Least square problem for polynomial regression

import library

In [1]:

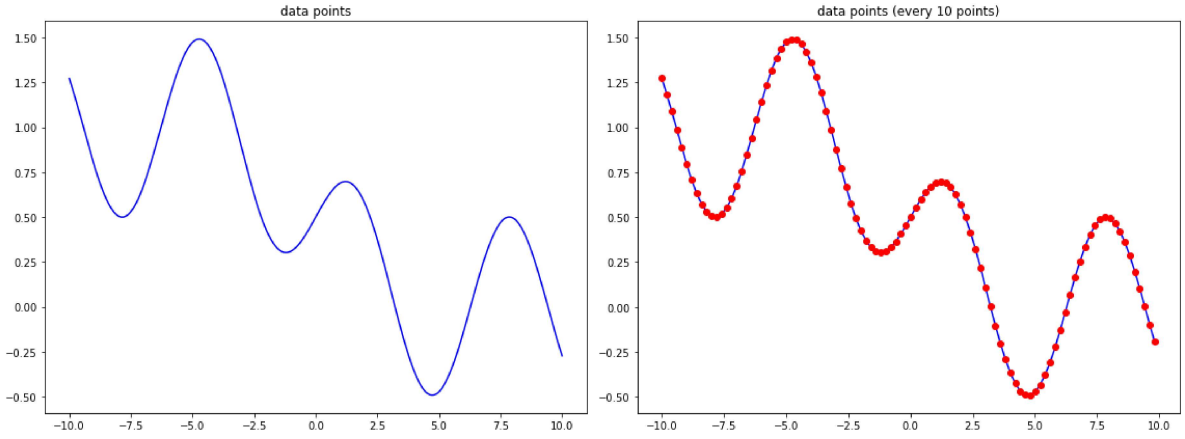
```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
import matplotlib.colors as colors
```

load point data

In [2]:

```
filename    = 'assignment_05_data.csv'
data        = np.loadtxt(filename, delimiter = ',')
print(data)
x   = data[0, :]    # independent variable
y   = data[1, :]    # dependent variable
x2  = x[::10]
y2  = y[::10]

plt.figure(figsize=(16,6))

plt.subplot(121)
plt.plot(x, y, '-', color = 'blue')
plt.title('data points')

plt.subplot(122)
plt.plot(x2, y2, '-', color = 'blue')
plt.plot(x2, y2, 'o', color = 'red')
plt.title('data points (every 10 points)')

plt.tight_layout()
plt.show()
```

```
[[-10.         -9.97998    -9.95996   ...   9.95996    9.97998   10.        ]
 [  1.271965    1.263511    1.254952  ...  -0.254952  -0.263511  -0.271965]]
```



In [3]:

```
#
# A : [ x_1^0, x_1^1, ... , x_1^{p-1} ; x_2^0, x_2^1, ... , x_2^{p-1} ; ... ; x_n^0, x_n^1, ... , x_
#
def construct_matrix_A(x, p):
    A = [ [ np.power(x, i) for i in range(p) ] for x in x ]
    return A

#
# x : independent variable
# y : dependent variable
# p : power of the polinomial (theta_0 * x^0, theta_1 * x^1, ..., theta_{p-1} * x^{p-1})
# alpha : coefficient for the regularization term : ₩| theta ₩|_2^2
#
def solve_polynomial_regression(x, y, p, alpha = 0):
    A = construct_matrix_A(x, p)
    A_t = np.transpose(A)

    # use functions including : np.identity -> 대각성분은 '1'이고 나머지 성분은 '0'으로 구성된 정방행
    # np.matmul -> 곱셈, np.linalg.inv -> 역행렬
    A_tdoty = np.matmul(A_t, y)

    A_tdotA = np.matmul(A_t, A)
    A_tdotA_alpha = A_tdotA + alpha
    alpha_i = np.linalg.inv(A_tdotA_alpha)

    theta = np.matmul(alpha_i, A_tdoty)

    ###################
#     t1 = np.matmul(np.transpose(theta) , A_t) - np.transpose(y)
#     t2 = np.matmul(A, theta) - y
#     t3 = np.matmul(np.transpose(theta), theta)
#     t3 = t3 * alpha
#     Ltheta = np.matmul(t1, t2) + t3

    h = np.matmul(A, theta)

    return h

h_01        = solve_polynomial_regression(x, y, 1)
h_02        = solve_polynomial_regression(x, y, 2)
h_04        = solve_polynomial_regression(x, y, 4)
h_08        = solve_polynomial_regression(x, y, 8)
h_16        = solve_polynomial_regression(x, y, 16)
h_32        = solve_polynomial_regression(x, y, 32)

h_24_0      = solve_polynomial_regression(x, y, 24, 0)
h_24_00001  = solve_polynomial_regression(x, y, 24, 0.0001)
h_24_0001   = solve_polynomial_regression(x, y, 24, 0.001)
h_24_001    = solve_polynomial_regression(x, y, 24, 0.01)
h_24_01     = solve_polynomial_regression(x, y, 24, 0.1)
h_24_1      = solve_polynomial_regression(x, y, 24, 1)
```

In [4]:

```
def plot_polynomial_regression_3x2(x, y, h_01, title_01, h_02, title_02, h_04, title_04, h_08, title

    plt.figure(figsize=(16,18))

    plt.subplot(321)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_01, '-', color='red')
    plt.title(title_01)

    plt.subplot(322)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_02, '-', color='red')
    plt.title(title_02)

    plt.subplot(323)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_04, '-', color='red')
    plt.title(title_04)

    plt.subplot(324)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_08, '-', color='red')
    plt.title(title_08)

    plt.subplot(325)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_16, '-', color='red')
    plt.title(title_16)

    plt.subplot(326)
    plt.plot(x, y, '-', color='blue')
    plt.plot(x, h_32, '-', color='red')
    plt.title(title_32)

    plt.tight_layout()
    plt.show()
```

*** ***

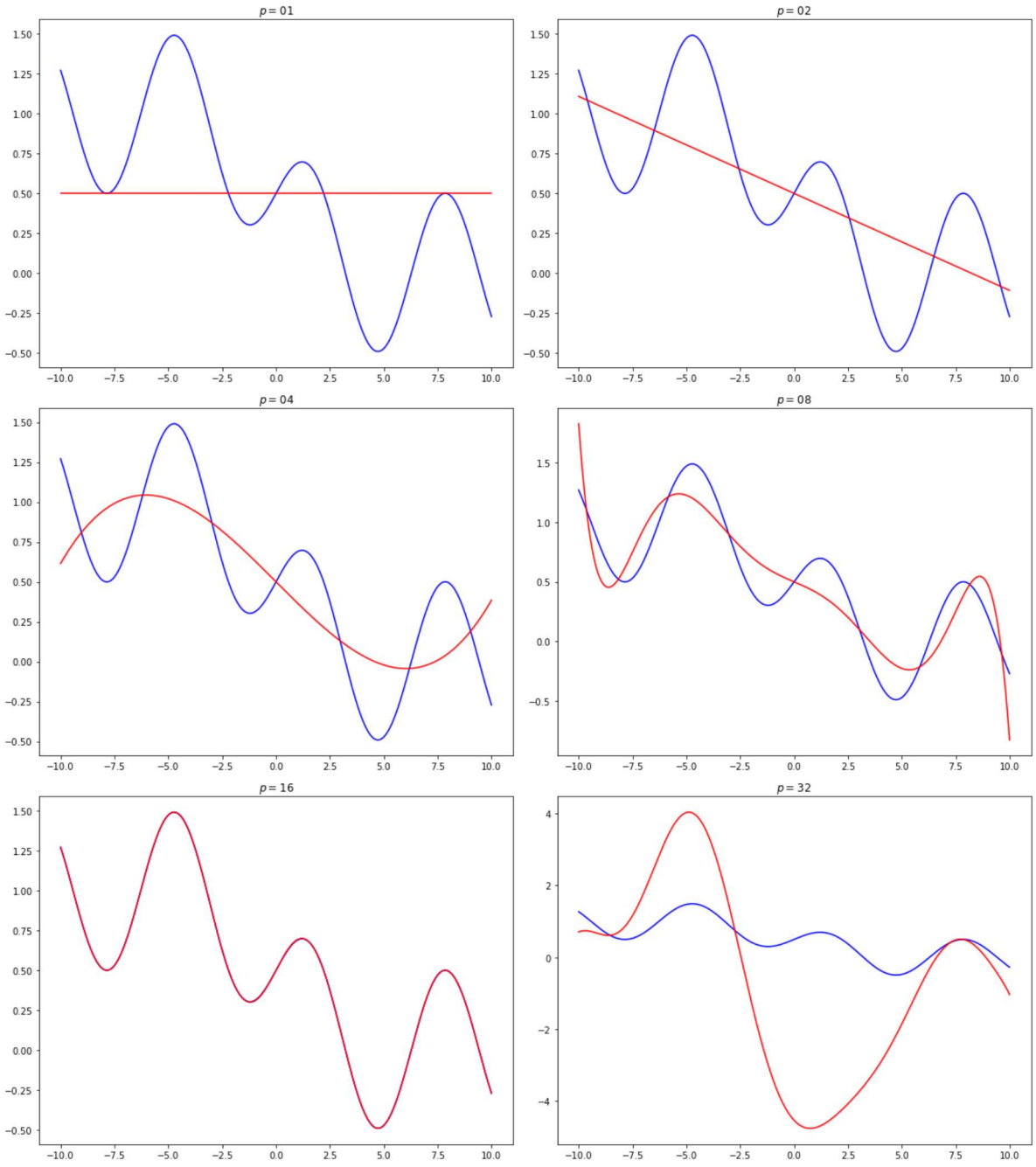
* results

*** ***

01. plot the input data in blue and the polynomial approximations with varying degrees in red ($p = 1, 2, 4, 8, 16, 32$)

In [5]:

```
plot_polynomial_regression_3x2(x, y, h_01, '$p = 01$', h_02, '$p = 02$', h_04, '$p = 04$', h_08, '$p
```



 # 02. plot the input data in blue and the polynomial approximations with varying regularization parameters at $p = 24$ ($\alpha = 0, 0.0001, 0.001, 0.01, 0.1, 1$)

In [6]:

```
plot_polynomial_regression_3x2(x, y, h_24_0, '$p = 24, alpha = 0$', h_24_00001, '$p = 24, alpha = 0.
```