darren Liemantoro

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INTRODUCTION

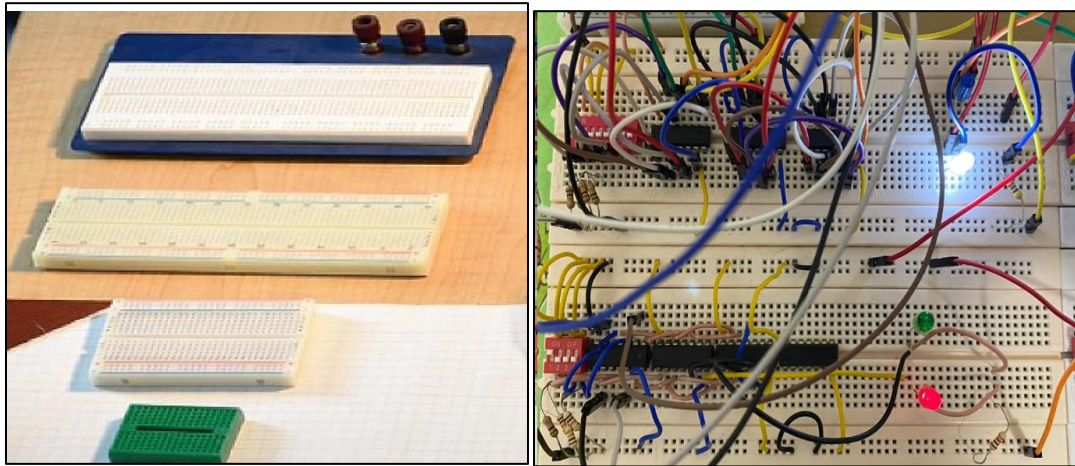


Figure 1 - Breadboard Circuits and their Ranging Sizes (CircuitBread, 2023)

In aims of building circuits or prototyping, a breadboard serves as a medium for attaching components & connections without the requirement of soldering. The breadboard holes safeguard the connection by physically holding wires & parts and providing means of electrical connection. The ease of speed & use is useful for learning and simple circuit prototyping. Higher complex and frequency circuits are less suited for this type of circuit breadboards. However, in digital electronics, these breadboards are compact and finetuned for working on personal projects.

In the field of digital electronics, breadboard circuits have an in depth working based on digital signals for performing numerous tasks to meet specific requirements. In the case of an input signal applied to circuits in digital form, it is given as 0's & 1's based on the generic binary language format. In terms of this assignment, utilizing the basic fundamentals of digital electronics, a password authentication system can be simulated and constructed on a physical breadboard.

Utilized in everyday aspects of life such as computers, phones, doors, bank-accounts, etc. Passcodes are a string of letters/numbers to prevent any individual/corporation from stealing or confiscating personal information. The devices consist of passcodes which have a certain extent of tries before disallowing access because of numerous failed login attempts. **(Descope, 2023)**

Thereby, the main aim of this digital electronics project is to deliver a password-based security system exhibiting the provision to alter the passcode by sole authority, utilizing logic gates as comparators and controlled inverters. Furthermore, the proposed system will prove to be

a user-friendly security system for homes and organizations (if commercialized). In the condition when the passcode is input correctly, the green L.E.D will switch on. Besides, any incorrect means of inputting the password through the switches, the red L.E.D will initiate, indicating to re-enter the password or seek security assistance.

OBJECTIVES

The objective of this group assignment is to construct a passcode authentication system on a physical breadboard utilizing minimum components and restricted to logic gates only, without the utilization of ICs.

1. **Investigate** the digital circuit operation which have the ability to perform code-conversion and data transmission functionalities.
2. **Exemplify** group collaboration and work effectively on digital circuit design problems

PROPOSED APPLICATION

Passcode Authentication System with LED Status Indication:



Figure 2 - Password Authentication Infographic (Descope, 2023)

In general, designing the passcode system involves the users to input a specified passcode through the switches on the circuit. Afterwards, the electronic system compares the input passcode directly based on the stored passcode utilizing the logic gates. Moving on with the authentication process, once the passcode is entered, the system automatically compares each bit

of the input passcode. This segment is where the green and red L.E.Ds come into essence. In terms of the logic gate operations, in the password authentication system:

1. AND Gates - responsible for the comparison of the corresponding bits of stored & entered passcodes.
2. OR Gates - in charge of the combination of the AND gate outputs for determination of an entire successful passcode match.
3. NOT gates - have the capability of signal generation for illuminating LEDs on the basis of success or failure in authentication.

This passcode authentication system was implemented without the consideration of flip-flops adhering to the requirements of the assignment. The efficiency within the logic gate operations is vital for ensuring the LED indication reflects the status of authentication accurately and seamlessly. The circuit should be carefully designed for the proper functionality of the L.E.Ds. Integration of the power supply is considered through a 9V battery. Switches provide means of pull-up / pull-down configurations for passcode authentication system activation to complete the setup in the physical breadboard.

In terms of the troubleshooting and testing, it is conducted by the data validation method of inputting incorrect and correct passwords and observing the two L.E.D.s placed in the circuit. To ensure that no safety is compromised in construction of the physical breadboard circuit, the wires will be cut from both ends by an automatic wire stripping tool.

EQUIPMENT UTILIZED

Equipment Utilized – Passcode Authentication System			
S/No.	Name of Component	Quantity	Model
1	XOR Gate	4x	IC 74LS86
2	NOT Gate	4x	IC 74LS04
3	AND Gate	4x	IC 74LS08
4	DIP Switch	6x	4-Bit
5	DIP Switch	2x	8-Bit
6	Resistors	20x	10 k Ω
7	Resistors	4x	470 Ω
8	Battery	4x	1.5V
9	Breadboard	5x	4 Long, 1 Short (Solderless)
10	Connecting Wires	4x	Arduino-compatible and normal cable

Figure 3 - Equipment Utilized - Password Authentication System

Proposed Circuit Diagram for Passcode Authentication System

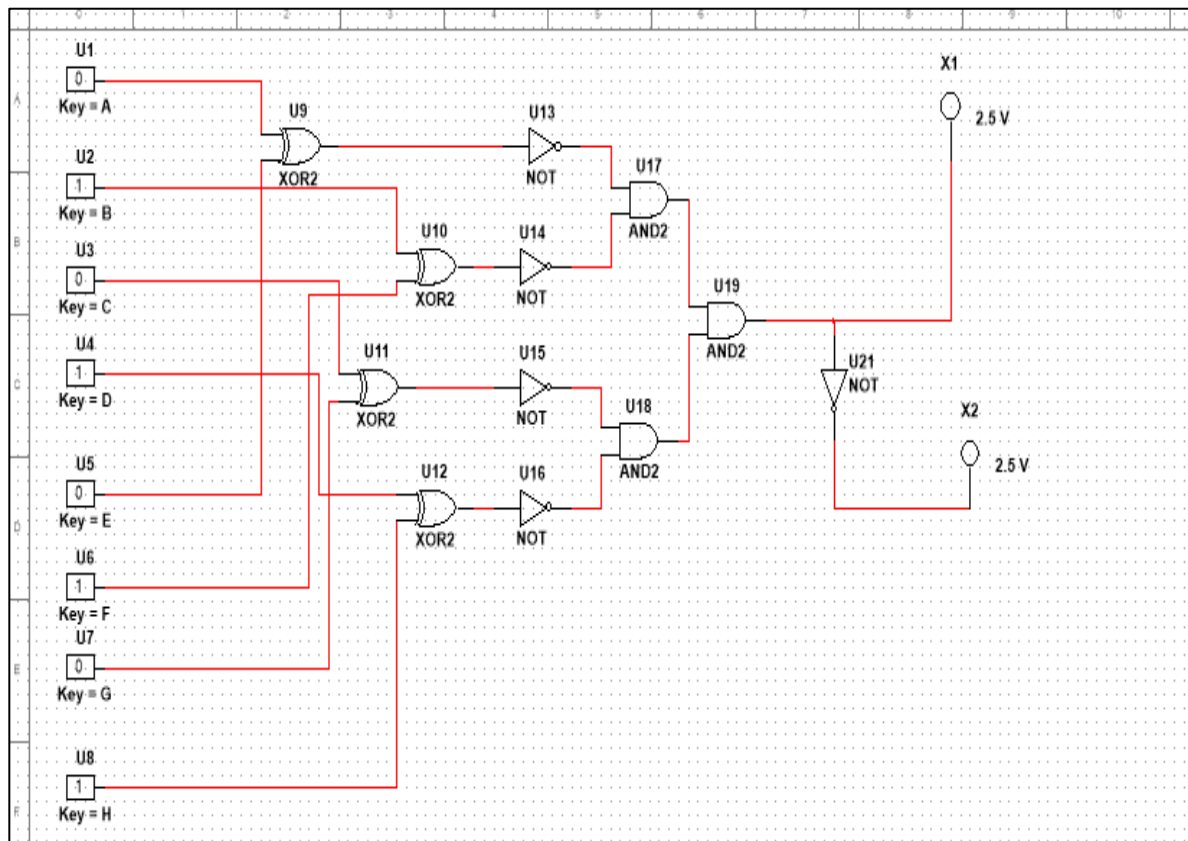


Figure 4 - Proposed Circuit Diagram - Password Authentication System

The circuit design will utilize XOR gate, NOT gate, AND gate, DIP switches, LEDs, and resistors. It is considered that the careful integration of 10k Ω resistors will safeguard the LEDs from excessive voltage, ensuring optimal functionality and preventing potential damage.

1. AND Gates - responsible for the comparison of the corresponding bits of stored & entered passcodes.
2. OR Gates - in charge of the combination of the AND gate outputs for determination of an entire successful passcode match.
3. NOT gates - have the capability of signal generation for illuminating LEDs on the basis of success or failure in authentication.

DESIGN OF THE SYSTEM

Truth Table

NO.	A	B	C	D	E	F	G	H	X
1	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	1	0
3	0	0	0	0	0	0	1	0	0
4	0	0	0	0	0	0	1	1	0
5	0	0	0	0	0	1	0	0	0
6	0	0	0	0	0	1	0	1	0
7	0	0	0	0	0	1	1	0	0
8	0	0	0	0	0	1	1	1	0
9	0	0	0	0	1	0	0	0	0
10	0	0	0	0	1	0	0	1	0
11	0	0	0	0	1	0	1	0	0
12	0	0	0	0	1	0	1	1	0
13	0	0	0	0	1	1	0	0	0
14	0	0	0	0	1	1	0	1	0
15	0	0	0	0	1	1	1	0	0
16	0	0	0	0	1	1	1	1	0
17	0	0	0	1	0	0	0	0	0
18	0	0	0	1	0	0	0	1	1
19	0	0	0	1	0	0	1	0	0
20	0	0	0	1	0	0	1	1	0
21	0	0	0	1	0	1	0	0	0
22	0	0	0	1	0	1	0	1	0
23	0	0	0	1	0	1	1	0	0
24	0	0	0	1	0	1	1	1	0
25	0	0	0	1	1	0	0	0	0
26	0	0	0	1	1	0	0	1	0
27	0	0	0	1	1	0	1	0	0
28	0	0	0	1	1	0	1	1	0
29	0	0	0	1	1	1	0	0	0
30	0	0	0	1	1	1	0	1	0
31	0	0	0	1	1	1	1	0	0
32	0	0	0	1	1	1	1	1	0
33	0	0	1	0	0	0	0	0	0
34	0	0	1	0	0	0	0	1	0
35	0	0	1	0	0	0	1	0	1
36	0	0	1	0	0	0	1	1	0
37	0	0	1	0	0	1	0	0	0
38	0	0	1	0	0	1	0	1	0
39	0	0	1	0	0	1	1	0	0
40	0	0	1	0	0	1	1	1	0
41	0	0	1	0	1	0	0	0	0
42	0	0	1	0	1	0	0	1	0
43	0	0	1	0	1	0	1	0	0
44	0	0	1	0	1	0	1	1	0
45	0	0	1	0	1	1	0	0	0

46	0	0	1	0	1	1	0	1	0
47	0	0	1	0	1	1	1	0	0
48	0	0	1	0	1	1	1	1	0
49	0	0	1	1	0	0	0	0	0
50	0	0	1	1	0	0	0	1	0
51	0	0	1	1	0	0	1	0	0
52	0	0	1	1	0	0	1	1	1
53	0	0	1	1	0	1	0	0	0
54	0	0	1	1	0	1	0	1	0
55	0	0	1	1	0	1	1	0	0
56	0	0	1	1	0	1	1	1	0
57	0	0	1	1	1	0	0	0	0
58	0	0	1	1	1	0	0	1	0
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61	0	0	1	1	1	1	0	0	0
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68	0	1	0	0	0	0	1	1	0
69	0	1	0	0	0	1	0	0	0
70	0	1	0	0	0	1	0	1	0
71	0	1	0	0	0	1	1	0	0
72	0	1	0	0	0	1	1	1	0
73	0	1	0	0	1	0	0	0	0
74	0	1	0	0	1	0	0	1	0
75	0	1	0	0	1	0	1	0	0
76	0	1	0	0	1	0	1	1	0
77	0	1	0	0	1	1	0	0	0
78	0	1	0	0	1	1	0	1	0
79	0	1	0	0	1	1	1	0	0
80	0	1	0	0	1	1	1	1	0
81	0	1	0	1	0	0	0	0	0
82	0	1	0	1	0	0	0	1	0
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86	0	1	0	1	0	1	0	1	0
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88	0	1	0	1	0	1	1	1	0
89	0	1	0	1	1	0	0	0	0
90	0	1	0	1	1	0	0	1	0
91	0	1	0	1	1	0	1	0	0

92	0	1	0	1	1	0	1	1	0
93	0	1	0	1	1	1	0	0	0
94	0	1	0	1	1	1	0	1	0
95	0	1	0	1	1	1	1	0	0
96	0	1	0	1	1	1	1	1	0
97	0	1	1	0	0	0	0	0	0
98	0	1	1	0	0	0	0	1	0
99	0	1	1	0	0	0	1	0	0
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108	0	1	1	0	1	0	1	1	0
109	0	1	1	0	1	1	0	0	0
110	0	1	1	0	1	1	0	1	0
111	0	1	1	0	1	1	1	0	0
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114	0	1	1	1	0	0	0	1	0
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116	0	1	1	1	0	0	1	1	0
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126	0	1	1	1	1	1	0	1	0
127	0	1	1	1	1	1	1	0	0
128	0	1	1	1	1	1	1	1	0
129	1	0	0	0	0	0	0	0	0
130	1	0	0	0	0	0	0	1	0
131	1	0	0	0	0	0	1	0	0
132	1	0	0	0	0	0	1	1	0
133	1	0	0	0	0	1	0	0	0
134	1	0	0	0	0	1	0	1	0
135	1	0	0	0	0	1	1	0	0
136	1	0	0	0	0	1	1	1	0
137	1	0	0	0	1	0	0	0	0

The truth table for the passcode authentication system encompasses a meticulous 256 K-Maps columns of digital signals, yielding the proper functionality of green and red L.E.D.. This effectively proves the complexity and accuracy of the authentication project.

Karnaugh (K-MAP)

	EF GH	00 00	00 01	00 10	00 11	01 00	01 01	01 10	01 11	11 00	11 01	11 10	11 11	10 00	10 01	10 10	10 11
ABCD																	
0000		1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0001		0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0010		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0011		0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0100		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0101		0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0110		0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0111		0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
1100		0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1101		0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
1110		0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
1111		0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
1000		0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
1001		0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
1010		0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
1011		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 5 - K Map - Password Authentication System

The system is designed with 8 input bits (A, B, C, D, E, F, G, H), so it will use 2 to the power of 8 or 256 different input combinations, as each bit can be either 0 or 1. The unique aspect of our system is its validation for a 'valid' password and we can change the password, which is based on the condition that the values of the (ABCD) must be the same as the values of (EFGH). In other words, for an input combination to produce an output of "1", A must equal E, B must equal F, C must equal G, and D must equal H. All other combinations yield an output of "0", indicating an invalid password.

In the truth table for our system, there are 256 rows, reflecting all possible combinations of the inputs. However, only 16 of these combinations will have an output of "1". This specificity in the validation logic ensures a secure yet customizable password setup, allowing for 16 valid password combinations out of the 256 possibilities.

When visualizing this logic using a Karnaugh map (K-map), which is conceptually a tool for simplifying Boolean expressions, the map would have 256 cells corresponding to the 256 combinations. In our case, the K-map would display '1's where the input values of A, B, C, and D are the same by E, F, G, and H, as the same with the truth table. This approach to password validation provides a robust and straightforward mechanism for ensuring security, with a reasonably large space of valid password combinations enhancing the system's effectiveness.

This is the Boolean expression simplification that had been done from the Karnaugh map:

$$\begin{aligned} &A'B'C'D'E'F'G'H' + A'B'C'DE'F'G'H' + A'B'CD'E'F'GH' + A'B'CDE'F'GH' + A'BC'D'E'FG'H' \\ &+ A'BC'DE'FG'H' + A'BCD'E'FGH' + A'BCDE'FGH' + ABC'D'EFG'H' + ABC'DEFG'H' + ABC \\ &D'EFGH' + ABCDEFGH' + AB'C'D'EF'G'H' + AB'C'DEF'G'H' + AB'CD'EF'GH' + AB'CDEF'G \\ &H. \end{aligned}$$

And the simplified expression:

$$((A \oplus E)'(B \oplus F)'(C \oplus G)'(D \oplus H)')$$

RESULTS

Simulation Results

Green L.E.D (Valid Inputted Passcode)

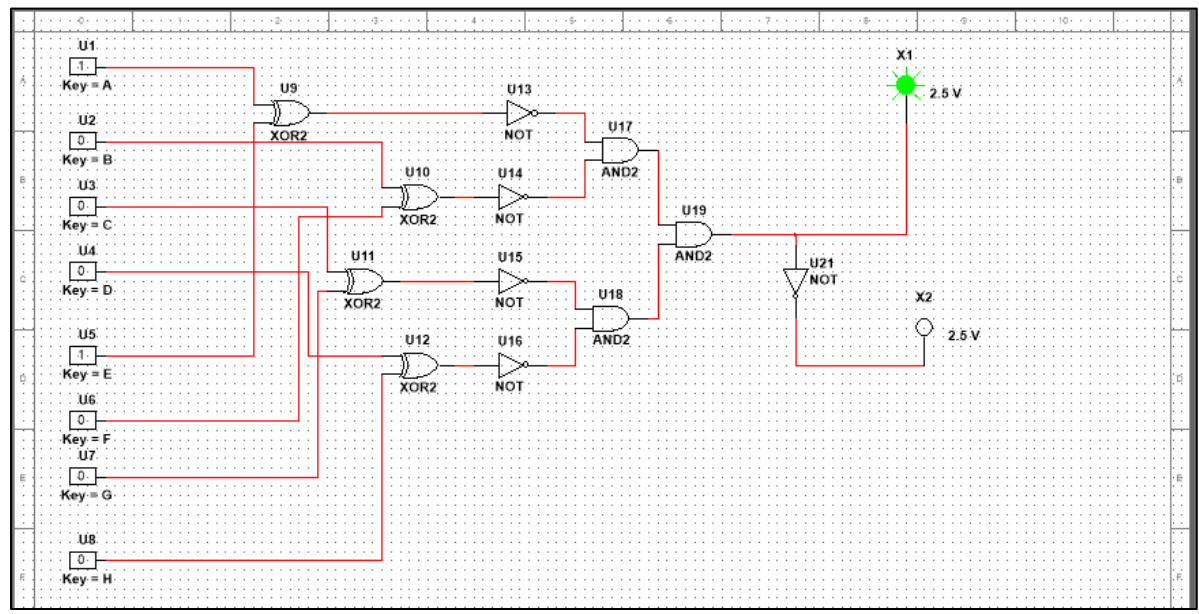


Figure 6 - Green L.E.D Switching Mechanism Circuit – Password Authentication System

Red L.E.D (Incorrect Inputted Passcode)

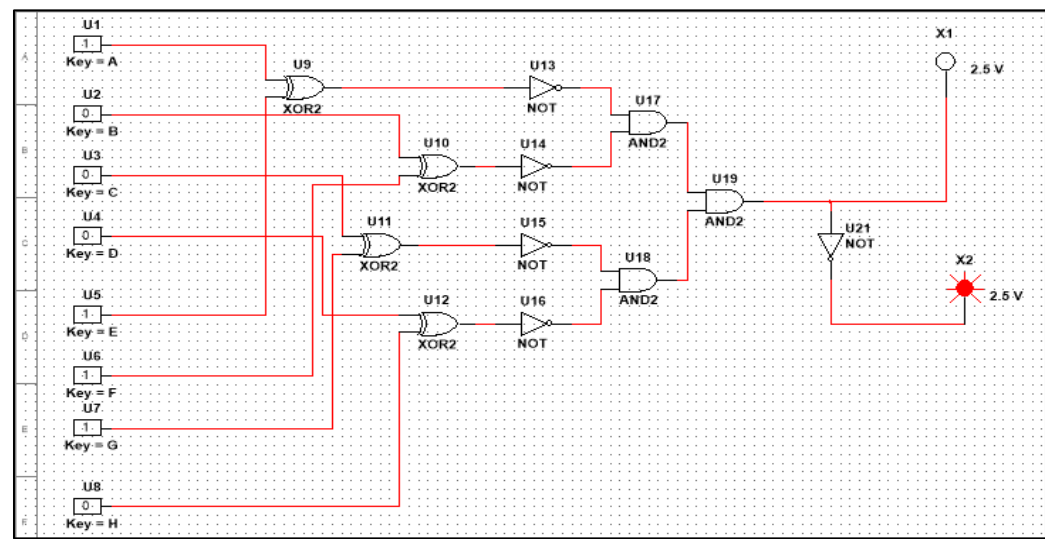


Figure 7 - Green L.E.D Switching Mechanism Circuit - Password Authentication System

PIN CONFIGURATION AND TRUTH TABLE

XOR GATE

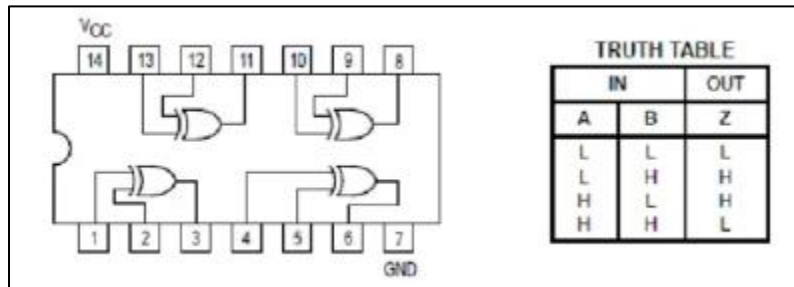


Figure 8 - XOR Gate Pin Configuration & Truth Table

- Input pins (1,2,4,5,9,10,12,13)
- Output pins (3,6,8,11)

NOT GATE

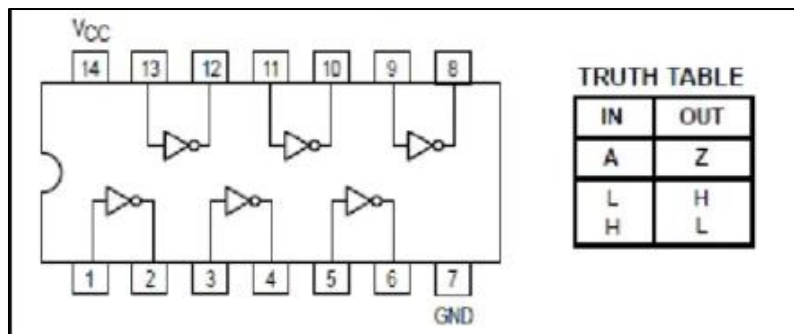


Figure 9 - NOT Gate Pin Configuration & Truth Table

- Input pins (1,3,5,13,11,9)
- Output pins (2,4,6,8,10,12)

AND GATE

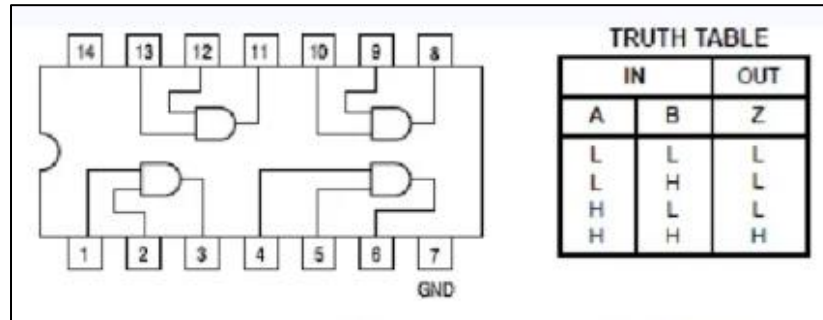


Figure 10 - AND Gate Pin Configuration & Truth Table

- Input pins (1,2,4,5,9,10,12,13)
- Output pins (3,6,8,11)

Physical Circuit Pictures

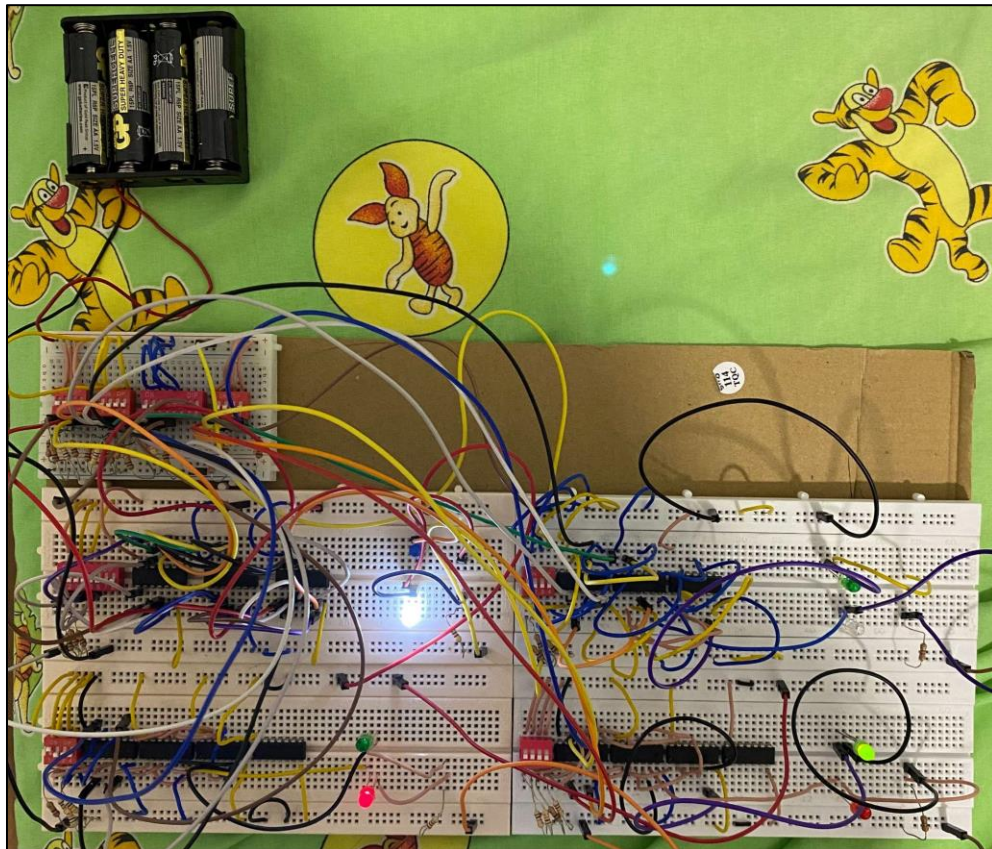


Figure 11 - Final Breadboard Circuit - Passcode Authentication System

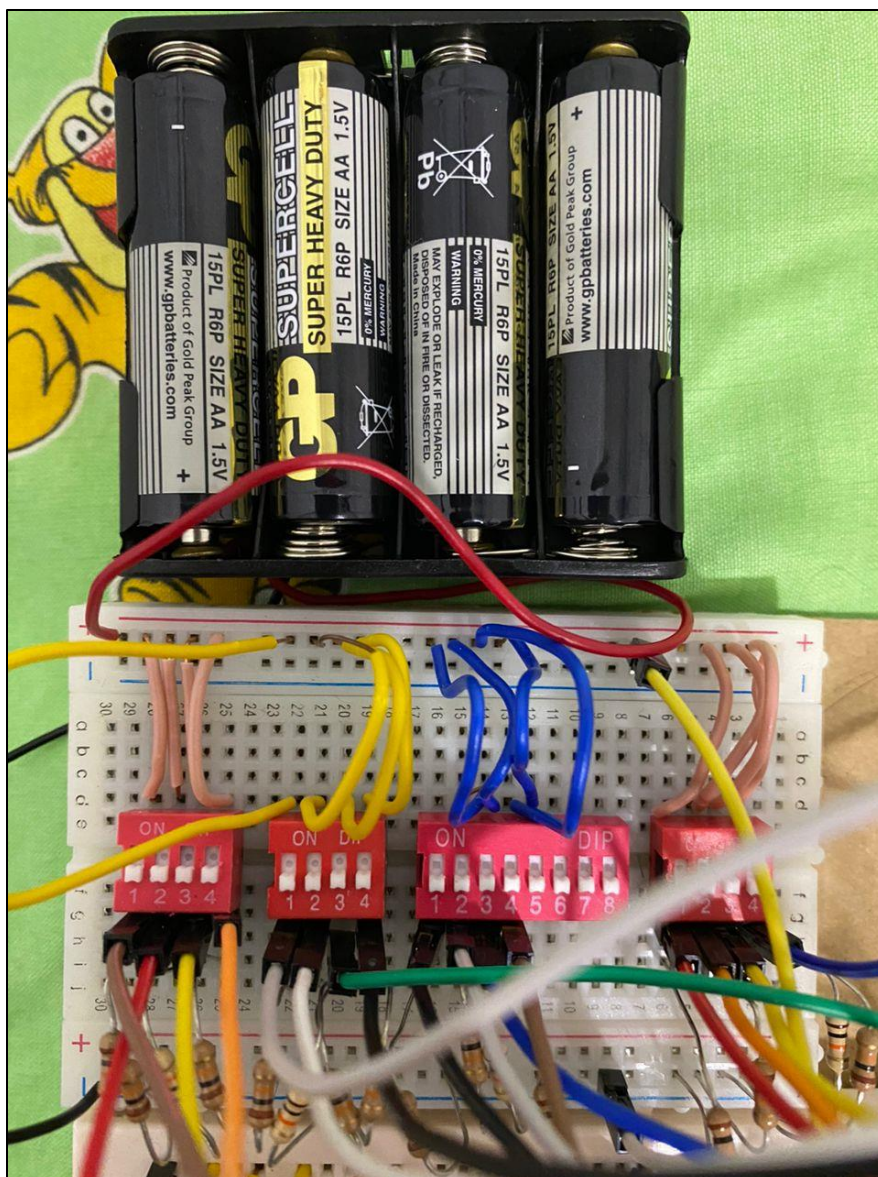


Figure 12 - Standalone Power Supply and Switches for Storage of Saved Passwords

DISCUSSION

The fundamental components involved in the passcode authentication circuit involved 4 XOR IC 74LS86, 4 NOT IC 74LS04, 4 AND IC 74LS08, 4 DIP Switches (4-bit/8-bit), 10k Ω resistors, 470 Ω resistors, 5 Breadboards, and connecting wires. The passcode is stored as an input within the breadboard circuit, further allowing potential for safe storage methodologies such as encryption and hashing.

The red and green L.E.Ds highlight and visually portray the successful and unsuccessful authentication attempts. The passcode authentication system importance signifies that its circuit design is well-fit for offering safety mechanisms in the application of access control in digital electronics.

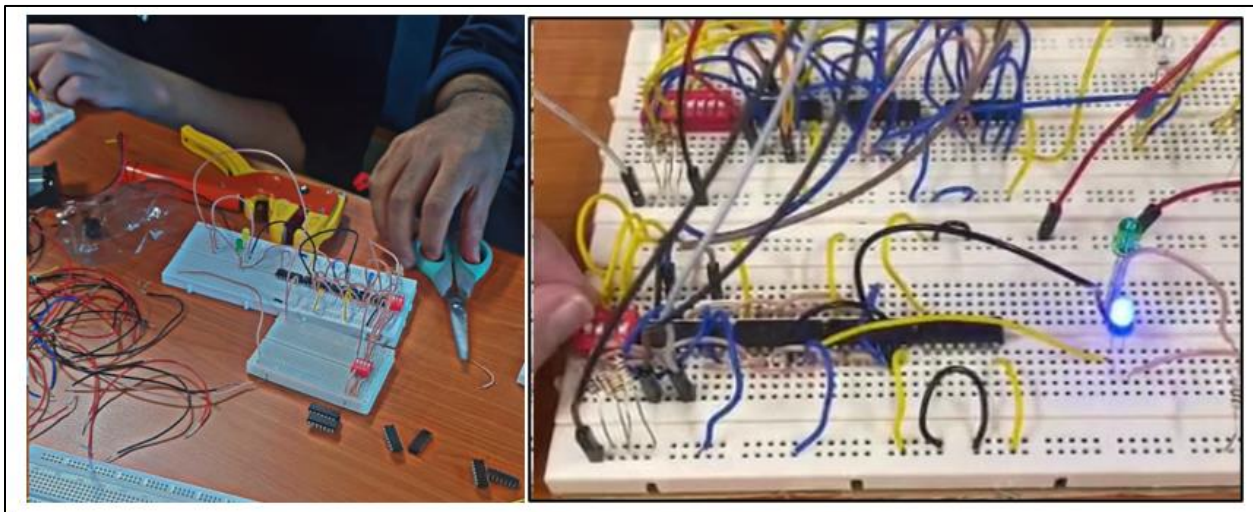


Figure 13 - Breadboard Setup Progress Throughout 4 Weeks (Password Authentication System)

Firstly, the breadboard was painstakingly set up to smoothly include the crucial code and data entering elements. Two Light Emitting Diodes (LEDs) were used as the primary inputs to commence a verification procedure that was made possible by the combination of key code switches and data entry switches.

The passcode authentication system required the following components for the breadboard:

1. 4 XOR IC 74LS86
2. 4 NOT IC 74LS04
3. 4 AND IC 74LS08
4. 6, DIP Switch 4-bit.
5. 2, DIP Switch 8-bit.
6. (20) Twenty 10 k Ω resistors and (4) Four 470 Ω resistors
7. One 6-volt battery
8. Five breadboards
9. Connecting wires

Hence, in this configuration, the Red LED signaled a wrong password entry, and the Green LED functioned as a visual cue for successful password authentication. This result was only possible through the clever use of logic gates, including AND, and XOR, in conjunction with the well-planned integration of DIP switches to facilitate the authentication process. Resistors provided the current-limiting functionalities and enhanced the safety aspect of the passcode authentication system (10 K Ω)

Moreover, these resistors were purposefully added in order to keep the LEDs from being over-driven, which could have led to an output that was too brilliant and possibly damaged the components. This preventive measure also guaranteed the LEDs' longevity and best performance by reducing the possibility of an early burnout or malfunction brought on by exposure to too much voltage.

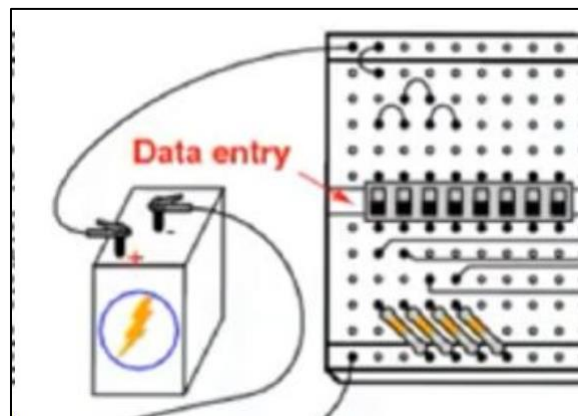


Figure 14 - Data Entry Mechanism - Passcode Authentication System

Subsequently, by taking these safety precautions and taking into account the technical subtleties of the circuitry, the system was able to preserve the integrity and functionality of the LED displays in addition to accurately verifying passwords, which added to the overall dependability and durability of the authentication setup.

The simulation results, created with Multisim, reveal an eight-switch passcode authentication system. Switches A, B, C, and D are set for the saved password, while switches E, F, G, and H are for users to enter their password. The system also includes LED indicators—a green light comes on when the password is correct, showing it's unlocked, and a red light indicates a locked state when the password is wrong. This smart setup relies on XOR, NOT, and AND gates to ensure a secure and straightforward password verification process.

Last of all, as shown and justified in the previous segments, it is quite evident that the circuit design coincides with the truth table, meaning that the generated circuit proves to be sufficient in passcode authentication, thus proving that the control circuit which checks whether the password entered is valid or not, is created in an accurate manner.

CONCLUSION

Overall, the password authentication project adhered to the objectives & guidelines of the assignment. The project investigated a real-life application of a digital circuit utilizing logic gates only, resistors, LEDs, switches, and a 6V battery supply. The circuit design utilized 4 XOR IC 74LS86, 4 NOT IC 74LS04, 4 AND IC 74LS08, 4 DIP Switches (4-bit/8-bit), 10k Ω resistors, 470 Ω resistors, 5 Breadboards, and connecting wires. The physical breadboard demonstrated the authentication process where green and red L.E.Ds signaled correct & incorrect password outcome(s) respectively. The 10k Ω resistors were implemented for the sole purposes of safeguarding the components involved in the password authentication circuit, especially the LEDs from excessive upsurge of voltage once the circuit is closed, which prevented damage to the component(s). The circuit's overall design only relied on XOR, NOT, and AND gates, indicating the simplicity and successive simulation results utilizing the Multisim software which incorporated an eight-switch password security system with visual LED indicators for assisting in the password verification process. The reliance on logic gates deemed useful and adhered to the group assignment requirements. The password authentication system demonstrated technical understanding of the digital electronic circuit and showcased the conceptual understanding of the basic fundamental knowledge of logic gates and circuit theory. The project was well-collaborated among all the groupmates where each member justified their task with proper assumption(s), factual statement(s), educated opinion(s), ensuring a safe mechanism for digital electronics access control.

REFERENCES

(CircuitBread, 2023) CircuitBread. (2023, February 1). *What is a Breadboard?*

<https://www.circuitbread.com/ee-faq/what-is-a-breadboard>

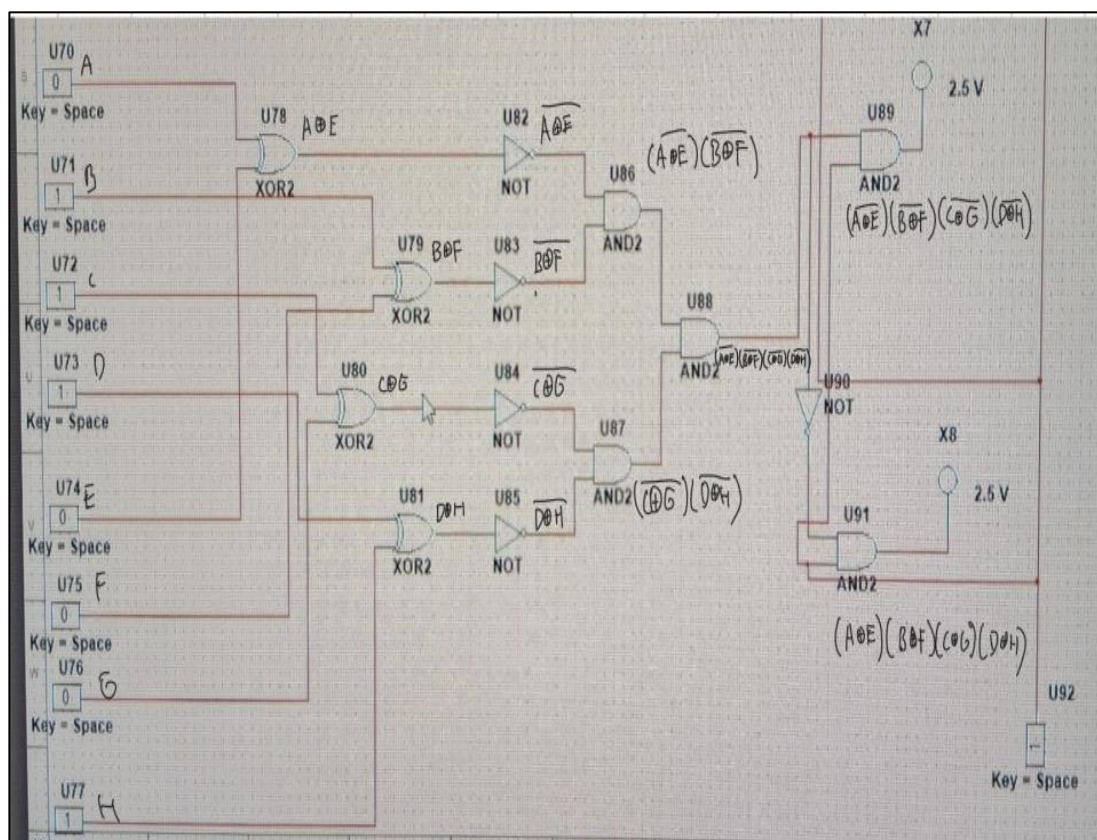
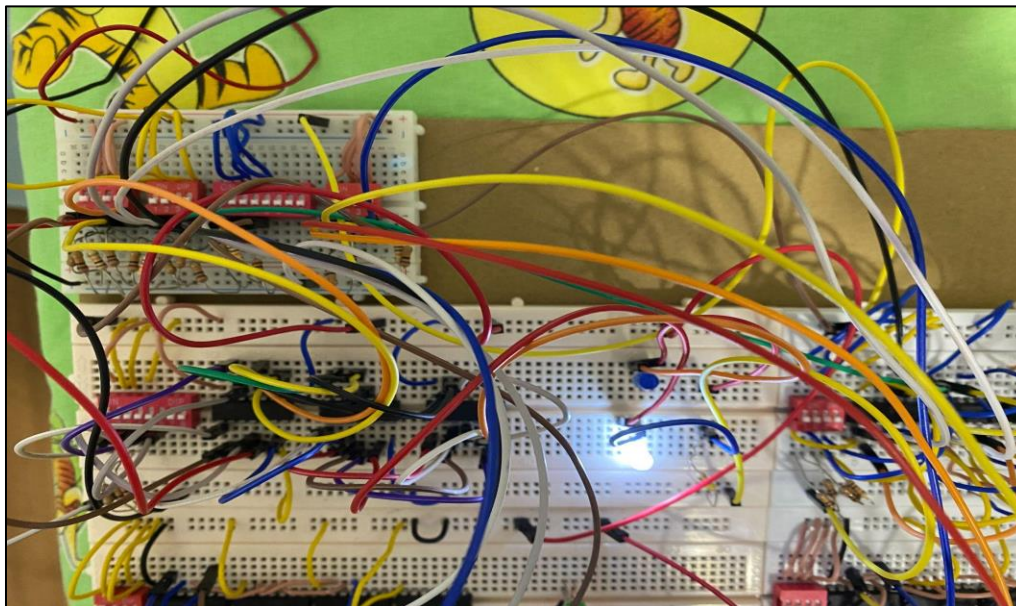
(Ayesha, n.d.) “6-Bit Password Security System Using (XOR, and & NOT) | PDF | Electrical Resistance and Conductance | Resistor.” Scribd, www.scribd.com/document/458333823/G-4. Accessed 6 Dec. 2023.

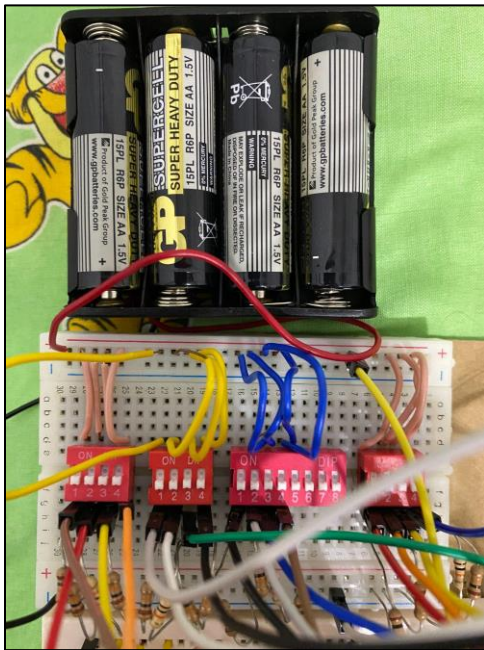
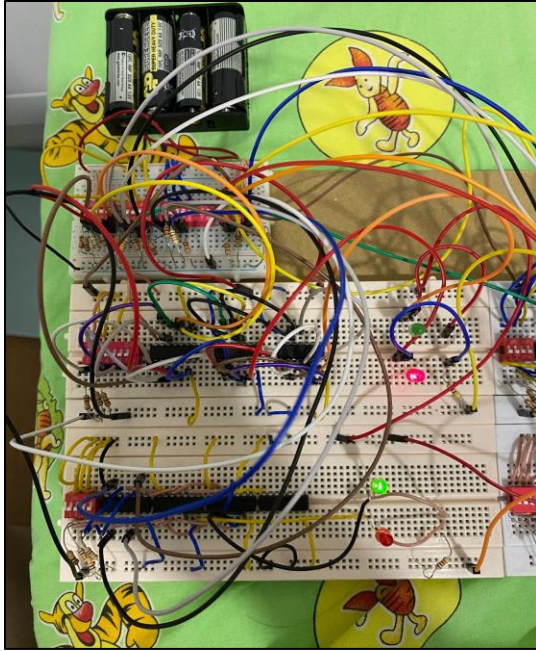
(Descope, 2023) Descope. (2023). *What is password-based authentication?*. What is Password-Based Authentication? <https://www.descope.com/learn/post/password-authentication>

(Prakash, 2020) Prakash, A. (2020, September 19). *Passcode security system*. PASSCODE SECURITY SYSTEM. <https://medium.com/@abhijnanprakash/passcode-security-system-347f52a654a2>

APPENDIX

Prototype and Preliminary Hand Sketch





$$\begin{array}{lll}
 A'B'C'(D'E'F'G'H' + D'E'F'G'H) & A'B'C'(D'E'F'G'H' + D'E'F'G'H) & A'B'C'(D'E'F'G'H' + D'E'F'G'H) \\
 A'B'C'(E'F'G'(D \cdot H)) & A'B'C'E'F'G'(D \cdot H + D \cdot H) & A'B'C'E'F'G'(D \cdot H) \\
 A'B'C'E'F'G'(D \cdot H) & A'B'C'E'F'G'(D \cdot H) & A'B'C'E'F'G'(D \cdot H) \\
 A'B'C'(D'E'F'G'H' + D'E'F'G'H) & A'B'C'(D'E'F'G'H' + D'E'F'G'H) & A'B'C'(D'E'F'G'H' + D'E'F'G'H) \\
 A'B'C'E'F'G'(D \cdot H) & A'B'C'E'F'G'(D \cdot H) & A'B'C'E'F'G'(D \cdot H) \\
 A'B'C'(D'E'F'G'H' + D'E'F'G'H) & A'B'C'(D'E'F'G'H' + D'E'F'G'H) & A'B'C'(D'E'F'G'H' + D'E'F'G'H) \\
 A'B'C'E'F'G'(D \cdot H) & A'B'C'E'F'G'(D \cdot H) & A'B'C'E'F'G'(D \cdot H)
 \end{array}$$

$$\begin{aligned}
 & ((D \cdot H)(A'B'C'E'F'G' + A'B'C'E'F'G' + A'B'C'E'F'G' + \\
 & \quad A'B'C'E'F'G' + A'B'C'E'F'G' + A'B'C'E'F'G' + \\
 & \quad A'B'C'E'F'G' + A'B'C'E'F'G')) \\
 & (D \cdot H) A'B'E'F'(C \cdot G) + A'B'E'F'(C \cdot G) + \\
 & \quad ABEF(C \cdot G) + AB'E'F'(C \cdot G) \\
 & (D \cdot H)(C \cdot G) A'B'E'F' + A'B'E'F' + ABEF + AB'E'F' \\
 & (D \cdot H)(C \cdot G) A'E'(B \cdot F) + AE(B \cdot F) \\
 & (A \cdot E)(B \cdot F)(C \cdot G)(D \cdot H) \\
 & \rightarrow (\overline{A \cdot E})(\overline{B \cdot F})(\overline{C \cdot G})(\overline{D \cdot H})
 \end{aligned}$$