

MAT 110

Practice Sheet # 3

Techniques of Differentiation

1. Find the differential coefficients of the following functions with respect to x

(i.e. $\frac{dy}{dx}$).

(i) $y = \sin x \sin 2x \sin 3x$, (ii) $y = \sec^3 x$, (iii) $y = \cos 2x \cos 3x$, (iv) $y = \sin^{-1}(x^2)$,

(v) $y = \tan(\sin^{-1} x)$, (vi) $\cot^{-1}\left(\frac{1+x}{1-x}\right)$, (vii) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, (viii) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$,

(ix) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, (x) $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, (xi) $\sin\left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}}\right)$, (xii) $\ln \sqrt{\frac{1-\cos x}{1+\cos x}}$.

2. Find the differential coefficients of:

(i) $(\sin x)^{\ln x}$, (ii) $(\sin x)^{\cos x} + (\cos x)^{\sin x}$.

3. Find $\frac{dy}{dx}$ in the following cases:

(i) $3x^4 - x^2 y + 2y^3 = 0$, (ii) $x^3 + y^3 + 4x^2 y - 25 = 0$, (iii) $x^y = y^x$.

4. Find $\frac{dy}{dx}$ when

(i) $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, (ii) $x = \sin^2 \theta$, $y = \tan \theta$, (iii) $x = a \sec^2 \theta$, $y = a \tan^2 \theta$.

5. Differentiate the left-side functions with respect to the right-side ones:

(i) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ (ii) $x^{\sin^{-1}(x)}$ with respect to $\sin^{-1} x$.