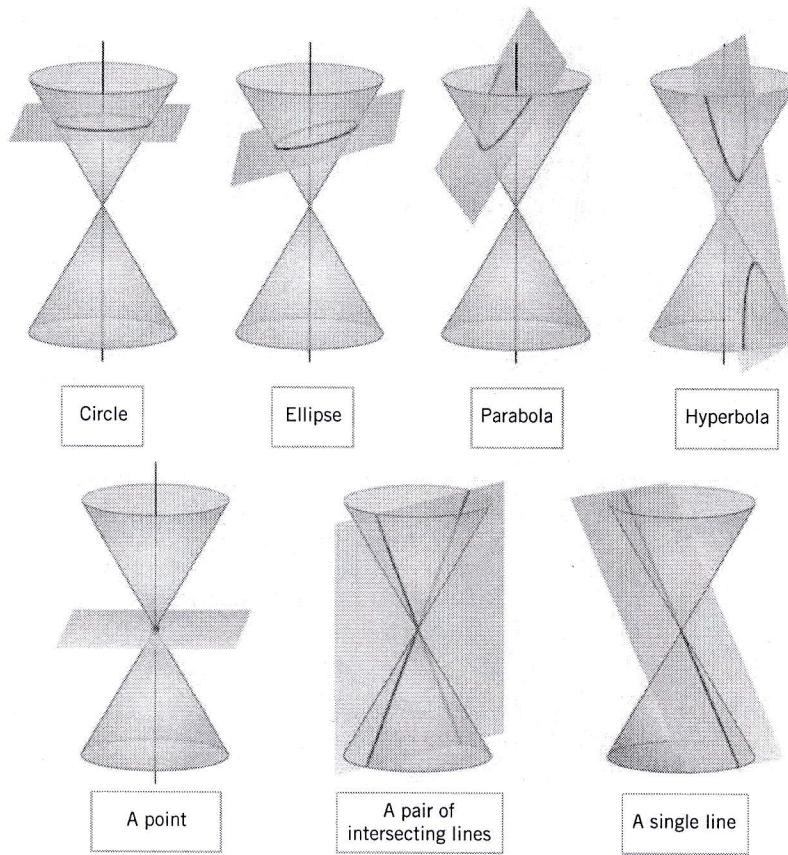
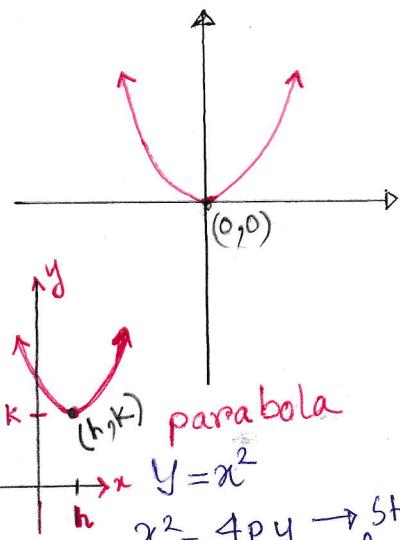


Conic Section**CONIC SECTIONS**

Circles, ellipses, parabolas, and hyperbolas are called **conic sections** or **conics** because they can be obtained as intersections of a plane with a double-napped circular cone (Figure 10.4.1). If the plane passes through the vertex of the double-napped cone, then the intersection is a point, a pair of intersecting lines, or a single line. These are called **degenerate conic sections**.



▲ Figure 10.4.1

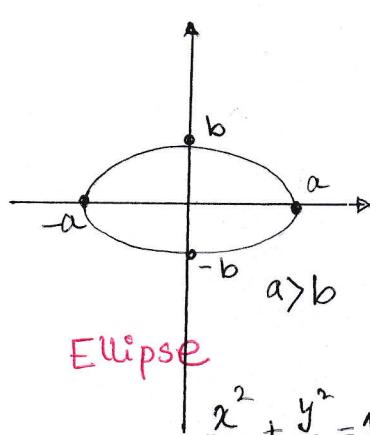


$$x^2 = 4py \rightarrow \text{Standard form}$$

Vertex: $(0,0)$ (turning point)

$$(x-h)^2 = 4p(y-k) \quad ; \quad p > 0 \quad \text{or} \quad p < 0$$

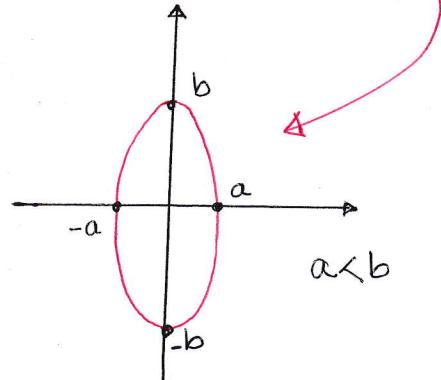
Vertex: (h,k)



Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow \text{Center: } (0,0)$$

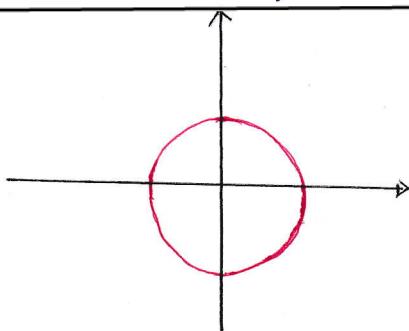
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$



$a < b$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \rightarrow \text{Center: } (h, k)$$

If it was not centered
at the origin ↑

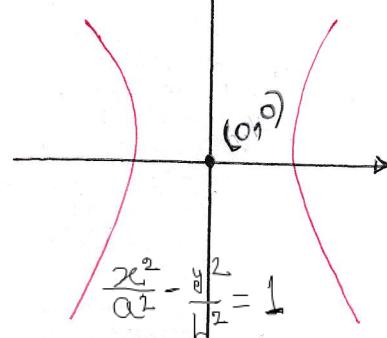


$$x^2 + y^2 = r^2$$

Center: $(0,0)$

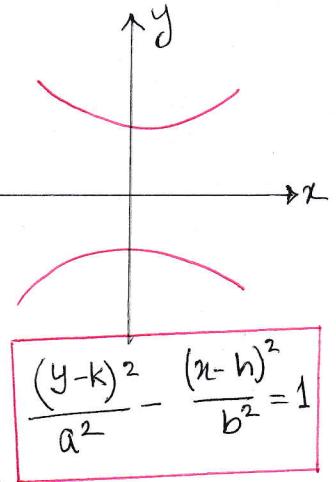
$$(x-h)^2 + (y-k)^2 = r^2$$

Center: (h,k)



Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



Center: (h, k)

General Equation of Second Degree

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad \text{--- (1)}$$

or
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

A=a ; B=2h ; C=b
D=2g ; E=2f ; F=c

We need to find the following invariants

$$\Delta = abct + 2fg - af^2 - bg^2 - ch^2$$

discriminant of Eqn (1)

$$C = h^2 - ab$$

$$I = a+b$$

Invariants are some unchanged constants which indicates the locus of conic.

Locus — a set of points, the position of which satisfy a set of algebraic condition, such as circle, ellipse etc.

" Δ " identifies whether the conic is "Degenerate" or not (weak conic, quality of conic reduces)

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = a(bc-f^2) - h(hc-fg) + g(hf-bg)$$

$$= abc - af^2 - ch^2 + fgh + fgh - bg^2$$

$$= abc + 2fgh - af^2 - bg^2 - ch^2$$

For degenerate case $\Delta = 0$; For proper conic $\Delta \neq 0$

$$B^2 - 4AC$$

$$(2h)^2 - 4ab$$

$$4h^2 - 4ab$$

$$4(h^2 - ab)$$

$$(h^2 - ab)$$

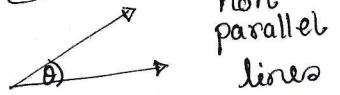
we can ignore the constant "4"

Degenerate Case : $\Delta=0$

If $h^2-ab=0$ then Parallel lines or straight line

If $a+b=0$ then perpendicular line 

If $\Delta=0$ only but the above conditions do not fulfil then pair of st. lines.



Non-Degenerate Case : $\Delta \neq 0$

$h^2-ab=0 \longrightarrow$ Parabola

$h^2-ab > 0 \longrightarrow$ hyperbola

$h^2-ab < 0 \longrightarrow$ Ellipse

$h^2-ab < 0$
 $a=b$
 $h=0$

What is represented by the following equation(s) ?

$$\textcircled{i} \quad 4x^2 - 24xy - 6y^2 + 4x - 12y + 1 = 0$$

$\begin{matrix} 4 \\ 2h \end{matrix}$ $\begin{matrix} -24xy \\ b \end{matrix}$ $\begin{matrix} -6y^2 \\ 2g \end{matrix}$ $\begin{matrix} +4x \\ 2f \end{matrix}$ $\begin{matrix} -12y \\ c \end{matrix}$ $\begin{matrix} 1 \\ \end{matrix}$

$$\begin{array}{r} 2h = -24 \\ h = -12 \end{array}$$

$$2g = 4$$

$$g = 2$$

$$2f = -12$$

$$f = -6$$

$$\Delta = abc + 2fg h - af^2 - bg^2 - ch^2$$

$$= (4)(-6)(1) + 2(-6)(2)(-12) - 4(-6)^2 - (-6)(2)^2$$

$$= (4)(-6)(1) + 2(-6)(2)(-12) - (1)(-12)^2$$

$$\begin{cases} h^2-ab = (-12)^2 - (4)(-6) = 168 \neq 0 \\ a+b = 4+(-6) = -2 \neq 0 \end{cases}$$

The eqn represents
a pair of st. lines

$= 0$

Degenerate Case

ii) $x^2 - 4xy + y^2 + 8x + 2y - 5 = 0$

$$\begin{array}{l} a=1 \\ b=1 \\ c=-5 \end{array} \quad \left\{ \begin{array}{l} h=-4 \Rightarrow h=-2 \\ c=-5 \end{array} \right\} \quad \left\{ \begin{array}{l} g=8 \\ g=4 \end{array} \right\} \quad \left\{ \begin{array}{l} 2f=2 \\ f=1 \end{array} \right\}$$

$$h^2 - ab = (-2)^2 - (1)(1) = 3 > 0$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= -5 + (-16) - 1 - 16 - (-20)$$

The eqn represents a hyperbola

iii) $8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0$

$$\begin{array}{l} a=8 \\ b=5 \\ c=13 \end{array} \quad \left\{ \begin{array}{l} 2h=4 \\ h=2 \end{array} \right\} \quad \left\{ \begin{array}{l} 2g=-16 \\ g=-8 \end{array} \right\} \quad \left\{ \begin{array}{l} 2f=-14 \\ f=-7 \end{array} \right\}$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 520 + 224 - 392 - 320 - 52$$

$$h^2 - ab = 2^2 - (8)(13) = -100 < 0$$

$$= -20 \neq 0$$

The eqn represents an ellipse

iv) $x^2 - 6xy + 9y^2 + 4x + 8y + 15 = 0$

$$\begin{array}{l} a=1 \\ b=9 \\ c=15 \end{array} \quad \left\{ \begin{array}{l} 2h=-6 \\ h=-3 \end{array} \right\} \quad \left\{ \begin{array}{l} 2g=4 \\ g=2 \end{array} \right\} \quad \left\{ \begin{array}{l} 2f=8 \\ f=4 \end{array} \right\}$$

$$h^2 - ab = (-3)^2 - (1)(9) = 0$$

$\therefore h^2 - ab = 0$, It could be degenerate or parabola

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= (1)(9)(15) + 2(4)(2)(-3) - (1)(4)^2 - (9)(2)^2 - (15)(-3)^2$$

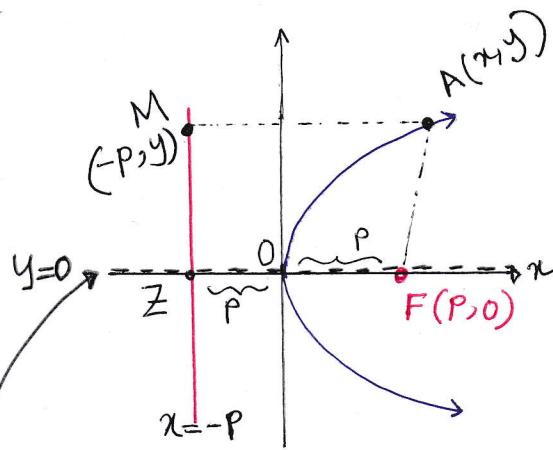
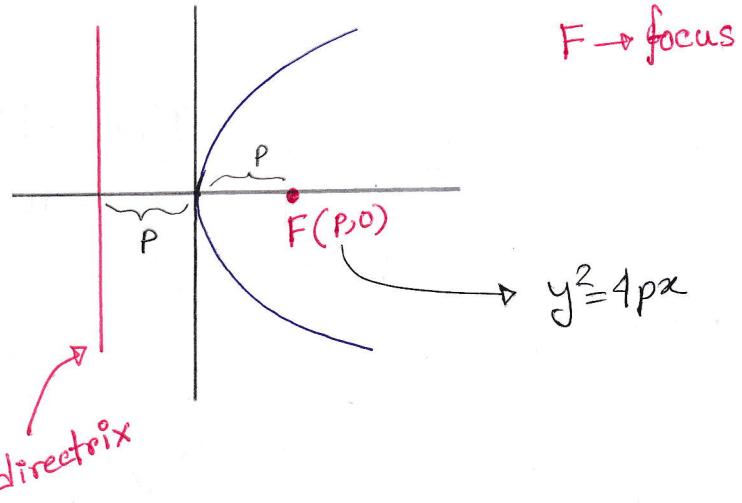
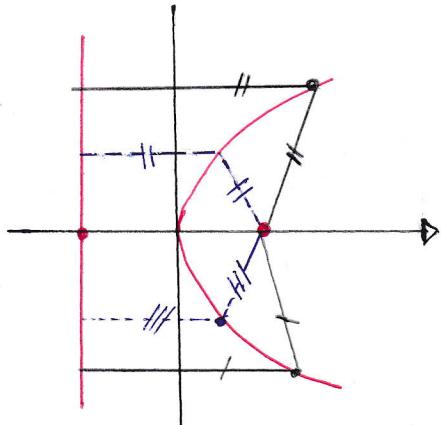
$$= -100$$

$\therefore \Delta \neq 0$

Hence the eqn represents a parabola.

Parabola

Set of points in a plane that are equidistant from a fixed point F (focus) and a fixed line (directrix)



$$FA = AM$$

$$\sqrt{(x-p)^2 + (y-0)^2} = \sqrt{(x-(-p))^2 + (y-y)^2}$$

$$\sqrt{(x-p)^2 + y^2} = \sqrt{(x+p)^2}$$

$$(x-p)^2 + y^2 = (x+p)^2$$

square both sides

$$x^2 - 2xp + p^2 + y^2 = x^2 + 2xp + p^2$$

$$y^2 = 4px$$

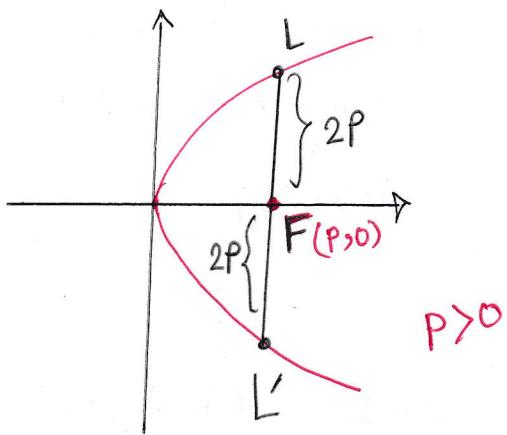
Focus, $F = (p, 0)$

Eqn of directrix, $x = -p$

$$(y-k)^2 = 4p(x-h)$$

→ if the vertex shifts from the origin $(0,0)$

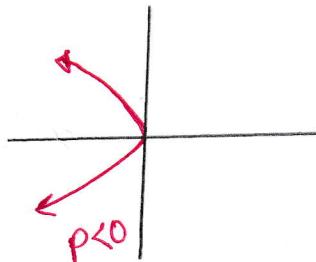
Axis of symmetry:
the line that divides the parabola into two equal parts: $y=0$



LL' : Latus Rectum

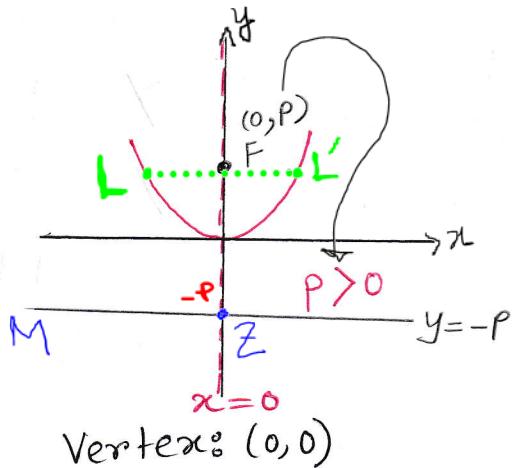
$$\therefore LL' = 4p$$

$$p > 0 \text{ or } p < 0$$



Eqn of Latus Rectum: $x = p$

Length of Latus Rectum: $LL' = |4p| \rightarrow$ length is always +ve



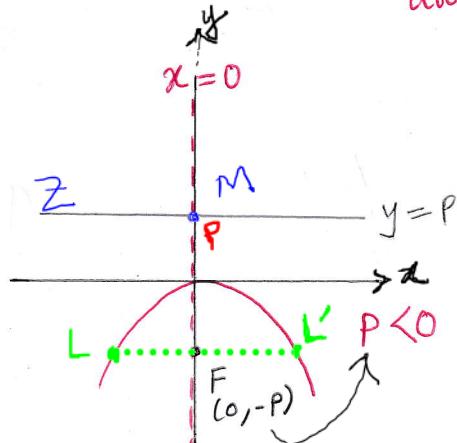
Vertex: $(0, 0)$

Focus: $(0, p)$

(directrix) MZ: $y = -p$

axis of symmetry: $x = 0$

Latus Rectum, $LL' = |4p|$



Vertex: $(0, 0)$

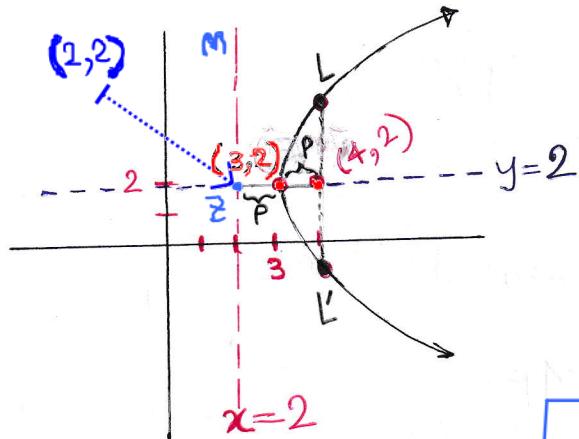
Focus: $(0, -p)$

MZ: $y = p$

axis of symmetry: $x = 0$

Latus Rectum, $LL' = |4p|$

Find the vertex, focus, equation of directrix, equation of axis of symmetry, length of Latus rectum of the parabola: $(y-2)^2 = 4(x-3)$



$$\text{Vertex: } (3, 2)$$

$$\text{Focus: } (4, 2)$$

$$\text{Eqn of directrix: } x=2$$

$$x=2$$

$$\text{Eqn of axis: }$$

$$y=0$$

$$y-2=0$$

$$y=2$$

$$\begin{aligned} \text{Length of Latus Rectum: } & |4P| \\ & (LL') \\ & = |4(1)| \\ & = 4 \end{aligned}$$

$$(y-2)^2 = 4(x-3)$$

$$Y^2 = 4pX$$

$$4p = 4$$

$$p = 1$$

$$Y=0 \Rightarrow y-2=0 \\ y=2$$

$$X=0 \Rightarrow x-3=0 \\ x=3$$

$$\text{Vertex: } (3, 2)$$

$$\text{Focus: } (p, 0)$$

$$= (4, 2)$$

$$X=p$$

$$Y=0$$

$$x-3=p$$

$$y-2=0$$

$$x-3=1$$

$$y=2$$

$$x=4$$

$$\begin{aligned} \text{Eqn of directrix: } & X=-p \\ & \Rightarrow X+p=0 \\ & \Rightarrow (x-3)+p=0 \\ & \Rightarrow x=2 \end{aligned}$$

Find the vertex, focus, directrix, axis and length of latus rectum for the parabola, $y^2 - 2y - 4x - 3 = 0$

$$y^2 - 2y - 4x - 3 = 0 \rightarrow \text{General form}$$

$$y^2 - 2 \cdot y \cdot 1 + 1^2 - 1^2 - 4x - 3 = 0 \quad \text{completing square}$$

$$(y-1)^2 - 4x - 4 = 0$$

$$(y-1)^2 = 4(x+1) \rightarrow \text{Standard form}$$

(i) Vertex: $(-1, 1)$

(ii) Focus: $(P, 0) = (0, 1)$

$$X = P$$

$$x+1 = 1$$

$$x = 0$$

$$Y = 0$$

$$y-1 = 0$$

$$y = 1$$

$$(y-1)^2 = 4p(x+1)$$

$$y^2 = 4pX$$

$$4p = 4$$

$$\therefore P = 1$$

(iii) Eqn of directrix: $x = -2$

$$X = -P$$

$$x+1 = -1$$

$$x = -2$$

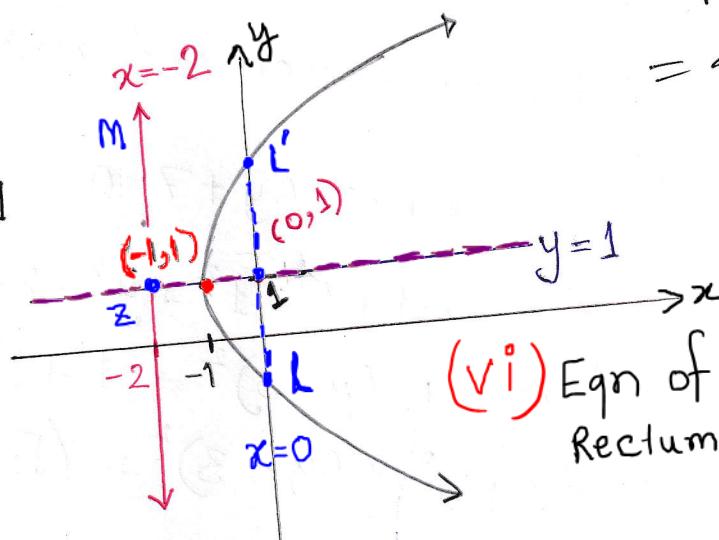
(iv) Eqn of axis: $y = 1$

$$Y = 0$$

$$y-1 = 0$$

$$y = 1$$

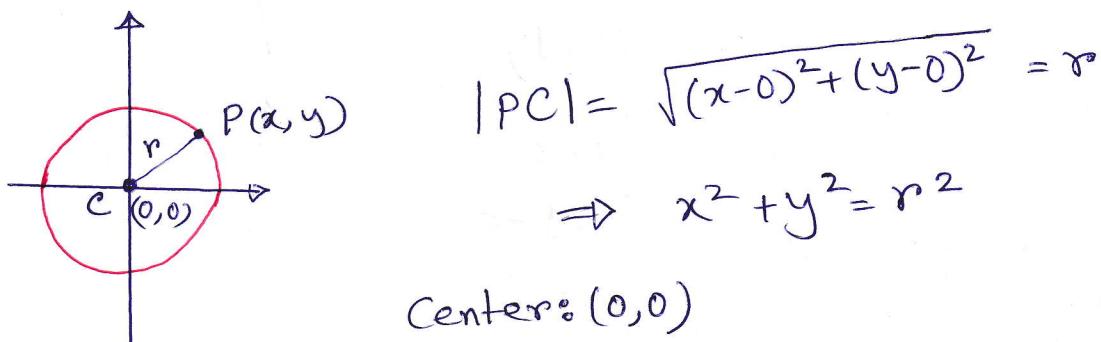
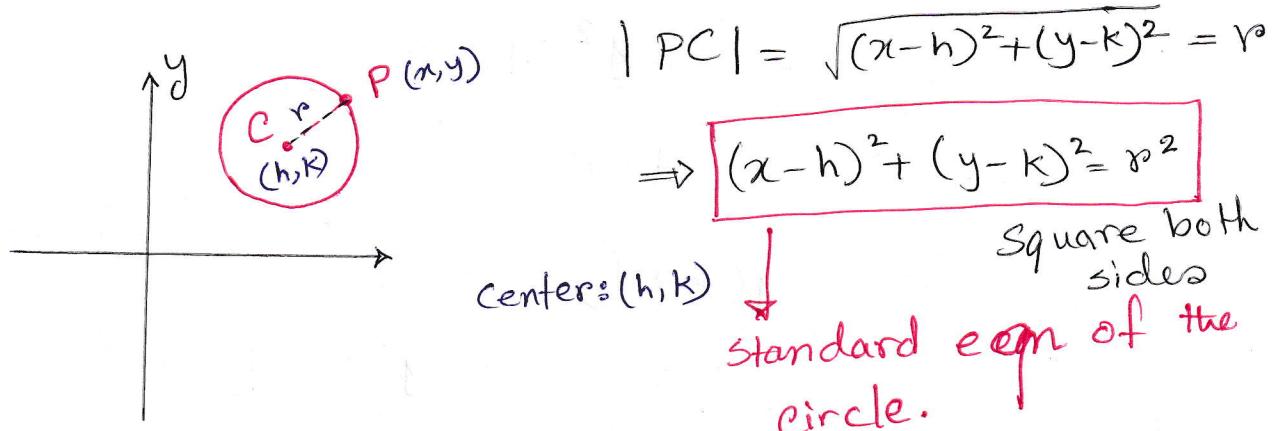
(v) Length of Latus Rectum: $|4p|$
 $= |4(1)|$
 $= 4$



(vi) Eqn of Latus Rectum: $x = 0$

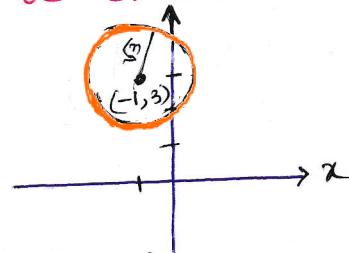
Circle

Set of points in a plane that are equidistant from a fixed point C (center).



Question: Sketch the graph with the equation $x^2 + y^2 + 2x - 6y + 7 = 0$
by first showing that it represents a circle and
then finding its center and radius.

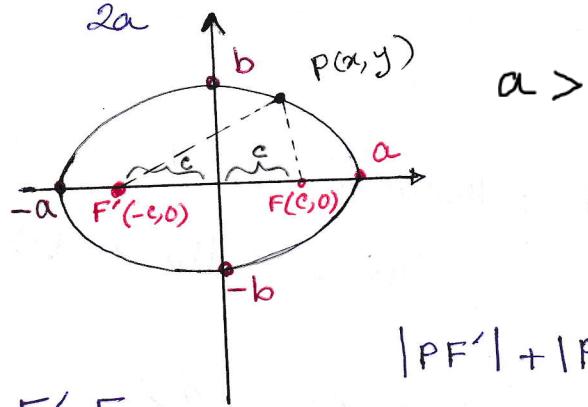
$$\begin{aligned}
 & x^2 + 2x + y^2 - 6y + 7 = 0 \\
 & \Rightarrow x^2 + 2 \cdot x \cdot 1 + 1^2 - 1^2 + y^2 - 2 \cdot y \cdot 3 + 3^2 - 3^2 + 7 = 0 \\
 & \Rightarrow (x+1)^2 + (y-3)^2 = 1^2 + 3^2 - 7 = 3 \\
 & \Rightarrow (x+1)^2 + (y-3)^2 = (\sqrt{3})^2
 \end{aligned}$$



Center: $(-1, 3)$
radius: $\sqrt{3}$

Ellipse

An ellipse is the set of all points in the plane the sum of whose distances from two fixed points (the foci) is a constant. foci \rightarrow plural of focus



$$|PF'| + |PF| = 2a$$

F' , F
foci

$$\Rightarrow \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

$$\Rightarrow \sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2}$$

$+x^2 + 2cx + c^2 + y^2$
(sq both sides)

$$\Rightarrow a\sqrt{(x+c)^2 + y^2} = a^2 + cx$$

$$\Rightarrow a^2(x^2 + 2cx + c^2 + y^2) = a^4 + 2a^2cx + c^2x^2$$

(sq both sides)

$$\Rightarrow a^2x^2 + 2a^2cx + a^2c^2 + a^2y^2 = a^4 + 2a^2cx + c^2x^2$$

$$\Rightarrow a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$$

$$\Rightarrow (a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

$$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2$$

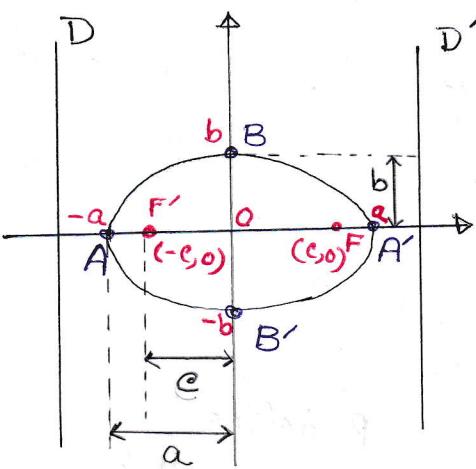
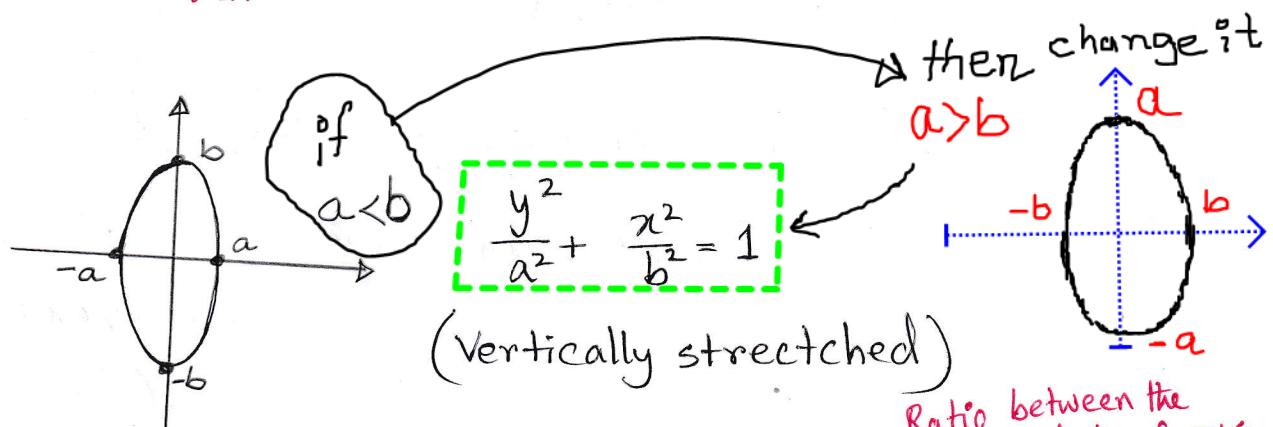
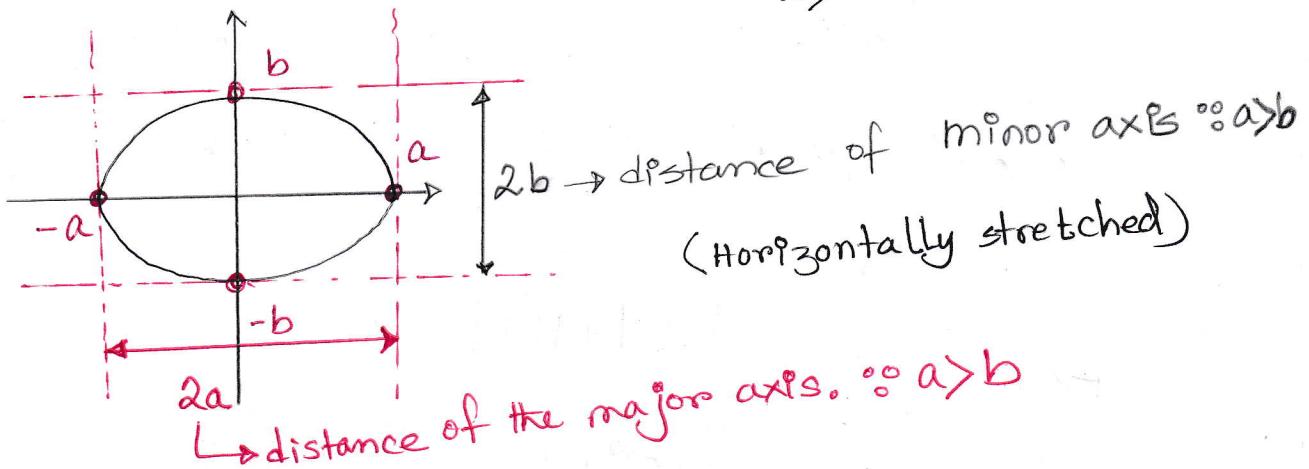
Redefine the constant
 $a^2 - c^2 = b^2$

$$\Rightarrow \frac{b^2x^2}{a^2b^2} + \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2} \quad (\text{div by } a^2b^2)$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \rightarrow \text{Eqn of Ellipse}$$

Center $(0, 0)$

$a > b$



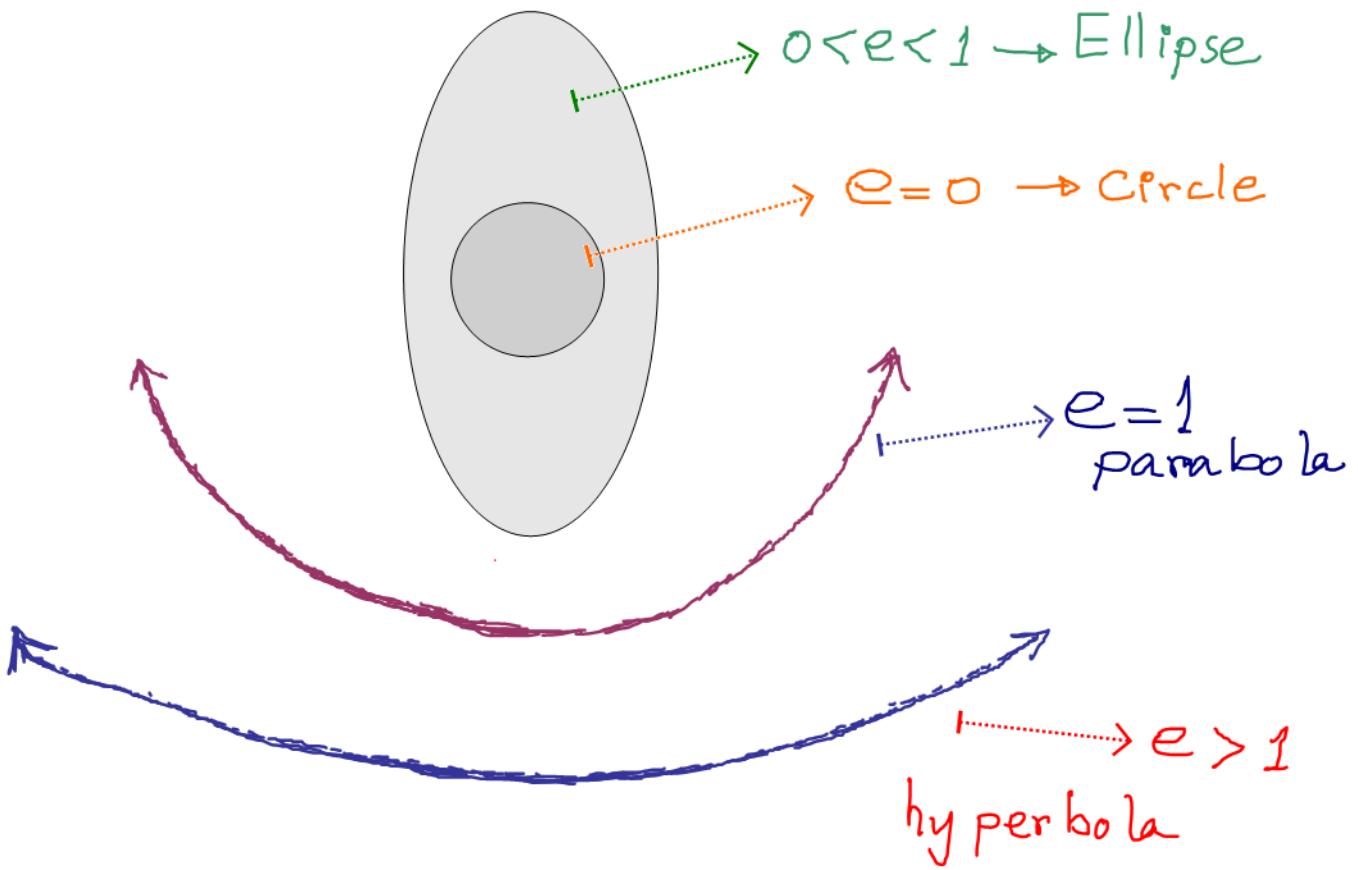
Eccentricity, $e = \frac{c}{a}$ **axis (one of the vertices)**

It is a constant

$0 < e < 1$

{ It defines how circular or uncircular the shape of the conic is }

$c < a$



$$e = \frac{c}{a}$$

$a^2 - c^2 = b^2 \quad \because \text{we redefined}$
 this constant

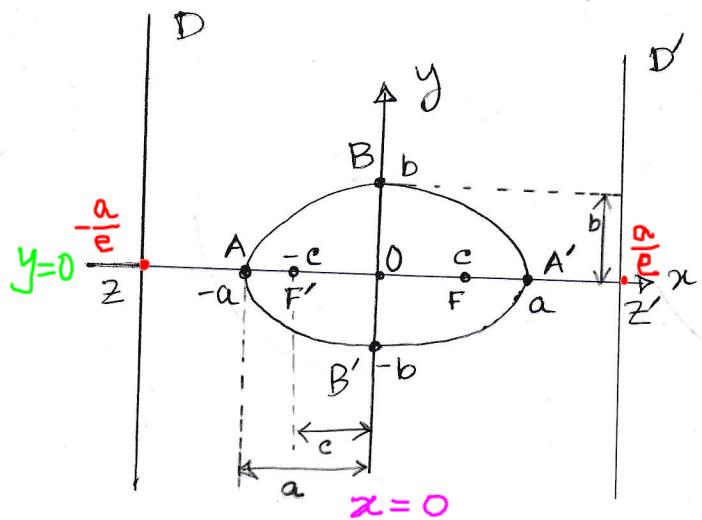
$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$\Rightarrow c^2 = a^2 - b^2$$

$$\Rightarrow c = \sqrt{a^2 - b^2}$$

$$= \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$= \sqrt{1 - \frac{b^2}{a^2}}$$



Centre, $O : (0,0)$
 Foci, $(\pm c, 0)$ or $(\pm ae, 0)$

$\hookrightarrow F' \& F$

$$\begin{cases} e = \frac{c}{a} \\ c = ae \end{cases}$$

Vertices $(\pm a, 0) \rightarrow A \& A'$

Co-vertices $(0, \pm b) \rightarrow B \& B'$

Equation of major axis: $y=0 \rightarrow AA'$ (the line)

Equation of minor axis: $x=0 \rightarrow BB'$ (the line)

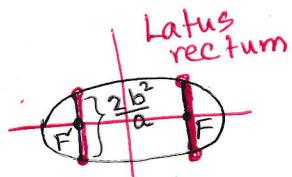
Length of major axis: $2a$ (AA')

Length of minor axis: $2b$ (BB')

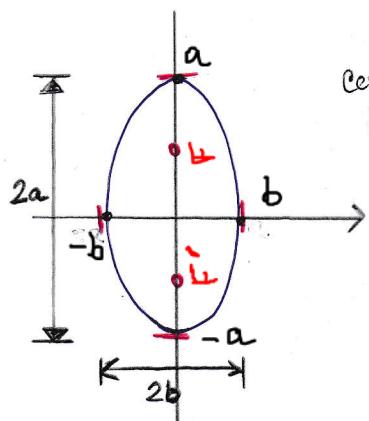
Equation of directrices: DZ & $D'Z'$

$$x = \pm \frac{a}{e}$$

$$\begin{cases} x = \pm c \\ x = \pm ae \end{cases} \quad \because c = ae$$



Length of Latus rectum: $\frac{2b^2}{a}$



center: $(0,0)$

Foci: $(0, \pm ae)$

Vertices: $(0, \pm a)$ \rightarrow major axis

Co-vertices: $(\pm b, 0)$ \rightarrow minor axis

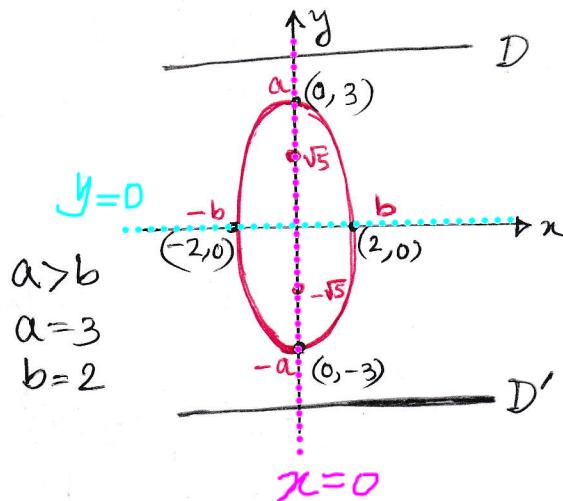
Examples: Find all the characteristics of the graph with equation: $9x^2 + 4y^2 = 36$

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad (\div \text{ by } 36)$$

$$\Rightarrow \frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

$$\Rightarrow \frac{y^2}{3^2} + \frac{x^2}{2^2} = 1$$



center: $(0,0)$

Vertices: $(0, \pm a) = (0, \pm 3)$ \rightarrow major axis

Covertices: $(\pm b, 0) = (\pm 2, 0)$ \rightarrow minor axis

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

Foci: $(0, \pm ae) = (0, \pm \sqrt{5})$

$$ae = 3 \cdot \frac{\sqrt{5}}{3} = \sqrt{5}$$

Equation of major axis: $x=0$

\Leftarrow eqn of minor axis: $y=0$

Length of major axis: $2a = 2(3) = 6$

Length of minor axis: $2b = 4$

Equation of directrices: $y = \pm \frac{a}{e} = \pm \frac{3}{\sqrt{5}}$ $\left\{ \begin{array}{l} a=3 \\ e=\frac{\sqrt{5}}{3} \\ b=2 \end{array} \right.$

Eqn of latus rectum: $y = \pm ae = \pm \sqrt{5}$

Length of latus rectum: $\frac{2b^2}{a} = \frac{8}{3}$

Compare

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\frac{Y^2}{a^2} + \frac{X^2}{b^2} = 1$$

if center is not
at the origin;
Center: (h, k)

Standard forms of the eqn of an ellipse with center: (h, k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

- $a > b$

- length of major axis = $2a$

- " " minor " = $2b$

- Coordinates of the vertices: $(h \pm a, k)$

- Coordinates of the covertices: $(h, k \pm b)$

- Coordinate of the foci: $(h \pm c, k)$

$$(\pm a, 0)$$

$$x = \pm a$$

$$y = 0$$

$$x - h = \pm a$$

$$y - k = 0$$

$$x = h \pm a$$

$$y = k$$

$$(0, \pm b)$$

$$x = 0 \quad y = \pm b$$

$$x - h = 0 \quad y - k = \pm b$$

$$x = h \quad y = k \pm b$$

$$y = k \pm b$$

$$(\pm c, 0)$$

$$x = \pm c$$

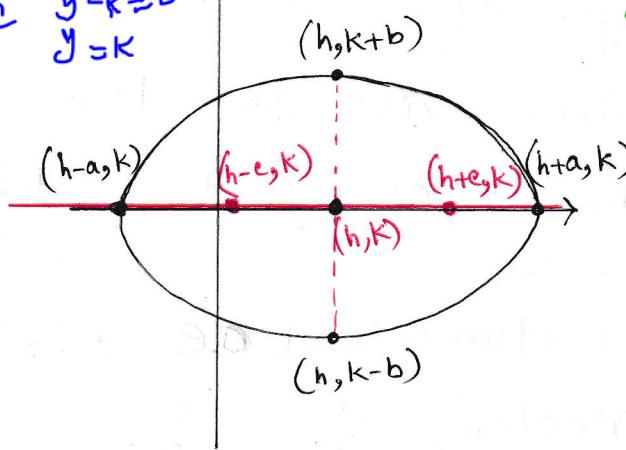
$$x - h = \pm c$$

$$x = h \pm c$$

$$y = 0$$

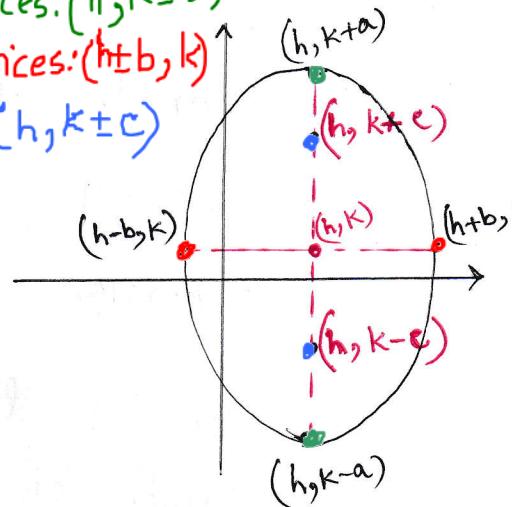
$$y - k = 0$$

$$y = k$$



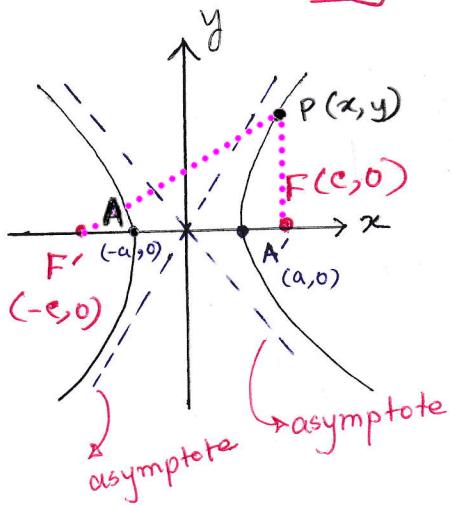
Horizontally stretched

Vertices: $(h, k \pm a)$
Cofvertices: $(h \pm b, k)$
foci: $(h, k \pm c)$



Vertically stretched

Hyperbola



set of all points in a plane the difference of whose distances from two fixed points (the foci) is a constant.

$F \& F'$

$$|PF| - |PF'| = \pm 2a$$

$$PF > PF' = 2a$$

$$PF < PF' = -2a$$

$$\Rightarrow \sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2} = \pm 2a$$

$$\Rightarrow \sqrt{(x-c)^2 + y^2} = \pm 2a + \sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow x^2 - 2cx + c^2 + y^2 = 4a^2 \pm 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$$

$$\Rightarrow -4cx = 4a^2 \pm 4a\sqrt{(x+c)^2 + y^2}$$

$$\Rightarrow \pm a\sqrt{(x+c)^2 + y^2} = a^2 + cx$$

$$\Rightarrow a^2(x^2 + 2cx + c^2 + y^2) = a^4 + 2a^2cx + c^2x^2$$

$$\Rightarrow a^2x^2 + 2a^2cx + a^2c^2 + y^2 = a^4 + 2a^2cx + c^2x^2$$

$$\Rightarrow a^2x^2 + a^2c^2 + y^2 = a^4 + c^2x^2$$

$$\Rightarrow a^2x^2 + a^2(a^2 + b^2) + a^2y^2 = a^4 + c^2x^2$$

$$\Rightarrow a^2x^2 + a^4 + a^2b^2 + a^2y^2 = a^4 + a^2x^2 + b^2x^2$$

$$\Rightarrow a^2b^2 = b^2x^2 - a^2y^2$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

standard form

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Redefine the constant

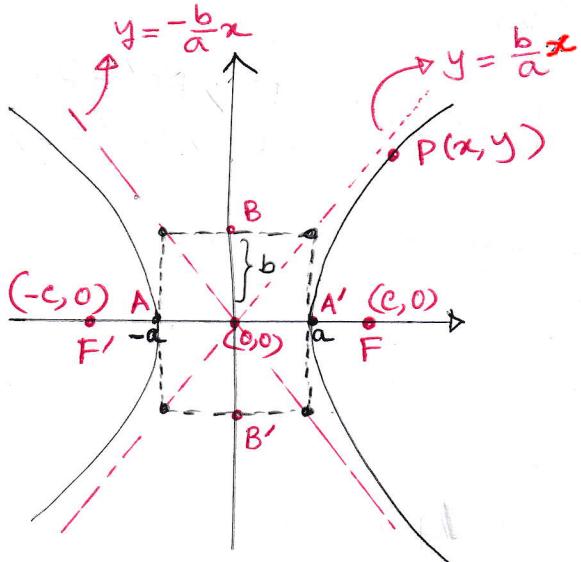
$$c^2 = a^2 + b^2$$

In Ellipse we relabel

$$c^2 = a^2 - b^2$$

$$\text{Center: } (0,0) \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Center: } (h,k) \Rightarrow \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



Center: $C(0,0)$

Vertices: $A \& A'$

$$\Rightarrow (\pm a, 0)$$

Foci: $F \& F'$

$$\Rightarrow (\pm c, 0) \text{ or } (\pm ae, 0)$$

$$e > 1$$

See graph at pg [13]

Eccentricity: $e = \sqrt{1 + \frac{b^2}{a^2}}$: $e = \frac{c}{a}$, $e > 1$
 The sign is opposite to ellipse

Eqn of Asymptotes: $y = \pm \left(\frac{b}{a}\right)x$

Length of transverse axis: $2a$ (AA')

Length of conjugate axis: $2b$ (BB')

Eqn of directrices: $x = \pm \frac{a}{e}$
 (D & D')

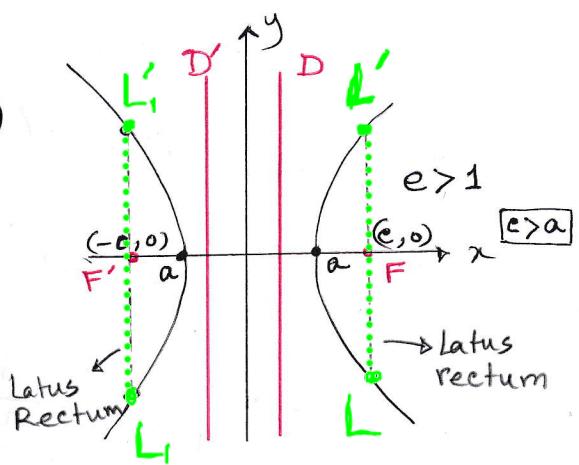
same
as
ellipse

Length of Latus rectum: $\frac{2b^2}{a}$

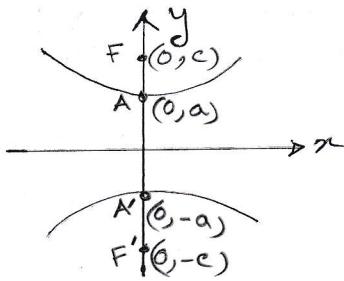
Eqn of Latus rectum:

$$x = -\frac{c}{ae}, x = \frac{c}{ae}$$

Foci



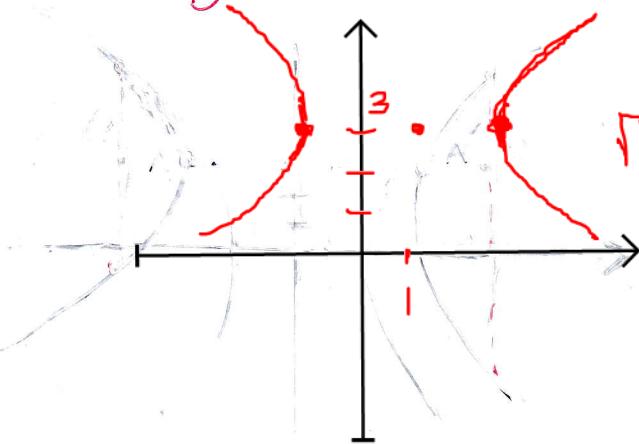
If we have: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



Vertices: $(0, \pm a)$

Foci: $(0, \pm c)$ or $(0, \pm ae)$

Example: Find all the characteristics of the hyperbola with equation $2(x-1)^2 - 3(y-3)^2 = 30$



$$2(x-1)^2 - 3(y-3)^2 = 30$$

$$\frac{(x-1)^2}{15} - \frac{(y-3)^2}{10} = 1$$

$$a = \sqrt{15}, b = \sqrt{10}$$

Center, $C = (1, 3)$

Vertices = $(1 \pm \sqrt{15}, 3)$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices: $(\pm a, 0)$

$$\begin{aligned} X = \pm a \\ \Rightarrow x-1 = \pm a \\ \Rightarrow x = 1 \pm a \end{aligned} \quad \begin{aligned} Y = 0 \\ \Rightarrow y-3 = 0 \\ \Rightarrow y = 3 \end{aligned}$$

$$x = 1 \pm \sqrt{15}$$

$$e = \sqrt{\frac{25}{15}} = \frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$$

Foci: $(\pm ae, 0)$

$$\begin{aligned} X = \pm ae \\ x-1 = \pm \sqrt{15} \left(\frac{\sqrt{15}}{3} \right) = \pm \frac{15}{3} = \pm 5 \\ x = 1 \pm 5 = -4, 6 \end{aligned}$$

Foci: $(6, 3)$ and $(-4, 3)$

Eqn of directrices: $X = \pm \frac{a}{e}$

$$x-1 = \pm \frac{\sqrt{15}}{\sqrt{15}/3} = \pm 3$$

$$x = 1 \pm 3$$

$$x = 4, x = -2$$

Eqn of asymptotes: $y = \pm \frac{b}{a}x$

$$y-3 = \pm \frac{\sqrt{10}}{\sqrt{15}}(x-1)$$

$$y = 3 \pm \frac{\sqrt{2}}{\sqrt{3}}(x-1)$$

$$= 3 \pm \frac{\sqrt{6}}{3}(x-1)$$

$$y = \frac{\sqrt{6}}{3}(x-1) + 3; \quad y = -\frac{\sqrt{6}}{3}(x-1) + 3$$

Eqn of latus rectum:

$$x=6 \quad \text{and} \quad x=-4$$

$\underbrace{\hspace{10em}}$ Foci

Length of Latus rectum: $\frac{2b^2}{a} = \frac{2(\sqrt{10})^2}{\sqrt{15}}$

$$= \frac{20}{\sqrt{15}}$$

$$= \frac{20\sqrt{15}}{15}$$

$$= \frac{4\sqrt{15}}{3}$$

Length of transverse axis: $2a = 2\sqrt{15}$
 " " conjugate axis: $2b = 2\sqrt{10}$

