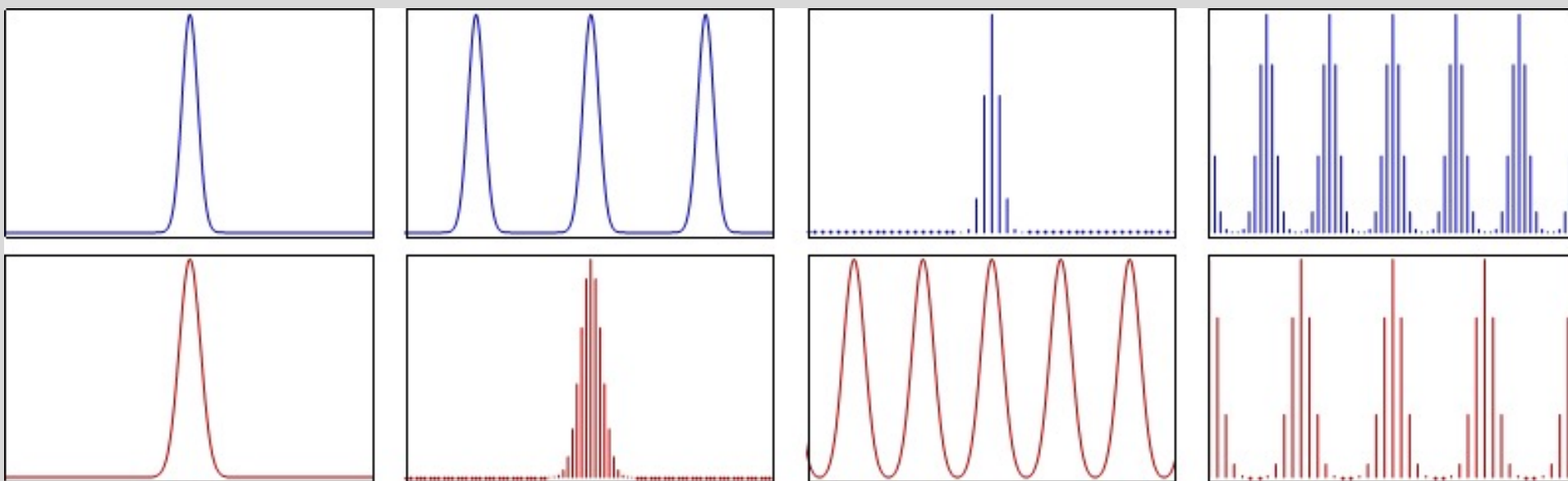


Trasformata di Fourier Discreta - DFT

Fondamenti Elaborazione dei Segnali e Immagini
(FESI)

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DIGEST FROM WIKIPEDIA



Trasformata di Fourier Discreta

Consideriamo $f(t)$ funzione di periodo T discreta (o discretizzata) con N punti nell'intervallo $[0, T]$

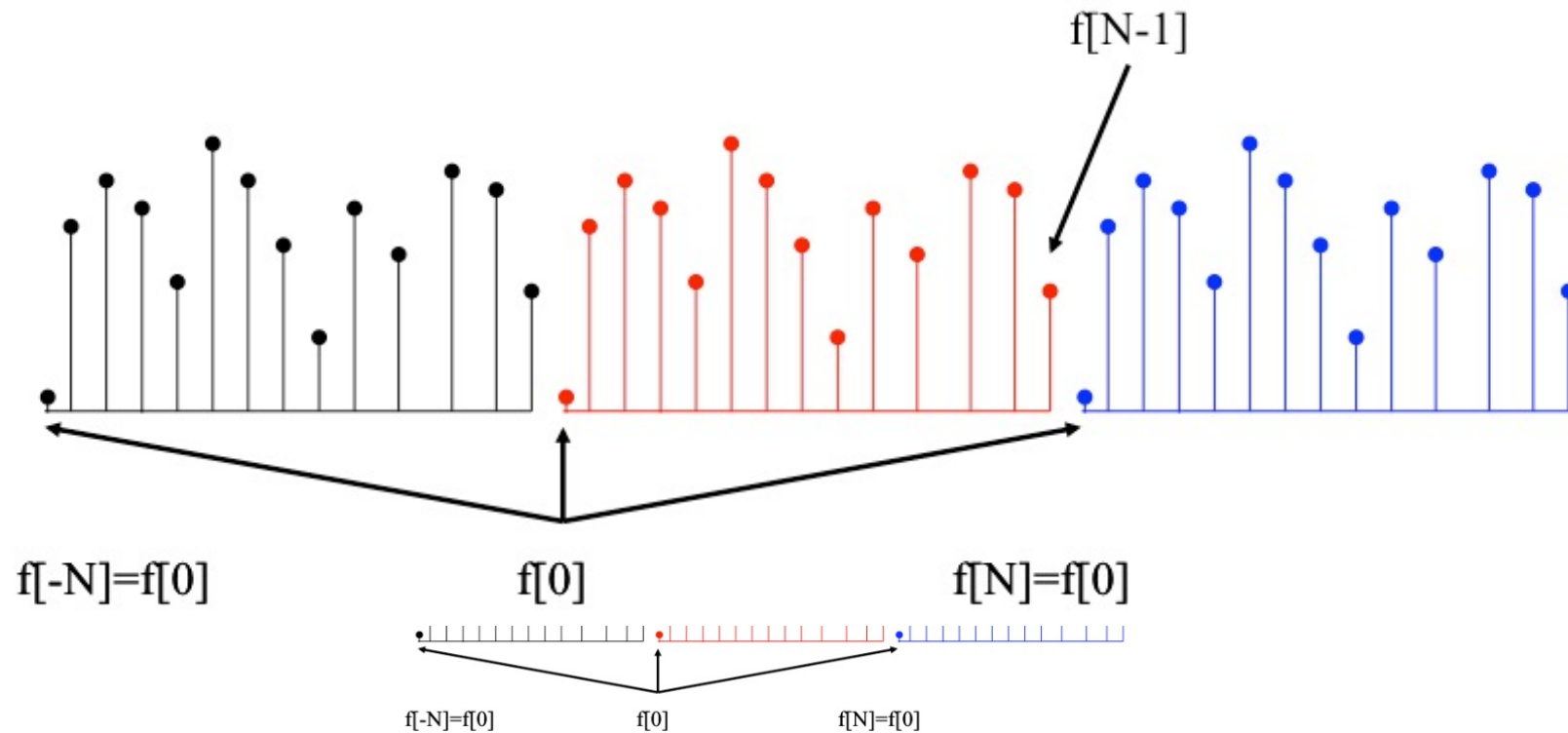
Chiamiamo $f[n]$ l'array che contiene i valori campionati

Definiamo la **Trasformata di Fourier Discreta (DFT)**

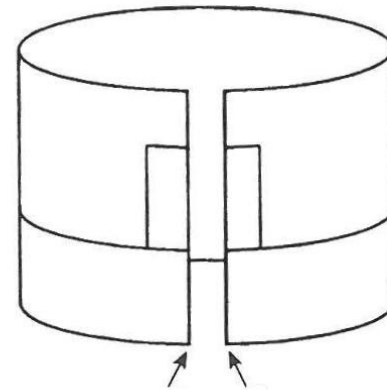
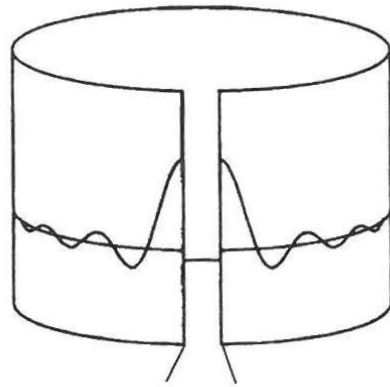
$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-j \frac{2\pi}{N} kn}$$

Trasformata di Fourier Discreta

Let f a signal sampled by N points. In the following let consider N



Periodicità



Inversa della DFT

Inversa della Trasformata di Fourier Discreta

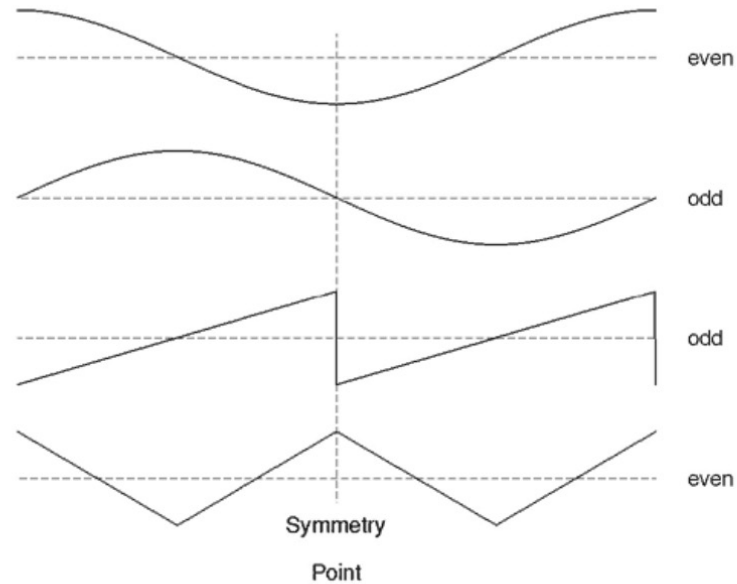
$$f[n] = \sum_{k=0}^{N-1} F[k] e^{j \frac{2\pi}{N} kn}$$

Importante: restituisce esattamente i valori originali!

Segnali pari e dispari

Even signal: $f[-m] = f[m]$

Odd signal: $f[-m] = -f[m]$



DFT di segnali a valori reali

La DFT di un segnale discreto a valori reali ha una simmetria speciale:

- La parte reale è simmetrica pari
- La parte immaginaria è simmetrica dispari

Nota: la maggior parte dei segnali provenienti dal “mondo” sono reali

DFT proprietà di simmetria

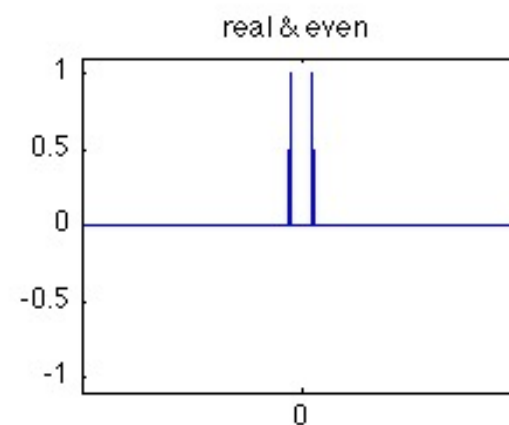
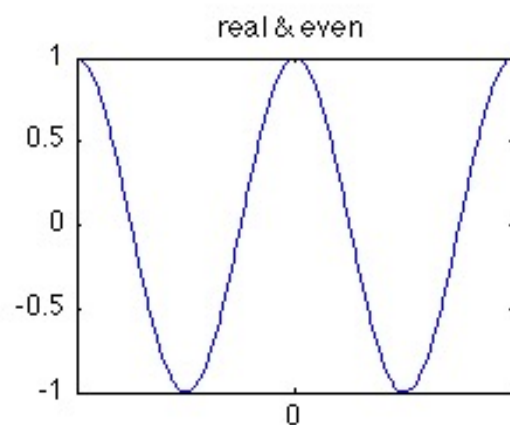
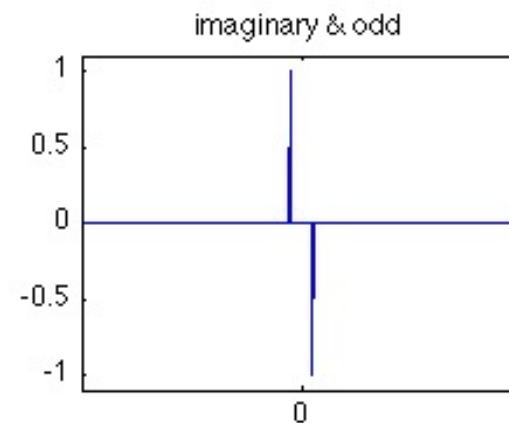
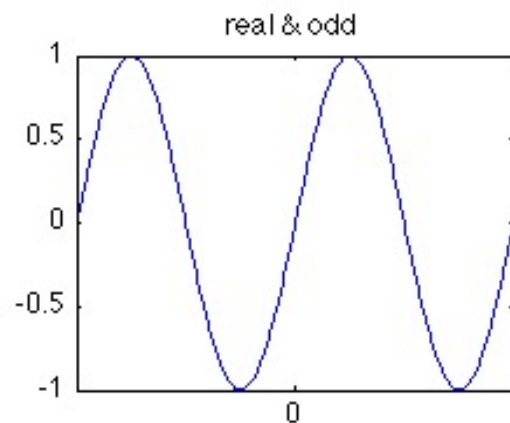
- ▷ if signal is real-valued and even, DFT is real-valued and even

$$f[n] = f^*[n] = f[-n] \quad , \quad F[k] = F^*[k] = F[-k]$$

- ▷ if signal is real-valued and odd, DFT is imaginary and odd

$$f[n] = f^*[n] = -f[-n] \quad , \quad F[k] = -F^*[k] = -F[-k]$$

Esempi : DFT di seni e coseni



F[0]

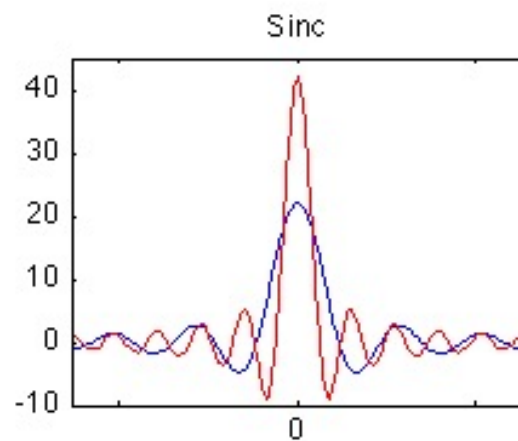
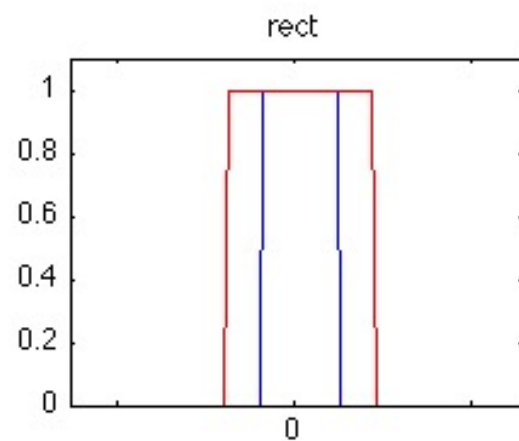
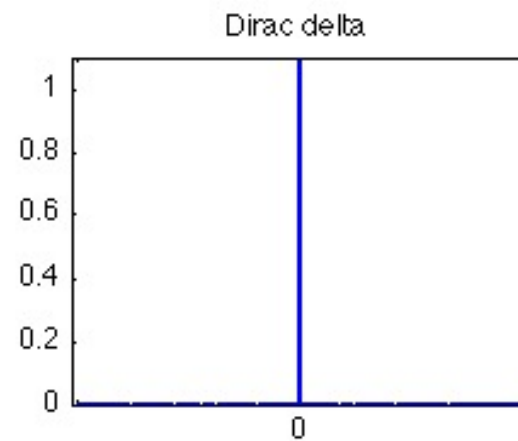
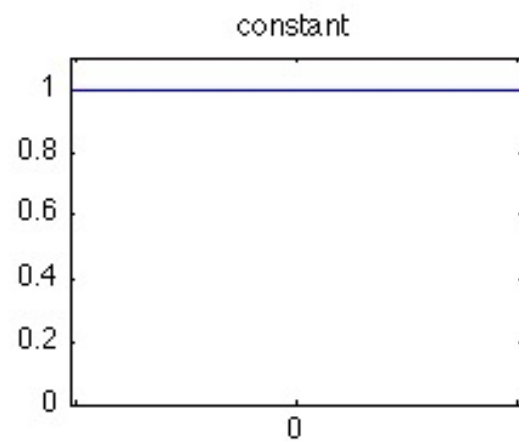
$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-i \frac{2\pi}{N} nk}$$

K=0

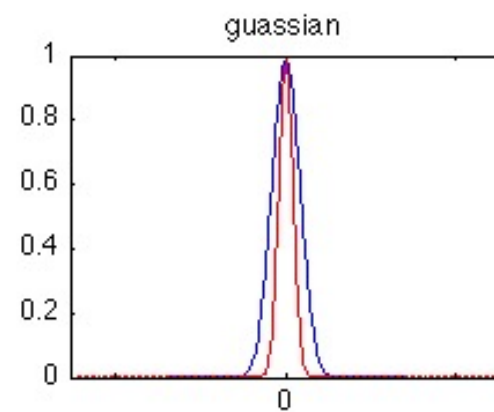
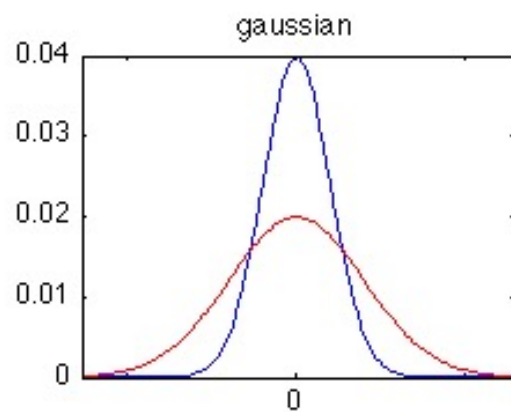
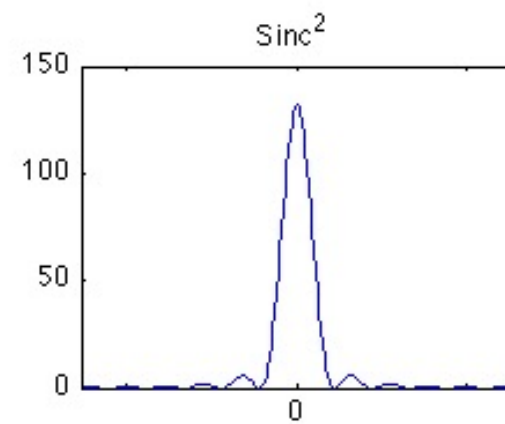
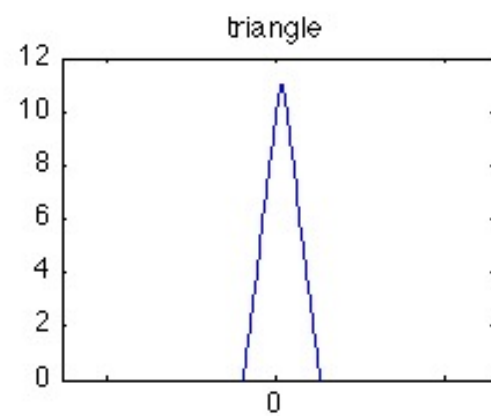
$$F[0] = \sum_{n=0}^{N-1} f[n]$$

Se f è reale $F[0]$ è reale

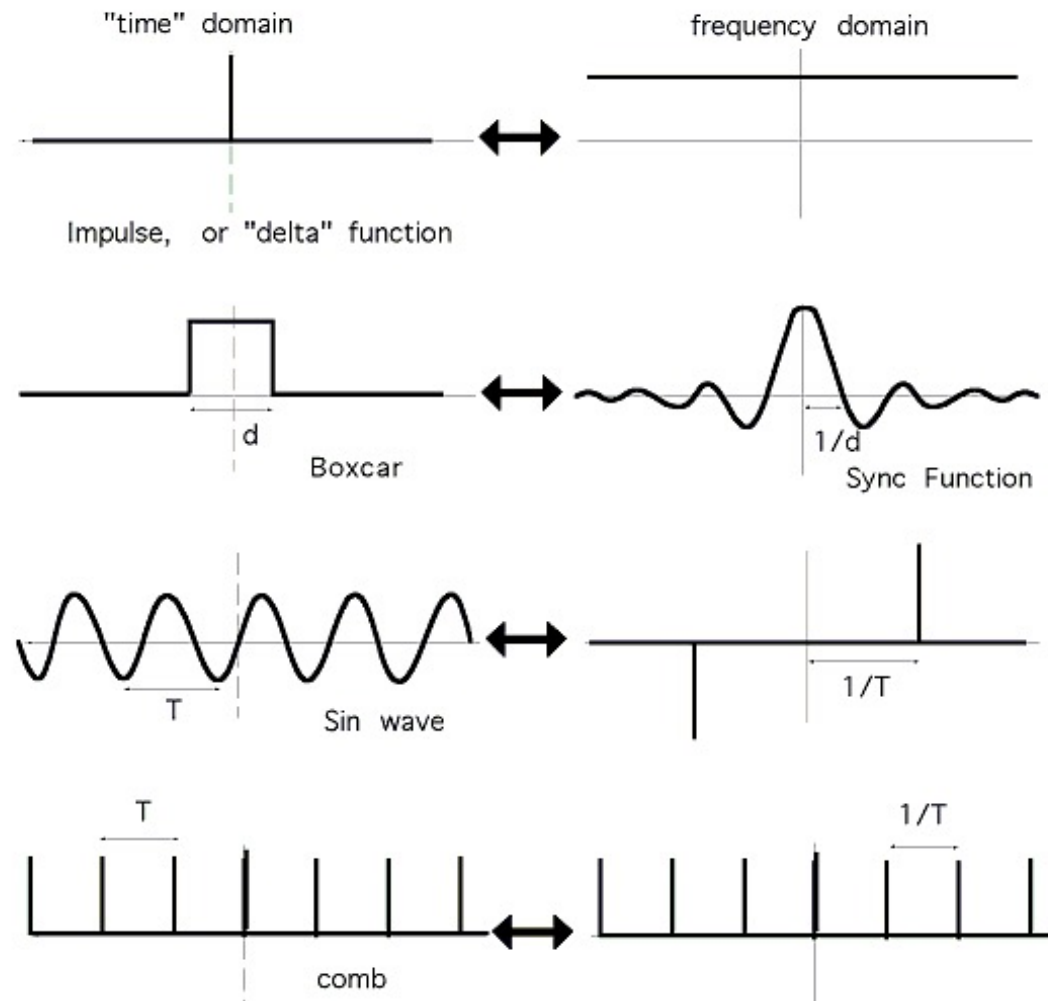
Coppie famose



Coppie famosa



Ampiezze nel tempo e nelle frequenze



DFT e shifting: osservazioni operative

Una proprietà importante: la DFT di un segnale shiftato di p è

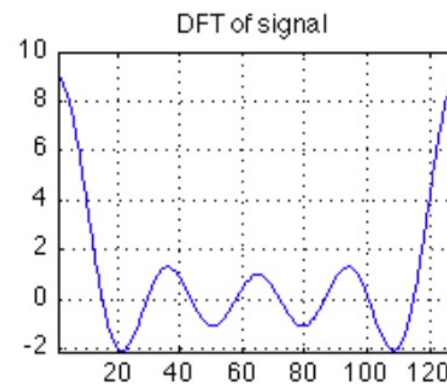
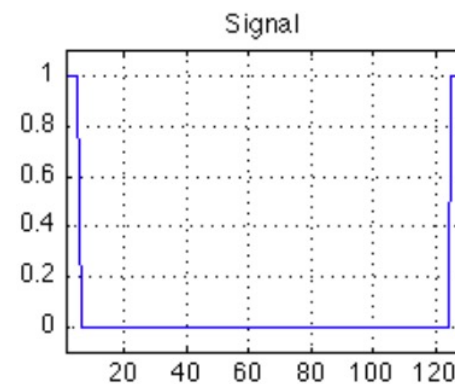
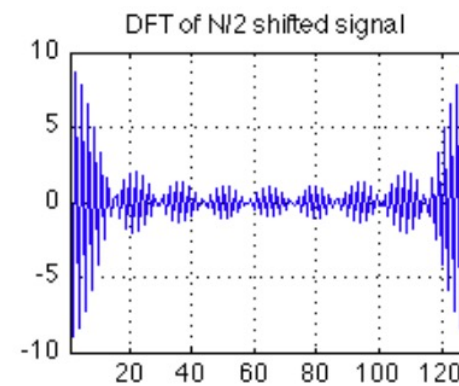
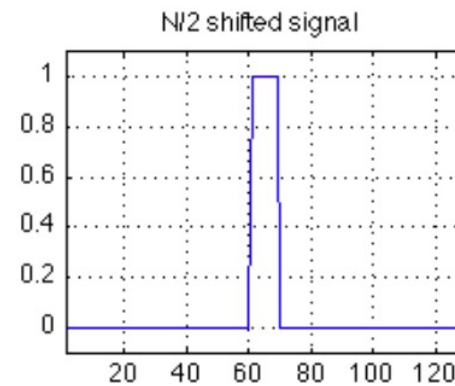
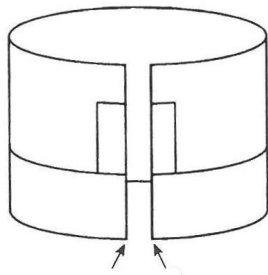
$$F_p[k] = e^{-i\frac{2\pi}{N}pk} F[k]$$

Se $p=N/2$ (N pari) abbiamo

$$F_{N/2}[k] = (-1)^k F[k]$$

DFT e shifting: esempio

Example of a DFT of a signal and a $N/2$ ($N = 128$) shifted signal



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