Binary classification based on Logistic Regression using non-linear regression function

import library

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as colors
```

load data

In [2]:

```
fname_data1 = 'assignment_09_data1.txt'
fname_data2 = 'assignment_09_data2.txt'

data1 = np.genfromtxt(fname_data1, delimiter=',')
data2 = np.genfromtxt(fname_data2, delimiter=',')

num_data1 = data1.shape[0]
num_data2 = data2.shape[0]

x1 = np.zeros(num_data1)
y1 = np.zeros(num_data1)
label1   = np.zeros(num_data1)

for i in range(num_data1):
    x1[i]  = data1[i,0]
    y1[i]  = data1[i,1]
    label1[i]    = data1[i,2]


x2 = np.zeros(num_data2)
y2 = np.zeros(num_data2)
label2   = np.zeros(num_data2)

for i in range(num_data2):
    x2[i]  = data2[i,0]
    y2[i]  = data2[i,1]
    label2[i]    = data2[i,2]

xy_data1 = np.vstack((x1, y1)).T
xy_data2 = np.vstack((x2, y2)).T

# data[:,0] : x
# data[:,1] : y
# data[:,2] : label {0, 1}
```

define the feature function for each data to obtain the best accuracy

In [3]:

```
def feature_function1(x, y):
    feature = np.array([1, x, y, x*x, x*y, y*y], dtype = object)
    return feature
```

In [4]:

```
def feature_function2(x, y):
    feature = np.array([1, x, y, x*x, x*y, x*x*x], dtype = object)
    return feature
```

define regression function with a vector θ model parameters and input data

In [5]:

```
def regression_function(theta, feature):
    value = np.matmul(np.transpose(theta), feature)
    return value
```

define sigmoid function with input x

In [6]:

```
def logistic_function(x):
#     for _ in range(len(x)) :
    z = 1/(1+np.exp(-x))
    return z
```

define loss function with feature and label based on the logistic regression

In [7]:

```
def compute_loss_feature(theta, feature, label):
    z = regression_function(theta, feature)
    h = logistic_function(z)
    loss = (-label * np.log(h) - (1 - label) * np.log(1 - h)).mean()
    return loss
```

define gradient vector for the model parameters θ

In [8]:

```
def compute_gradient_feature(theta, feature, label):
    z = regression_function(theta, feature)
    h = logistic_function(z)
    num_data = feature[1].shape[0]
    one = np.ones(num_data)
    X = np.column_stack([one, feature[1], feature[2], feature[3], feature[4], feature[5]])
    gradient = np.dot(X.T, (h - label)) / feature[1].shape[0]
    return gradient
```

compute the accuracy

In [9]:

```
def compute_accuracy(theta, feature, label):
    num_data = feature[1].shape[0]
    correctNum = 0
    f = regression_function(theta, feature)
    for i in range(0, num_data)  :
        if f[i]>=0 :
            ans = 1
        else :
            ans = 0
        if ans == label[i] :
            correctNum += 1
    accuracy = correctNum / num_data
    return accuracy
```

gradient descent for the model parameters θ

In [10]:

```
num_iteration   = 30000         # USE THIS VALUE for the number of gradient descent iterations
learning_rate   = 0.3           # USE THIS VALUE for the learning rate
theta1           = np.array((0, 0, 0, 0, 0, 0))   # USE THIS VALUE for the initial condition of the
theta2           = np.array((0, 0, 0, 0, 0, 0))
theta1_iteration = np.zeros((num_iteration, theta1.size))
loss1_iteration  = np.zeros(num_iteration)
theta2_iteration = np.zeros((num_iteration, theta2.size))
loss2_iteration  = np.zeros(num_iteration)
```

In [11]:

```
for i in range(num_iteration):
    theta1 = theta1 - learning_rate * compute_gradient_feature(theta1, feature_function1(x1, y1), la
    loss1 = compute_loss_feature(theta1, feature_function1(x1, y1), label1)
    theta1_iteration[i] = theta1
    loss1_iteration[i] = loss1
    print("iteration = %4d, loss = %5.5f" % (i, loss1))
print("############################################################")
for i in range(num_iteration):
    theta2 = theta2 - learning_rate * compute_gradient_feature(theta2, feature_function2(x2, y2), la
    loss2 = compute_loss_feature(theta2, feature_function2(x2, y2), label2)
    theta2_iteration[i] = theta2
    loss2_iteration[i] = loss2
    print("iteration = %4d, loss = %5.5f" % (i, loss2))

theta1_optimal = theta1
theta2_optimal = theta2
```

```
iteration =    0, loss = 0.72428
iteration =    1, loss = 0.57369
iteration =    2, loss = 0.54468
iteration =    3, loss = 0.52980
iteration =    4, loss = 0.51470
iteration =    5, loss = 0.50205
iteration =    6, loss = 0.48955
iteration =    7, loss = 0.47822
iteration =    8, loss = 0.46716
iteration =    9, loss = 0.45680
iteration =   10, loss = 0.44675
iteration =   11, loss = 0.43718
iteration =   12, loss = 0.42794
iteration =   13, loss = 0.41906
iteration =   14, loss = 0.41050
iteration =   15, loss = 0.40224
iteration =   16, loss = 0.39427
iteration =   17, loss = 0.38657
iteration =   18, loss = 0.37913
```

compute accuracy of the classifiers

In [12]:

```
accuracy_classifier1 = compute_accuracy(theta1_optimal, feature_function1(x1, y1), label1)
accuracy_classifier2 = compute_accuracy(theta2_optimal, feature_function2(x2, y2), label2)
print(accuracy_classifier1)
print(accuracy_classifier2)
```

```
0.999
0.993
```

plot the results

In [13]:

```
def plot_loss_curve(loss_iteration):

    plt.figure(figsize=(8,6))
    plt.title('loss')

    plt.plot(loss_iteration, '-', color = 'red')

    plt.xlabel('iteration')
    plt.ylabel('loss')

    plt.tight_layout()
    plt.show()
```

In [14]:

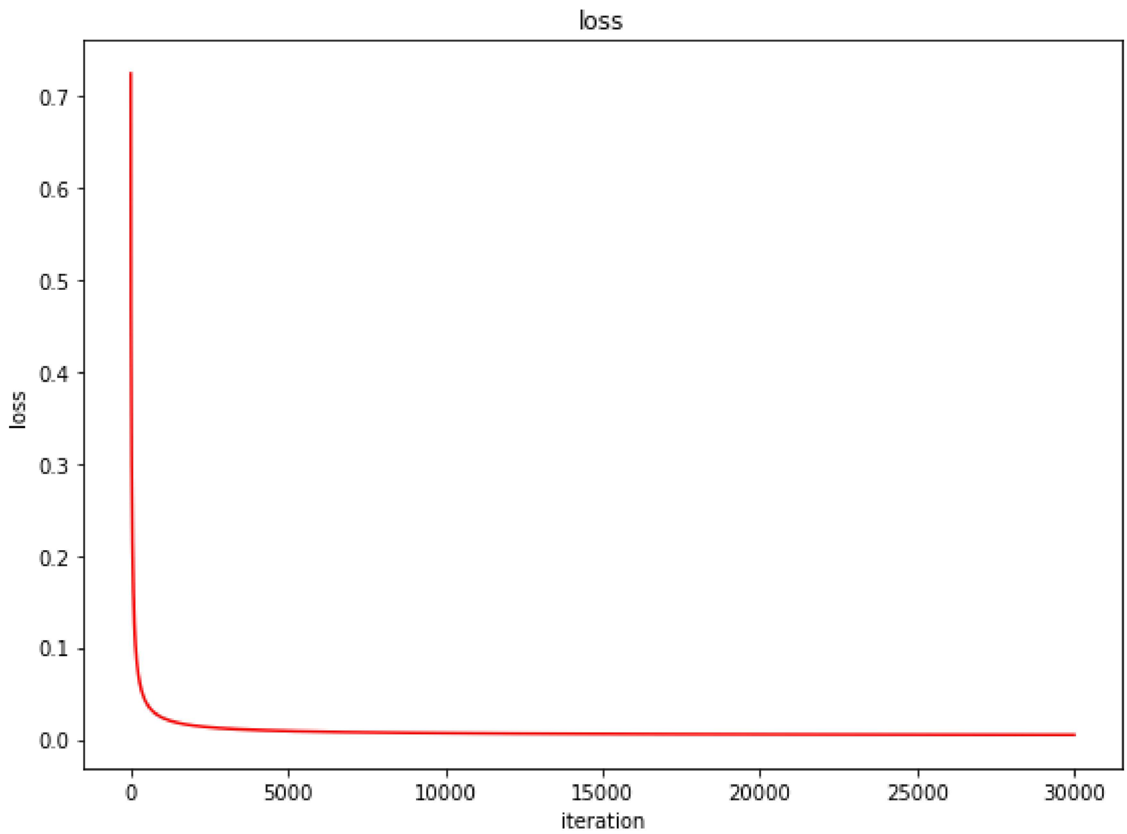
```
def plot_data(point_x, point_y, label):
    plt.figure(figsize=(8,8))

    plt.title('data')
    xx = []
    yy = []
    xxx = []
    yyy = []
    for i in range(0, num_data1) :
        x = point_x[i]
        y = point_y[i]
        if label[i] == 0 :
            xx.append(x)
            yy.append(y)
        elif label[i] == 1 :
            xxx.append(x)
            yyy.append(y)
    plt.scatter(xx,yy,c='blue', label = 'Class = 0')
    plt.scatter(xxx,yyy,c='red', label = 'Class = 1')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

In [15]:

```
def plot_model_parameter(theta_iteration):

    plt.figure(figsize=(8,6))
    plt.title('model parameter')
    thetaT = theta_iteration.T
    plt.plot(thetaT[0], '-', color = 'red', label = r'$\theta_0$')
    plt.plot(thetaT[1], '-', color = 'green', label = r'$\theta_1$')
    plt.plot(thetaT[2], '-', color = 'blue', label = r'$\theta_2$')
    plt.plot(thetaT[3], '-', color = 'black', label = r'$\theta_3$')
    plt.plot(thetaT[4], '-', color = 'orange', label = r'$\theta_4$')
    plt.plot(thetaT[5], '-', color = 'skyblue', label = r'$\theta_5$')

    plt.xlabel('iteration')
    plt.legend()

    plt.tight_layout()
    plt.show()
```

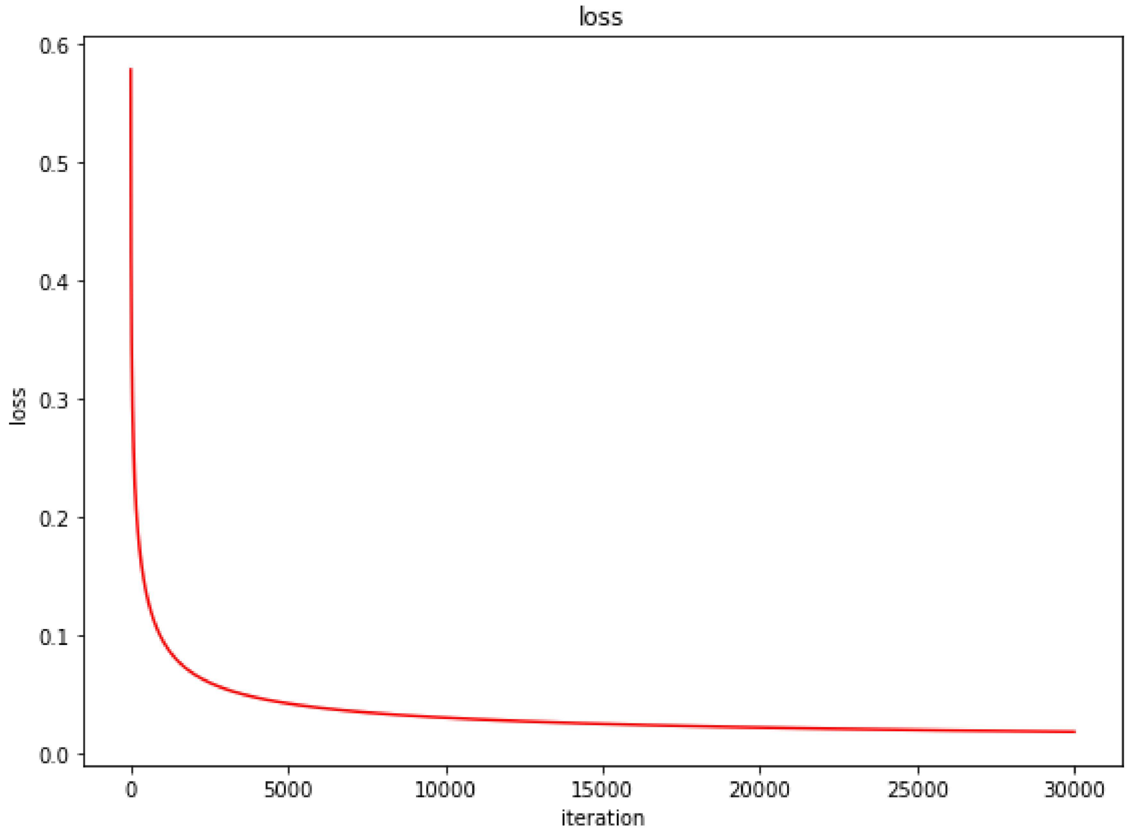
In [16]:

```
plot_loss_curve(loss1_iteration)
```



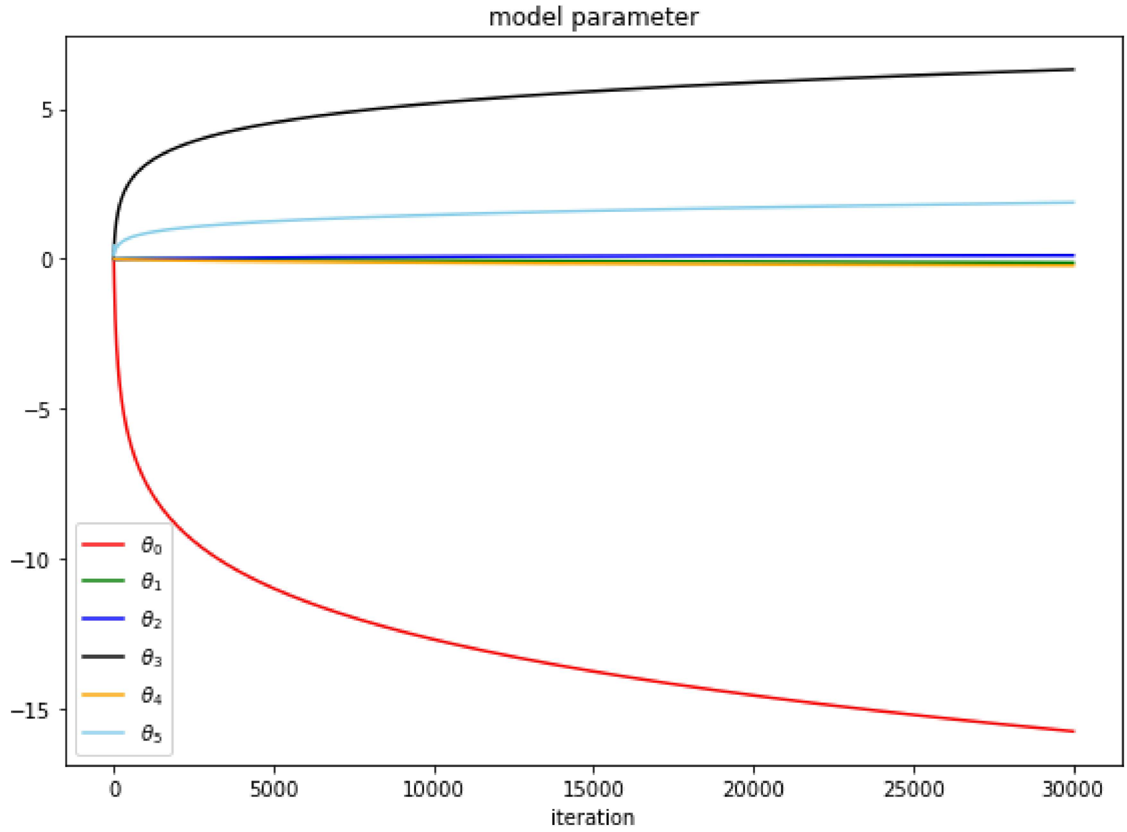
In [17]:

```
plot_loss_curve(loss2_iteration)
```



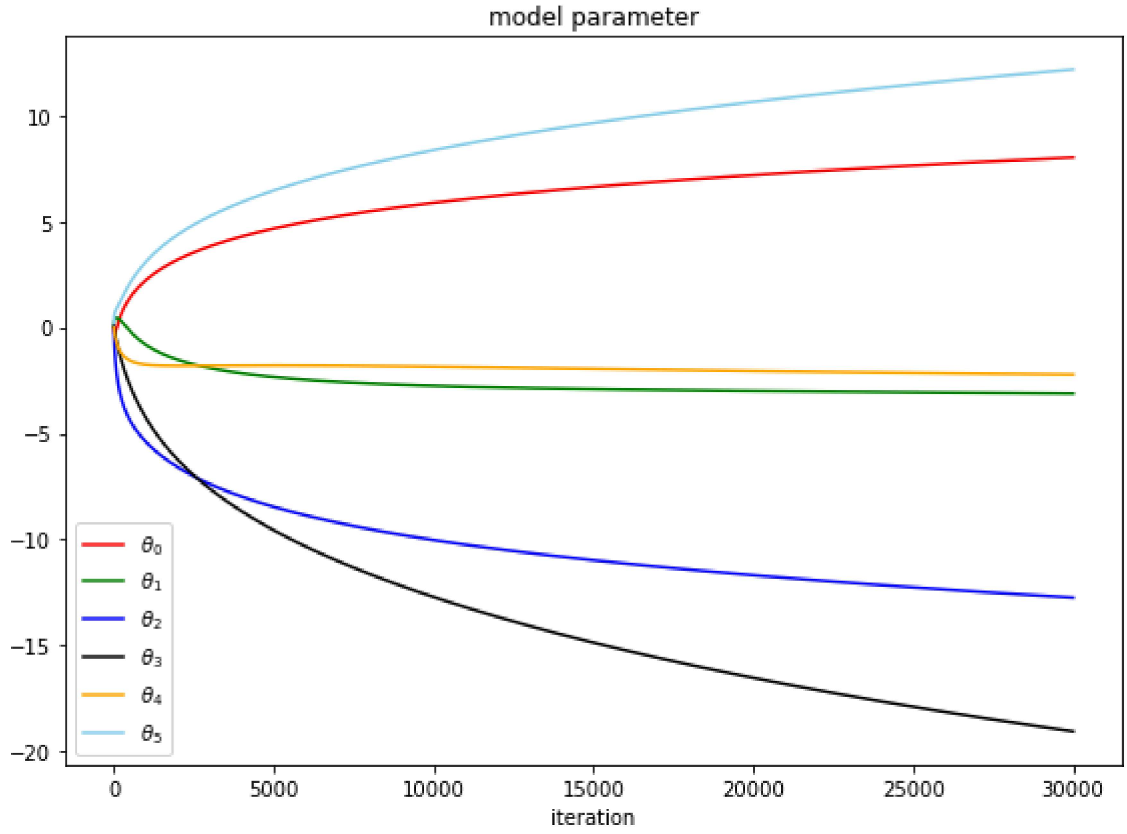
In [18]:

```
plot_model_parameter(theta1_iteration)
```



In [19]:

```
plot_model_parameter(theta2_iteration)
```



In [20]:

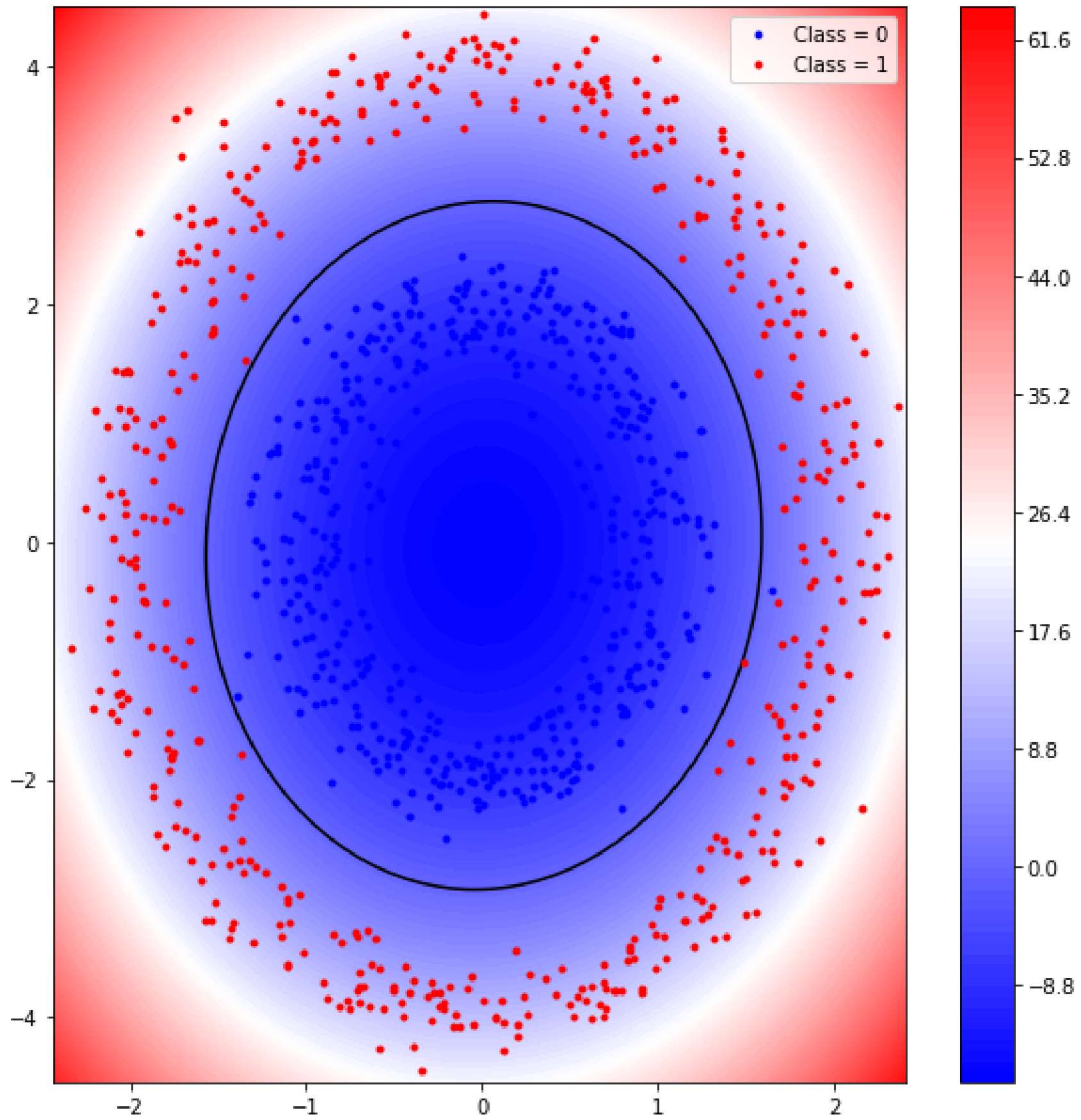
```
def plot_classifier1(data, theta):
    plt.figure(figsize=(8,8))
    xx = []
    yy = []
    xxx = []
    yyy = []
    X = np.arange(x1.min()-0.1, x1.max()+0.1, 0.05)
    Y = np.arange(y1.min()-0.1, y1.max()+0.1, 0.05)
    gX, gY = np.meshgrid(X, Y)
    Z = regression_function(theta1, feature_function1(gX, gY))
    for i in range(0, num_data1) :
        x = x1[i]
        y = y1[i]
        if label1[i] == 0 :
            xx.append(x)
            yy.append(y)
        elif label1[i] == 1 :
            xxx.append(x)
            yyy.append(y)
    plt.plot(xx, yy, 'o',color='blue', label='Class = 0', markersize = 3)
    plt.plot(xxx, yyy, 'o',color='red', label='Class = 1', markersize = 3)
    plt.contourf(gX, gY, Z, levels=100, cmap = 'bwr')
    plt.colorbar()
    plt.contour(gX,gY, Z, levels = [0], colors = 'black')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

In [21]:

```
def plot_classifier2(data, theta):

    plt.figure(figsize=(8,8))
    xx = []
    yy = []
    xxx = []
    yyy = []
    X = np.arange(x2.min()-0.1, x2.max()+0.1, 0.05)
    Y = np.arange(y2.min()-0.1, y2.max()+0.1, 0.05)
    gX, gY = np.meshgrid(X, Y)
    Z = regression_function(theta2, feature_function2(gX, gY))
    for i in range(0, num_data2) :
        x = x2[i]
        y = y2[i]
        if label2[i] == 0 :
            xx.append(x)
            yy.append(y)
        elif label2[i] == 1 :
            xxx.append(x)
            yyy.append(y)
    plt.plot(xx, yy, 'o',color='blue', label='Class = 0', markersize = 3)
    plt.plot(xxx, yyy, 'o',color='red', label='Class = 1', markersize = 3)
    plt.contourf(gX, gY, Z, levels=100, cmap = 'bwr')
    plt.colorbar()
    plt.contour(gX,gY, Z, levels = [0], colors = 'black')

    plt.legend()
    plt.tight_layout()
    plt.show()
```

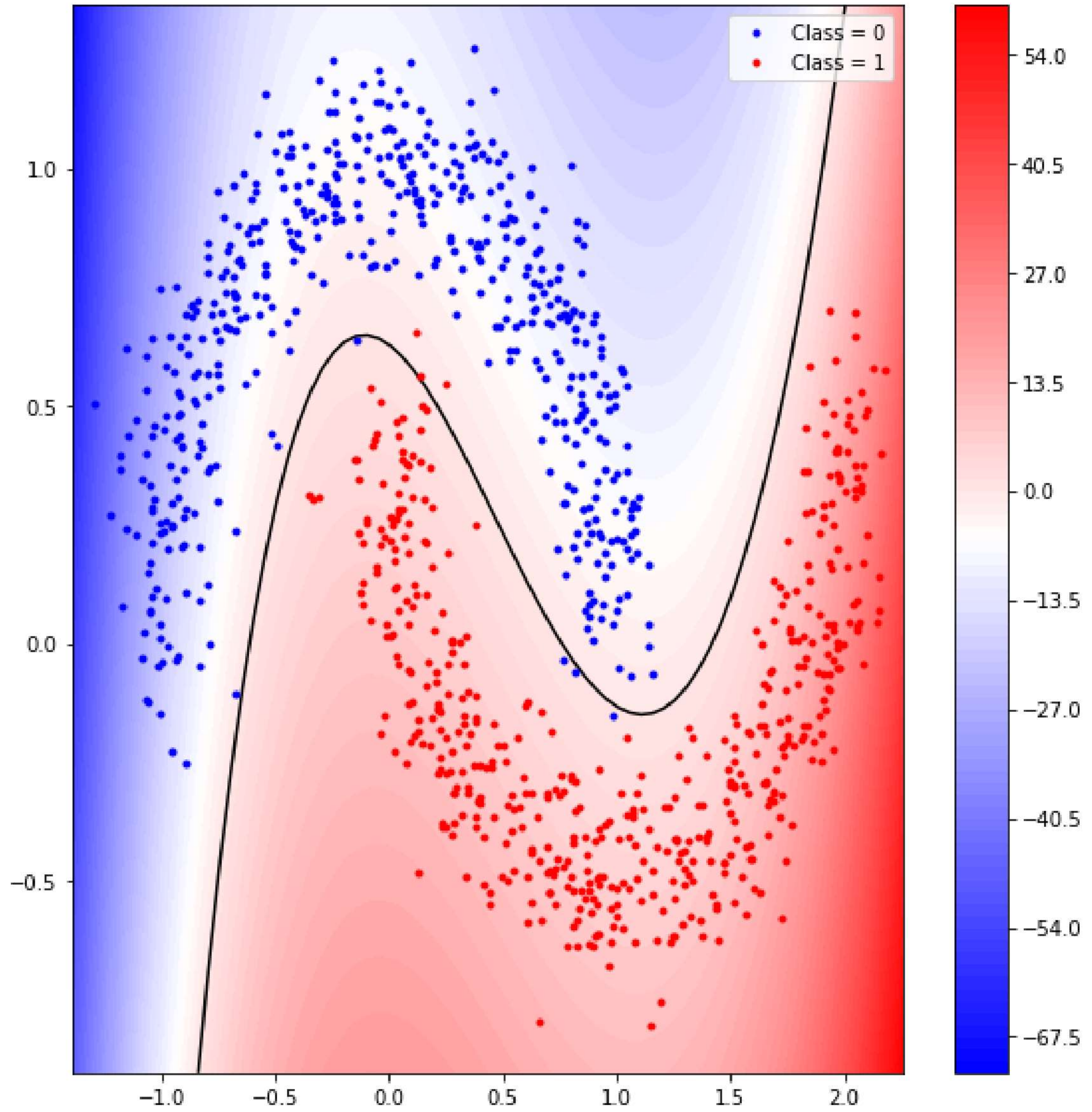
In [22]:

```
plot_classifier1(data1, theta1_optimal)
```



In [23]:

```
plot_classifier2(data2, theta2_optimal)
```



* *

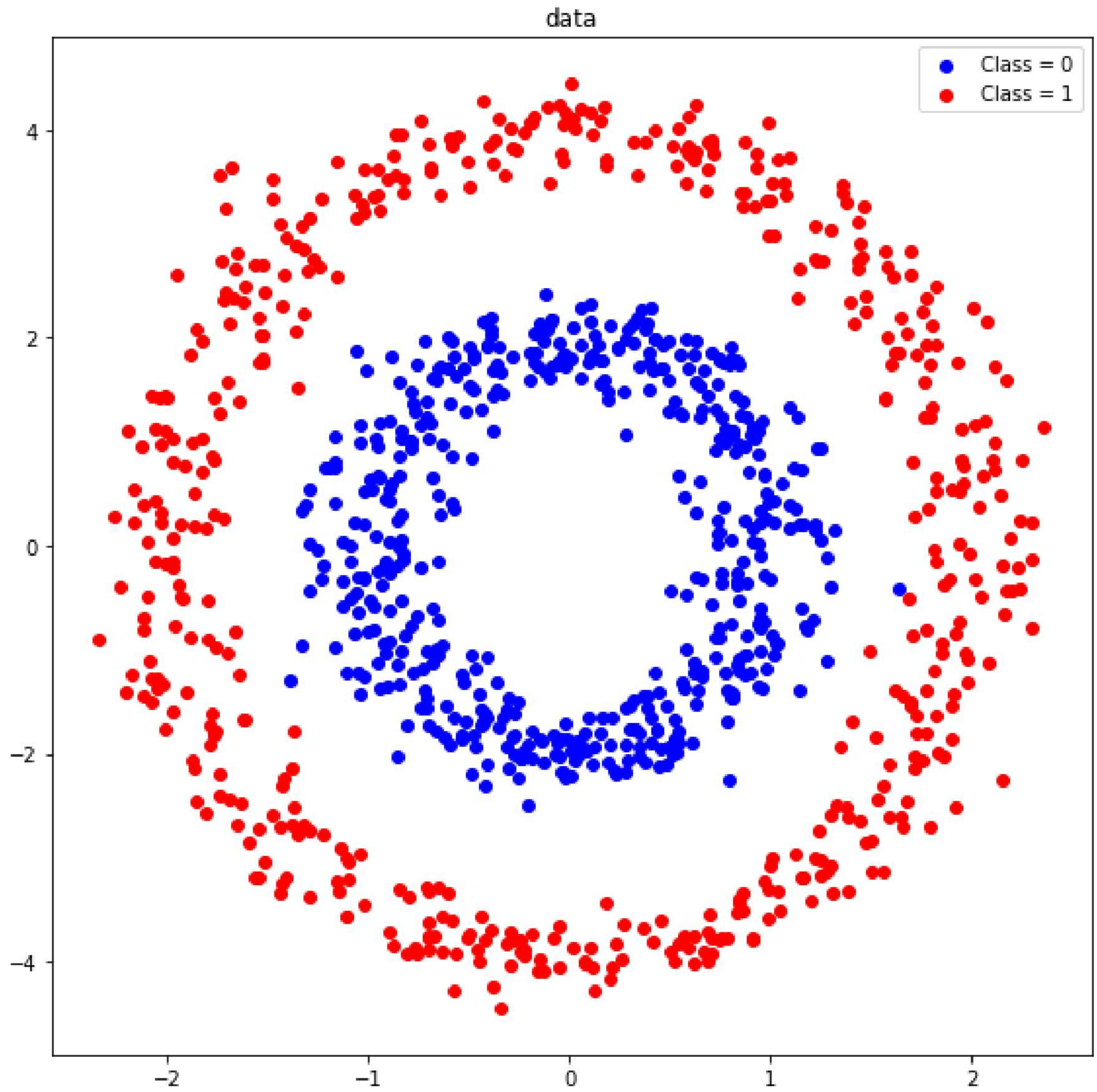
* results

* *

01. plot the input data (data1) from the file [assignment_09_data1.txt] in blue for class 0 and in red for class 1

In [24]:

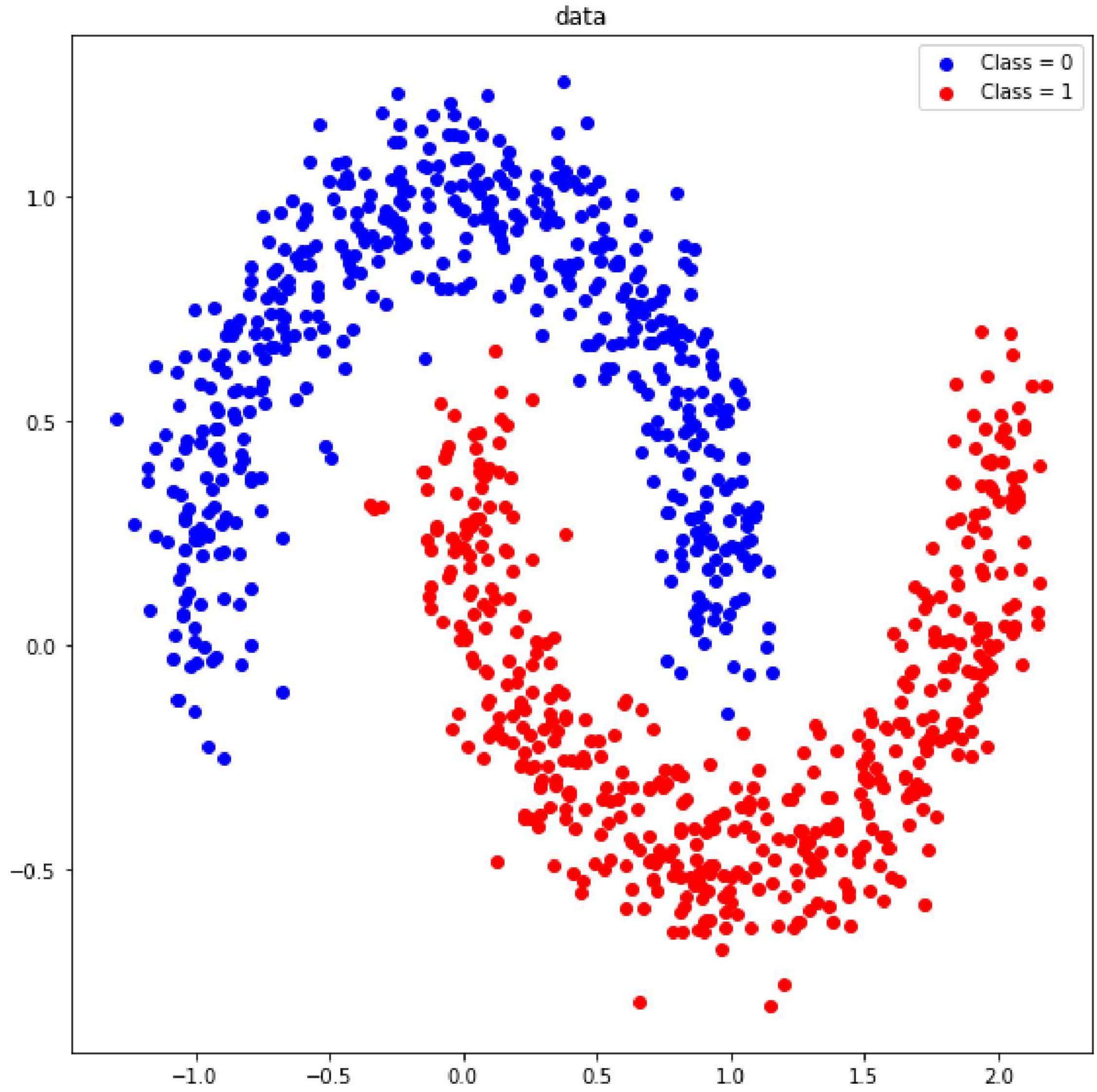
```
plot_data(x1,y1,label1)
```



02. plot the input data (data2) from the file [assignment_09_data2.txt] in blue for class 0 and in red for class 1

In [25]:

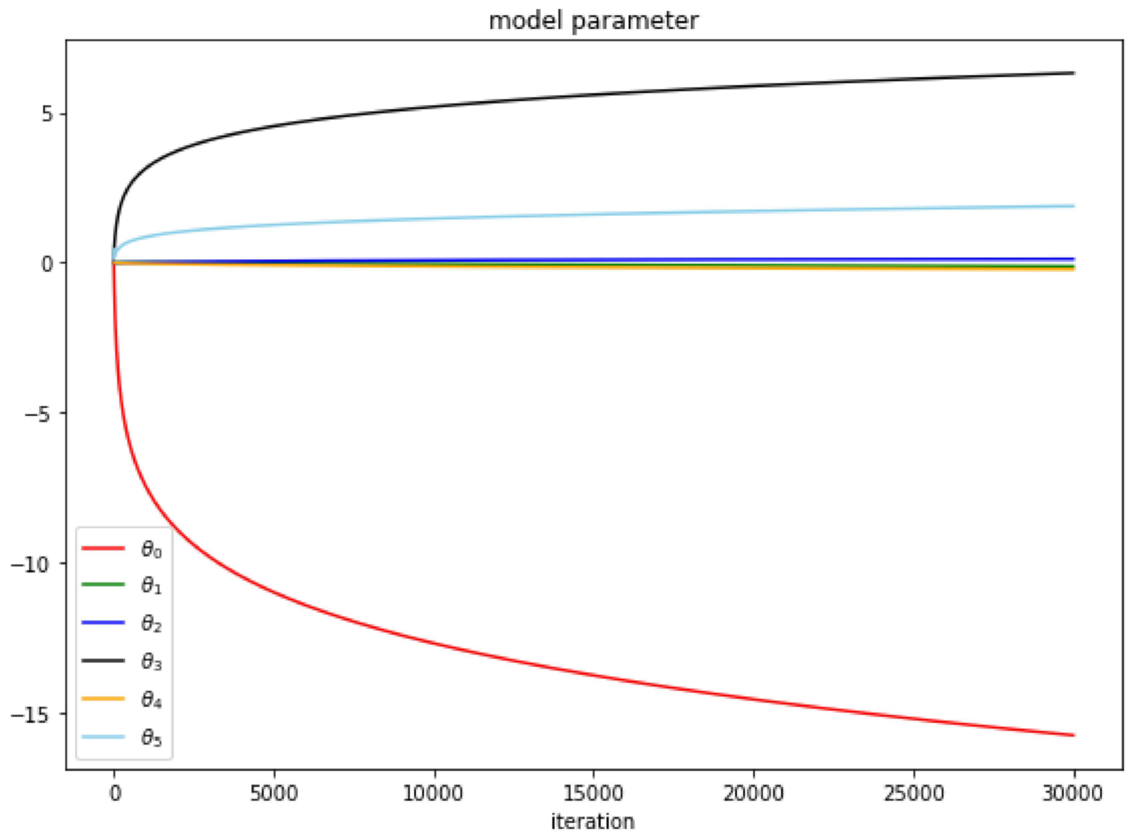
```
plot_data(x2,y2,label2)
```



03. plot the values of the model parameters θ as curves over the gradient descent iterations using different colors for data1

In [26]:

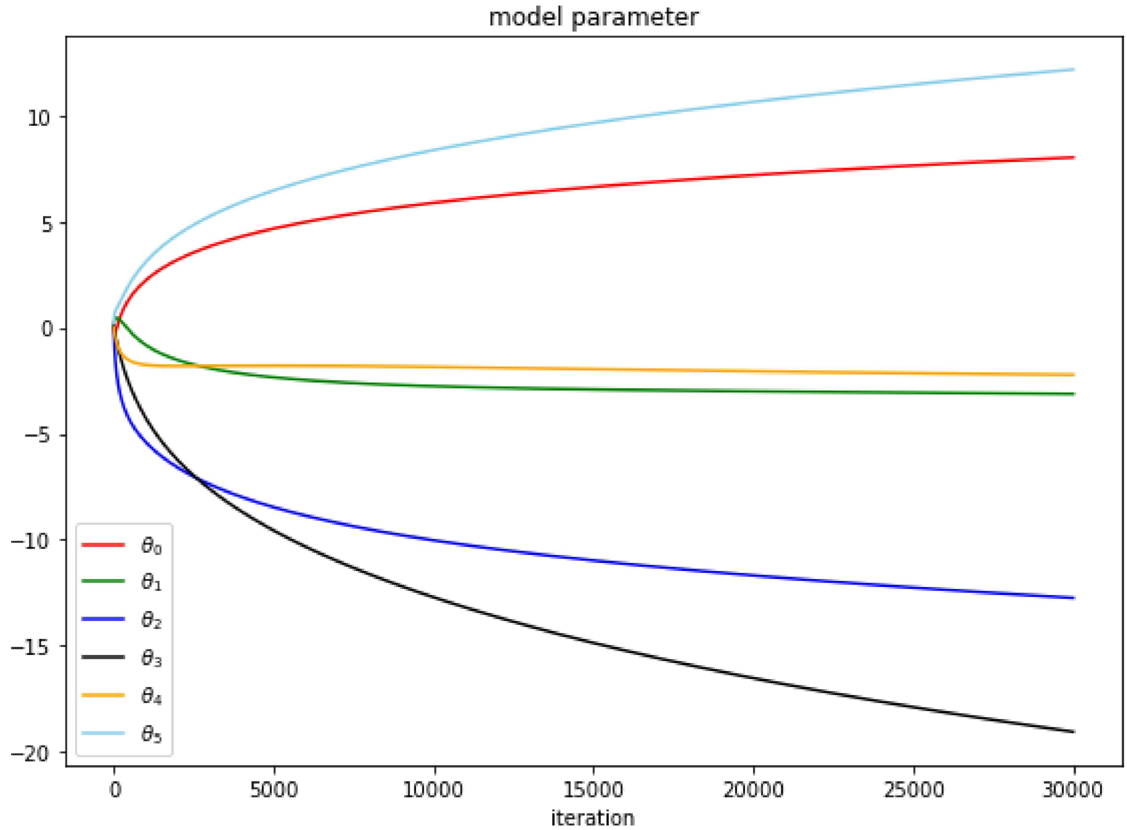
```
plot_model_parameter(theta1_iteration)
```



04. plot the values of the model parameters θ as curves over the gradient descent iterations using different colors for data2

In [27]:

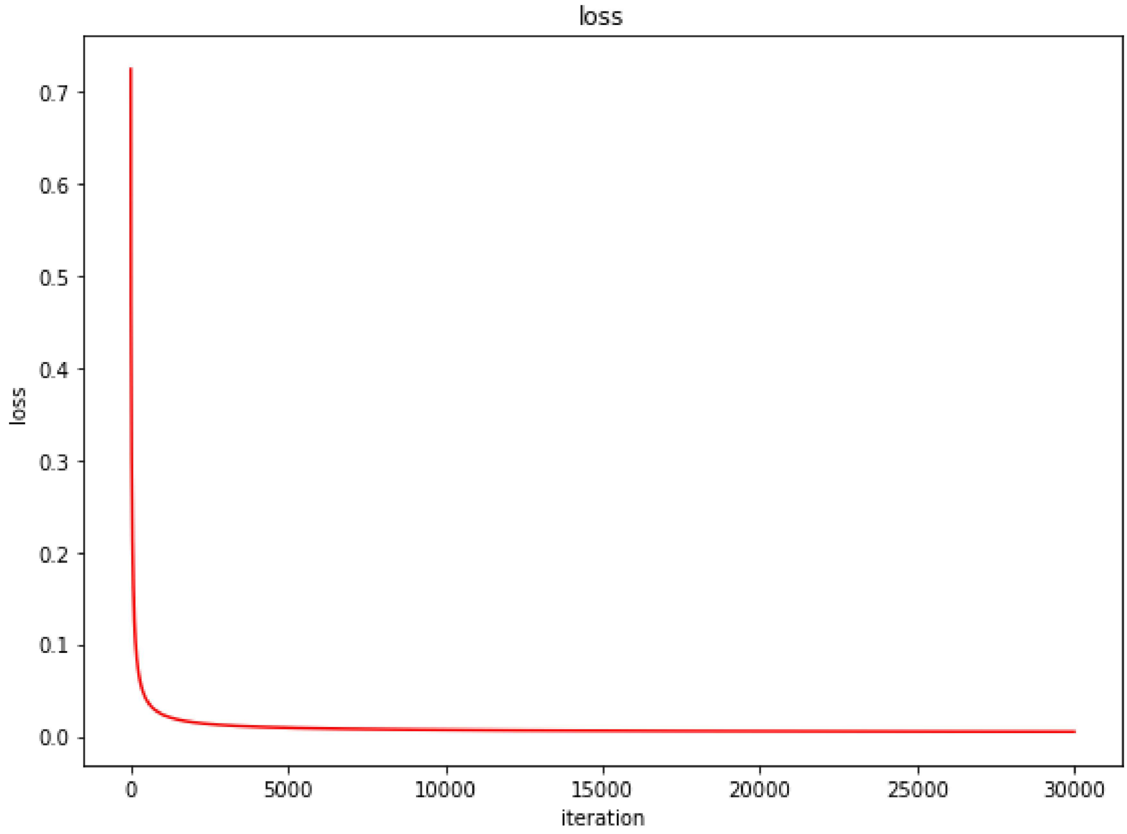
```
plot_model_parameter(theta2_iteration)
```



05. plot the loss values in red curve over the gradient descent iterations for data1

In [28]:

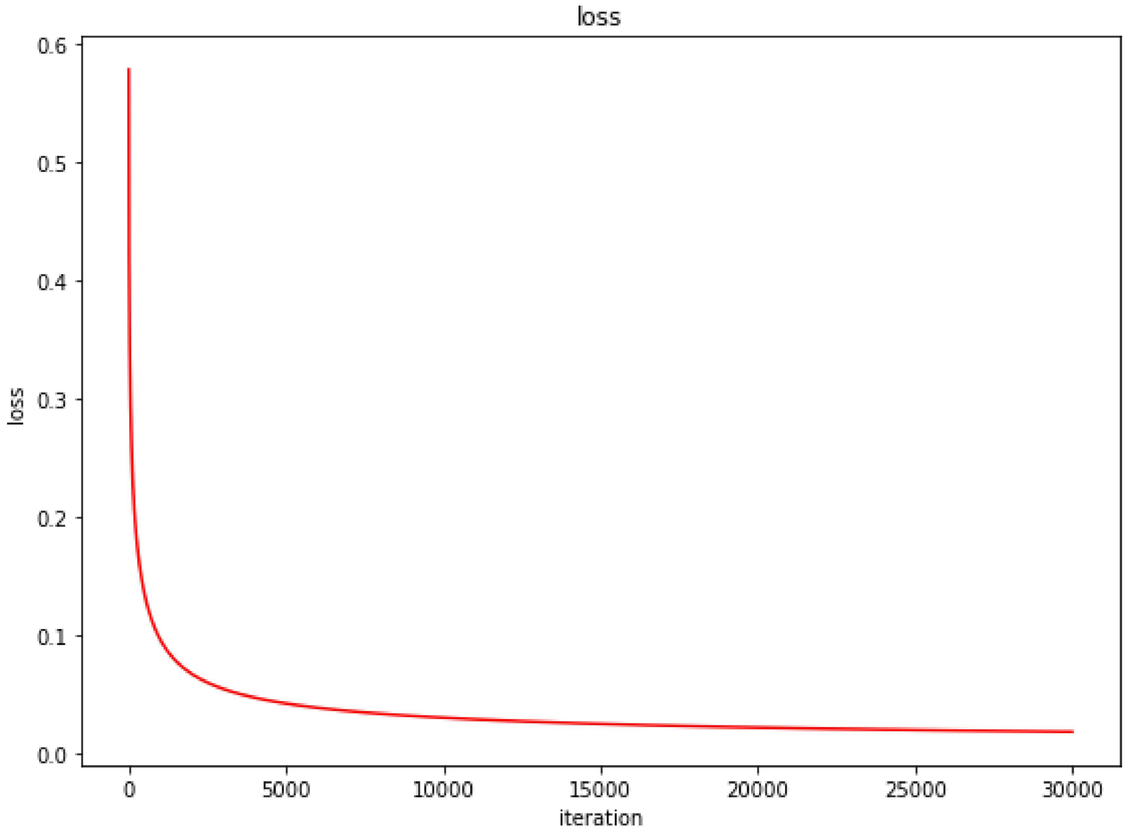
```
plot_loss_curve(loss1_iteration)
```



06. plot the loss values in red curve over the gradient descent iterations for data2

In [29]:

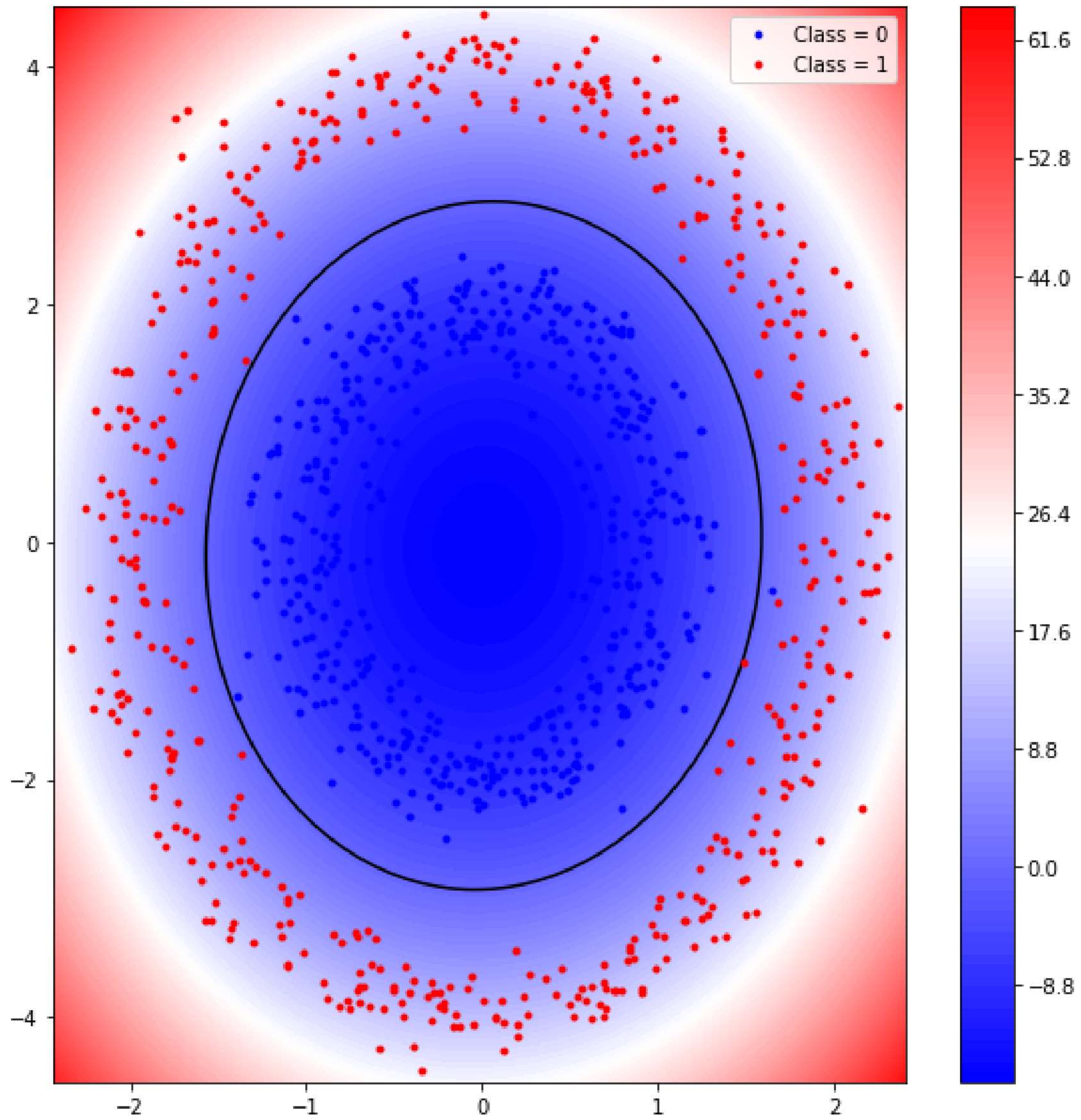
```
plot_loss_curve(loss2_iteration)
```



07. plot the classifier with the given data points superimposed for data1

In [30]:

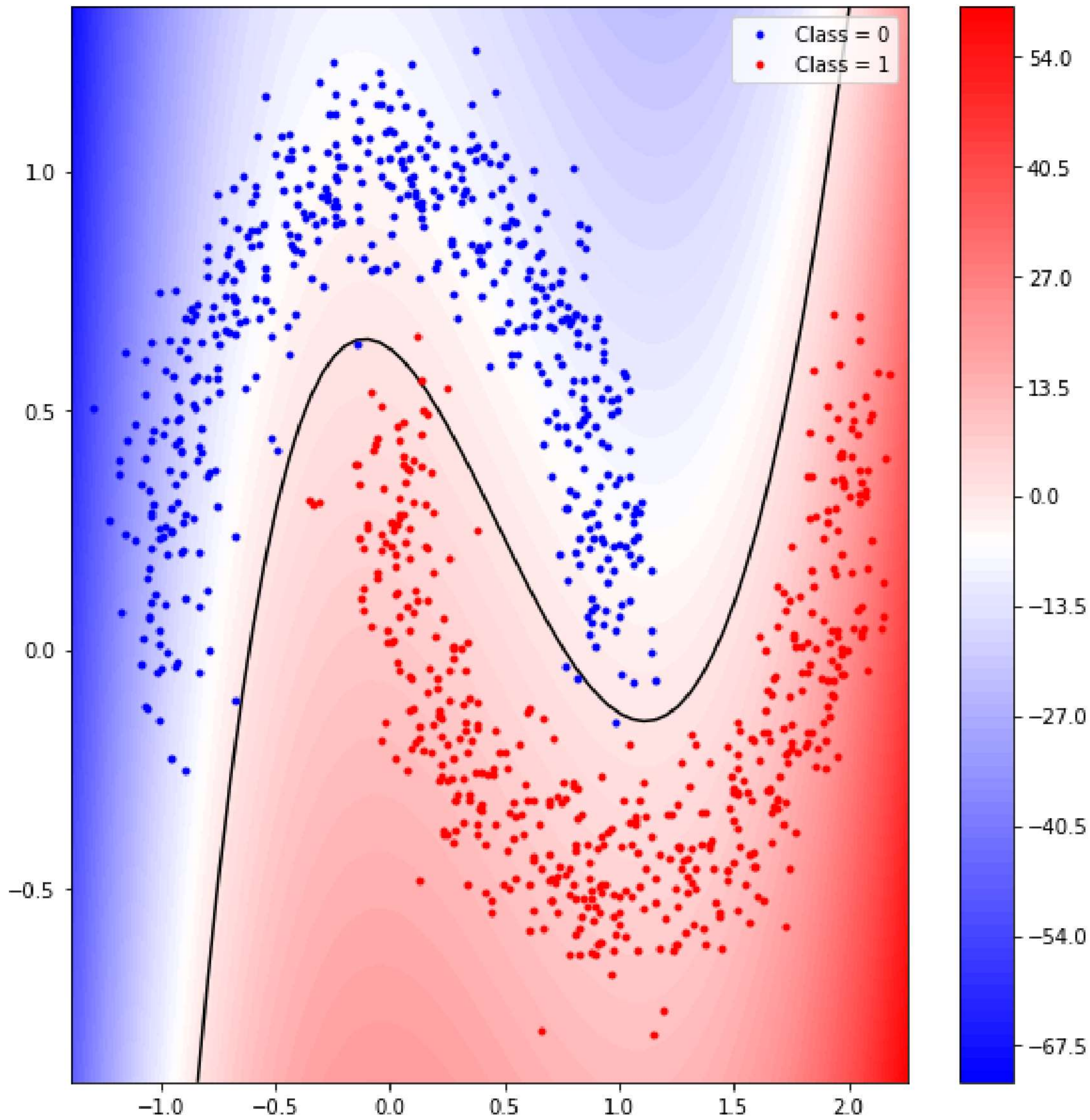
```
plot_classifier1(data1, theta1_optimal)
```



08. plot the classifier with the given data points superimposed for data2

In [31]:

```
plot_classifier2(data2, theta2_optimal)
```



09. print out the accuracy of the obtained classifier1 for data1

In [32]:

```
print(accuracy_classifier1)
```

0.999

10. print out the accuracy of the obtained classifier2 for data1

In [33]:

```
print(accuracy_classifier2)
```

0.993