

- Maxima Minima of function of Several Variables
- Taylor Expansion of a two variable function

## Maxima Minima of function of Several Variables

### Critical Points

A point  $(x_0, y_0)$  in the domain of a function  $f(x, y)$  is called a critical point of the function.

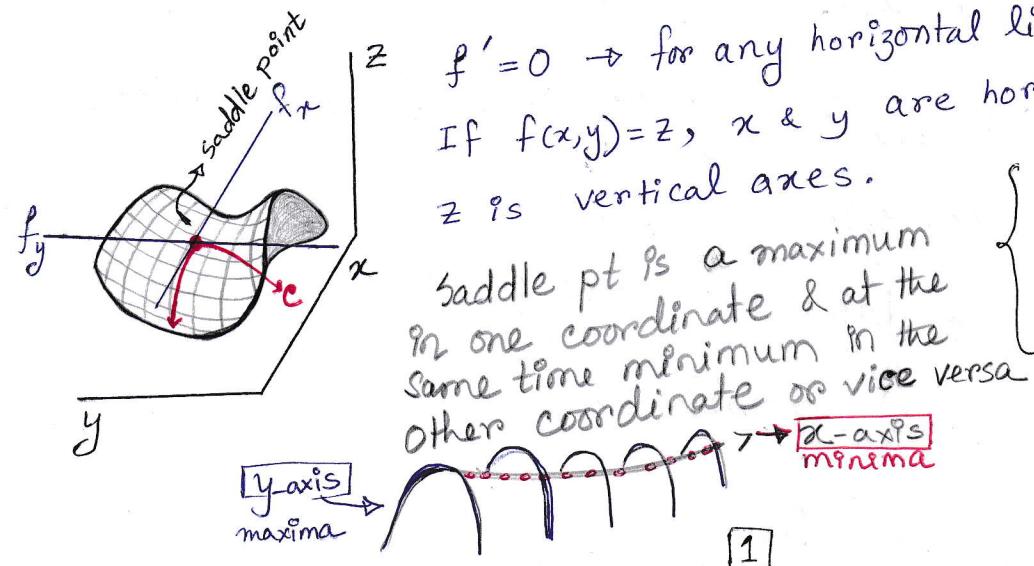
if  $f_x(x_0, y_0) = 0$  and  $f_y(x_0, y_0) = 0$

OR

if  $f_x(x_0, y_0) = \infty$  and  $f_y(x_0, y_0) = \infty$

### Saddle Point:

A saddle point is a point in the domain of a function that is a stationary point but not a local extremum on both axes (x & y axes).



$f' = 0 \rightarrow$  for any horizontal line  
If  $f(x, y) = z$ ,  $x$  &  $y$  are horizontal axes.  
 $z$  is vertical axes.

saddle pt is a maximum in one coordinate & at the same time minimum in the other coordinate or vice versa

→  $\text{y-axis maxima}$       →  $\text{x-axis minima}$

[1]

$$\left\{ \begin{array}{l} f_x(x_0, y_0) = \infty \\ f_y(x_0, y_0) = \infty \\ \text{or both } \infty \end{array} \right.$$

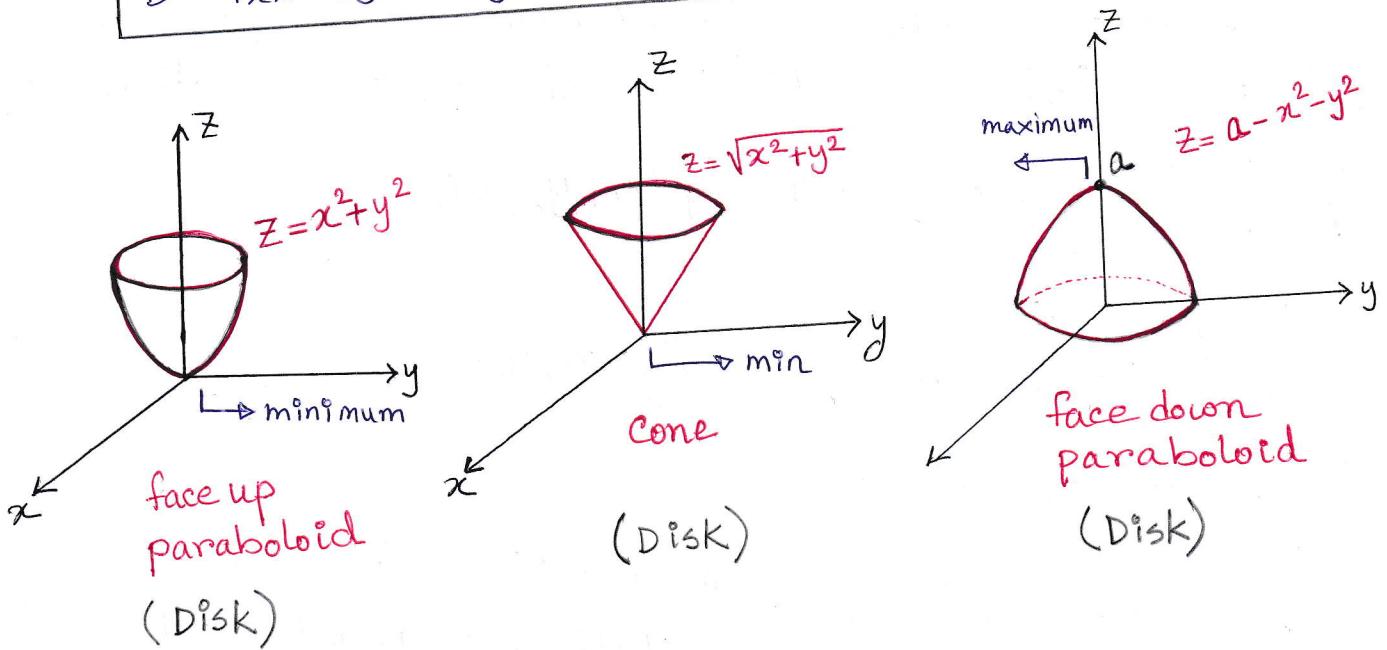
These conditions are not covered in saddle point.

## The Second Partial Test

Let  $f$  be a function of two variables with continuous second-order partial derivatives in some disk centered at a critical point  $(x_0, y_0)$  and

let  $D(x_0, y_0) = f_{xx}(x_0, y_0)f_{yy}(x_0, y_0) - f_{xy}^2(x_0, y_0)$

$$D = f_{xx}f_{yy} - f_{xy}^2 \text{ at } (x_0, y_0)$$



- a) If  $D > 0$  and  $f_{xx}(x_0, y_0) > 0$  then  $f$  has a relative minimum at  $(x_0, y_0)$ .
- b) If  $D > 0$  and  $f_{xx}(x_0, y_0) < 0$  then  $f$  has a relative maximum at  $(x_0, y_0)$ .
- c) If  $D < 0$  then  $f$  has a saddle point at  $(x_0, y_0)$ .
- d) If  $D = 0$  then no conclusion

Ch 13.8

Locate all relative maxima, relative minima, and saddle points, if any.

$$9. f(x, y) = y^2 + xy + 3y + 2x + 3$$

$$f_y(x, y) = 2y + x + 3$$

$$f_y = 0 \Rightarrow 2y + x + 3 = 0$$

$$\Rightarrow 2(-2) + x + 3 = -3$$

$$\Rightarrow x = -3 + 4$$

$$\Rightarrow x = 1$$

$$\left\{ \begin{array}{l} f_x(x, y) = y + 2 \\ f_x = 0 \Rightarrow y + 2 = 0 \end{array} \right.$$

$$y = -2$$

(1, -2) is the only critical point

$$f_x(x, y) = y + 2$$

$$f_{xx}(x, y) = 0$$

$$\rightarrow f_{xy}(x, y) = 1$$

$$f_y(x, y) = 2y + x + 3$$

$$f_{yy}(x, y) = 2$$

$$D = f_{xx}(1, -2) f_{yy}(1, -2) - f_{xy}^2(1, -2)$$

$$= (0)(2) - 1^2 = -1 < 0$$

$\therefore D < 0$  then  $f$  has a saddle pt at (1, -2)

$$10. f(x, y) = x^2 + xy - 2y - 2x + 1$$

$$f_x(x, y) = 2x + y - 2$$

$$f_x = 0 \Rightarrow 2x + y - 2 = 0$$

$$2x + y = 2$$

$$\therefore 2(2) + y = 2$$

$$\boxed{y = -2}$$

$$f_y(x, y) = x - 2$$

$$f_y = 0 \Rightarrow x - 2 = 0$$

$$\boxed{x = 2}$$

$\therefore (2, -2)$  is the only critical point

$$f_x(x, y) = 2x + y - 2$$

$$f_y(x, y) = x - 2$$

$$f_{xx}(x, y) = 2$$

$$f_{yy}(x, y) = 0$$

$$\rightarrow f_{xy}(x, y) = 1$$

$$D = f_{xx}(2, -2) f_{yy}(2, -2) - f_{xy}^2(2, -2)$$

$$= (2)(0) - 1^2 = -1 < 0$$

$\therefore D < 0$  then  $f$  has a saddle point at  $(2, -2)$ .

$$11. f(x,y) = x^2 + xy + y^2 - 3x$$

$$f_x(x,y) = 2x + y - 3$$

$$f_x = 0 \Rightarrow 2x + y - 3 = 0$$

$$\Rightarrow 2x + y = 3 \quad \text{---} \textcircled{i}$$

$$f_y(x,y) = x + 2y$$

$$f_y = 0 \Rightarrow x + 2y = 0 \quad \text{---} \textcircled{ii}$$

$$\textcircled{i} - 2\textcircled{ii} \Rightarrow -3y = 3$$

$$\boxed{y = -1}$$

substitute  $y = -1$  into  $\textcircled{i}$

$$2x + (-1) = 3$$

$$2x = 3 + 1 = 4$$

$$\boxed{x = 2}$$

$\therefore (2, -1)$  is the only critical point

$$f_x = 2x + y - 3$$

$$f_y = x + 2y$$

$$f_{xx} = 2$$

$$f_{yy} = 2$$

$$f_{xy} = 1$$

$$D = f_{xx}(2, -1) f_{yy}(2, -1) - f_{xy}^2(2, -1)$$

$$= (2)(2) - 1^2 = 3 > 0$$

$\therefore D > 0$  and  $f_{xx} > 0$ ,

hence  $f$  has a relative minimum at  $(2, -1)$ .

$$12. f(x, y) = xy - x^3 - y^2$$

$$f_x = y - 3x^2$$

$$f_x = 0 \Rightarrow y - 3x^2 = 0$$

$$\therefore y - 3(2y)^2 = 0$$

$$y - 12y^2 = 0$$

$$y(1 - 12y) = 0$$

$$y = 0 \quad y = \frac{1}{12}$$

$$\rightarrow f_{xx} = -6x$$

$$\rightarrow f_{xy} = 1$$

$$f_y = x - 2y$$

$$f_y = 0 \Rightarrow x - 2y = 0$$

$$x = 2y$$

$$y = 0 \Rightarrow x = 2(0) = 0$$

$$y = \frac{1}{12} \Rightarrow x = 2\left(\frac{1}{12}\right) = \frac{1}{6}$$

$\therefore (0, 0)$  and  $(\frac{1}{6}, \frac{1}{12})$  are critical points

$$\rightarrow f_{yy} = -2$$

At  $(0, 0)$  we have:

$$D = f_{xx}(0, 0) f_{yy}(0, 0) - f_{xy}^2(0, 0)$$

$$= [-6(0)][-2] - [1]^2$$

$= -1 < 0$ ;  $\therefore D < 0$  hence  $f$  has a saddle pt at  $(0, 0)$ .

At  $(\frac{1}{6}, \frac{1}{12})$

$$D = f_{xx}(\frac{1}{6}, \frac{1}{12}) f_{yy}(\frac{1}{6}, \frac{1}{12}) - f_{xy}^2(\frac{1}{6}, \frac{1}{12})$$

$$= [-6(\frac{1}{6})][-2] - [1]^2$$

$$= 2 - 1 = 1 > D$$

$$f_{xx}(\frac{1}{6}, \frac{1}{12}) = -6(\frac{1}{6}) \\ = -1$$

$$\therefore f_{xx}(\frac{1}{6}, \frac{1}{12}) < 0$$

$$D > 0$$

$\therefore D > 0$  &  $f_{xx}(\frac{1}{6}, \frac{1}{12}) < 0$  hence  $f$  has relative maximum at  $(\frac{1}{6}, \frac{1}{12})$ .

$$13. f(x, y) = x^2 + y^2 + \frac{2}{xy}$$

$$f_x = 2x + \frac{2}{y} (-1)x^{-2} = 2x - \frac{2}{x^2 y}$$

$$f_x = 0 \Rightarrow 2x - \frac{2}{x^2 y} = 0$$

$$\frac{2x^3 y - 2}{x^2 y} = 0$$

$$2x^3 y - 2 = 0; \boxed{x \neq 0 \\ y \neq 0}$$

$$x^3 y - 1 = 0$$

$$x^3 y = 1$$

$$\boxed{y = \frac{1}{x^3}}$$

$$x = -1 \Rightarrow y = \frac{1}{(-1)^3} = -1$$

$$x = +1 \Rightarrow y = \frac{1}{(+1)^3} = 1$$

$$f_y = 2y + \frac{2}{x} (-1)y^{-2} = 2y - \frac{2}{x y^2}$$

$$f_y = 0 \Rightarrow 2y - \frac{2}{x y^2} = 0$$

$$\frac{2x y^3 - 2}{x y^2} = 0$$

$$2x y^3 - 2 = 0$$

$$x y^3 - 1 = 0$$

$$x y^3 = 1$$

$$x \left(\frac{1}{x^3}\right)^3 = 1$$

$$\therefore y = \frac{1}{x^3}$$

$$x \cdot \frac{1}{x^9} = 1$$

$$\frac{1}{x^8} = 1$$

$$x^8 = 1$$

$$x = \pm 1$$

$\therefore$  Critical points are  $(1, 1), (-1, -1), (0, 0)$

Critical points $(x_0, y_0)$	$f_{xx}(x_0, y_0)$	$f_{yy}(x_0, y_0)$	$f_{xy}(x_0, y_0)$	$D = f_{xx} f_{yy} - f_{xy}^2$	Decision
$(0, 0)$	$\infty$	$\infty$	$\infty$	$D = \infty$	
$(1, 1)$	$6 > 0$	$6$	$2$	$D = 32 > 0$	minimum
$(-1, -1)$	$6 > 0$	$6$	$2$	$D = 32 > 0$	minimum

$$14. f(x,y) = xe^y$$

$$f_x = e^y$$

$$f_x = 0 \Rightarrow e^y = 0 \quad \textcircled{1}$$

$$f_y = xe^y$$

$$f_y = 0 \Rightarrow xe^y = 0 \quad \textcircled{11}$$

compare \textcircled{1} & \textcircled{11}:  $e^y = xe^y$

$$e^y - xe^y = 0$$

$$e^y(1-x) = 0$$

$$e^y = 0$$

$$\begin{aligned} 1-x &= 0 \\ \boxed{x &= 1} \end{aligned}$$

$$\ln e^y = \ln 0$$

$$y \ln e = \ln 0$$

$$y = \infty$$

no critical point.

$$15. f(x,y) = x^2 + y - e^y$$

$$f_x = 2x$$

$$f_x = 0 \Rightarrow 2x = 0$$

$$\boxed{x = 0}$$

$$f_y = 1 - e^y$$

$$f_y = 0 \Rightarrow 1 - e^y = 0$$

$$e^y = 1$$

$$\ln e^y = \ln 1$$

$$y \ln e = \ln 1$$

$$\boxed{y = 0}$$

\infty critical point at (0, 0)

$$\therefore f_y = 1 - e^y$$

$$f_{yy} = -e^y$$

$$D = f_{xx}(0,0)f_{yy}(0,0) - f_{xy}^2(0,0)$$

$$= (2)(-1) - 0^2 = -2 < 0$$

\therefore D < 0, f has a saddle pt at (0,0).

$$17. f(x, y) = e^x \sin y$$

$$f_x = e^x \sin y$$

$$f_x = 0 \Rightarrow e^x \sin y = 0$$

(i)

$$f_y = e^x \cos y$$

$$f_y = 0 \Rightarrow e^x \cos y = 0$$

(ii)

Compare (i) & (ii)

$$e^x \sin y = e^x \cos y$$

$$e^x \sin y - e^x \cos y = 0$$

$$e^x (\sin y - \cos y) = 0$$

$$e^x = 0$$

$$\ln e^x = \ln 0$$

$$x = \infty$$

$$\sin y - \cos y = 0$$

$$\sin y = \cos y$$

$$\frac{\sin y}{\cos y} = 0$$

$$\tan y = 0$$

$$y = \tan^{-1} 0$$

$$y = 0$$

no critical point.

$$16. f(x, y) = xy + \frac{2}{x} + \frac{4}{y}$$

$$f_x = y - 2x^{-2}$$

$$f_x = 0 \Rightarrow y - \frac{2}{x^2} = 0$$

$$\frac{x^2 y - 2}{x^2} = 0$$

$$x^2 y - 2 = 0 \quad \boxed{x \neq 0}$$

$$x^2 y = 2$$

$$\boxed{y = \frac{2}{x^2}}$$

$$y = \frac{2}{x^2} \quad \therefore x = 1$$

$$\therefore (0, 0) \text{ & } (1, 2)$$

are critical points  
and (1, 2) is the stationary point

$$\boxed{y = 2}$$

$$f_y = x - 4y^{-2}$$

$$f_y = 0 \Rightarrow x - 4y^{-2} = 0$$

$$\Rightarrow x - \frac{4}{y^2} = 0$$

$$\frac{xy^2 - 4}{y^2} = 0$$

$$xy^2 - 4 = 0 ; \quad \boxed{y \neq 0}$$

$$x \left(\frac{2}{x^2}\right)^2 - 4 = 0 \quad \therefore y = \frac{2}{x^2}$$

$$x \left(\frac{4}{x^4}\right) - 4 = 0$$

$$\frac{4}{x^3} = 4 \Rightarrow \frac{1}{x^3} = 1$$

$$\Rightarrow x^3 = 1$$

$$\Rightarrow \boxed{x = 1}$$

$$\begin{aligned}
 f_x &= y - 2x^{-2} & f_y &= x - 4y^{-2} \\
 f_{xx} &= 4x^{-3} & f_{yy} &= 8y^{-3} = \frac{8}{y^3} \\
 \rightarrow f_{xy} &= 1
 \end{aligned}$$

considering the stationary pt  $(1, 2)$

$$\begin{aligned}
 D &= f_{xx}(1, 2) f_{yy}(1, 2) - f_{xy}^2(1, 2) \\
 &= [4(1)^{-3}] \left[ \frac{8}{(2^3)} \right] - [1]^2 \\
 &= (4)(1) - 1 = 3 > 0
 \end{aligned}$$

$$f_{xx}(1, 2) = \frac{4}{x^3} = \frac{1}{1^3} = 4 > 0$$

$\therefore D > 0$  &  $f_{xx}(1, 2) > 0 \therefore f$  has relative minimum at  $(1, 2)$ .

$$18. f(x, y) = y \sin x$$

$$f_x = y \cos x$$

$$f_x = 0 \Rightarrow y \cos x = 0$$

$$\Rightarrow y \cos 0^\circ = 0 \quad \because x = 0$$

$$\Rightarrow y = 0$$

$\therefore$  The critical pt is  $(0, 0)$

$$\rightarrow f_{xx} = -y \sin x$$

$$\rightarrow f_{xy} = \cos x$$

$$f_y = \sin x$$

$$f_y = 0 \Rightarrow \sin x = 0$$

$$\Rightarrow x = \sin^{-1} 0$$

$$\Rightarrow \boxed{x = 0}$$

$$D = f_{xx}(0, 0) f_{yy}(0, 0) - f_{xy}^2(0, 0)$$

$$= [-(0)\sin 0] [\cos 0] - [\cos(0)]^2 = (0)(1) - 1^2 = -1 < 0$$

$\therefore D < 0$ , then  $f$  has a saddle point at  $(0, 0)$

$$19. f(x, y) = e^{-(x^2+y^2+2x)}$$

$$f_x = e^{-(x^2+y^2+2x)} (-2x-2)$$

$$f_x = 0 \Rightarrow e^{-(x^2+y^2+2x)} (-2x-2) = 0$$

$$(2x+2)e^{-(x^2+y^2+2x)} = 0$$

(1)

$$\text{Compare (1) \& (ii)} \quad \begin{cases} 2x+2 = 2y \\ x+1 = y \end{cases}$$

The solution can be written in the form  $(x, x+1)$ ,  $x \in \mathbb{R}$   
 Consider two possible solutions  $x+1 = y$   
 $(-1, 0), (0, 1)$ .

$$\text{at } (-1, 0) \begin{cases} (i) e^{-(1-2)} (-2+2) = 0 & \checkmark \\ (ii) e^{-(1-2)} (0) = 0 & \checkmark \end{cases} \quad (-1, 0) \text{ satisfies eqn (i) \& (ii)}$$

$$\text{at } (0, 1) \begin{cases} (i) e^{-1} 2 \neq 0 \\ (ii) e^{-1} 2 \neq 0 \end{cases} \quad (0, 1) \text{ does not satisfy either (i) or (ii)}$$

$\therefore (-1, 0)$  is the only critical point.

$$f_{xx} = -2e^{-(x^2+y^2+2x)} + (-2x-2)e^{-(x^2+y^2+2x)} (-2x-2)$$

$$= -2e^{-(x^2+y^2+2x)} + (2x+2)^2 e^{-(x^2+y^2+2x)}$$

$$f_{xx}(-1, 0) = -2e^{-(-1)} + 0 = -2e$$

$$f_{yy} = -2e^{-(x^2+y^2+2x)} + (-2y)e^{-(x^2+y^2+2x)} (-2y)$$

$$= -2e^{-(x^2+y^2+2x)} + 4y^2 e^{-(x^2+y^2+2x)}$$

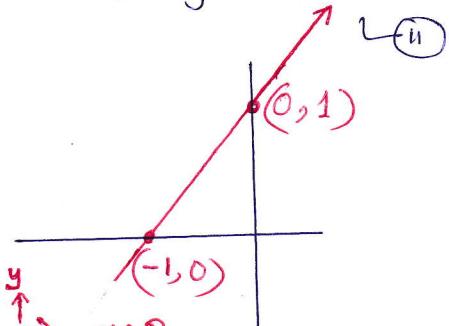
$$f_{yy}(-1, 0) = -2e^{-(1-2)} = -2e$$

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$$f_y = e^{-(x^2+y^2+2x)} (-2y)$$

$$f_y = 0 \Rightarrow -2y e^{-(x^2+y^2+2x)} = 0$$

$$\Rightarrow 2y e^{-(x^2+y^2+2x)} = 0$$



$$f_{xy} = (-2x-2)e^{-x^2-y^2-2x} [-2y]$$

$$f_{xy}(-1,0) = 0$$

$$D = f_{xx}(-1,0) f_{yy}(-1,0) - f_{xy}^2(-1,0)$$

$$= (-2e)(-2e) - 0^2$$

$$= 4e^2 > 0$$

$$\therefore D > 0 \text{ and } f_{xx}(-1,0) = -2e < 0$$

$\therefore f$  has relative maximum at  $(-1,0)$ .

$$(20) \quad f(x,y) = xy + \frac{a^3}{x} + \frac{b^3}{y} \quad (a \neq 0, b \neq 0)$$

$$f_x = y - \frac{a^3}{x^2}$$

$$f_x = 0 \Rightarrow y - \frac{a^3}{x^2} = 0$$

$$\frac{x^2y - a^3}{x^2} = 0$$

$$x^2y - a^3 = 0 ; \boxed{x \neq 0}$$

$$\boxed{y = \frac{a^3}{x^2}}$$

$$y = \frac{a^3}{\left(\frac{a^2}{b}\right)^2} \quad \therefore x = \frac{a^2}{b}$$

$$= a^3 \cdot \frac{b^2}{a^4} \Rightarrow \boxed{y = \frac{b^2}{a}}$$

$\therefore$  Critical pts are  $(0,0), \left(\frac{a^2}{b}, \frac{b^2}{a}\right)$

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$$f_y = x - \frac{b^3}{y^2}$$

$$f_y = 0 \Rightarrow x - \frac{b^3}{y^2} = 0$$

$$\frac{xy^2 - b^3}{y^2} = 0$$

$$xy^2 - b^3 = 0 ; \boxed{y \neq 0}$$

$$x\left(\frac{a^3}{x^2}\right)^2 - b^3 = 0 \quad \therefore y = \frac{a^3}{x^2}$$

$$x\left(\frac{a^6}{x^4}\right) = b^3$$

$$\frac{a^6}{x^3} = b^3 \Rightarrow x^3 = \frac{a^6}{b^3}$$

$$\boxed{x = \frac{a^2}{b}}$$

$$f_x = y - \frac{a^3}{x^2}$$

$$f_{xx} = -a^3(-2)x^{-3} = \frac{2a^3}{x^3}$$

$$f_{xy} = 1$$

$$f_y = x - \frac{b^3}{y^2}$$

$$\begin{aligned} f_{yy} &= -b^3(-2)y^{-3} \\ &= \frac{2b^3}{y^3} \end{aligned}$$

Considering stationary No.	$f_{xx}$	$f_{yy}$	$f_{xy}$	$D = f_{xx}f_{yy} - f_{xy}^2$
$(\frac{a^2}{b}, \frac{b^2}{a})$	$\frac{2a^3}{a^6/b^3} = 2a^3 \cdot \frac{b^3}{a^6} = \frac{2b^3}{a^3}$	$\frac{2b^3}{(\frac{b^2}{a})^3} = 2b^3 \cdot \frac{a^3}{b^6} = \frac{2a^3}{b^3}$	1	$\frac{2b^3}{a^3} \cdot \frac{2a^3}{b^3} - 1^2 = 4 - 1 = 3 > 0$

$$D > 0$$

$f$  has relative maximum at  $(\frac{a^2}{b}, \frac{b^2}{a})$  either if  $a, b$  is -ve or  $a$  is -ve.

$f$  has relative minimum at  $(\frac{a^2}{b}, \frac{b^2}{a})$  if  $a, b$  both +ve or if  $a, b$  both -ve

Week 7, MAT 110 Differential Calculus & Coordinate Geometry  
 (Continued) =

Taylor Expansion of a two variable function

Taylor expansion of single variable functions:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \dots + \frac{(x-a)^n}{n!}f^{(n)}(a)$$

Taylor expansion around  $a=0$

$$f(x, y) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{n,m} x^n y^m$$

$$= c_{0,0} + c_{1,0}x + c_{0,1}y + c_{2,0}x^2 + c_{1,1}xy + c_{0,2}y^2 \\ + \dots$$

The coefficients would be

$$c_{0,0} = f(0, 0)$$

$$c_{1,0} = \frac{\partial f}{\partial x}$$

$$c_{0,1} = \frac{\partial f}{\partial y}$$

$$c_{1,1} = \frac{\partial f}{\partial x \partial y}$$

$$c_{2,0} = \frac{1}{2!} \frac{\partial^2 f}{\partial x^2}$$

$$c_{n,0} = \frac{1}{n!} \frac{\partial^n f}{\partial x^{(n)}}$$

$$c_{0,n} = \frac{1}{n!} \frac{\partial^n f}{\partial y^{(n)}}$$

Taylor expansion around  $(0, 0)$ :  
 (Also known as Maclaurine's expansion)

$$f(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} \left( x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n f(x, y)$$

Taylor expansion around the point  $(a, b)$ :

$$f(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} \left( (x-a) \frac{\partial}{\partial x} + (y-b) \frac{\partial}{\partial y} \right)^n f(x, y)$$

Example Compute the 1<sup>st</sup> and 2<sup>nd</sup> degree Taylor Polynomials,  
 L(x, y) and Q(x, y) for  $f(x, y) = xe^y + 1$  for  $(x, y)$  near  
 the point  $(1, 0)$

Note  $\frac{\partial}{\partial x} = f_x$ ;  $\frac{\partial}{\partial y} = f_y$ ;  $\frac{\partial^2}{\partial x \partial y} = f_{xy}$ ;  $\frac{\partial^2}{\partial x^2} = f_{xx}$ ;  $\frac{\partial^2}{\partial y^2} = f_{yy}$

$$f(x, y) = xe^y + 1 \rightarrow f_{xx}(x, y) = 0$$

$$f_x(x, y) = e^y \rightarrow f_{yy}(x, y) = xe^y$$

$$f_y(x, y) = xe^y$$

$$f_x(1, 0) = e^0 = 1$$

$$f_y(1, 0) = 1 \cdot e^0 = 1$$

$$f_{xx}(1, 0) = 0$$

$$f_{yy}(1, 0) = 1 \cdot e^0 = 1$$

$$f_{xy}(1, 0) = e^0 = 1$$

$$f(1, 0) = 1 \cdot e^0 + 1 = 2$$

1<sup>st</sup> degree :

$$L(x, y) = f(1, 0) + f_x(1, 0)(x-1) + f_y(1, 0)(y-0)$$
$$= 2 + 1(x-1) + 1(y)$$

$$= 2 + x - 1 + y$$

$$L(x, y) = 1 + x + y \rightarrow 1^{\text{st}} \text{ order Taylor Polynomial}$$

Approximation

2<sup>nd</sup> degree :

$$Q(x, y) = L(x, y) + \frac{f_{xx}(1, 0)}{2!} (x-1)^2 + \frac{f_{xy}(1, 0)}{2!} (x-1)(y-0)$$
$$+ \frac{f_{yy}(1, 0)}{2!} (y-0)^2$$

$$= (1+x+y) + \frac{0}{2!} (x-1)^2 + 1(x-1)(y-0) + \frac{1}{2!} (y-0)^2$$

$$= 1 + x + y + 0 + xy - y + \frac{1}{2} y^2$$

$$Q(x, y) = 1 + x + xy + \frac{1}{2} y^2 \rightarrow 2^{\text{nd}} \text{ order Taylor Polynomial}$$

Approximation