

Week 4

Linear Approximation

Recall equation of a tangent line at $x=a$
[pt slope form eqn of a line:]

$$y - y_1 = m(x - x_1)$$

$$m = f'(x)$$

at $x=a$
we have

$$y = f(a)$$

$$\& m = f'(a)$$

$$\therefore (x_1, y_1) = (a, f(a))$$

$$y - f(a) = f'(a)(x - a)$$

$$y = f(a) + f'(a)(x - a)$$

$$L(x) = f(a) + f'(a)(x - a)$$

Linear approximation calculation method.

Linear approximation uses the fact that every curve will always look like a line if we zoom in small enough.

Example

① Find the linearization of the function $f(x) = 3x^2$ at $a=1$
& use it to approximate $f(0.9)$

step 1

$$\begin{cases} f(a) = f(1) = 3(1)^2 = 3 \\ \therefore (a, f(a)) = (1, 3) \end{cases}$$

step 2

$$\begin{cases} f(x) = 3x^2 \\ f'(x) = 6x \\ f'(a) = f'(1) = 6(1) = 6 = m \end{cases}$$

point slope form: \rightarrow (optional)

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y &= y_1 + m(x - x_1) \\ &= 3 + 6(x - 1) \end{aligned}$$

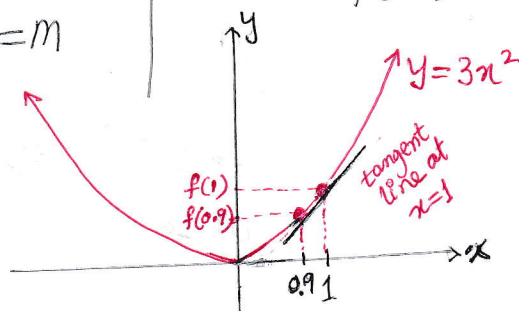
step 3

step 1

$$\begin{cases} L(x) = f(a) + f'(a)(x - a) \\ L(0.9) = 3 + 6(0.9 - 1) \\ = 2.4 \end{cases}$$

Using calculator

$$f(0.9) = 3(0.9)^2 = 2.43$$



② Find $f(-8.1)$ given $f(x) = \sqrt[3]{x}$

Let $a = -8$ $\because -8$ is close to -8.1

$$\sqrt[3]{-8.1} = ?$$

$$\sqrt[3]{-8} = -2$$

step 1) $f(a) = f(-8) = \sqrt[3]{-8} = -2$
 $\therefore (x_1, y_1) = (a, f(a)) = (-8, -2)$

$$L(x) = f(a) + f'(a)(x-a)$$

step 2) $f(x) = \sqrt[3]{x} = x^{1/3}$

$$f'(x) = \frac{1}{3} x^{1/3 - 1} = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$f'(-8) = \frac{1}{3(-8)^{2/3}} = \frac{1}{3(-8^{1/3})^2} = \frac{1}{3(-2)^2} = \frac{1}{3(4)} = \frac{1}{12}$$

$$\therefore m = f'(a) = f'(-8) = \frac{1}{12}$$

step 3)

point slope form:

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{1}{12}(x - (-8))$$

$$y + 2 = \frac{1}{12}(x + 8)$$

$$y = -2 + \frac{1}{12}(x + 8)$$

step 4) Linear Approximation:

$$L(x) = f(a) + f'(a)(x-a)$$

$$L(-8.1) = -2 + \frac{1}{12}(-8.1 + 8)$$

$$= -2.008$$

compare

Using calculator $f(-8.1) = \sqrt[3]{-8.1} = -2.008299 \dots$

